ECE 220

Lecture x000C

Slides based on material originally by: Yuting Chen & Thomas Moon

Review

- Last time we discussed sorting & searching
  - C • Selection sort
  - A • Insertion sort
  - D • Quick sort
  - B • Bubble sort

A. Select next element and move things to place it in proper spot
B. Keep comparing pairs and swapping them till no more swaps
C. Find minimum in each pass and bring to appropriate spot
D. Pick pivot & move elements to left (lesser) or right (greater) of pivot

Lesson objectives

- Understand recursion as a *divide-and-conquer* programming technique.
    - Understand difference between recursive cases and terminating/base cases.
    - Understand recursion implicitly uses stack data structure.
- Be able to implement recursive functions in C.
- Be (in principle) be able to convert recursive functions to iterative ones.
- Be able to lower recursive C functions to LC3 assembly.

Recap binary search

Key = 109

Pick middle
Is middle > key?

Is middle == key?
Go left

Pick middle
Is middle > key?

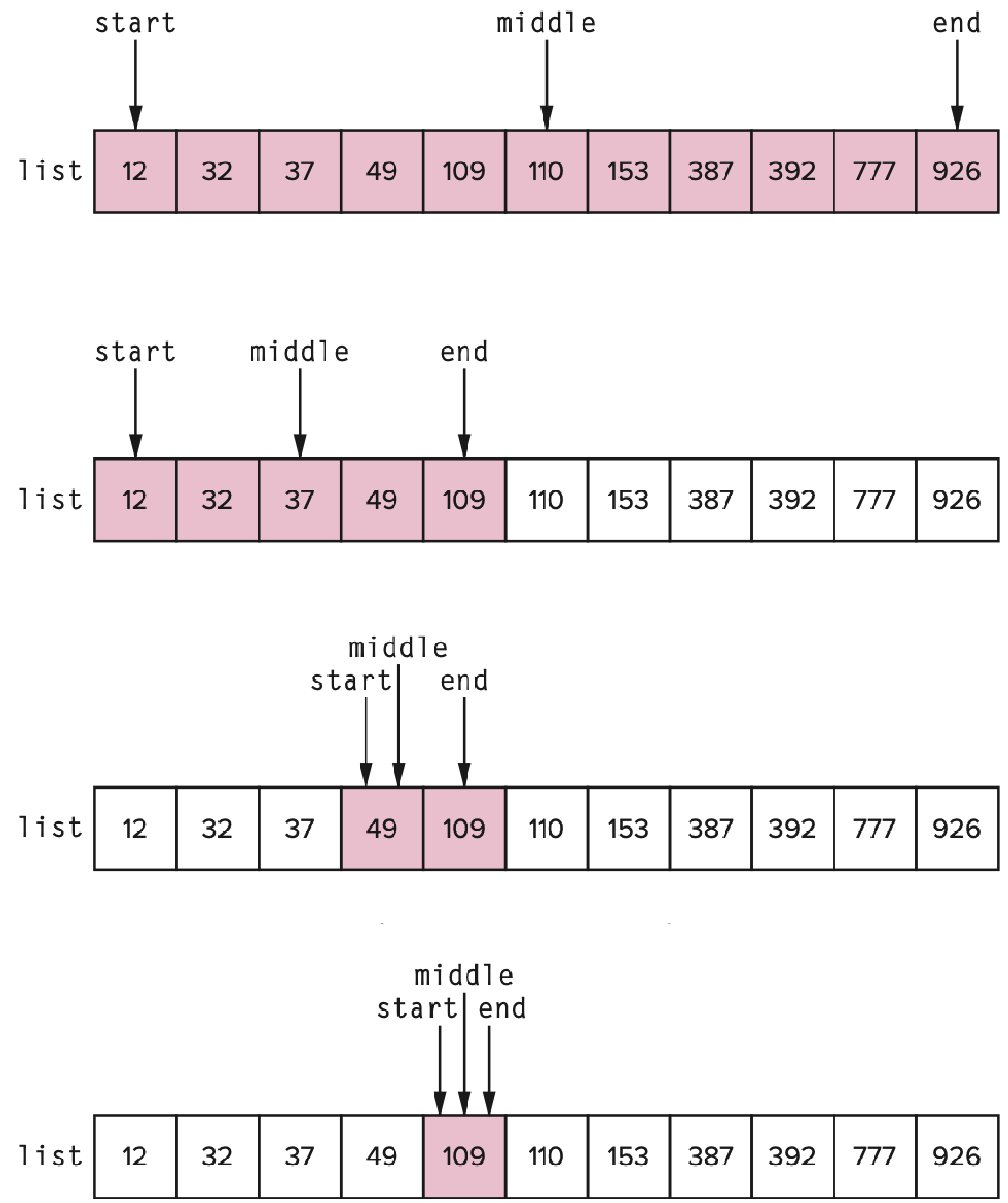
Is middle == key?
Is middle < key?
Go right

Pick middle
Is middle > key?

Is middle == key?
Is middle < key?
Go right

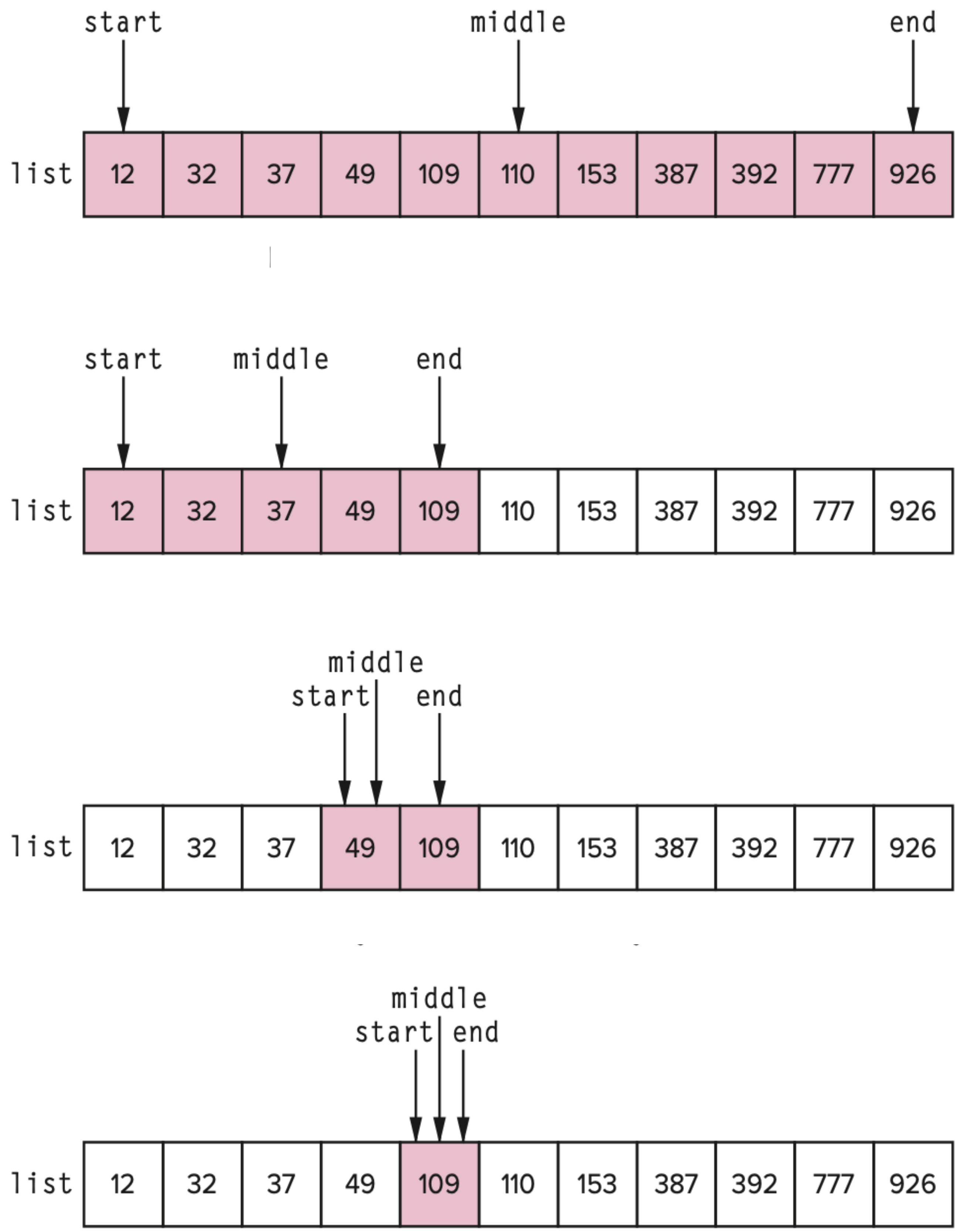
Pick middle

Is middle == key?



Recap binary search code

```
int binary(int arr[], int n, int key){
  int start = 0;        // Left pointer
  int end = _____; // Right pointer

  while (end >= start){
    int mid = (___________) / 2; // Pick middle
element

    // Logic to focus search on left or right of mid
    if (key == arr[mid])
      return mid;
    else if (key < arr[mid])
      end = _______;
    else
      start = ________;
  }
  return -1; // Loop exited, element not present.
}
```



Binary search

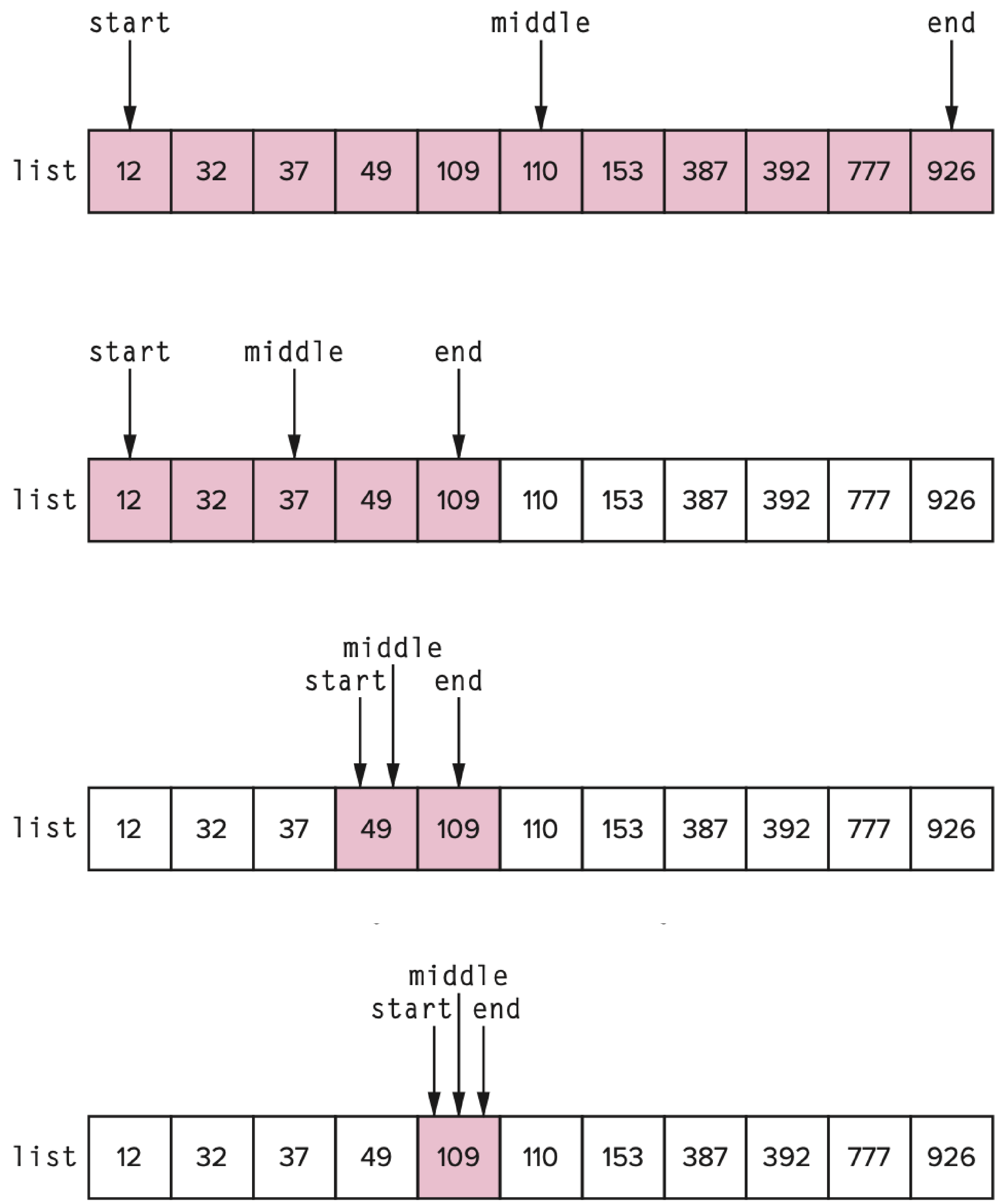
- We are repeating the same process of finding `mid` and going left or right of `mid` on each *subarray.*

- Can we apply

```
binary(arr[], n, key)
```

on each subarray?

- Idea is called *recursion*.
    - We already saw this in QuickSort!



Recursion

A **recursive function** is one that solves its task by **calling itself** on smaller pieces of data.

- Similar to recurrence function in mathematics
- Like iteration — can be used interchangeably; sometimes recursion results in simpler solution … but not always!
- Must have **at least** *one* base case (terminal case) that ends the recursive process; similar to loop needing condition to exit.

Examples: Factorial function, Fibonacci series, binary search, etc.

Recursive function

- Base case (a.k.a terminating case)
  - This case is **required** so the recursion can terminate.
  - The base case must provide a condition that will *eventually become true* and returns from the function. **Otherwise, the run-time stack will overflow**.
- Recursive case (a.k.a inductive case)
  - This case returns a (recursive) call to the function itself. It breaks down the problem into ***smaller*** chunks that can be solved by the *same* function.
  - The input to the next call gets reduced gradually to the terminating case.

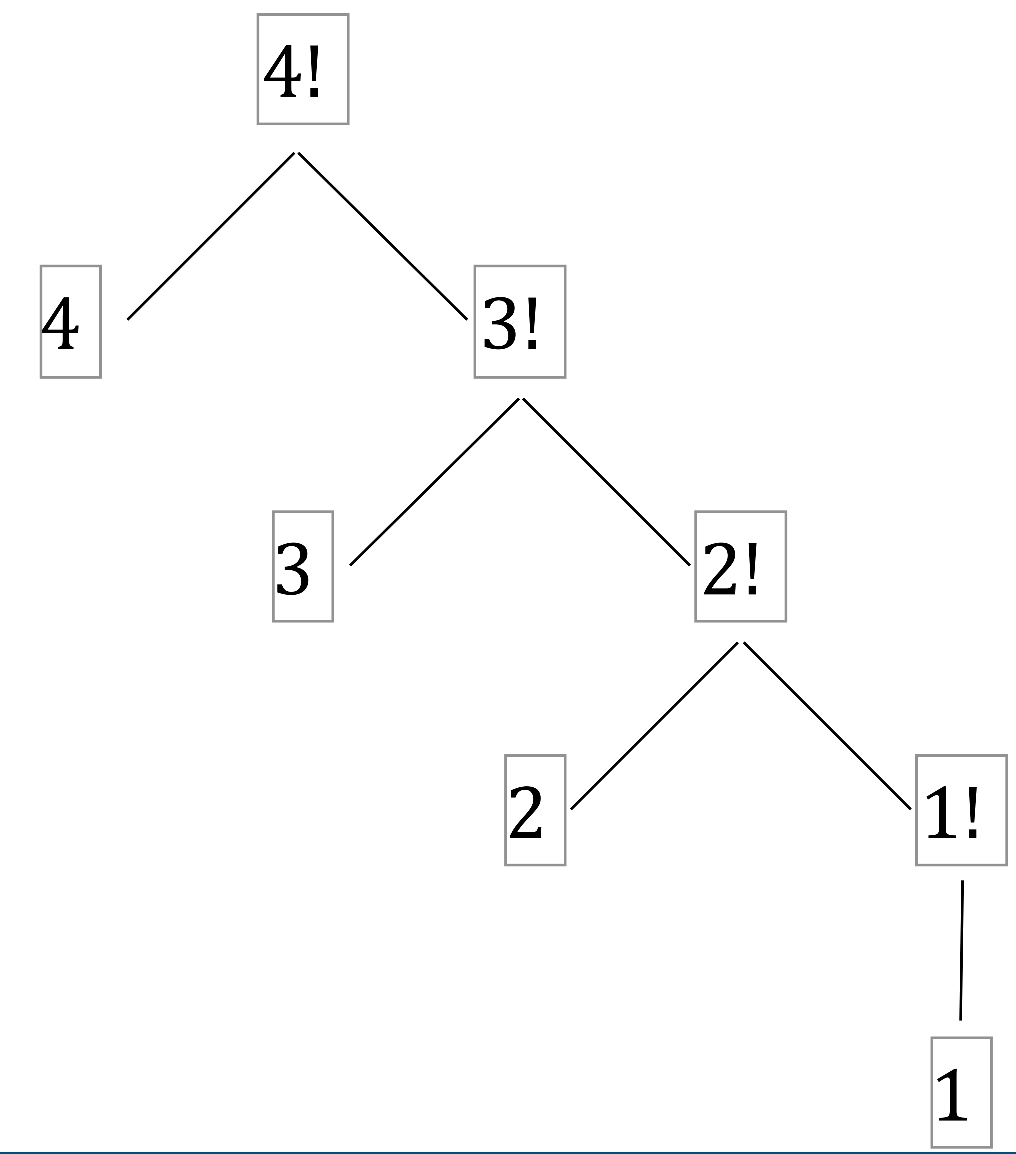
Example: Factorial

- Mathematical definition

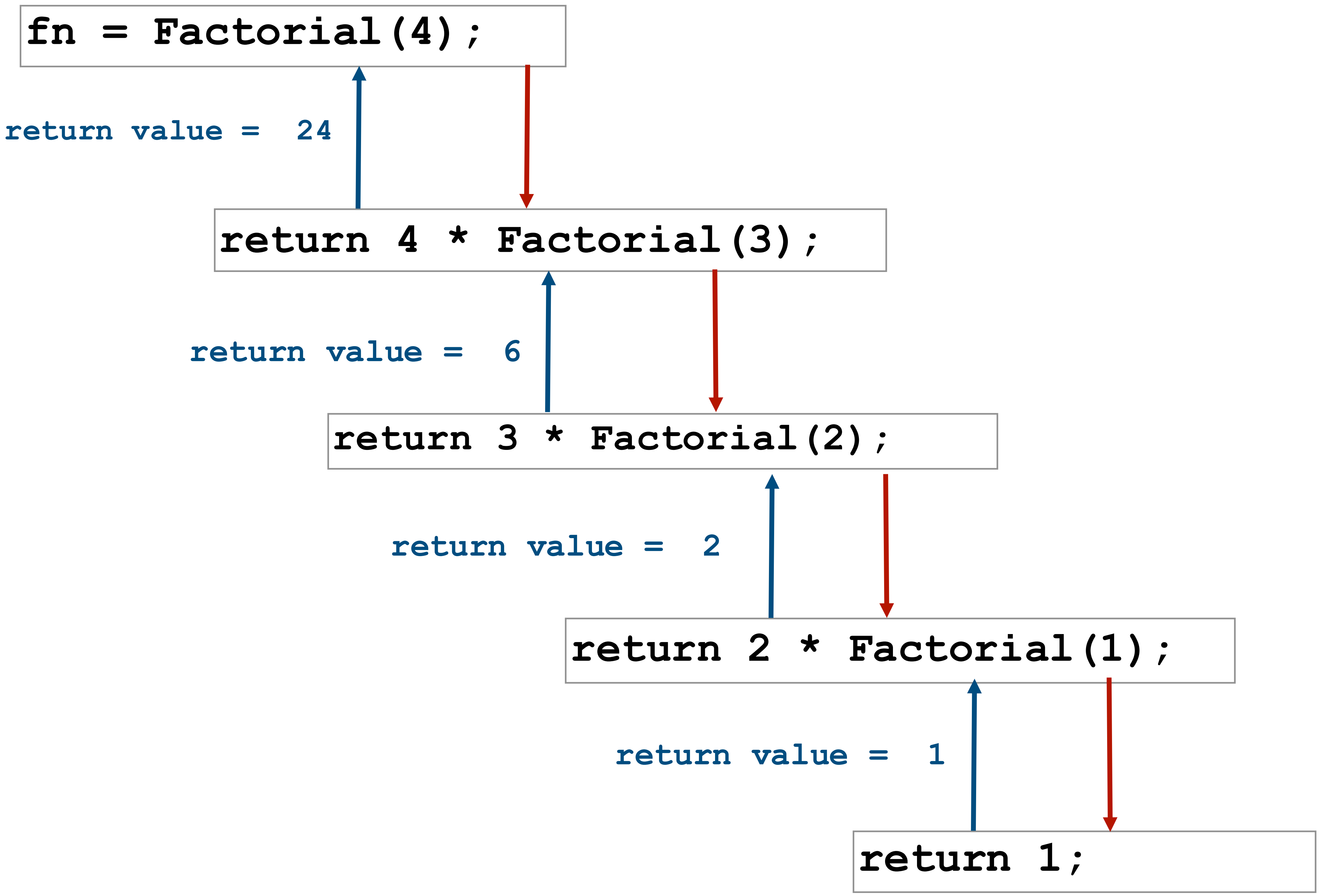
$$n! = n \cdot (n-1) \cdot (n-2) \ldots \cdot 2 \cdot 1 \qquad for\,n > 0$$

- Recursive form

$$n! = \begin{cases} 1, & if\,n = 1 \\ n \cdot (n-1)!, & else \end{cases}$$

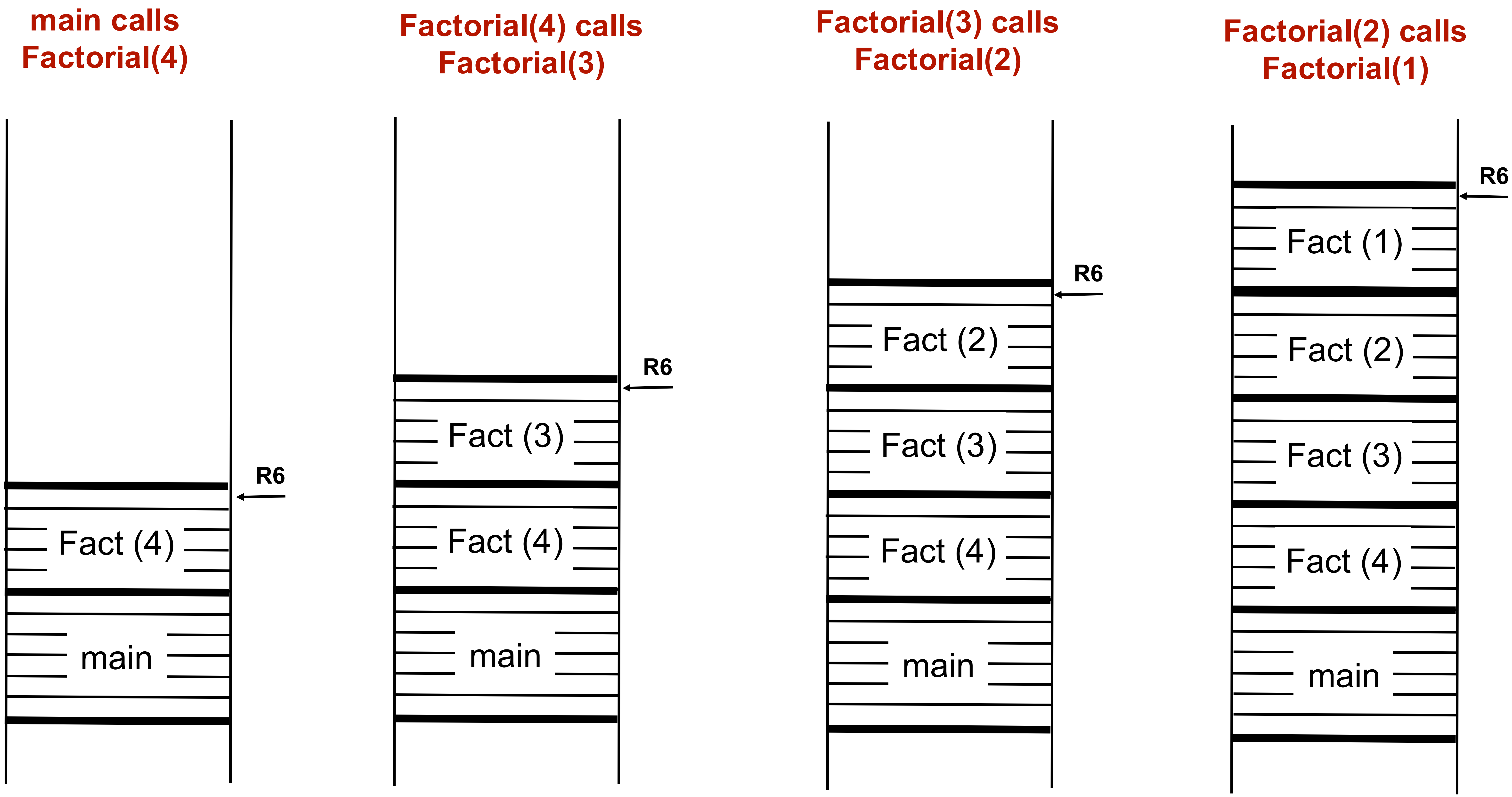


Example: Factorial C-code

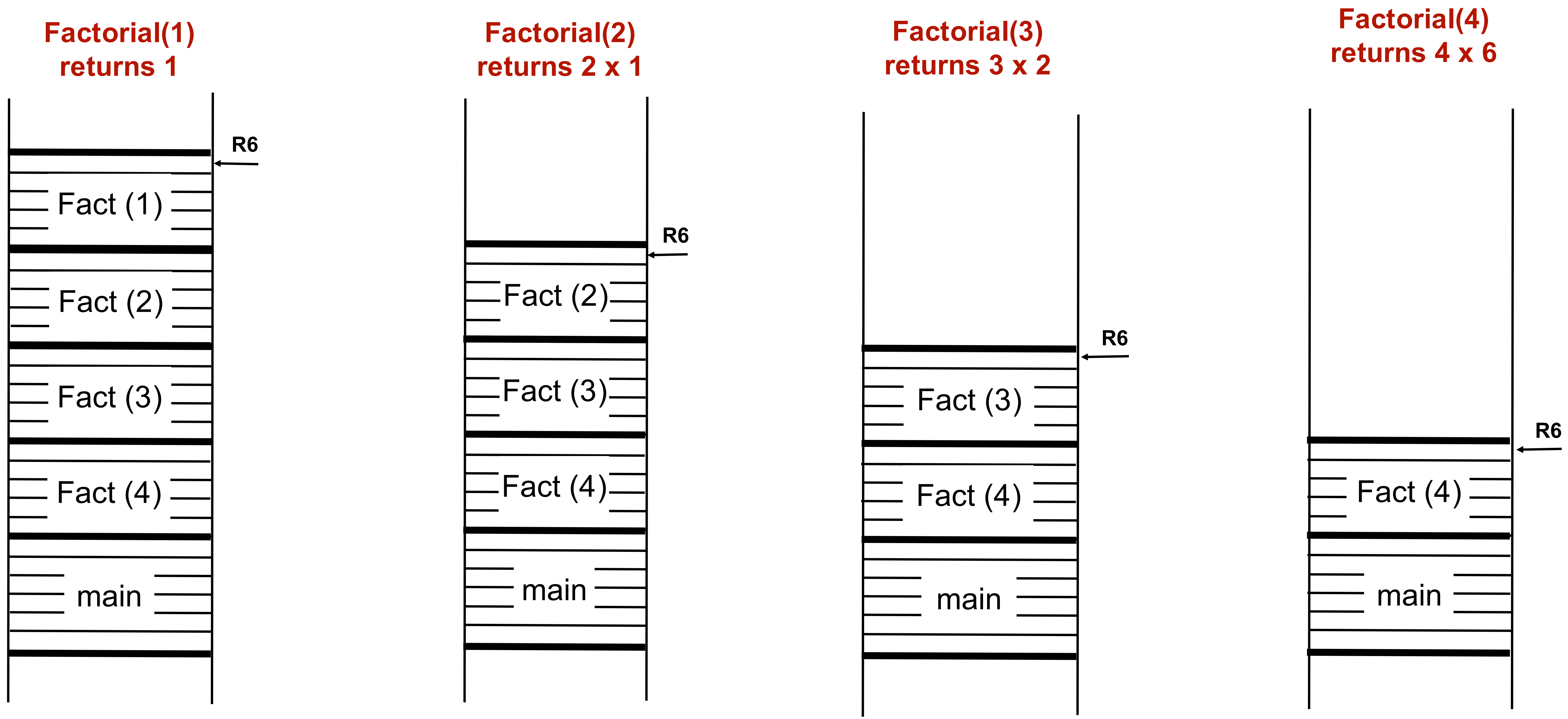



```c
int Factorial(int n){
    if (n == 1)
        return 1;
    else
        return n * Factorial(n - 1);
}
```

RTS During Factorial – build up



RTS During Factorial - teardown



Example: Fibonacci series

Mathematical definition:

Fibonacci Series: 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, ……

$$f(n) = f(n-1) + f(n-2)$$
$$f(1) = 1$$
$$f(0) = 1$$

Fibonacci (3) = Fibonacci(2) + Fibonacci(1)

= (Fibonacci(1) + Fibonacci(0)) + Fibonacci(1)

= 1 + 1 + 1 = 3

Fibonacci series C code

```
int Fibonacci (int n) {
    int sum;

    if (n == 0 || n == 1)
        return 1;
    else {
        sum = (Fibonacci(n-1) + Fibonacci(n-2));
        return sum;
}
}
```

Example: Binary search

```
int binary(int arr[], int n, int key){
  int start = 0;       // Left pointer
  int end = n - 1; // Right pointer

  while (end >= start){
    int mid = (start + end) / 2; // Pick middle element

    // Logic to focus search on left or right of mid
    if (key == arr[mid])
      return mid;
    else if (key < arr[mid])
      end = mid - 1;
    else
      start = mid + 1;
  }
  return -1; // Loop exited, element not present.
}
```

Can we implement binary search in *recursive* way?

- **Need**: A mechanism to keep track of the `start` and `end` indices across recursive calls; local variables won't do (why?).

Binary search C code

Mechanism

```
int binary_search(int item, int list[], int start, int end){
    int middle = (end + start)/2;
    if (end < start)
      return -1;                     // Did not find key
    else if (_____________________) // Found item!
        return middle;
    else if (____________________)  // Search left half
        return binary_search(item, list, start, middle-1);
    else                            // Search right half
        return binary_search(__________________________);
}
```

Binary search C code filled

```
int binary_search(int item, int list[], int start, int end){
    int middle = (end + start)/2;
    if (end < start)
      return -1;                          // Did not find key
    else if (list[middle] == item) // Found item!
        return middle;
    else if (item < list[middle])  // Search left half
        return binary_search(item, list, start, middle-1);
    else                                // Search right half
        return binary_search(item, list, middle+1, end);
}
```

Concept check 1

- What are the base/terminating cases in binary search?
- What are the recursive cases in binary search?

Example: Quicksort

- We already saw Quicksort last time and remarked it was recursive … but provided an iterative solution.

- Recall the steps ….

1. Choose first element of given array as pivot.
2. Maintain pointers from the left and right.
3. Increment left pointer while the element it points to is less than pivot
4. Decrement right pointer while the element it points to is greater than pivot.
    1. If pointers cross/overlap, **split array**, then **repeat** on each subarray.
5. If neither pointers can move swap elements.
6. Repeat 3-5 while left pointer < right pointer.

And a loop to repeat on subarrays after split

We wrote one <u>function</u> to split array

Recursive implementation

```
void Swap(int* one, int* two){
  int temp = *one;
  *one = *two;
  *two = temp;
}

void QuickSort(int arr[], int start, int end){
  if (start < end){
    int pivotVal = partition(arr, start, end);

    // Now sort left half
    QuickSort(arr, start, pivotVal);

    // And right half
    QuickSort(arr, pivotVal + 1, end);
  }
}
```

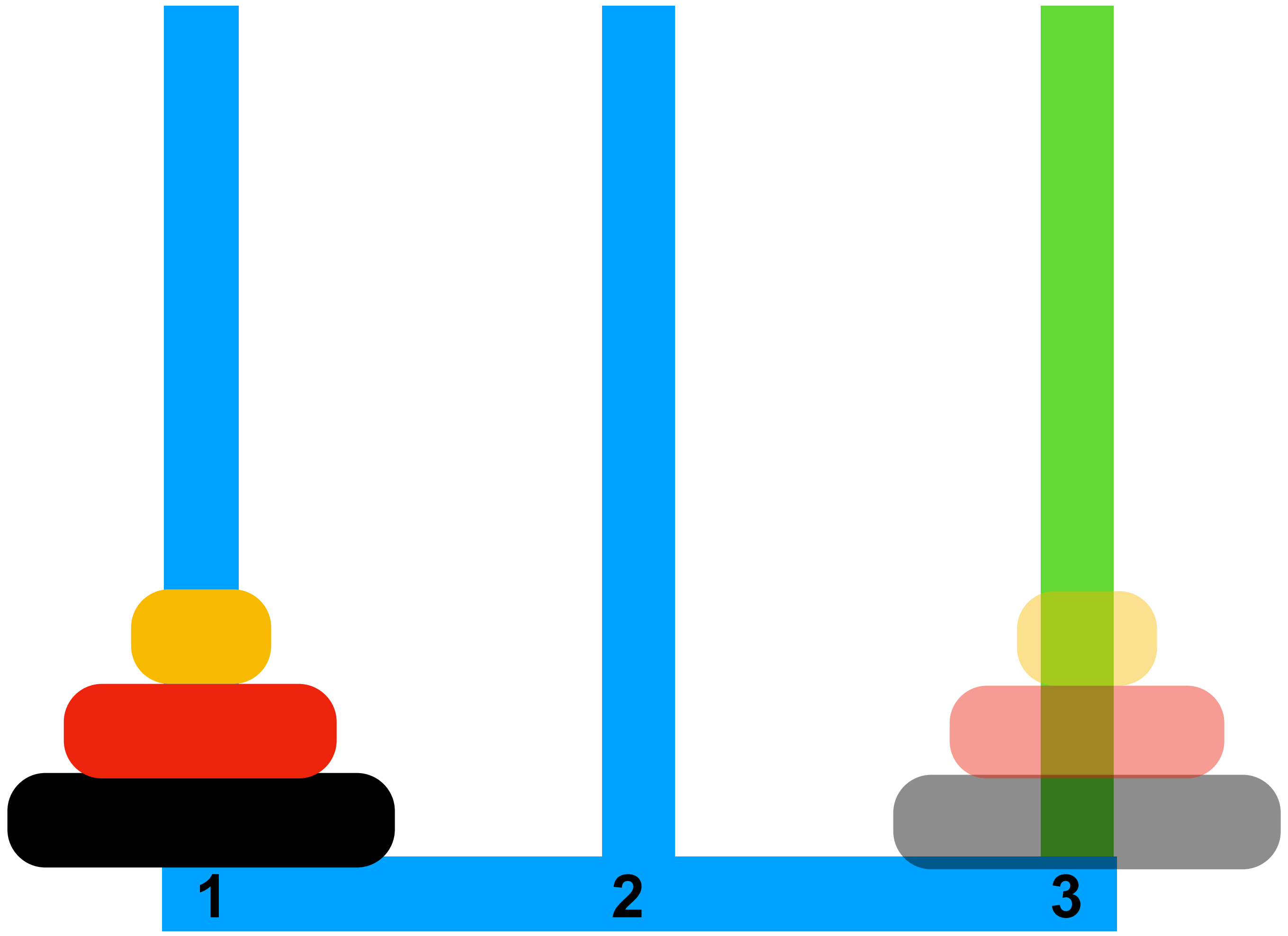
```
int partition(int arr[], int start, int end){

  int pivotVal = arr[start];
  int left = start - 1; // Initialize left
  int right = end + 1;    // Initialize right

  while(1){
    do left++; // Increment left till …
    while (arr[left] < pivotVal);

    do right--; // Decrement right till …
    while (arr[right] > pivotVal);

    if (left >= right) // Split if overlap
      return right;

    Swap(&arr[left], &arr[right]);
  }
}
```

Check Github for reference material

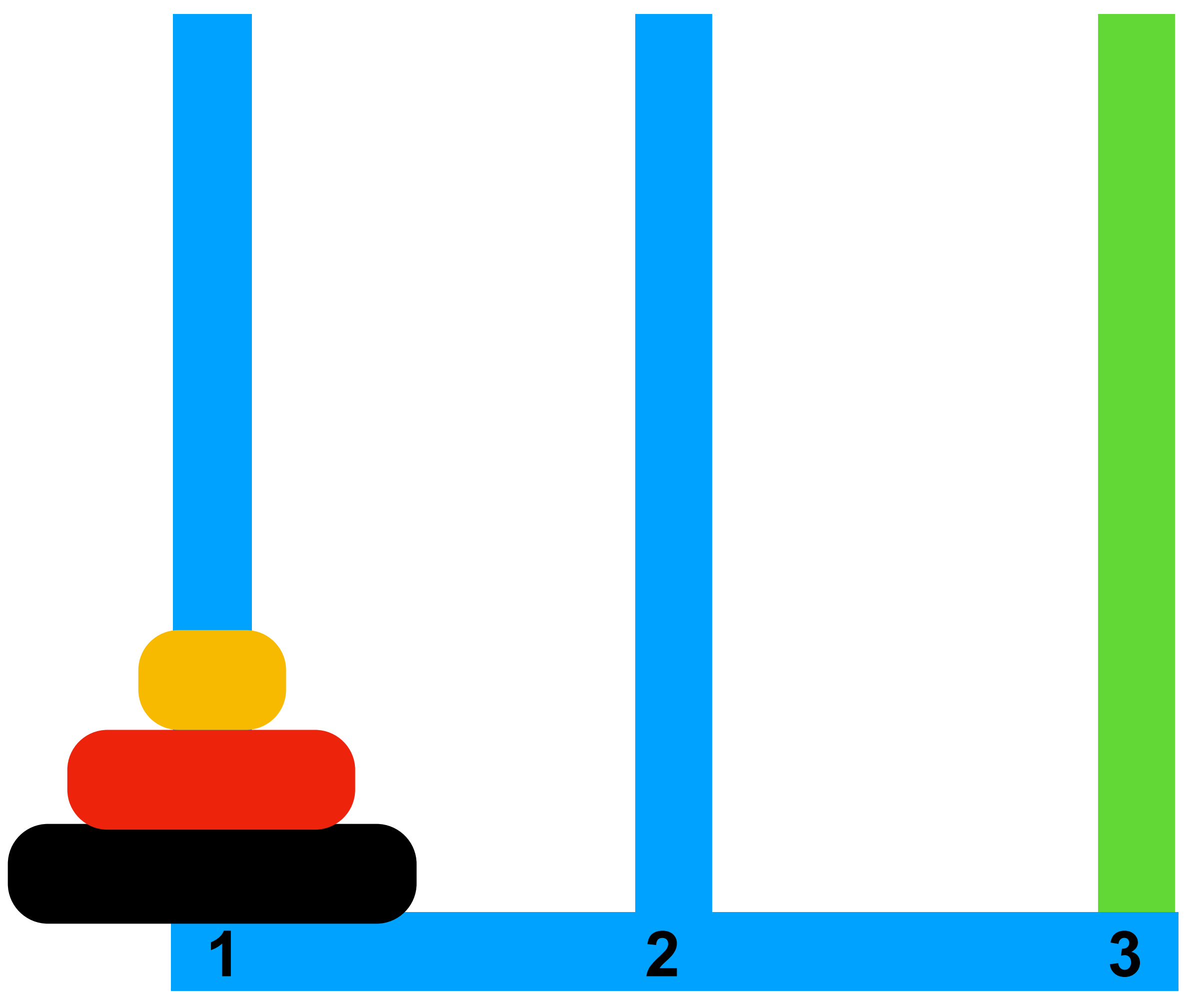
Concept check 2

- Why did iterative version of Quicksort need a stack to work but iterative binary search did not?

Example: Towers of Hanoi



Towers of Hanoi - pseudocode



move(diskStacks, src, dest, temp) // Move 2+ disks
dmove(disk_num, src, dest) // Move single disk

move(disks3, r1, r3, r2)

↓

move(disks2, r1, r2, r3)
+
dmove(disk_3, r1, r3)
+
move(disks2, r2, r3, r1)

dmove(disk_1, r1, r3)
dmove(disk_2, r1, r2)
dmove(disk_1, r3, r2)

dmove(disk_1, r2, r1)
dmove(disk_2, r2, r3)
dmove(disk_1, r1, r3)

Recursion in Towers of Hanoi

move(disks3, r1, r3, r2)

move(disks2, r1, r2, r3)

dmove(disk_1, r1, r3)

dmove(disk_2, r1, r2)

dmove(disk_1, r3, r2)

+

dmove(disk_3, r1, r3)

+

move(disks2, r2, r3, r1)

dmove(disk_1, r2, r1)

dmove(disk_2, r2, r3)

dmove(disk_1, r1, r3)

move(disks4, r1, r3, r2)

move(disks3, r1, r2, r3)

move(disks2, r1, r3, r2)

dmove(disk_3, r1, r2)

move(disks2, r3, r2, r1)

+

dmove(disk_4, r1, r3)

+

move(disks3, r2, r3, r1)

move(disks2, r2, r1, r3)

dmove(disk_3, r2, r3)

move(disks2, r1, r3, r2)

Towers of Hanoi: General formula?

```
void move(disknum, src, dest, temp){
  if (disknum > 1){
    move(disknum - 1, src, temp, dest);

    printf("Moved disk %d from rod %d to %d\n", disknum, src, dest);

    move(disknum - 1, temp, dest, src);
  }
  else
    printf("Moved disk 1 from rod %d to %d\n", src, dest);
}
```

Move a sub-stack to from src to intermediate rod

Direct move of single disk.

Move the sub-stack from intermediate to dest rod

Base case; all moves involving sub-stacks end-up here.

Recursion in LC3

```
int running_sum(int n){
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}

int main(void){
    int n = 5;
    running_sum(5);
}
```

How can we write equivalent LC3 code?

Recall function calls are implemented using the run time stack.

Recursive calls need not be treated any different from normal function calls!

Generating an activation record

1. *Caller* build-up: Push callee's arguments onto stack
2. Pass control to callee (`JSR/JSRR`)

Caller

3. *Callee* build-up: (push bookkeeping info and local variables onto stack)
4. Execute function
5. *Callee* tear-down (update return value, pop local variables, caller's frame pointer and return address from stack)
6. Return to caller (`RET`)

Callee

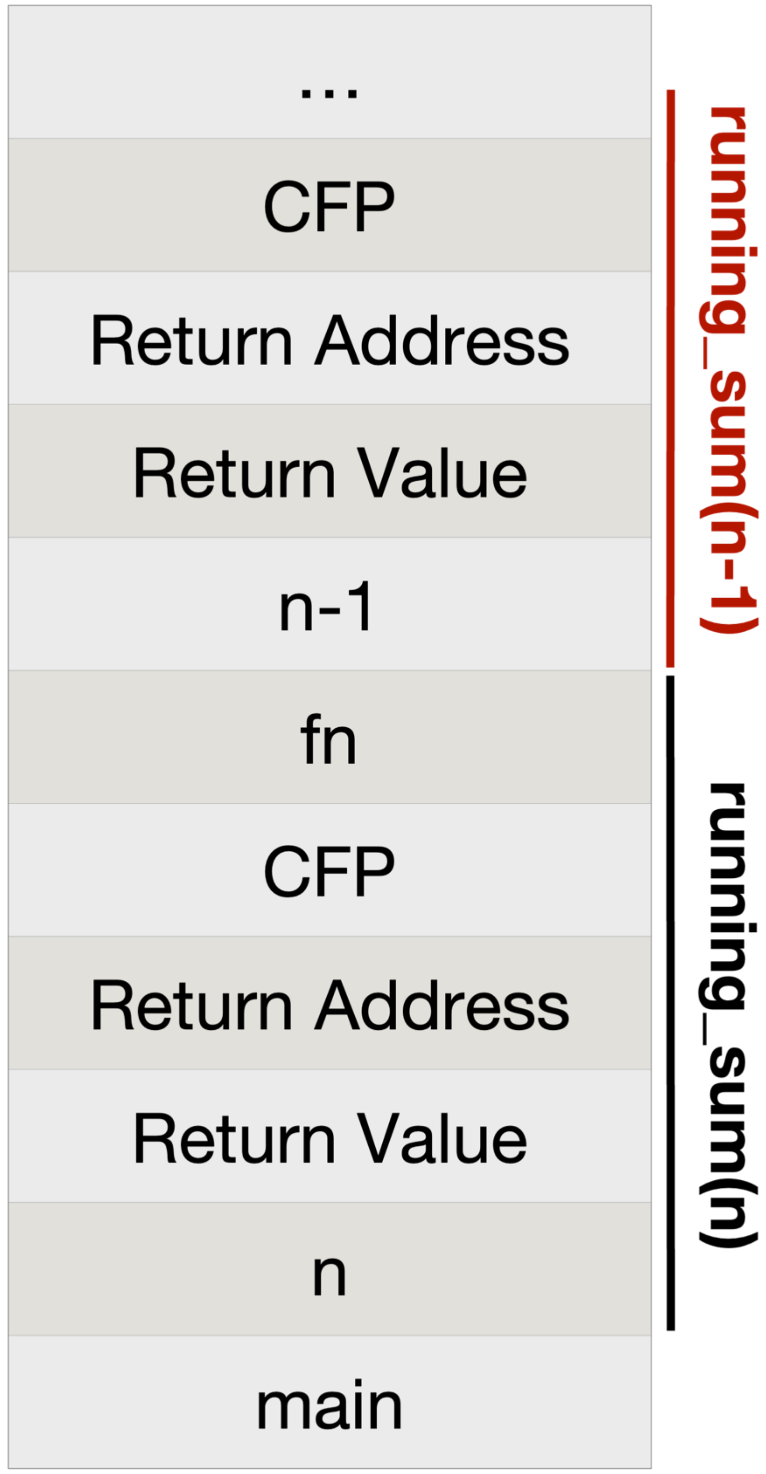
7. *Caller* tear-down (pop callee's return value and arguments from stack)

Caller

Check Github for reference material: https://github.com/iabraham/ece220-sp25/blob/main/lec12/README.md

Recursion in LC3 – set up

```c
int running_sum(int n){
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}

int main(void){
    int n = 4;
    int answer;
    answer = running_sum(n);
}
```

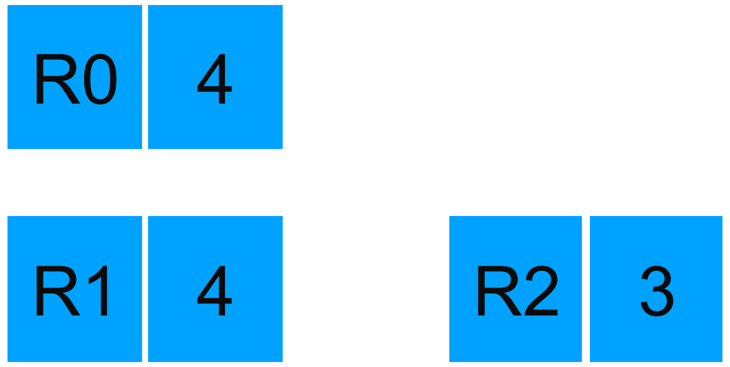


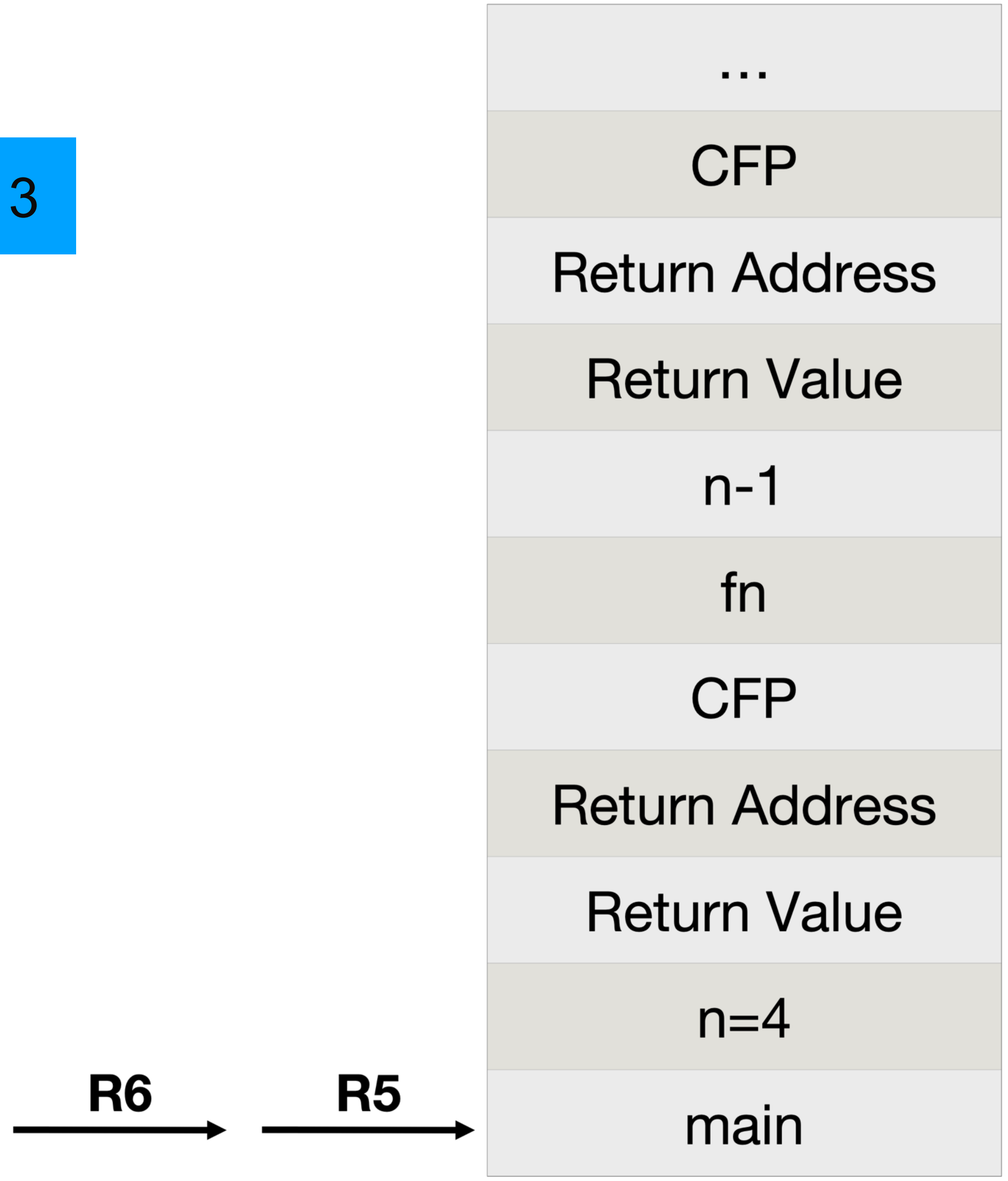
```
int running_sum(int n){
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}
```

```
int main(void){
    int n = 4;
    int answer;
    answer = running_sum(n);
}
```

Recursion in LC3 – build up

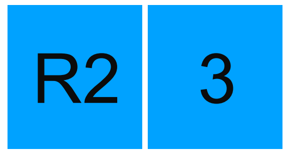
```
;Caller set-up
LDR R0, R5, #0
ADD R6, R6, #-1
STR R0, R6, #0
JSR RUNNING

RUNNING
;callee set-up of Running(n)'s activation record
;push return value, return address & caller's frame pointer
ADD R6, R6, #-3
STR R7, R6, #1
STR R5, R6, #0

;update frame pointer & push local variables
ADD R5, R6, #-1
ADD R6, R6, #-1

;function logic
;base case (n==1)
LDR R1, R5, #4
ADD R2, R1, #-1
BRz BASE_CASE
```





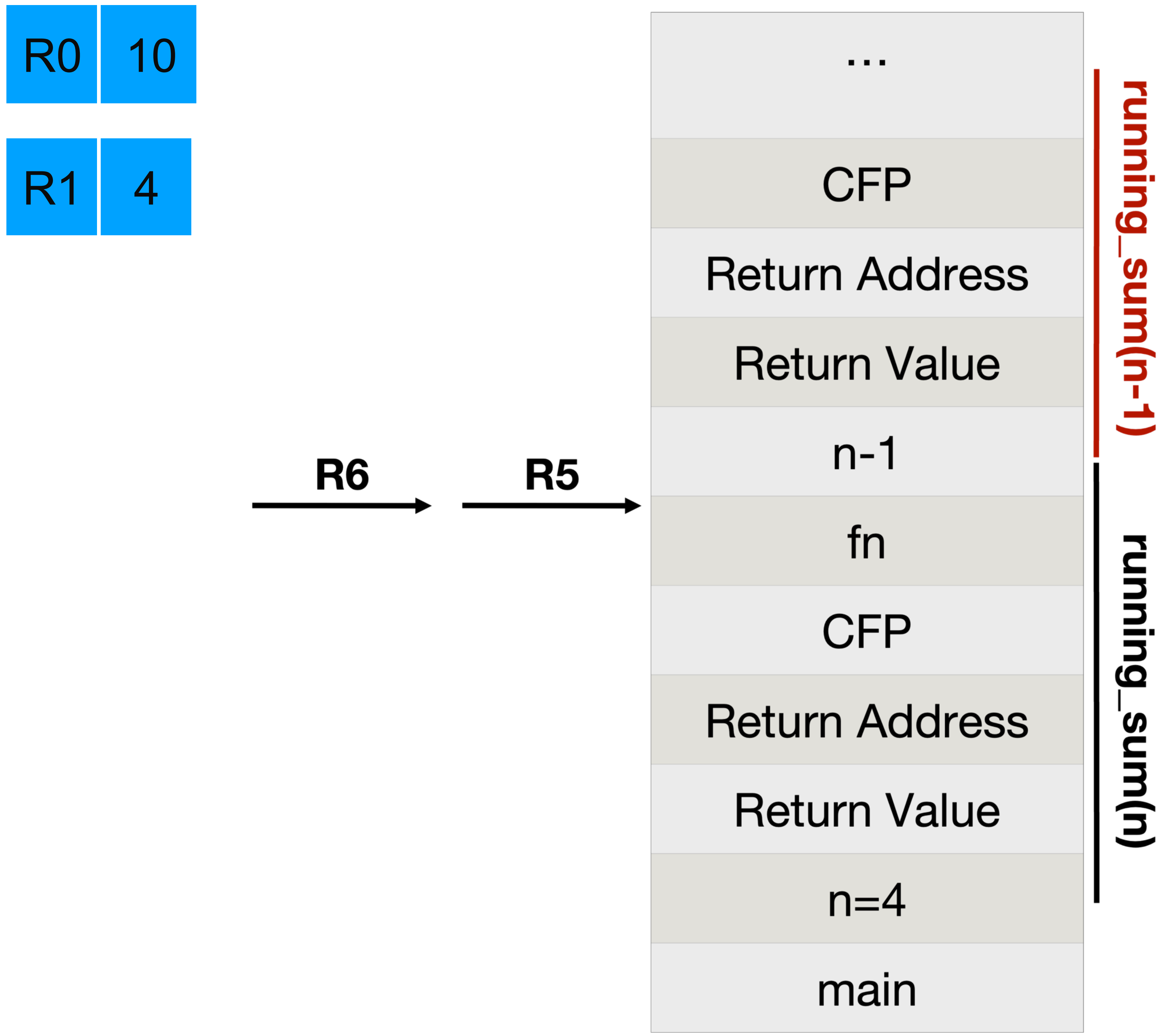
```c
int running_sum(int n){
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}
```

```c
int main(void){
    int n = 4;
    int answer;
    answer = running_sum(n);
}
```



Recursion in LC3 - logic

```
;Recursive case
;Caller setup: push argument n-1 onto RTS
ADD R6, R6, #-1
STR R2, R6, #0 ; R2 = n - 1
JSR RUNNING ; call Running(n-1)



;Caller tear-down for Running(n-1)
;pop Running(n-1)'s return value to R0
LDR R0, R6, #0
ADD R6, R6, #1

;pop Running(n-1)'s argument
ADD R6, R6, #1

;calculate n + Running(n-1)
LDR R1, R5, #4
ADD R0, R1, R0
STR R0, R5, #0 ;store result in fn

;ready to return
BRnzp RETURN
```

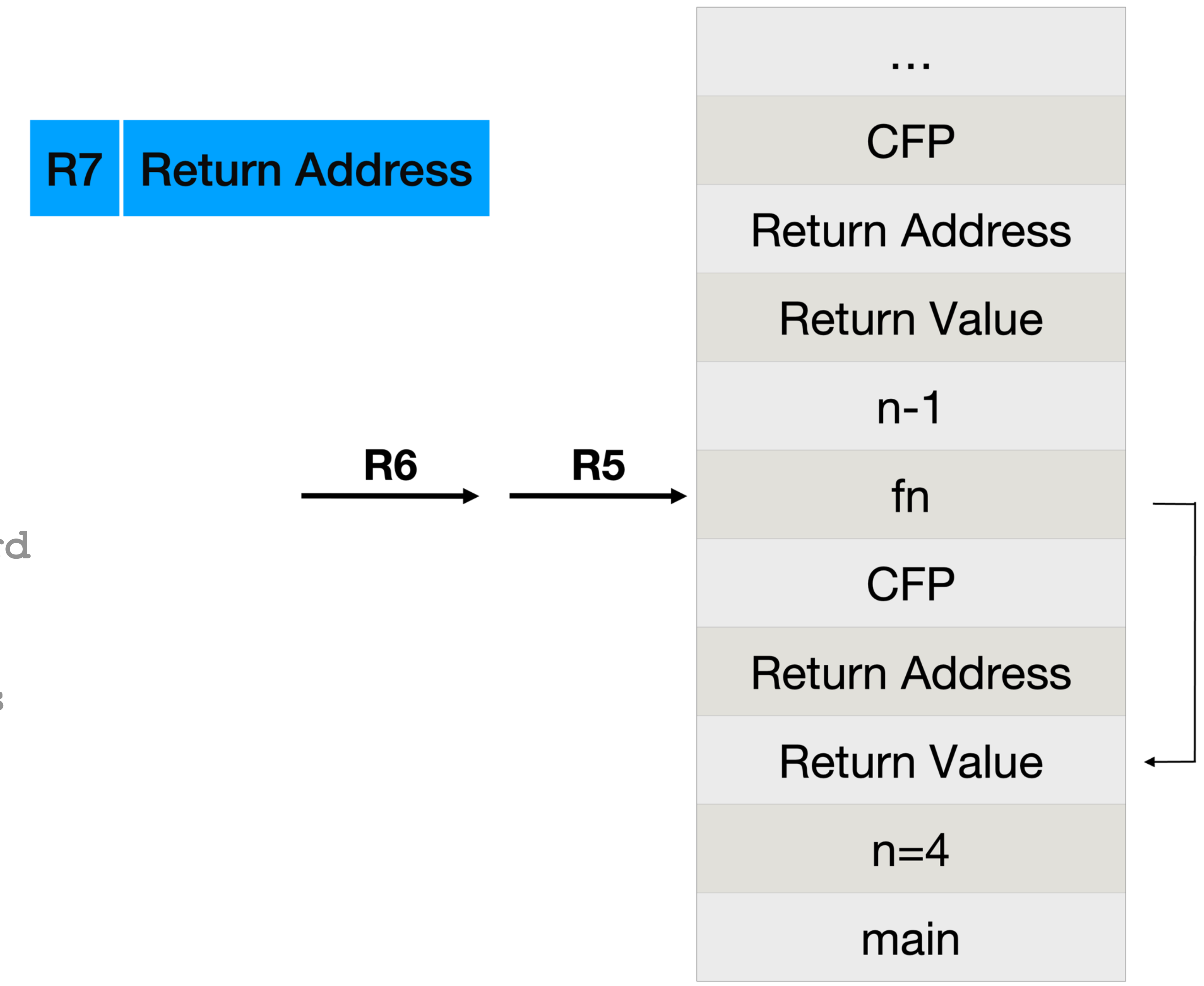


```
int running_sum(int n){
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}
```

```
int main(void){
    int n = 4;
    int answer;
    answer = running_sum(n);
}
```

Recursion in LC3 - teardown

```
BASE_CASE
AND R2, R2, #0
ADD R2, R2, #1
STR R2, R5, #0 ;set fn = 1

RETURN
;set return value
LDR R0, R5, #0
STR R0, R5, #3

;callee tear-down of Running(n)'s activation record
ADD R6, R5, #3 ;pop local variables

;restore caller's frame pointer and return address
LDR R5, R6, #-2
LDR R7, R6, #-1


;return to caller
RET
```



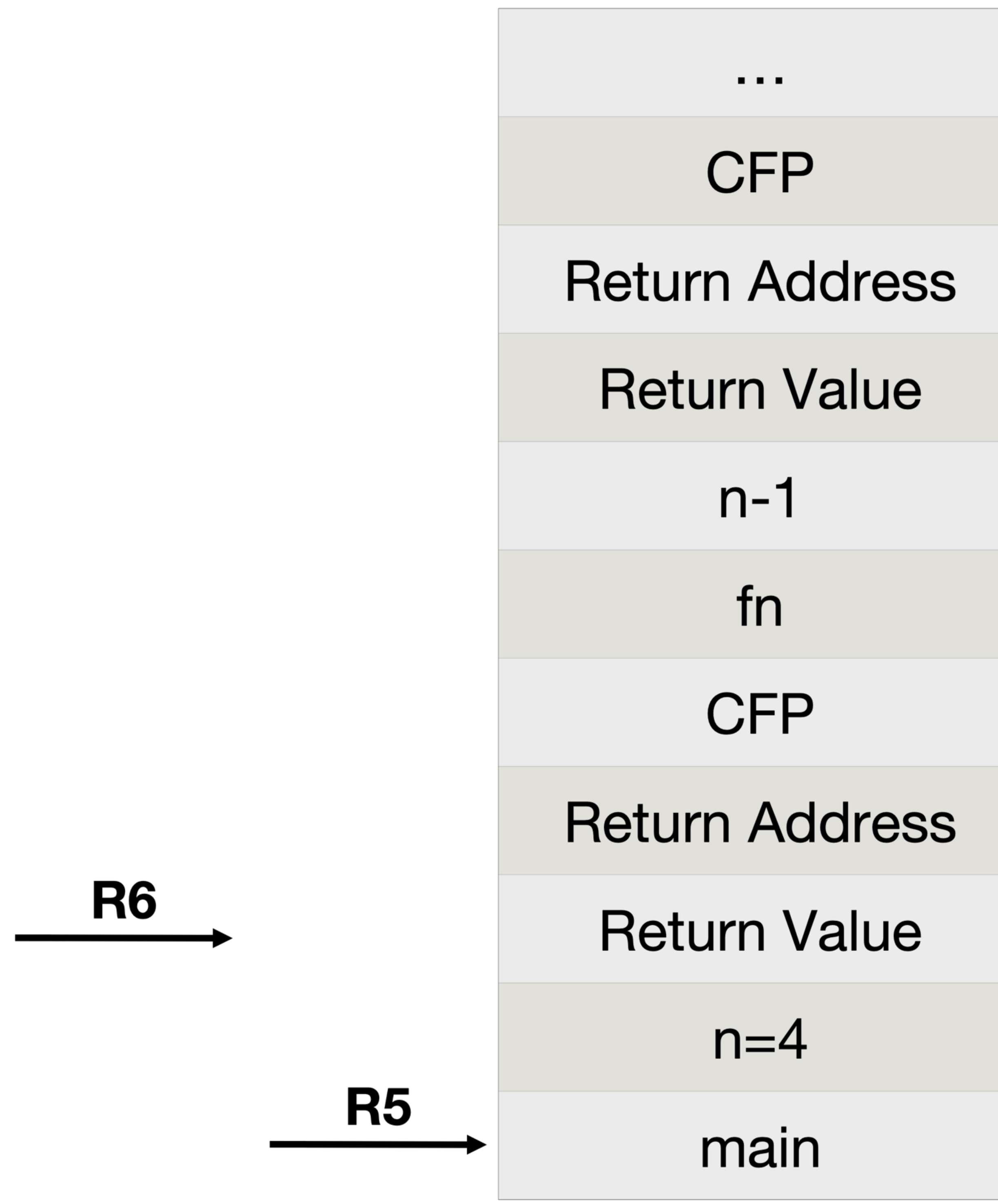
```
int running_sum(int n){
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}
```

```
int main(void){
    int n = 4;
    int answer;
    answer = running_sum(4);
}
```

Recursion in LC3 – wrapup

```
;Caller stack Tear-down for Running(n)
LDR R0, R6, #0   ;copy return value to R0
STR R0, R5, #-1  ;save return value to answer
ADD R6, R6, #1   ;pop return value from stack
ADD R6, R6, #1   ;pop argument from stack
```

Inside main’s activation frame, answer is the second local variable



Practice practice practice!

Back to where we started!

Next time

- More problem solving with recursion.
  - A small chess problem
  - Solving a maze
- When is recursion good vs. bad?