UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN
Department of Electrical and Computer Engineering

ECE 310 Digital Signal Processing – Spring 2026

Homework 11

Profs. Snyder, Shomorony **Due: Friday, April 24, 2026, 11:59pm on Gradescope**

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1. For each of the following scenarios, decide which digital filter type (low-pass, high-pass, band-pass, or band-stop) should be used. Specify the cutoff frequencies for the passband(s) and stopband(s) of each filter allowing for the largest possible transition band. State all cutoff frequencies as digital radial frequencies between $0 \leq \omega \leq \pi$.

   (a) A piece of music has been recorded with high-pitch background noise present. We know the music was recorded with ideal sampling at 24 kHz, the maximum frequency of the music is 4 kHz, and the background noise has a minimum frequency of 8 kHz. Specify the digital filter to remove the noise from the music.

   (b) Three people with distinct voices are speaking at the same time. We know the first person speaks in the range of 50 Hz – 200 Hz, the second person speaks in the range of 300 Hz – 500 Hz, and the third person speaks in the range of 600 Hz – 800 Hz. We record the speakers by sampling all three speaking at the same time with an ideal A/D converter at sampling frequency 2 kHz. Specify the digital filter to remove the first and third speakers and only listen to the second speaker.

2. Consider a causal LTI system implemented by the below LCCDE:

$$y[n] = -\frac{1}{2}y[n-1] + y[n-2] + 2x[n] - \frac{1}{4}x[n-1] + x[n-2].$$

   (a) Determine the transfer function $H(z)$ of this LTI system.

   (b) Sketch the Direct Form-I block diagram for this system.

   (c) Sketch the Direct Form-II block diagram for this system.

3. The frequency resposne of a generalized linear phase (GLP) filter can be expressed as

$$H_d(\omega) = A(\omega)e^{j(-\alpha\omega+\beta)},$$

   where $A(\omega)$ is a real-valued function and α and β are real-valued constants. For each of the following filters, determine whether it is a GLP filter. If the filter has generalized linear phase, find $A(\omega)$, α, β, and indicate whether the filter also has linear phase.

   (a) $\{h[n]\}_{n=0}^{1} = \{1, -1\}$

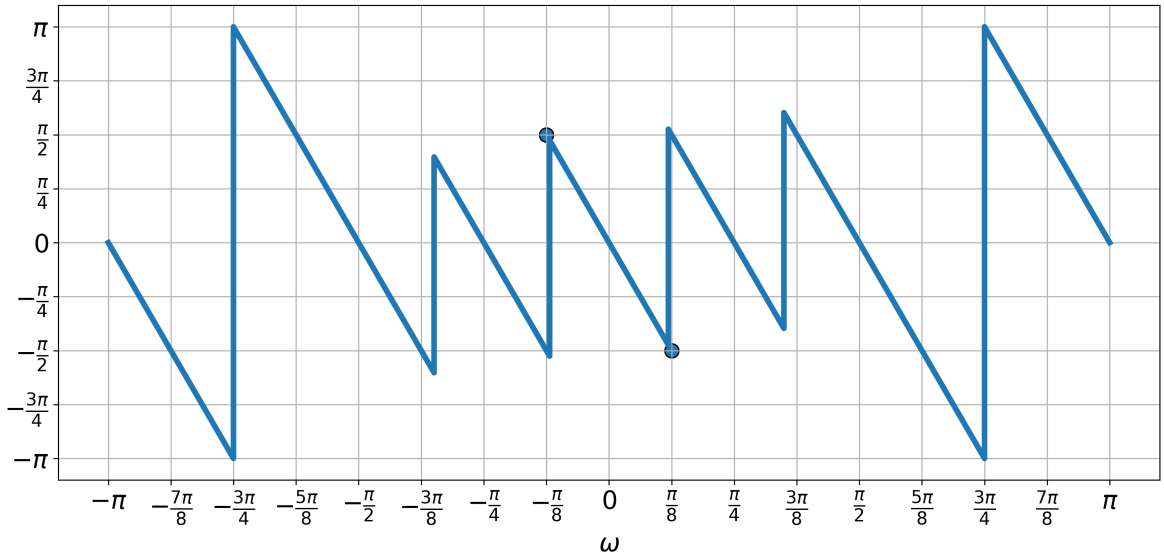
   (b) $\{h[n]\}_{n=0}^{2} = \{1, 2, 3\}$

   (c) $\{h[n]\}_{n=0}^{4} = \{1, 0, 0, 0, 1\}$

   (d) $\{h[n]\}_{n=0}^{3} = \{1, 1, 1, 1\}$

   (e) $\{h[n]\}_{n=0}^{2} = \{2, -1, -2\}$

4. We are given that a GLP FIR filter is obtained by shifting the impulse response of an ideal filter and applying a rectangular window. The phase response $\angle H_d(\omega)$ of this filter is shown below.



(a) Does this filter have strictly linear phase? Briefly justify your reasoning.

(b) Is it possible for this filter to be a Type–III or Type–IV GLP filter? Briefly justify your reasoning.

(c) What is the length N of this filter?

(d) What is the value of $\left|H_d\left(\frac{\pi}{8}\right)\right|$?