HW Solutions

ECE 310: Digital Signal Processing, Spring 2026 Version: **1.0**

Due Date: 03/13/2026
Prepared by: Muzaffer & Lucas

1

Solution:

a)The magnitude of the frequency response is

$$|H_d(\omega)| = |(1 + 2\cos(2\omega))e^{-j2\omega}|.$$

Using the property that the magnitude of a product equals the product of magnitudes

$$|H_d(\omega)| = |1 + 2\cos(2\omega)| \cdot |e^{-j2\omega}|.$$

Since

$$|e^{-j2\omega}| = 1,$$

the magnitude response simplifies to

$$|H_d(\omega)| = |1 + 2\cos(2\omega)|.$$

b)

The phase of a product equals the sum of the phases:

$$\angle H_d(\omega) = \angle(1 + 2\cos(2\omega)) + \angle(e^{-j2\omega}).$$

Since

$$\angle(e^{-j2\omega}) = -2\omega,$$

we only need to determine the phase of the real term $1+2\cos(2\omega)$. Because $1+2\cos(2\omega)$ is real-valued, its phase is

$$\angle(1 + 2\cos(2\omega)) = \begin{cases} 0, & 1 + 2\cos(2\omega) > 0 \\ \pi, & 1 + 2\cos(2\omega) < 0 \end{cases}$$

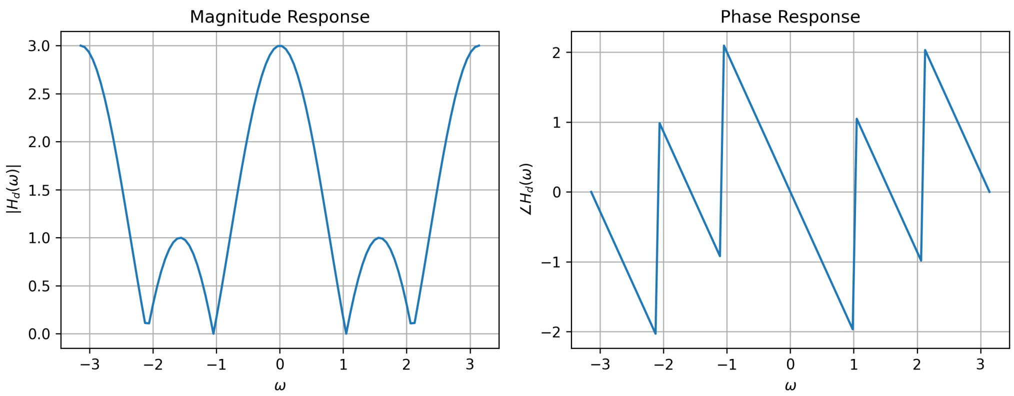
The sign change occurs when

$$1 + 2\cos(2\omega) = 0 \rightarrow \cos(2\omega) = -\frac{1}{2} \rightarrow \omega = \pm\frac{\pi}{3}, \pm\frac{2\pi}{3}$$

These frequencies correspond to zeros of the magnitude response and phase discontinuities. Therefore, the phase response is

$$\angle H_d(\omega) = \begin{cases} -2\omega, & |\omega| < \frac{\pi}{3} \\ -2\omega + \pi, & \frac{\pi}{3} < |\omega| < \frac{2\pi}{3} \\ -2\omega, & \frac{2\pi}{3} < |\omega| < \pi \end{cases}$$

in the following figure, the phase is defined modulo 2π over $[-\pi, \pi]$ range. When the linear phase -2ω exceeds the principal interval $[-\pi, \pi]$, it is wrapped back into this range, producing the additional jumps observed in the plotted phase response.



Grading: *20 points*

- +10 for each part

2

Solution:

The sampled discrete-time signal is obtained by evaluating the continuous-time signal at integer multiples of the sampling period T:

$$x[n] = x_a(nT).$$

Given the continuous-time signal

$$x_a(t) = \cos(40\pi t),$$

the sampled signal becomes

$$x[n] = x_a(nT) = \cos(40\pi nT).$$

a) $T = 1/100s$**:**

$$x[n] = \cos\left(40\pi n \cdot \frac{1}{100}\right) = \cos\left(\frac{4\pi}{10}n\right).$$

b) $T = 1/40s$**:**

$$x[n] = \cos\left(40\pi n \cdot \frac{1}{40}\right) = \cos(\pi n) = (-1)^n.$$

c) $T = 1/30s$**:**

$$x[n] = \cos\left(40\pi n \cdot \frac{1}{30}\right) = \cos\left(\frac{4\pi}{3}n\right)$$

d)

Given that the sampled signal is

$$x[n] = \cos\left(\frac{\pi}{2}n\right),$$

and from the sampling relation

$$x[n] = x_a(nT) = \cos(40\pi nT),$$

Then match them

$$40\pi T = \frac{\pi}{2} + 2\pi k, \quad k \in \mathbb{Z}.$$

or

$$40\pi T = -\frac{\pi}{2} + 2\pi k, \quad k \in \mathbb{Z}.$$

Solving for T,

$$T = \frac{\pm\frac{\pi}{2} + 2\pi k}{40\pi}$$

for $k = 0, 1$ 2 possible solutions are $T = \frac{1}{80}s, T = \frac{5}{80}s$

Grading: *20 points*

- +4 pts correct part (a)
- +4 pts correct part (b)
- +4 pts correct part (c)
- +8 pts correct part (d)

3

Solution:

By definition of sampling, the discrete signal after sampling on $x_a(t) = sin(\Omega_o t)$ is:

$$x[n] = sin(\Omega_o nT)$$

With $T = 1/2000$s, to make $x[n] = sin(\frac{\pi}{8}n)$ we must satisfy:

$$\Omega_o T = \frac{\pi}{8} + 2\pi k, k \in \mathcal{Z} \rightarrow \Omega_o = \frac{1}{T}(\frac{\pi}{8} + 2\pi k), k \in \mathcal{Z}$$

or since $sin(\theta) = sin(\pi - \theta)$

$$\Omega_o T = \pi - \frac{\pi}{8} + 2\pi k, k \in \mathcal{Z} \rightarrow \Omega_o = \frac{1}{T}(\frac{7\pi}{8} + 2\pi k), k \in \mathcal{Z}$$

Grading: *20 points*

4

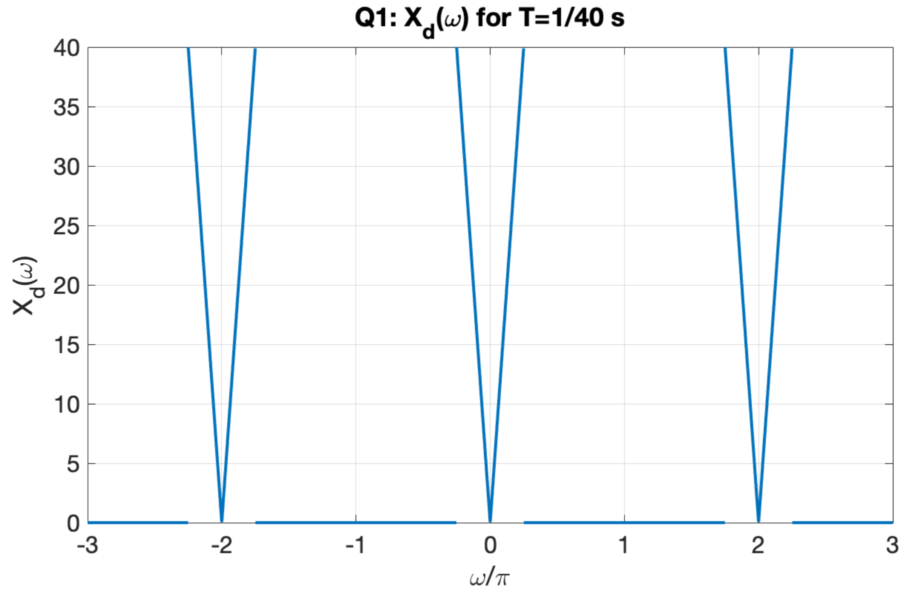
Solution:

a) The continuous-time signal $x_a(t)$ has spectrum $X_a(\Omega)$ band-limited to $|\Omega| \leq 10\pi$ rad/s (i.e. $f_{\max} = 5$ Hz). The Nyquist rate equals twice the highest frequency:

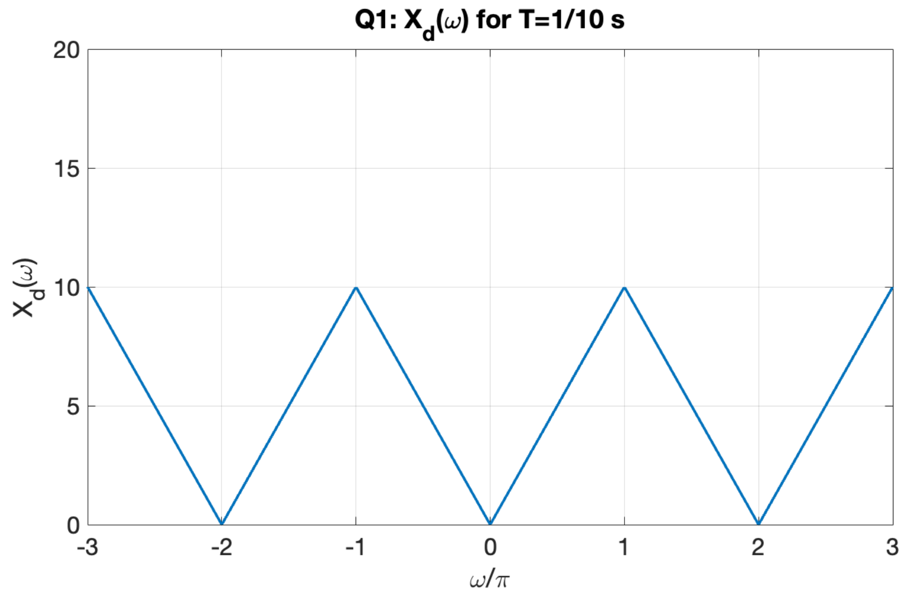
$$f_{\text{nyq}} = 2f_{\max} = 10 \text{ Hz}.$$

Thus, the minimum sampling period to avoid aliasing is $T_{\min} = 0.1$ s.

b) $T = \frac{1}{40}$ **s: (Oversampling)** Here $f_s = 40$ Hz $= 4 \times$ Nyquist, no aliasing. Frequency mapping: $\omega = \Omega T \Rightarrow |\omega| \leq 10\pi T = \pi/4$. Hence, within $[-\pi, \pi]$, $X_d(\omega)$ is nonzero only in $[-\pi/4, \pi/4]$.



c) $T = \frac{1}{10}$ **s: (Nyquist sampling)** $f_s = 10$ Hz $=$ Nyquist rate. Mapping gives $|\omega| \leq 10\pi T = \pi$. Thus, $X_d(\omega)$ exactly fills the main $[-\pi, \pi]$ range, touching at edges but not overlapping.



Grading: *20 points*

- +8 pts for part a
- +6 pts for part b
- +6 pts for part c