UNIVERSITY OF ILLINOIS URBANA-CHAMPAIGN
Department of Electrical and Computer Engineering

ECE 310 Digital Signal Processing – Spring 2026

Homework 4

Profs. Snyder, Shomorony **Due: Friday, Feb 20, 11:59pm on Gradescope**

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1. Suppose a signal $x[n] = (1/2)^n u[n-1]$ is the input to an LTI system with impulse response $h[n] = -2^n u[-n-1]$. Compute the output $y[n]$ by computing the z-transforms of $x[n]$ and $h[n]$ and the inverse z-transform of $H(z)X(z)$.

2. Determine whether each of the following systems that map input signal $\{x[n]\}$ to output signal $\{y[n]\}$ is bounded-input bounded-output (BIBO) stable.
   (a) $y[n] = (x[n])^2$.
   (b) $y[n] = nx[n]$.
   (c) $y[n] = (x * u)[n]$.

3. Consider a causal system described by the LCCDE
$$y[n] = \frac{1}{6}y[n-1] + \frac{1}{6}y[n-2] + 2x[n] + x[n-1],$$
and assume that the system is initially at rest ($y[n] = 0$ for $n < 0$).
   (a) Find the transfer function and its ROC.
   (b) Identify all the zeros and poles of the system on the ROC plot.
   (c) Find the impulse response of the system.
   (d) Is the system BIBO stable?
   (e) Determine the output $\{y[n]\}$ given the input $\{x[n]\} = \{\underset{\uparrow}{3}, 1\}$.

4. Suppose two LTI systems with impulse responses
$$h_1[n] = -\delta[n+1] + 2\delta[n] - \delta[n-1] \qquad h_2[n] = (n+1)u[n]$$
are connected in series. Determine the transfer functions $H_1(z)$ and $H_2(z)$ and use them to determine (i) whether each individual system is BIBO stable and (ii) whether the overall system (obtained by connecting both systems in series) is BIBO stable.

5. Consider an LTI system described by the following input-output relationship:
$$y[n] = x[n] + \frac{5}{2}x[n-1] + x[n-2].$$
   (a) Find the transfer function and the impulse response of the system $H(z)$.
   (b) Derive a causal inverse system $G(z)$ and the corresponding difference equation that inputs $\{y[n]\}$. and outputs $\{s[n]\}$ such that $s[n] = x[n]$ for all n.
   (c) Is the system $H(z)$ BIBO stable? How about $G(z)$?
   (d) Now factor the system $G(z)$ into a cascade of two simpler single-pole systems $G_1(z)$ and $G_2(z)$. Describe the implementation of each of them as either a causal or an anti-causal system so that the overall system is BIBO stable. Explain your answer carefully.