HW6 Solutions

ECE 310: Digital Signal Processing, Spring 2026 Version: **1.0**

Due Date: 10/11 @ 11:59pm
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1

Solution:

a) $y[n] = x[-n+2]$

We first use the *Property of Time-Shift* to get DTFT of $x[n+2]$, which is $X_d(w)e^{j2w}$, followed by *Property of Time Reversal* to get:

$$Y_d(w) = X_d(-w)e^{-j2w}$$

b) $y[n] = x[n] + x^*[n]$

Here we can use the *Property of Linearity* and the *Property of Conjugation*:

$$Y_d(w) = X_d(w) + X_d^*(-w)$$

c) $Y[n] = \cos(\pi n/6)x[n-1]$ Since

$$\cos(\pi n/6)x[n-1] = \frac{e^{-j\pi n/6} + e^{j\pi n/6}}{2}x[n-1]$$

We can first use the *Property of Time-Shift* to get DTFT of $x[n-1]$, which is $X_d(w)e^{-jw}$, and then do *Property of Frequency Shift* to get:

$$Y_d(w) = \frac{X_d(w+\pi/6)e^{-j(w+\pi/6)} + X_d(w-\pi/6)e^{-j(w-\pi/6)}}{2}$$

d) $y[n] = (x[n-3] + x[n+3])/2$

We can use the *Property of Linearity* and the *Property of Time-Shift*:

$$\begin{aligned} Y_d(w) &= \frac{X_d(w)e^{-j3w} + X_d(w)e^{j3w}}{2} \\ &= X_d(w)\left(\frac{e^{-j3w} + e^{j3w}}{2}\right) \\ &= X_d(w)\cos(3w) \end{aligned}$$

Grading: *40 points*

- +9 each
  - -0: Correct
  - -3: Minor mistake
  - -6: Major mistake
  - -9: Empty or invalid

2

Solution:

a) $y[n] = x[n+2] + x[n-2]$

First, find the frequency response:

$$H_d(w) = e^{+j2\omega} + e^{-j2\omega} = 2\cos(2\omega)$$

From this, we get:

$$|H_d(\omega)| = |2cos(2\omega)|$$
$$\angle H_d(\omega) = \angle cos(2\omega)$$

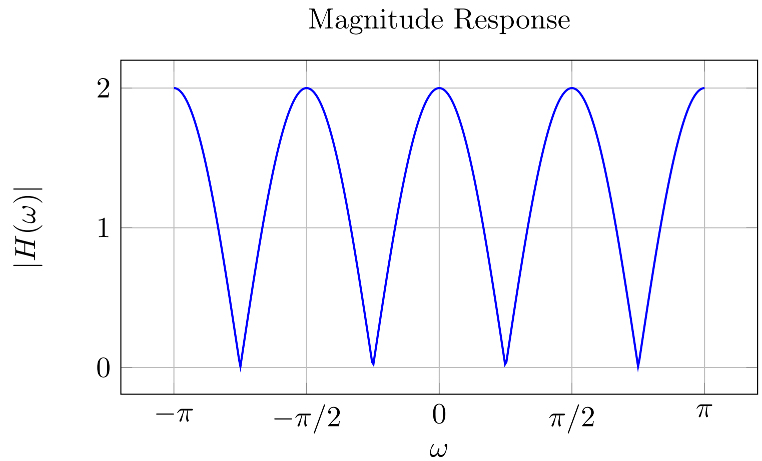
The phase has a term $\angle cos(2\omega)$ as the sign of $cos(\omega)$ also contributes to the phase. Since $cos(2\omega)$ is a real value, we can get its phase as

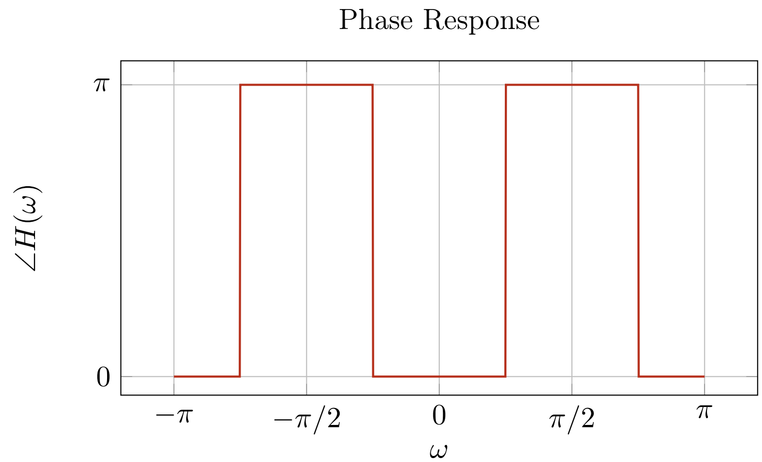
$$\angle cos(2\omega) = \begin{cases} 0 & \text{, when } cos(2\omega) > 0 \\ \pi & \text{, else} \end{cases} \tag{1}$$

Thus the phase is

$$\angle H_d(\omega) = \begin{cases} 0 & , -\frac{\pi}{2} < \omega < \frac{\pi}{2} \\ \pi & \text{, else} \end{cases} \tag{2}$$

The magnitude plot and phase plot are given below:





Please note that in the phase plot, we keep the value in the range $[-\pi, \pi]$ by adding $2k\pi$ to it where $k \in Z$. In DTFT, because of the 2π period, the phase values are equivalent if adding $2k\pi$ to it.

b) $y[n] = x[n] + x[n-2]$

First, find the frequency response:

$$H_d(w) = 1 + e^{-j2\omega}$$

This can be written as:

$$H_d(\omega) = e^{-jw}(e^{jw} + e^{-jw}) = 2cos(\omega)e^{-jw}$$

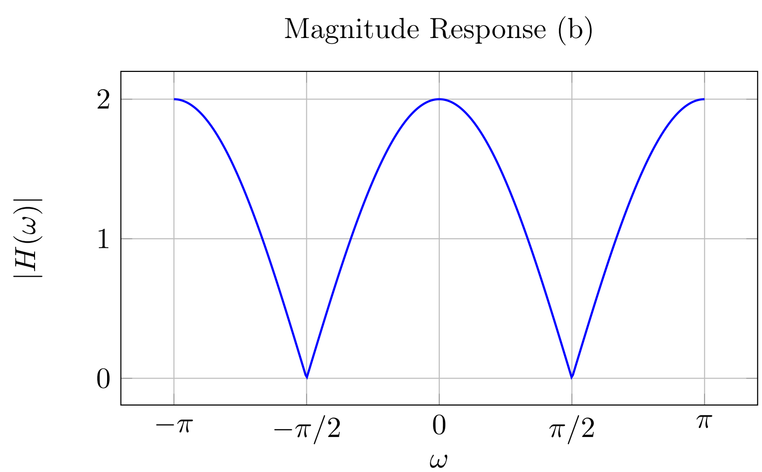
From this, we get:

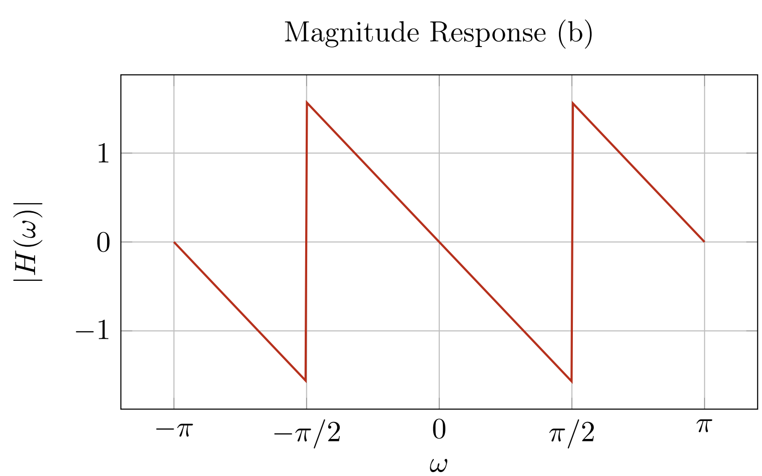
$$|H_d(\omega)| = |2cos(\omega)| = -w + \angle cos(\omega)$$

Where the phase term is Thus the phase is

$$\angle H_d(\omega) = \begin{cases} -w & , -\frac{\pi}{2} < \omega < \frac{\pi}{2} \\ -w + \pi & , \text{else} \end{cases} \quad (3)$$

The magnitude plot and phase plot are given below:





Please note that in the phase plot, we keep the value in the range $[-\pi, \pi]$ by adding $2k\pi$ to it where $k \in Z$. In DTFT, because of the 2π period, the phase values are equivalent if adding $2k\pi$ to it.

c) $y[n] = x[n+1] - x[n]$

First, find the transfer function:

$$H_d(w) = e^{j\omega} - 1$$

This can be written as:

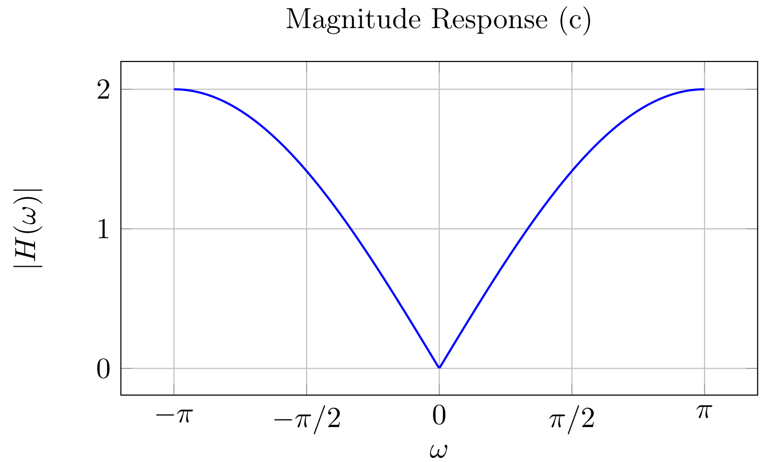
$$H_d(\omega) = e^{j\frac{w}{2}}(e^{j\frac{w}{2}} - e^{-j\frac{w}{2}})$$
$$H_d(\omega) = 2jsin(\frac{\omega}{2})e^{j\frac{w}{2}}$$
$$H_d(\omega) = 2sin(\frac{\omega}{2})e^{j(\frac{\pi}{2}+\frac{w}{2})}$$

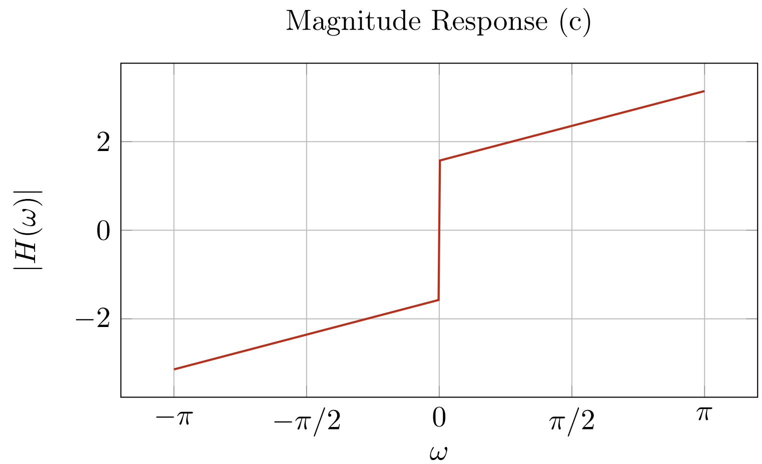
From this, we get:

$$|H_d(\omega)| = |2sin(\frac{\omega}{2})|$$
$$\angle H_d(\omega) = \frac{\pi}{2} + \frac{w}{2} + \angle sin(\frac{\omega}{2})$$

$$\angle H_d(\omega) = \begin{cases} \frac{\pi}{2} + \frac{\omega}{2} - \pi = -\frac{\pi}{2} + \frac{\omega}{2} & ,\omega < 0 \\ \frac{\pi}{2} + \frac{\omega}{2} & ,\omega > 0 \end{cases} \tag{4}$$

The magnitude plot and phase plot are given below:





Grading: *30 points*

- +10 points each
  - -0: Correct
  - -1: Phase response: Minor mistake
  - -3: Phase response: major mistake, empty, or invalid
  - -1: Phase plot: minor mistake
  - -2: Phase plot: major mistake, empty, or invalid
  - -1: Magnitude response: Minor mistake
  - -3: Magnitude response: major mistake, empty, or invalid
  - -1: Magnitude plot: minor mistake
  - -2: Magnitude plot: major mistake, empty, or invalid
  - -10: Empty, Wrong, or invalid

3

Solution:

(a)

To check if $h[n]$ is real-valued, we need to check whether $H_d(-w) = H_d^*(w)$. Equivalently, we can check whether $|H_d(w)| = |H_d(-w)|$ and $\angle H_d(w) = -\angle H_d(-w)$ both are satisfied.

From the figure, we can see that $|H_d(w)|$ is even symmetric, so $|H_d(w)| = |H_d(-w)|$ is satisfied. Additionally, $\angle H_d(w)$ is odd symmetric, so $\angle H_d(w) = -\angle H_d(-w)$ is also satisfied. Then we can conclude that $h[n]$ is real-valued.

(b)

To find the output to $x[n]$, we can use the following input-output relationship:

$$Ae^{w_0 n} \rightarrow H_d(w) \rightarrow H_d(w_0)Ae^{w_0 n}$$

Since $h[n]$ is real-valued, we can use the following input-output relationship for sinusoidal signals:

$$A\cos(w_0 n + \theta) \rightarrow H_d(w) \rightarrow |H_d(w_0)|A\cos(w_0 n + \theta + \angle H_d(w_0))$$

The input $x[n] = 3 + 2\cos(\frac{\pi}{4}n)$ has two components with different frequencies.

The first component, 3, has $w_0 = 0$, so the corresponding output is $3H_d(0) = 0$.

The second component $2\cos(\frac{\pi}{4}n)$ has $w_0 = \frac{\pi}{4}$, so the corresponding output is $|H_d(\frac{\pi}{4})|2\cos(\frac{\pi}{4}n + \angle H_d(\frac{\pi}{4})) = \frac{1}{2}\cos(\frac{\pi}{4}n + \frac{\pi}{2})$.

Then the output to $x[n]$ is $y[n] = \frac{1}{2}\cos(\frac{\pi}{4}n + \frac{\pi}{2})$.

Grading: *16 points*

- +8 points for part (a)
  - -0: Correct answer and reasoning
  - -4: Correct answer without valid reasoning
  - -8: Incorreect answer or invalid response
- +8 points for part (b)

– -0: Correct
– -2: Minor mistake
– -5: Major mistake
– -8: Wrong, empty, or invalid

4

Solution:

$y[n] + \frac{1}{3}y[n-1] = x[n]$

We start by finding $H_d(\omega)$:

$$H_d(\omega) = \frac{1}{1 + \frac{1}{3}e^{-j\omega}}$$

To find the output to $x[n]$, we can use the following input-output relationship:

$$Ae^{j\omega_0 n} \rightarrow H_d(\omega) \rightarrow H_d(\omega_0)Ae^{j\omega_0 n}$$

From the difference equation, we can conclude that $h[n]$ is real-valued. Therefore, we can use the following input-output relationship for sinusoidal signals:

$$A\cos(\omega_0 n + \theta) \rightarrow H_d(\omega) \rightarrow |H_d(\omega_0)|A\cos(\omega_0 n + \theta + \angle H_d(\omega_0))$$

(a) $x[n] = 1 + \cos\left(\frac{\pi}{2}n + \frac{\pi}{4}\right)$

The first component, 1, has $\omega_0 = 0$. As $|H_d(0)| = \frac{3}{4}$ and $\angle H_d(0) = 0$, the corresponding output is

$$|H_d(0)| \times 1 = \frac{3}{4}.$$

The second component, $\cos\left(\frac{\pi}{2}n + \frac{\pi}{4}\right)$, has $\omega_0 = \frac{\pi}{2}$. So the output is

$$|H_d\left(\tfrac{\pi}{2}\right)|\cos\left(\frac{\pi}{2}n + \frac{\pi}{4} + \angle H_d\left(\tfrac{\pi}{2}\right)\right),$$

where

$$\left|H_d\left(\tfrac{\pi}{2}\right)\right| = \frac{3}{\sqrt{10}} \qquad \text{and} \qquad \angle H_d\left(\tfrac{\pi}{2}\right) = \arctan\left(\frac{1}{3}\right).$$

Therefore, the corresponding output is

$$\frac{3}{\sqrt{10}}\cos\left(\frac{\pi}{2}n + \frac{\pi}{4} + \arctan\left(\frac{1}{3}\right)\right).$$

By adding them together, we can obtain the output to $x[n]$:

$$y[n] = \frac{3}{4} + \frac{3}{\sqrt{10}}\cos\left(\frac{\pi}{2}n + \frac{\pi}{4} + \arctan\left(\frac{1}{3}\right)\right)$$

(b) $x[n] = (-1)^{n+3}$

First rewrite

$$x[n] = (-1)^{n+3} = e^{j\pi(n+3)} = -e^{j\pi n}.$$

We can see that $\omega_0 = \pi$.

Then

$$H_d(\pi) = \frac{1}{1 + \frac{1}{3}e^{-j\pi}} = \frac{1}{1 - \frac{1}{3}} = \frac{3}{2}.$$

So we can obtain the output:

$$y[n] = H_d(\pi)\left(-e^{j\pi n}\right) = -\frac{3}{2}e^{j\pi n} = \frac{3}{2}(-1)^{n+3}.$$

Grading: *18 points*

- +9 Each correct answer
  - -0: Correct
  - -3: Minor mistake
  - -6: Major mistake
  - -9: Wrong, empty, or invalid