UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN
Department of Electrical and Computer Engineering

ECE 310 Digital Signal Processing – Spring 2026

Homework 3

Profs. Snyder, Shomorony **Due: Friday, Feb 13, 11:59pm on Gradescope**

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1. For each of the following discrete-time signals, **(i)** determine the z-transform and **(ii)** state the region of convergence (ROC).

   (a) $x_1[n] = \delta[n+3] + 4\delta[n] - \delta[n-2]$

   (b) $x_2[n] = \left(\frac{3}{4}\right)^{n+3} u[n-2]$

   (c) $x_3[n] = 3^n u[-n] + 2^{-n} u[n]$

   (d) $x_4[n] = \left(\frac{1}{4}\right)^{|n|}$

   (e) $x_5[n] = n\left(\frac{1}{2}\right)^n (u[n] - u[n-5])$

2. We are given the following z-transform pair

$$x[n] \overset{\mathcal{Z}}{\longleftrightarrow} X(z) = \frac{1}{1-\frac{1}{3}z^{-1}}, \text{ ROC: } |z| > \frac{1}{3}.$$

   Using properties of the z-transform, determine the z-transform of each of the following signals and state the region of convergence (ROC).

   (a) $x_1[n] = x[n+3]$

   (b) $x_2[n] = 2^n x[n]$

   (c) $x_3[n] = \cos\left(\frac{\pi}{4}n\right) x[n]$

   (d) $x_4[n] = n(n-2)x[n-1]$

   (e) $x_5[n] = \left(\frac{1}{5}\right)^n u[n] * x[n]$

3. The z-transform $H(z)$ and ROC of multiple systems are given below. Determine the corresponding impulse response $h[n]$ of each system.

   (a) $H_1(z) = 1 - \frac{2}{3}z^{-1} + \frac{4}{9}z^{-2} - \frac{8}{27}z^{-3}$, ROC: $z \neq 0$

   (b) $H_2(z) = \frac{3z^{-2}}{1+2z^{-1}}$, ROC: $|z| > 2$

   (c) $H_3(z) = \frac{1}{1-\frac{1}{4}z^{-1}} + \frac{1}{1-\frac{3}{2}z^{-1}}$, ROC: $\frac{1}{4} < |z| < \frac{3}{2}$

   (d) $H_4(z) = \frac{1}{\left(1-\frac{2}{3}z^{-1}\right)\left(1-z^{-1}\right)}$, ROC: $|z| > 1$

4. Consider a **right-sided** LTI system with impulse response $h[n]$. The z-transform of $h[n]$, $H(z)$, is given below.

$$H(z) = \frac{1 - 3z^{-1}}{1 + \frac{1}{6}z^{-1} - \frac{1}{3}z^{-2}}$$

   (a) Determine the poles and zeros of $H(z)$.

   (b) Determine the ROC of $H(z)$ and sketch the pole-zero plot.

   (c) Determine the impulse response of this system $h[n]$.