UNIVERSITY OF ILLINOIS URBANA-CHAMPAIGN
Department of Electrical and Computer Engineering

ECE 310 DIGITAL SIGNAL PROCESSING – SPRING 2026

Homework 6

Profs. Snyder, Shomorony   **Due: Friday, March 6, 11:59pm on Gradescope**

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1. Let $\{x[n]\}$ be an arbitrary discrete-time signal (not necessarily real-valued) with DTFT $X_d(\omega)$. Express the DTFT of $\{y[n]\}$ in terms of $X_d(\omega)$:

    (a) $y[n] = x[-n+2]$.

    (b) $y[n] = x[n] + x^*[n]$

    (c) $y[n] = \cos\left(\frac{\pi n}{6}\right) x[n-1]$

    (d) $y[n] = (x[n-3] + x[n+3])/2$

2. For each of the following LTI systems, compute and sketch its magnitude response $|H_d(\omega)|$ and phase response $\angle H_d(\omega)$, for $-\pi \le \omega \le \pi$:

    (a) $y[n] = x[n+2] + x[n-2]$.

    (b) $y[n] = x[n] + x[n-2]$.

    (c) $y[n] = x[n+1] - x[n]$.

3. Suppose an LTI system has a frequency response $H_d(\omega)$ as shown below:
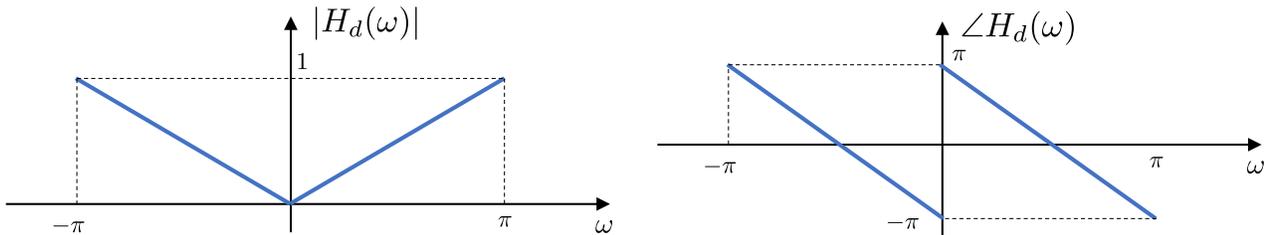
    (a) Is the impulse response $h[n]$ real-valued?

    (b) What is the output $y[n]$ when the input to this system is $x[n] = 3 + 2\cos\left(\frac{\pi}{4}n\right)$?

4. Consider a causal LTI system described by the difference equation

$$y[n] + \frac{1}{3}y[n-1] = x[n], \quad \text{for } n = 0, 1, 2, \dots$$

    Determine the output $y[n]$ when the input is

    (a) $x[n] = 1 + \cos\left(\frac{\pi}{2}n + \frac{\pi}{4}\right)$.

    (b) $x[n] = (-1)^{n+3}$.