UNIVERSITY OF ILLINOIS URBANA-CHAMPAIGN
Department of Electrical and Computer Engineering

ECE 310 Digital Signal Processing – Spring 2026

Homework 8

Profs. Snyder, Shomorony **Due: Friday, March 27, 11:59pm on Gradescope**

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1. Consider the continuous-time signal $x_a(t)$ with Fourier transform given by

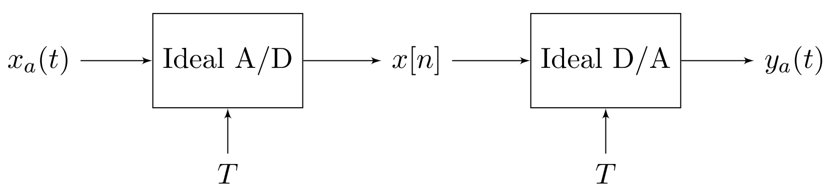
$$X_a(\Omega) = \begin{cases} 10 & \text{if } |\Omega| \leq 50\pi \\ 5 & \text{if } 50\pi < |\Omega| < 150\pi \\ 0 & \text{otherwise.} \end{cases}$$

   (a) Determine a closed-form expression for $x_a(t)$. Make sure you specify the value of $x_a(0)$.

   (b) Suppose $x_a(t)$ is sampled using sampling period $T = 1/50$. Determine the discrete-time signal $x[n]$. Make sure you simplify your answer.

   (c) Determine and sketch the discrete-time Fourier transform $X_d(\omega)$ of $x[n]$, for the value of T in part (b).

2. Consider the following system consisting of an ideal analog-to-digital converter followed by an ideal digital-to-analog converter.



   Suppose the sampling period is $T = \frac{1}{100}$s. For each of the following parts, determine $x[n]$ and $y_a(t)$ for the given input signal $x_a(t)$.

   (a) $x_a(t) = \cos(150\pi t)$.

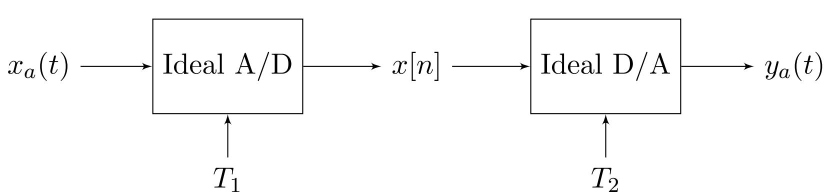
   (b) $x_a(t) = \cos(100\pi t)$.

   (c) $x_a(t) = \sin(100\pi t)$.

3. Suppose we record audio of a pure sinusoid at the frequency of the musical note A4, i.e., A in the fourth octave. This musical note occurs at frequency 440 Hz, thus, the signal $x_a(t)$ is

$$x_a(t) = \sin(2\pi \cdot 440t) = \sin(880\pi t).$$

   We would like to change the musical note/frequency of this audio using the below re-sampling system.



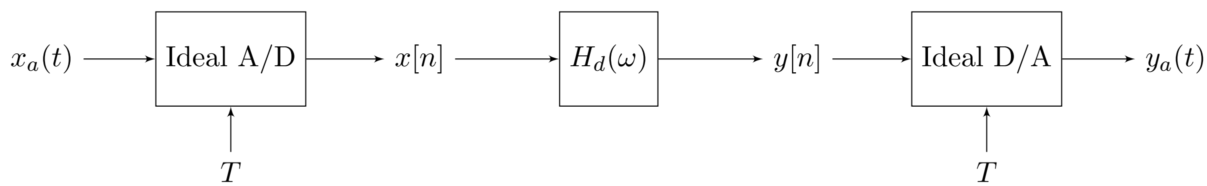
Note the sampling periods of the two converters are different! We sample $x_a(t)$ at $T_1 = \frac{1}{8000}$s for 4 seconds to obtain $x[n]$. You may round any decimals to three places or provide exact answers.

(a) Determine $x[n]$ and clearly state the values of n for which it is defined (i.e., the length of $x[n]$).

(b) Suppose we would like for $y_a(t)$ to sound like A5, A in the fifth octave and frequency 880 Hz, where $y_a(t) = \sin(2\pi \cdot 880t) = \sin(1760\pi t)$. Determine the necessary choice of T_2 and how many seconds the resampled audio clip $y_a(t)$ will be.

(c) Suppose instead we would like for $y_a(t)$ to sound like G3, G in the third octave and frequency 196 Hz, where $y_a(t) = \sin(2\pi \cdot 196t) = \sin(392\pi t)$. Determine the necessary choice of T_2 and how many seconds the resampling audio clip $y_a(t)$ will be.

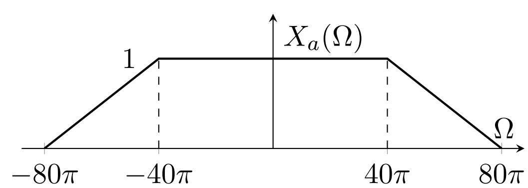
4. The below system consists of digital filter $H_d(\omega)$ fit between ideal A/D and D/A converters with shared sampling period T.



Suppose we know that $H_d(\omega)$ is given by the following low-pass filter.

$$H_d(\omega) = \begin{cases} 1, & |\omega| \le \frac{\pi}{3} \\ 0, & \frac{\pi}{3} < |\omega| \le \pi \end{cases}.$$

Let $X_a(\Omega)$, $X_d(\omega)$, $Y_d(\omega)$, and $Y_a(\Omega)$ be the Fourier transforms of $x_a(t), x[n], y[n]$ and $y_a(t)$, respectively. The CTFT $X_a(\Omega)$ is plotted below.



(a) Determine the Nyquist rate of $x_a(t)$, i.e. the Nyquist sampling frequency f_{nyq}.

(b) Let $T = \frac{1}{180}$s.

i. Sketch $X_d(\omega)$ and $Y_d(\omega)$ for $-3\pi \le \omega \le 3\pi$.

ii. Is there aliasing present in $X_d(\omega)$? Is there aliasing present in $Y_d(\omega)$?

(c) Let $T = \frac{1}{60}$s.

i. Sketch $X_d(\omega)$ and $Y_d(\omega)$ for $-3\pi \le \omega \le 3\pi$.

ii. Is there aliasing present in $X_d(\omega)$? Is there aliasing present in $Y_d(\omega)$?

(d) Determine the largest possible choice of T such that there is no aliasing present in $Y_d(\omega)$. **Hint**: some amount of aliasing may be allowable in $X_d(\omega)$!