HW1 Solutions

ECE 310: Digital Signal Processing, Spring 2026 Version: **1.0**

Due Date: 01/30/2026
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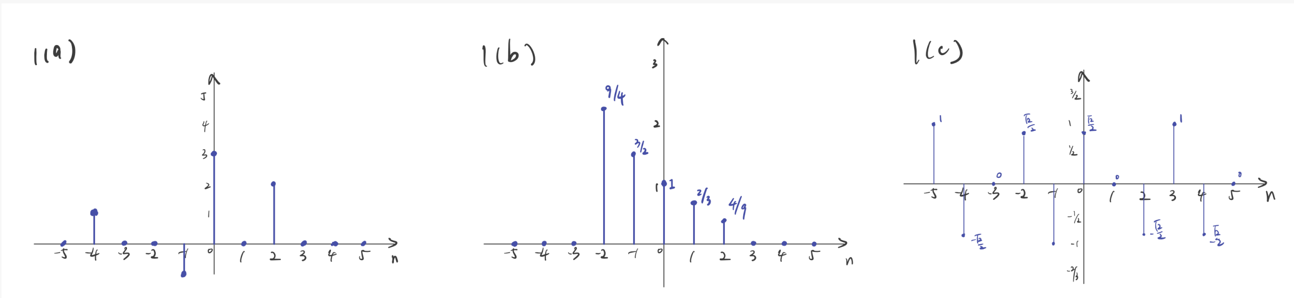
1

Solution:



Grading: *18 points, 6 points each*

- (a), (b), and (c):
  - +6: correct answer
  - +4: minor mistake (axis labeling, etc)
  - +2: major mistake (plot is significantly off from the given solution, attempts shown)
  - +0: empty or invalid

2

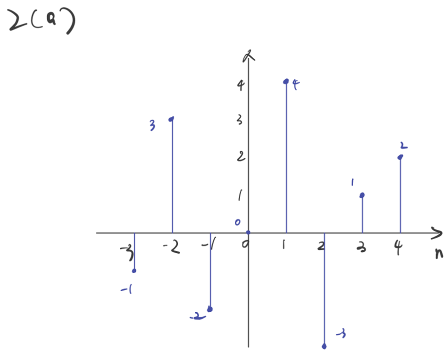
Solution:

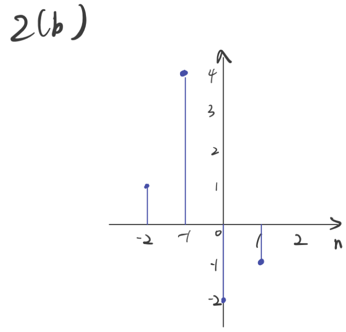
a)

$$y[n] = \{-1, 3, -2, \underset{\uparrow}{0}, 4, -3, 1, 2\}$$

b)

$$y[n] = \{1, 4, \underset{\uparrow}{-2}, -1\}$$





Grading: *16 points, 8 for each (a) and (b)*

- (a) and (b):
  - +8: plot correct
  - +6: minor numerical mistake in plot
  - +3: major mistake in plot, effort shown in student's work
  - +0: empty or invalid

3

Solution:

a)

- delta expression: $x[n] = 3(\delta[n+3] + \delta[n+2] + \delta[n+1] + \delta[n] + \delta[n-1])$
- step function expression: $3(u[n+3] - u[n-2])$

b)

- delta expression: $x[n] = 2(\delta[n+3]+\delta[n+2]+\delta[n+1]+\delta[n])-(\delta[n-1]+\delta[n-2]+\delta[n-3]+\delta[n-4])$
- step function expression: $2u[n+3] - 3u[n-1] + u[n-5]$

Grading: *20 points, 10 for each (i) and (ii)*

- (i) and (ii):
  - +5: correct Kronecker delta expression
  - +4: minor mistake in Kronecker delta expression
  - +2: major mistake in Kronecker delta expression
  - +0: empty or invalid in Kronecker delta expression
  - +5: correct unit-step expression
  - +4: minor mistake in unit-step expression
  - +2: major mistake in unit-step expression
  - +0: empty or invalid in unit-step expression

4

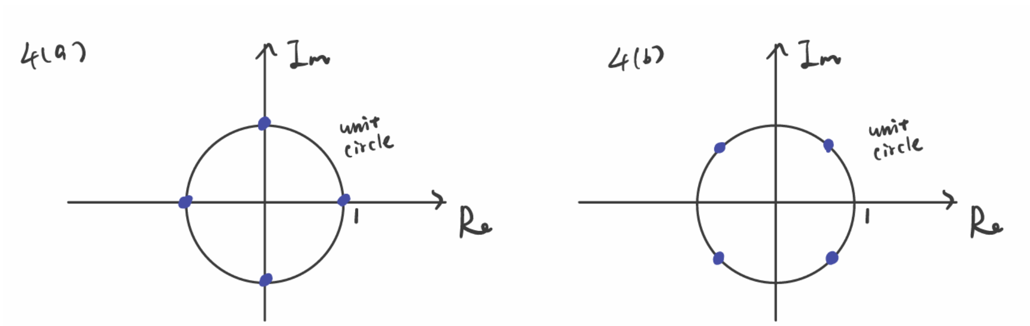
Solution:

a)

$$z^4 - 1 = 0$$
$$z^4 = e^{j(2\pi)k}, \quad k \in \mathbb{Z}$$
$$z = e^{j\frac{1}{2}\pi\cdot k}$$

b)

$$z^4 + 1 = 0$$
$$z^4 = e^{j(-\pi+2\pi k)}, \quad k \in \mathbb{Z}$$
$$z = e^{j\left(-\frac{\pi}{4}+\frac{1}{2}\pi k\right)}$$



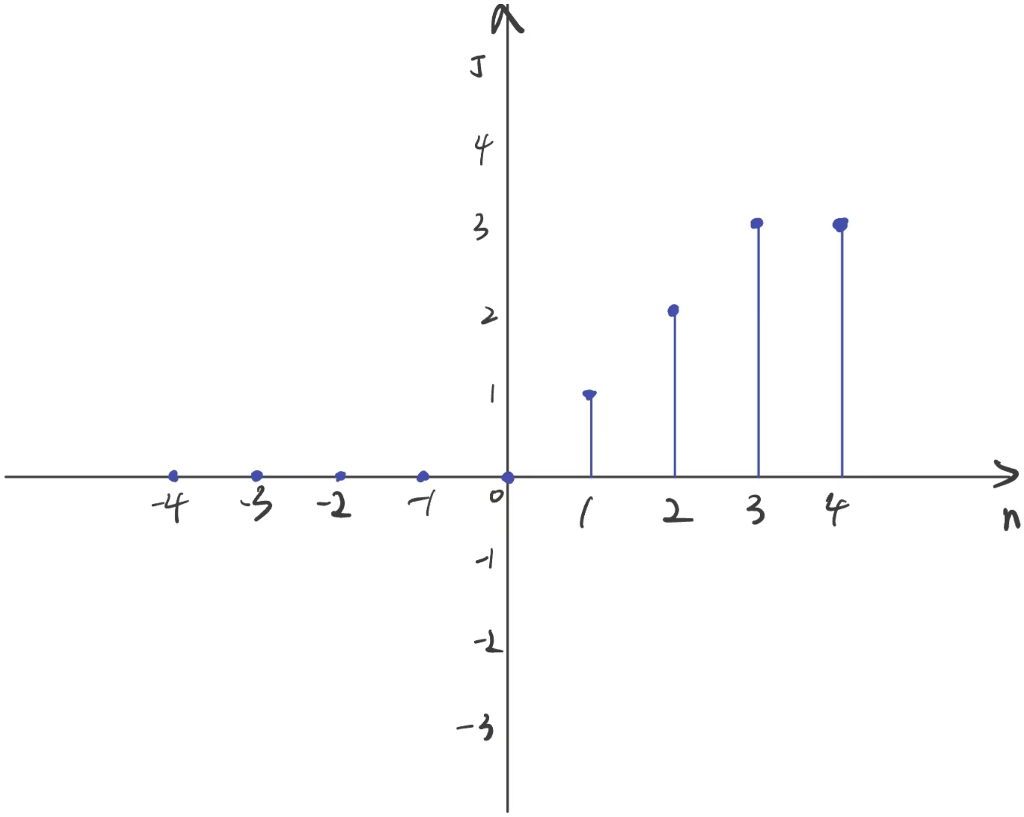
Grading: *20 points, 10 for each (i) and (ii)*

- for (a) and (b):
  - +5: correct z values
  - +4: minor mistake in z values
  - +2: major mistake in z values
  - +0: empty or invalid in z values
  - +5: correct z values plot
  - +4: minor z values plot
  - +2: major z values plot
  - +0: z values plot

5

Solution:

a)



b) It's **NOT LINEAR**. We can take a counter example by letting $x_1[n] = 1, x_2[n] = n^2, a = 0, b = 4$, then we can find

$$T(x_1[n] + x_2[n]) = \{\cdots, 4, 4, 2, \underset{\uparrow}{1}, 2, 4, 4, \cdots\}$$
$$T(x_1[n]) + T(x_2[n]) = \{\cdots, 5, 4, 2, \underset{\uparrow}{1}, 2, 4, 5, \cdots\}$$
$$T(x_1[n] + x_2[n]) \neq T(x_1[n]) + T(x_2[n])$$

c) It's **SHIFT INVARIANT**:

$$T(x[n-n_0]) = \begin{cases} b, & x \geq b \\ x[n-n_0], & a < x[n-n_0] < b \\ a, & x[n-n_0] \leq a \end{cases}$$

$$T(x)[n-n_0] = \begin{cases} b, & x \geq b \\ x[n-n_0], & a < x[n-n_0] < b \\ a, & x[n-n_0] \leq a \end{cases}$$

We can find that the clip operation will shift together with the input signal, so the system is shift-invariant.

Grading: *19 points, 6 for (a), 10 for each (b) and (c)*

- for (a):
  - +6: correct plot
  - +4: minor mistake in plot
  - +2: major mistake in proof
  - +0: empty or invalid
- for (b) and (c):
  - +10: correct proof
  - +7: minor mistake in proof
  - +4: major mistake in proof
  - +0: empty or invalid