

UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN
Department of Electrical and Computer Engineering
ECE 310 DIGITAL SIGNAL PROCESSING – SPRING 2026

Homework 7

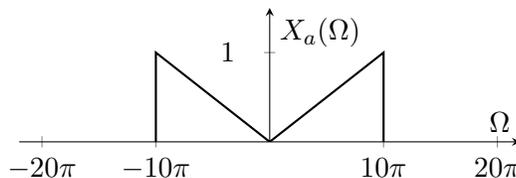
Prof. Snyder, Shomorony

Due: Friday, Mar. 13, 11:59pm on Gradescope

1. Let the frequency response of an LTI system be given by

$$H_d(\omega) = (1 + 2 \cos(2\omega))e^{-j2\omega}.$$

- (a) Plot the magnitude response $|H_d(\omega)|$ for $-\pi \leq \omega \leq \pi$.
 - (b) Plot the phase response $\angle H_d(\omega)$ for $-\pi \leq \omega \leq \pi$.
2. Consider the continuous-time signal $x_a(t) = \cos(40\pi t)$. We sample $x_a(t)$ with sampling period T to obtain $x[n]$.
- (a) Determine $x[n]$ when $T = \frac{1}{100}$ s.
 - (b) Determine $x[n]$ when $T = \frac{1}{40}$ s.
 - (c) Determine $x[n]$ when $T = \frac{1}{30}$ s.
 - (d) Suppose we are given that $x[n] = \cos(\frac{\pi}{2}n)$. Determine **two** possible values of T that would produce this signal $x[n]$.
3. Let $x[n] = \sin(\frac{\pi}{8}n)$ be a discrete-time signal that is obtained by sampling the continuous-time signal $x_a(t) = \sin(\Omega_0 t)$ at sampling period $T = \frac{1}{2000}$ s. Determine a general formula for the values of Ω_0 that could result in the same given sequence $x[n]$.
4. The continuous-time signal $x_a(t) \xleftrightarrow{\mathcal{F}} X_a(\Omega)$ has the continuous-time Fourier transform shown below.



The signal $x_a(t)$ is sampled with sampling period T to get the discrete-time signal $x[n] = x_a(nT)$ with DTFT $X_d(\omega)$.

- (a) Determine the Nyquist rate of $x_a(t)$, i.e. the Nyquist sampling frequency f_{nyq} .
- (b) Sketch the DTFT $X_d(\omega)$ when $T = \frac{1}{40}$ s for $-3\pi \leq \omega \leq 3\pi$.
- (c) Sketch the DTFT $X_d(\omega)$ when $T = \frac{1}{10}$ s for $-3\pi \leq \omega \leq 3\pi$.