ECE 220

Lecture x000C - 02/27

Slides based on material originally by: Yuting Chen & Thomas Moon

Recap

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- Last time we discussed sorting & searching

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Recap

- Last time we discussed sorting & searching
  - Selection sort

Recap

- Last time we discussed sorting & searching
    - Selection sort
    - Insertion sort

Recap

- Last time we discussed sorting & searching
  - Selection sort
  - Insertion sort
  - Quick sort

Recap

- Last time we discussed sorting & searching
  - Selection sort
  - Insertion sort
  - Quick sort
  - Bubble sort

Recap

- Last time we discussed sorting & searching
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A. Select next element and move things to place it in proper spot

Recap

- Last time we discussed sorting & searching
  - Selection sort
  - Insertion sort
  - Quick sort
  - Bubble sort

A. Select next element and move things to place it in proper spot

B. Keep comparing pairs and swapping them till no more swaps

Recap

- Last time we discussed sorting & searching
  - Selection sort
  - Insertion sort
  - Quick sort
  - Bubble sort

A. Select next element and move things to place it in proper spot

B. Keep comparing pairs and swapping them till no more swaps

C. Find minimum in each pass and bring to appropriate spot

Recap

- Last time we discussed sorting & searching
  - Selection sort
  - Insertion sort
  - Quick sort
  - Bubble sort

A. Select next element and move things to place it in proper spot

B. Keep comparing pairs and swapping them till no more swaps

C. Find minimum in each pass and bring to appropriate spot

D. Pick pivot & move elements to left (lesser) or right (greater) of pivot

Recap

- Last time we discussed sorting & searching
  - Selection sort
  - A Insertion sort
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A. Select next element and move things to place it in proper spot

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Recap

- Last time we discussed sorting & searching
  - Selection sort
  - A • Insertion sort
  - Quick sort
  - B • Bubble sort

A. Select next element and move things to place it in proper spot
B. Keep comparing pairs and swapping them till no more swaps
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- Last time we discussed sorting & searching
  - C • Selection sort
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C • Selection sort

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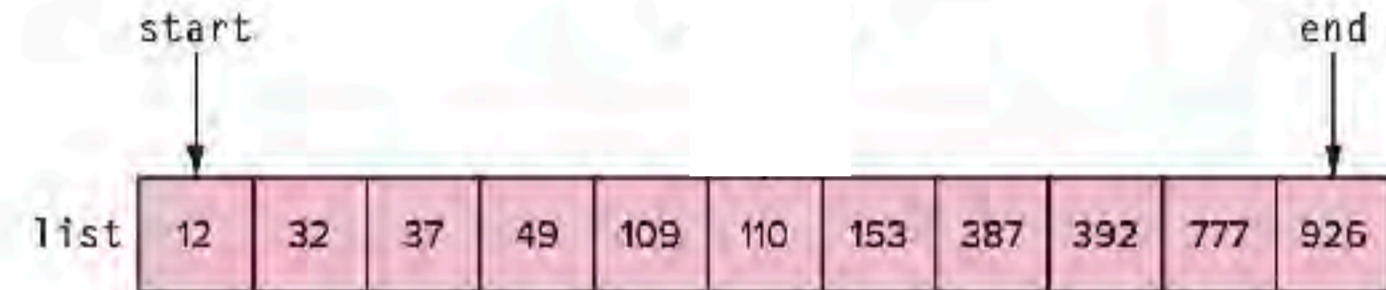
Complete codes for A, C, D available on Gitlab: https://gitlab.engr.illinois.edu/itabrah2/ece220-sp24

Recap binary search

Key = 109



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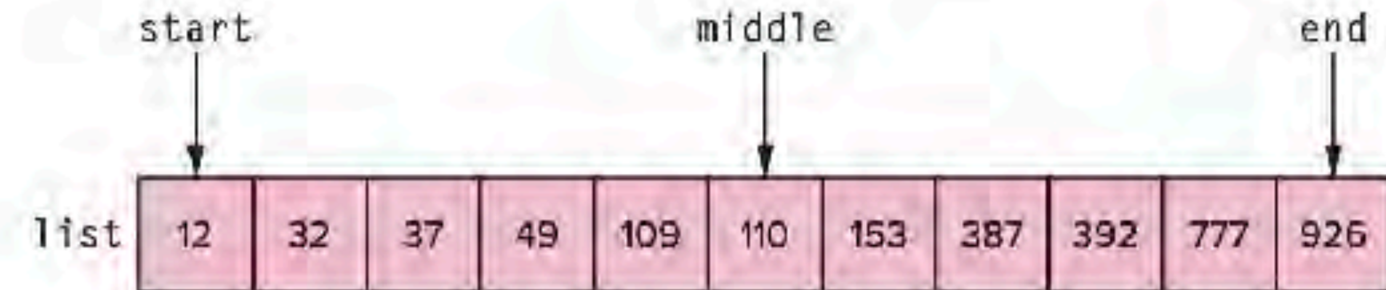
Recap binary search

Key = 109 **Pick middle**



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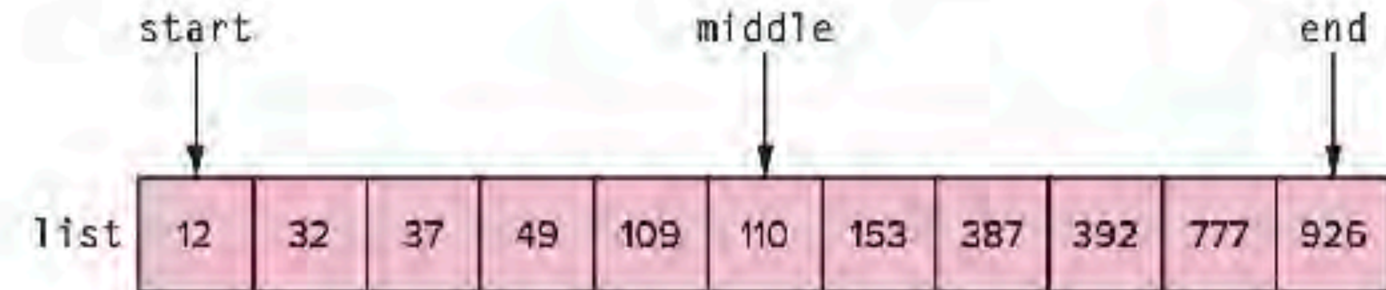
Recap binary search

Key = 109

Pick middle

Is middle == key?



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Recap binary search

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Pick middle
Is middle > key?

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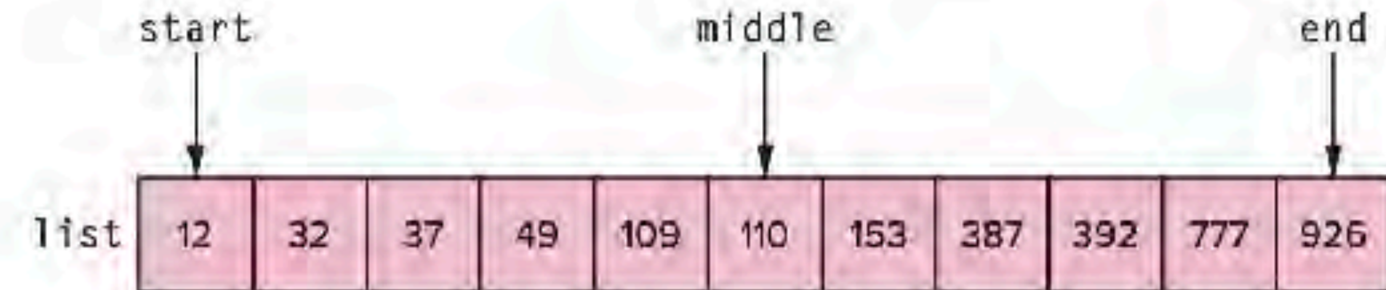

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Recap binary search

Key = 109

Pick middle
Is middle > key?

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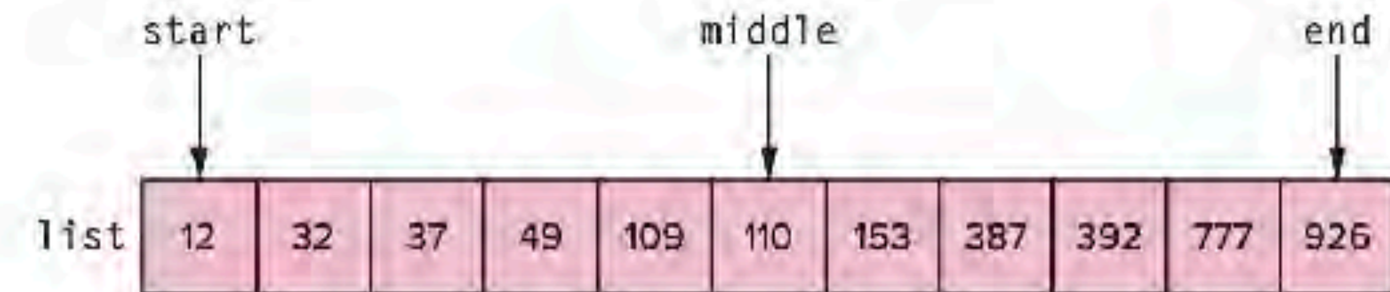
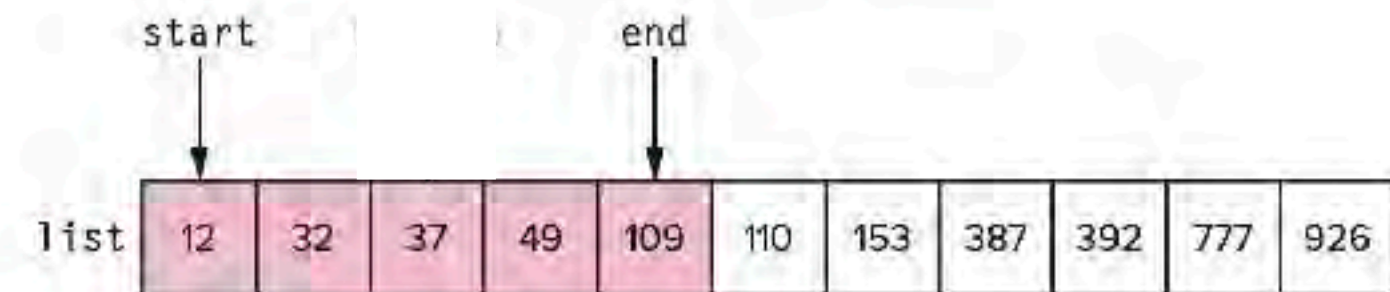
Go left





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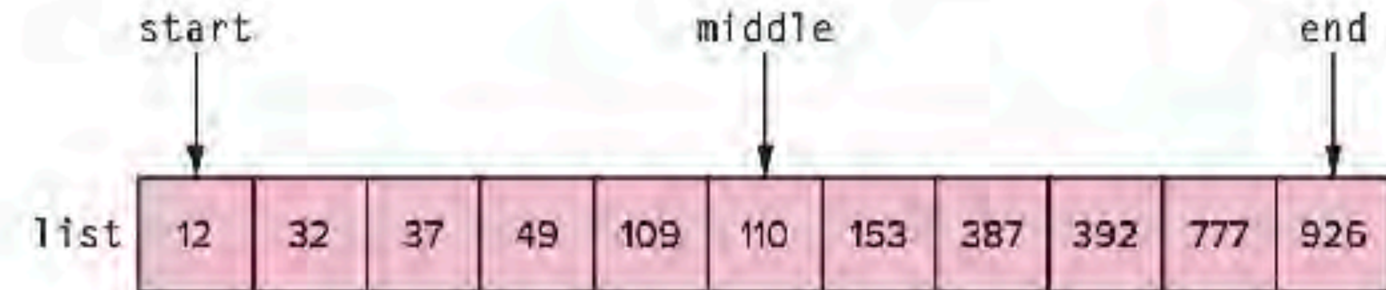

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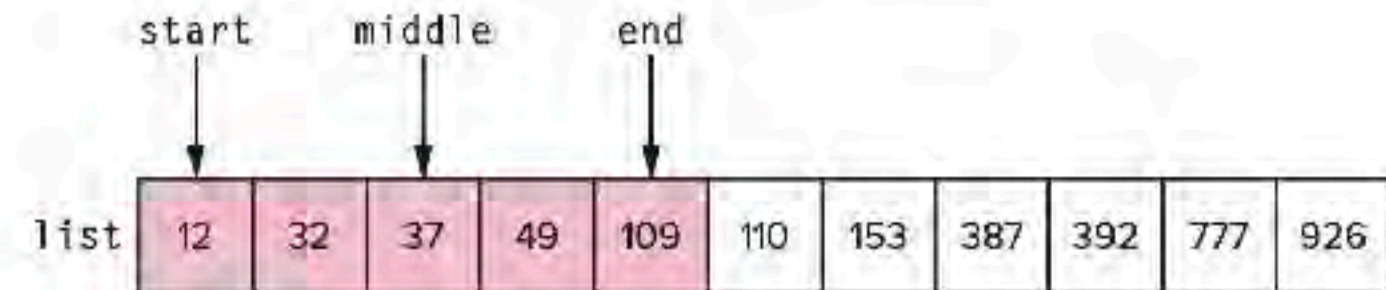

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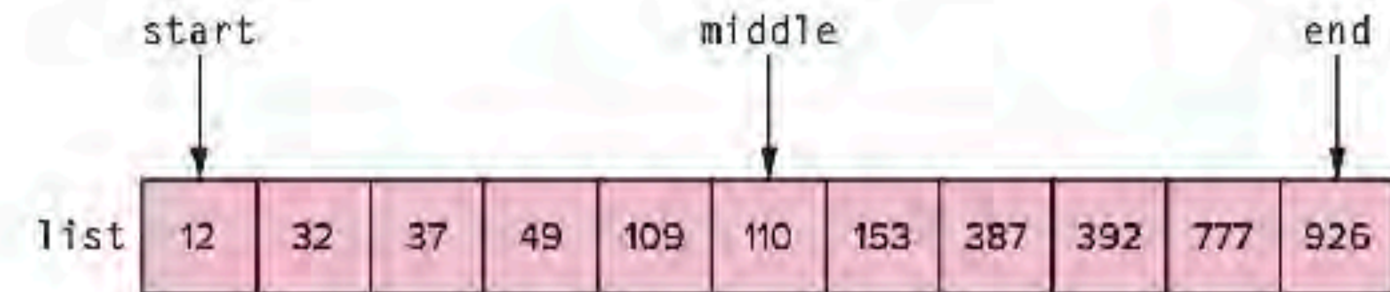

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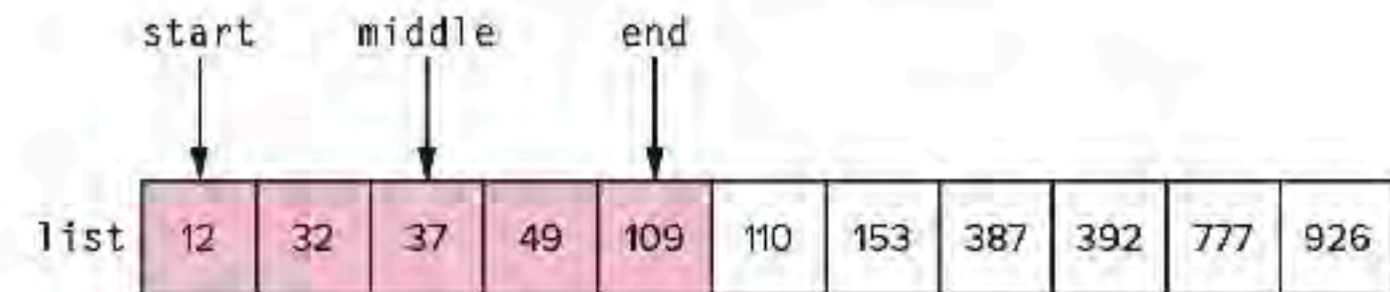

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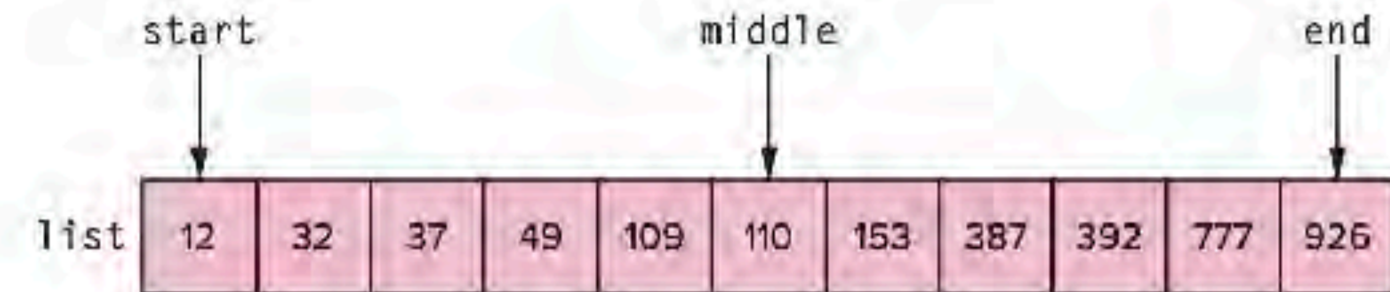

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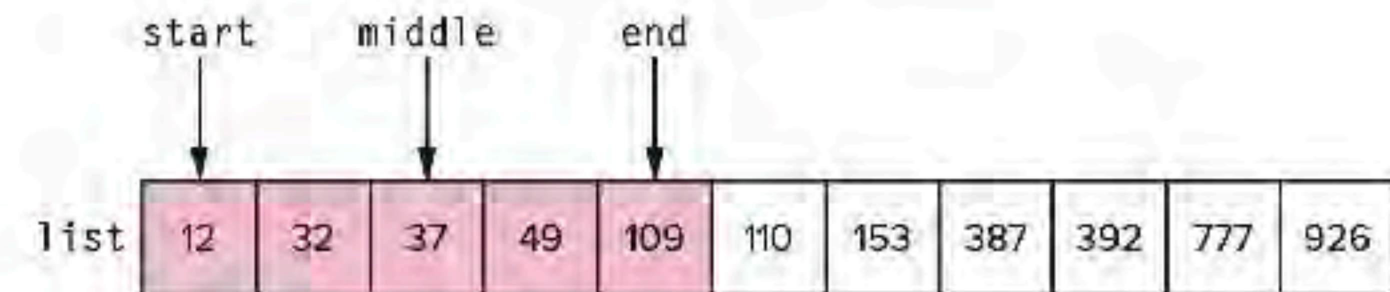

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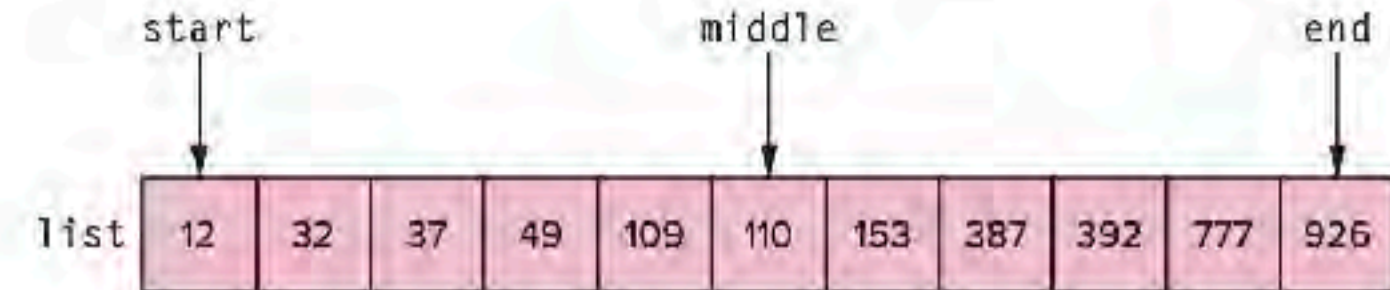

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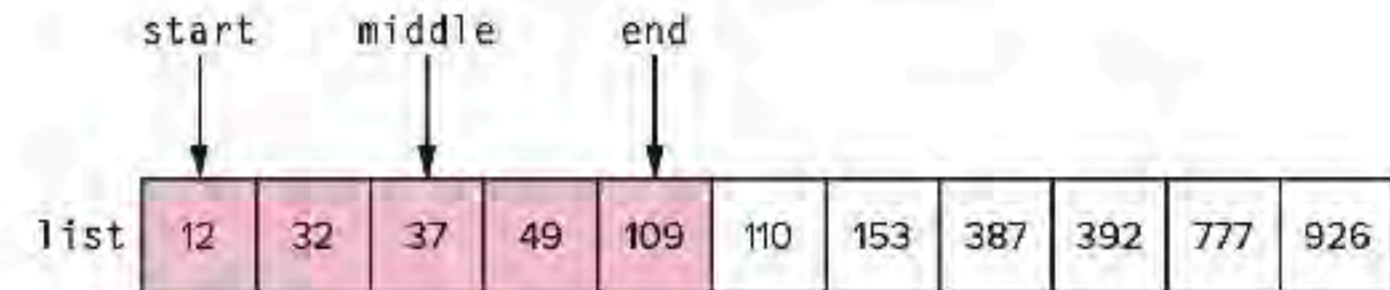

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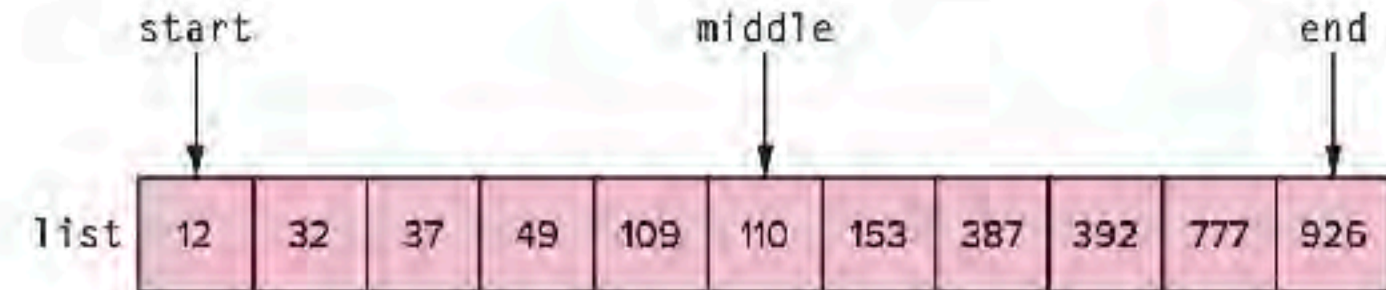

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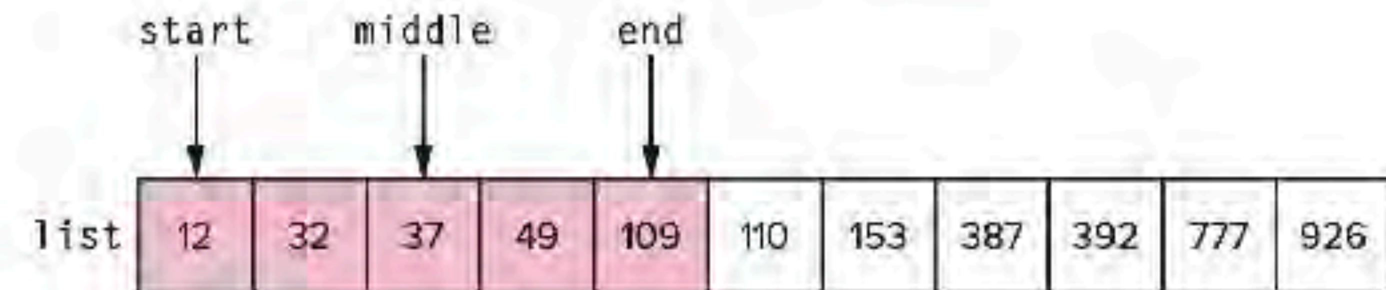

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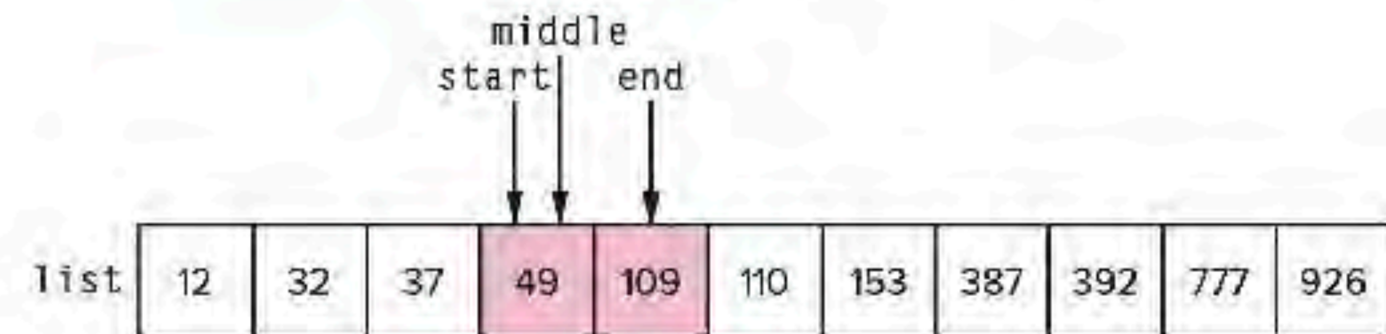

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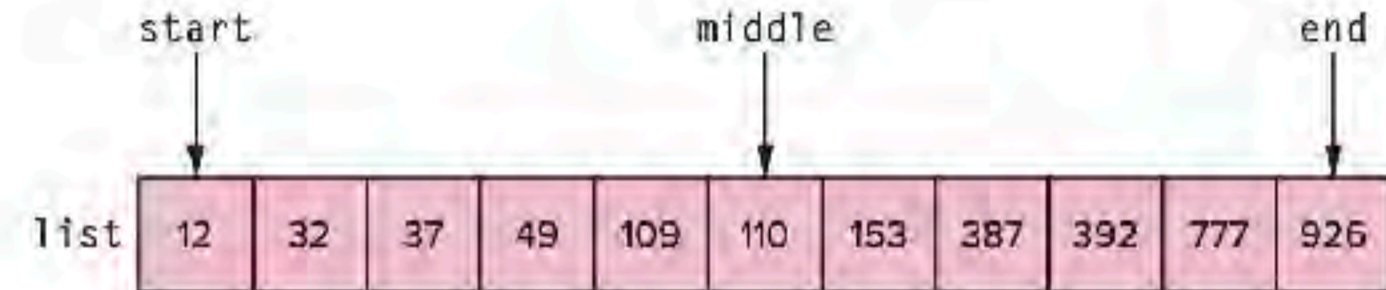

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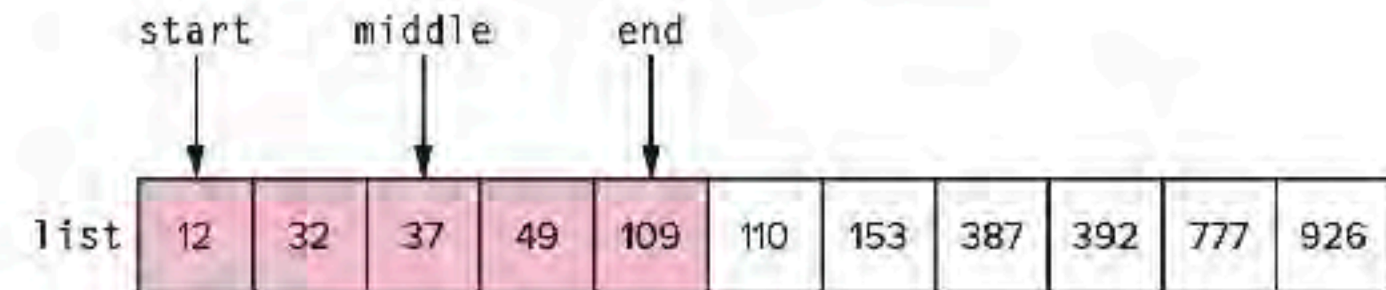

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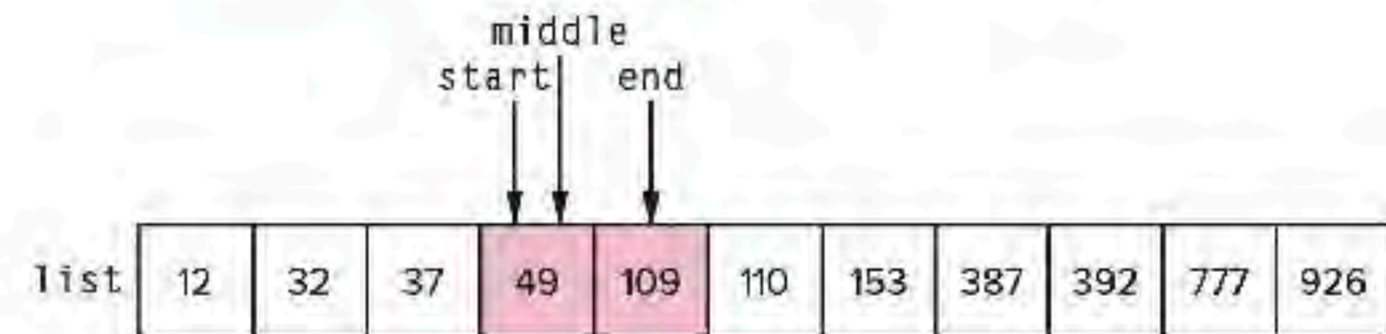

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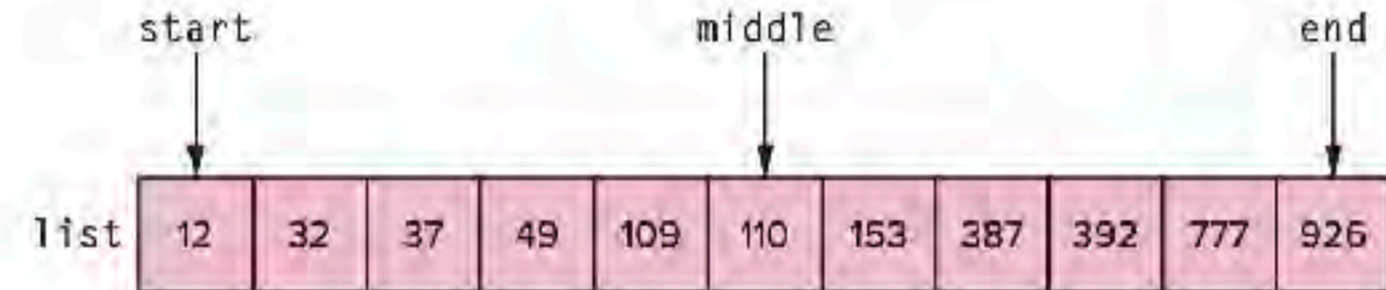

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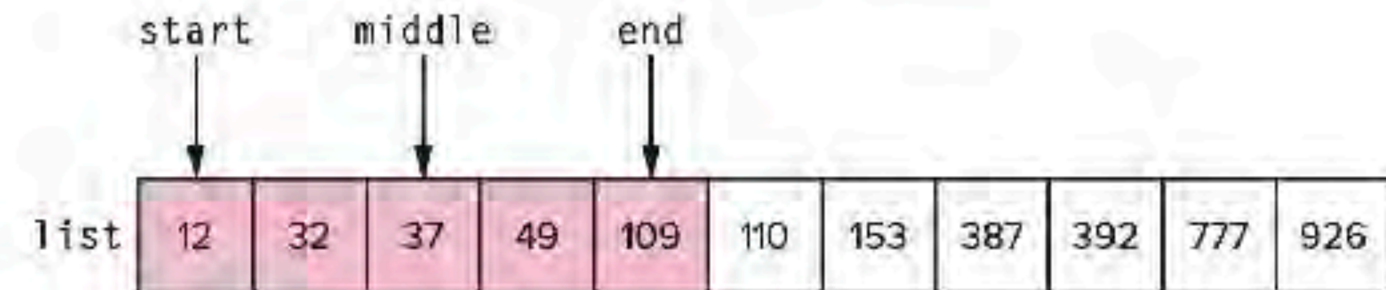

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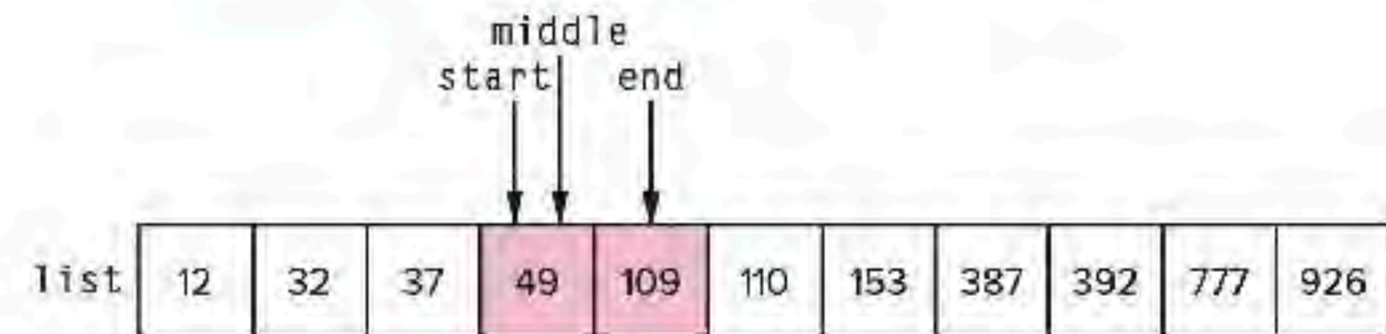

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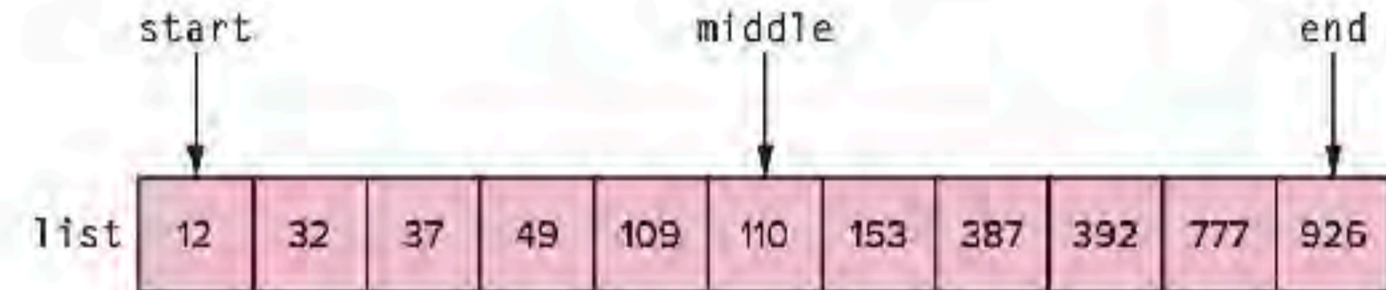

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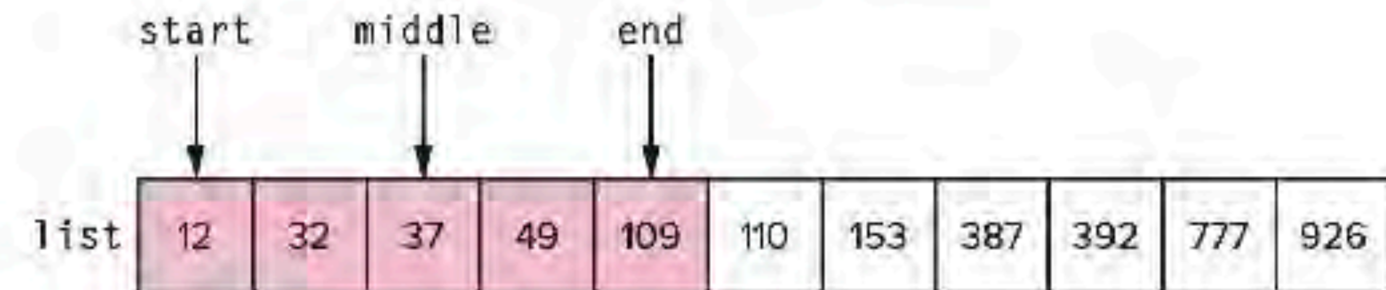

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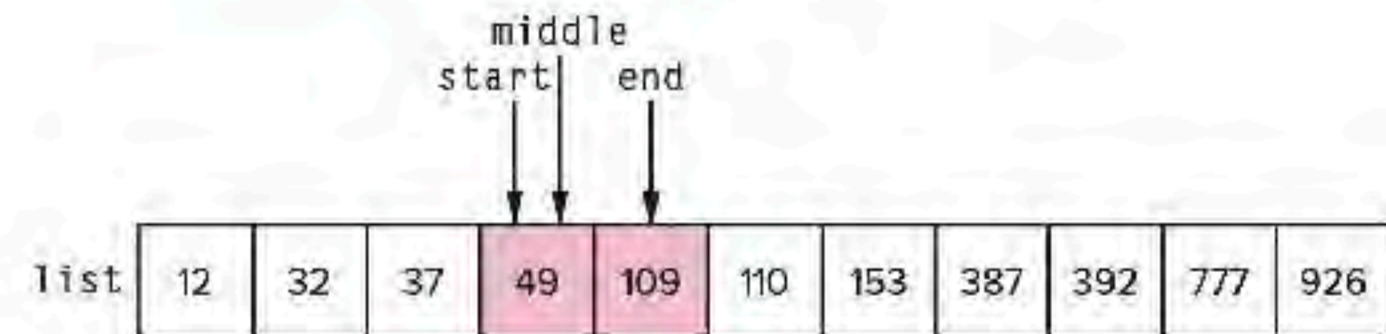

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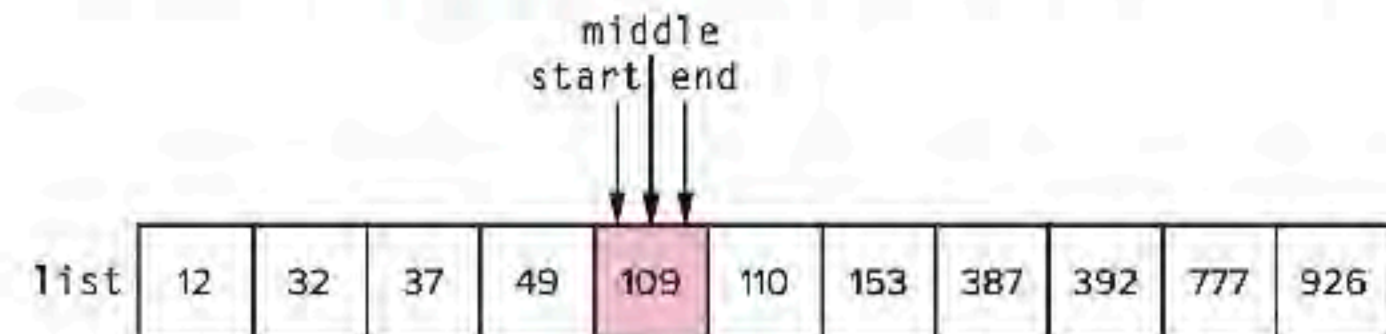

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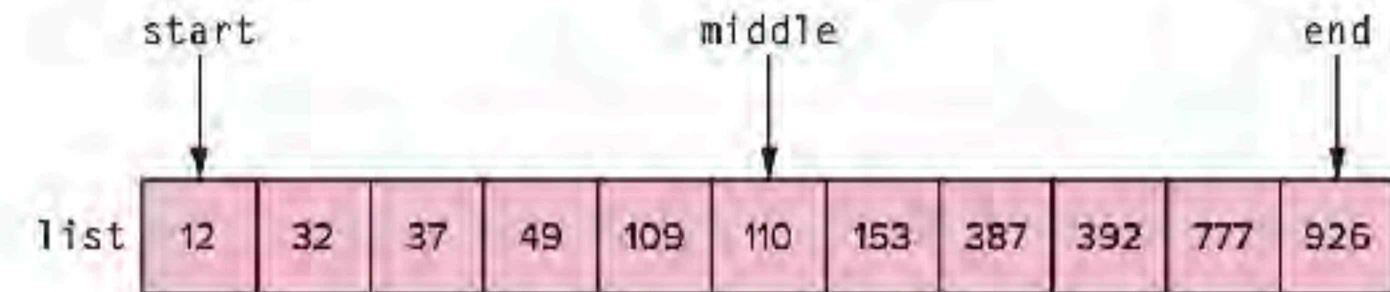

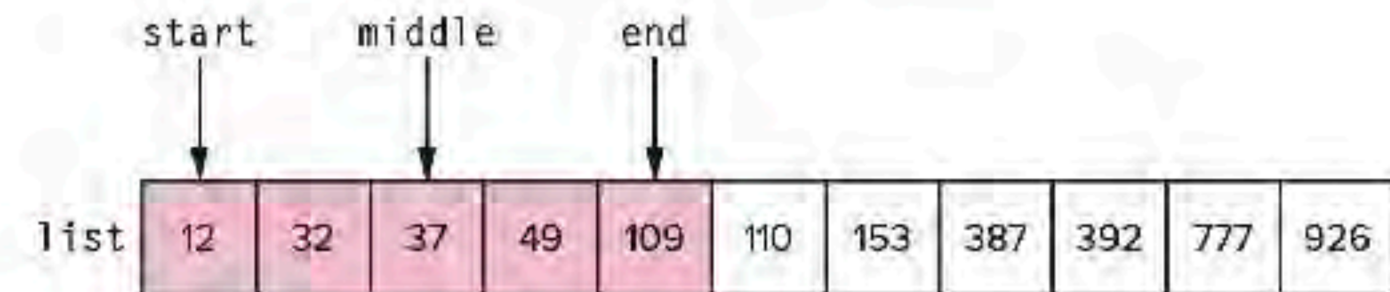

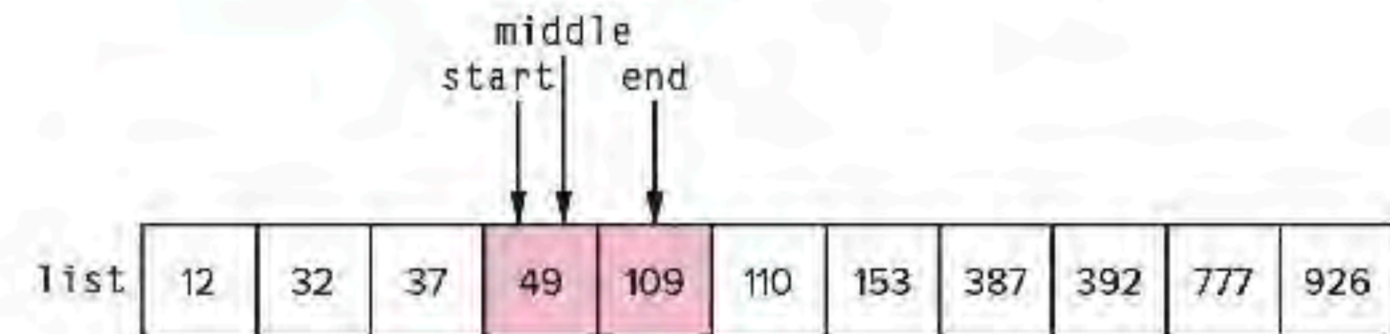

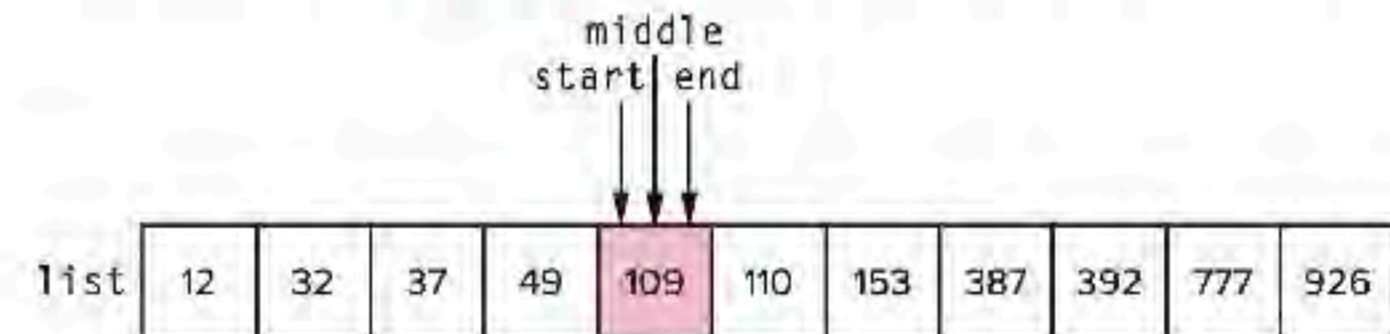

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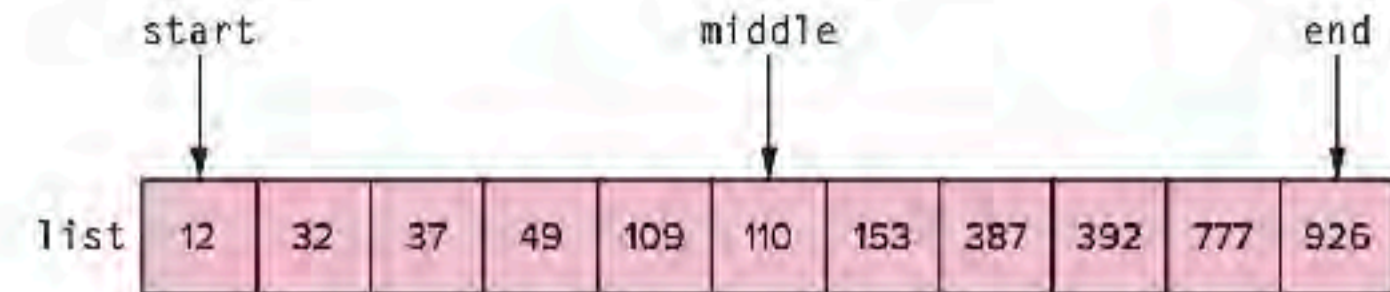
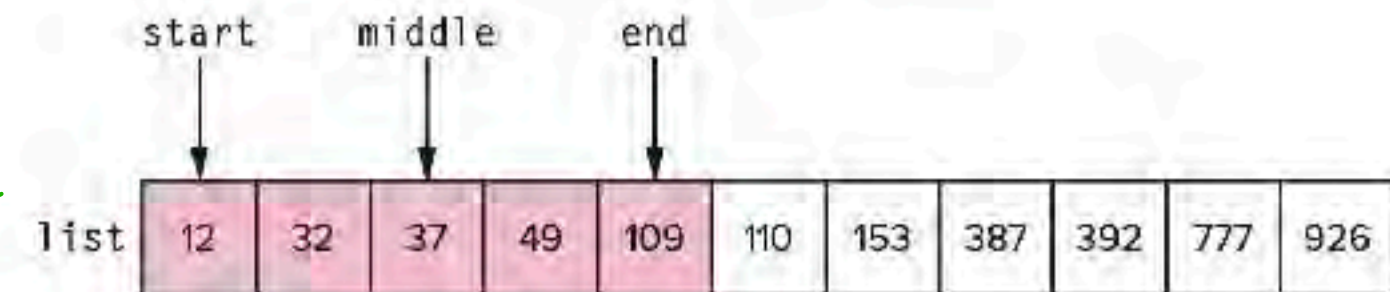
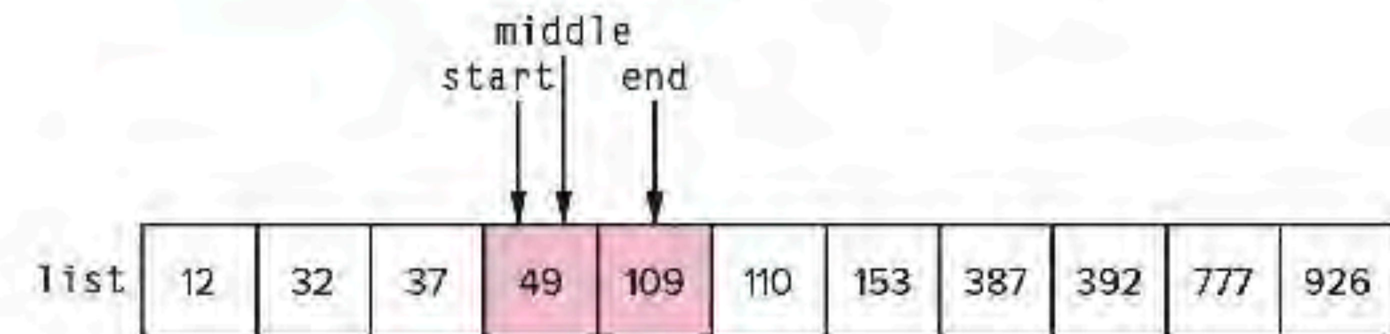
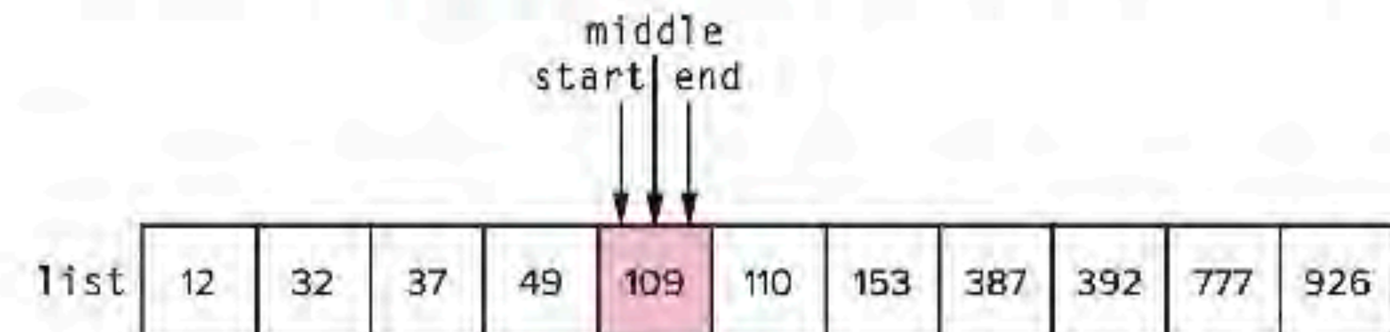
Recap binary search

```
int binary(int arr[], int n, int key){
  int start = 0;       // Left pointer
  int end = _____; // Right pointer

  while (end >= start){
    int mid = (___________) / 2; // Pick middle element

    // Logic to focus search on left or right of mid
    if (key == arr[mid])
      return mid;
    else if (key < arr[mid])
      end = _______;
    else
      start = ________;
  }
  return -1; // Loop exited, element not present.
}
```









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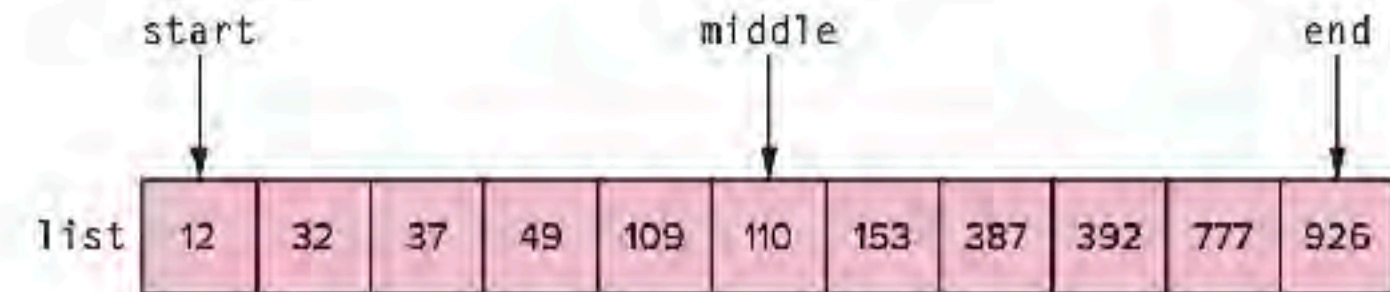
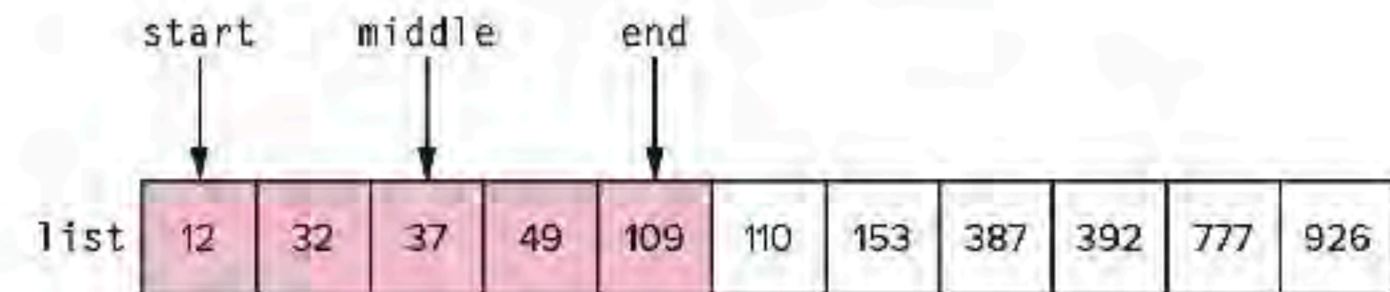
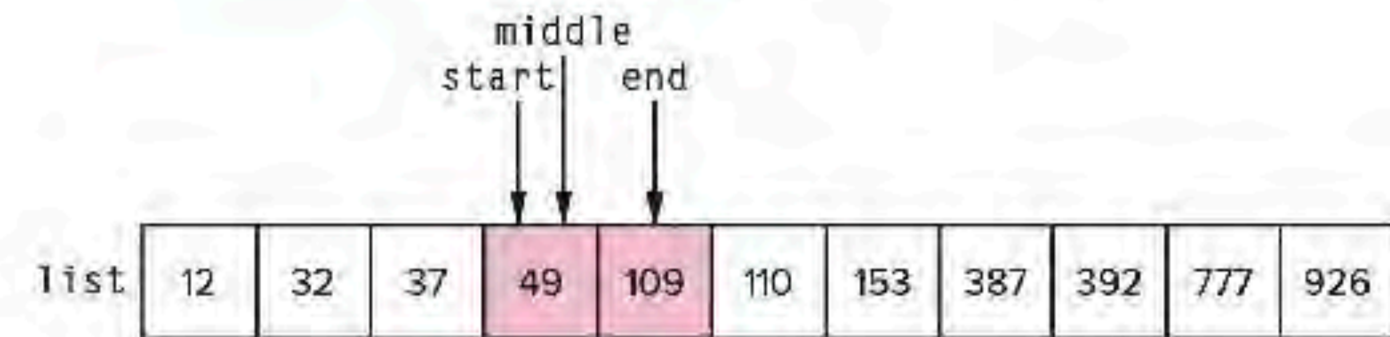
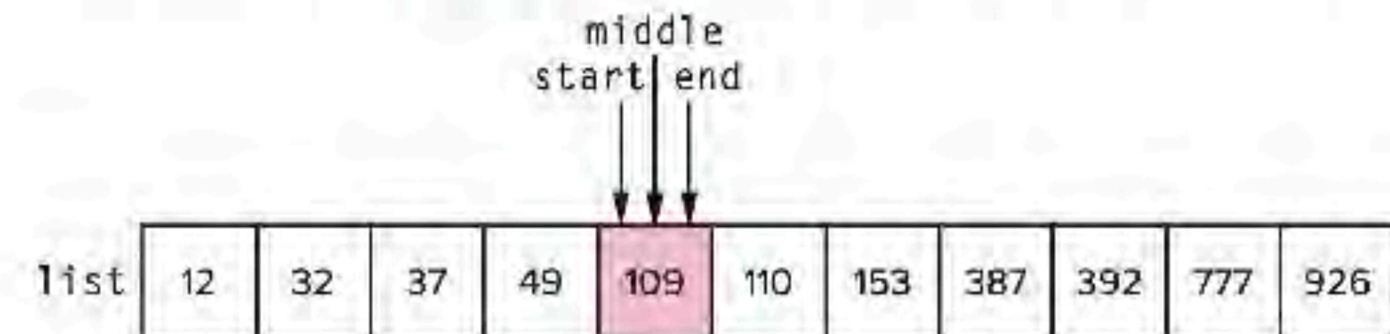
Binary search









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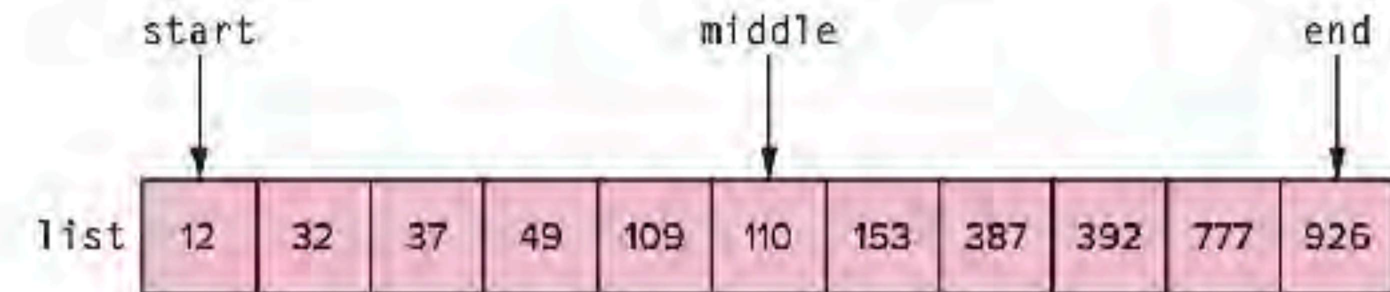
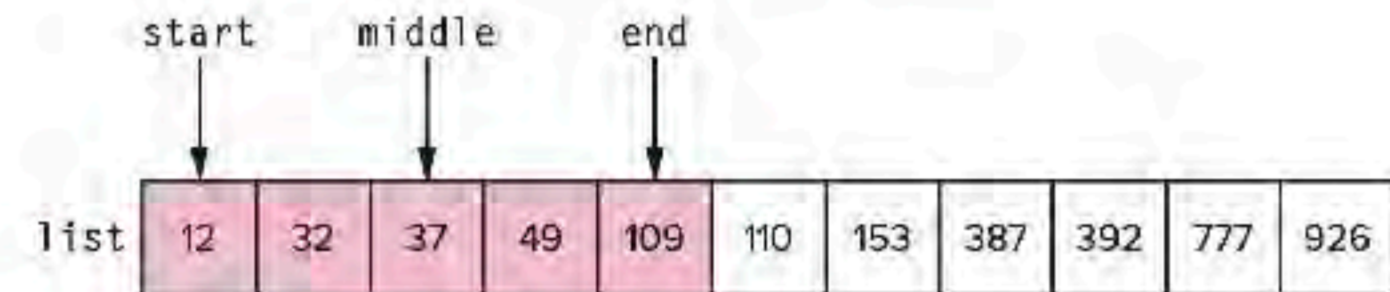
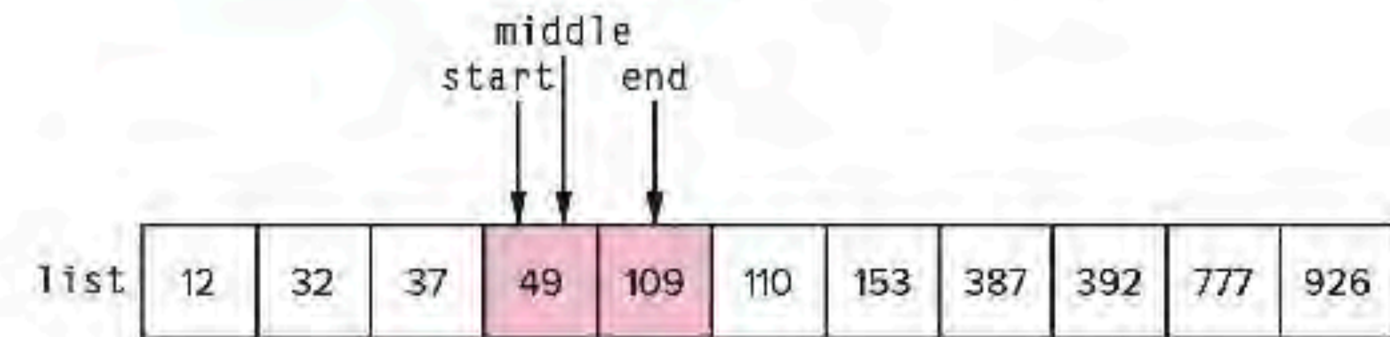
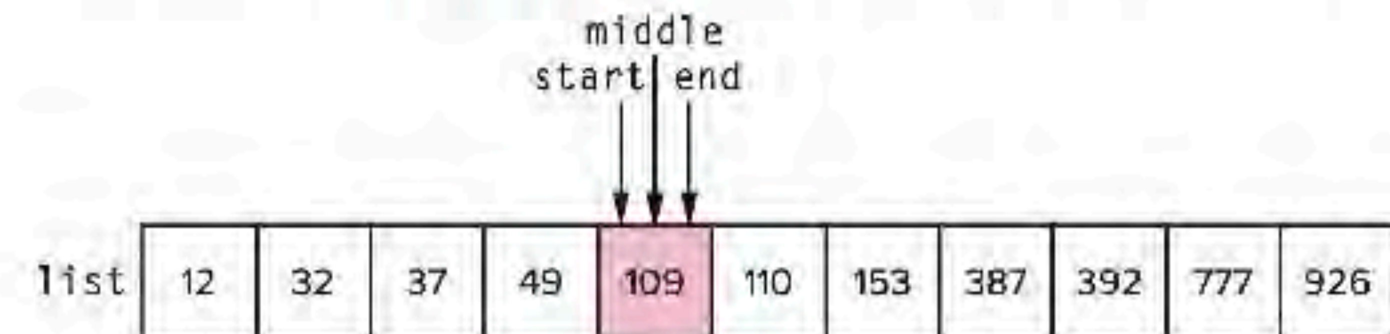
Binary search

- We are repeating the same process of finding `mid` and going left or right of `mid` on each *subarray*.









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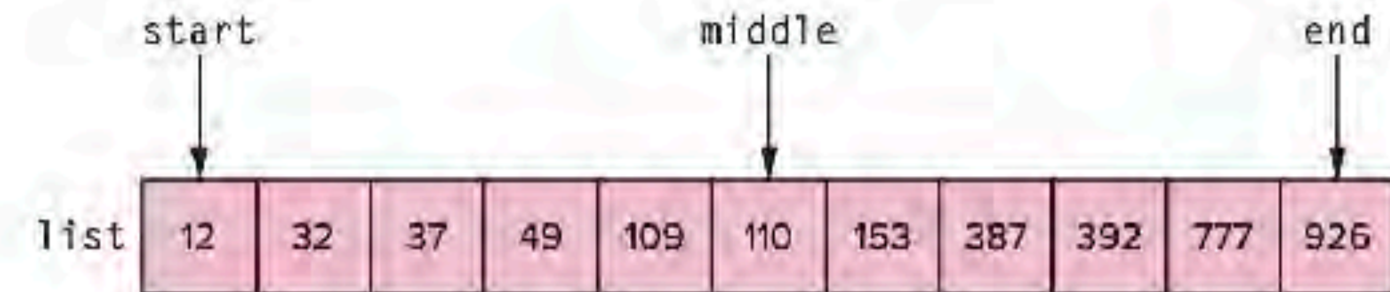

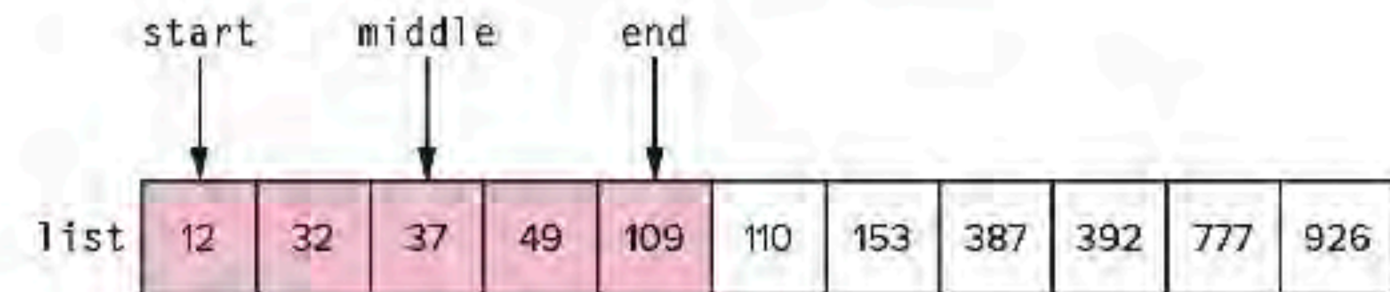

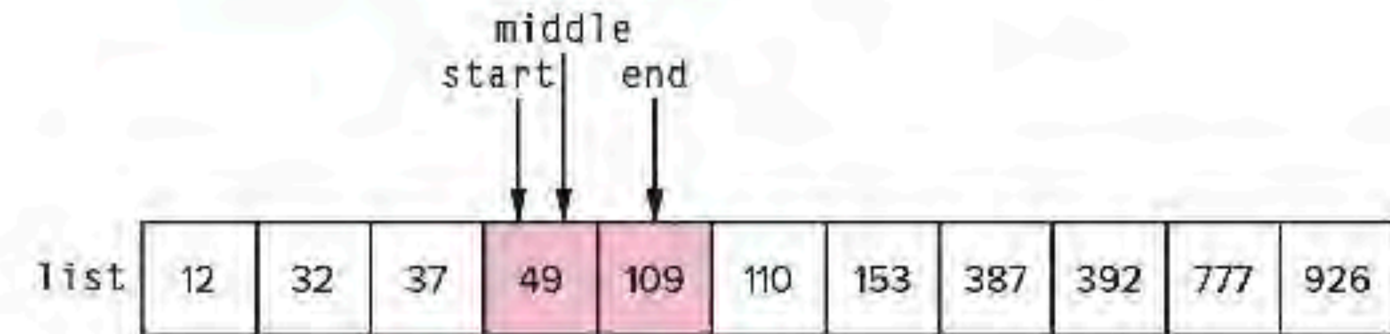

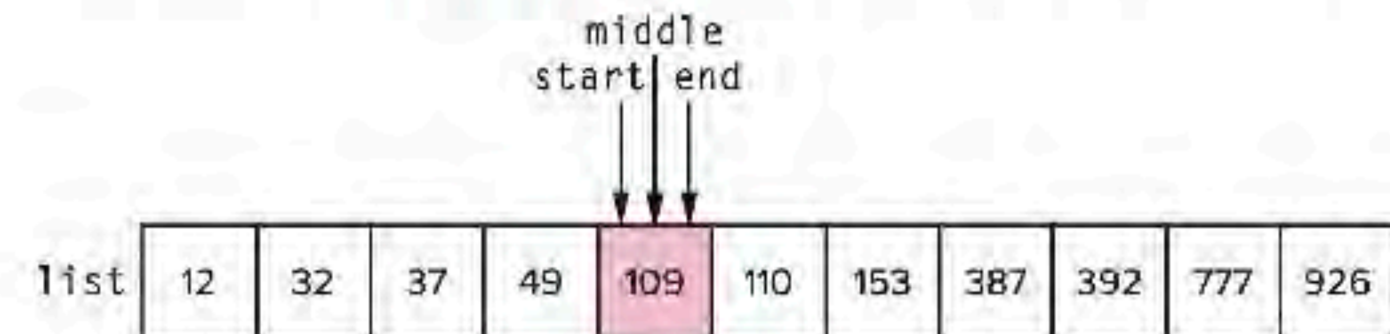

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binary(arr[], n, key)
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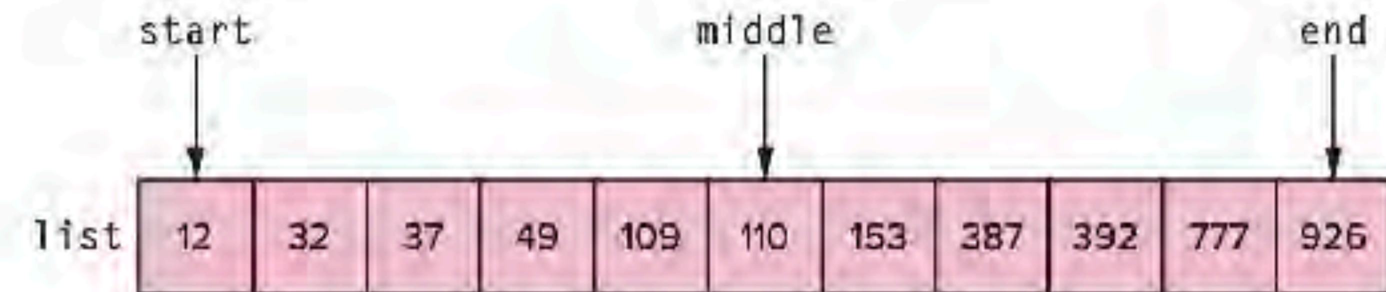

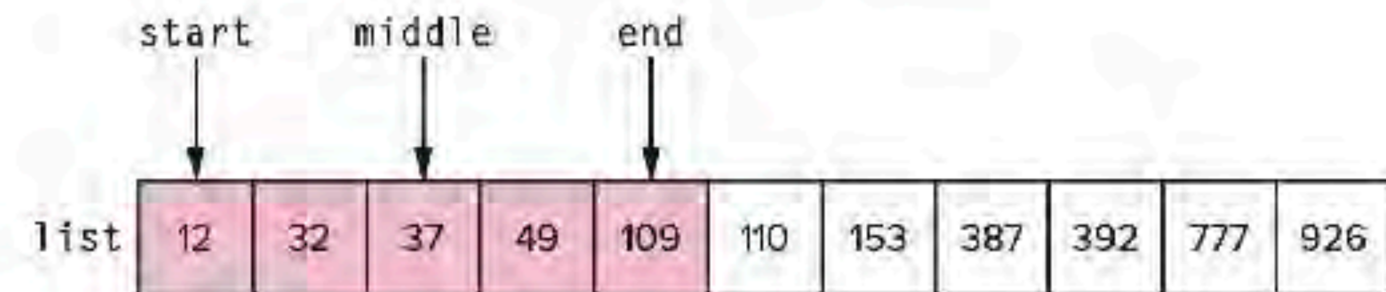

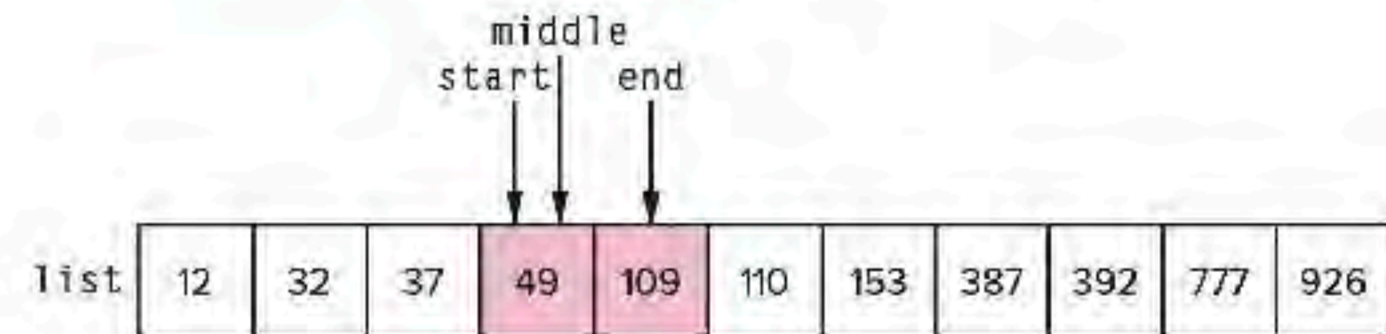

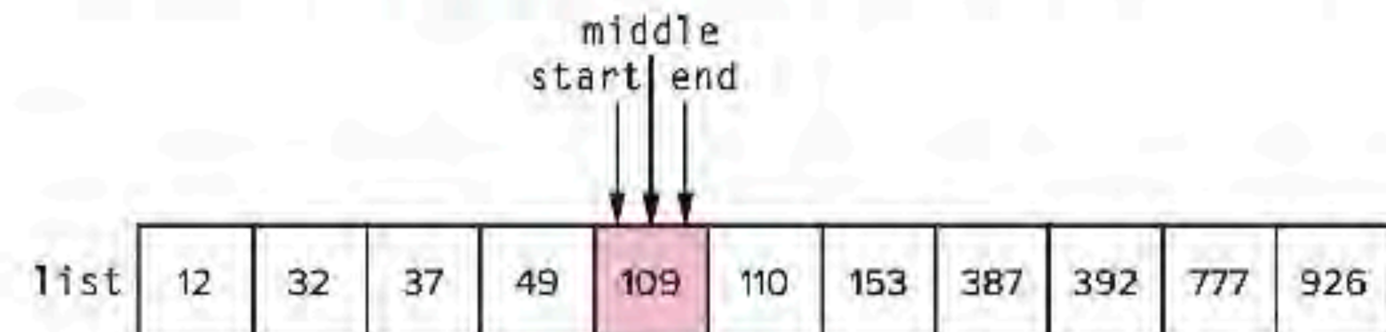

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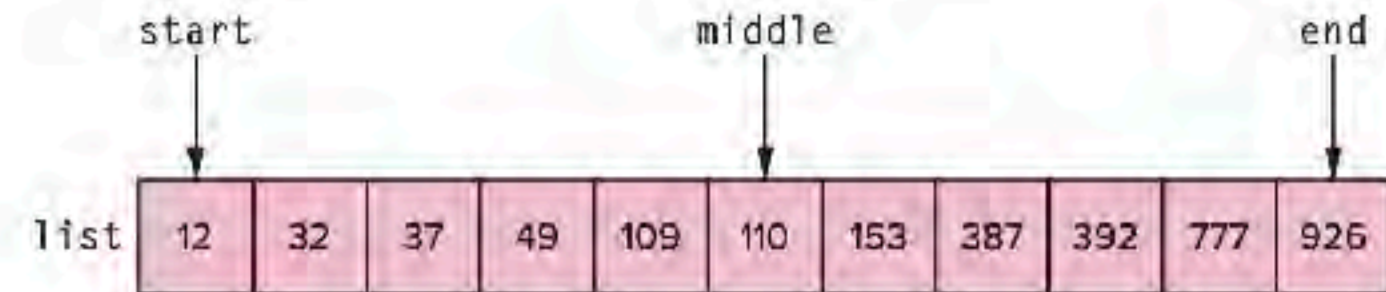

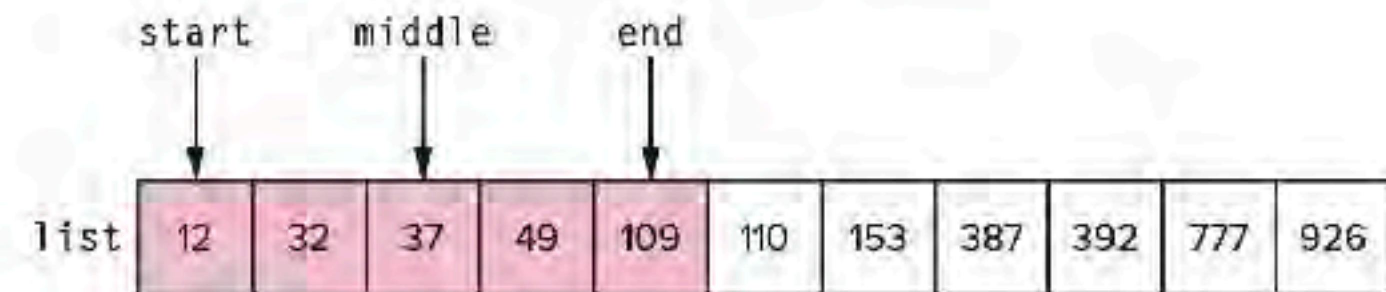

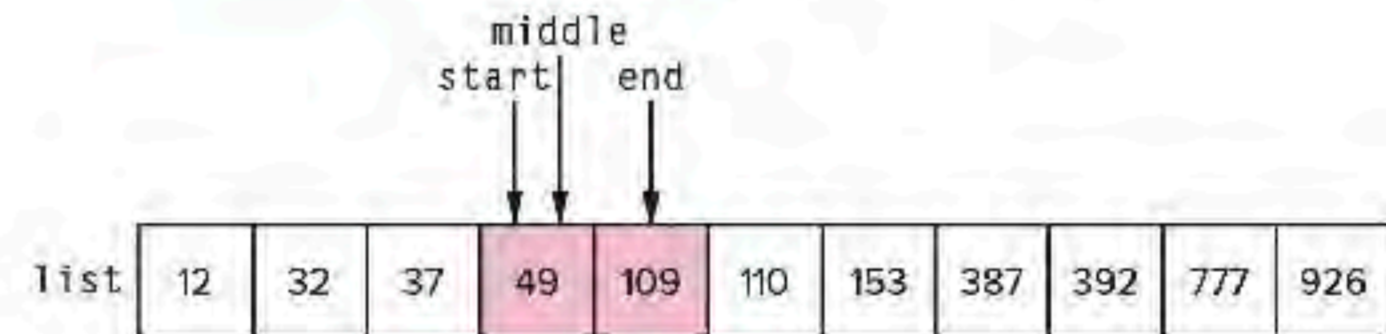

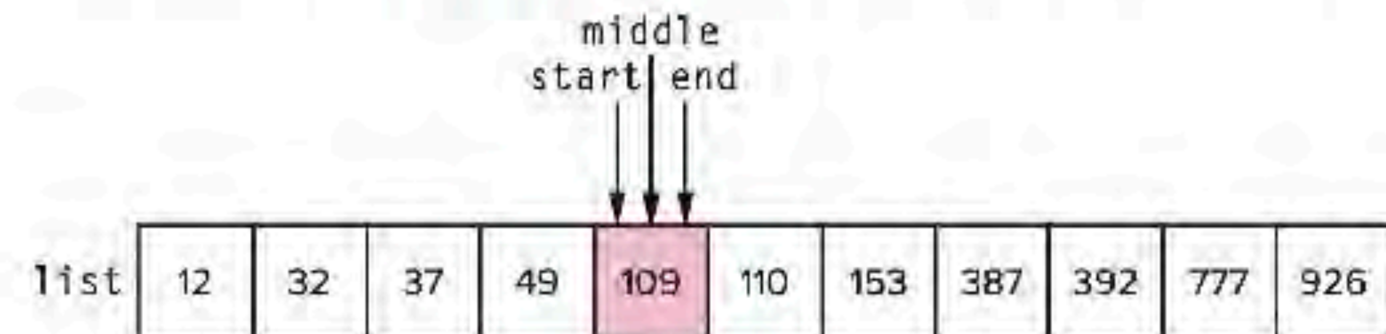

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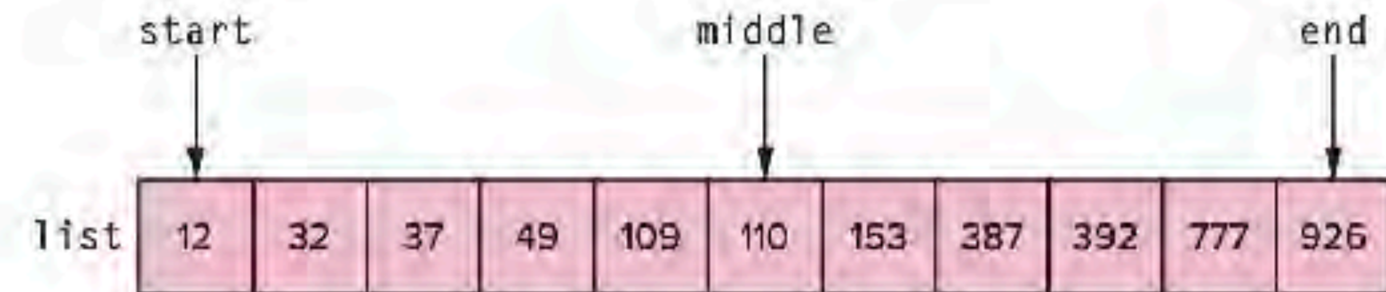
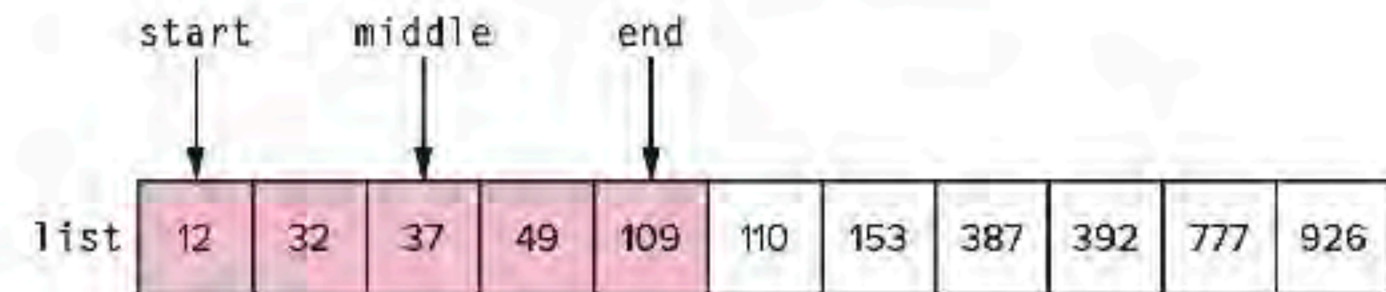
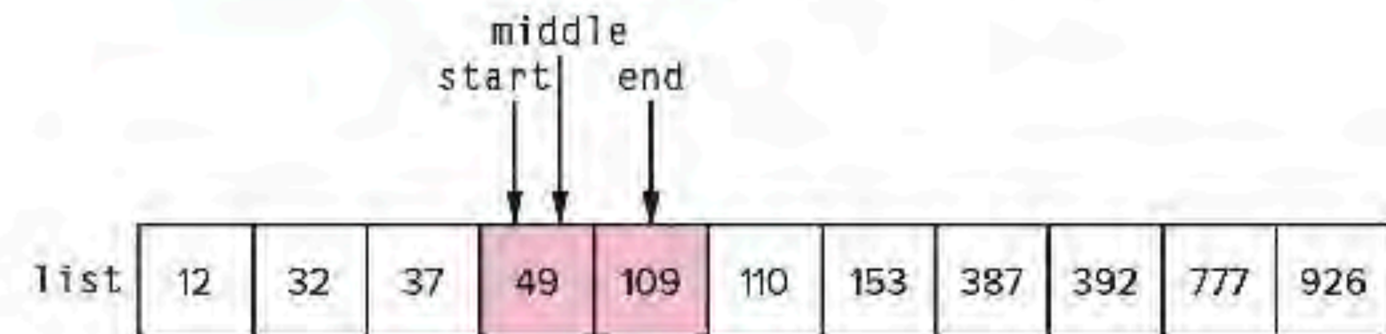
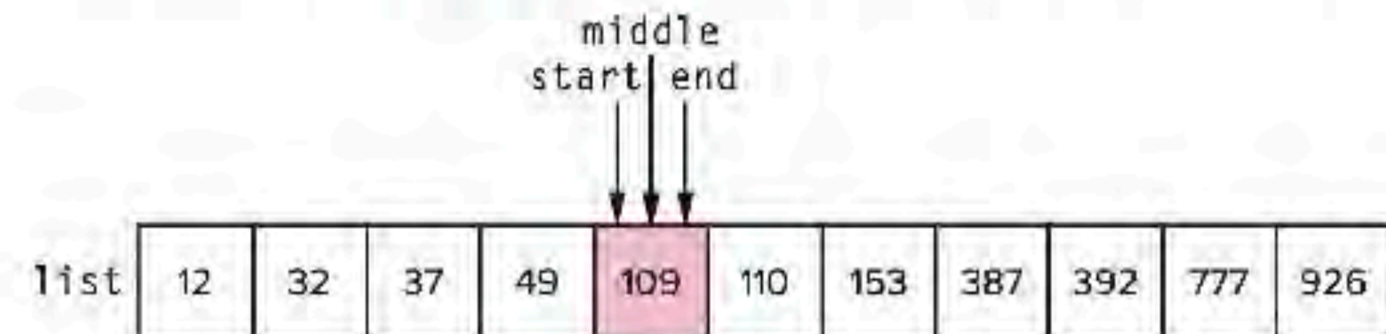
- Idea is called *recursion*.









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Recursion

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Examples: Factorial function, Fibonacci series, binary search, etc.

Recursive function

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  - This case returns a recursive call to function itself. It breaks down the problem into smaller chunks that can be solved over and over by the same function.
  - The input to the next call gets induced gradually.

Example: Factorial

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- Mathematical definition

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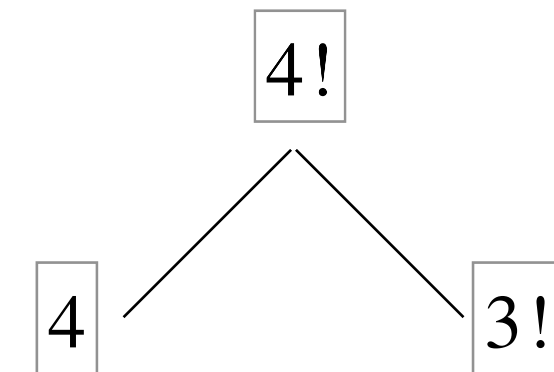

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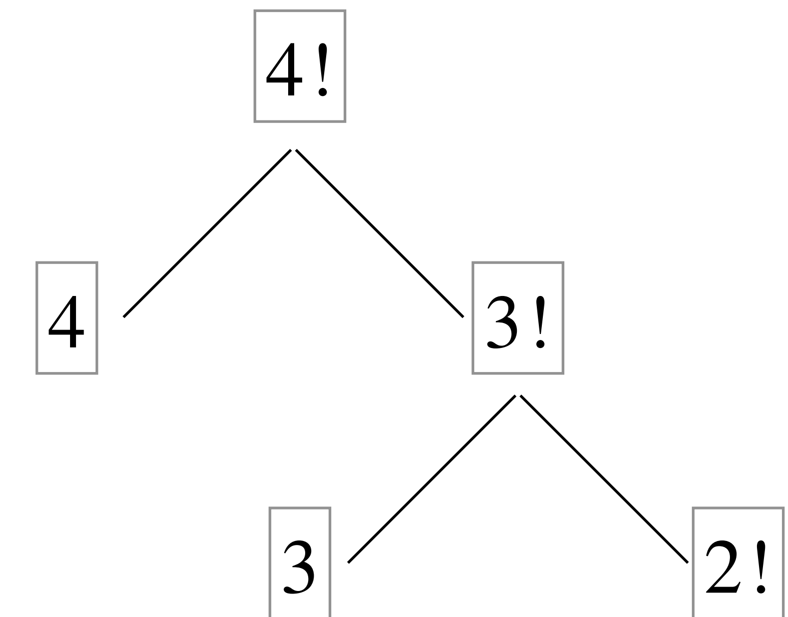

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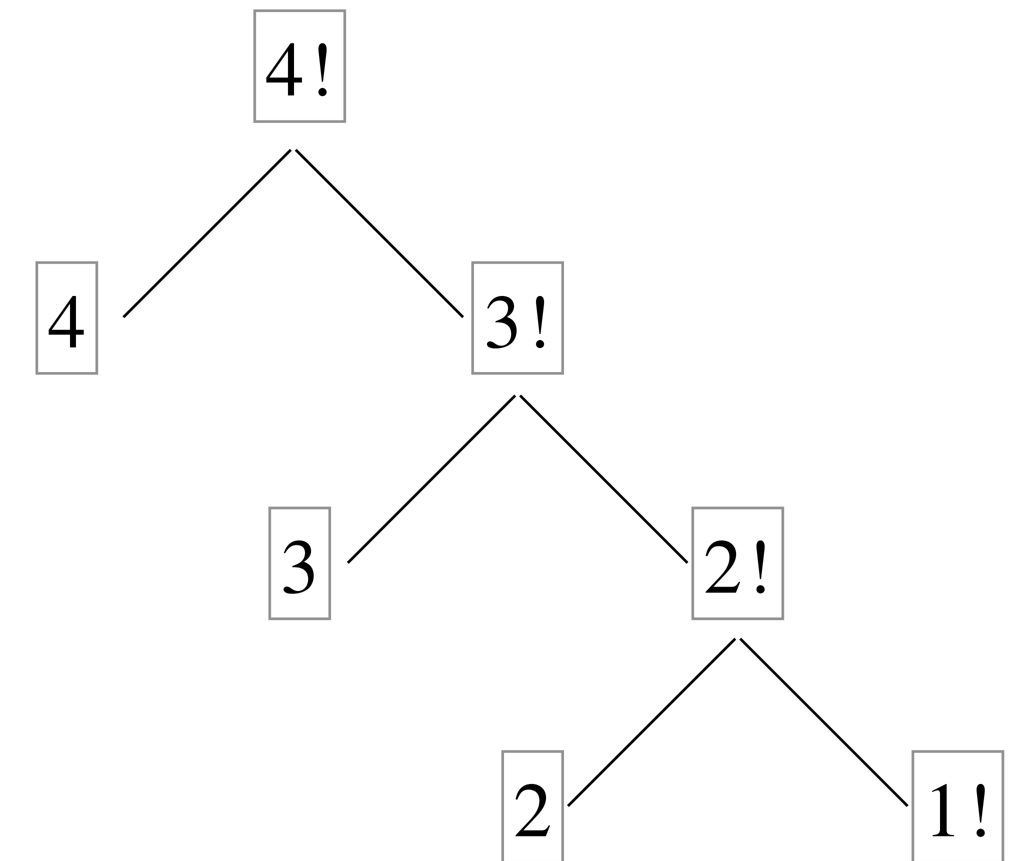

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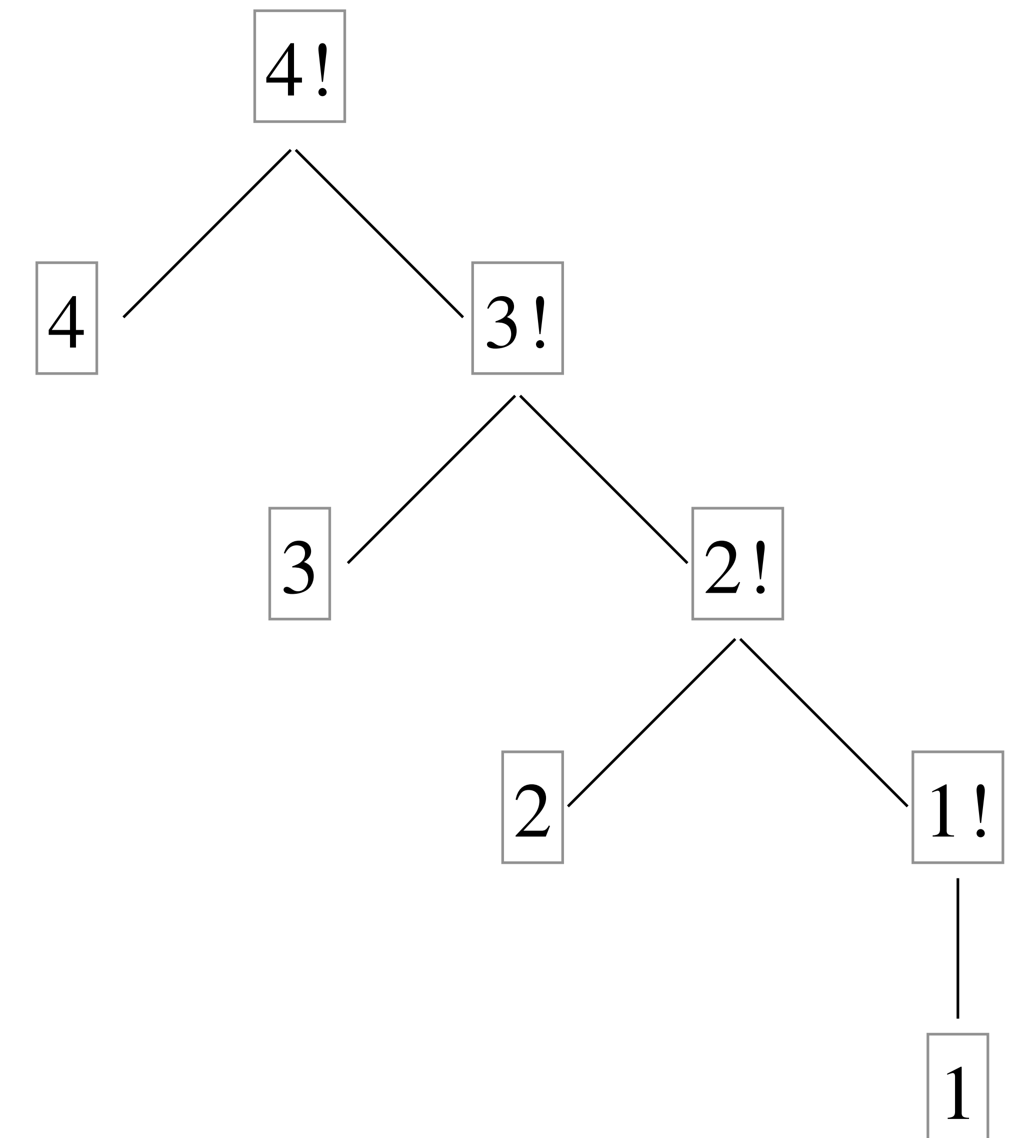

Example: Factorial

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```
int Factorial(int n){
    if (n == 1)
        return 1;
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        return n * Factorial(n - 1);
}
```

Example: Factorial

```
fn = Factorial(4);
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return 3 * Factorial(2);
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```
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```

return value = 1

```
return 1;
```

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Example: Factorial

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return value = 2

```
return 2 * Factorial(1);
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Example: Factorial

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fn = Factorial(4);
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```
return 4 * Factorial(3);
```

return value = 6

```
return 3 * Factorial(2);
```

return value = 2

```
return 2 * Factorial(1);
```

return value = 1

```
return 1;
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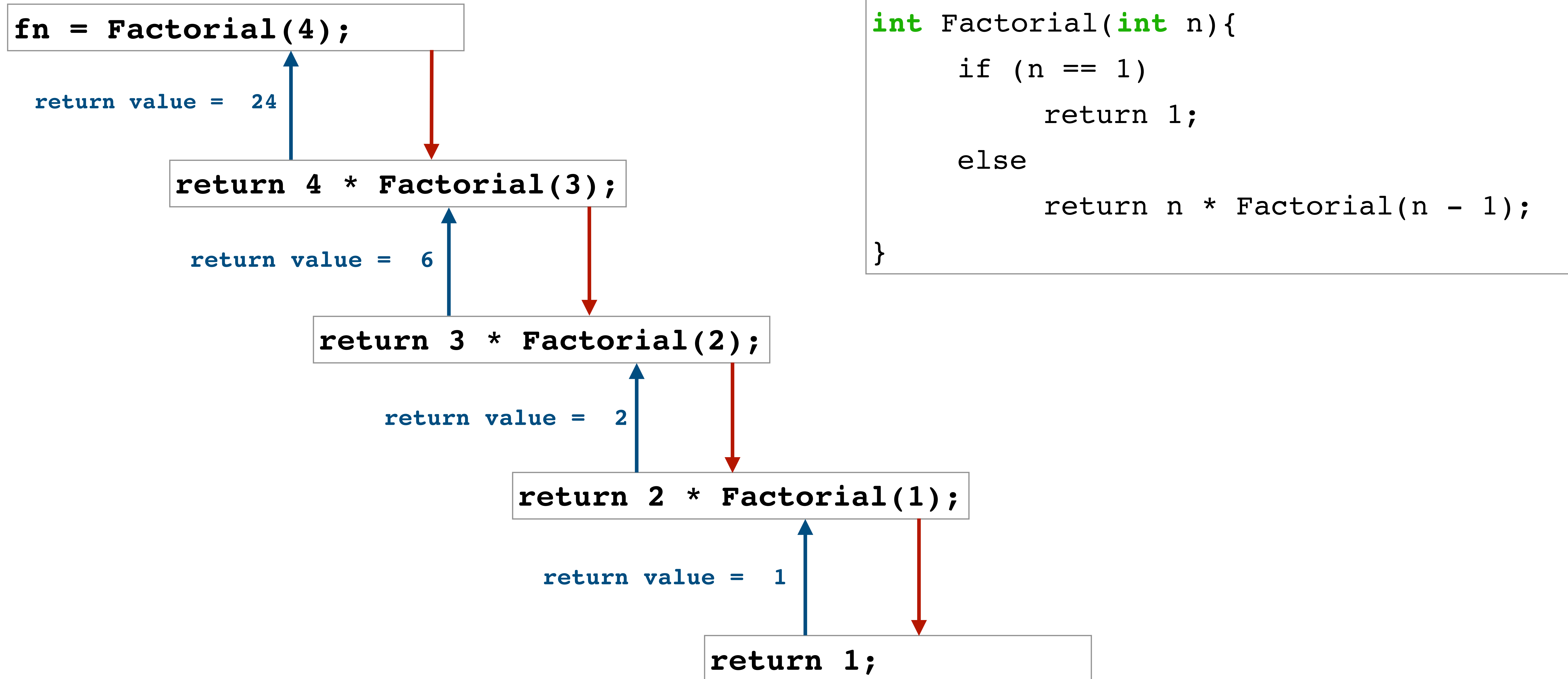
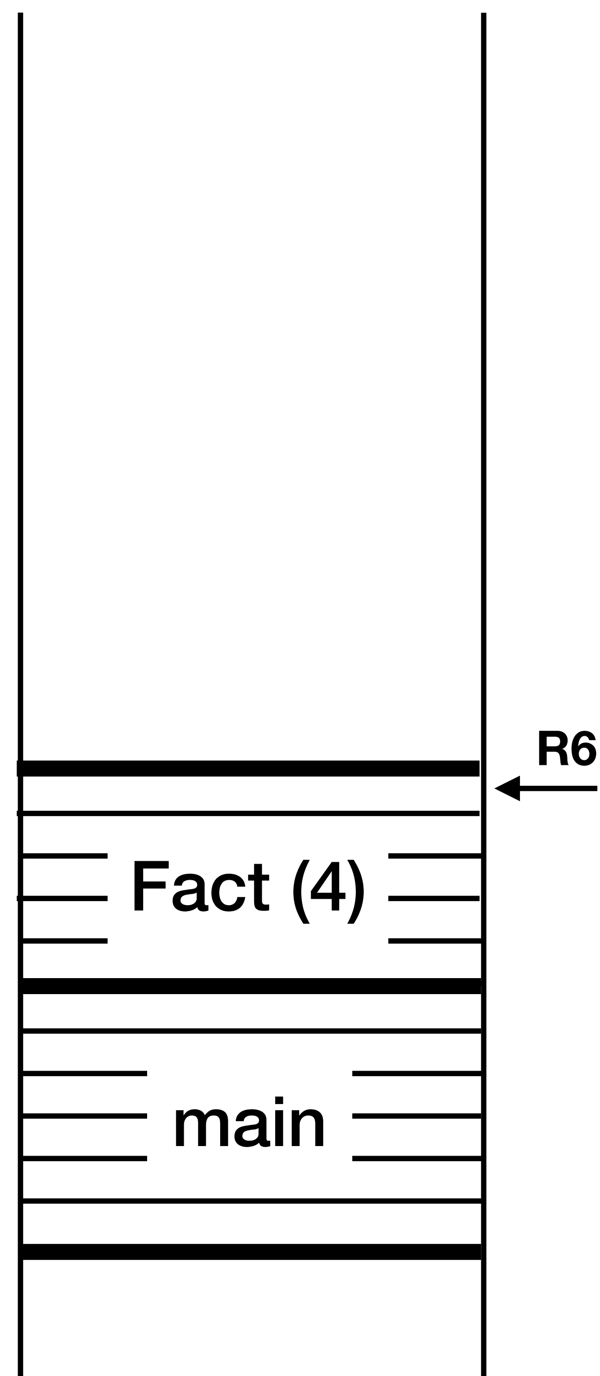


RTS During Execution of Factorial

main calls Factorial(4)



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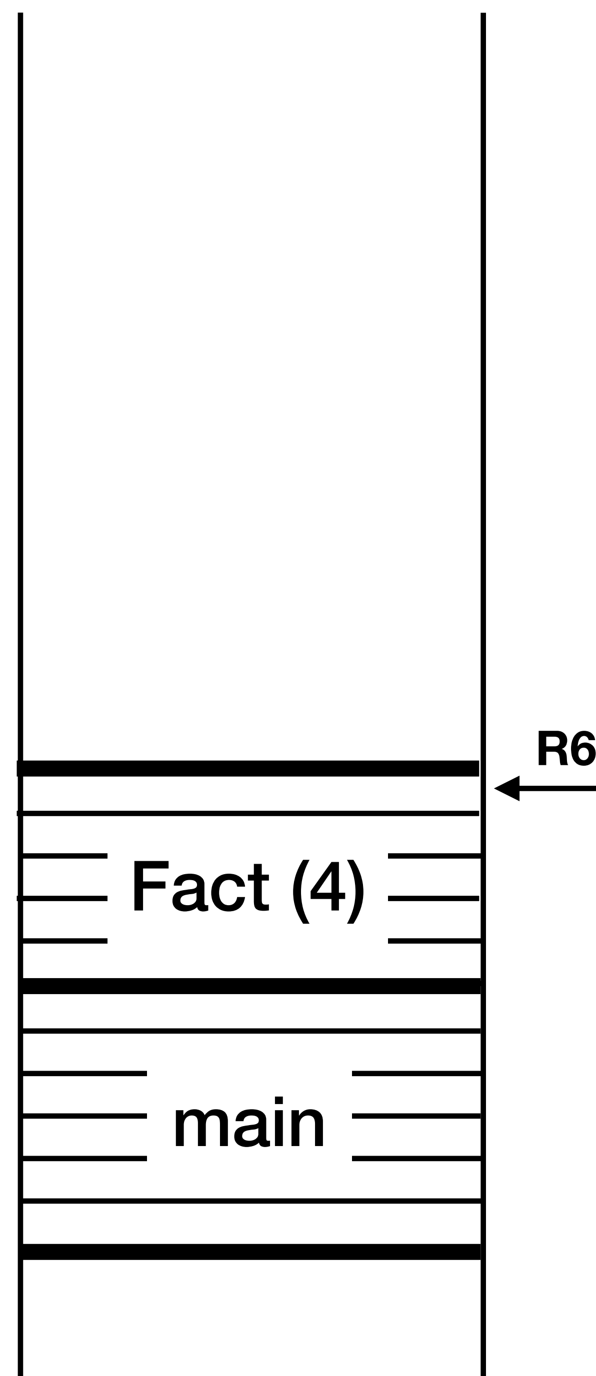
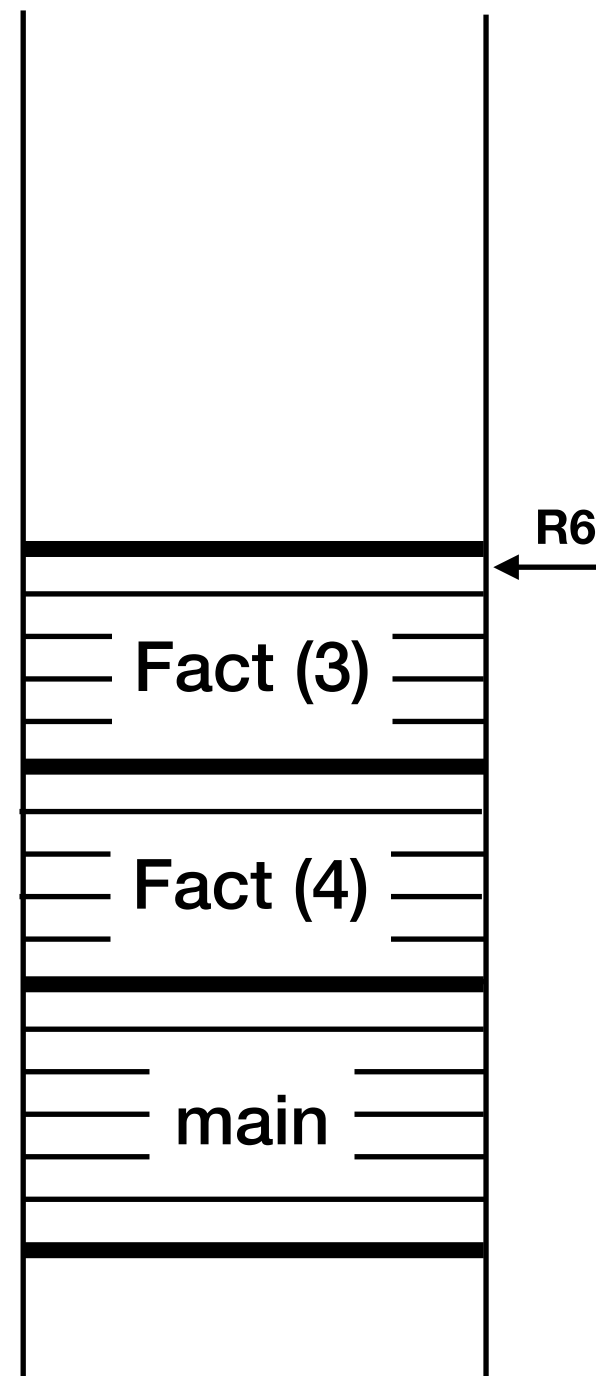


Factorial(4) calls Factorial(3)



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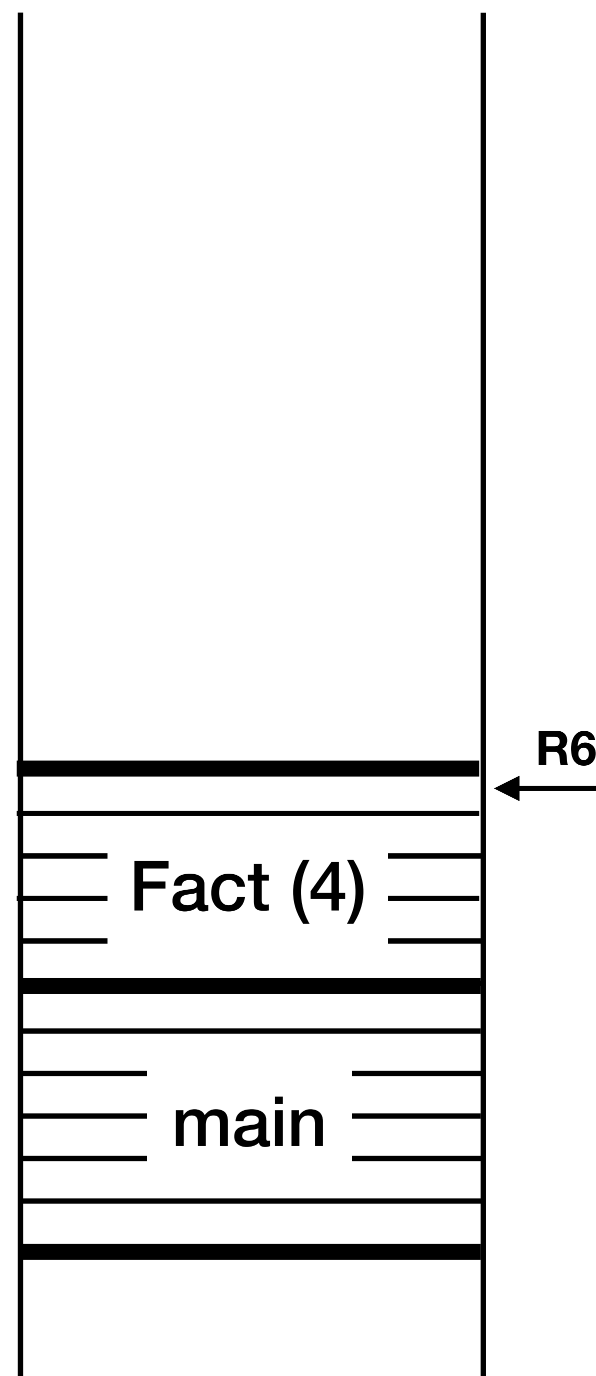

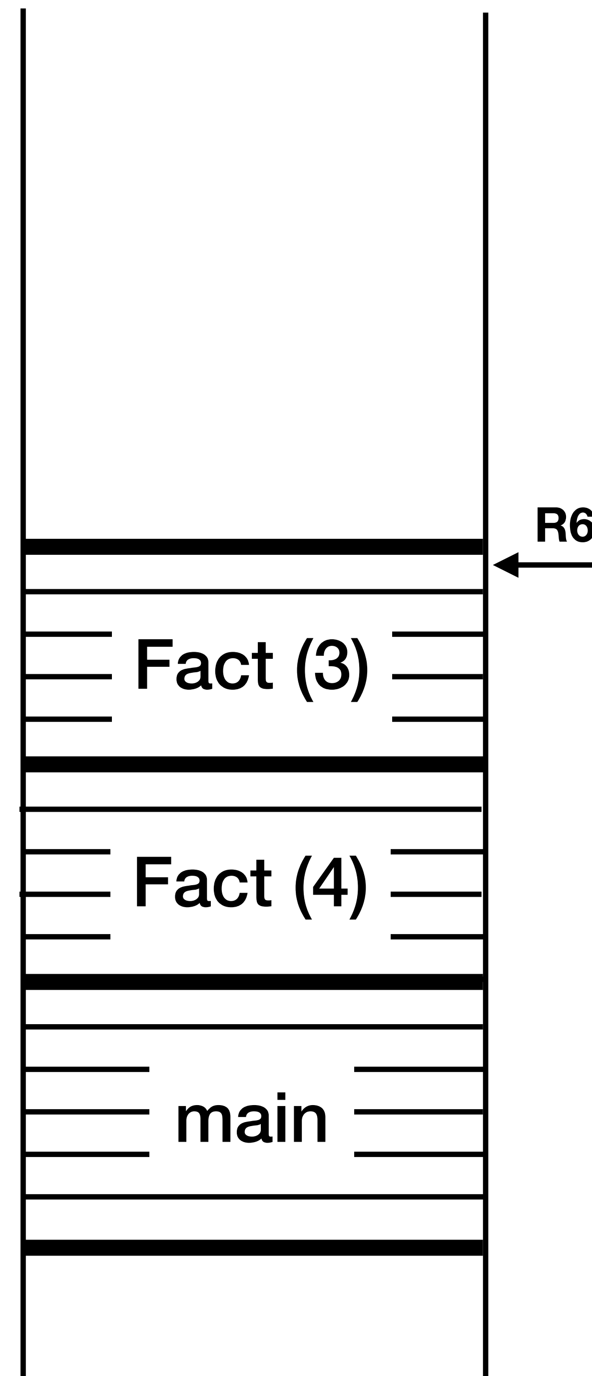

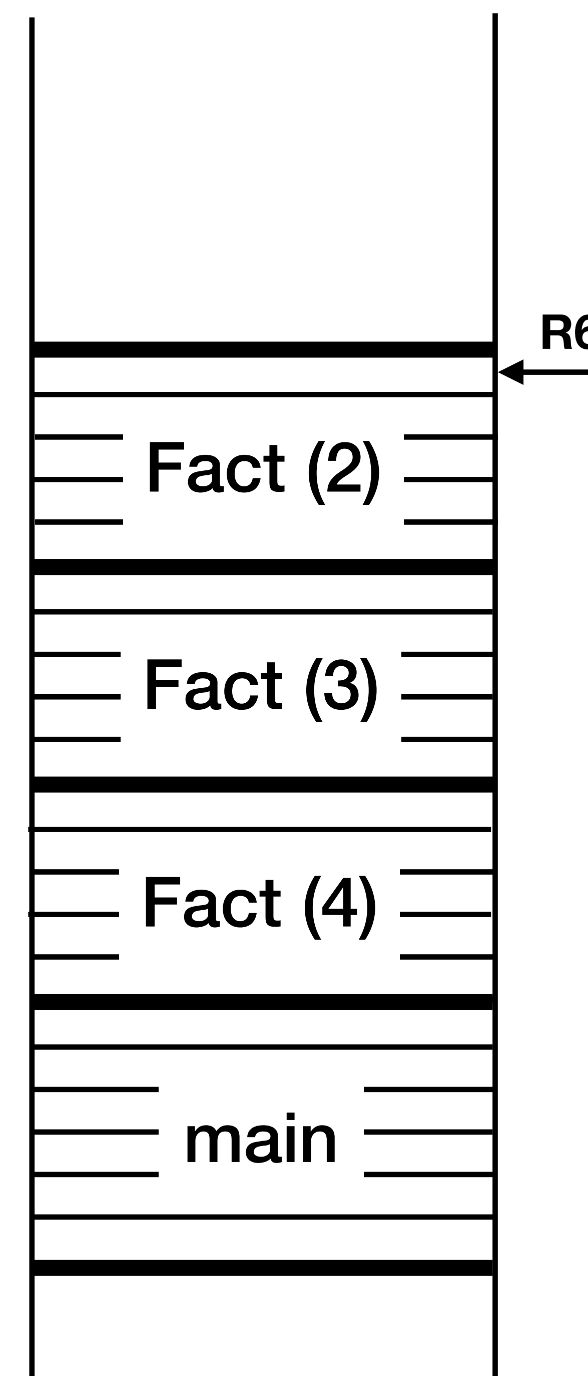

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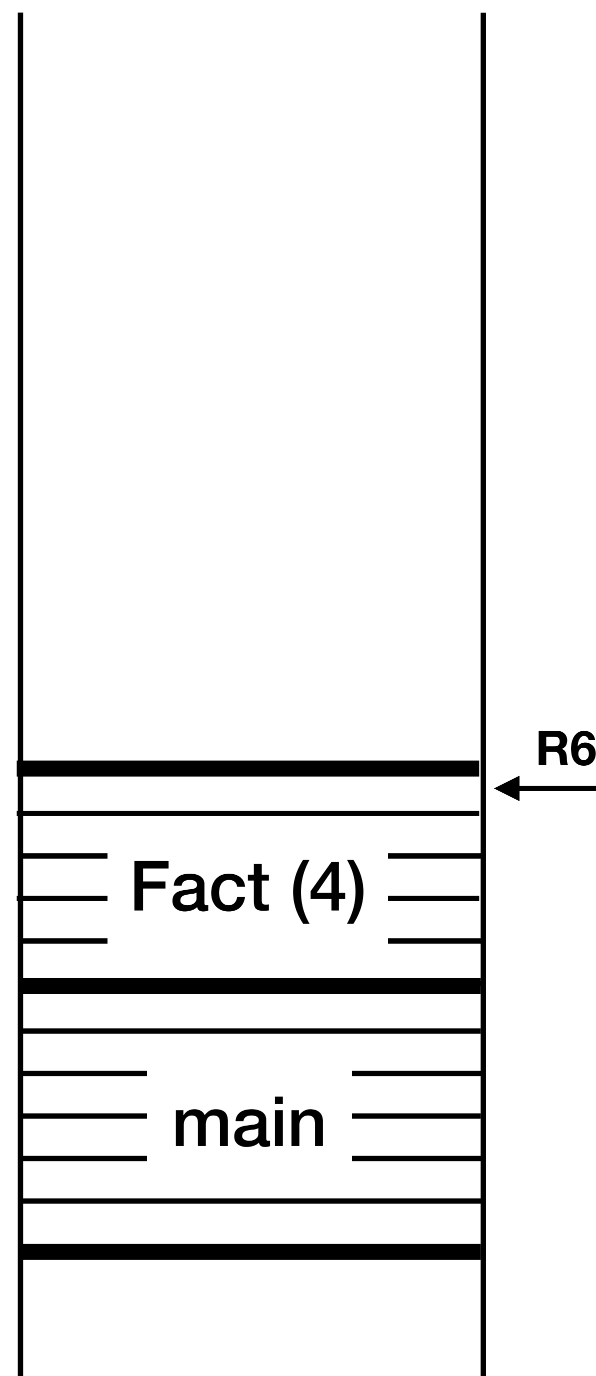

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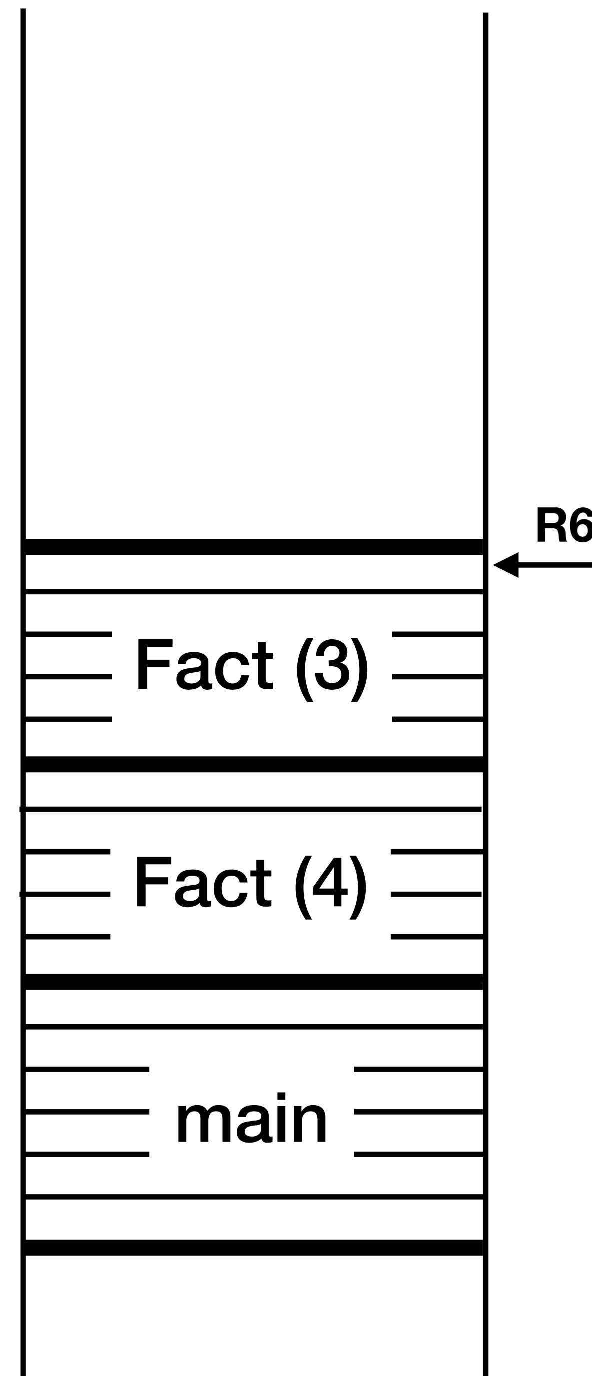
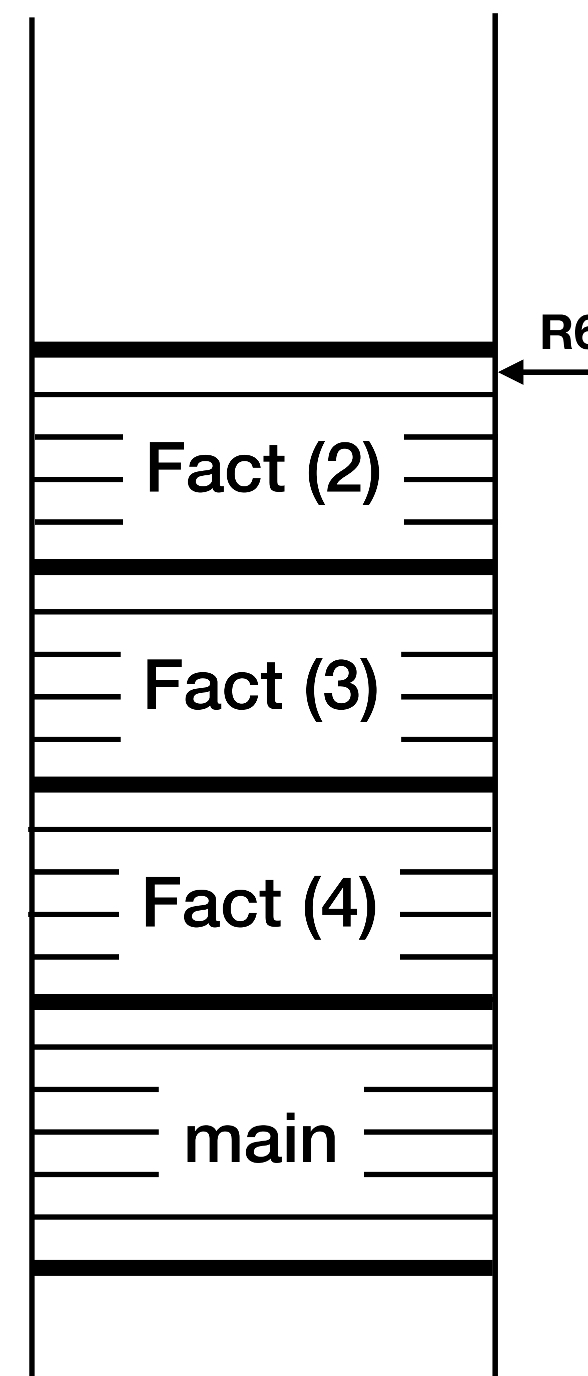
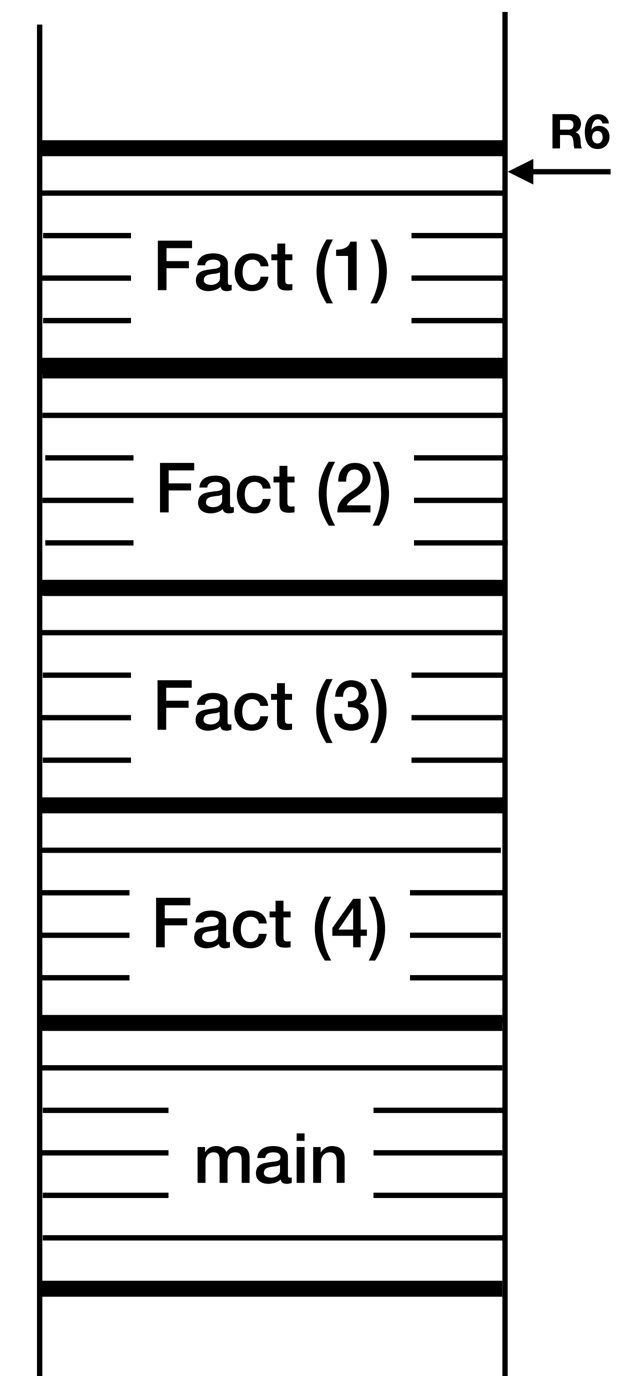


Factorial(3) calls Factorial(2)



Factorial(2) calls Factorial(1)



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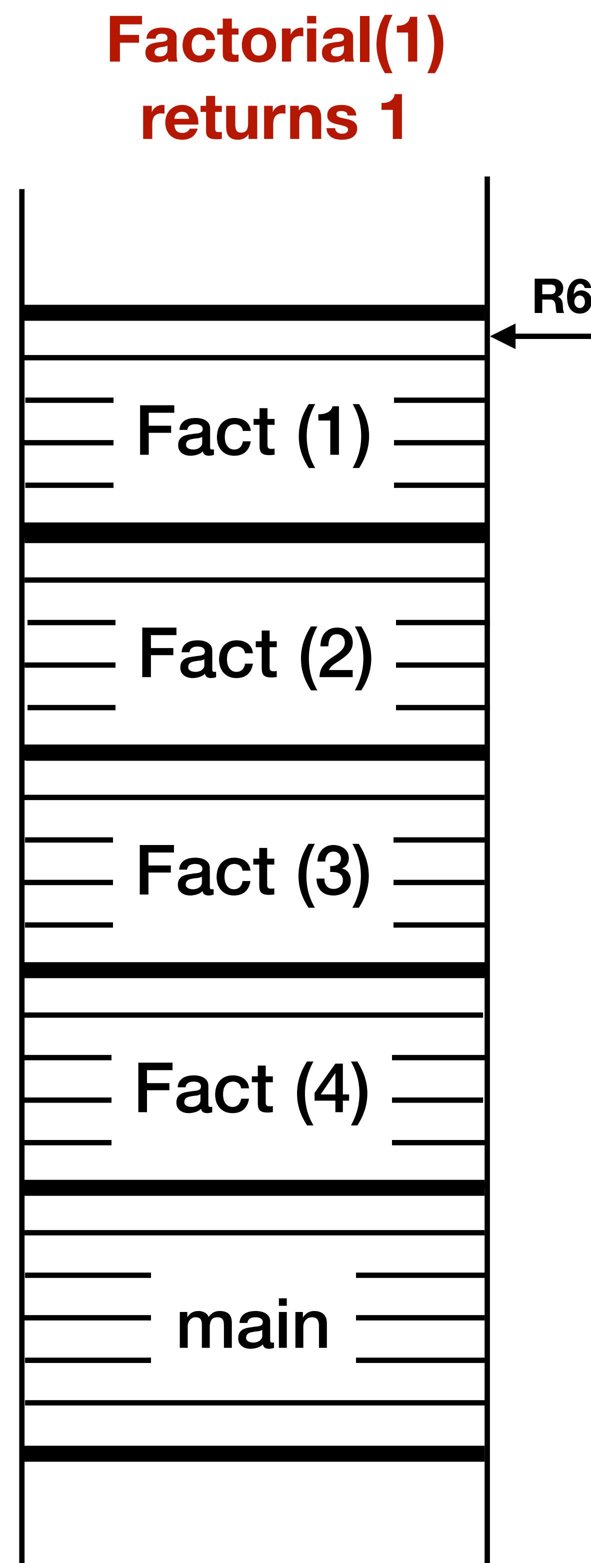

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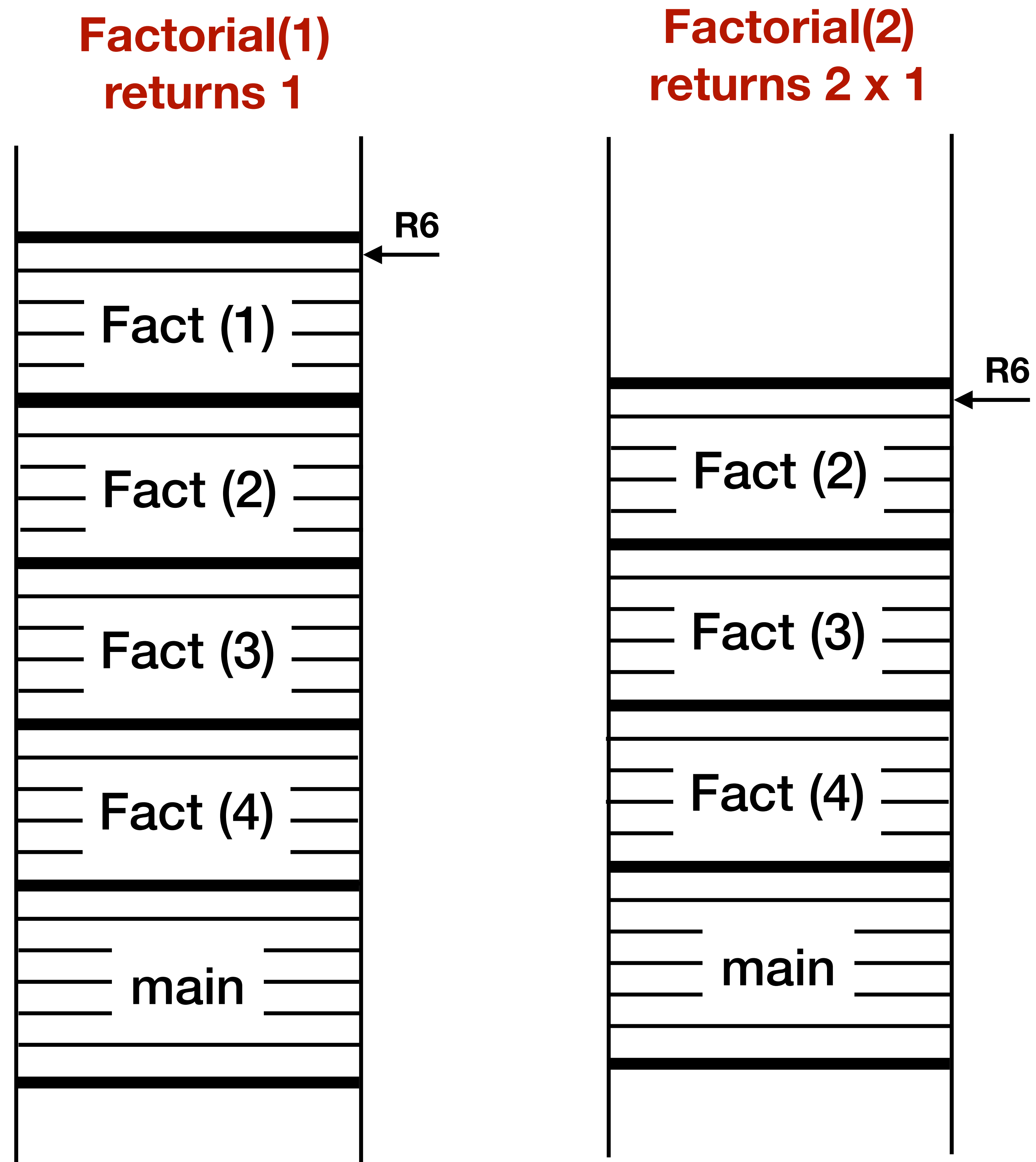
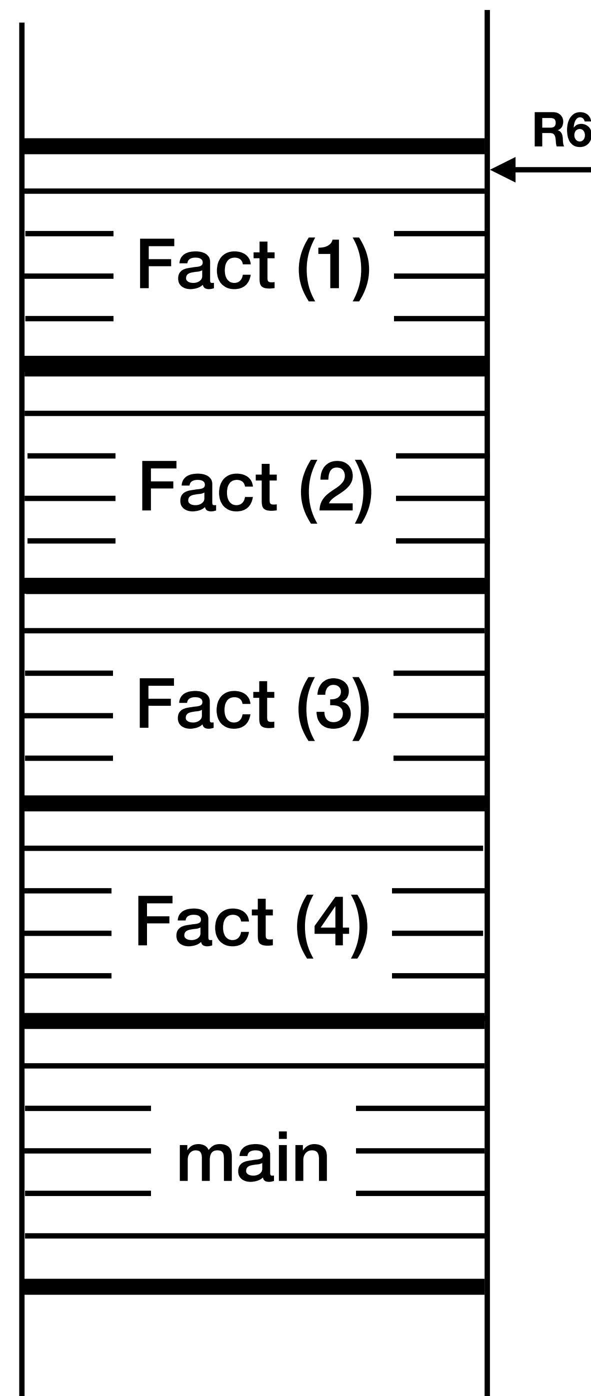
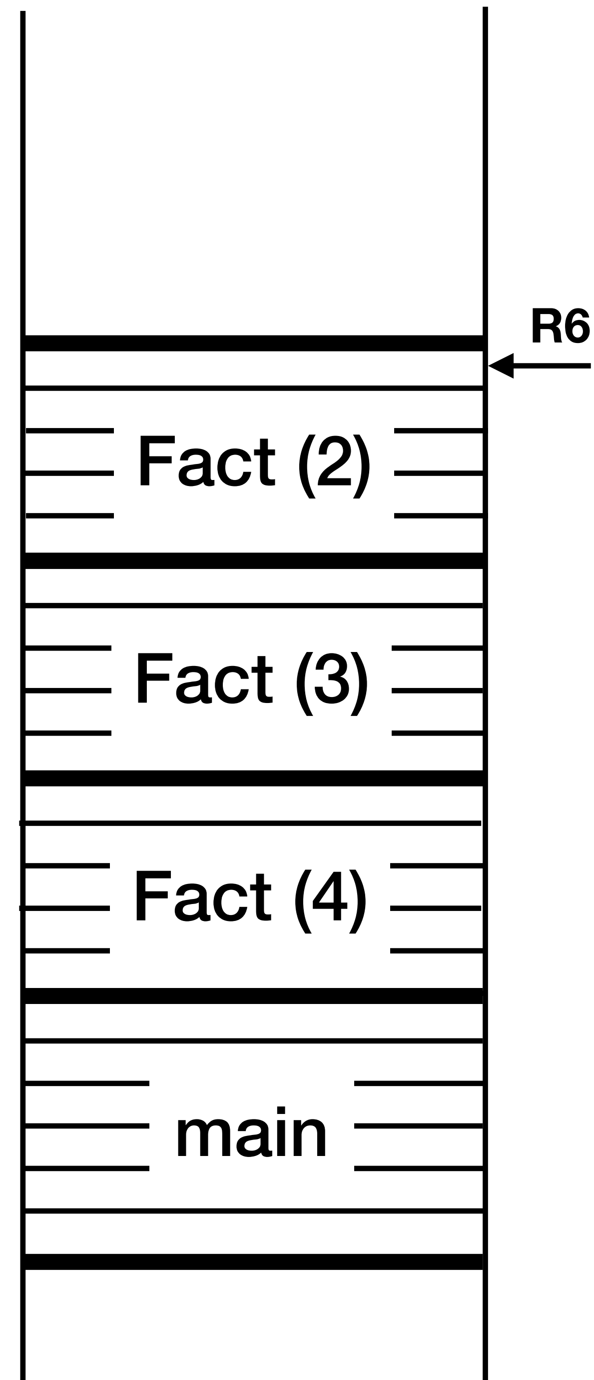


RTS During Execution of Factorial

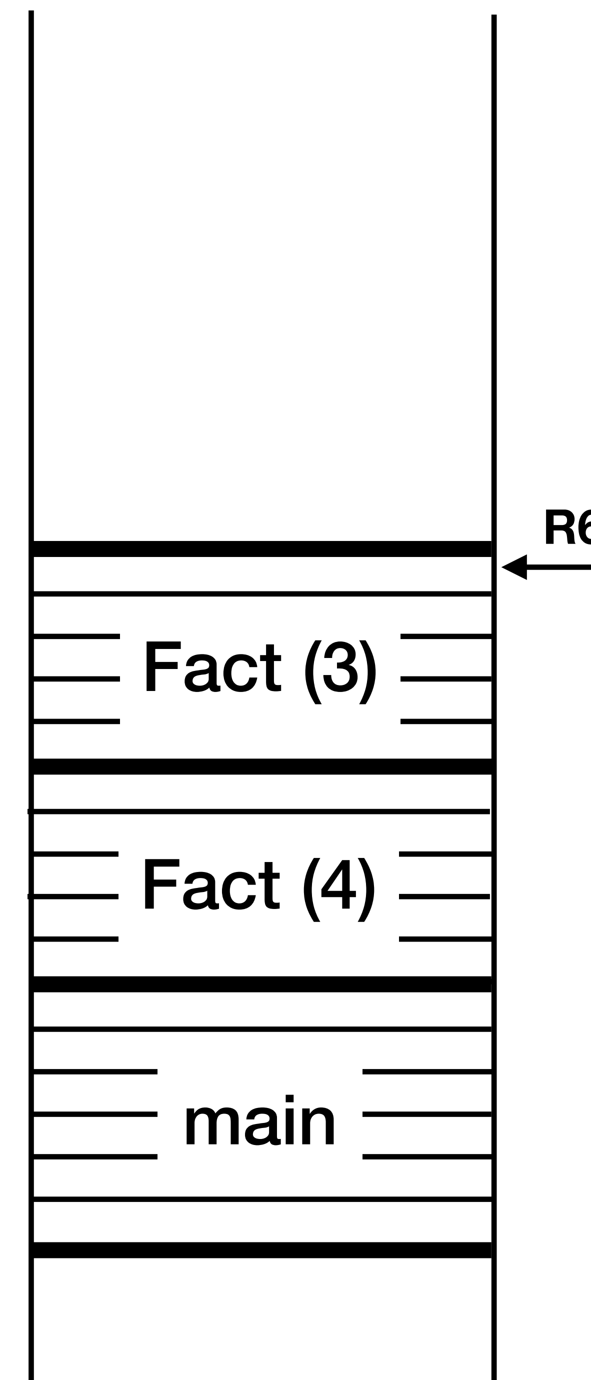
Factorial(1) returns 1



Factorial(2) returns 2 x 1



Factorial(3) returns 3 x 2



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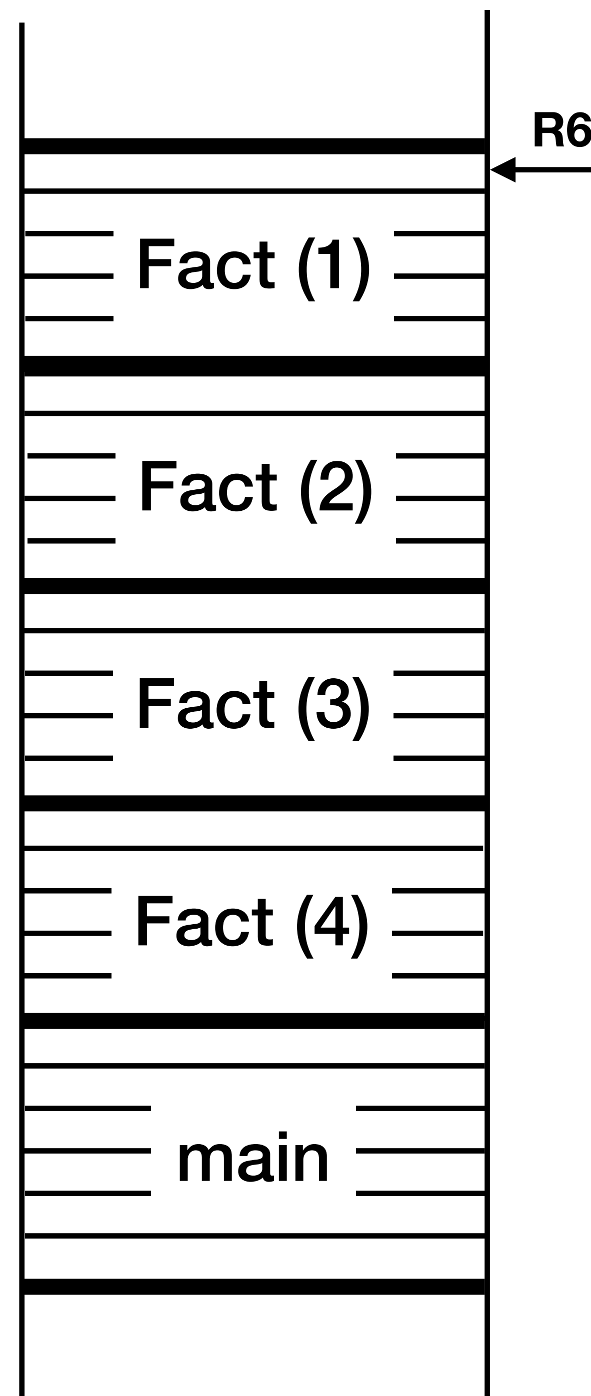

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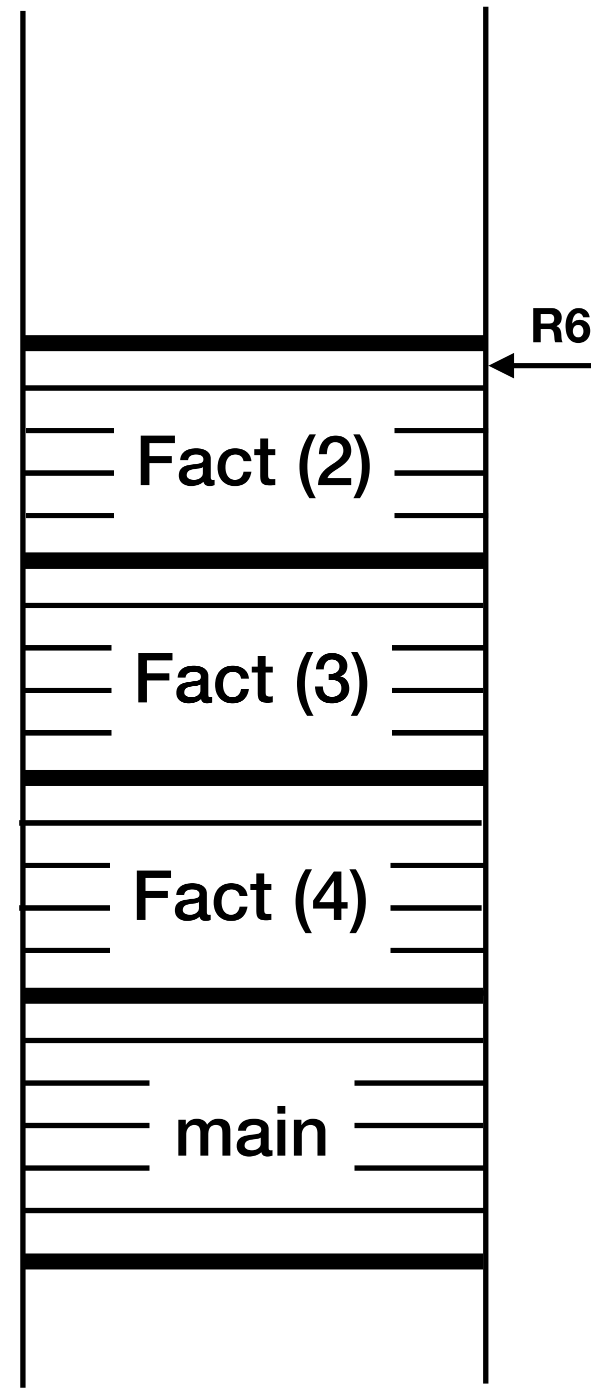

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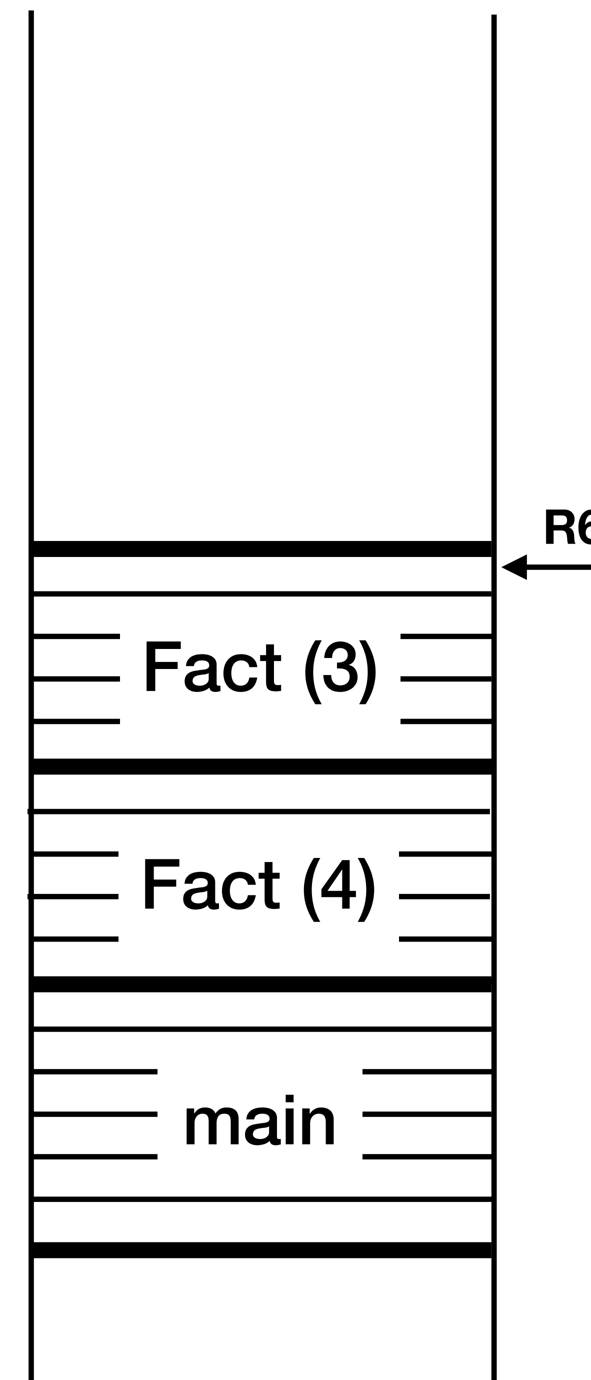
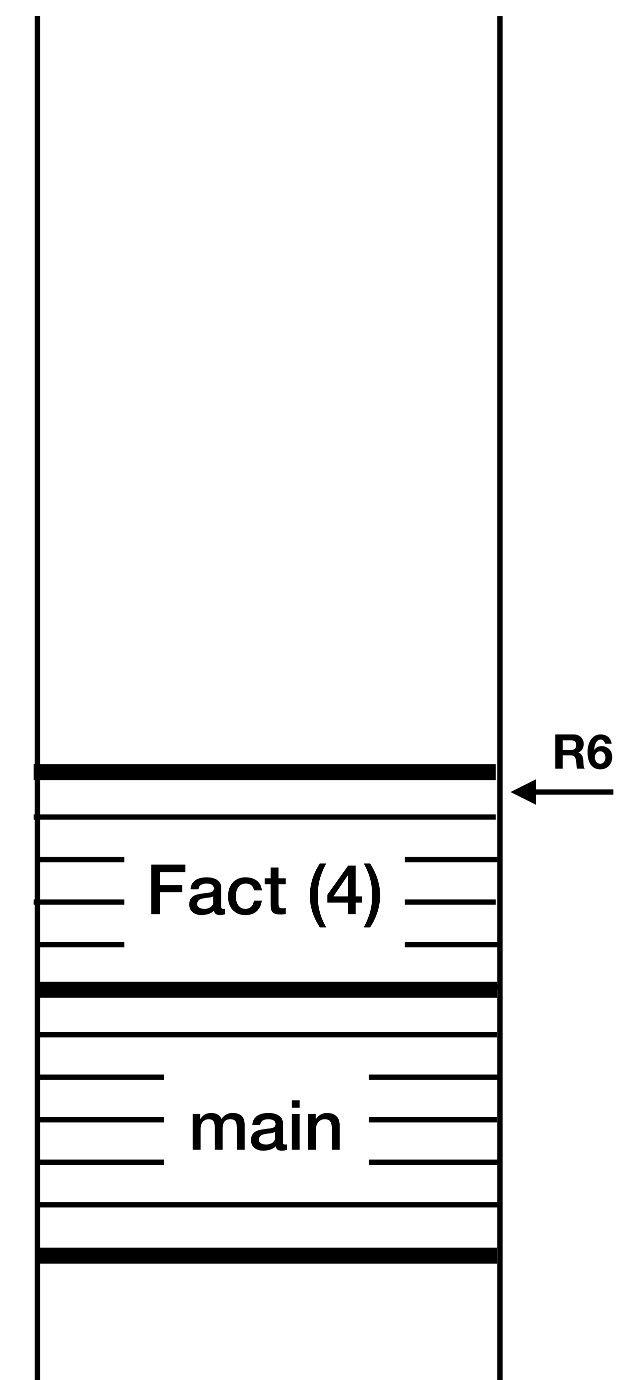


Factorial(4) returns 4 x 6



Example: Fibonacci series

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Mathematical definition:

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= (Fibonacci(1) + Fibonacci(0)) + Fibonacci(1)

= 1 + 1 + 1 = 3

Example: Fibonacci series

Example: Fibonacci series

```
int Fibonacci (int n) {
    int sum;

    if (n == 0 || n == 1)
        return 1;
    else {
        sum = (Fibonacci(n-1) + Fibonacci(n-2));
        return sum;
        }
}
```


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Example: Binary search

```
int binary(int arr[], int n, int key){
  int start = 0;       // Left pointer
  int end = n - 1; // Right pointer

  while (end >= start){
    int mid = (start + end) / 2; // Pick middle element

    // Logic to focus search on left or right of mid
    if (key == arr[mid])
      return mid;
    else if (key < arr[mid])
      end = mid - 1;
    else
      start = mid + 1;
  }
  return -1; // Loop exited, element not present.
}
```

Can we implement binary search in *recursive* way?

Example: Binary search

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- **Option 1**: Find a mechanism to keep track of the start and end indices across recursive calls; local variables won't do (why?).

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```

Can we implement binary search in *recursive* way?

- **Option 1**: Find a mechanism to keep track of the start and end indices across recursive calls; local variables won't do (why?).
- **Option 2**: Pass in subarrays & their lengths to `binary` itself.

Example: Binary search

Example: Binary search

```
int binary_opt1(int item, int list[], int start, int end){
    int middle = (end + start)/2;
    if (end < start)
      return -1;                      // Did not find key
    else if (_____________________) // Found item!
        return middle;
    else if (____________________)  // Search left half
        return binary_opt1(item, list, start, middle-1);
    else                            // Search right half
        return binary_opt1(__________________________);
}
```

Example: Binary search

Option A

```
int binary_opt1(int item, int list[], int start, int end){
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        return middle;
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        return binary_opt1(item, list, start, middle-1);
    else                                  // Search right half
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}
```


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}
```


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Example: Binary search

```
int binary_opt1(int item, int list[], int start, int end){
    int middle = (end + start)/2;
    if (end < start)
      return -1;                       // Did not find key
    else if (list[middle] == item) // Found item!
        return middle;
    else if (item < list[middle])  // Search left half
        return binary_opt1(item, list, start, middle-1);
    else                           // Search right half
        return binary_opt1(item, list, middle+1, end);
}
```

Example: Binary search

What if we *insist* the user shouldn't have to deal with the start, end pointers etc.?

We can always wrap `binary_opt1` in another function.

Below is *another* option.

```
int main() {
  int arr[] = {2, 3, 5, 7, 9, 11, 13, 17, 19, 23};
  int size = sizeof(arr) / sizeof(arr[0]);
  int index, key = 11;

  int *result = binary_opt2(arr, size, key);
  if (result == NULL)
    printf("Element not found");
  else{
    // Find index based on pointer
    index = (result-arr);
    printf("Element found at index: %d", index);
  }
}
```

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Index must be calculated manually.

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Below is *another* option.

Function returns a pointer

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  int arr[] = {2, 3, 5, 7, 9, 11, 13, 17, 19, 23};
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  int index, key = 11;

  int *result = binary_opt2(arr, size, key);
  if (result == NULL)
    printf("Element not found");
  else{
    // Find index based on pointer
    index = (result-arr);
    printf("Element found at index: %d", index);
  }
}
```

Index must be calculated manually.

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Example: Binary search

What if we *insist* the user shouldn't have to deal with the start, end pointers etc.?

```
int * binary(int arr[], int n, int key){
  int mid;
  if (n > 1){ // If not singleton do search
    mid=n/2;
    if (key == arr[mid]) // Check middle
      return &arr[mid];
    else if (key > arr[mid]) // Search right 1/2
      return binary(&arr[mid+1], n-mid, key);
    else
      return binary(arr, mid, key); // Left 1/2
  }
  else if (arr[0]==key) // If singleton check equality
    return arr;
  else // Didn't find element
    return NULL;
}
```

We can always wrap `binary_opt1` in another function.

Below is *another* option.

Function returns a pointer

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int main() {
  int arr[] = {2, 3, 5, 7, 9, 11, 13, 17, 19, 23};
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  int index, key = 11;

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  if (result == NULL)
    printf("Element not found");
  else{
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    index = (result-arr);
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}
```

Index must be calculated manually.

Example: Quicksort

- We already saw Quicksort last time and remarked it was recursive …
- Recall the steps ….

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Example: Quicksort

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1. Choose first element of given array as pivot.
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    1. If pointers cross/overlap, **split array** & **recurse** on each subarray.

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6. Repeat 3-5 while left pointer < right pointer.

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We will write one function to split array

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And another which will perform recursion after split

We will write one function to split array

Implementation

```
void Swap(int* one, int* two){
  int temp = *one;
  *one = *two;
  *two = temp;
}

void QuickSort(int arr[], int start, int end){
  if (start < end){
    int pivotVal = partition(arr, start, end);

    // Now sort left half
    QuickSort(arr, start, ________);

    // And right half
    QuickSort(arr, ____________, end);
  }
}
```

```
int partition(int arr[], int start, int end){

  int pivotVal = ___________;
  int i = __________; // Initialize left
  int j = ________;   // Initialize right

  while(1){
    do i++; // Increment left till …
    while (_________________);

    do j--; // Decrement right till …
    while (_________________);

    if (______) // If overlap need to split
      return j;

    Swap(&arr[i], &arr[j]);
  }
}
```

Check Gitlab for reference material: https://gitlab.engr.illinois.edu/itabrah2/ece220-sp24

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  int pivotVal = arr[start];
  int i = start - 1; // Initialize left
  int j = end + 1;   // Initialize right

  while(1){
    do i++; // Increment left till …
    while (arr[i] < pivotVal);

    do j--; // Decrement right till …
    while (arr[j] > pivotVal);

    if (i >= j) // If overlap need to split
      return j;

    Swap(&arr[i], &arr[j]);
  }
}
```

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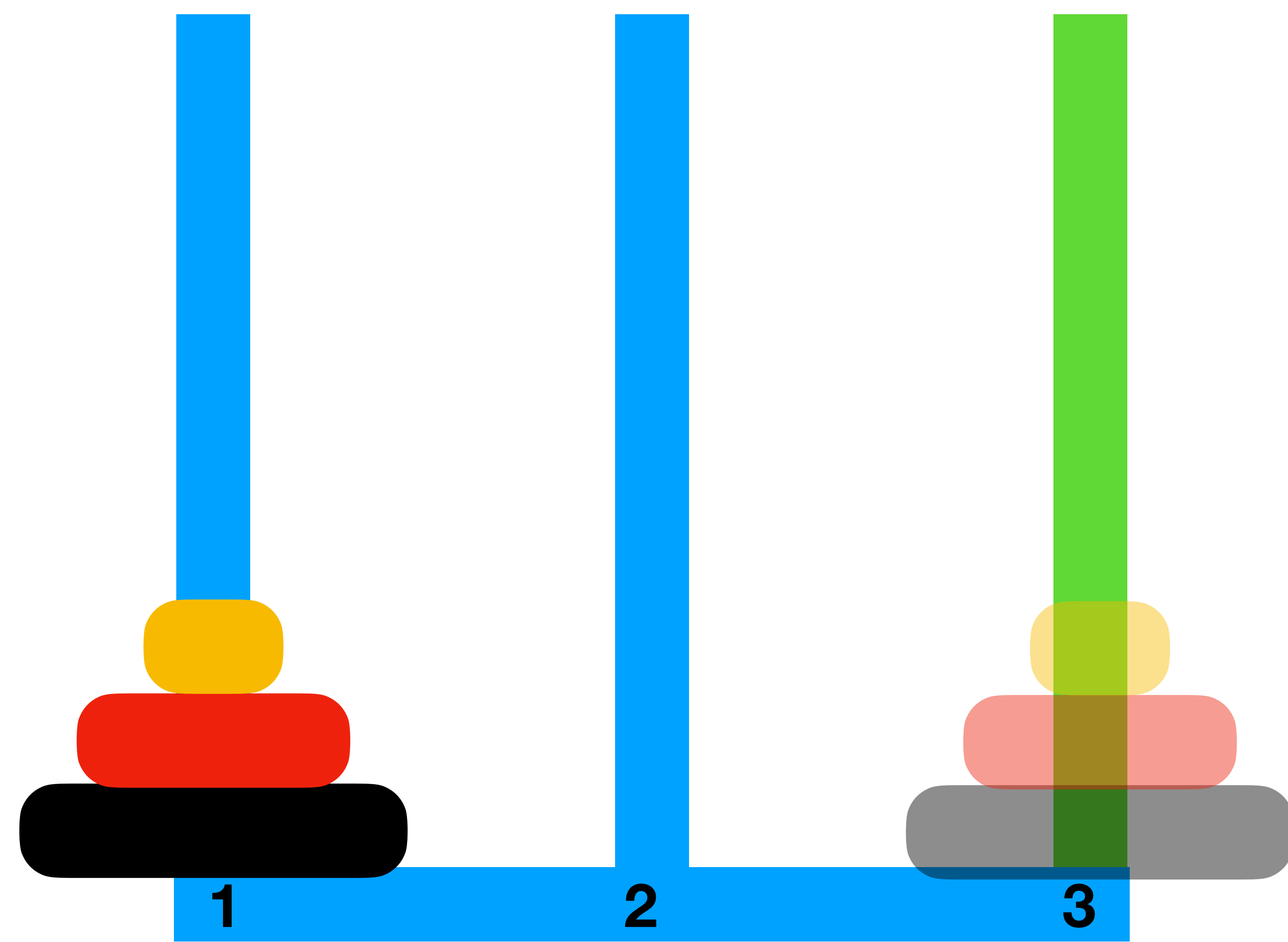
Example: Towers of Hanoi



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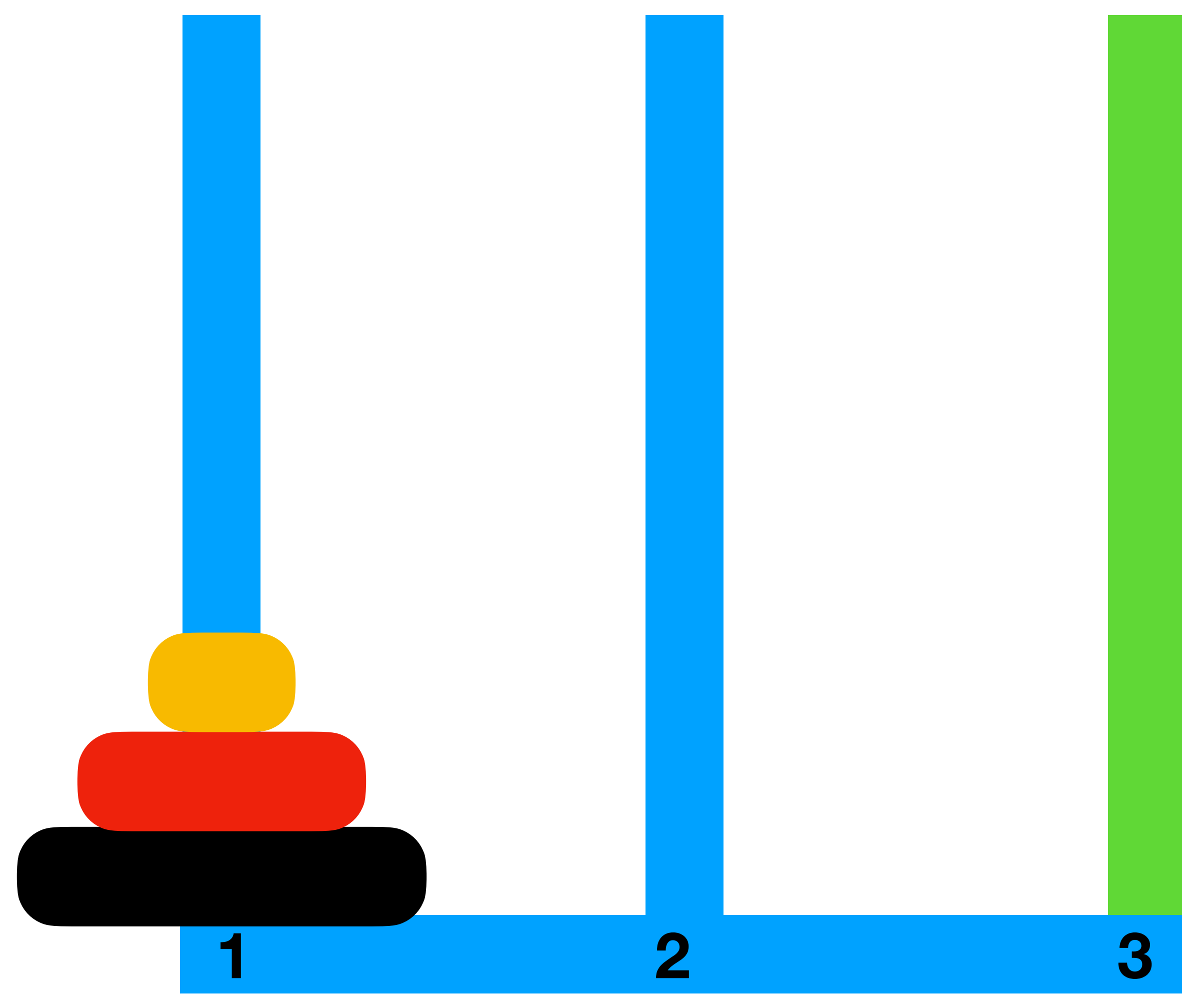

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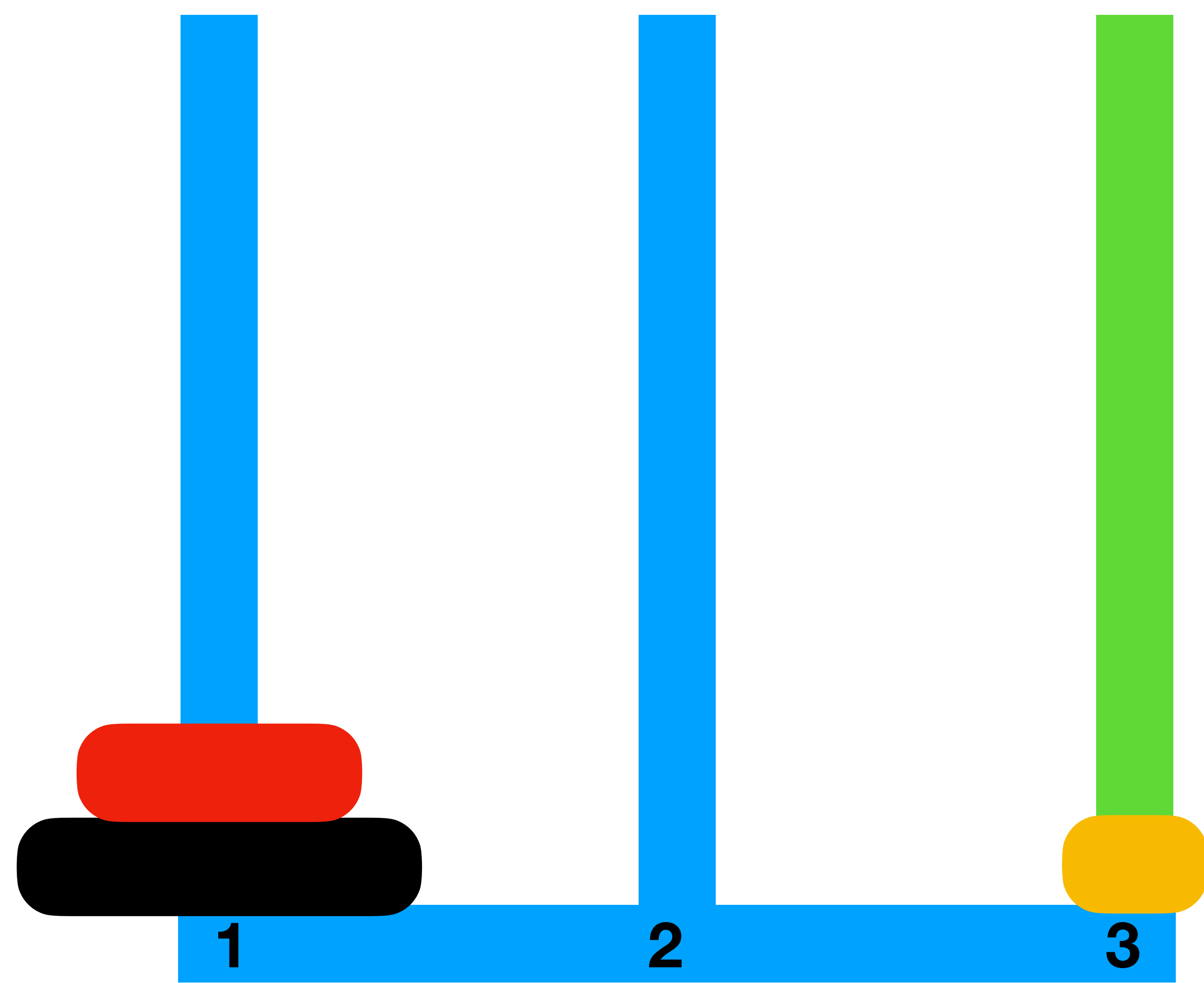

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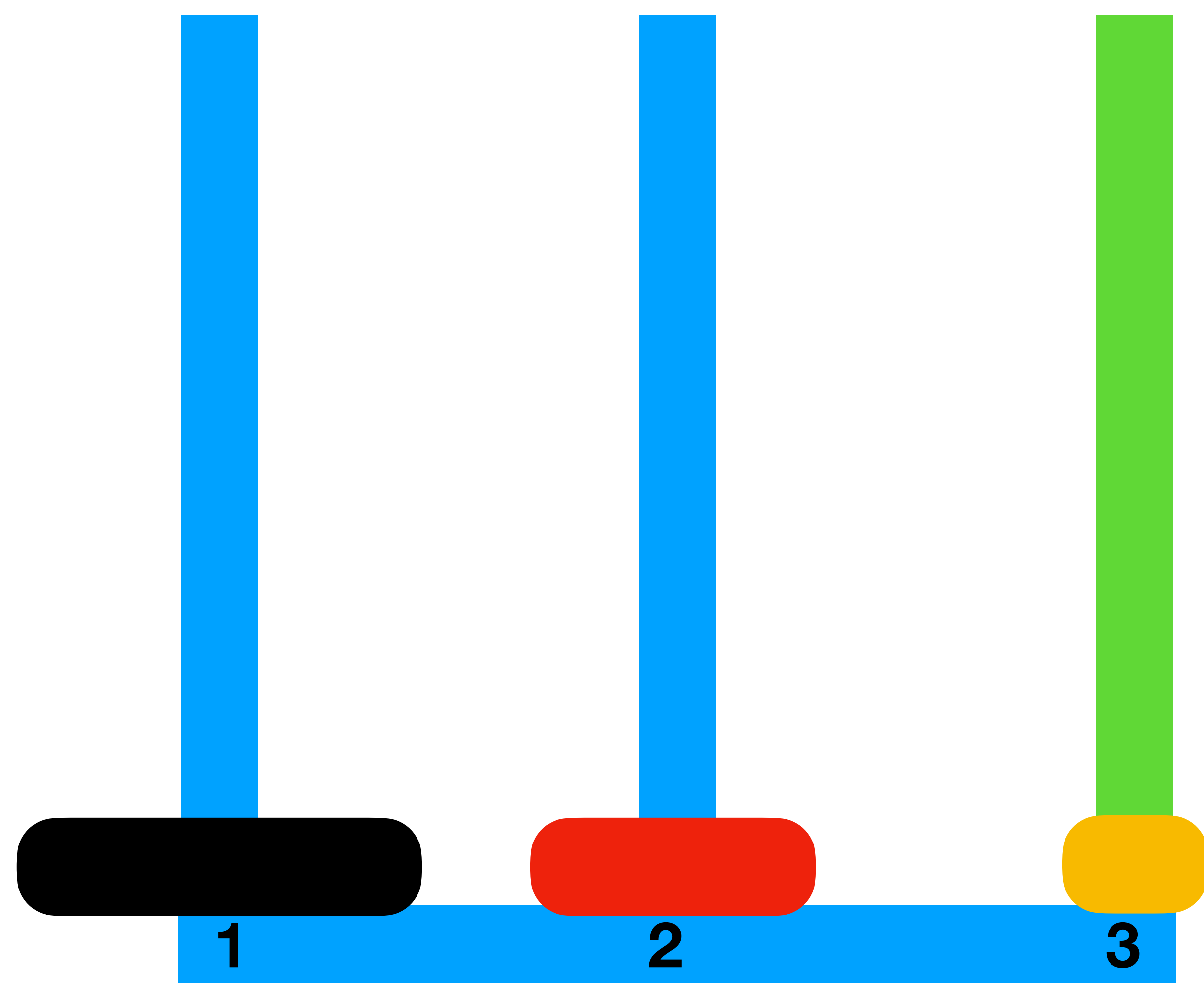

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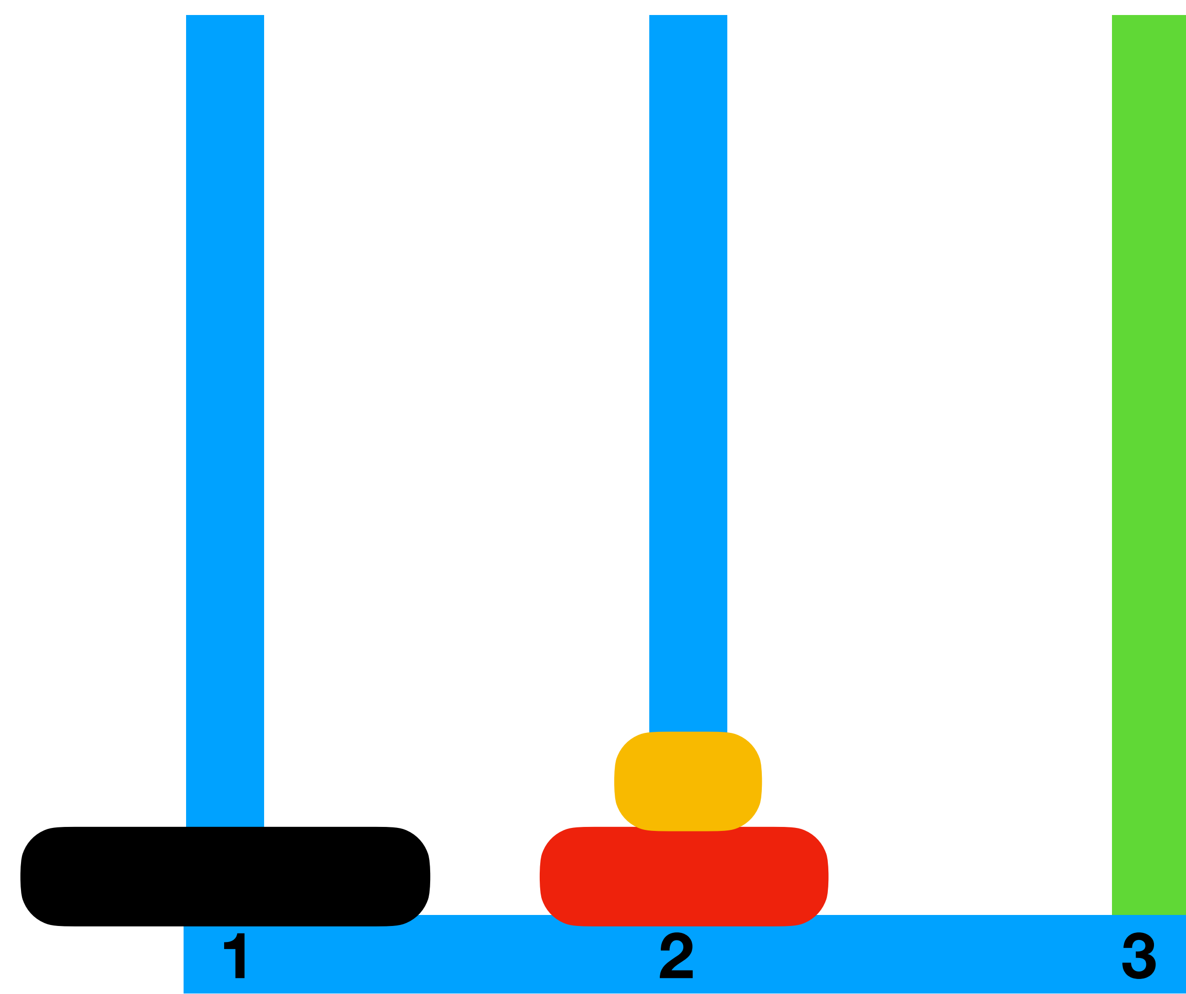

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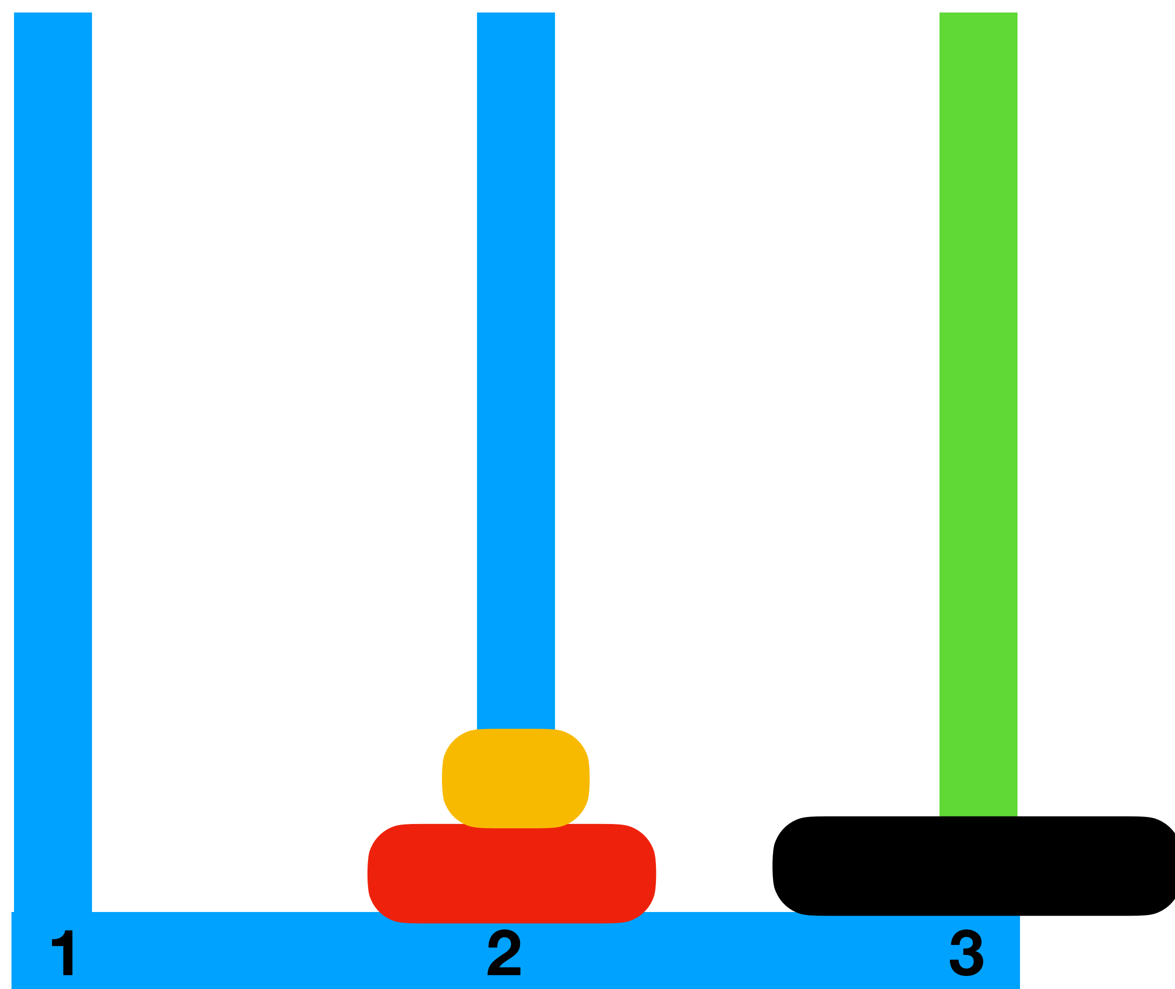

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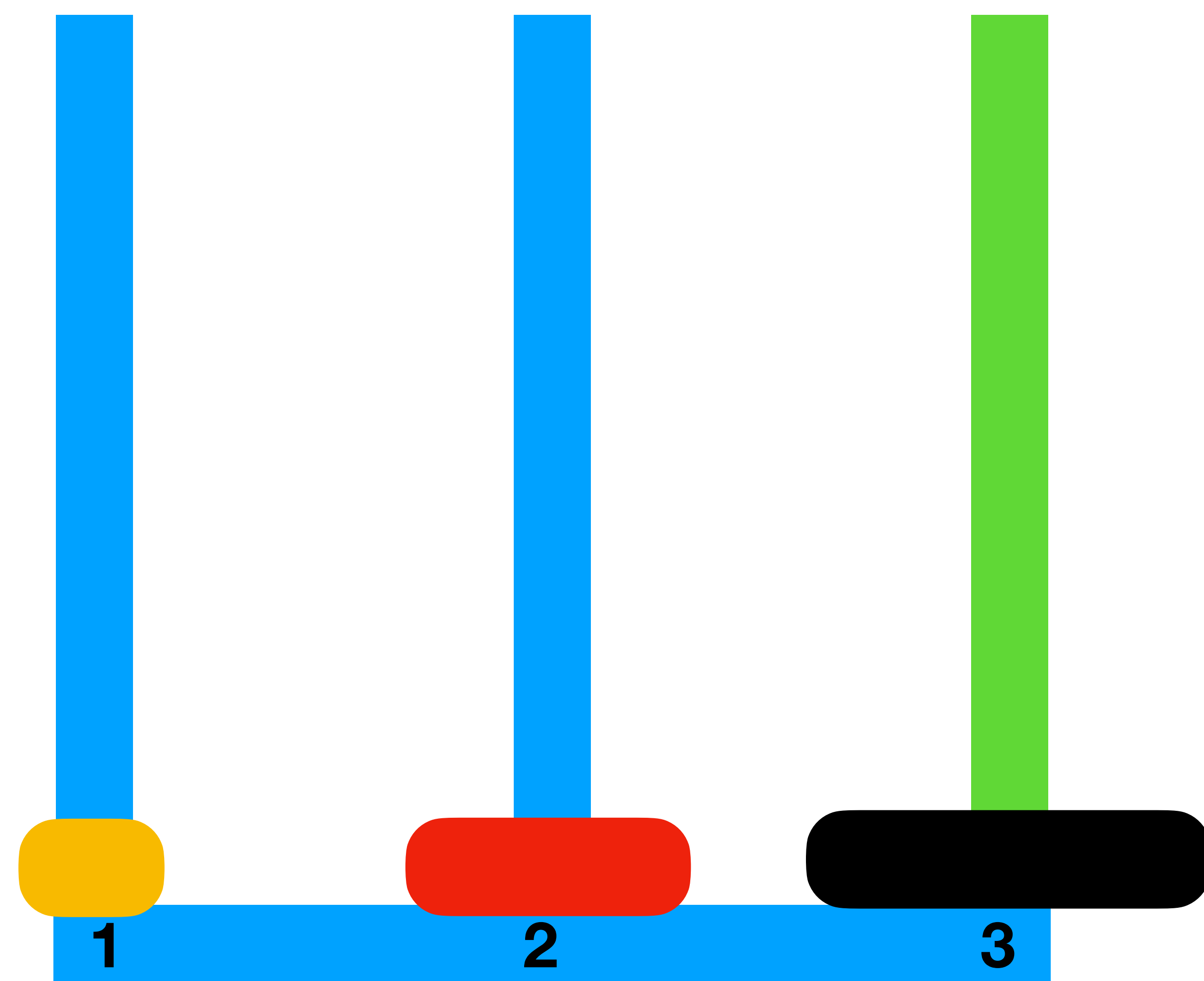

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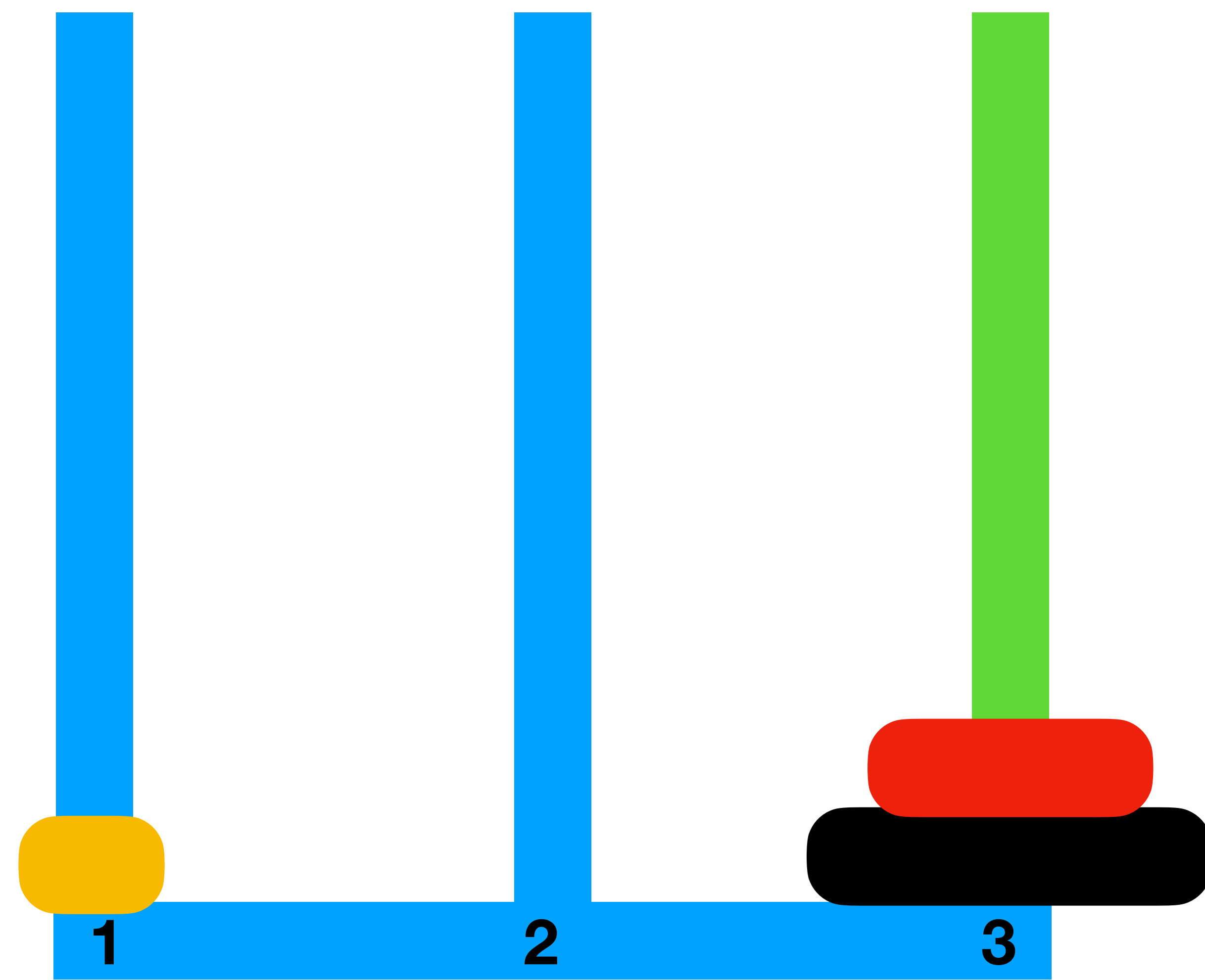

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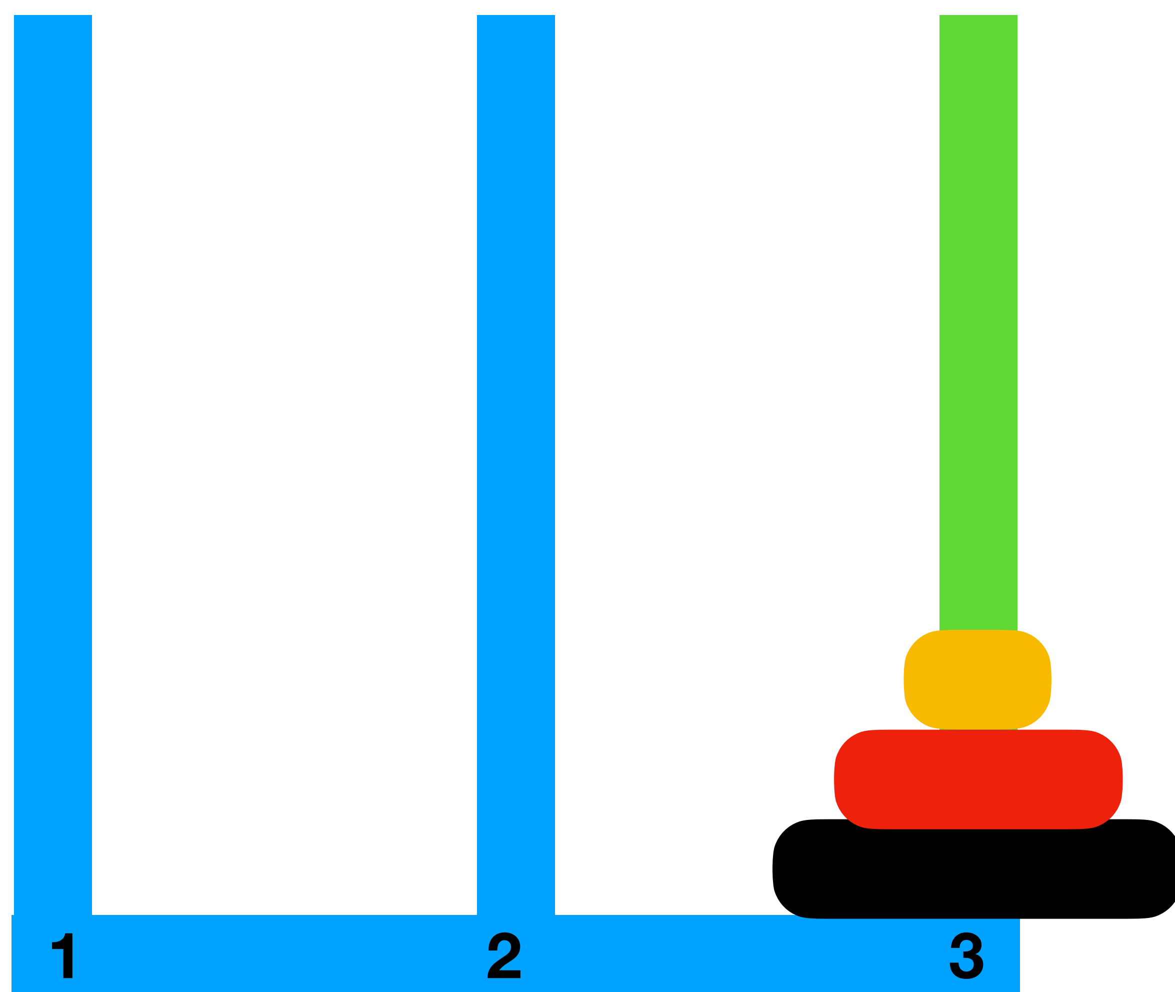

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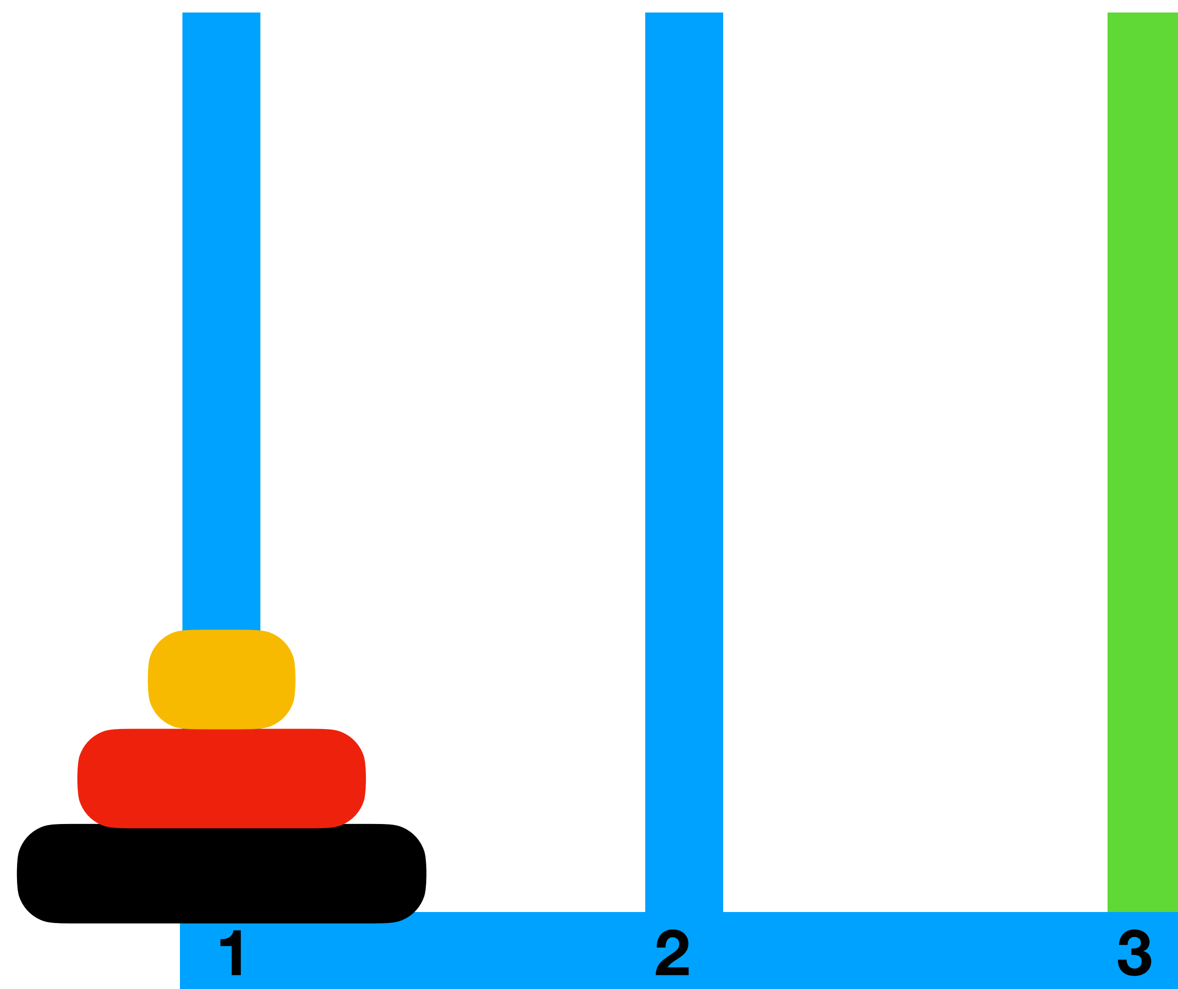

move(diskStacks, src, dest, temp) // Move 2+ disks
dmove(disk_num, src, dest) // Move single disk

move(disks3, r1, r3, r2)

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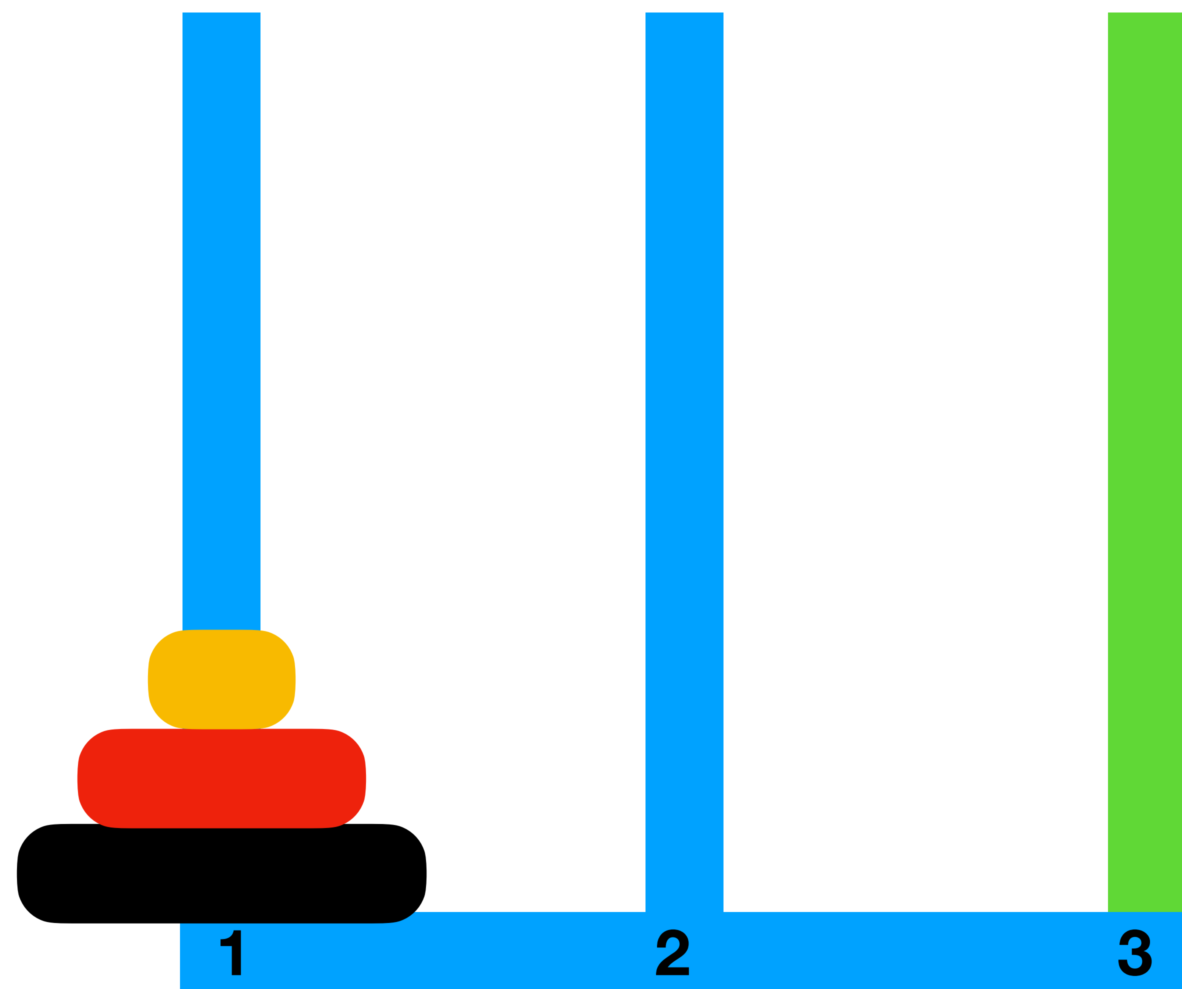

move(disks3, r1, r3, r2)

move(disks2, r1, r2, r3)

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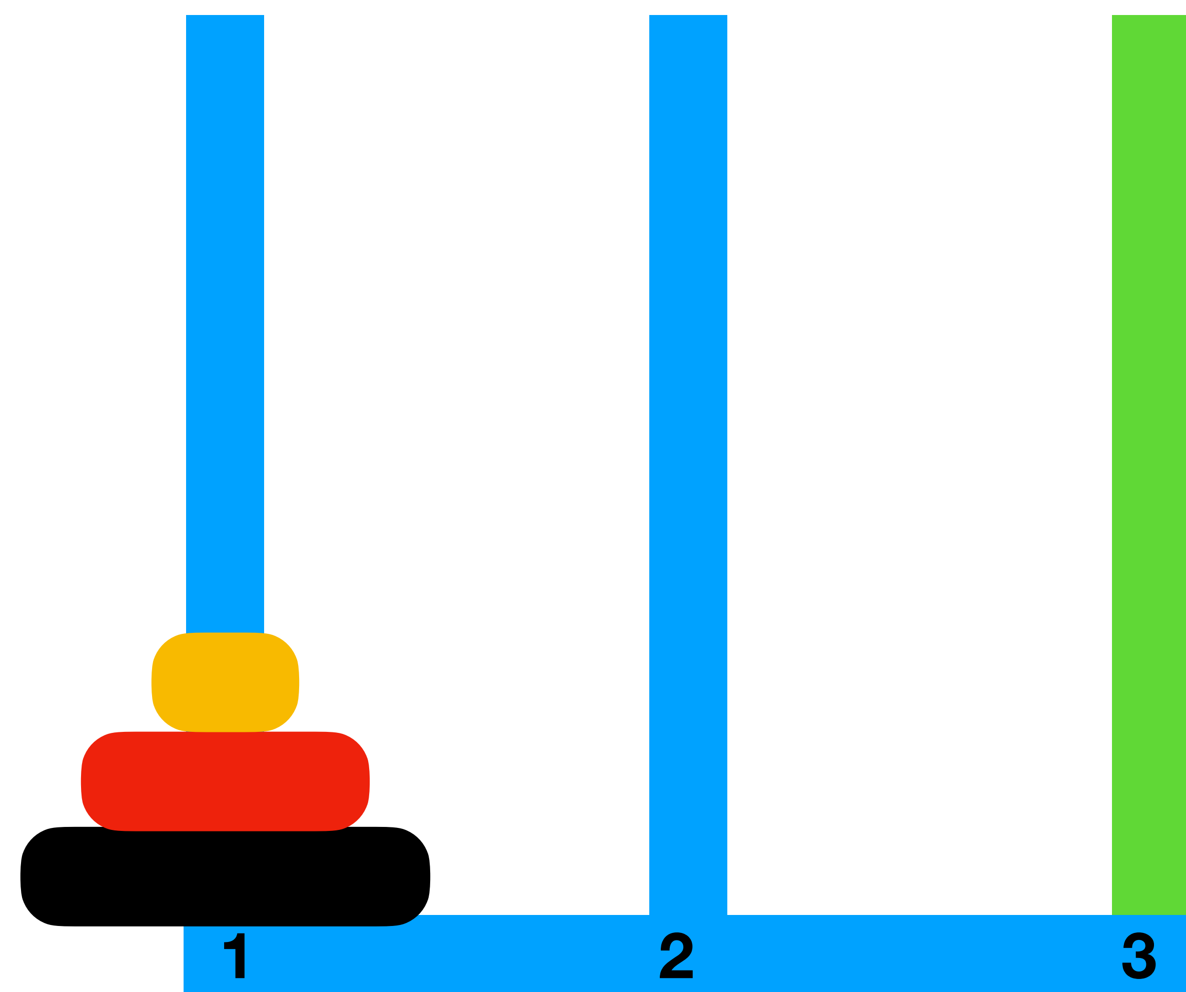
Example: Towers of Hanoi



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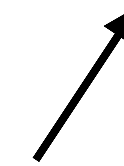
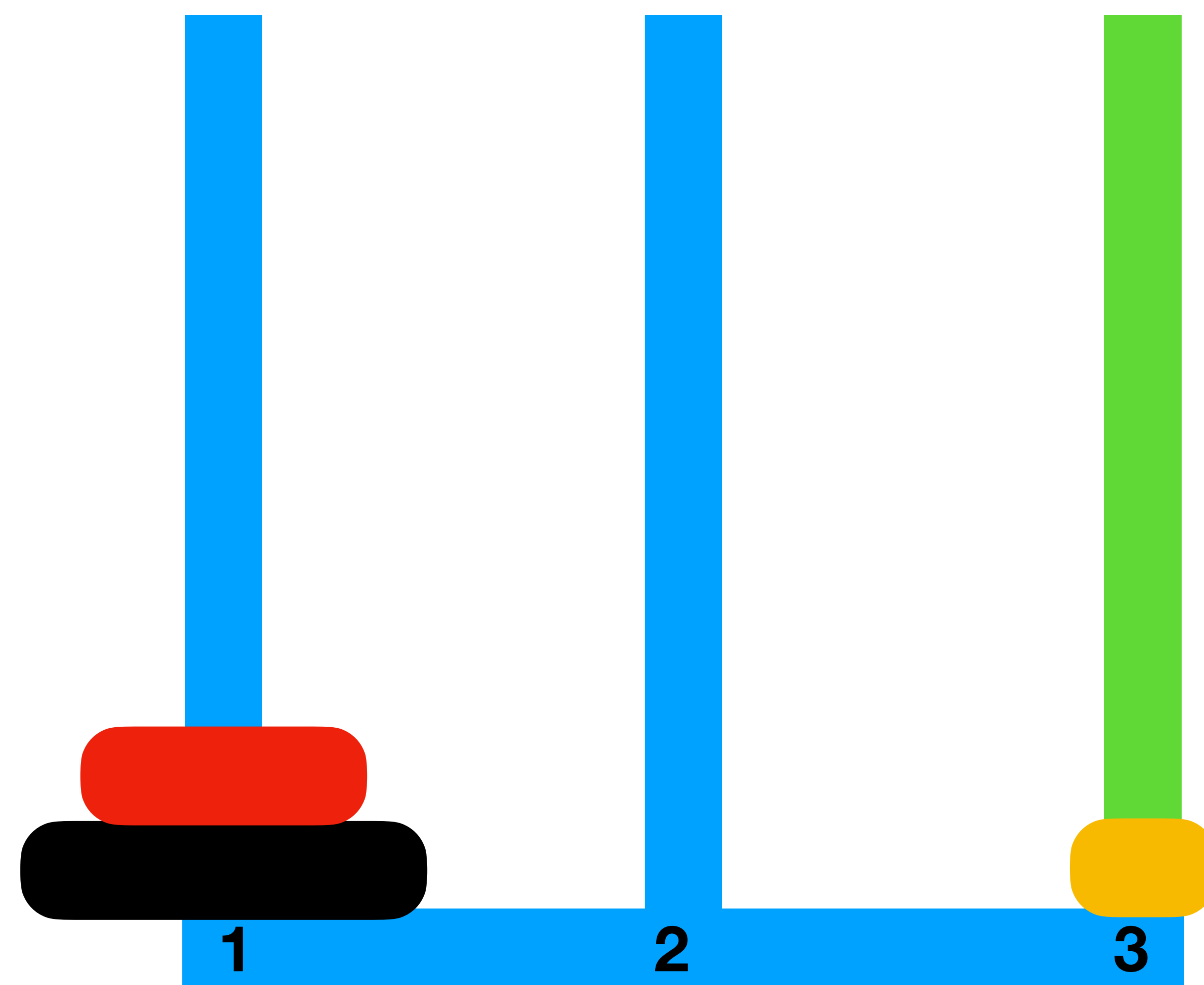


dmove(disk_1, r1, r3)

Example: Towers of Hanoi



move(diskStacks, src, dest, temp) // Move 2+ disks
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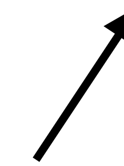
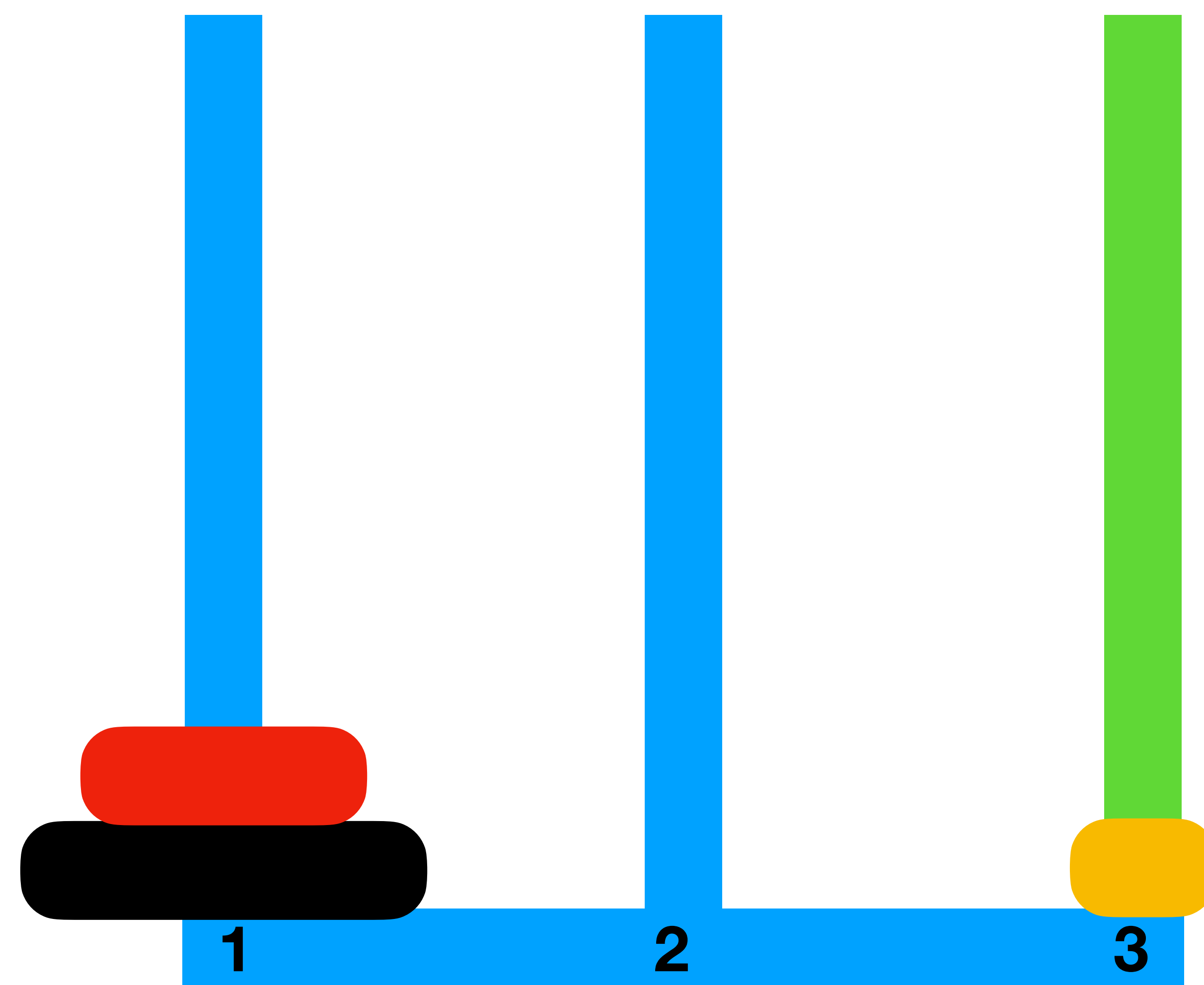


move(disks2, r1, r2, r3)

Example: Towers of Hanoi



move(diskStacks, src, dest, temp) // Move 2+ disks
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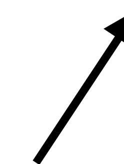
move(disks3, r1, r3, r2)

move(disks2, r1, r2, r3)



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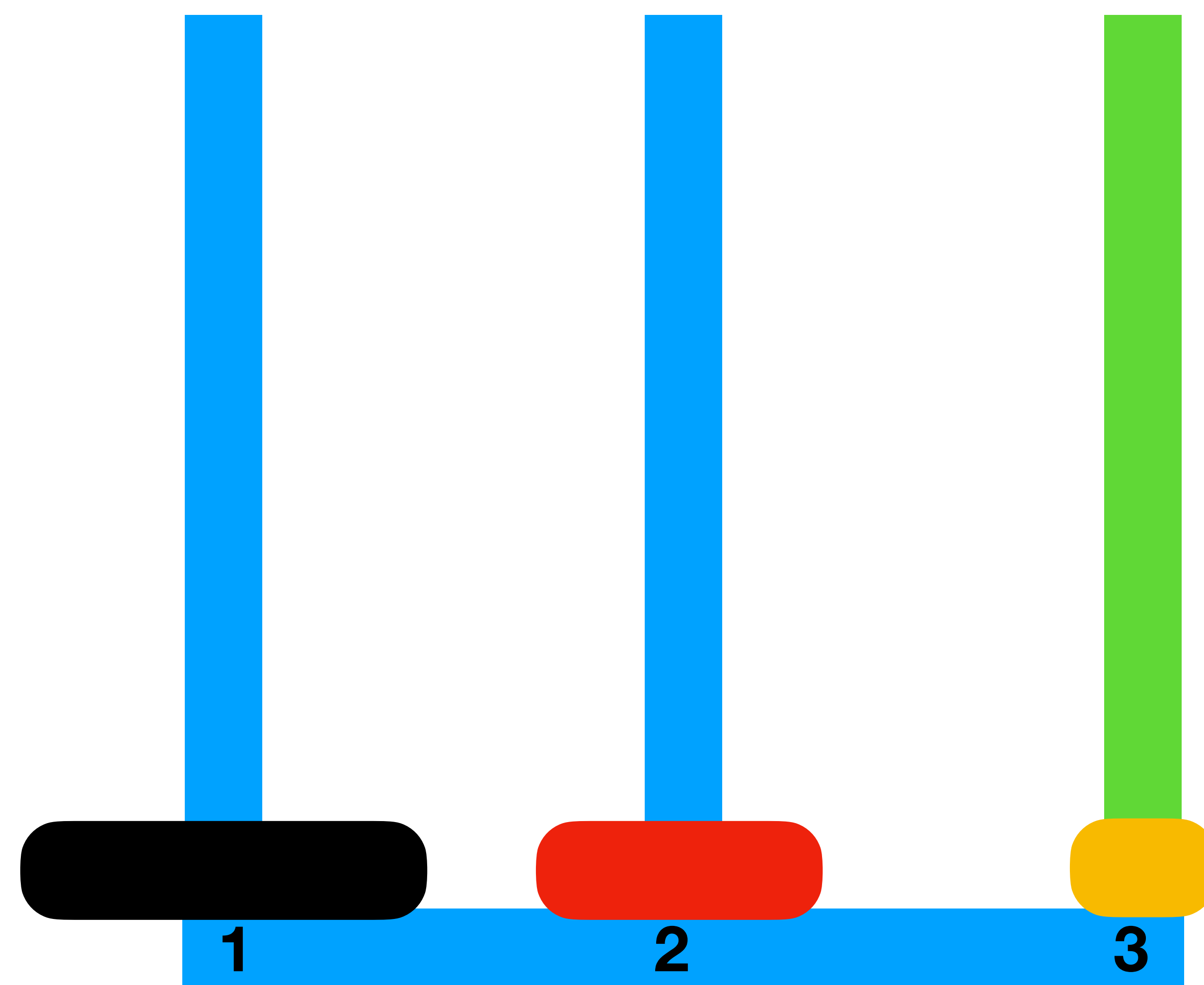

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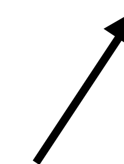

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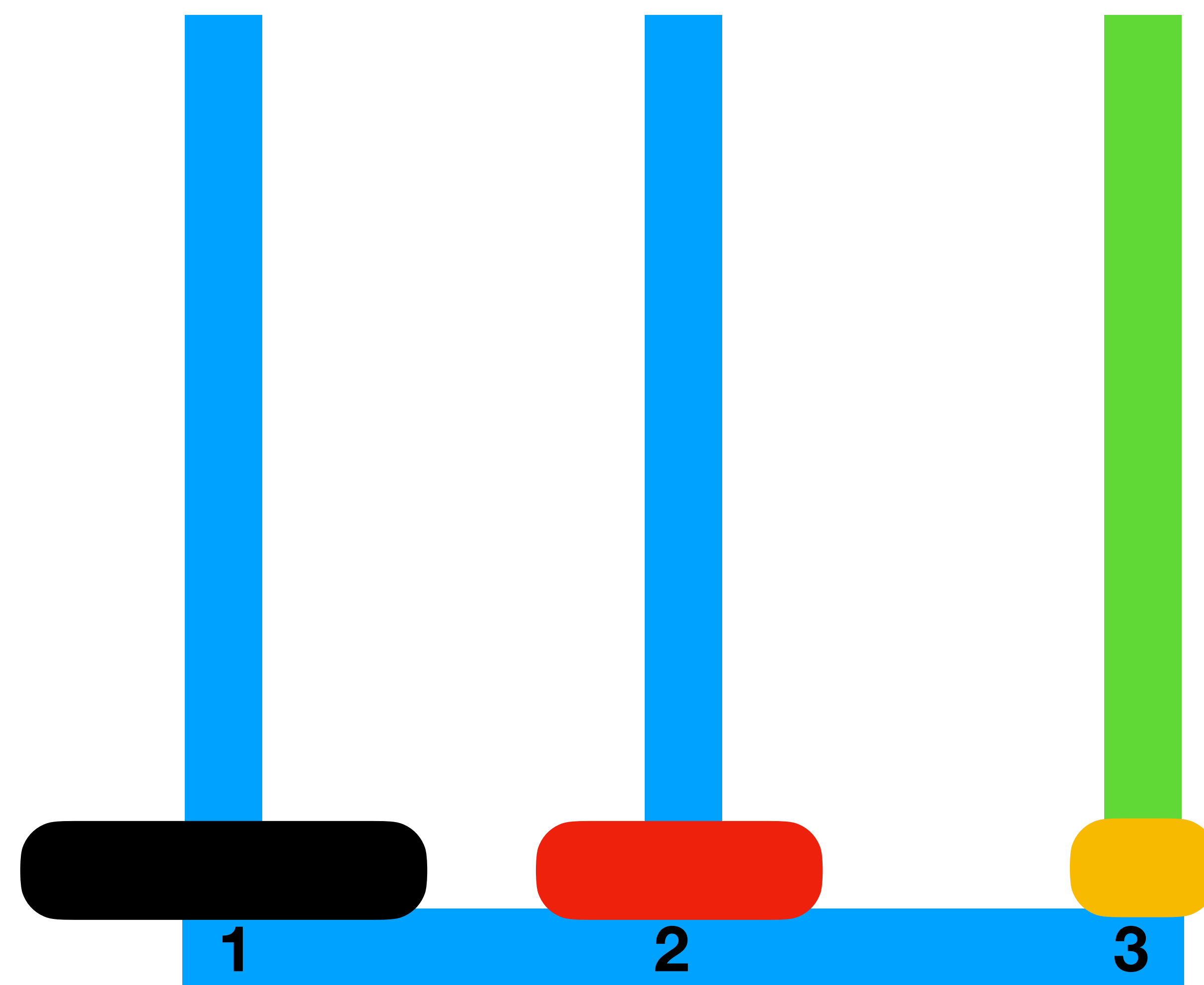

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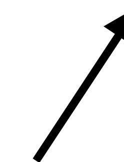
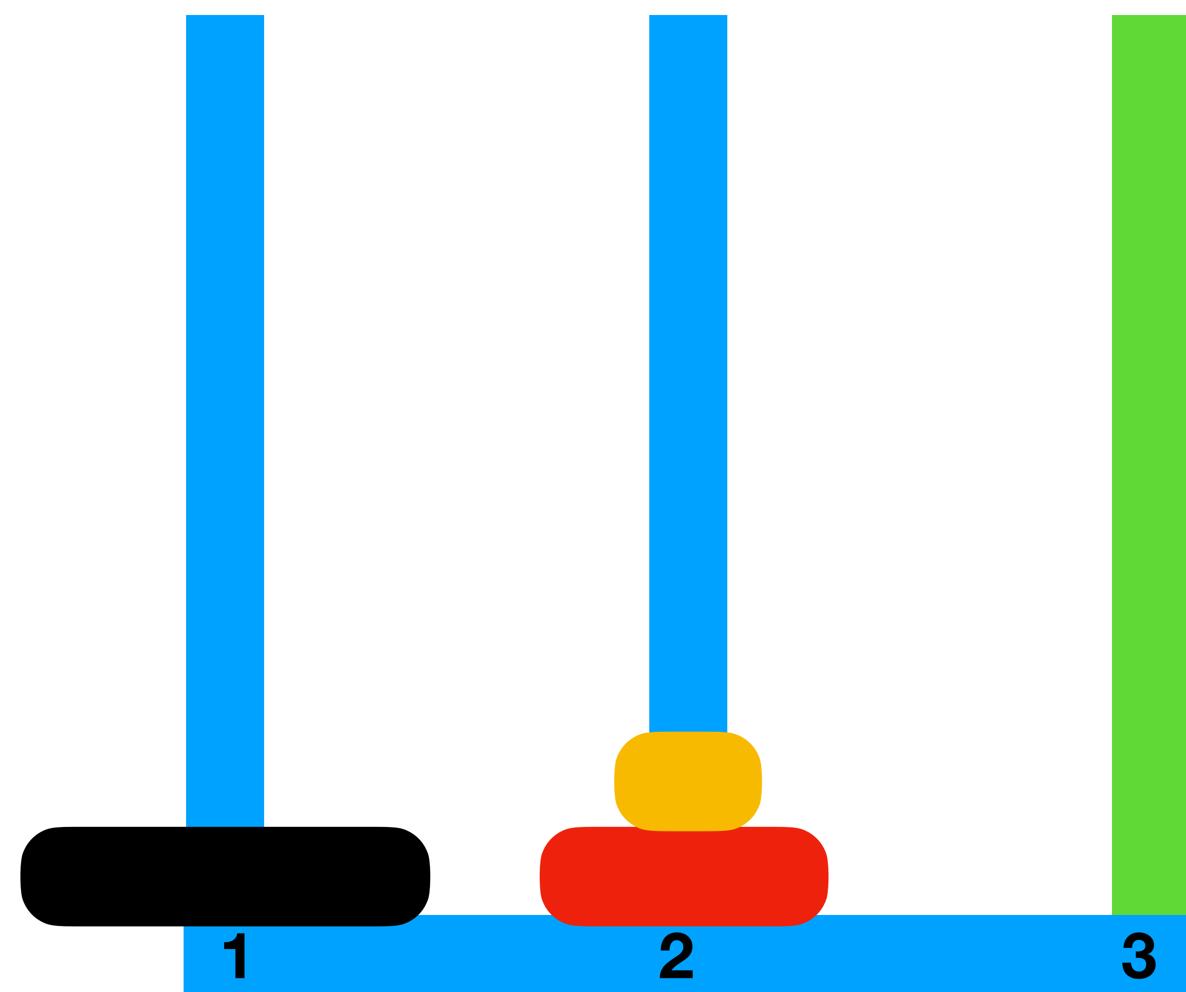
move(disks3, r1, r3, r2)

move(disks2, r1, r2, r3)



dmove(disk_1, r1, r3)
dmove(disk_2, r1, r2)
dmove(disk_1, r3, r2)

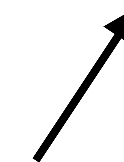
Example: Towers of Hanoi



move(diskStacks, src, dest, temp) // Move 2+ disks
dmove(disk_num, src, dest) // Move single disk

move(disks3, r1, r3, r2)

move(disks2, r1, r2, r3)



dmove(disk_1, r1, r3)
dmove(disk_2, r1, r2)
dmove(disk_1, r3, r2)

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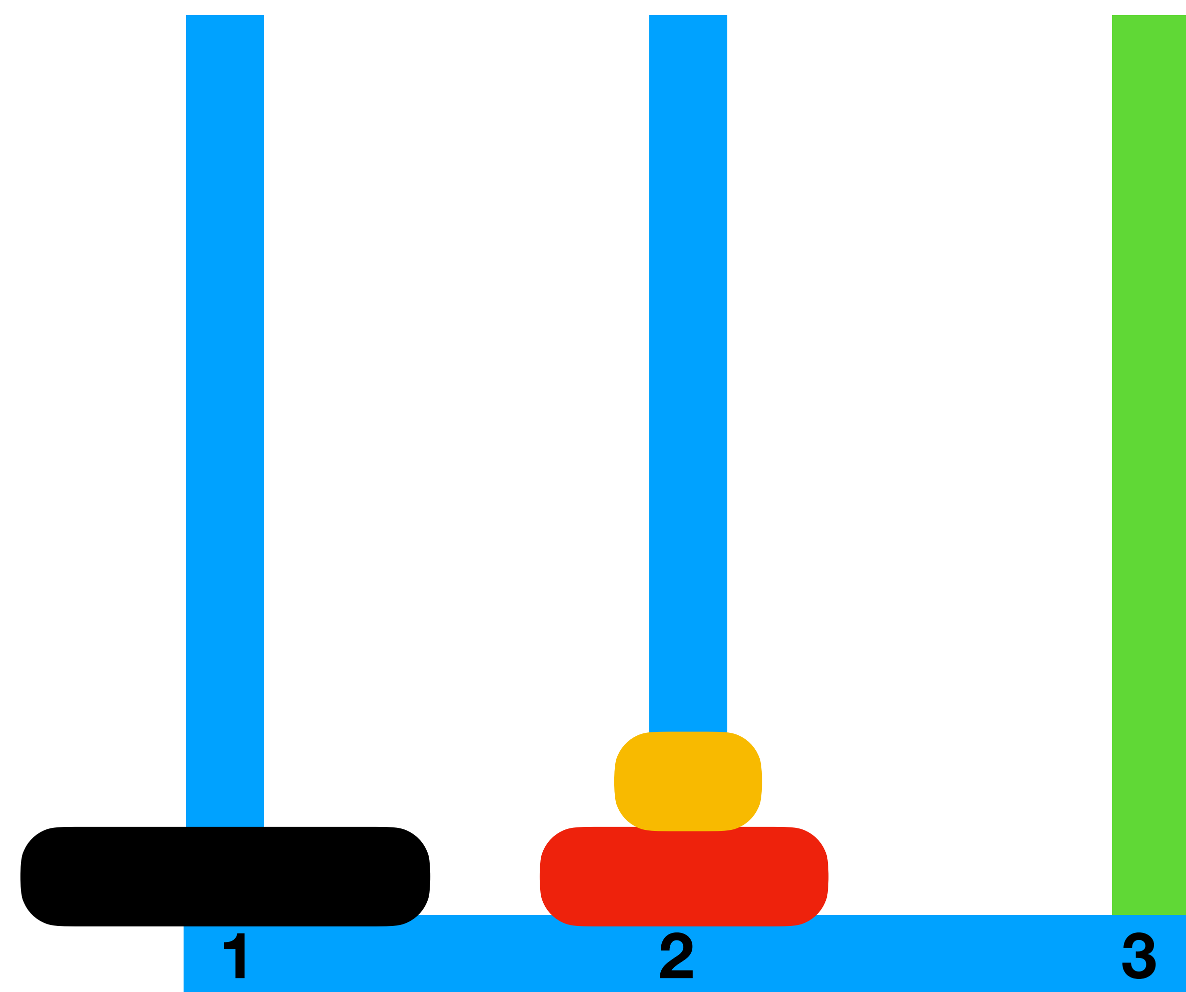
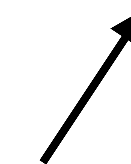
Example: Towers of Hanoi



move(diskStacks, src, dest, temp) // Move 2+ disks
dmove(disk_num, src, dest) // Move single disk

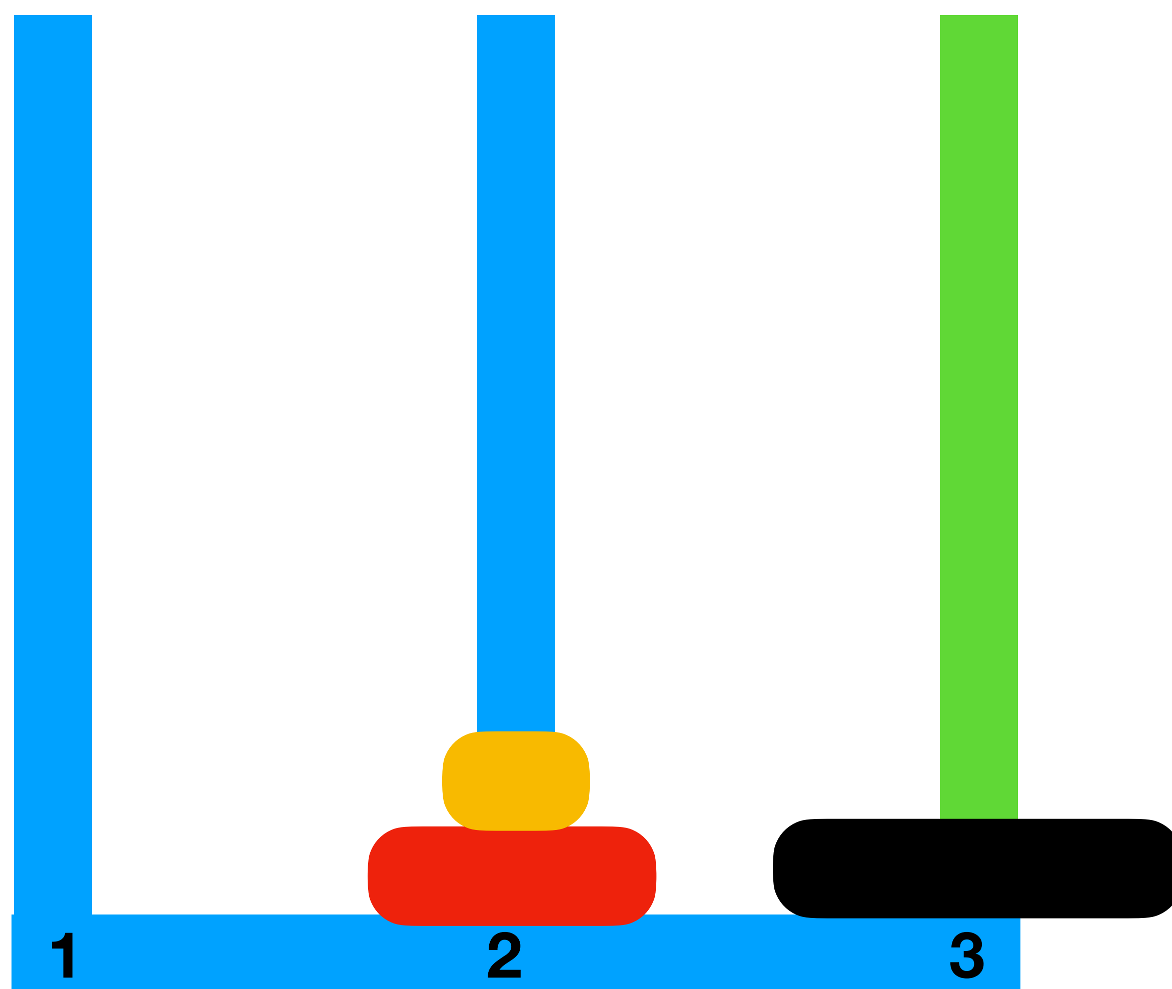
move(disks3, r1, r3, r2)

move(disks2, r1, r2, r3)
+
dmove(disk_3, r1, r3)



dmove(disk_1, r1, r3)
dmove(disk_2, r1, r2)
dmove(disk_1, r3, r2)

Example: Towers of Hanoi



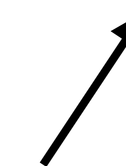
move(diskStacks, src, dest, temp) *// Move 2+ disks*
dmove(disk_num, src, dest) *// Move single disk*

move(disks3, r1, r3, r2)

move(disks2, r1, r2, r3)

+

dmove(disk_3, r1, r3)

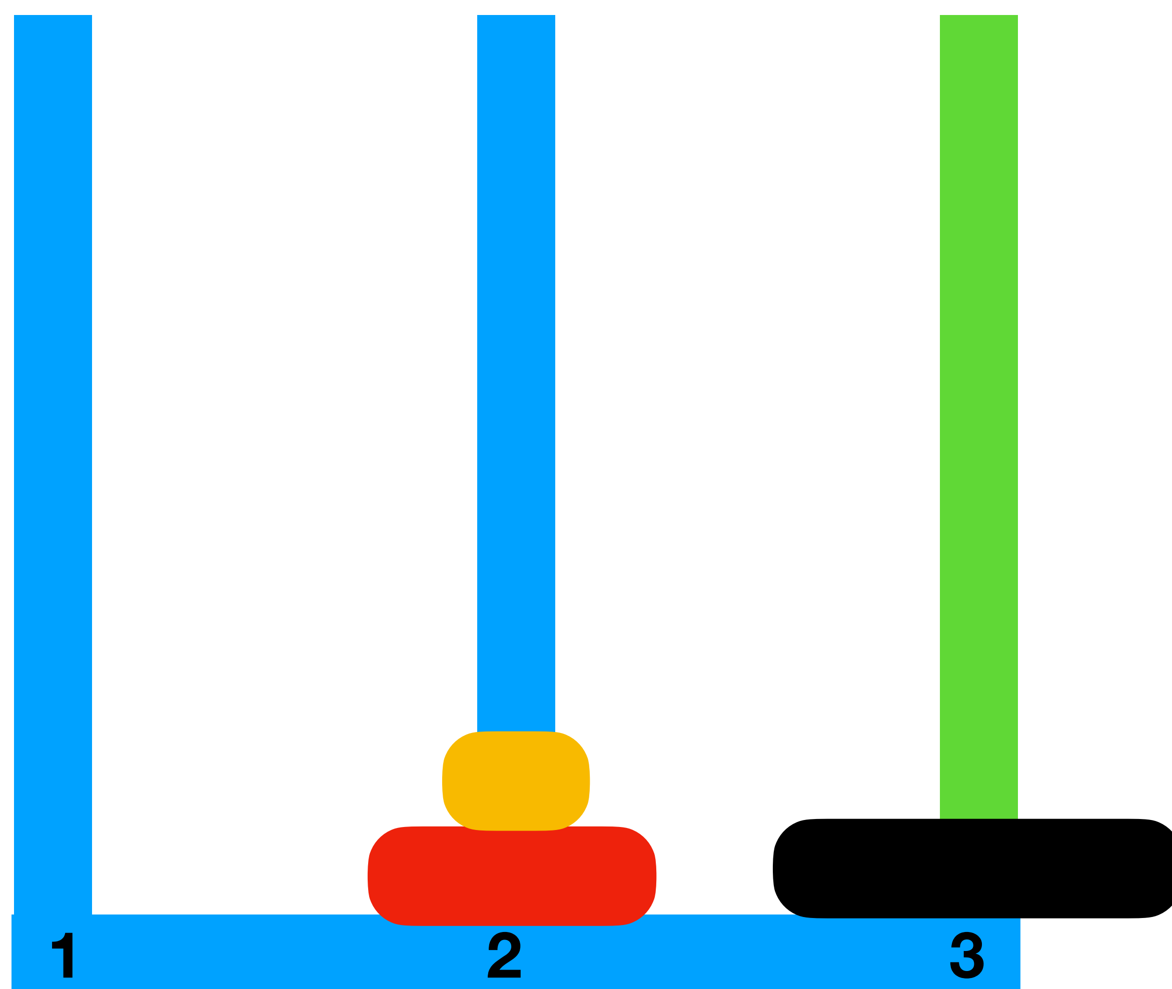


dmove(disk_1, r1, r3)

dmove(disk_2, r1, r2)

dmove(disk_1, r3, r2)

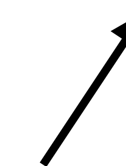
Example: Towers of Hanoi



move(diskStacks, src, dest, temp) // Move 2+ disks
dmove(disk_num, src, dest) // Move single disk

move(disks3, r1, r3, r2)

move(disks2, r1, r2, r3)
+
dmove(disk_3, r1, r3)
+
move(disks2, r2, r3, r1)



dmove(disk_1, r1, r3)
dmove(disk_2, r1, r2)
dmove(disk_1, r3, r2)

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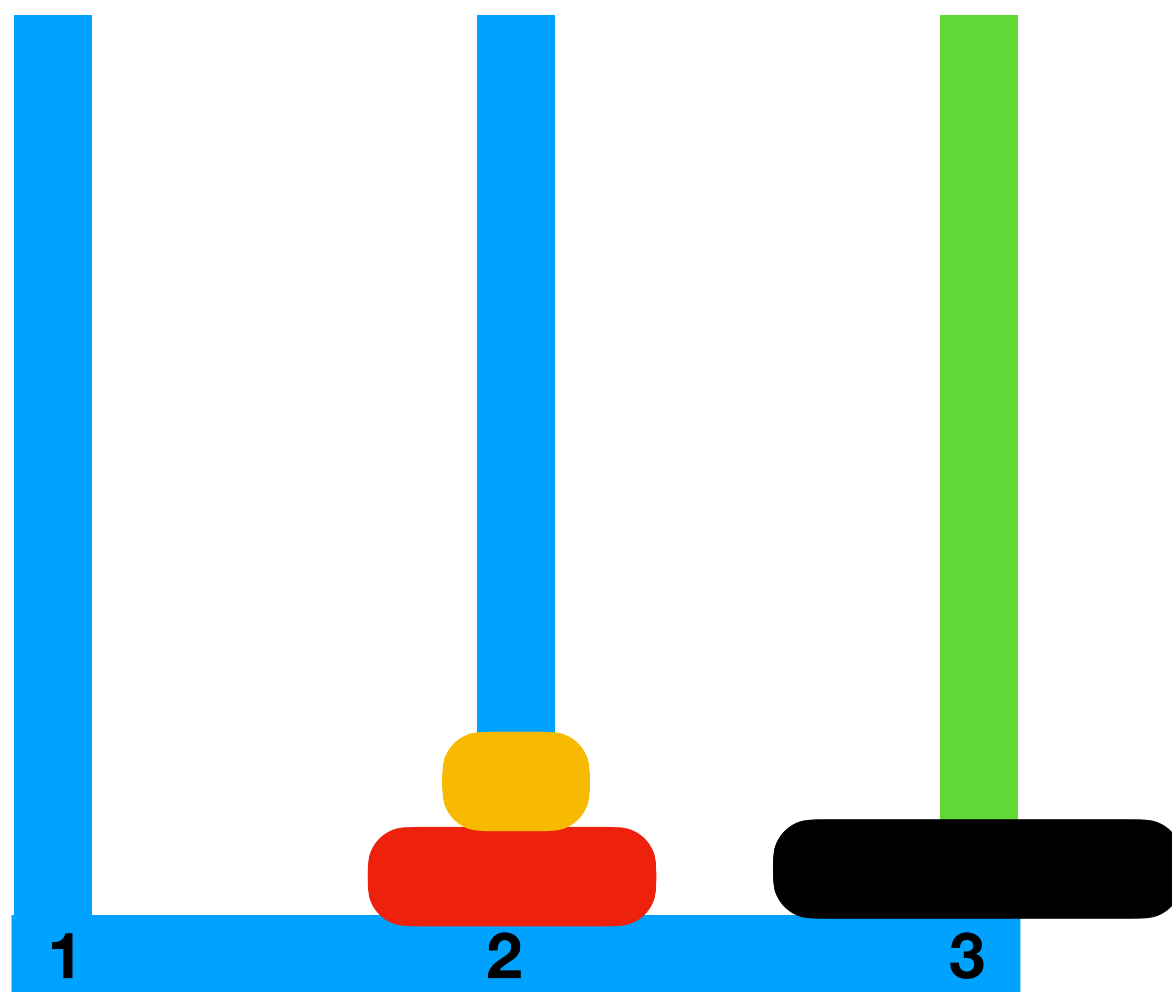
Example: Towers of Hanoi

move(diskStacks, src, dest, temp) // Move 2+ disks
dmove(disk_num, src, dest) // Move single disk



move(disks3, r1, r3, r2)

move(disks2, r1, r2, r3)
+
dmove(disk_3, r1, r3)
+
move(disks2, r2, r3, r1)

dmove(disk_1, r1, r3)
dmove(disk_2, r1, r2)
dmove(disk_1, r3, r2)

dmove(disk_1, r2, r1)

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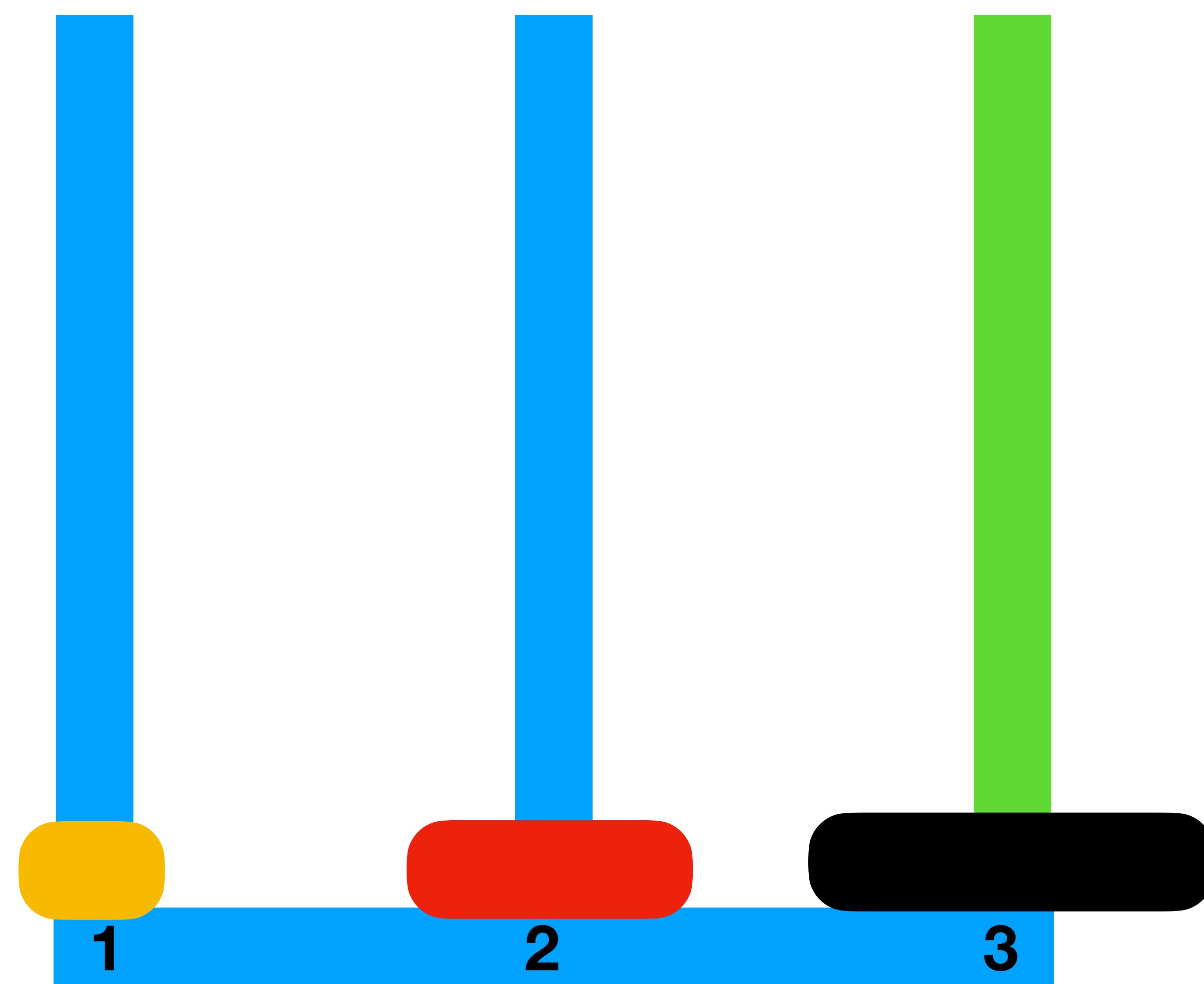
Example: Towers of Hanoi



move(diskStacks, src, dest, temp) // Move 2+ disks
dmove(disk_num, src, dest) // Move single disk

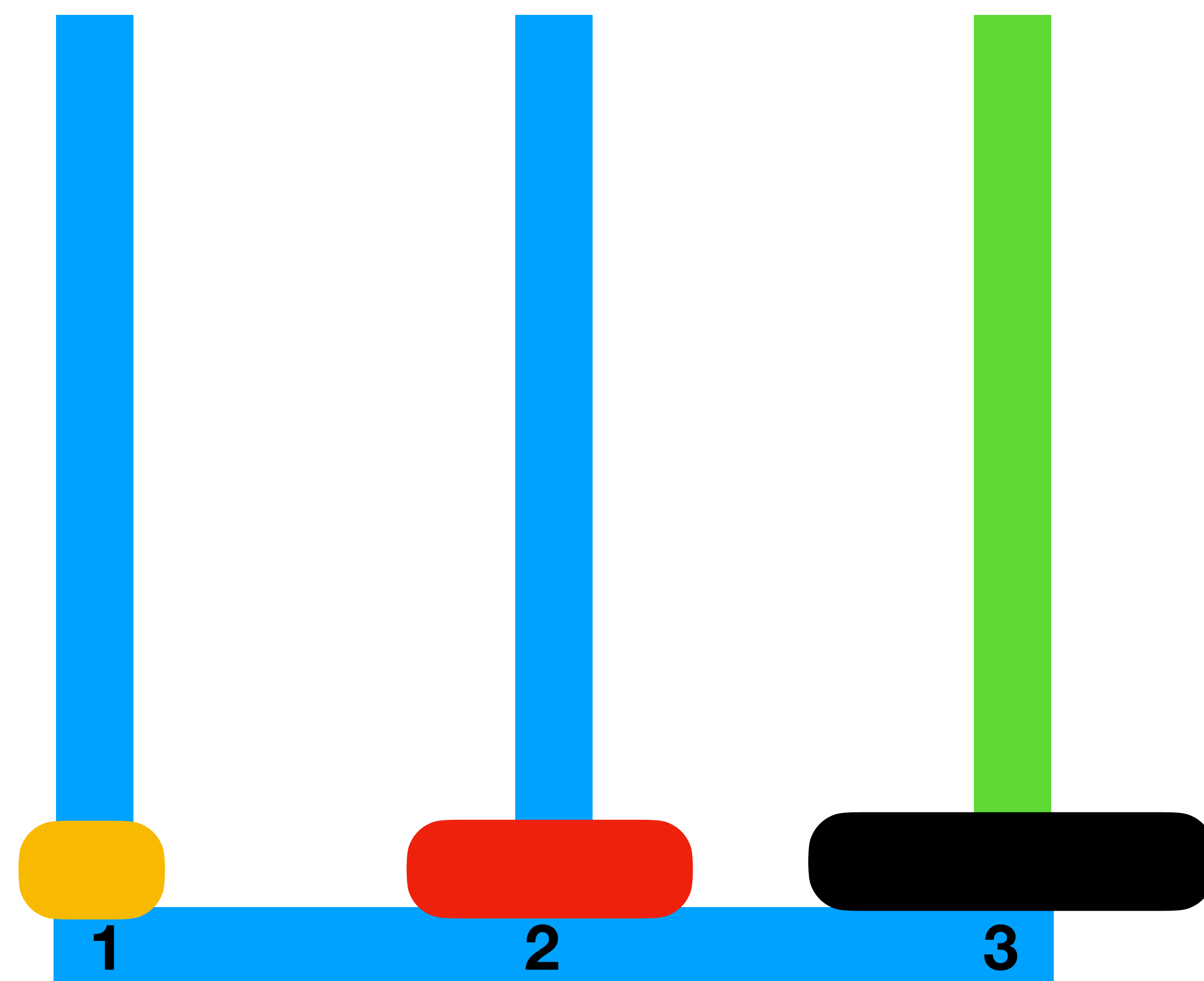
move(disks3, r1, r3, r2)

move(disks2, r1, r2, r3)
+
dmove(disk_3, r1, r3)
+
move(disks2, r2, r3, r1)

dmove(disk_1, r1, r3)
dmove(disk_2, r1, r2)
dmove(disk_1, r3, r2)

dmove(disk_1, r2, r1)

Example: Towers of Hanoi



move(diskStacks, src, dest, temp) // Move 2+ disks
dmove(disk_num, src, dest) // Move single disk

move(disks3, r1, r3, r2)

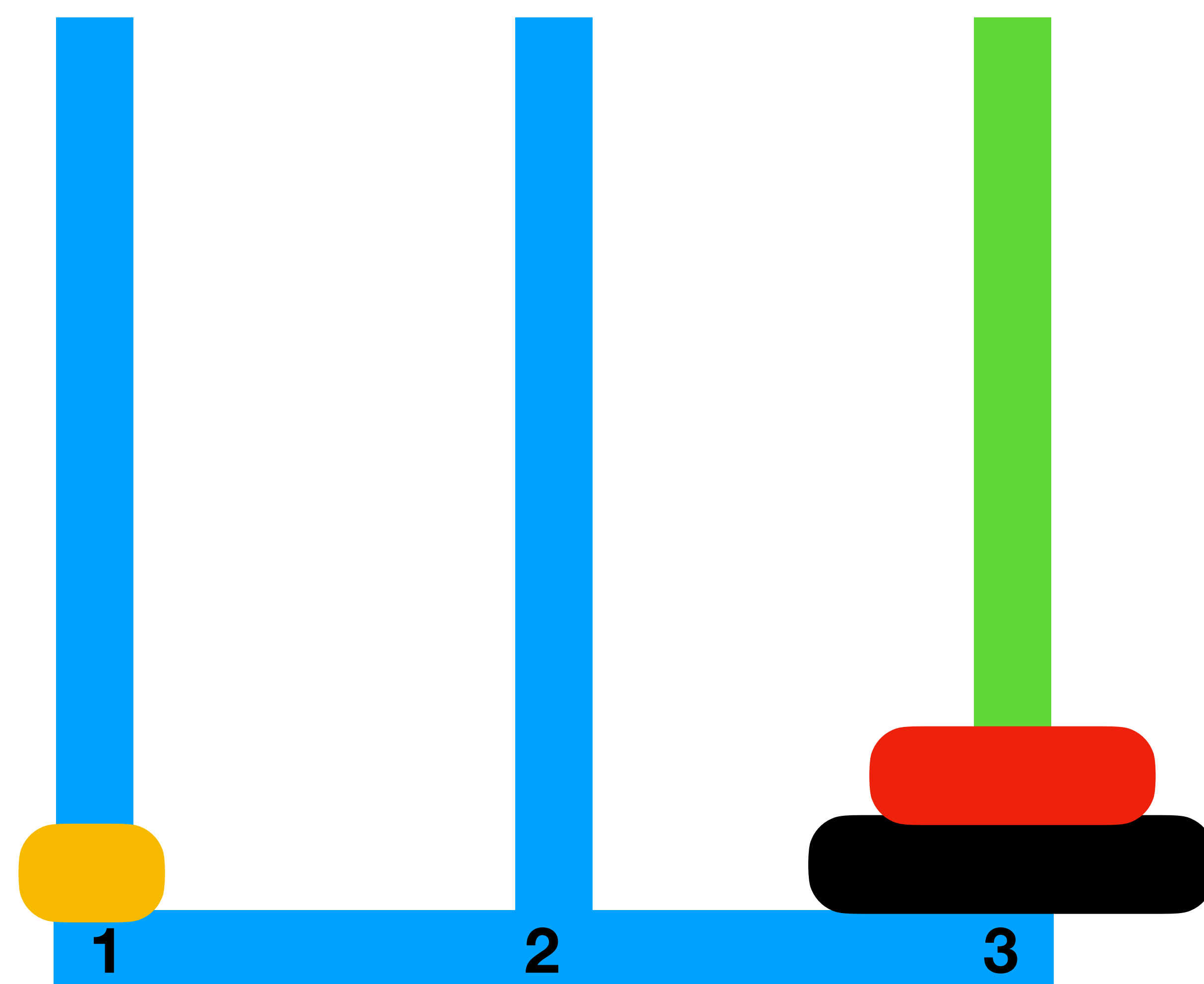
move(disks2, r1, r2, r3)
+
dmove(disk_3, r1, r3)
+
move(disks2, r2, r3, r1)

dmove(disk_1, r1, r3)
dmove(disk_2, r1, r2)
dmove(disk_1, r3, r2)

dmove(disk_1, r2, r1)
dmove(disk_2, r2, r3)

Example: Towers of Hanoi

move(diskStacks, src, dest, temp) // Move 2+ disks
dmove(disk_num, src, dest) // Move single disk



move(disks3, r1, r3, r2)

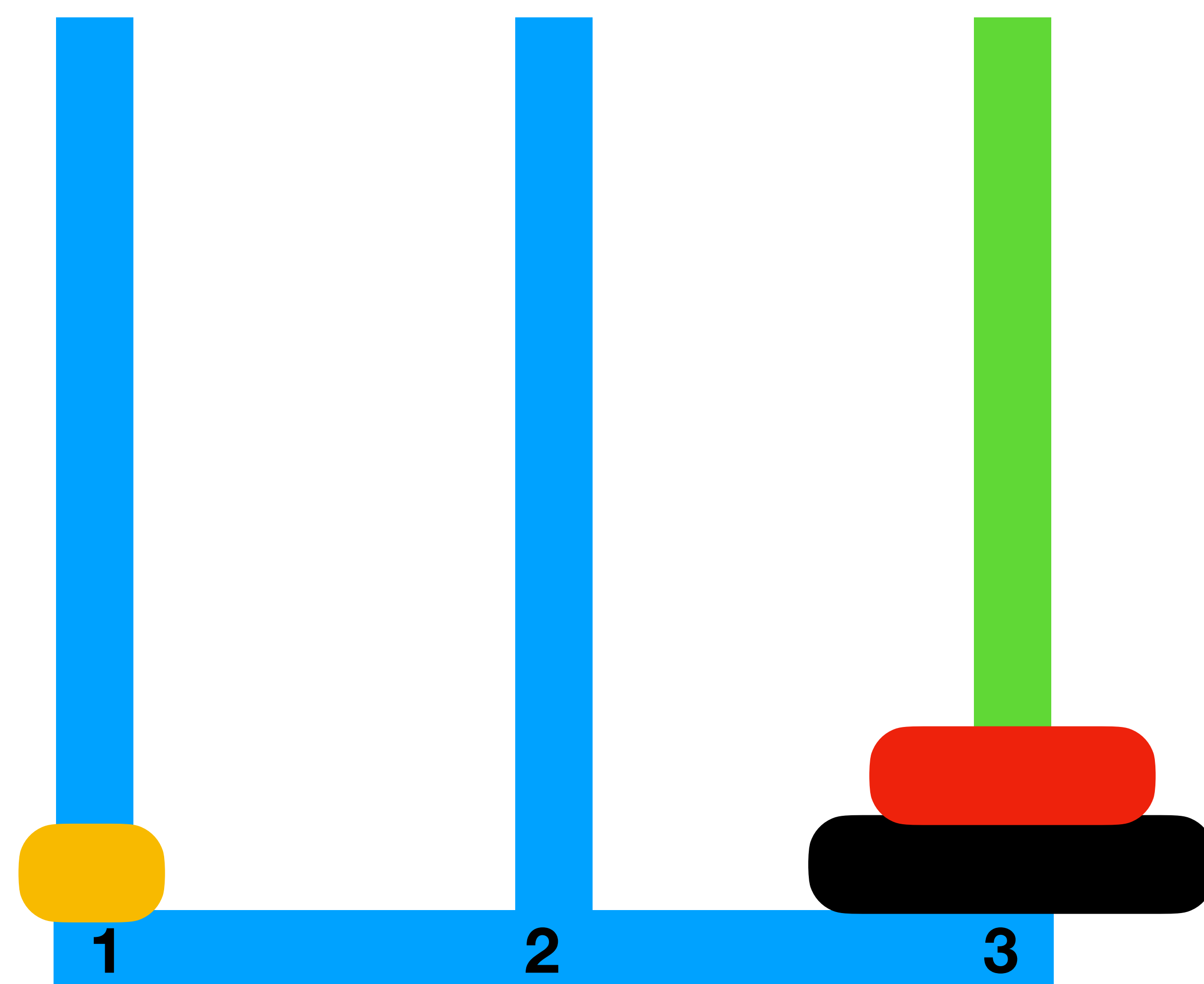
move(disks2, r1, r2, r3)
+
dmove(disk_3, r1, r3)
+
move(disks2, r2, r3, r1)

dmove(disk_1, r1, r3)
dmove(disk_2, r1, r2)
dmove(disk_1, r3, r2)

dmove(disk_1, r2, r1)
dmove(disk_2, r2, r3)

Example: Towers of Hanoi

move(diskStacks, src, dest, temp) // Move 2+ disks
dmove(disk_num, src, dest) // Move single disk



move(disks3, r1, r3, r2)

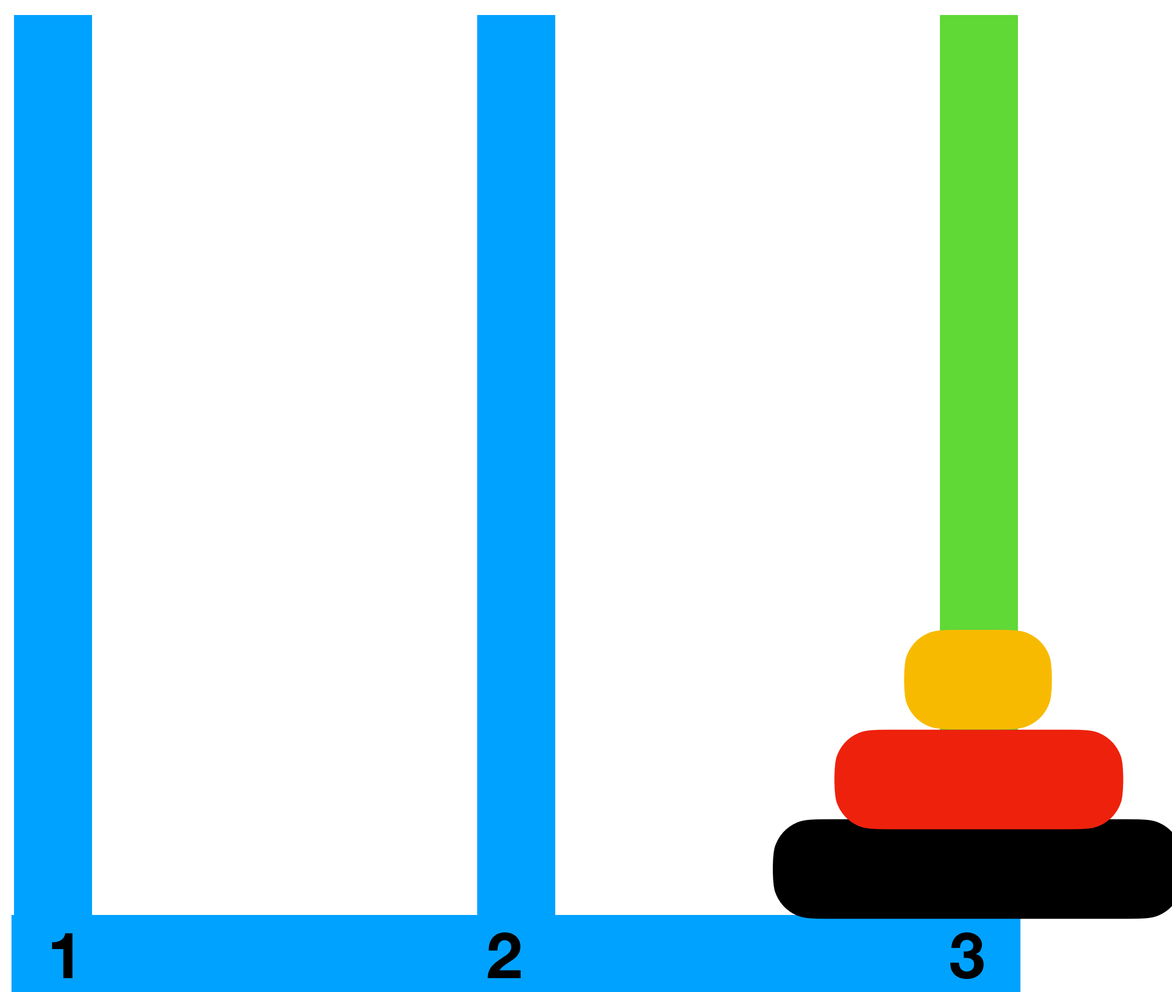
move(disks2, r1, r2, r3)
+
dmove(disk_3, r1, r3)
+
move(disks2, r2, r3, r1)

dmove(disk_1, r1, r3)
dmove(disk_2, r1, r2)
dmove(disk_1, r3, r2)

dmove(disk_1, r2, r1)
dmove(disk_2, r2, r3)
dmove(disk_1, r1, r3)

Example: Towers of Hanoi

move(diskStacks, src, dest, temp) // Move 2+ disks
dmove(disk_num, src, dest) // Move single disk



move(disks3, r1, r3, r2)

move(disks2, r1, r2, r3)
+
dmove(disk_3, r1, r3)
+
move(disks2, r2, r3, r1)

dmove(disk_1, r1, r3)
dmove(disk_2, r1, r2)
dmove(disk_1, r3, r2)

dmove(disk_1, r2, r1)
dmove(disk_2, r2, r3)
dmove(disk_1, r1, r3)

Recursion in Towers of Hanoi

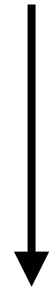
Recursion in Towers of Hanoi

move(disks3, r1, r3, r2)

move(disks4, r1, r3, r2)

Recursion in Towers of Hanoi

move(disks3, r1, r3, r2)



move(disks2, r1, r2, r3)

move(disks4, r1, r3, r2)

move(disks3, r1, r2, r3)

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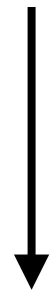
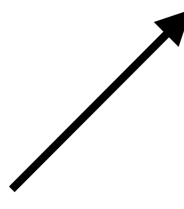
Recursion in Towers of Hanoi

move(disks3, r1, r3, r2)



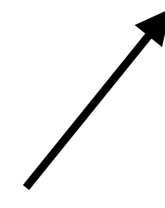
move(disks2, r1, r2, r3)



dmove(disk_1, r1, r3)

move(disks4, r1, r3, r2)

move(disks3, r1, r2, r3)

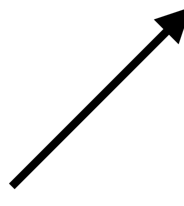


move(disks2, r1, r3, r2)

Recursion in Towers of Hanoi

move(disks3, r1, r3, r2)

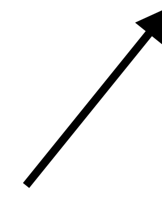
move(disks2, r1, r2, r3)



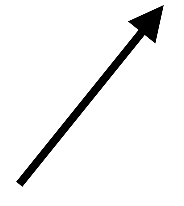
dmove(disk_1, r1, r3)

move(disks4, r1, r3, r2)

move(disks3, r1, r2, r3)



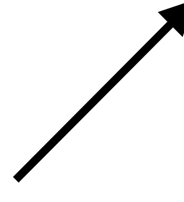
move(disks2, r1, r3, r2)



Recursion in Towers of Hanoi

move(disks3, r1, r3, r2)

move(disks2, r1, r2, r3)

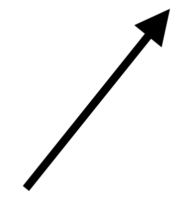


dmove(disk_1, r1, r3)

dmove(disk_2, r1, r2)

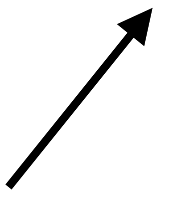
move(disks4, r1, r3, r2)

move(disks3, r1, r2, r3)



move(disks2, r1, r3, r2)

dmove(disk_3, r1, r2)



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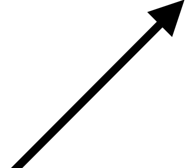
Recursion in Towers of Hanoi

move(disks3, r1, r3, r2)

move(disks2, r1, r2, r3)



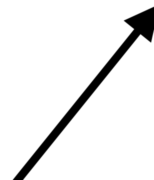
dmove(disk_1, r1, r3)

dmove(disk_2, r1, r2)

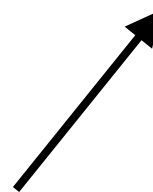
dmove(disk_1, r3, r2)

move(disks4, r1, r3, r2)

move(disks3, r1, r2, r3)



move(disks2, r1, r3, r2)



dmove(disk_3, r1, r2)

move(disks2, r3, r2, r1)

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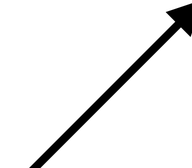
Recursion in Towers of Hanoi

move(disks3, r1, r3, r2)

move(disks2, r1, r2, r3)



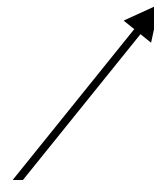
dmove(disk_1, r1, r3)

dmove(disk_2, r1, r2)

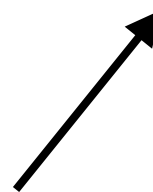
dmove(disk_1, r3, r2)

move(disks4, r1, r3, r2)

move(disks3, r1, r2, r3)

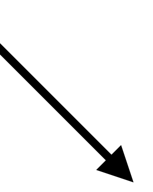


move(disks2, r1, r3, r2)



dmove(disk_3, r1, r2)

move(disks2, r3, r2, r1)



Recursion in Towers of Hanoi

move(disks3, r1, r3, r2)

move(disks2, r1, r2, r3)

\+

dmove(disk_3, r1, r3)

dmove(disk_1, r1, r3)

dmove(disk_2, r1, r2)

dmove(disk_1, r3, r2)

move(disks4, r1, r3, r2)

move(disks3, r1, r2, r3)

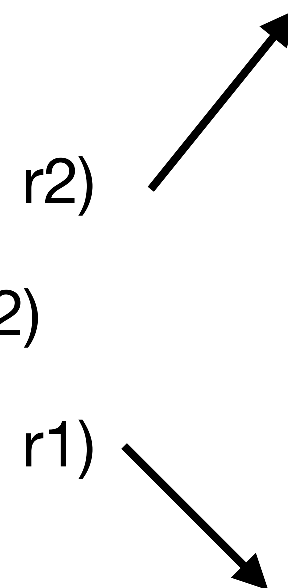
\+

dmove(disk_4, r1, r3)

move(disks2, r1, r3, r2)

dmove(disk_3, r1, r2)

move(disks2, r3, r2, r1)



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Recursion in Towers of Hanoi

move(disks3, r1, r3, r2)

move(disks2, r1, r2, r3)

\+

dmove(disk_3, r1, r3)

\+

move(disks2, r2, r3, r1)

dmove(disk_1, r1, r3)

dmove(disk_2, r1, r2)

dmove(disk_1, r3, r2)

move(disks4, r1, r3, r2)

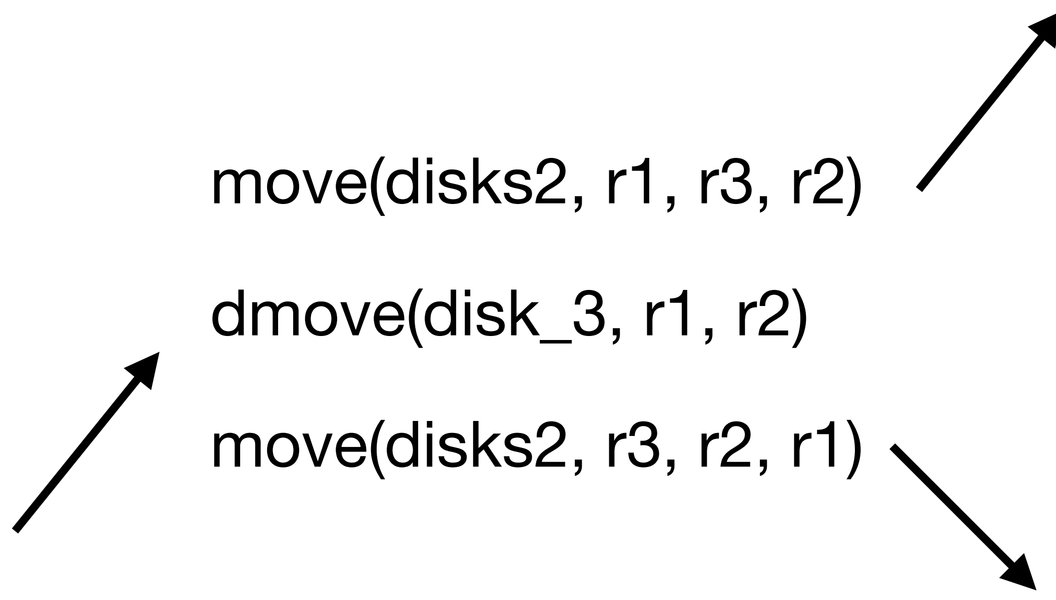
move(disks3, r1, r2, r3)

\+

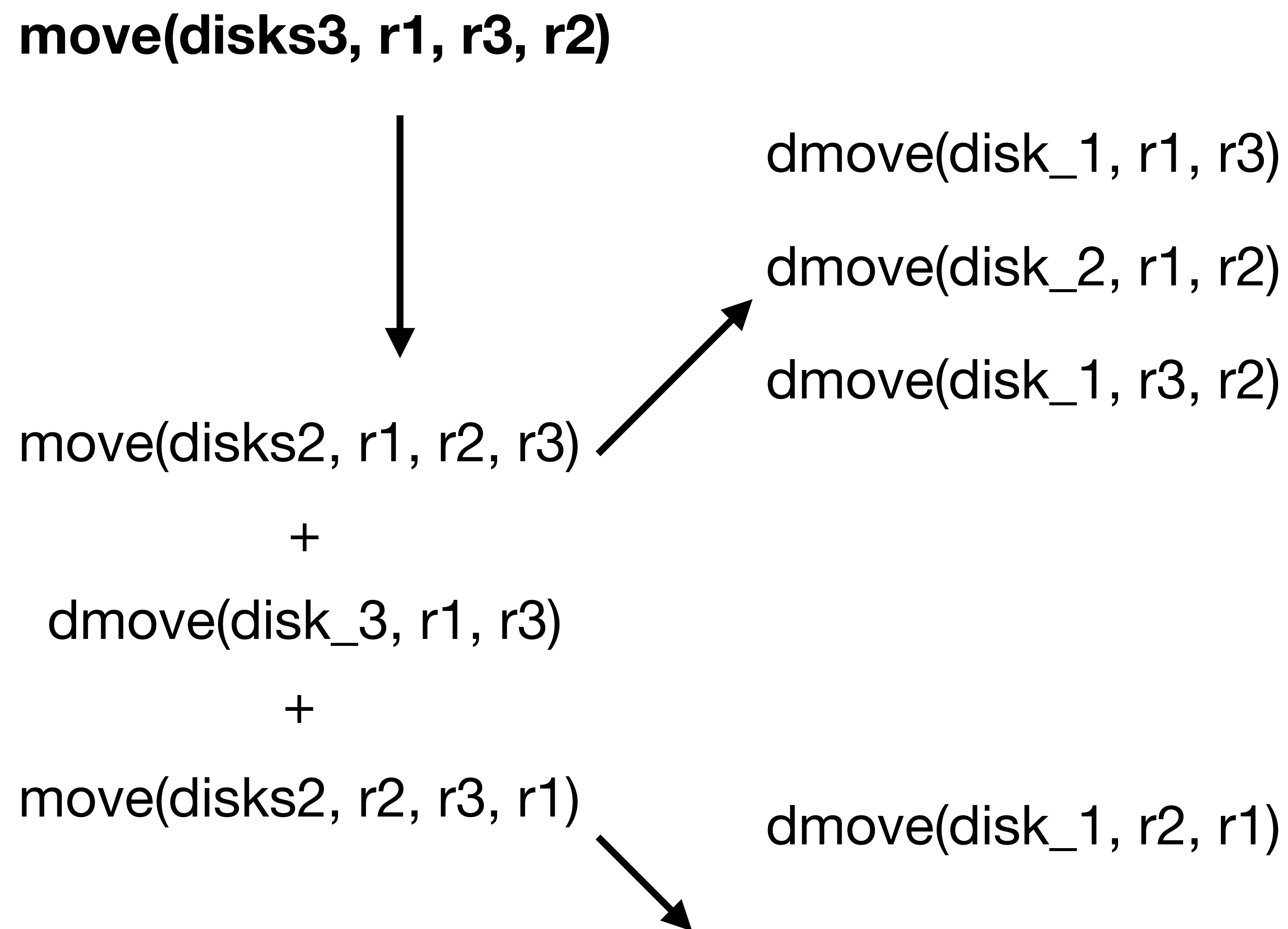
dmove(disk_4, r1, r3)

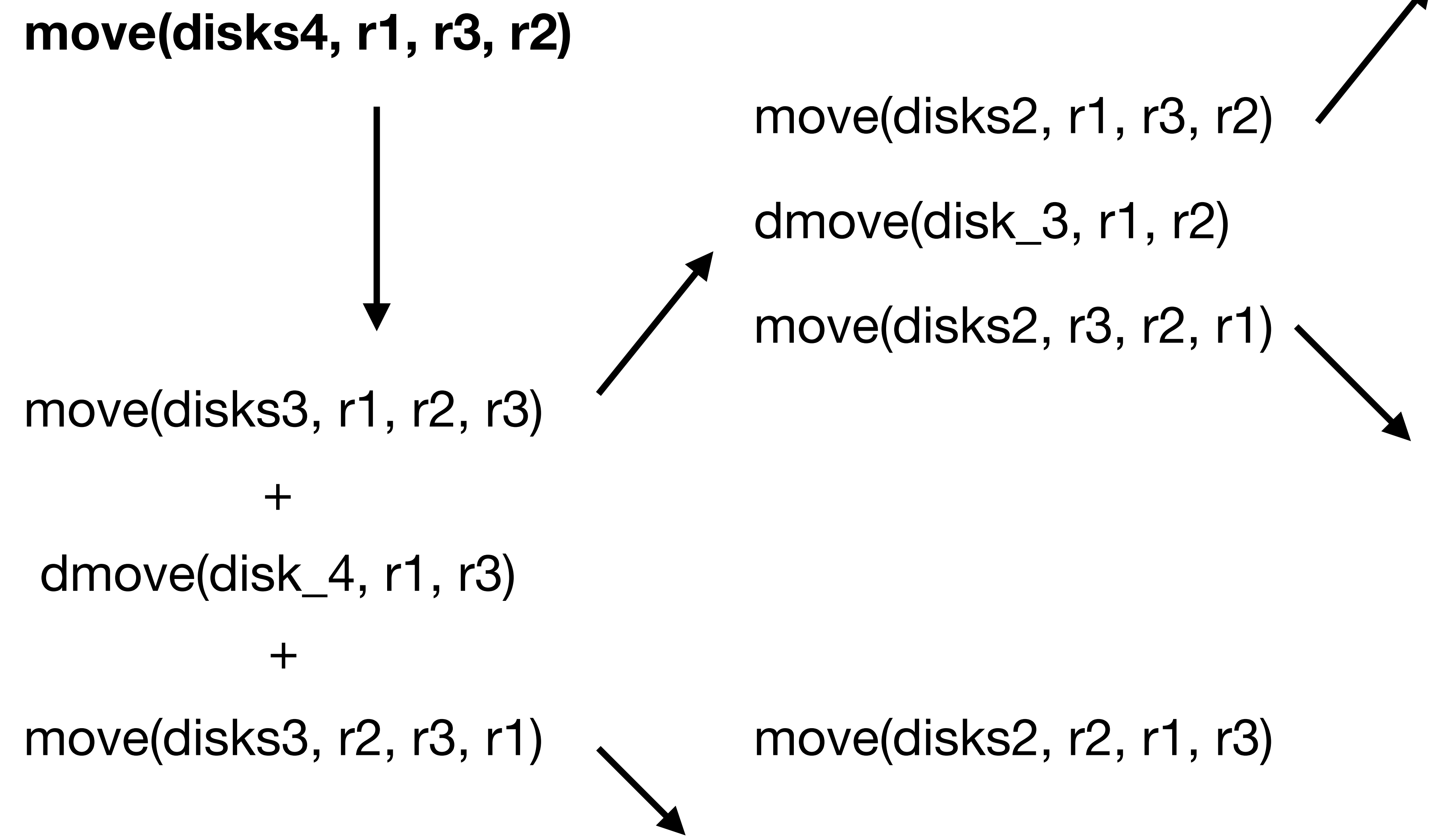
\+

move(disks3, r2, r3, r1)

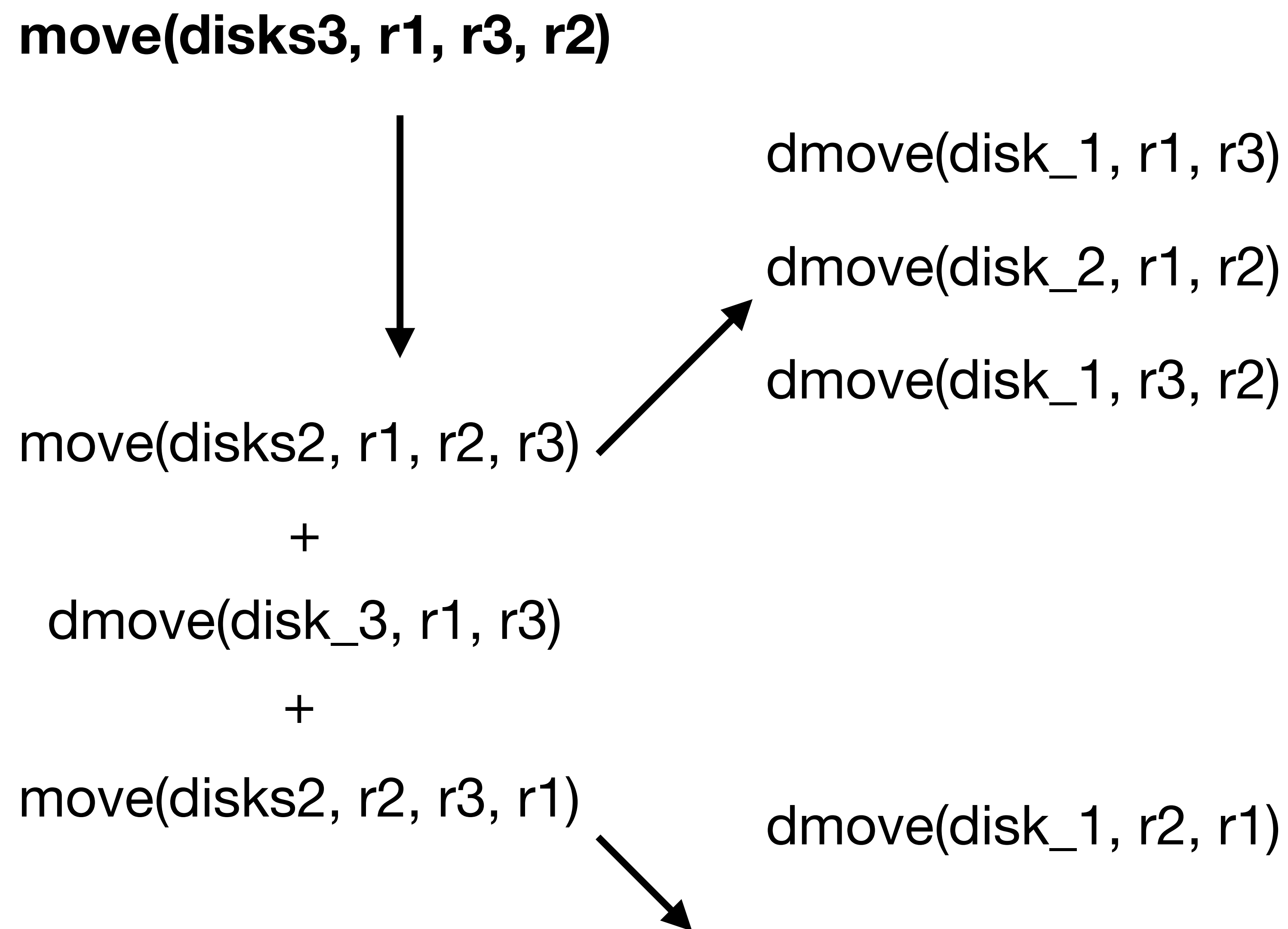


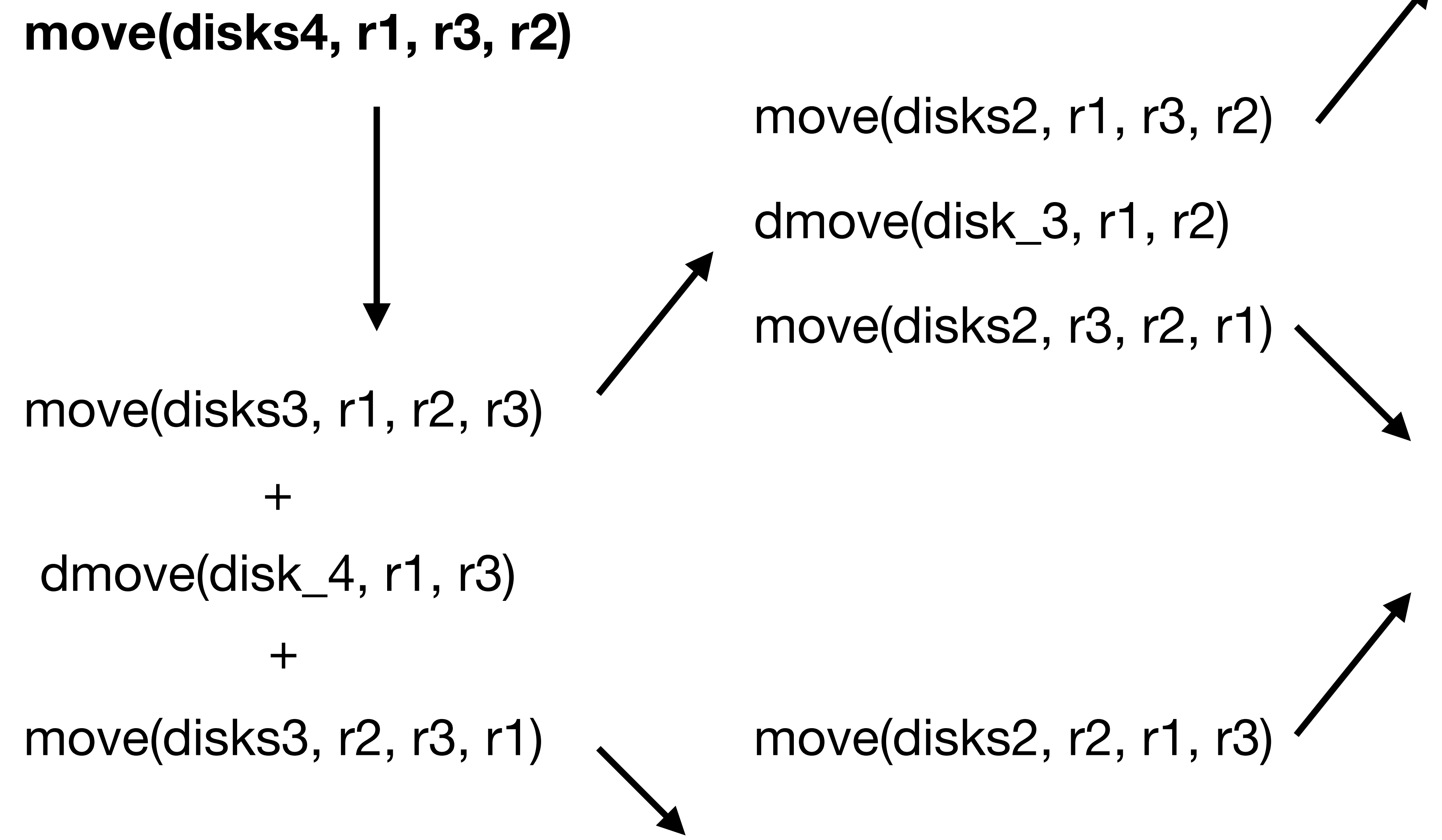
Recursion in Towers of Hanoi



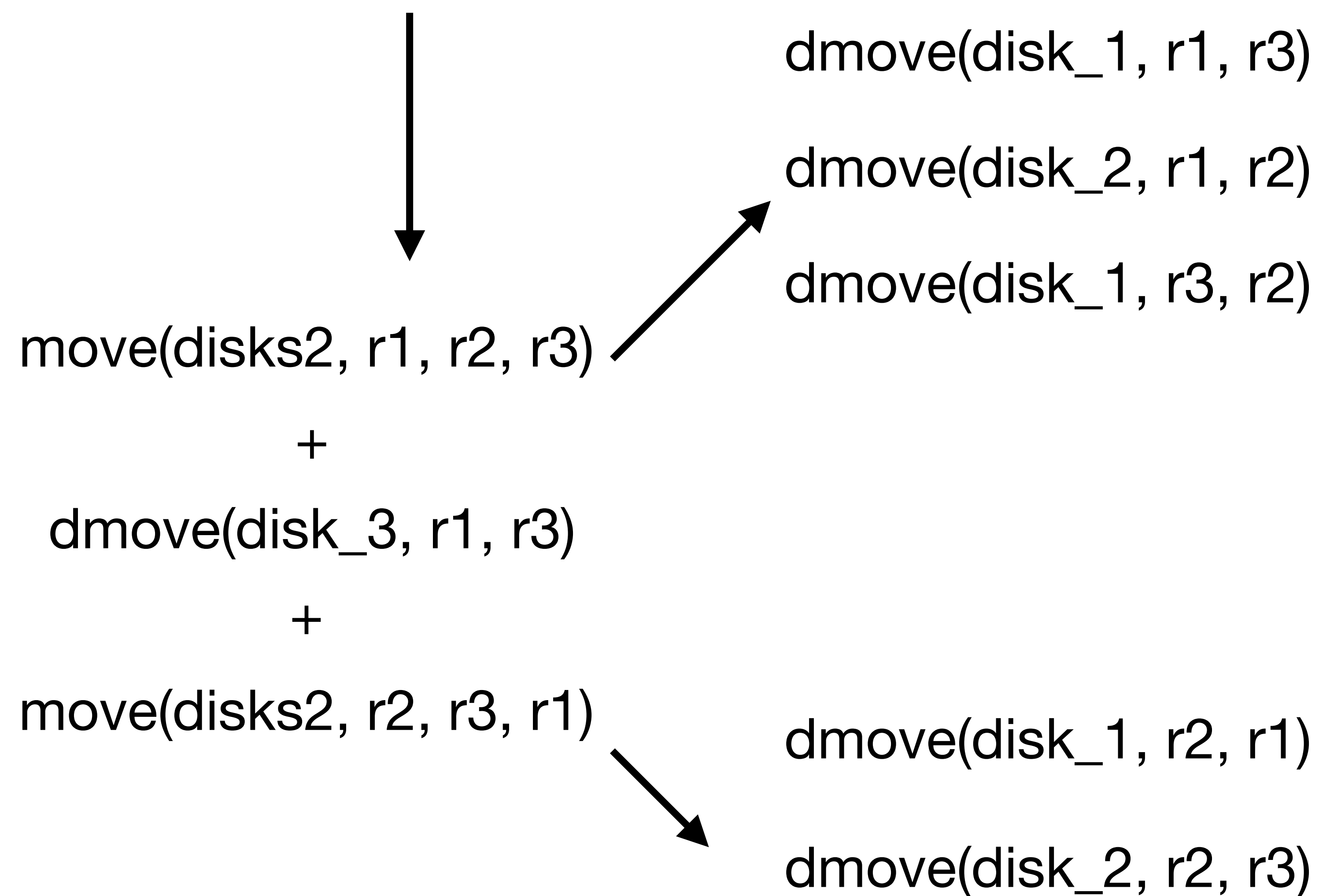


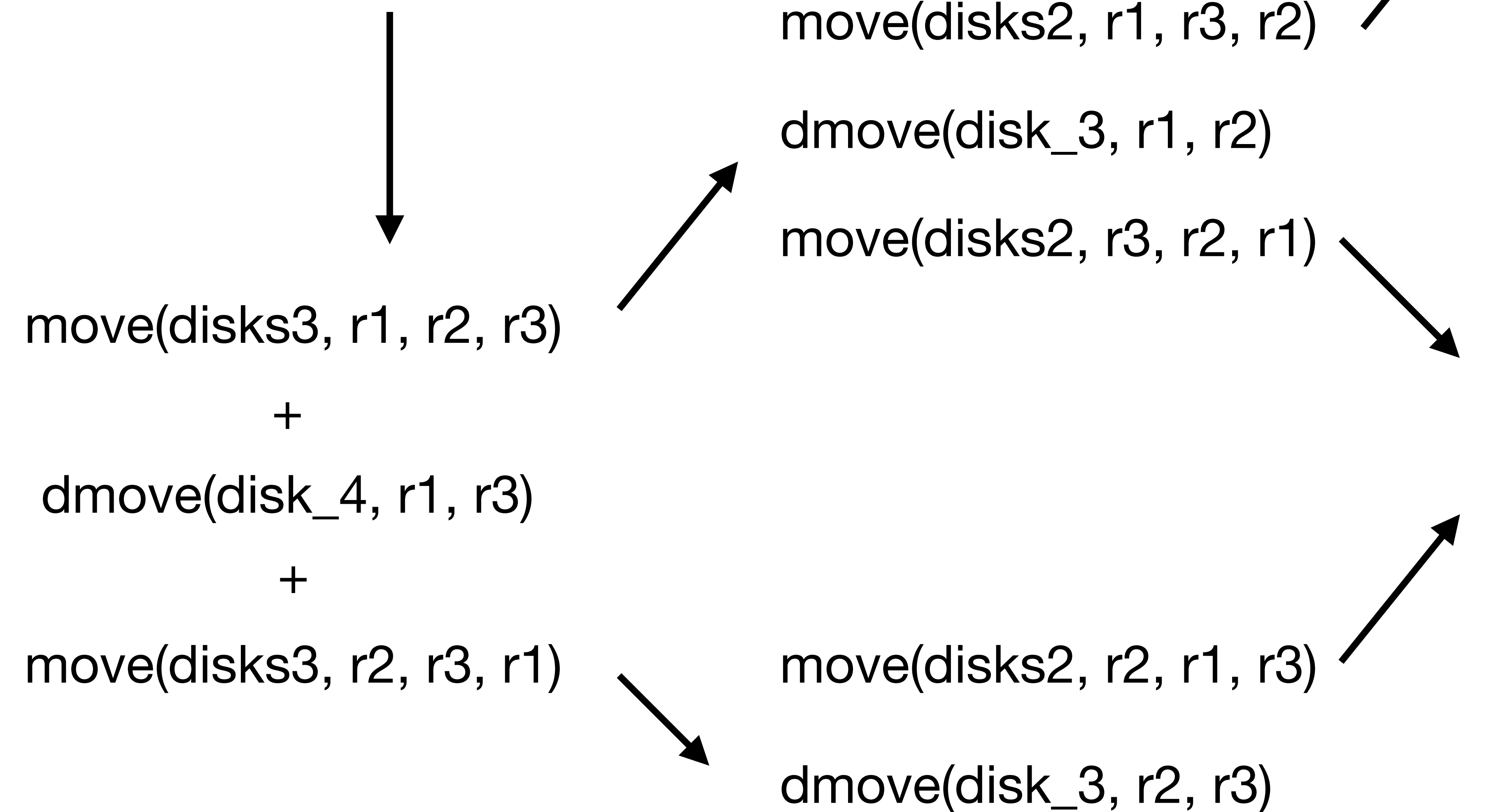
Recursion in Towers of Hanoi



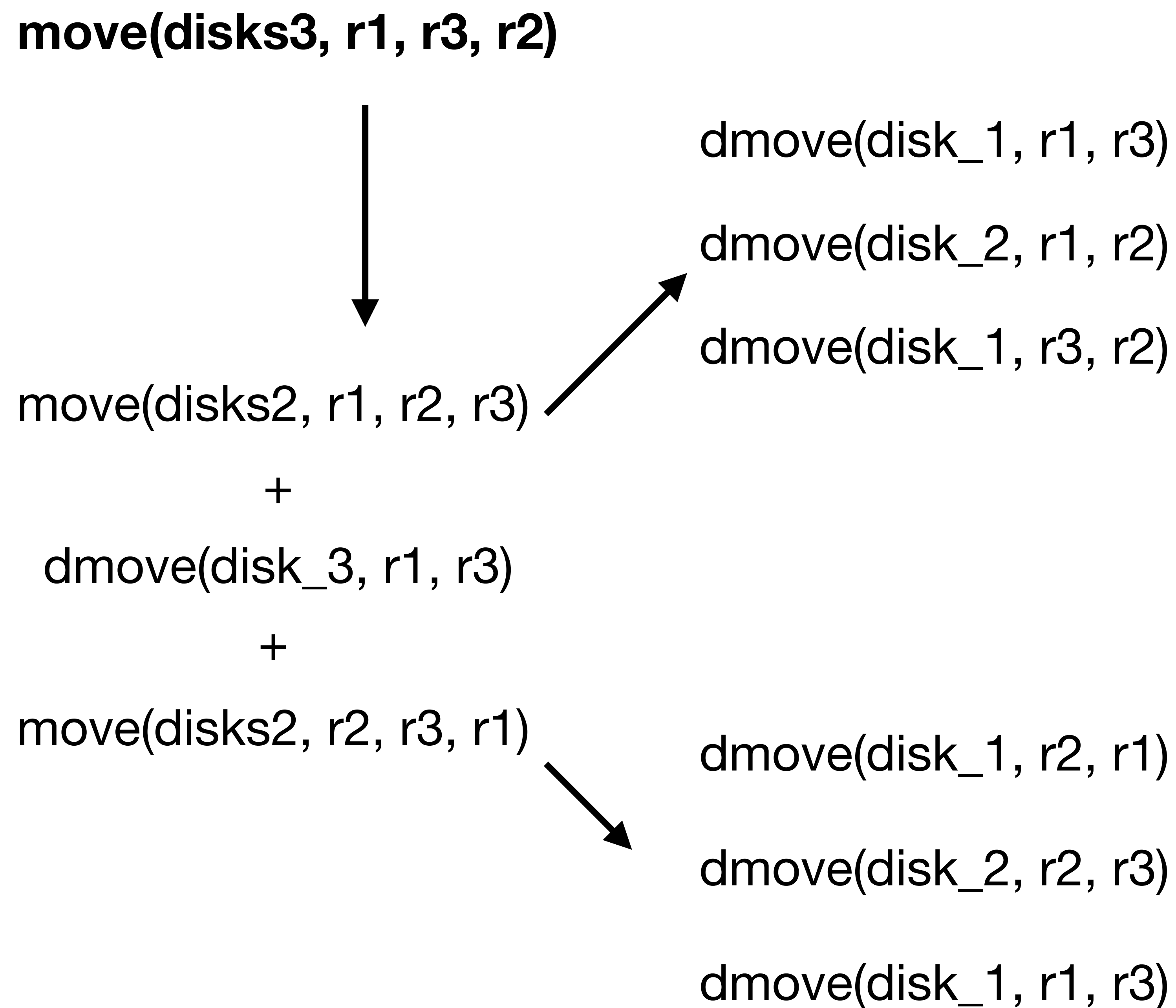


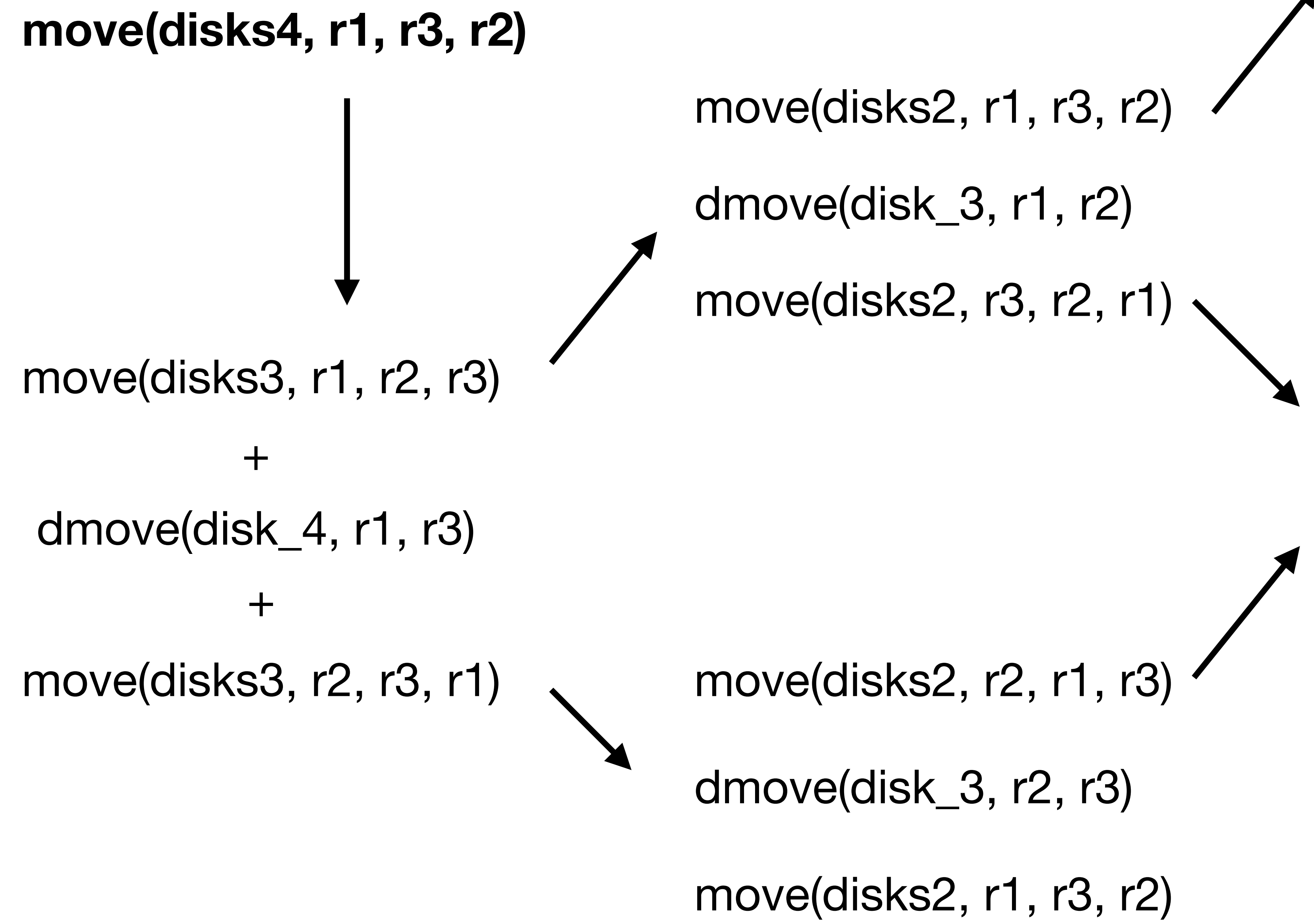
Recursion in Towers of Hanoi



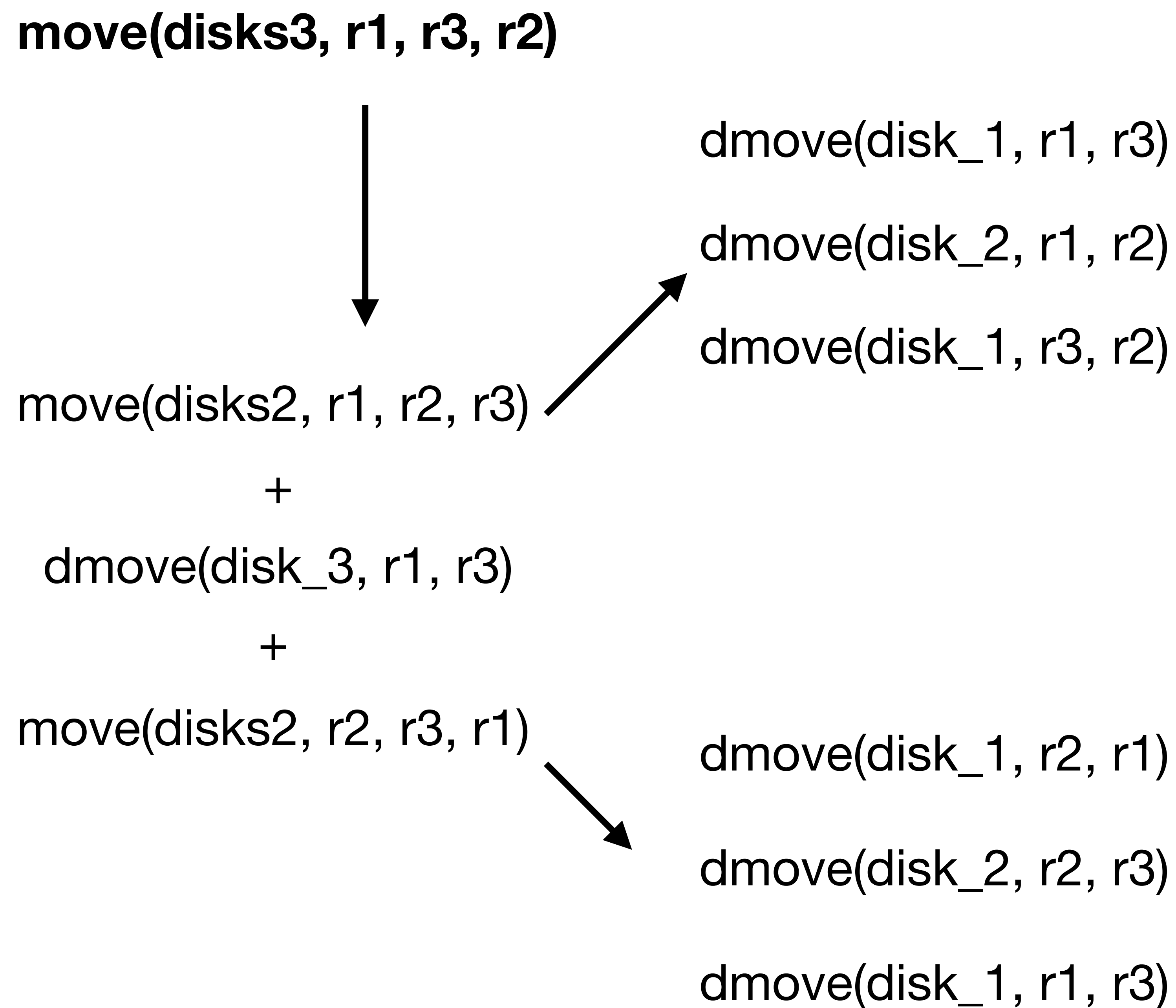


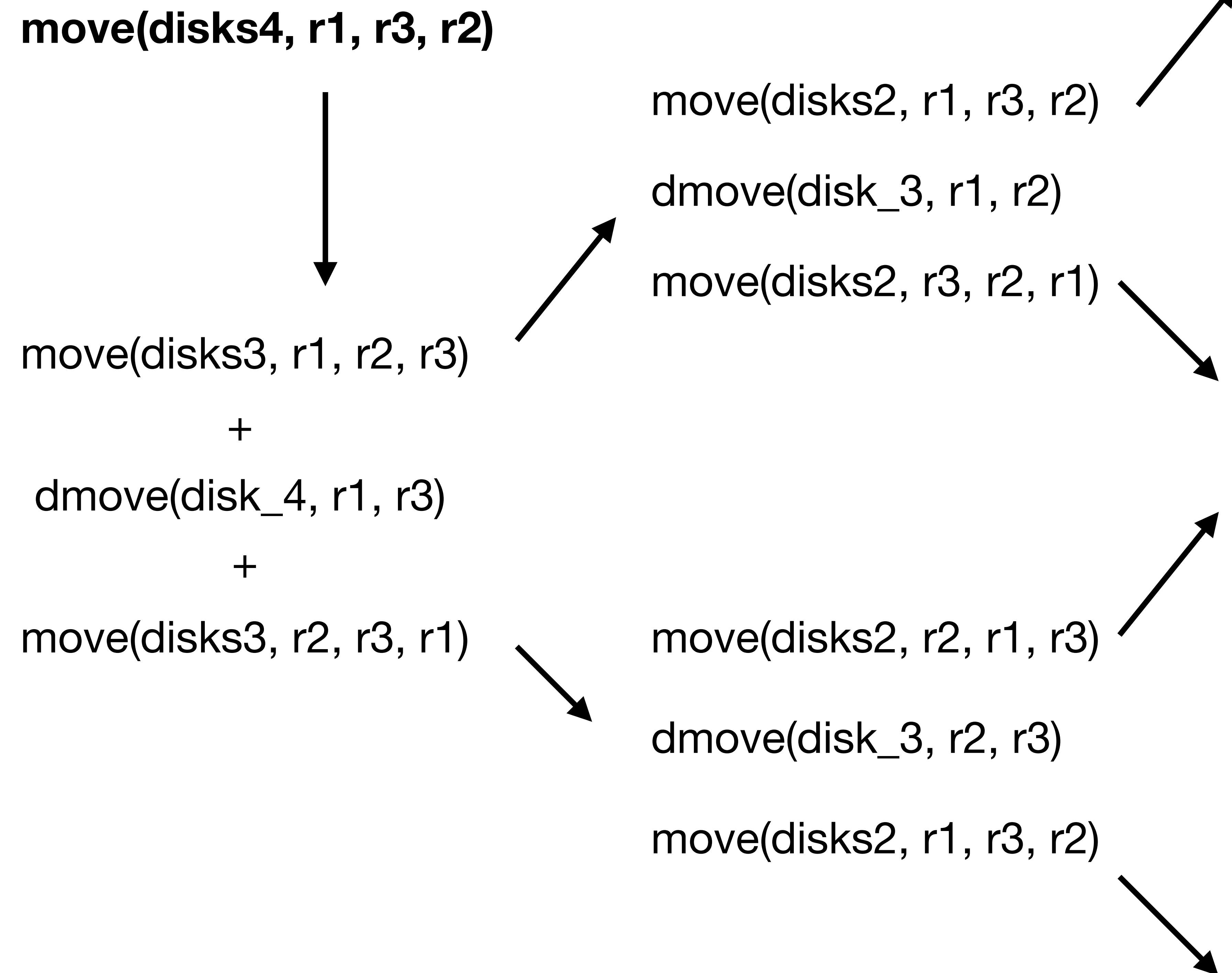
Recursion in Towers of Hanoi





Recursion in Towers of Hanoi





Towers of Hanoi: General formula?

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Towers of Hanoi: General formula?

```
void move(disknum, src, dest, temp){
  if (disknum > 1){
    move(disknum - 1, src, temp, dest);

    printf("Moved disk %d from rod %d to %d\n", disknum, src, dest);

    move(disknum - 1, temp, dest, src);
  }
  else
    printf("Moved disk 1 from rod %d to %d\n", src, dest);
}
```

Towers of Hanoi: General formula?

```
void move(disknum, src, dest, temp){
  if (disknum > 1){
    move(disknum - 1, src, temp, dest);

    printf("Moved disk %d from rod %d to %d\n", disknum, src, dest);

    move(disknum - 1, temp, dest, src);
  }
  else
    printf("Moved disk 1 from rod %d to %d\n", src, dest);
}
```

Move a sub-stack to from src to intermediate rod

Towers of Hanoi: General formula?

```
void move(disknum, src, dest, temp){
  if (disknum > 1){
    move(disknum - 1, src, temp, dest);

    printf("Moved disk %d from rod %d to %d\n", disknum, src, dest);

    move(disknum - 1, temp, dest, src);
  }
  else
    printf("Moved disk 1 from rod %d to %d\n", src, dest);
}
```

Move a sub-stack to from src to intermediate rod

Direct move of single disk.

Towers of Hanoi: General formula?

```
void move(disknum, src, dest, temp){
  if (disknum > 1){
    move(disknum - 1, src, temp, dest);

    printf("Moved disk %d from rod %d to %d\n", disknum, src, dest);

    move(disknum - 1, temp, dest, src);
  }
  else
    printf("Moved disk 1 from rod %d to %d\n", src, dest);
}
```

Move a sub-stack to from src to intermediate rod

Direct move of single disk.

Move the sub-stack from intermediate to dest rod

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Towers of Hanoi: General formula?

```
void move(disknum, src, dest, temp){
  if (disknum > 1){
    move(disknum - 1, src, temp, dest);

    printf("Moved disk %d from rod %d to %d\n", disknum, src, dest);

    move(disknum - 1, temp, dest, src);
  }
  else
    printf("Moved disk 1 from rod %d to %d\n", src, dest);
}
```

Move a sub-stack to from src to intermediate rod

Direct move of single disk.

Move the sub-stack from intermediate to dest rod

Base case; all moves involving sub-stacks end-up here.

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Recursion in LC3

Recursion in LC3

```
int running_sum(int n){
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}

int main(void){
    int n = 5;
    running_sum(5);
}
```


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Recursion in LC3

```
int running_sum(int n){
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}

int main(void){
    int n = 5;
    running_sum(5);
}
```

How can we write equivalent LC3 code?

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Recursion in LC3

```
int running_sum(int n){
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}

int main(void){
    int n = 5;
    running_sum(5);
}
```

How can we write equivalent LC3 code?

Recall function calls are implemented using the run time stack.

Recursion in LC3

```
int running_sum(int n){
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}

int main(void){
    int n = 5;
    running_sum(5);
}
```

How can we write equivalent LC3 code?

Recall function calls are implemented using the run time stack.

Recursive calls need not be treated any different from normal function calls!

Generating an activation record

Check Gitlab for reference material: https://gitlab.engr.illinois.edu/itabrah2/ece220-sp24

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Generating an activation record

1. *Caller* build-up: Push callee's arguments onto stack

Check Gitlab for reference material: https://gitlab.engr.illinois.edu/itabrah2/ece220-sp24

Generating an activation record

1. *Caller* build-up: Push callee's arguments onto stack
2. Pass control to callee (`JSR/JSRR`)

Check Gitlab for reference material: https://gitlab.engr.illinois.edu/itabrah2/ece220-sp24

Generating an activation record

1. *Caller* build-up: Push callee's arguments onto stack
2. Pass control to callee (`JSR/JSRR`)

Caller

Check Gitlab for reference material: https://gitlab.engr.illinois.edu/itabrah2/ece220-sp24

Generating an activation record

1. *Caller* build-up: Push callee's arguments onto stack
2. Pass control to callee (`JSR/JSRR`)

Caller

3. *Callee* build-up: (push bookkeeping info and local variables onto stack)

Check Gitlab for reference material: https://gitlab.engr.illinois.edu/itabrah2/ece220-sp24

Generating an activation record

1. *Caller* build-up: Push callee's arguments onto stack
2. Pass control to callee (`JSR/JSRR`)
3. *Callee* build-up: (push bookkeeping info and local variables onto stack)
4. Execute function

Caller

Check Gitlab for reference material: https://gitlab.engr.illinois.edu/itabrah2/ece220-sp24

Generating an activation record

1. *Caller* build-up: Push callee's arguments onto stack
2. Pass control to callee (`JSR`/`JSRR`)
3. *Callee* build-up: (push bookkeeping info and local variables onto stack)
4. Execute function
5. *Callee* tear-down (update return value, pop local variables, caller's frame pointer and return address from stack)

Caller

Check Gitlab for reference material: https://gitlab.engr.illinois.edu/itabrah2/ece220-sp24

Generating an activation record

1. *Caller* build-up: Push callee's arguments onto stack
2. Pass control to callee (`JSR/JSRR`)
3. *Callee* build-up: (push bookkeeping info and local variables onto stack)
4. Execute function
5. *Callee* tear-down (update return value, pop local variables, caller's frame pointer and return address from stack)
6. Return to caller (`RET`)

Caller

Check Gitlab for reference material: https://gitlab.engr.illinois.edu/itabrah2/ece220-sp24

Generating an activation record

1. *Caller* build-up: Push callee's arguments onto stack
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5. *Callee* tear-down (update return value, pop local variables, caller's frame pointer and return address from stack)
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Caller

Callee

Check Gitlab for reference material: https://gitlab.engr.illinois.edu/itabrah2/ece220-sp24

Generating an activation record

1. *Caller* build-up: Push callee's arguments onto stack
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3. *Callee* build-up: (push bookkeeping info and local variables onto stack)
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5. *Callee* tear-down (update return value, pop local variables, caller's frame pointer and return address from stack)
6. Return to caller (`RET`)
7. *Caller* tear-down (pop callee's return value and arguments from stack)

Caller

Callee

Check Gitlab for reference material: https://gitlab.engr.illinois.edu/itabrah2/ece220-sp24

Generating an activation record

1. *Caller* build-up: Push callee's arguments onto stack
2. Pass control to callee (`JSR/JSRR`)

Caller

3. *Callee* build-up: (push bookkeeping info and local variables onto stack)
4. Execute function
5. *Callee* tear-down (update return value, pop local variables, caller's frame pointer and return address from stack)
6. Return to caller (`RET`)

Callee

7. *Caller* tear-down (pop callee's return value and arguments from stack)

Caller

Check Gitlab for reference material: https://gitlab.engr.illinois.edu/itabrah2/ece220-sp24

Recursion in LC3

```c
int running_sum(int n){
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}

int main(void){
    int n = 4;
    running_sum(4);
}
```


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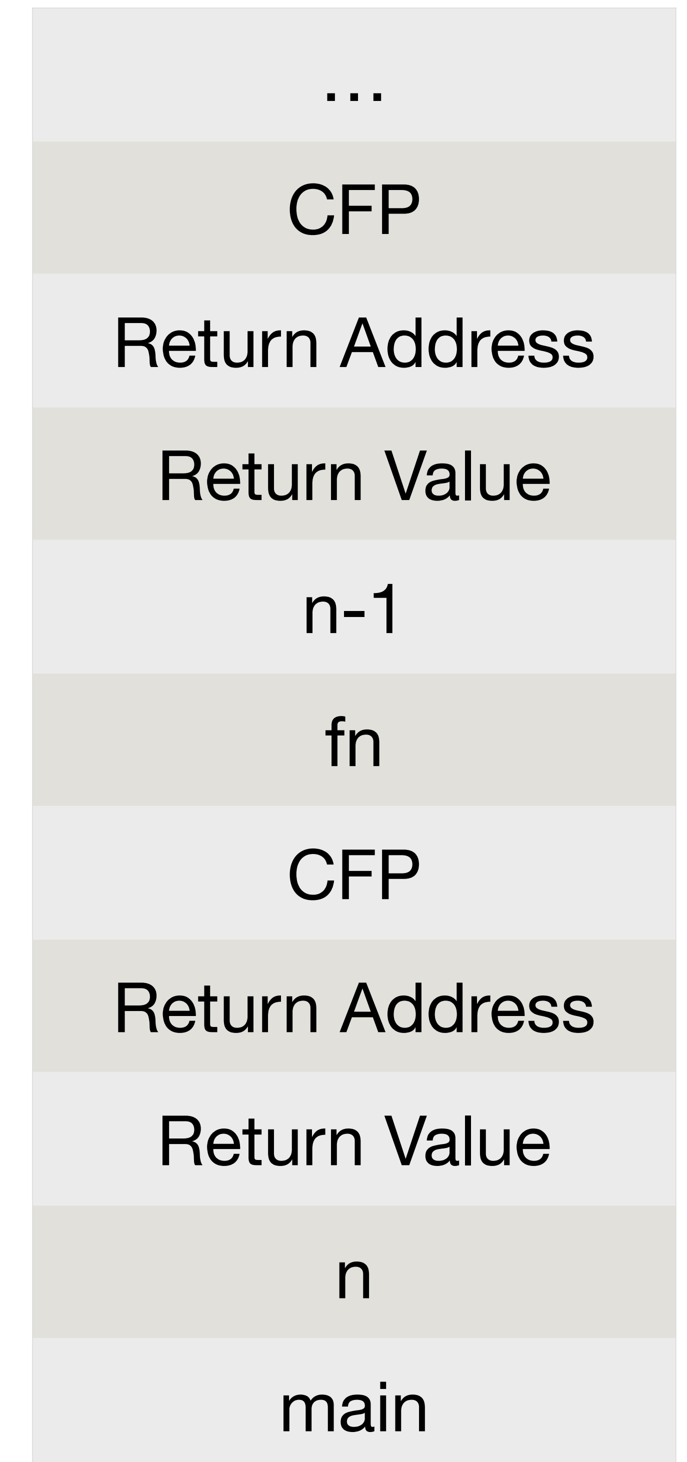
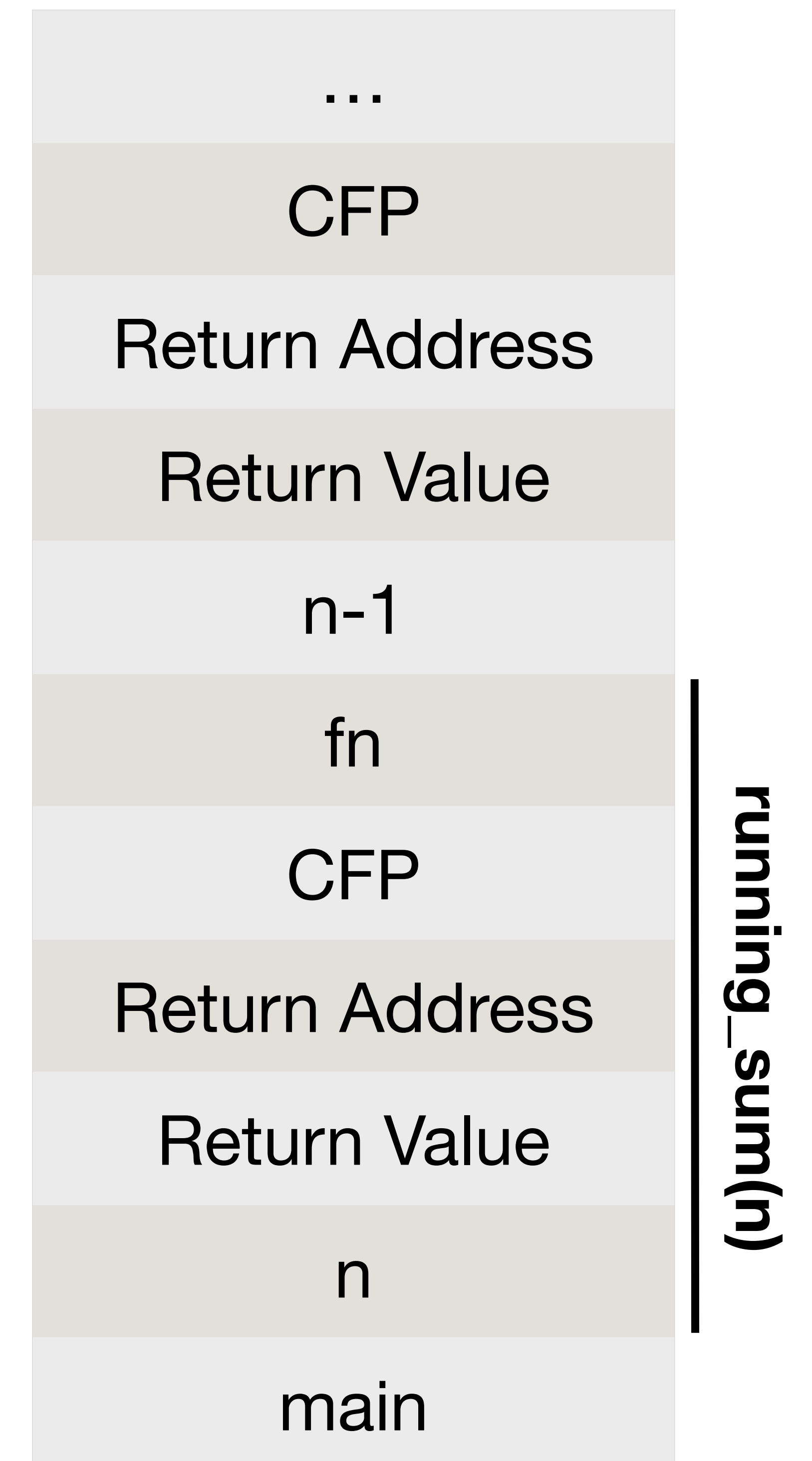
Recursion in LC3

```
int running_sum(int n){
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}

int main(void){
    int n = 4;
    running_sum(4);
}
```



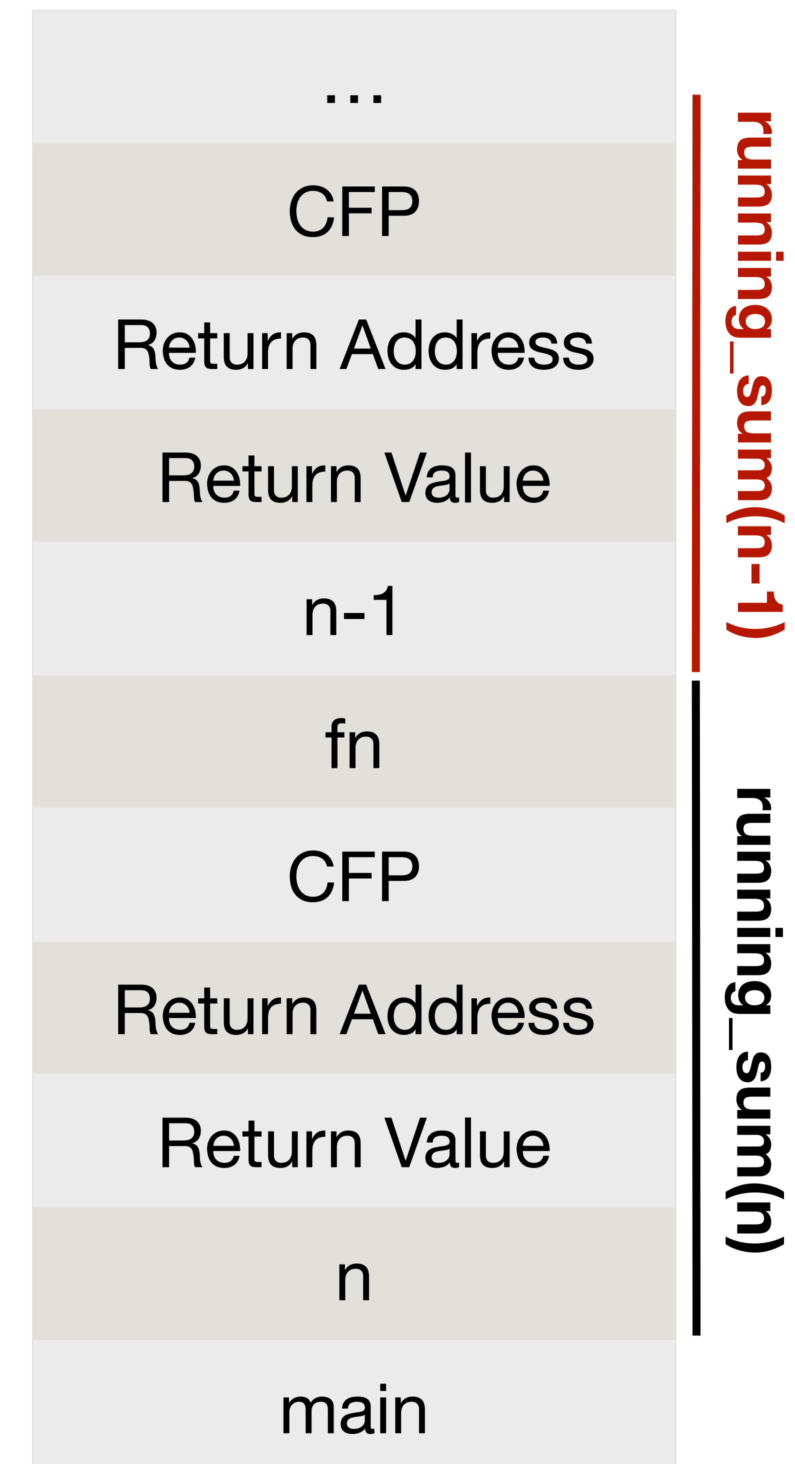
Recursion in LC3

```
int running_sum(int n){
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}

int main(void){
    int n = 4;
    running_sum(4);
}
```



Recursion in LC3

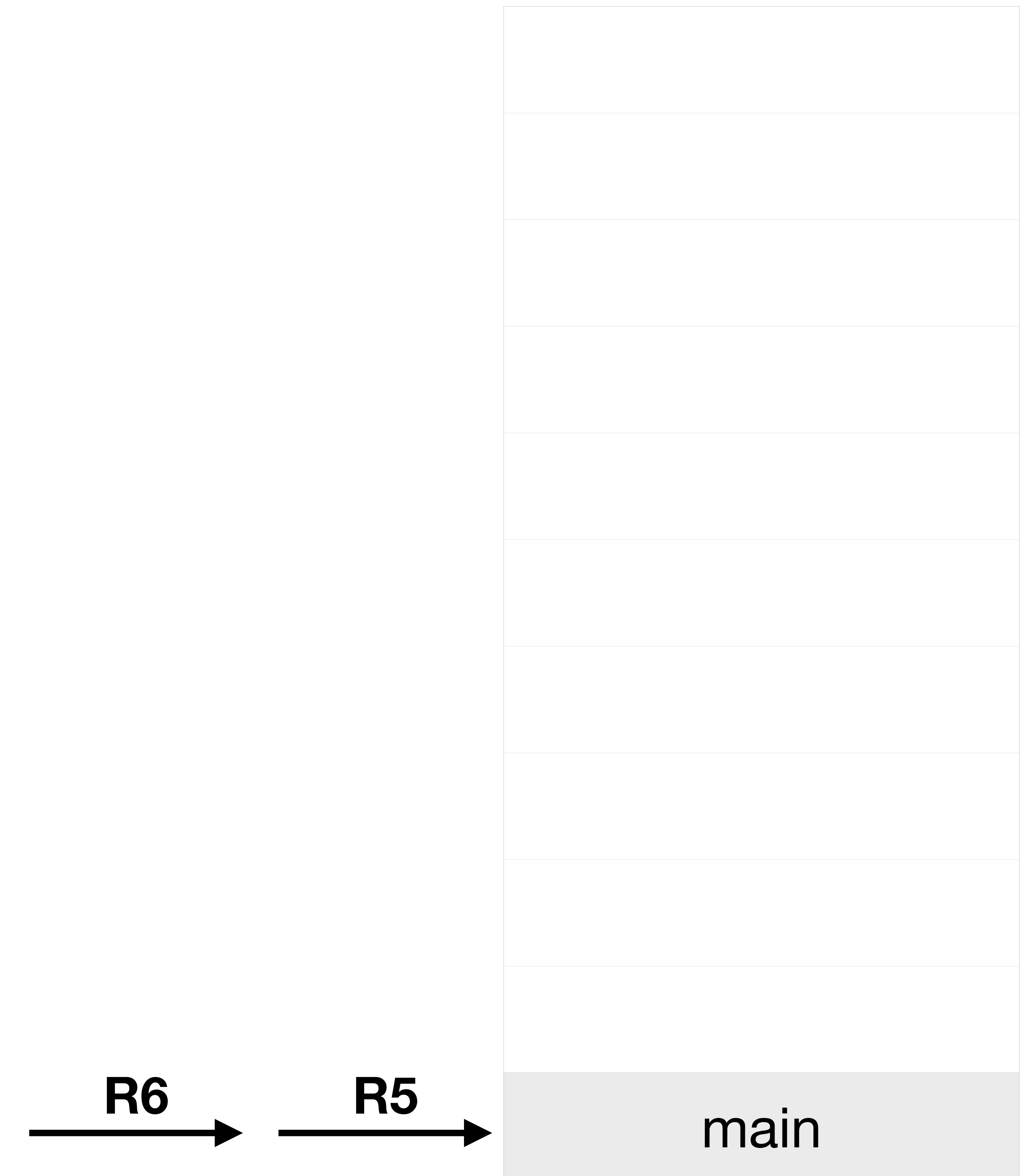
```
int running_sum(int n){
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}

int main(void){
    int n = 4;
    running_sum(4);
}
```



```c
int running_sum(int n){
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}
```

Recursion in LC3

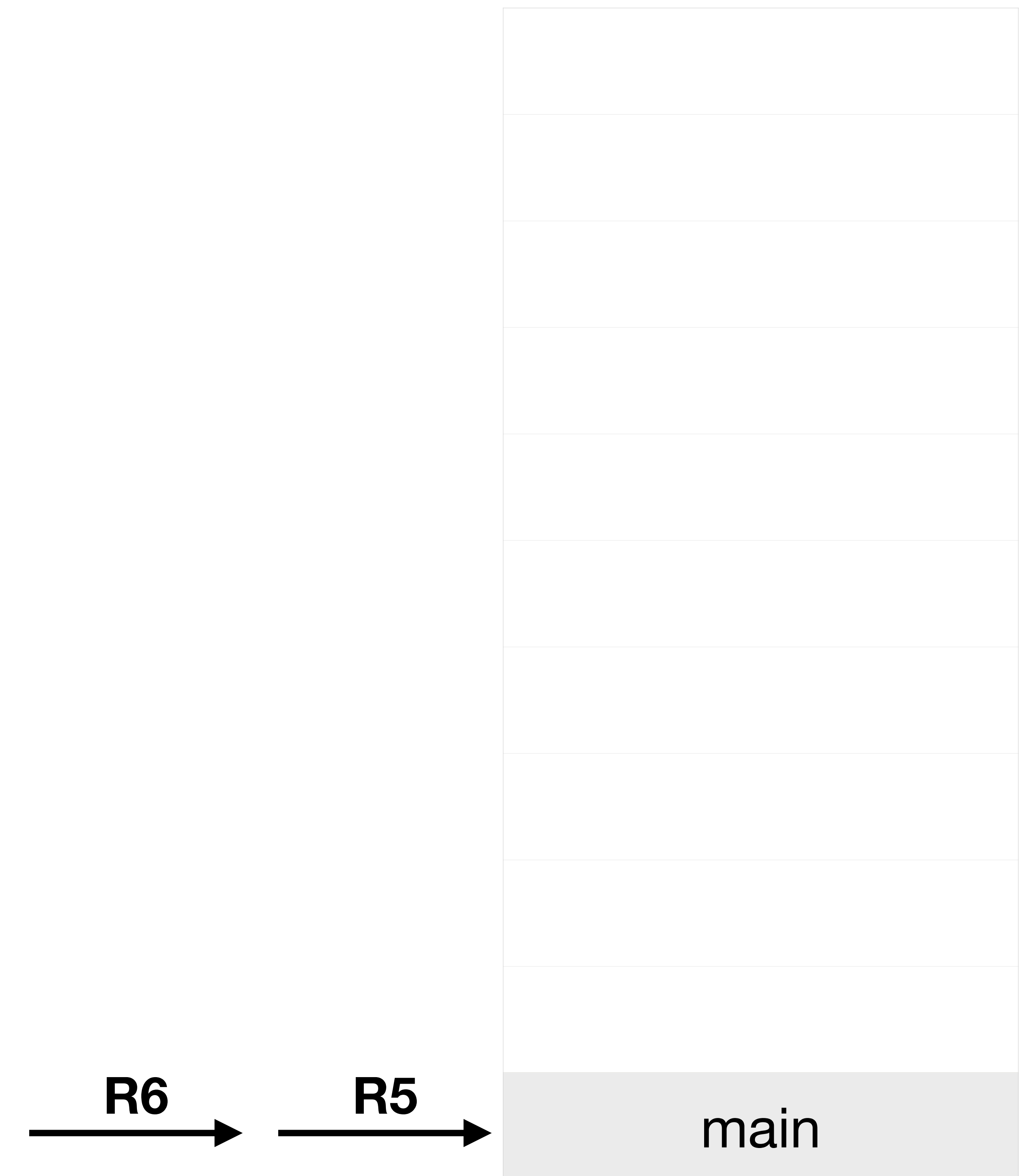
```
;Caller set-up
```



```c
int running_sum(int n){
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}
```

Recursion in LC3

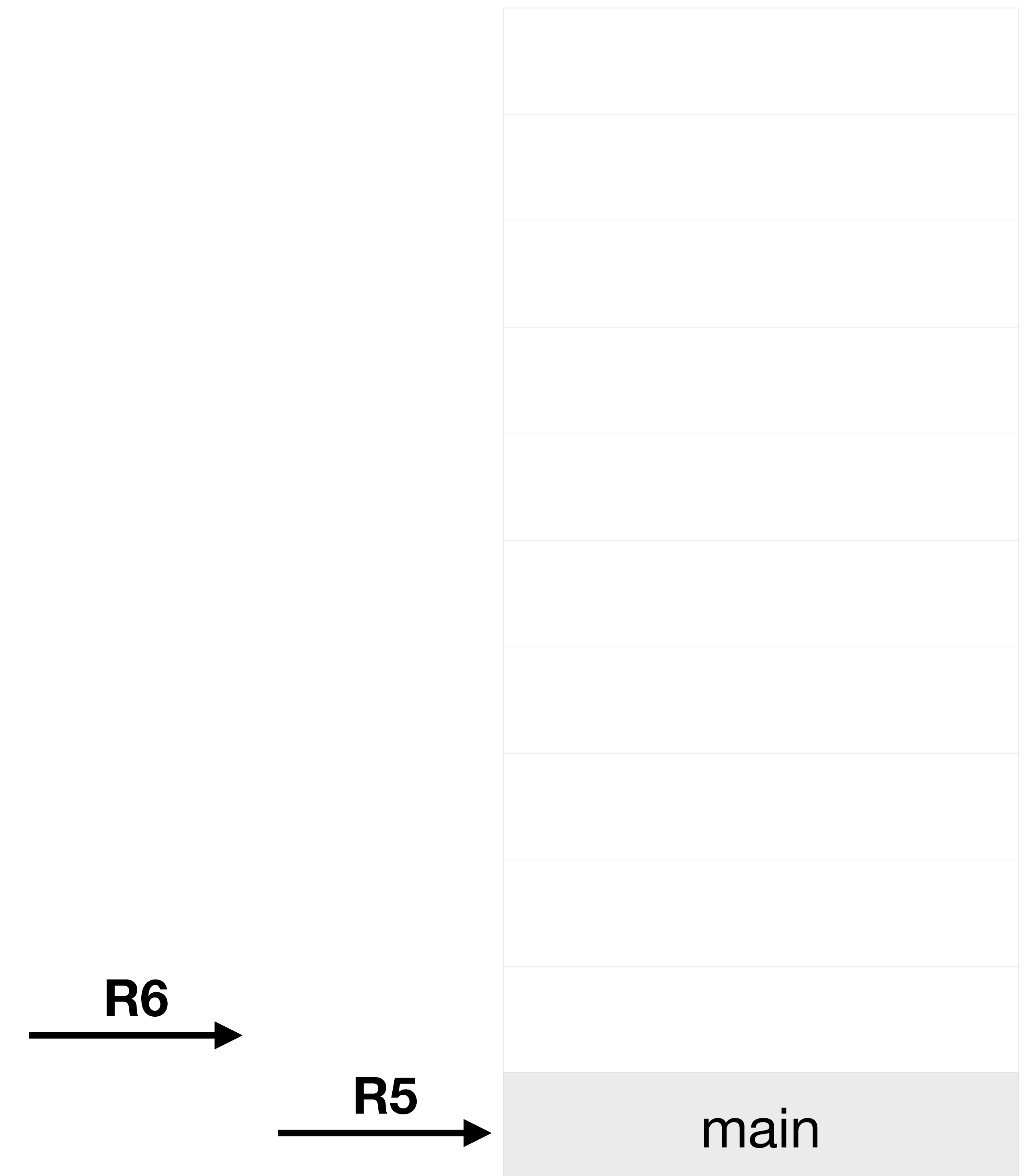
```
;Caller set-up
ADD R0, R0, #4
```



```c
int running_sum(int n){
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}
```

Recursion in LC3

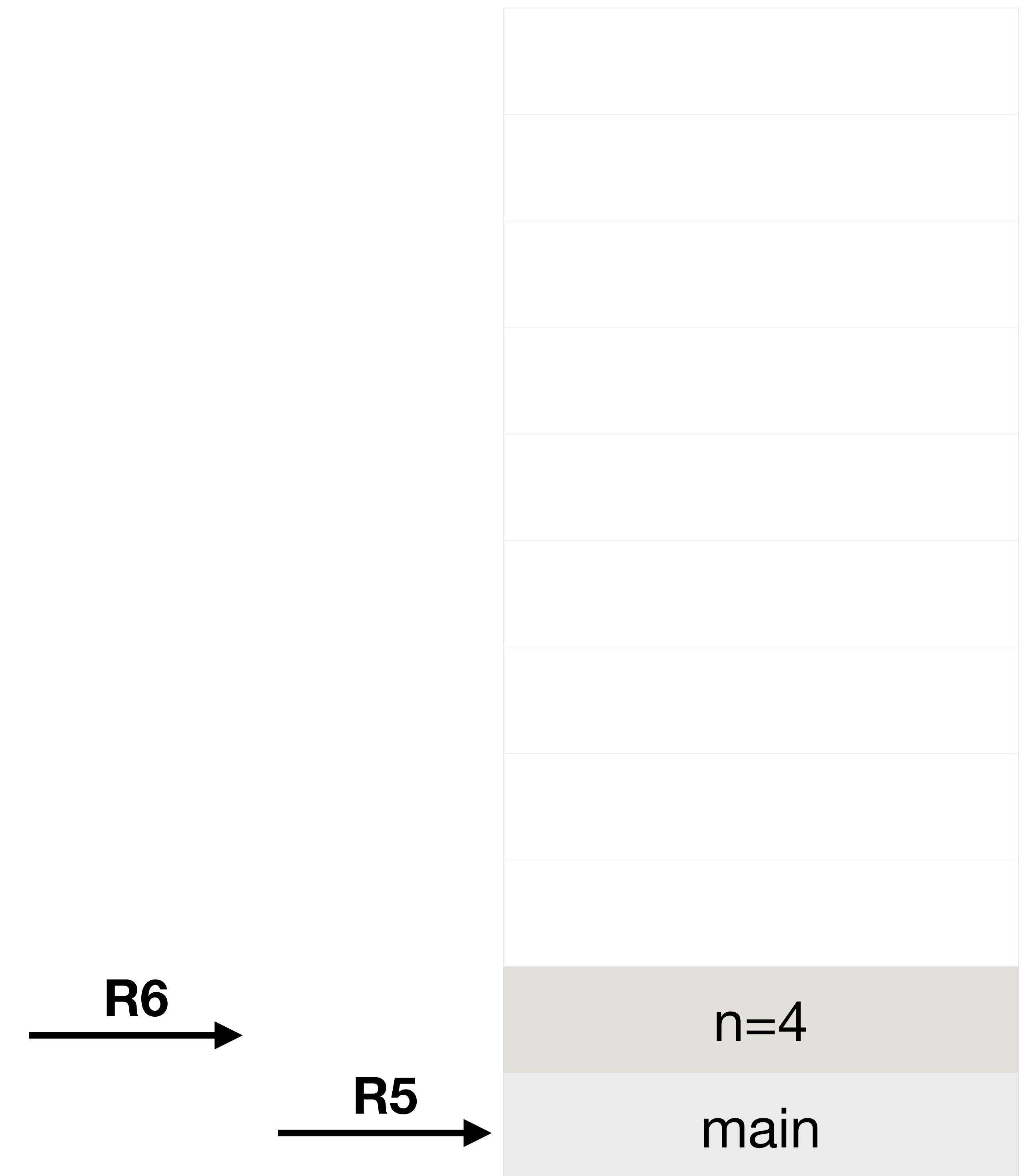
```
;Caller set-up
ADD R0, R0, #4
ADD R6, R6, #-1
```



```c
int running_sum(int n){
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}
```

Recursion in LC3

```
;Caller set-up
ADD R0, R0, #4
ADD R6, R6, #-1
STR R0, R6, #0
```

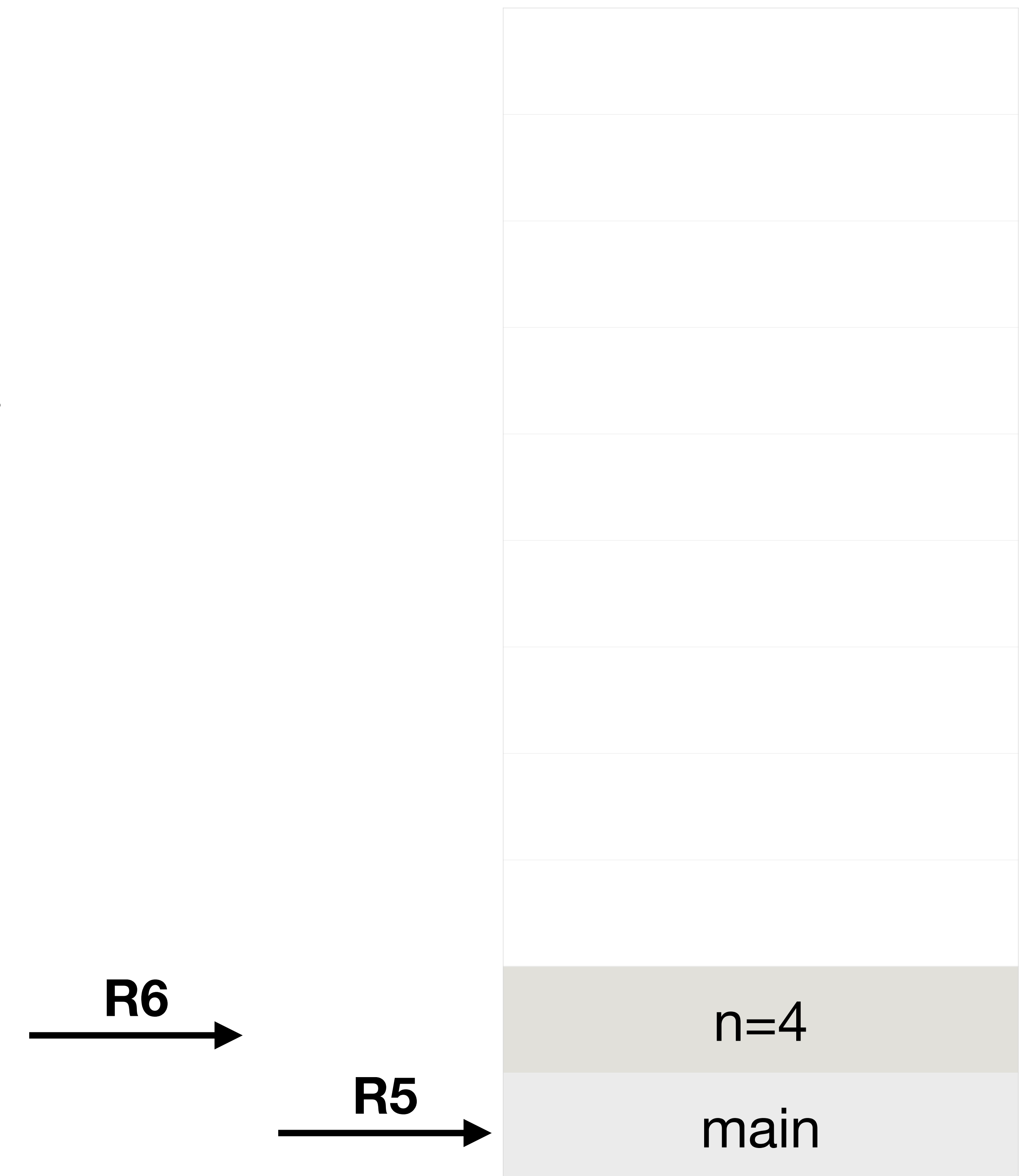


```c
int running_sum(int n){
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}
```

Recursion in LC3

```
;Caller set-up
ADD R0, R0, #4
ADD R6, R6, #-1
STR R0, R6, #0
```

```
RUNNING
;callee set-up of Running(n)'s activation record
;push return value, return address & caller's frame pointer
```

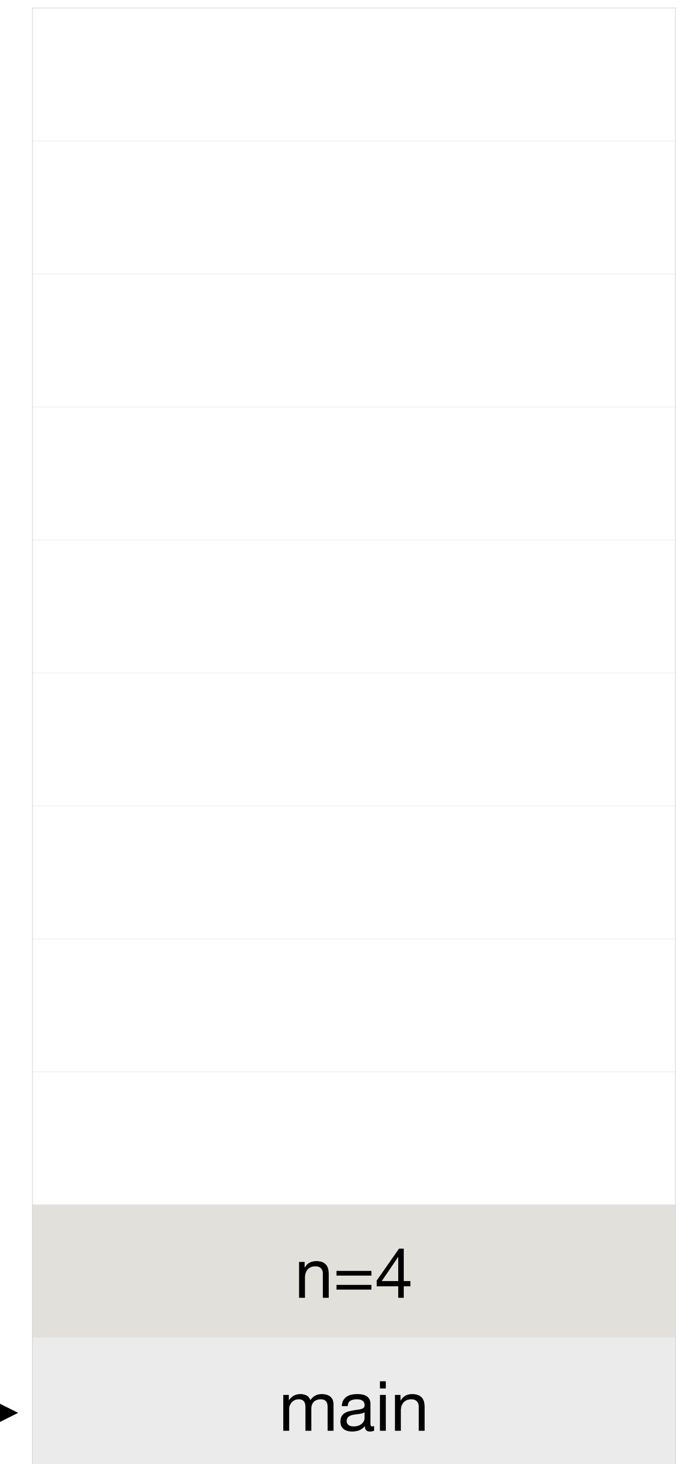


```c
int running_sum(int n){
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}
```

Recursion in LC3

R0 | 4

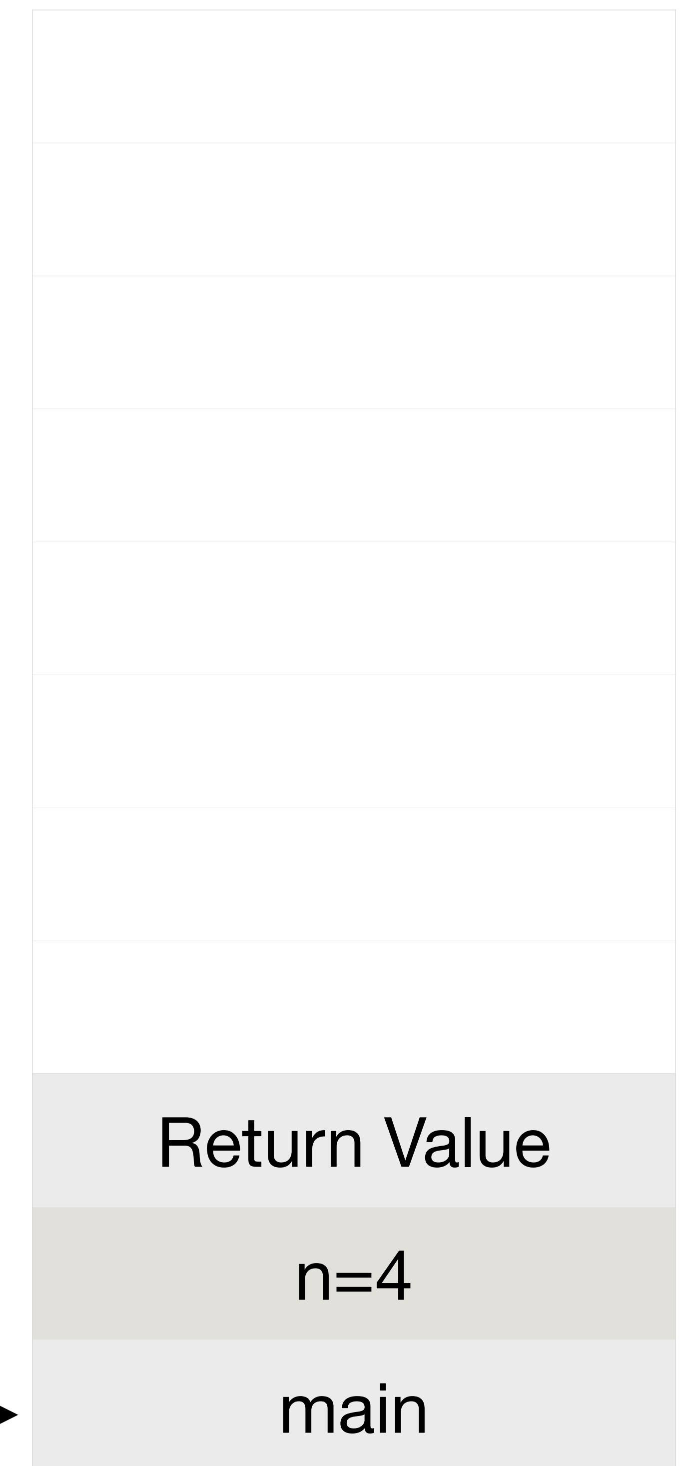
```
;Caller set-up
ADD R0, R0, #4
ADD R6, R6, #-1
STR R0, R6, #0

RUNNING
;callee set-up of Running(n)'s activation record
;push return value, return address & caller's frame pointer
ADD R6, R6, #-3
```



```c
int running_sum(int n){
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}
```

Recursion in LC3

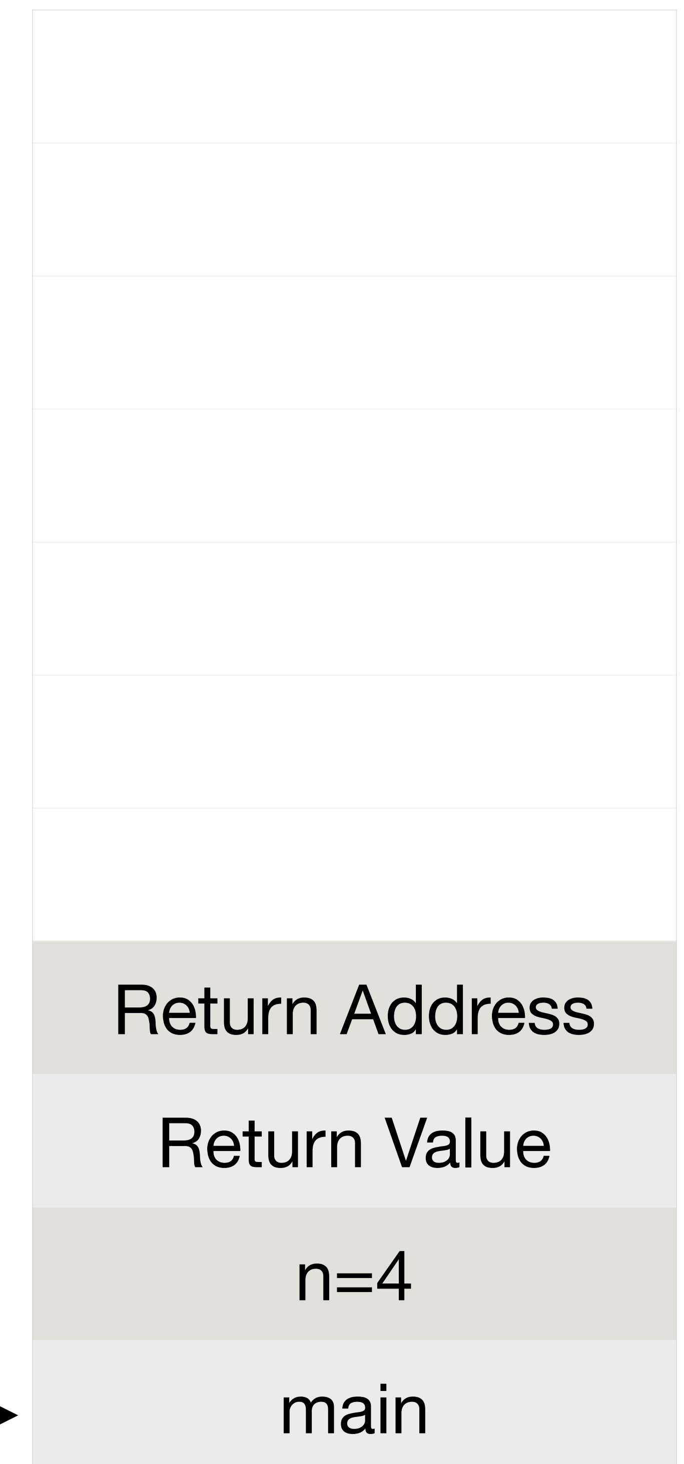
```
;Caller set-up
ADD R0, R0, #4
ADD R6, R6, #-1
STR R0, R6, #0

RUNNING
;callee set-up of Running(n)'s activation record
;push return value, return address & caller's frame pointer
ADD R6, R6, #-3
```



```c
int running_sum(int n){
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}
```

Recursion in LC3

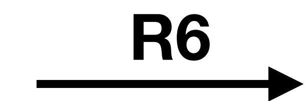
```
;Caller set-up
ADD R0, R0, #4
ADD R6, R6, #-1
STR R0, R6, #0

RUNNING
;callee set-up of Running(n)'s activation record
;push return value, return address & caller's frame pointer
ADD R6, R6, #-3
STR R7, R6, #1
```

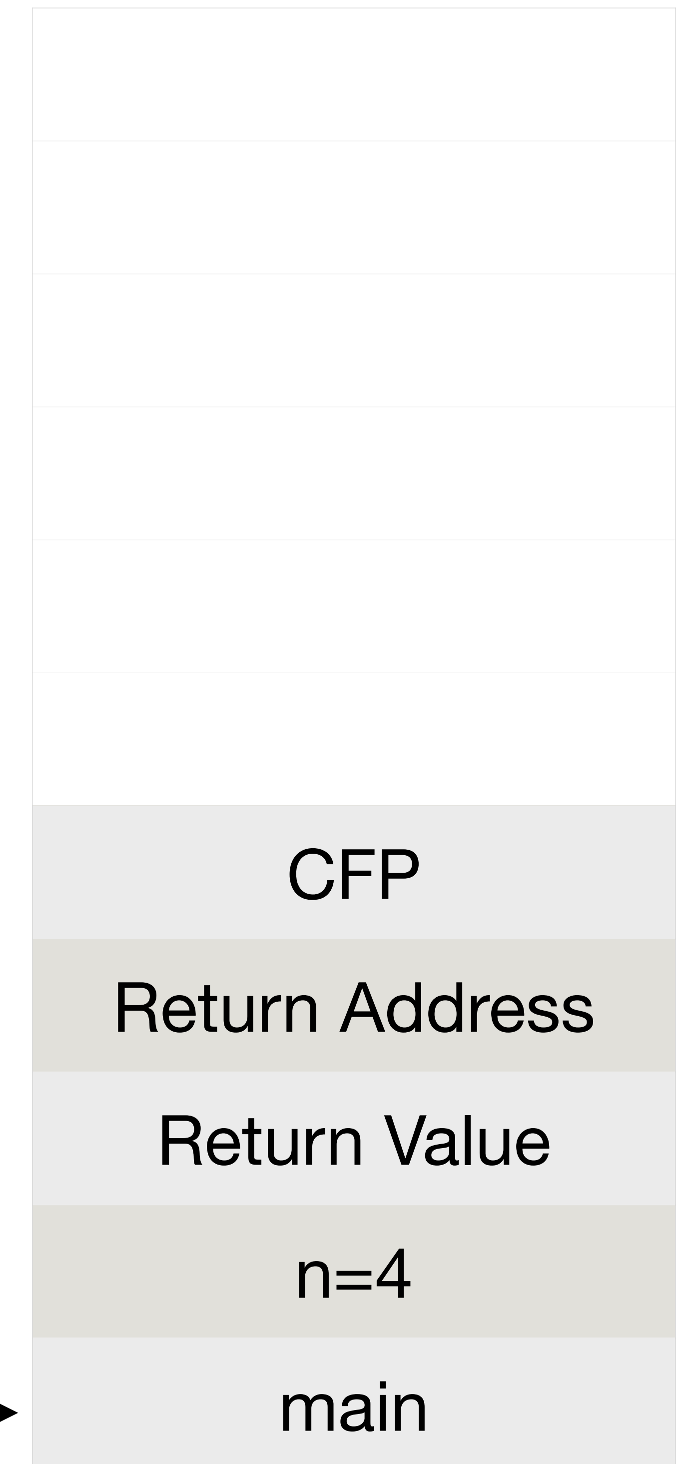


```c
int running_sum(int n){
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}
```

Recursion in LC3

```
;Caller set-up
ADD R0, R0, #4
ADD R6, R6, #-1
STR R0, R6, #0

RUNNING
;callee set-up of Running(n)'s activation record
;push return value, return address & caller's frame pointer
ADD R6, R6, #-3
STR R7, R6, #1
STR R5, R6, #0
```

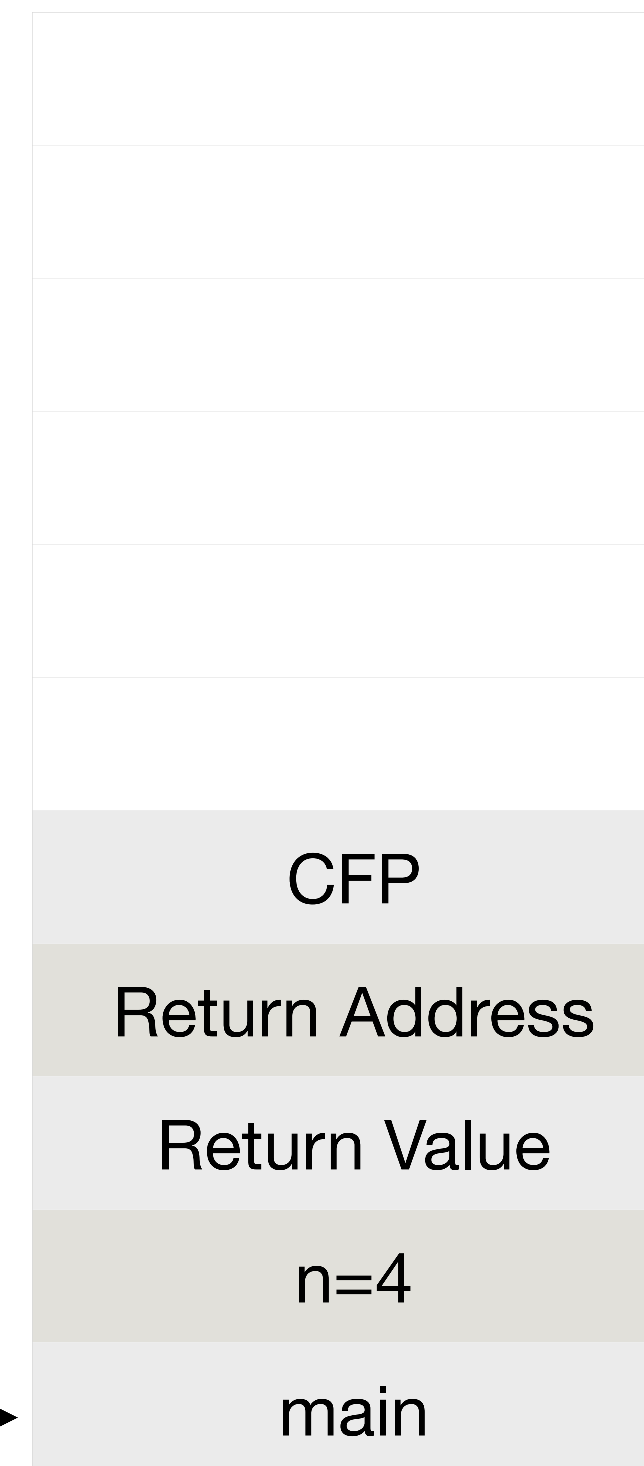




```c
int running_sum(int n){
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}
```

Recursion in LC3

```
;Caller set-up
ADD R0, R0, #4
ADD R6, R6, #-1
STR R0, R6, #0

RUNNING
;callee set-up of Running(n)'s activation record
;push return value, return address & caller's frame pointer
ADD R6, R6, #-3
STR R7, R6, #1
STR R5, R6, #0

;push local variables & update frame pointer
```



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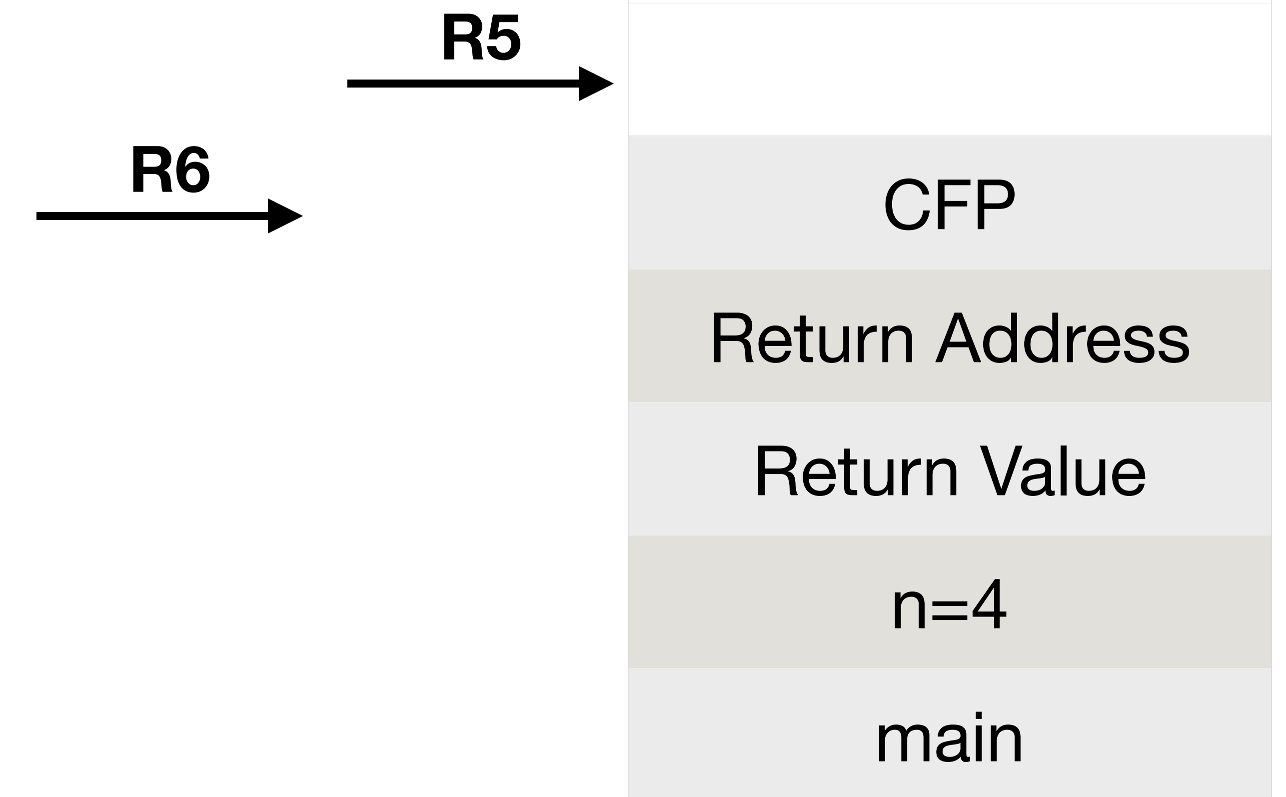
```c
int running_sum(int n){
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}
```

Recursion in LC3

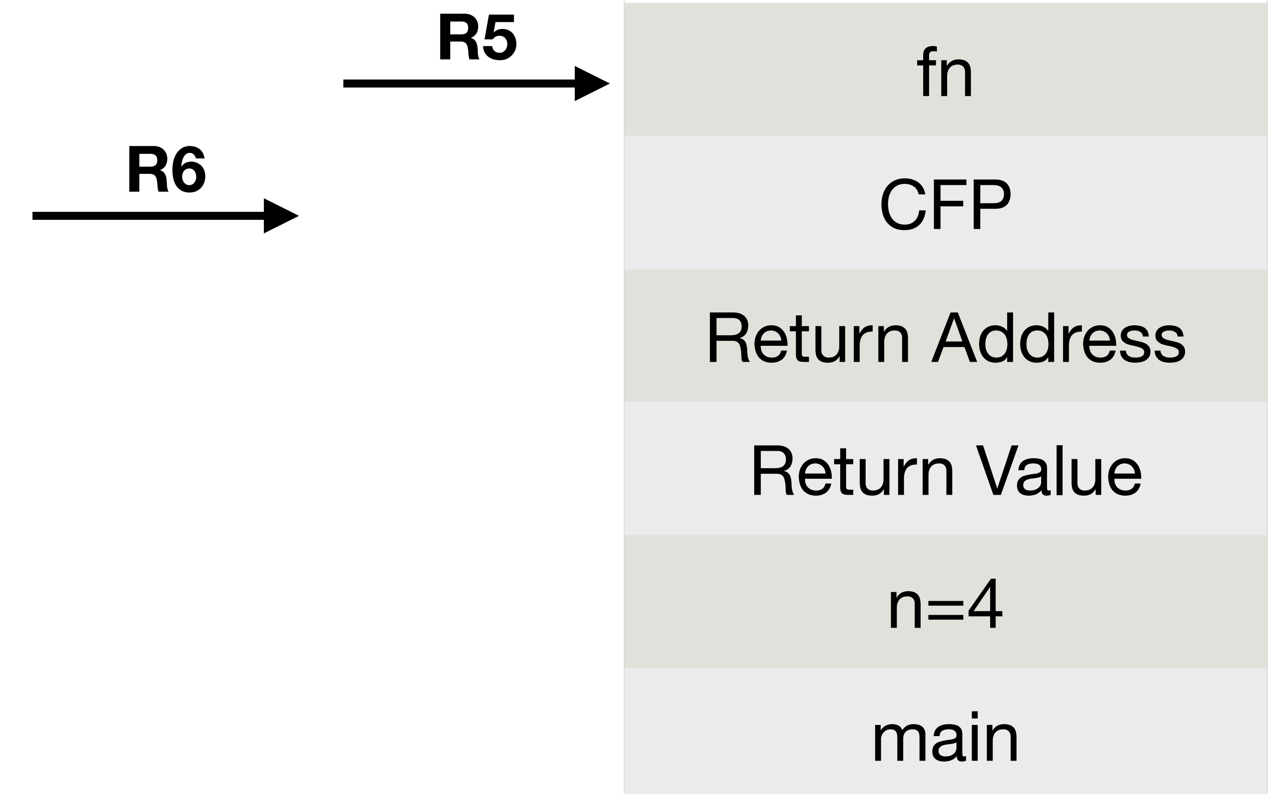
```
;Caller set-up
ADD R0, R0, #4
ADD R6, R6, #-1
STR R0, R6, #0

RUNNING
;callee set-up of Running(n)'s activation record
;push return value, return address & caller's frame pointer
ADD R6, R6, #-3
STR R7, R6, #1
STR R5, R6, #0

;push local variables & update frame pointer
ADD R5, R6, #-1
```



```c
int running_sum(int n){
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}
```

Recursion in LC3

```
;Caller set-up
ADD R0, R0, #4
ADD R6, R6, #-1
STR R0, R6, #0

RUNNING
;callee set-up of Running(n)'s activation record
;push return value, return address & caller's frame pointer
ADD R6, R6, #-3
STR R7, R6, #1
STR R5, R6, #0

;push local variables & update frame pointer
ADD R5, R6, #-1
```

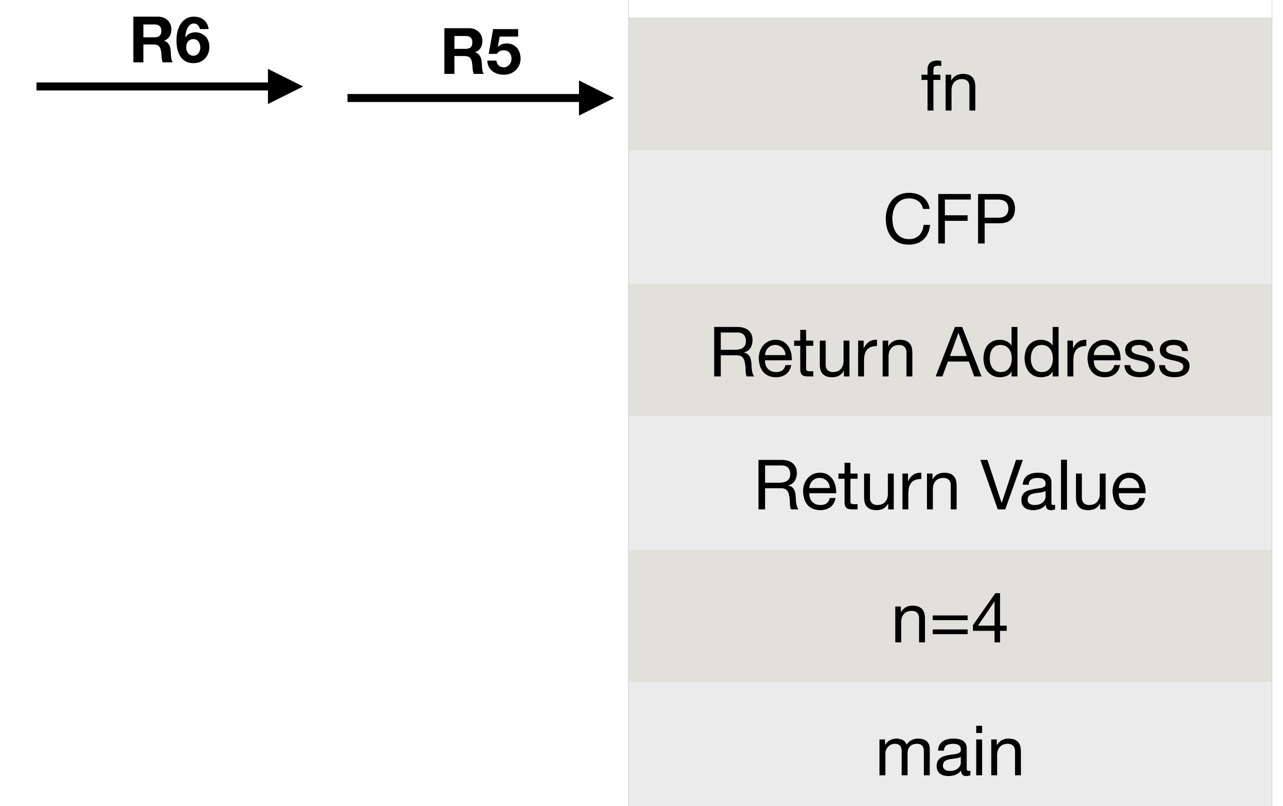


```c
int running_sum(int n){
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}
```

Recursion in LC3

```
;Caller set-up
ADD R0, R0, #4
ADD R6, R6, #-1
STR R0, R6, #0

RUNNING
;callee set-up of Running(n)'s activation record
;push return value, return address & caller's frame pointer
ADD R6, R6, #-3
STR R7, R6, #1
STR R5, R6, #0

;push local variables & update frame pointer
ADD R5, R6, #-1
ADD R6, R6, #-1
```

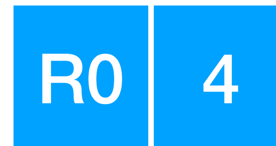
R0	4

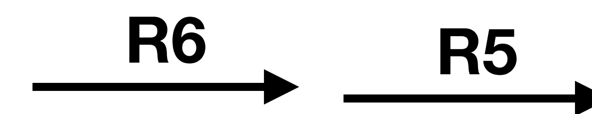


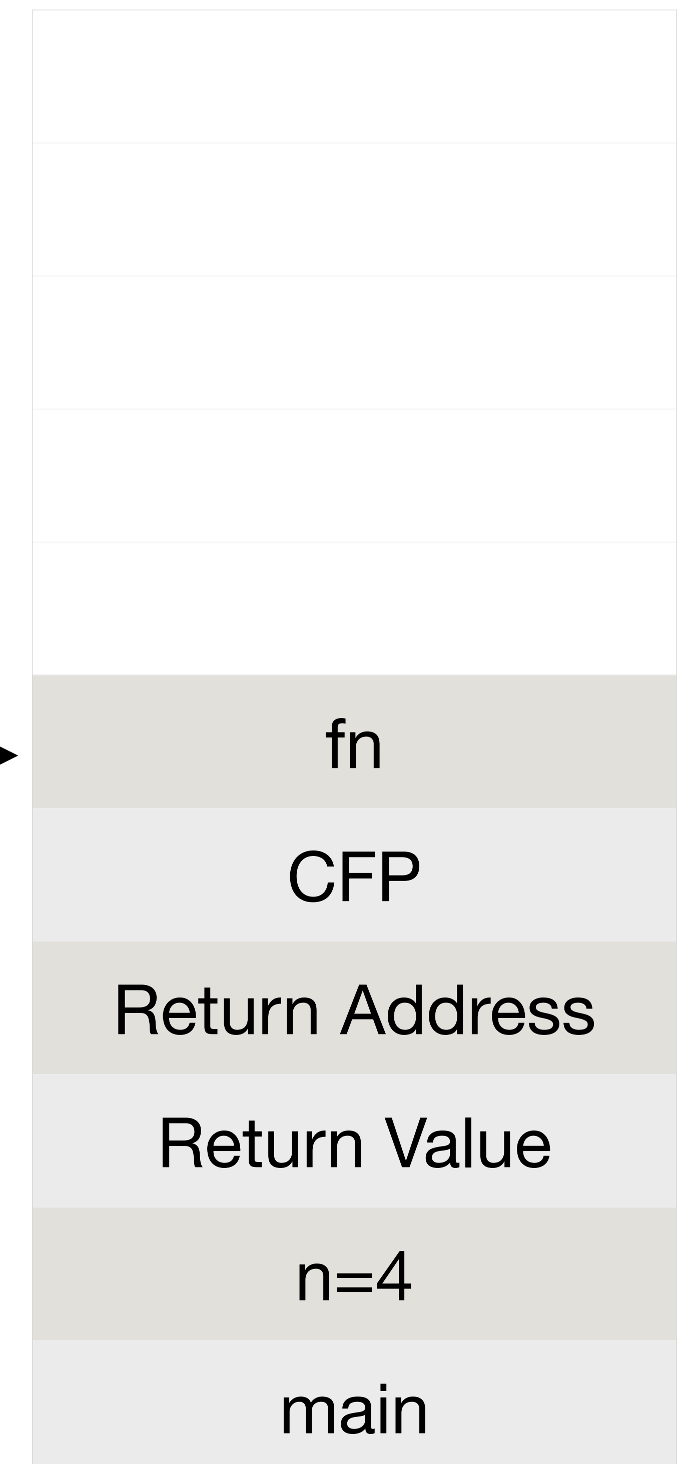
```c
int running_sum(int n){
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}
```

Recursion in LC3

```
;Caller set-up
ADD R0, R0, #4
ADD R6, R6, #-1
STR R0, R6, #0

RUNNING
;callee set-up of Running(n)'s activation record
;push return value, return address & caller's frame pointer
ADD R6, R6, #-3
STR R7, R6, #1
STR R5, R6, #0

;push local variables & update frame pointer
ADD R5, R6, #-1
ADD R6, R6, #-1

;function logic
```



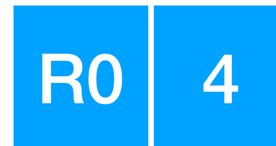


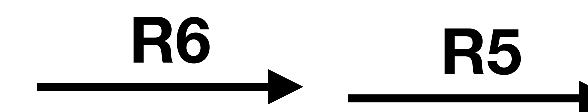


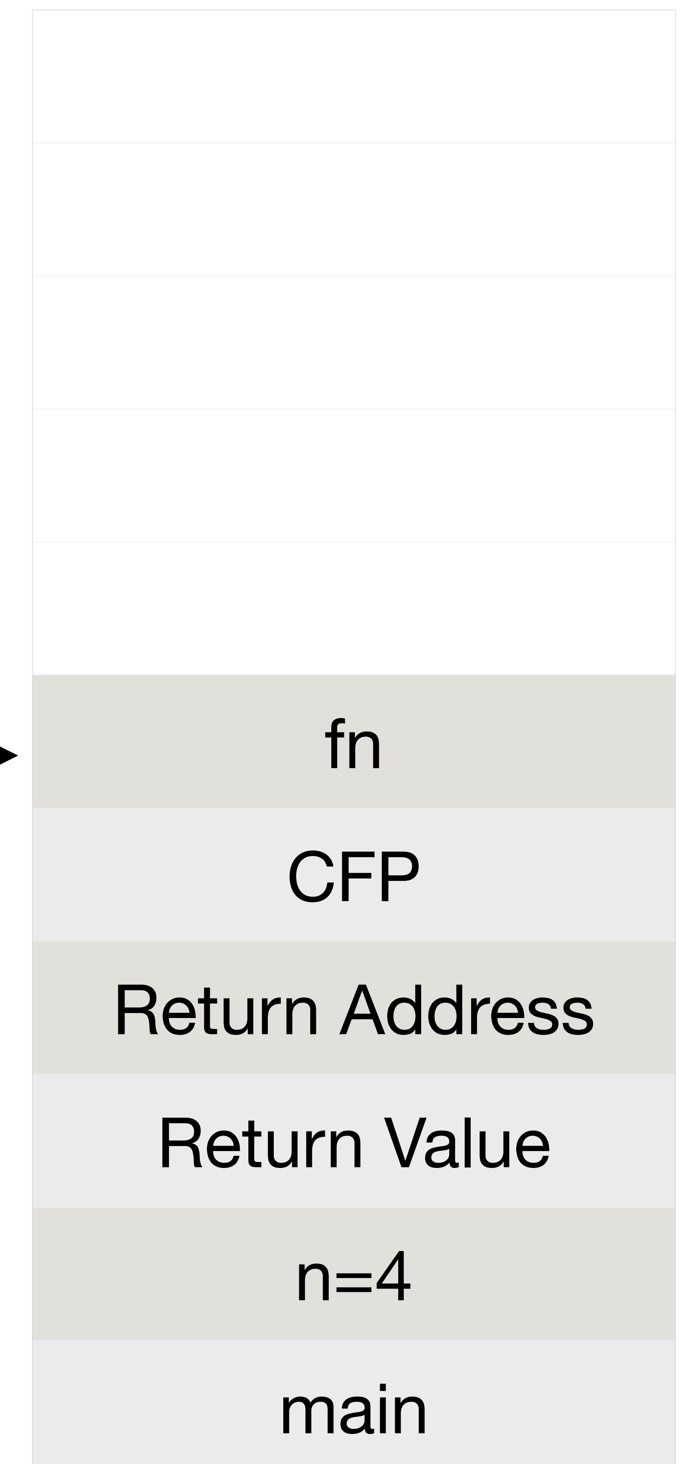
```c
int running_sum(int n){
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}
```

Recursion in LC3

```
;Caller set-up
ADD R0, R0, #4
ADD R6, R6, #-1
STR R0, R6, #0

RUNNING
;callee set-up of Running(n)'s activation record
;push return value, return address & caller's frame pointer
ADD R6, R6, #-3
STR R7, R6, #1
STR R5, R6, #0

;push local variables & update frame pointer
ADD R5, R6, #-1
ADD R6, R6, #-1

;function logic
;base case (n==1)
```





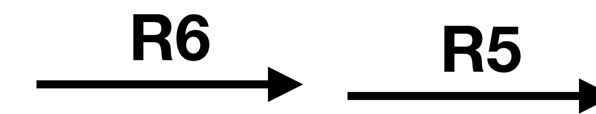


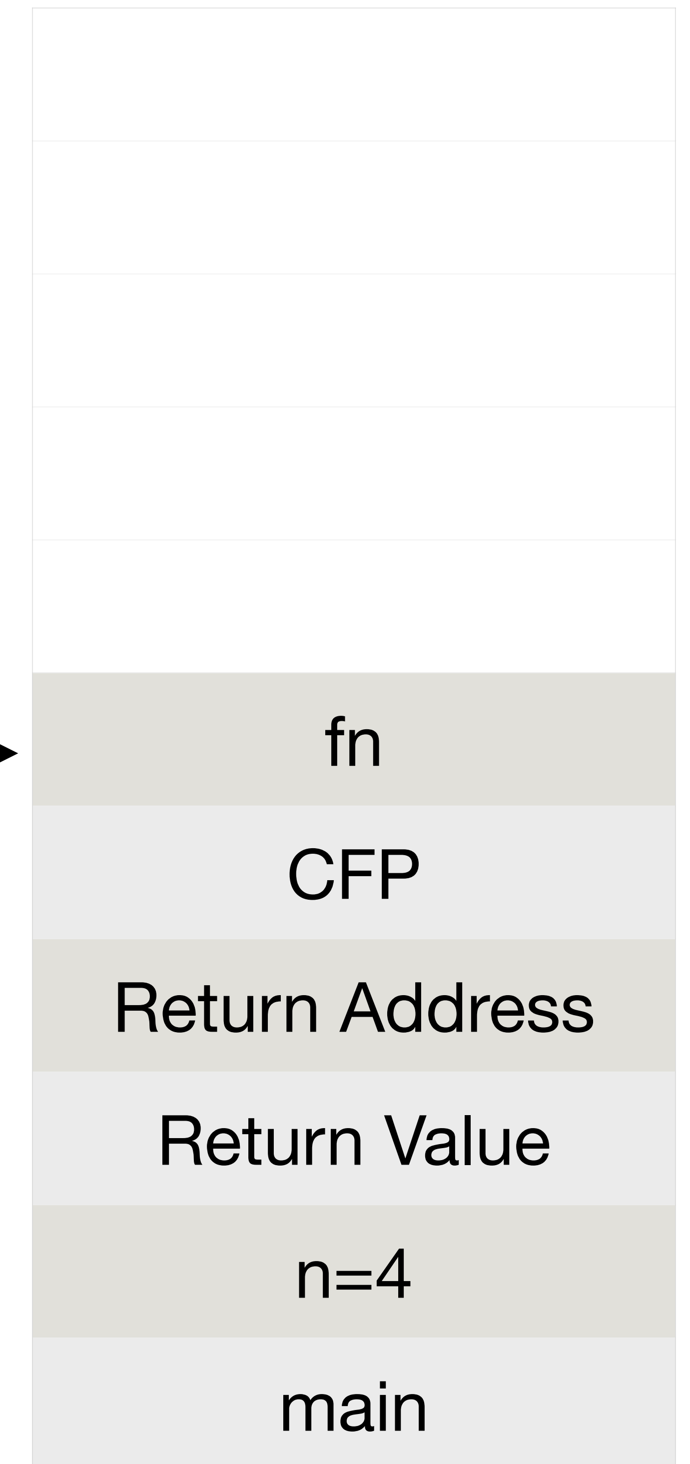
```c
int running_sum(int n){
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}
```

Recursion in LC3

```
;Caller set-up
ADD R0, R0, #4
ADD R6, R6, #-1
STR R0, R6, #0

RUNNING
;callee set-up of Running(n)'s activation record
;push return value, return address & caller's frame pointer
ADD R6, R6, #-3
STR R7, R6, #1
STR R5, R6, #0

;push local variables & update frame pointer
ADD R5, R6, #-1
ADD R6, R6, #-1

;function logic
;base case (n==1)
LDR R1, R5, #4
```

R0	4
R1	4

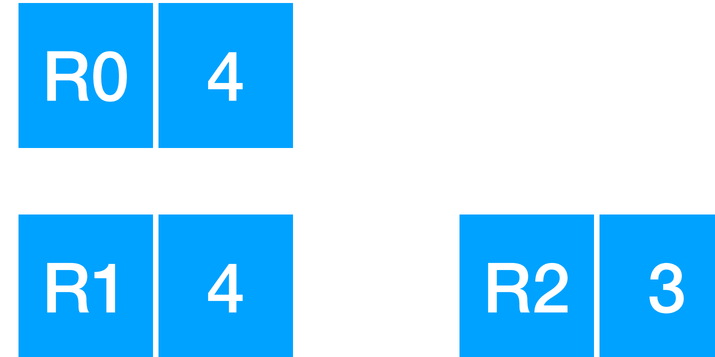


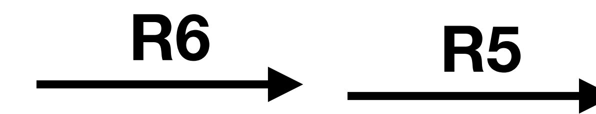


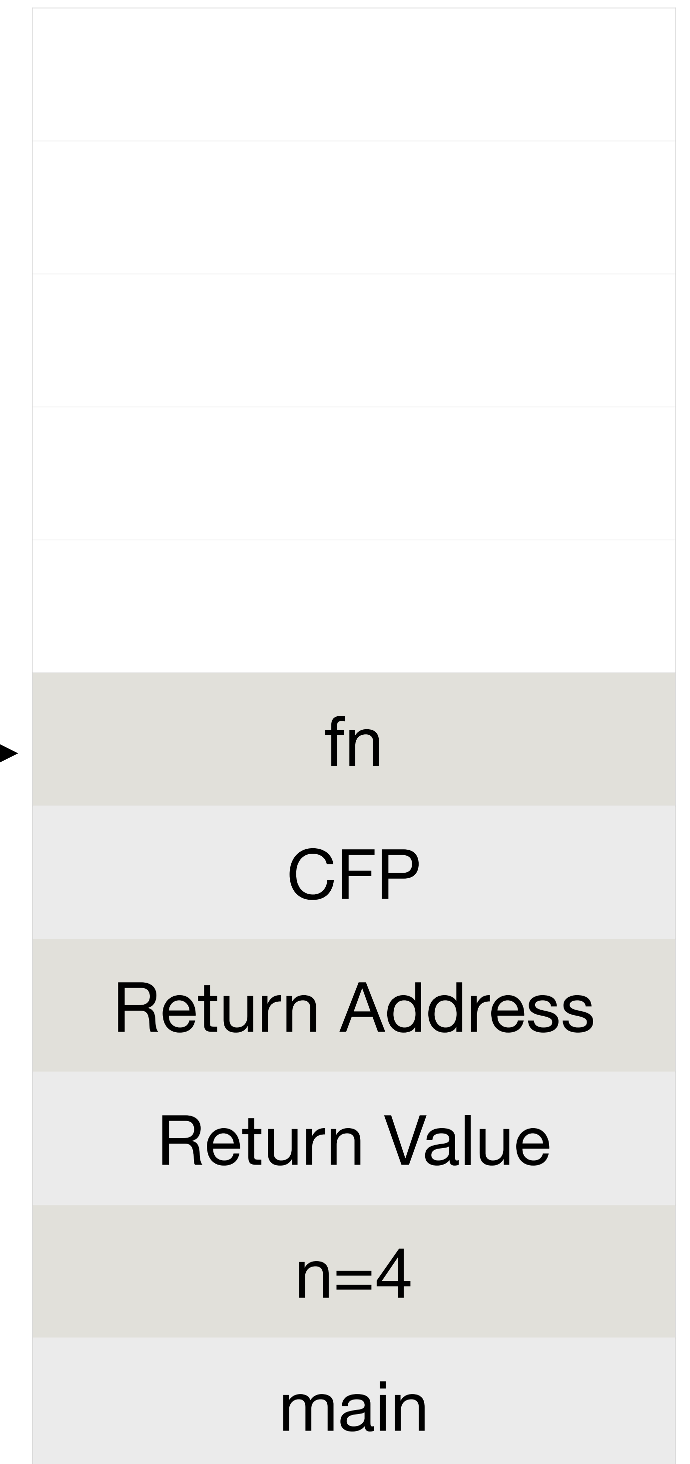
```c
int running_sum(int n){
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}
```

Recursion in LC3

```
;Caller set-up
ADD R0, R0, #4
ADD R6, R6, #-1
STR R0, R6, #0

RUNNING
;callee set-up of Running(n)'s activation record
;push return value, return address & caller's frame pointer
ADD R6, R6, #-3
STR R7, R6, #1
STR R5, R6, #0

;push local variables & update frame pointer
ADD R5, R6, #-1
ADD R6, R6, #-1

;function logic
;base case (n==1)
LDR R1, R5, #4
ADD R2, R1, #-1
```



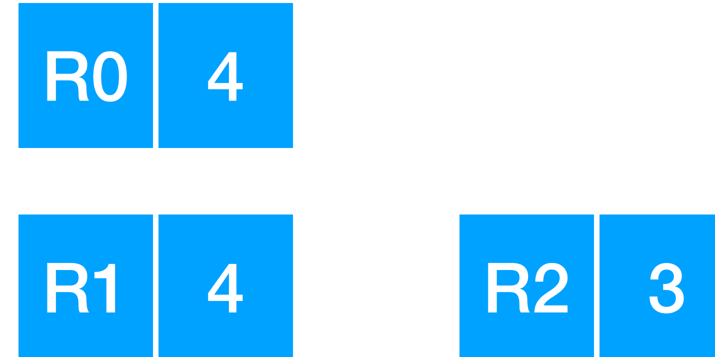




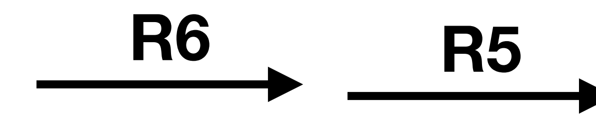
```c
int running_sum(int n){
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}
```

Recursion in LC3

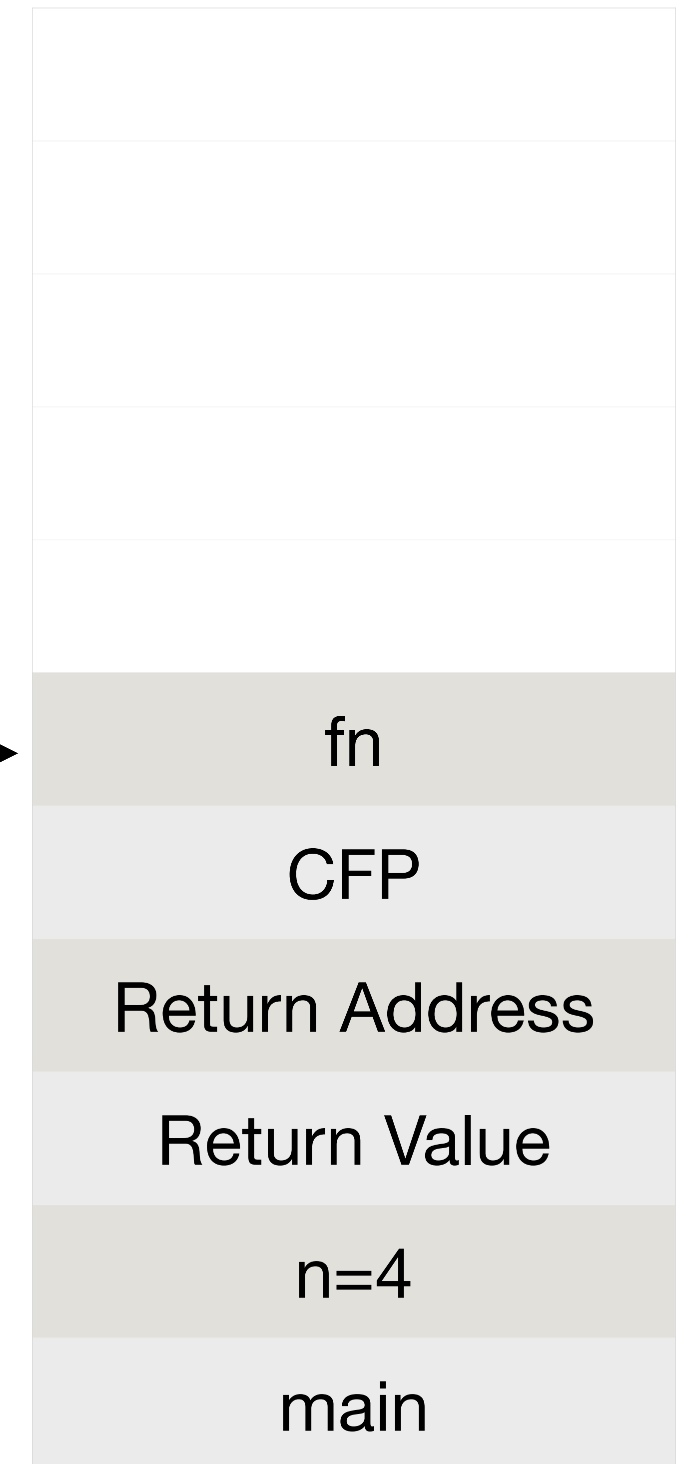
```
;Caller set-up
ADD R0, R0, #4
ADD R6, R6, #-1
STR R0, R6, #0

RUNNING
;callee set-up of Running(n)'s activation record
;push return value, return address & caller's frame pointer
ADD R6, R6, #-3
STR R7, R6, #1
STR R5, R6, #0

;push local variables & update frame pointer
ADD R5, R6, #-1
ADD R6, R6, #-1

;function logic
;base case (n==1)
LDR R1, R5, #4
ADD R2, R1, #-1
BRz BASE_CASE
```



R2 3

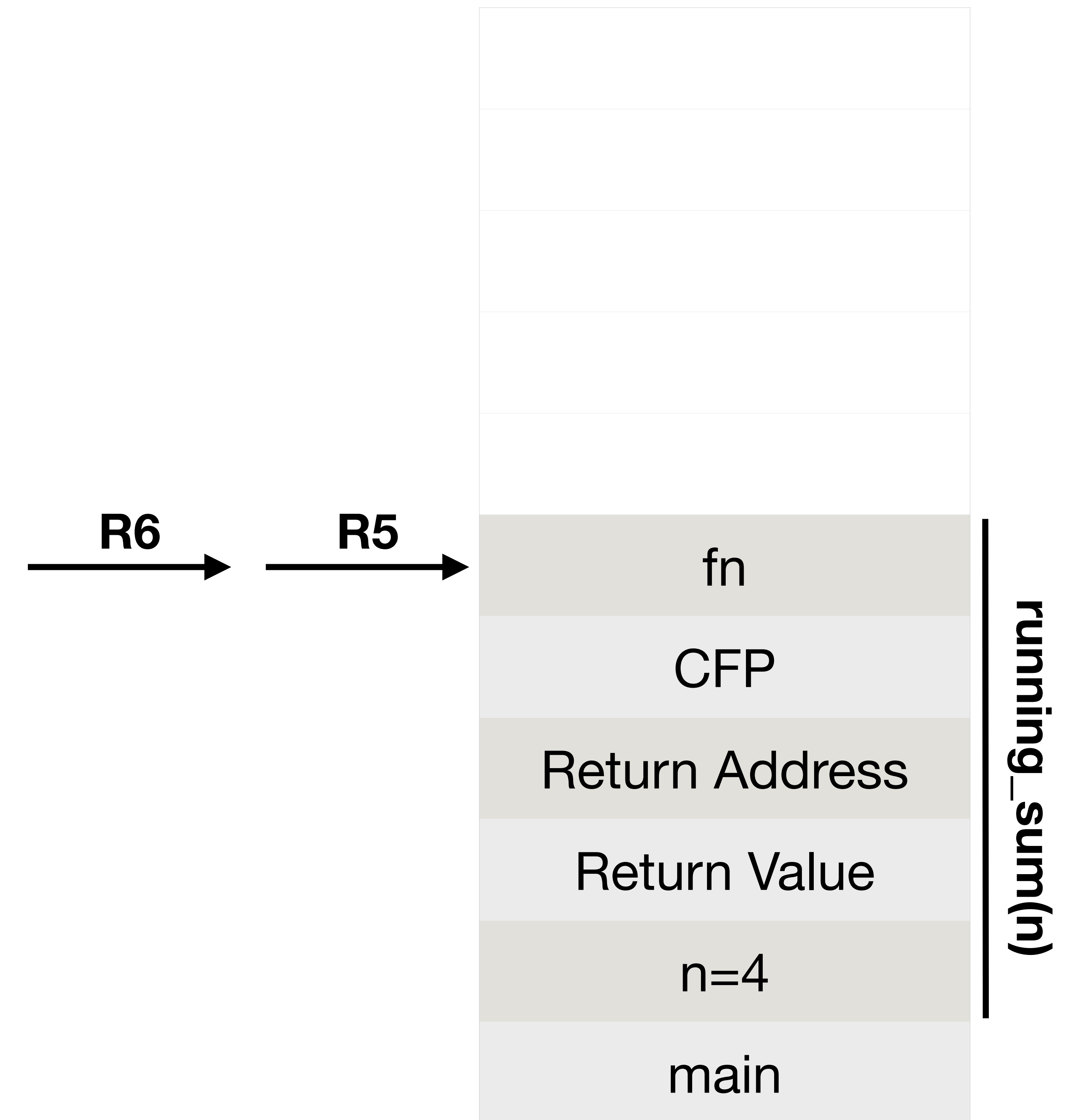


R5



```
int running_sum(int n){
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}
```

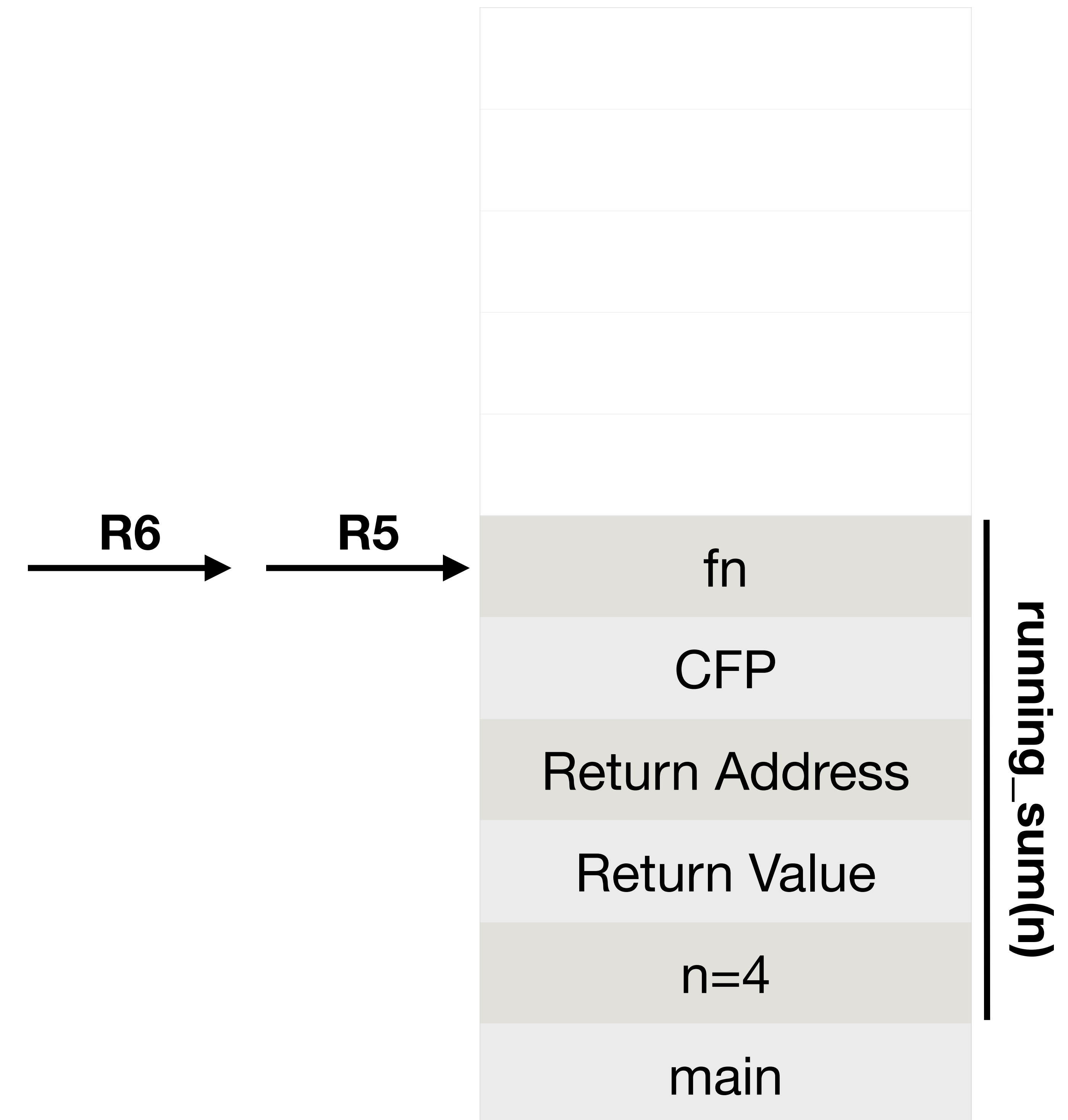
Recursion in LC3



```c
int running_sum(int n){
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}
```

Recursion in LC3

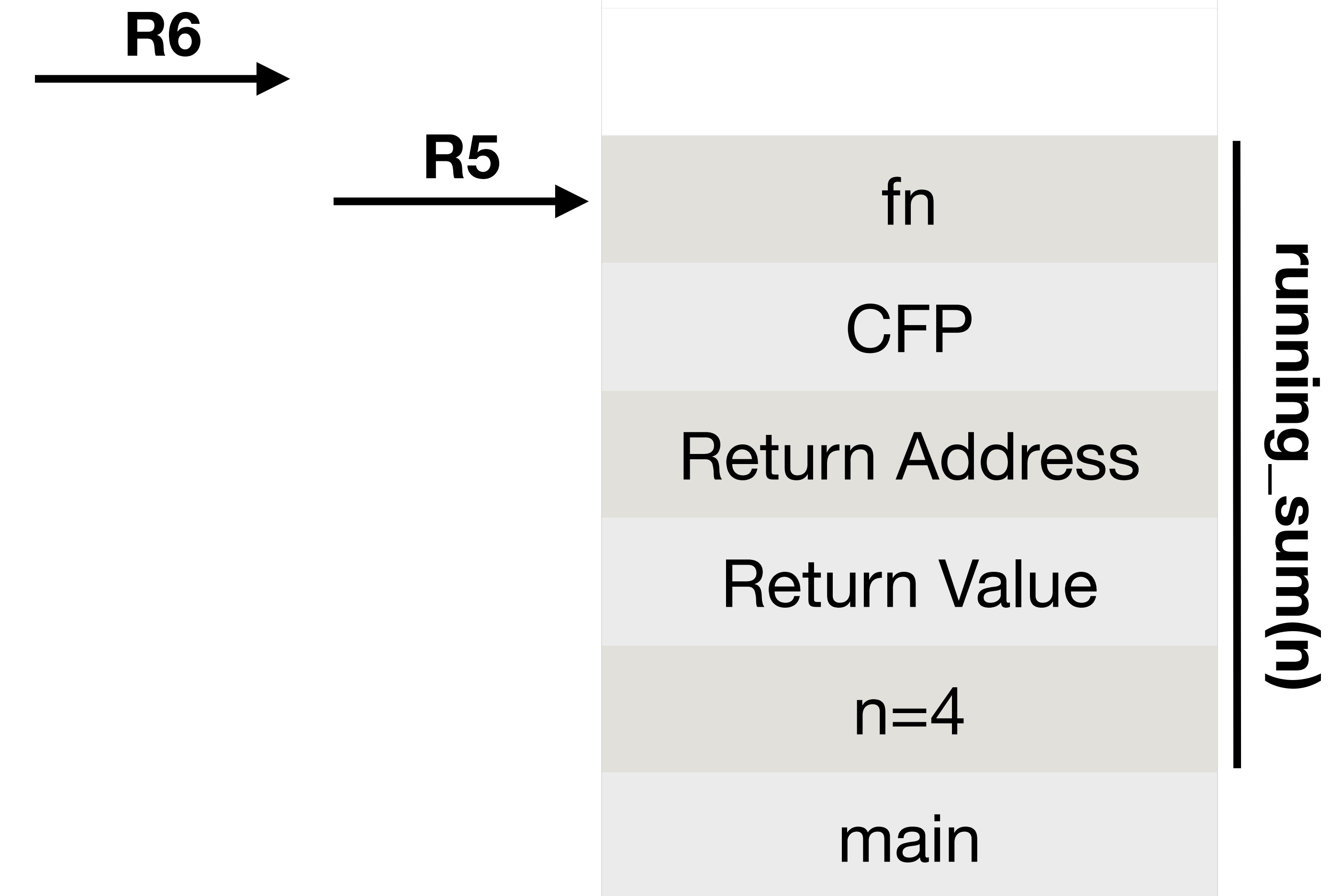
```
;Recursive case
;Caller setup: push argument n-1 onto RTS
ADD R6, R6, #-1
```



```c
int running_sum(int n){
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}
```

Recursion in LC3

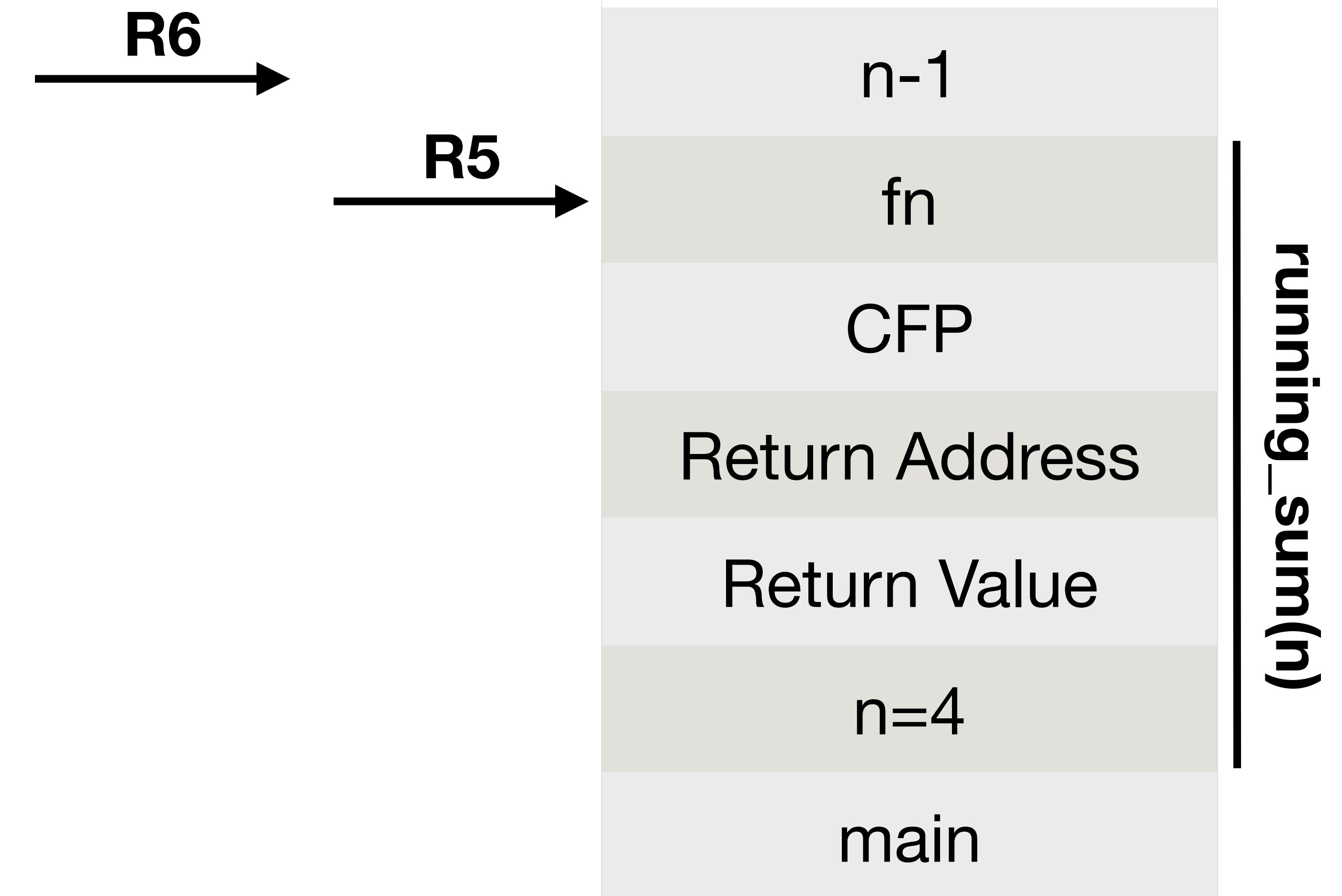
```
;Recursive case
;Caller setup: push argument n-1 onto RTS
ADD R6, R6, #-1
STR R2, R6, #0 ; R2 = n - 1
```



```c
int running_sum(int n){
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}
```

Recursion in LC3

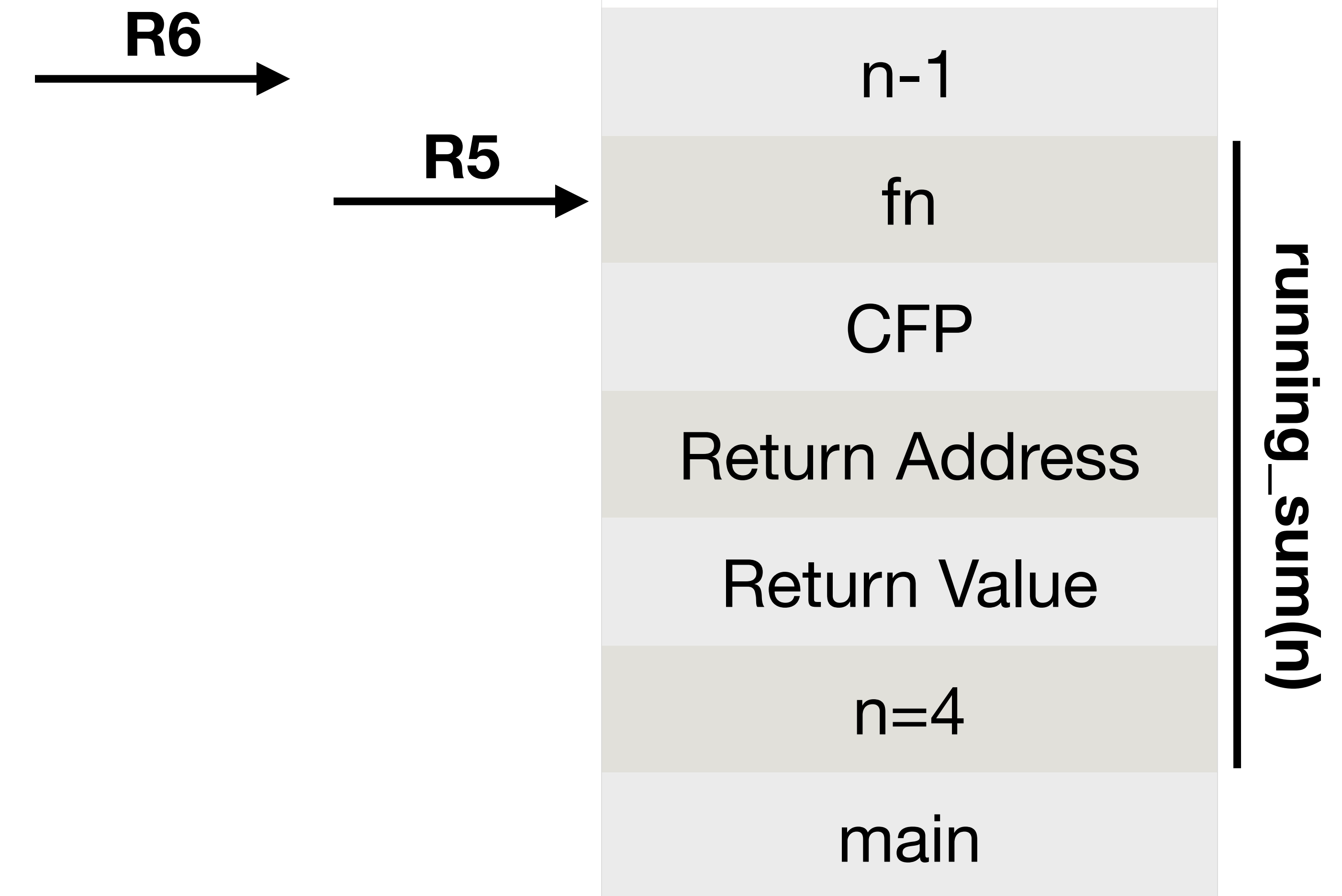
```
;Recursive case
;Caller setup: push argument n-1 onto RTS
ADD R6, R6, #-1
STR R2, R6, #0 ; R2 = n - 1
```



```c
int running_sum(int n){
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}
```

Recursion in LC3

```
;Recursive case
;Caller setup: push argument n-1 onto RTS
ADD R6, R6, #-1
STR R2, R6, #0 ; R2 = n - 1
JSR RUNNING ; call Running(n-1)
```



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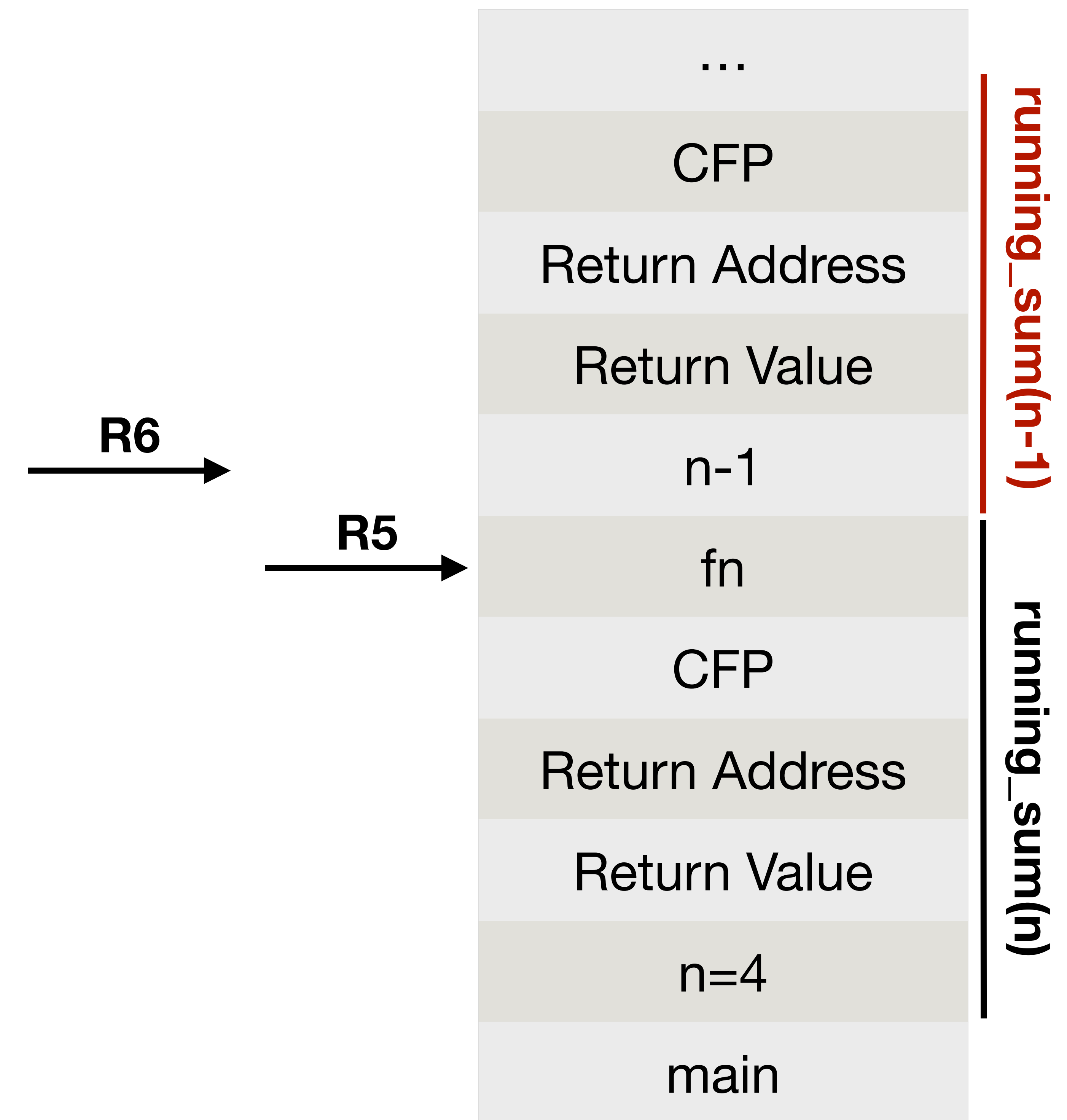
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}
```

Recursion in LC3

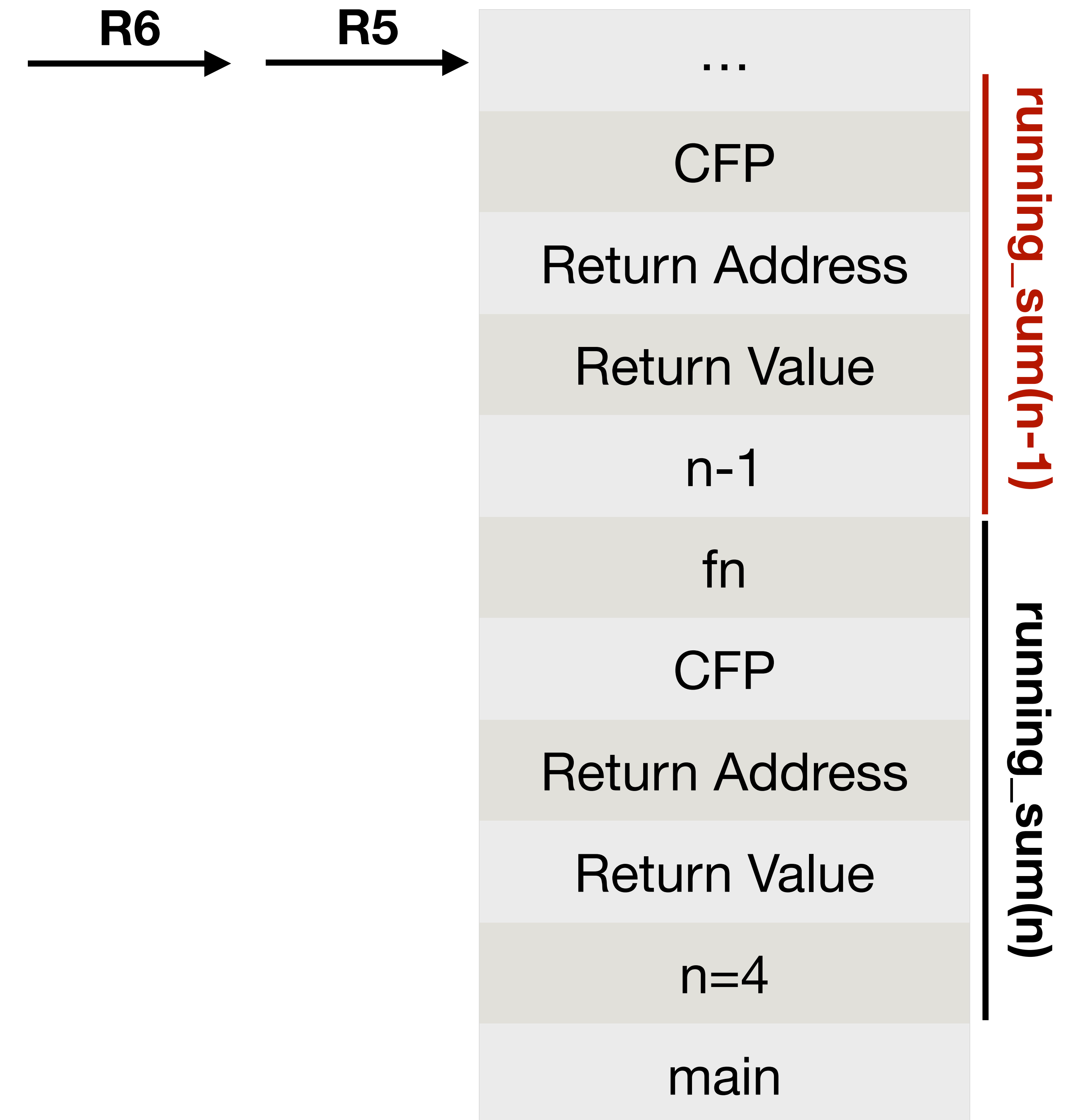
```
;Recursive case
;Caller setup: push argument n-1 onto RTS
ADD R6, R6, #-1
STR R2, R6, #0 ; R2 = n - 1
JSR RUNNING ; call Running(n-1)
```



```c
int running_sum(int n){
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}
```

Recursion in LC3

```
;Recursive case
;Caller setup: push argument n-1 onto RTS
ADD R6, R6, #-1
STR R2, R6, #0 ; R2 = n - 1
JSR RUNNING ; call Running(n-1)
```

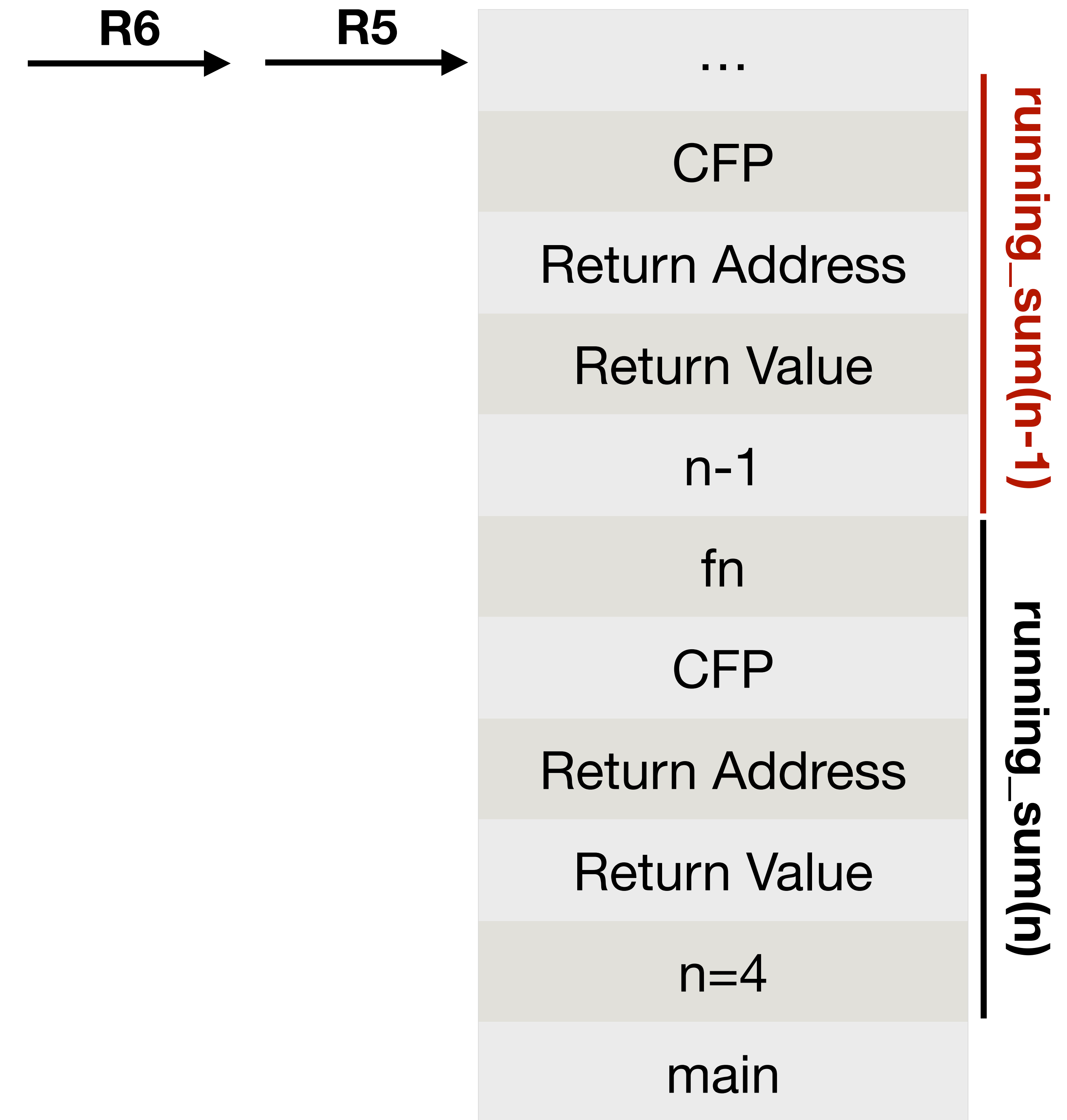


```c
int running_sum(int n){
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}
```

Recursion in LC3

```
;Recursive case
;Caller setup: push argument n-1 onto RTS
ADD R6, R6, #-1
STR R2, R6, #0 ; R2 = n - 1
JSR RUNNING ; call Running(n-1)

;Callee tear-down for Running(n-1) not shown
```

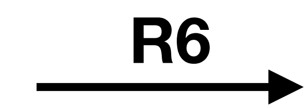


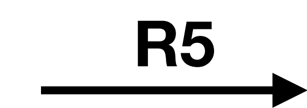
```c
int running_sum(int n){
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}
```

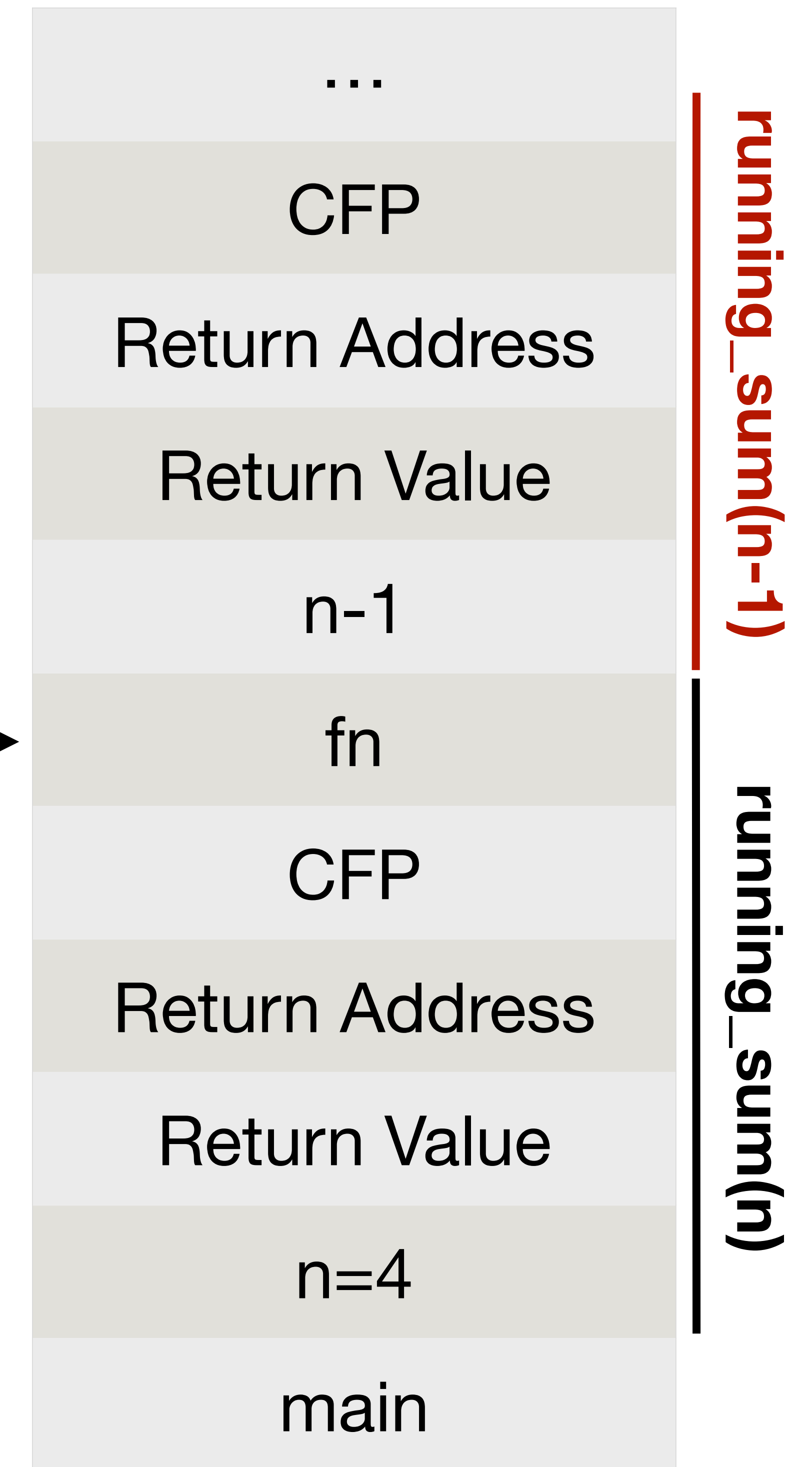
Recursion in LC3

```
;Recursive case
;Caller setup: push argument n-1 onto RTS
ADD R6, R6, #-1
STR R2, R6, #0 ; R2 = n - 1
JSR RUNNING ; call Running(n-1)

;Callee tear-down for Running(n-1) not shown
```







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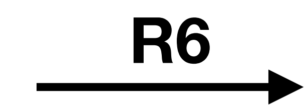
```c
int running_sum(int n){
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}
```

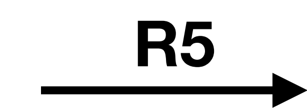
Recursion in LC3

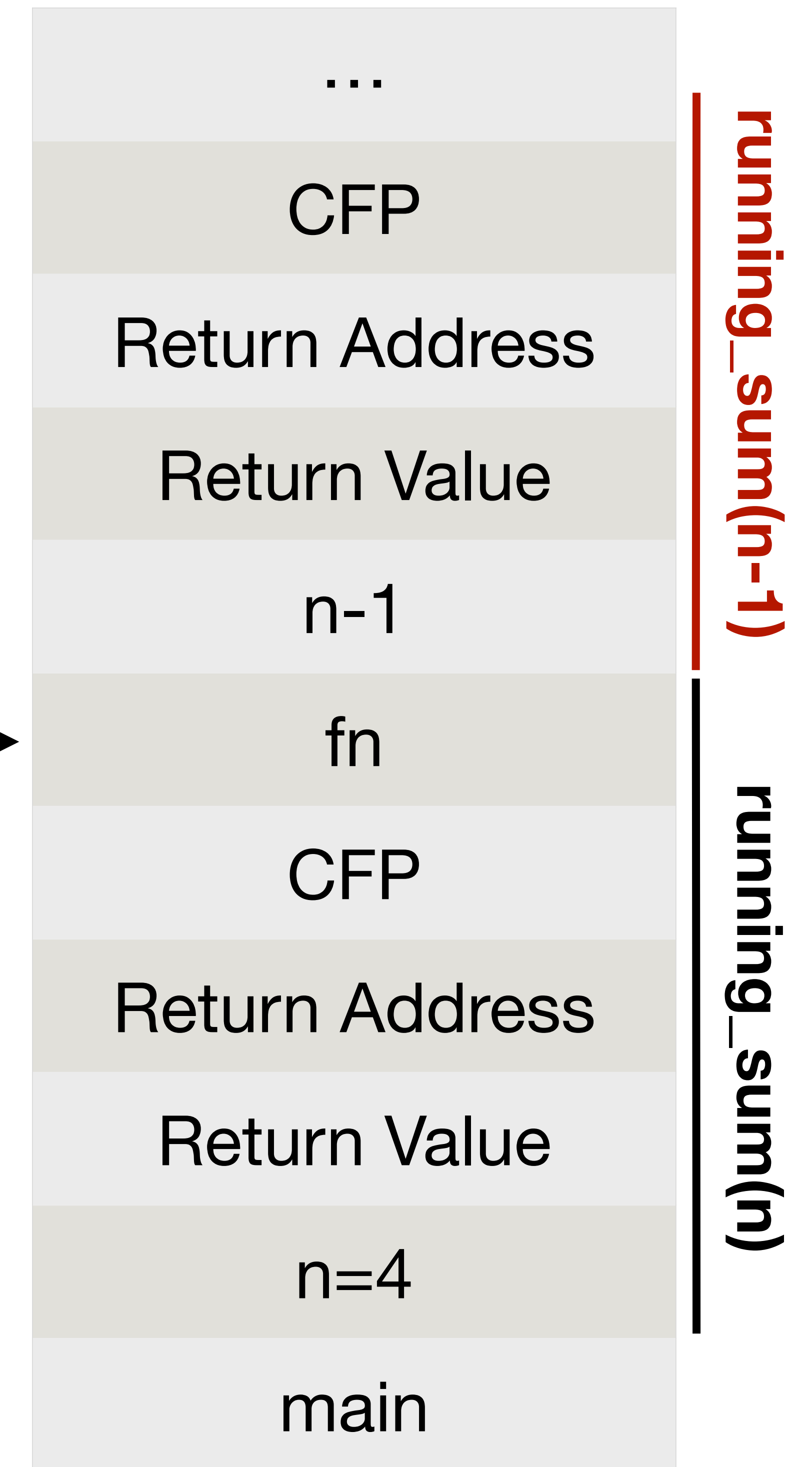
```
;Recursive case
;Caller setup: push argument n-1 onto RTS
ADD R6, R6, #-1
STR R2, R6, #0 ; R2 = n - 1
JSR RUNNING ; call Running(n-1)

;Callee tear-down for Running(n-1) not shown

;Caller tear-down for Running(n-1)
```







ILLINOIS

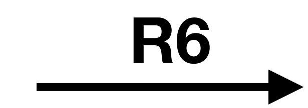
```c
int running_sum(int n){
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}
```

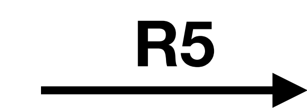
Recursion in LC3

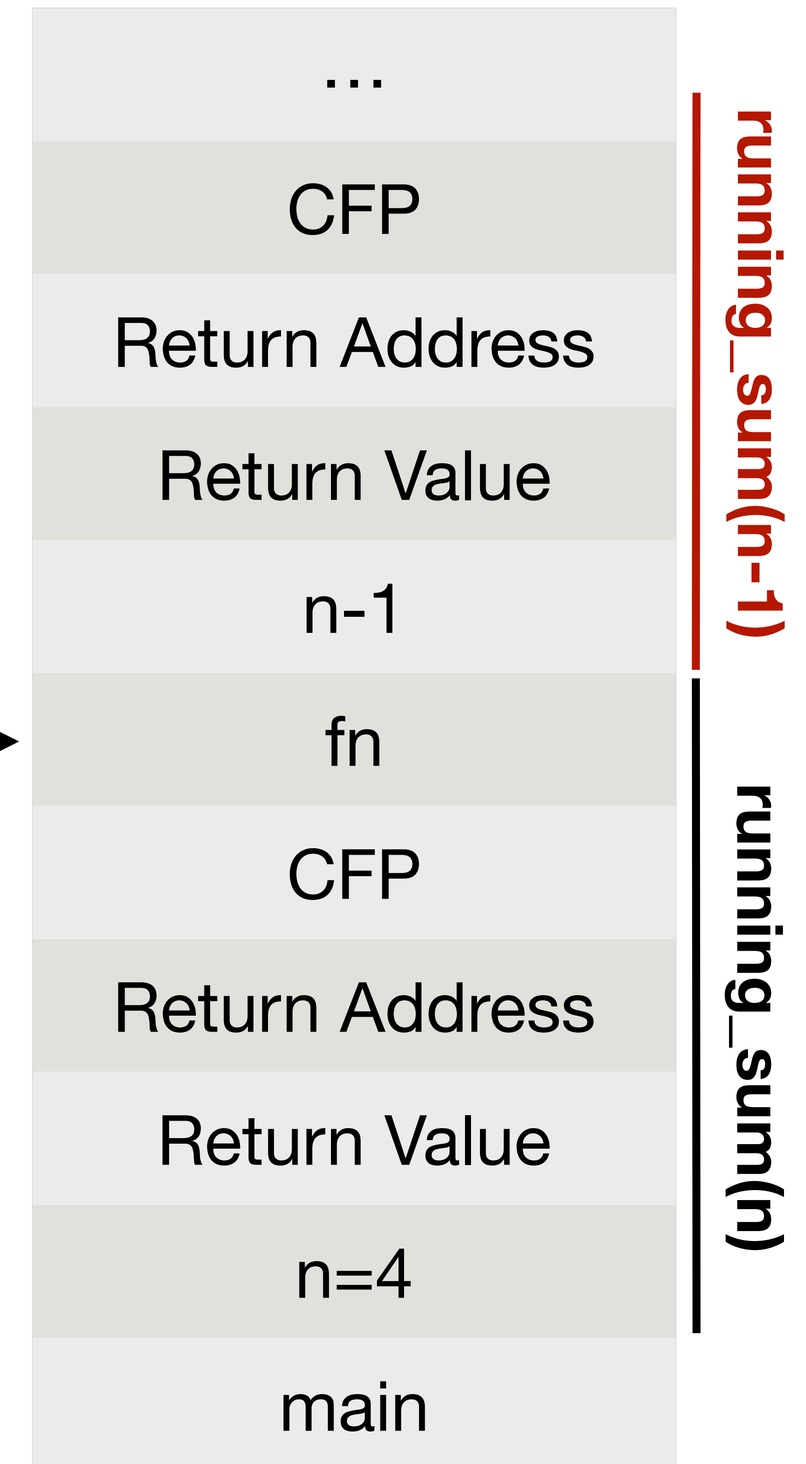
```
;Recursive case
;Caller setup: push argument n-1 onto RTS
ADD R6, R6, #-1
STR R2, R6, #0 ; R2 = n - 1
JSR RUNNING ; call Running(n-1)

;Callee tear-down for Running(n-1) not shown

;Caller tear-down for Running(n-1)
;pop Running(n-1)'s return value to R0
```







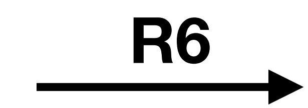
```c
int running_sum(int n){
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}
```

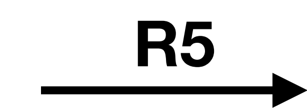
Recursion in LC3

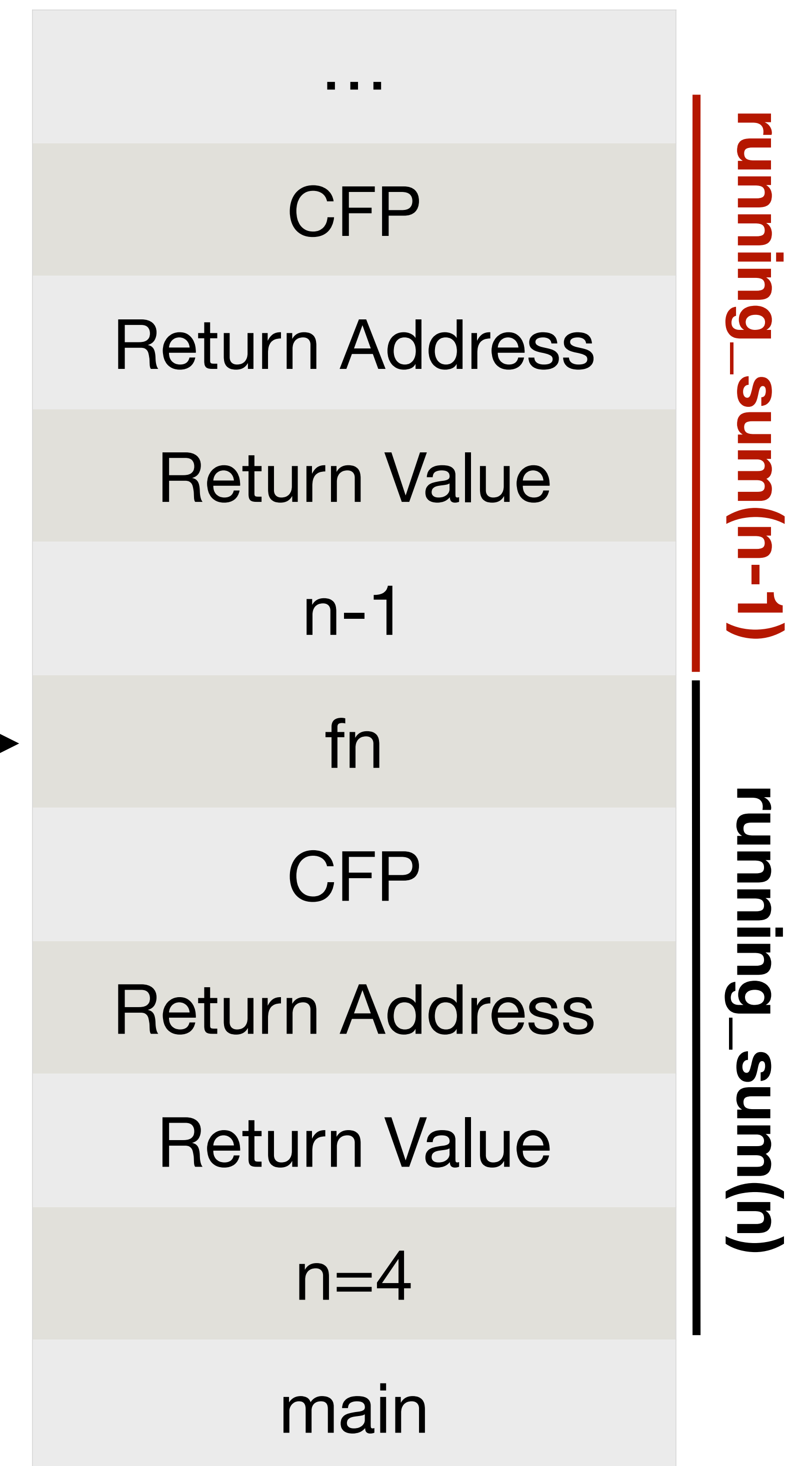
```
;Recursive case
;Caller setup: push argument n-1 onto RTS
ADD R6, R6, #-1
STR R2, R6, #0 ; R2 = n - 1
JSR RUNNING ; call Running(n-1)

;Callee tear-down for Running(n-1) not shown

;Caller tear-down for Running(n-1)
;pop Running(n-1)'s return value to R0
LDR R0, R6, #0
```







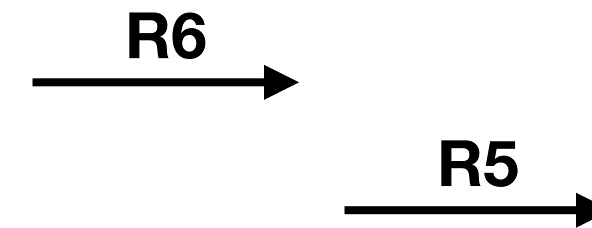
```c
int running_sum(int n){
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}
```

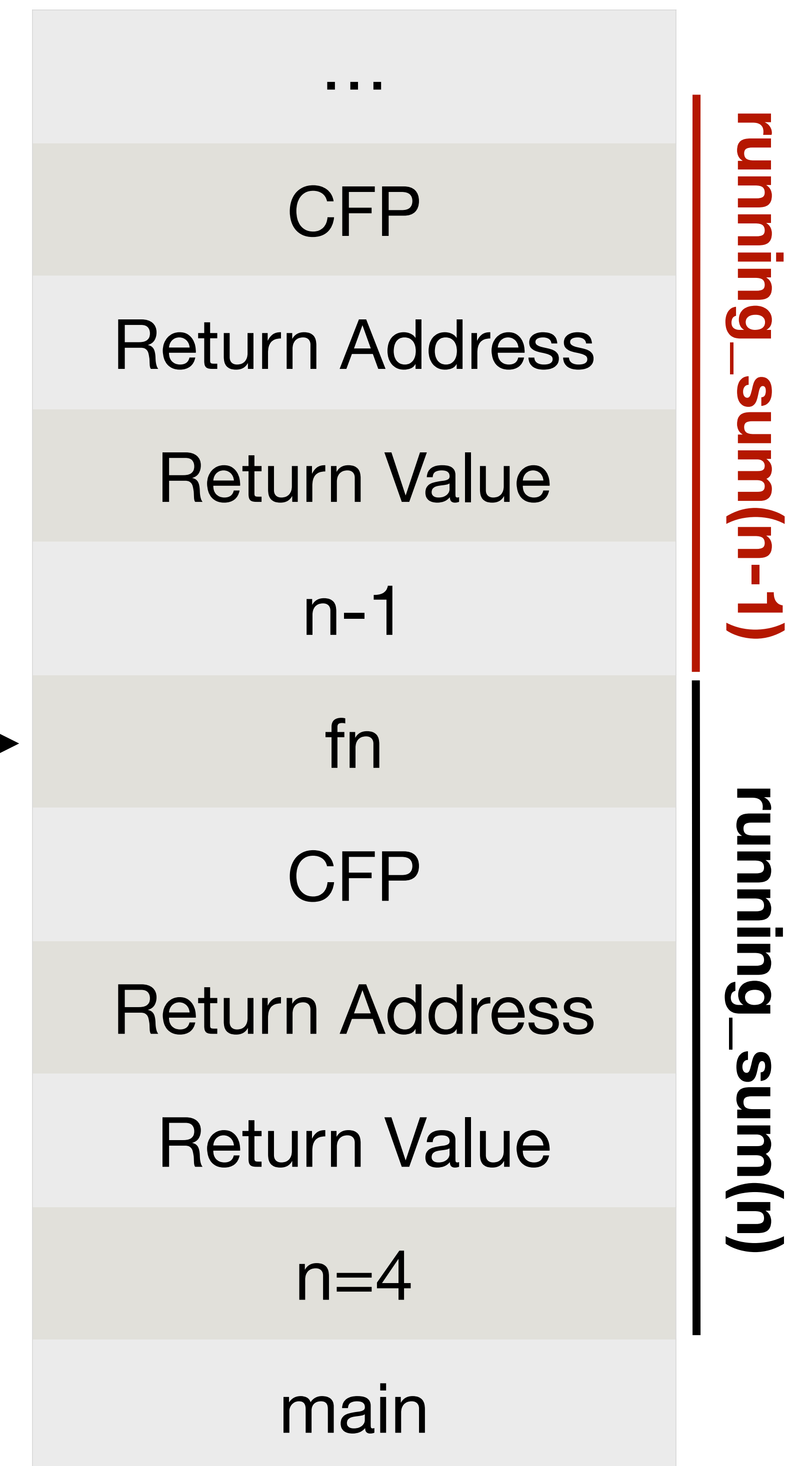
Recursion in LC3

```
;Recursive case
;Caller setup: push argument n-1 onto RTS
ADD R6, R6, #-1
STR R2, R6, #0 ; R2 = n - 1
JSR RUNNING ; call Running(n-1)

;Callee tear-down for Running(n-1) not shown

;Caller tear-down for Running(n-1)
;pop Running(n-1)'s return value to R0
LDR R0, R6, #0
ADD R6, R6, #1
```





```c
int running_sum(int n){
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}
```

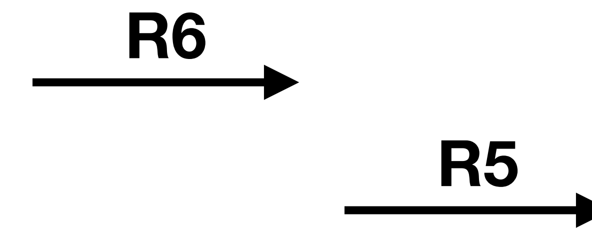
Recursion in LC3

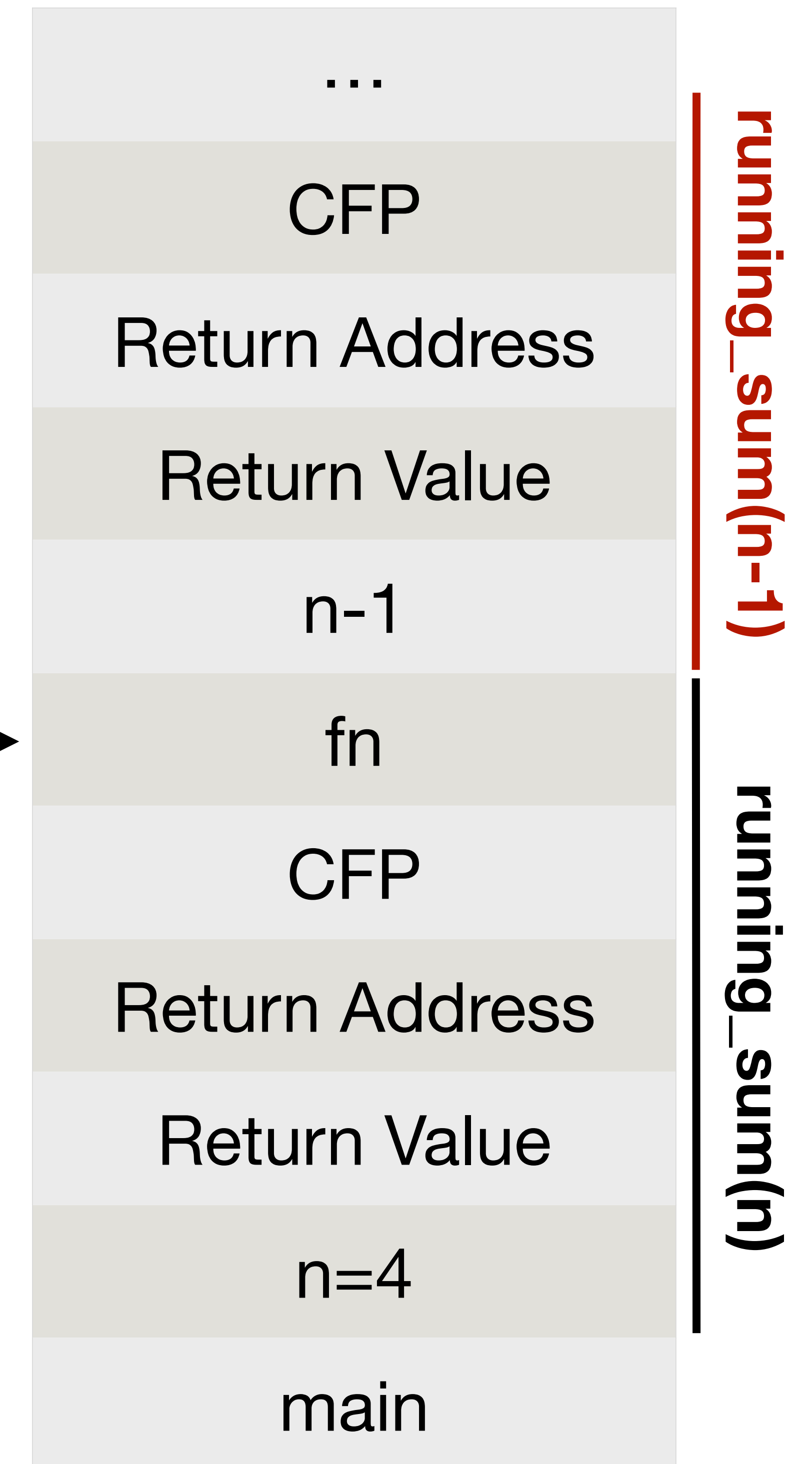
```
;Recursive case
;Caller setup: push argument n-1 onto RTS
ADD R6, R6, #-1
STR R2, R6, #0 ; R2 = n - 1
JSR RUNNING ; call Running(n-1)

;Callee tear-down for Running(n-1) not shown

;Caller tear-down for Running(n-1)
;pop Running(n-1)'s return value to R0
LDR R0, R6, #0
ADD R6, R6, #1

;pop Running(n-1)'s argument
```





```c
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    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}
```
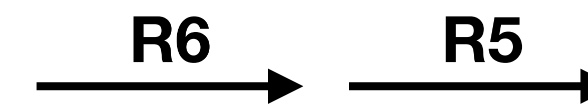
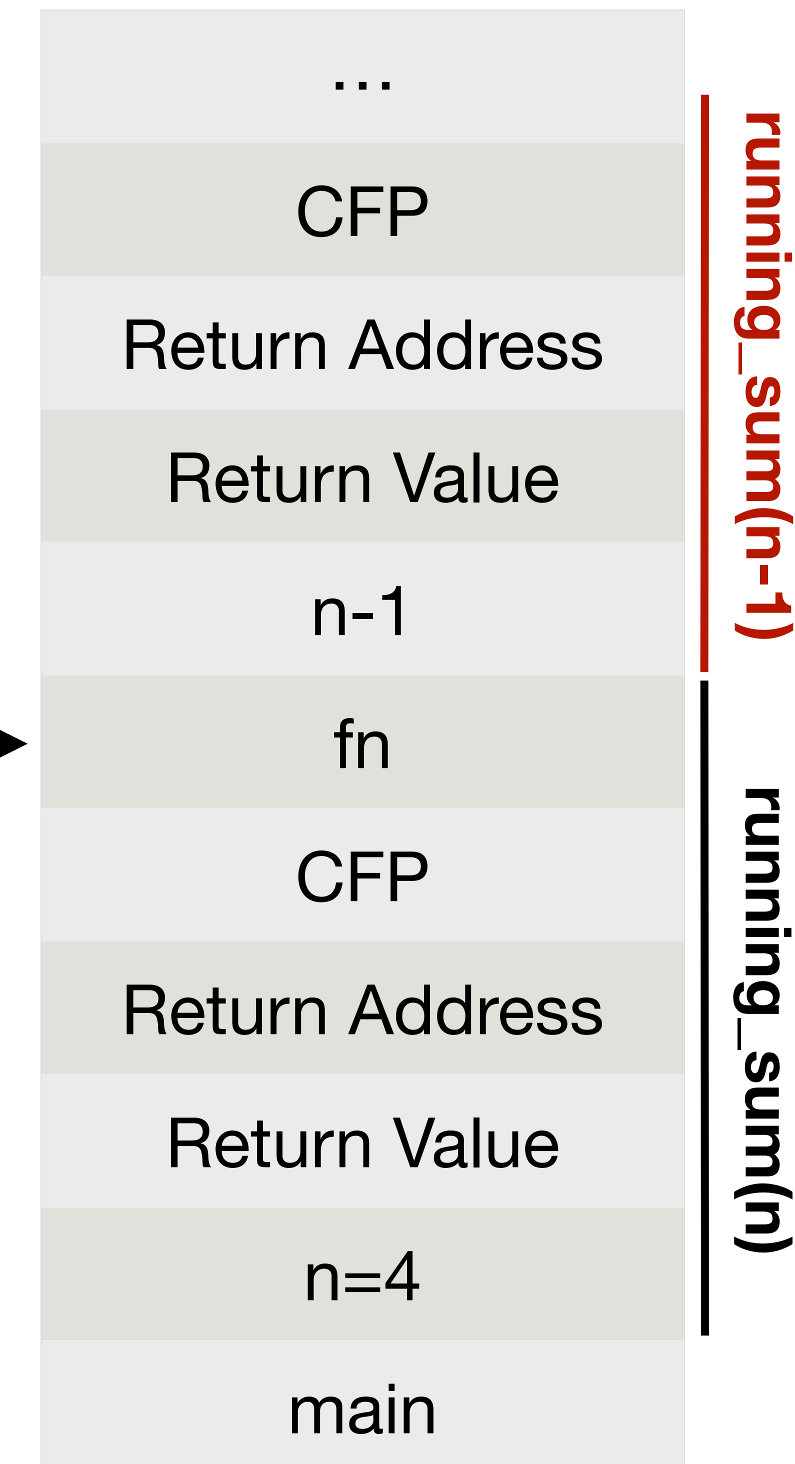

Recursion in LC3

```
;Recursive case
;Caller setup: push argument n-1 onto RTS
ADD R6, R6, #-1
STR R2, R6, #0 ; R2 = n - 1
JSR RUNNING ; call Running(n-1)

;Callee tear-down for Running(n-1) not shown

;Caller tear-down for Running(n-1)
;pop Running(n-1)'s return value to R0
LDR R0, R6, #0
ADD R6, R6, #1

;pop Running(n-1)'s argument
ADD R6, R6, #1
```

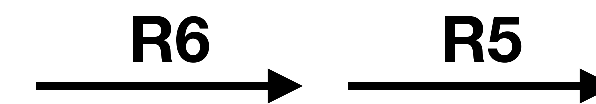


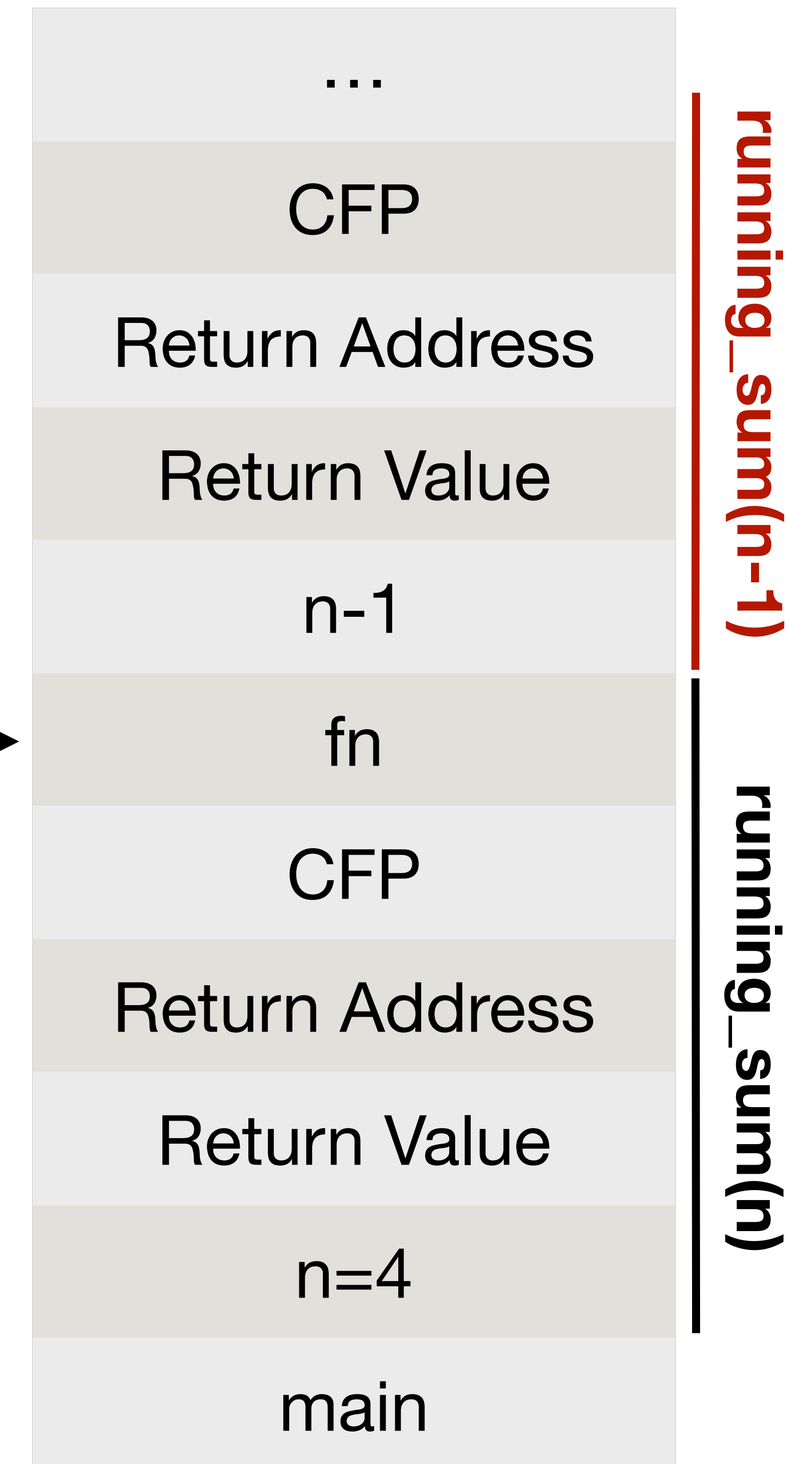


```c
int running_sum(int n){
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}
```

Recursion in LC3

```
;Recursive case
;Caller setup: push argument n-1 onto RTS
ADD R6, R6, #-1
STR R2, R6, #0 ; R2 = n - 1
JSR RUNNING ; call Running(n-1)

;Callee tear-down for Running(n-1) not shown

;Caller tear-down for Running(n-1)
;pop Running(n-1)'s return value to R0
LDR R0, R6, #0
ADD R6, R6, #1

;pop Running(n-1)'s argument
ADD R6, R6, #1

;calculate n + Running(n-1)
```





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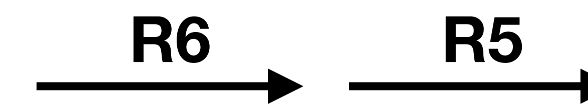
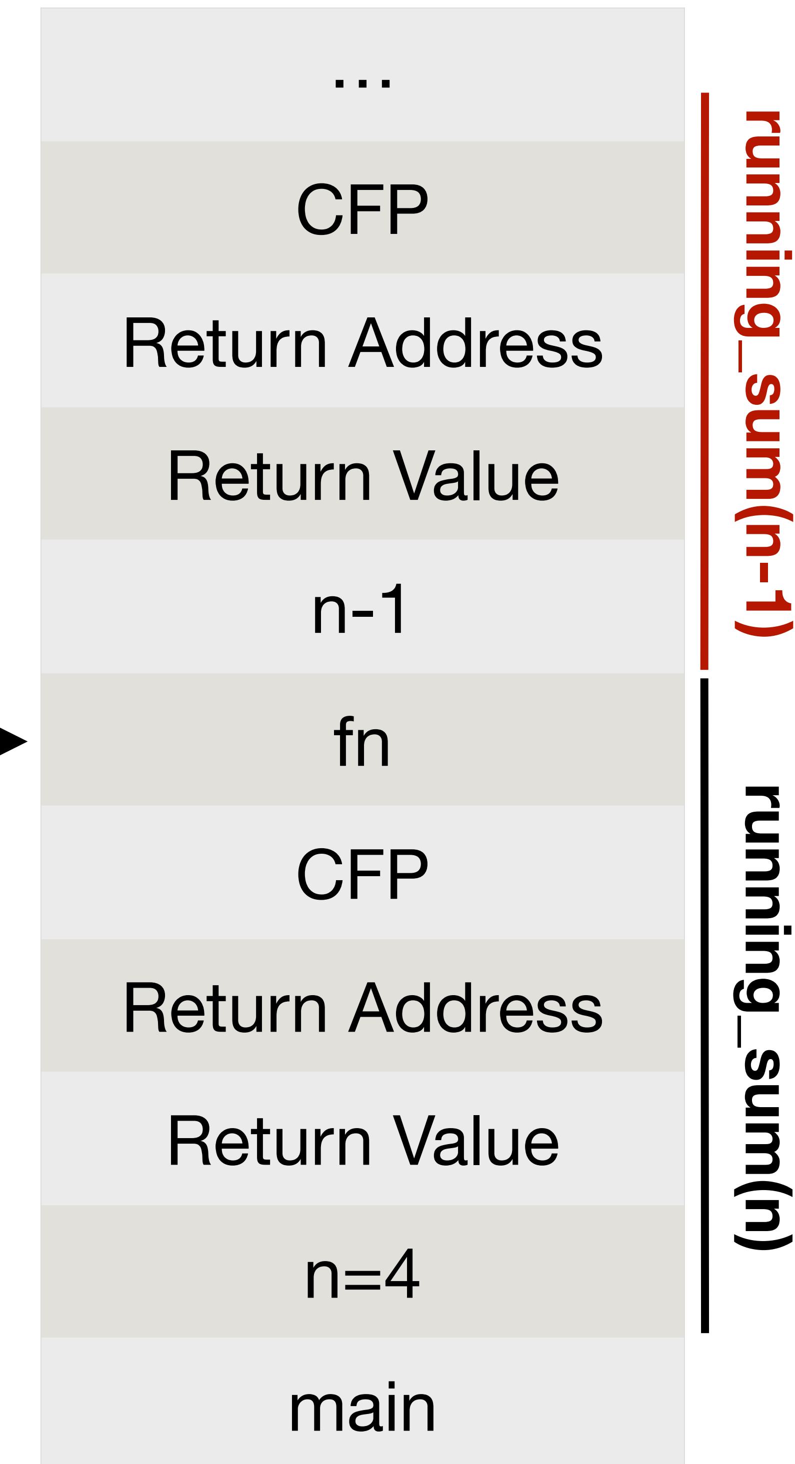
```c
int running_sum(int n){
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}
```

Recursion in LC3

```
;Recursive case
;Caller setup: push argument n-1 onto RTS
ADD R6, R6, #-1
STR R2, R6, #0 ; R2 = n - 1
JSR RUNNING ; call Running(n-1)

;Callee tear-down for Running(n-1) not shown

;Caller tear-down for Running(n-1)
;pop Running(n-1)'s return value to R0
LDR R0, R6, #0
ADD R6, R6, #1

;pop Running(n-1)'s argument
ADD R6, R6, #1

;calculate n + Running(n-1)
LDR R1, R5, #4
```

R0	6
R1	4



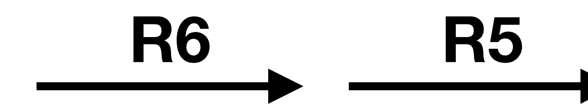


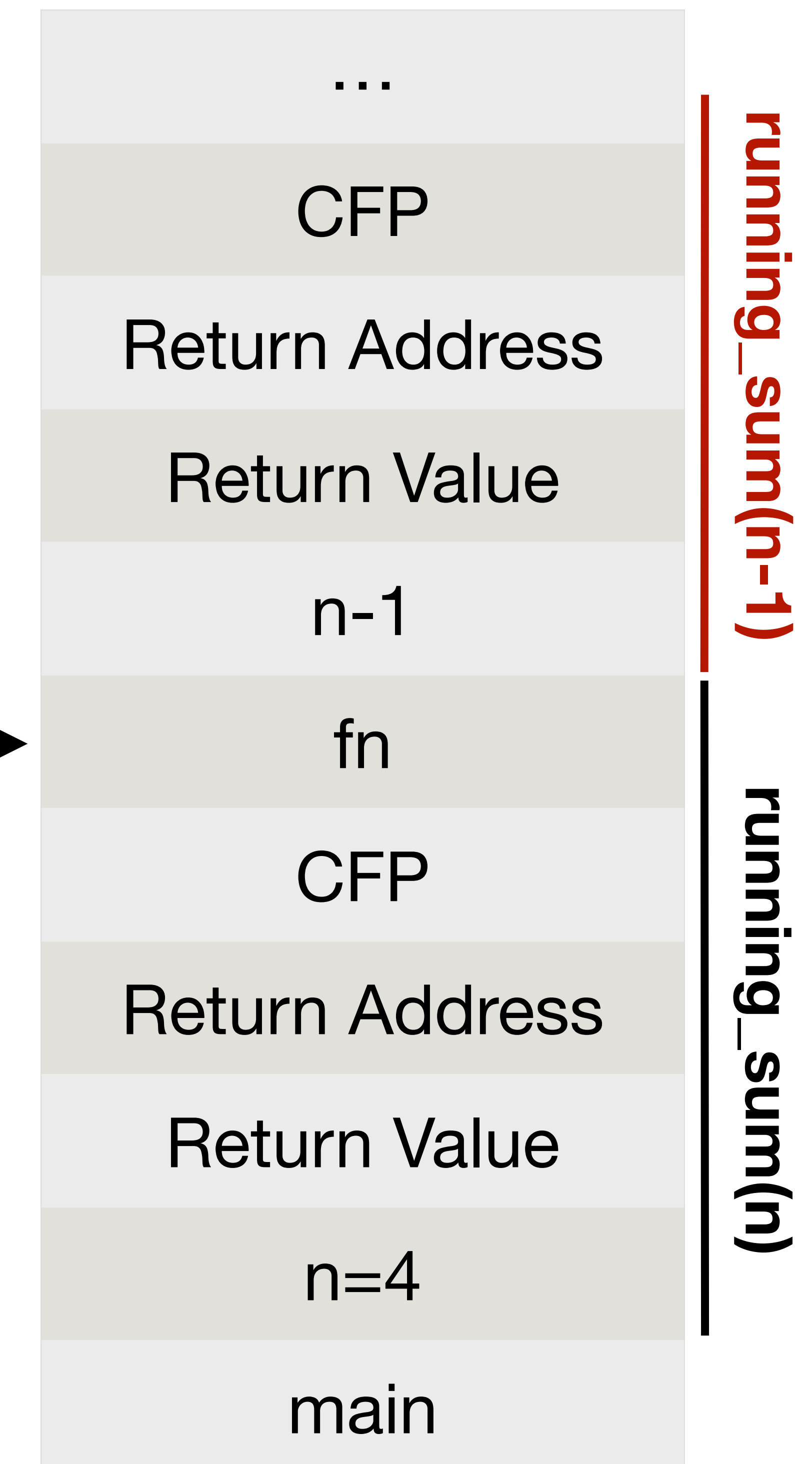
```c
int running_sum(int n){
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}
```

Recursion in LC3

```
;Recursive case
;Caller setup: push argument n-1 onto RTS
ADD R6, R6, #-1
STR R2, R6, #0 ; R2 = n - 1
JSR RUNNING ; call Running(n-1)

;Callee tear-down for Running(n-1) not shown

;Caller tear-down for Running(n-1)
;pop Running(n-1)'s return value to R0
LDR R0, R6, #0
ADD R6, R6, #1

;pop Running(n-1)'s argument
ADD R6, R6, #1

;calculate n + Running(n-1)
LDR R1, R5, #4
ADD R0, R1, R0
```

R0	10
R1	4



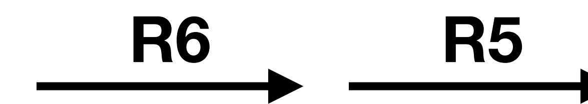


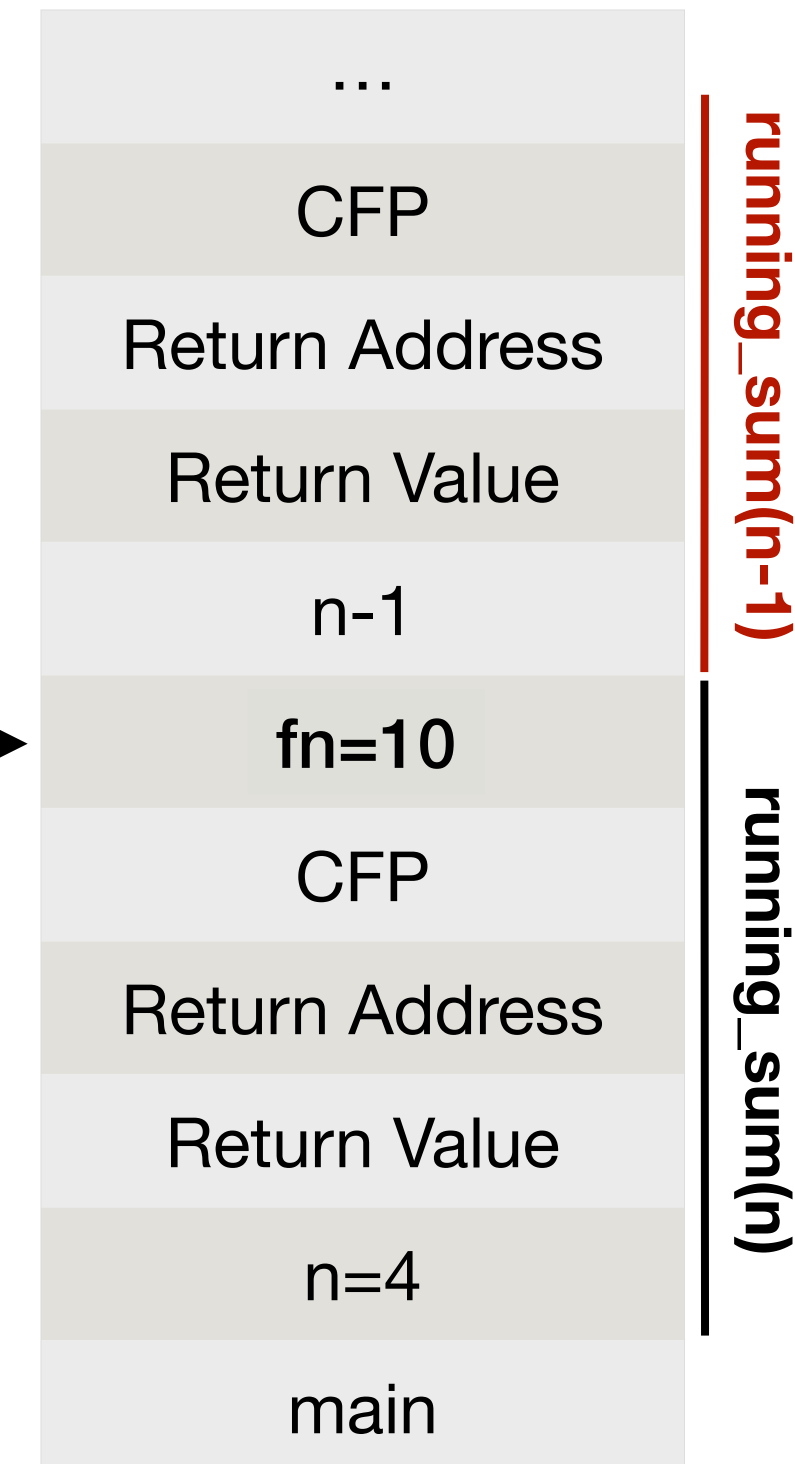
```c
int running_sum(int n){
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}
```

Recursion in LC3

```
;Recursive case
;Caller setup: push argument n-1 onto RTS
ADD R6, R6, #-1
STR R2, R6, #0 ; R2 = n - 1
JSR RUNNING ; call Running(n-1)

;Callee tear-down for Running(n-1) not shown

;Caller tear-down for Running(n-1)
;pop Running(n-1)'s return value to R0
LDR R0, R6, #0
ADD R6, R6, #1

;pop Running(n-1)'s argument
ADD R6, R6, #1

;calculate n + Running(n-1)
LDR R1, R5, #4
ADD R0, R1, R0
STR R0, R5, #0 ;store result in fn
```

R0	10
R1	4





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Recursion in LC3

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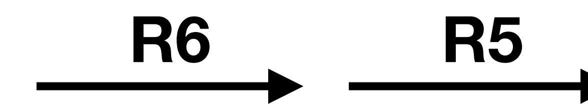
;pop Running(n-1)'s return value to R0
LDR R0, R6, #0
ADD R6, R6, #1

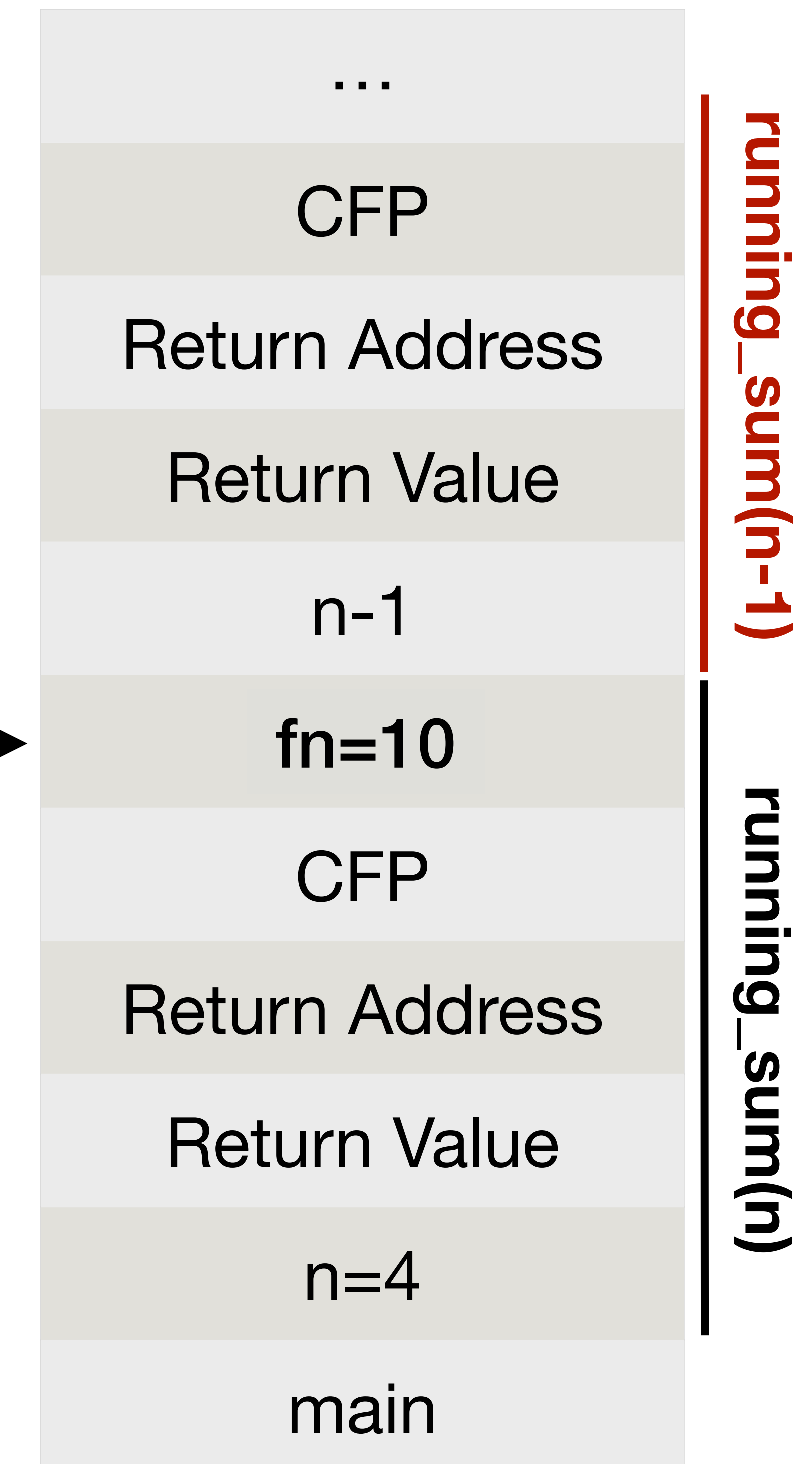
;pop Running(n-1)'s argument
ADD R6, R6, #1

;calculate n + Running(n-1)
LDR R1, R5, #4
ADD R0, R1, R0
STR R0, R5, #0 ;store result in fn

;ready to return
```

R0	10
R1	4





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Recursion in LC3

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;Caller tear-down for Running(n-1)
;pop Running(n-1)'s return value to R0
LDR R0, R6, #0
ADD R6, R6, #1

;pop Running(n-1)'s argument
ADD R6, R6, #1

;calculate n + Running(n-1)
LDR R1, R5, #4
ADD R0, R1, R0
STR R0, R5, #0 ;store result in fn

;ready to return
BRnzp RETURN
```
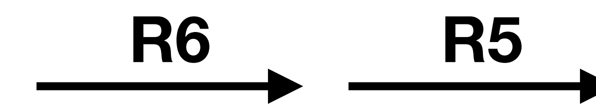
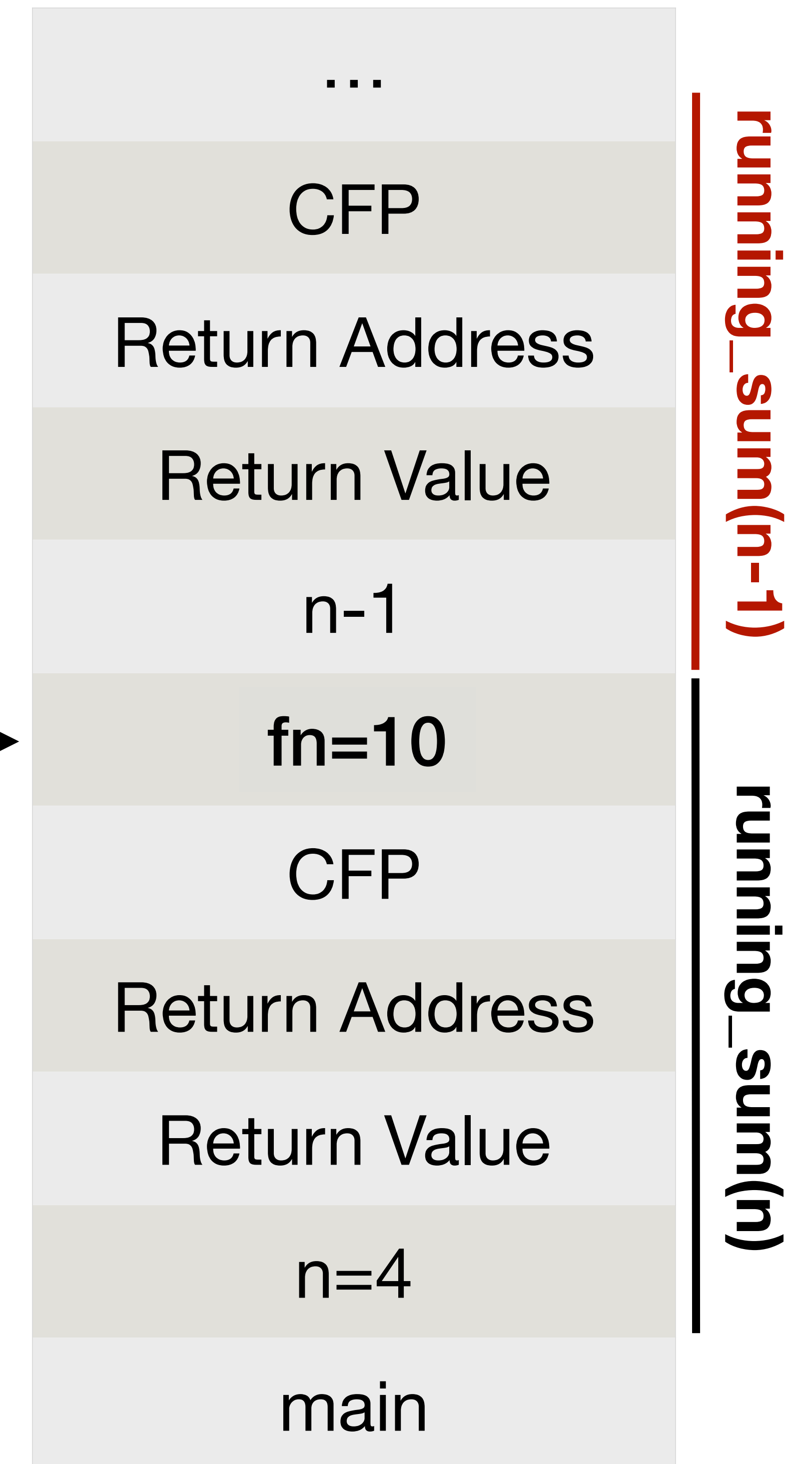

R0	10
R1	4





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    else
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    return fn;
}
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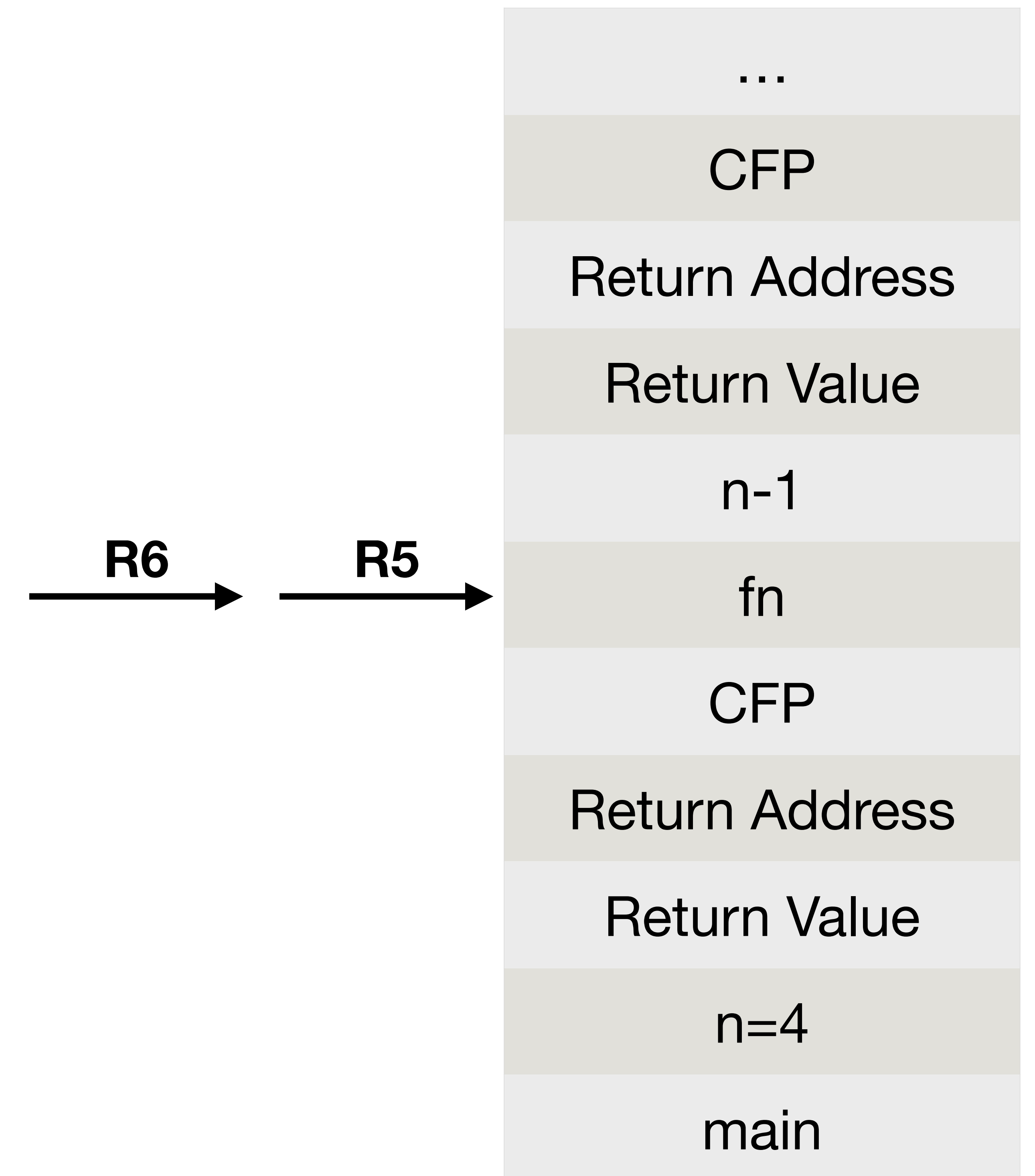
Recursion in LC3



```c
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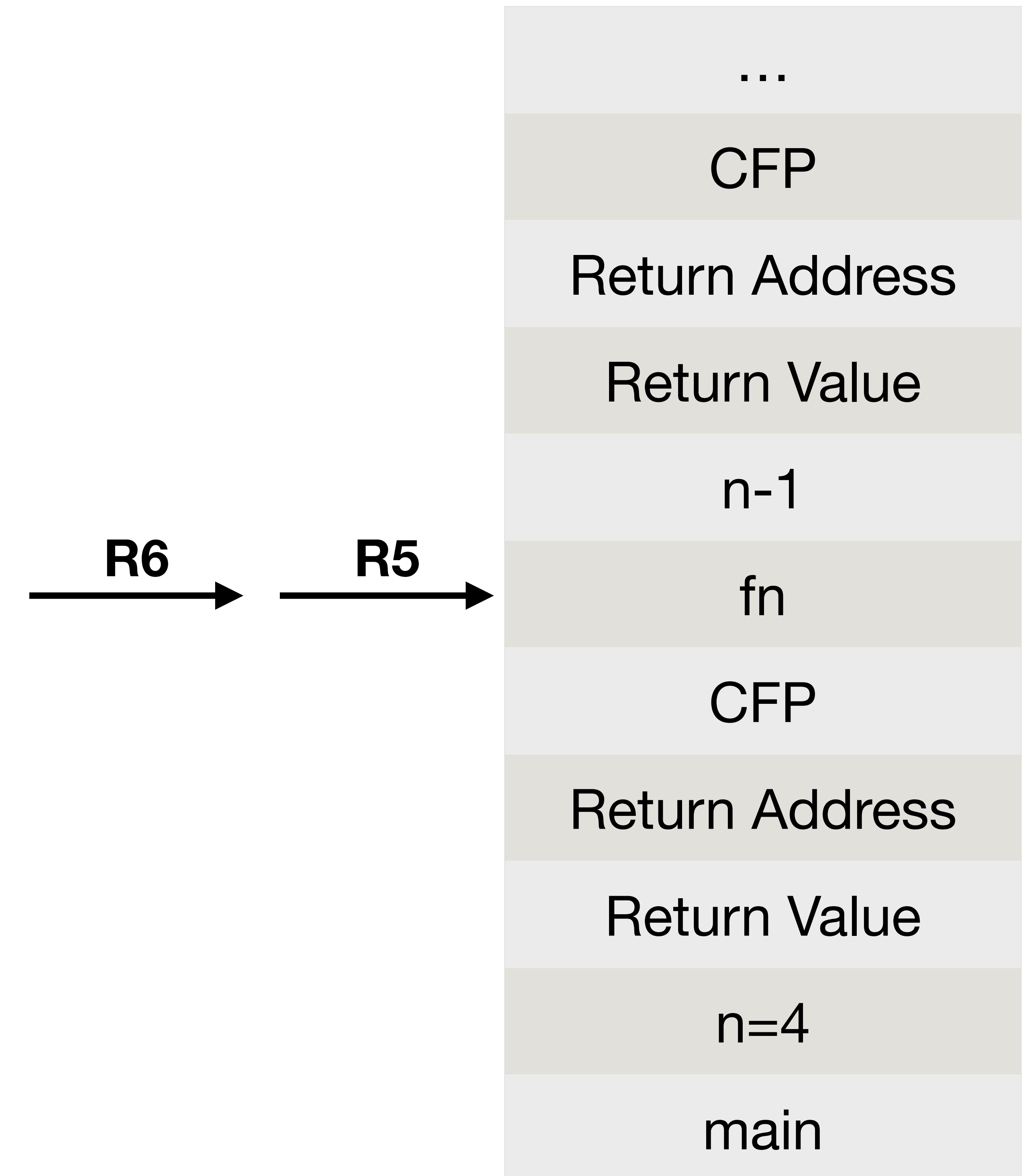
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}
```

Recursion in LC3

```
BASE_CASE
AND R2, R2, #0
ADD R2, R2, #1
```



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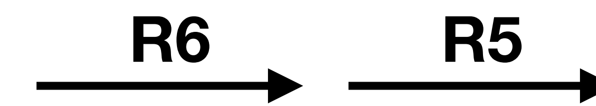
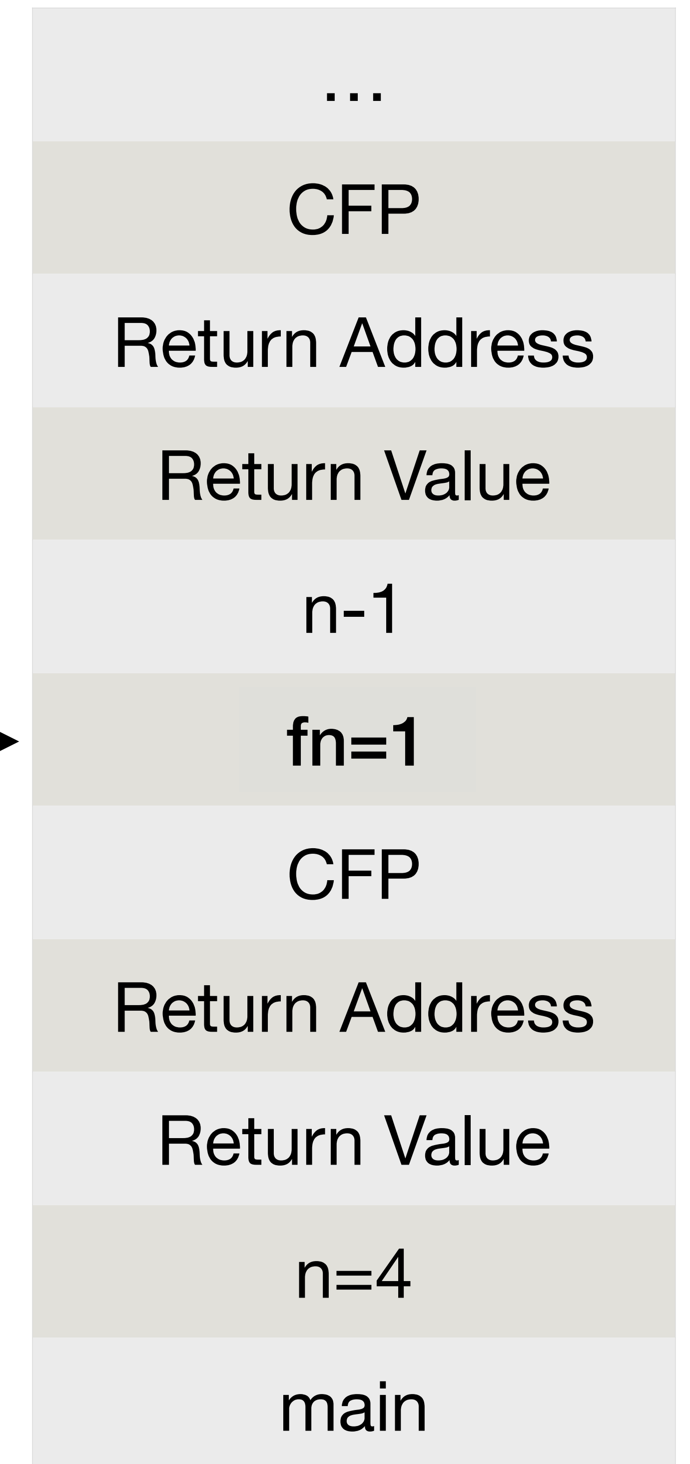
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}
```

Recursion in LC3

```
BASE_CASE
AND R2, R2, #0
ADD R2, R2, #1
STR R2, R5, #0 ;set fn = 1
```



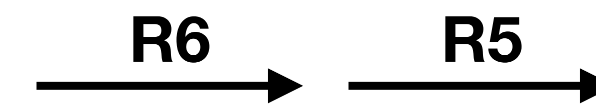


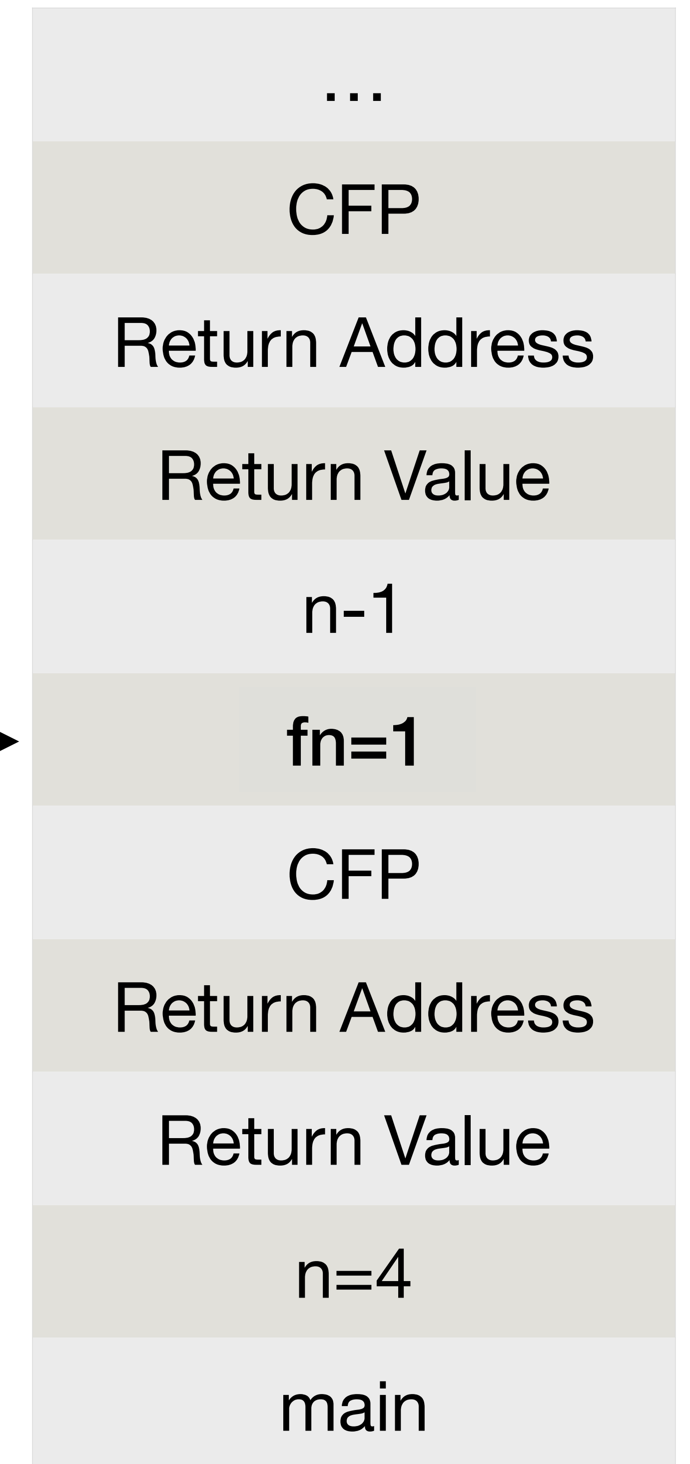
```c
int running_sum(int n){
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}
```

Recursion in LC3

```
BASE_CASE
AND R2, R2, #0
ADD R2, R2, #1
STR R2, R5, #0 ;set fn = 1

RETURN
;set return value
LDR R0, R5, #0
STR R0, R5, #3
```





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        fn = 1;
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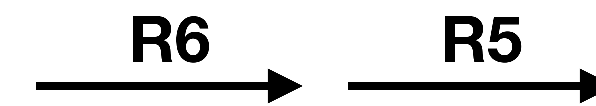
        fn = n + running_sum(n-1);
    return fn;
}
```

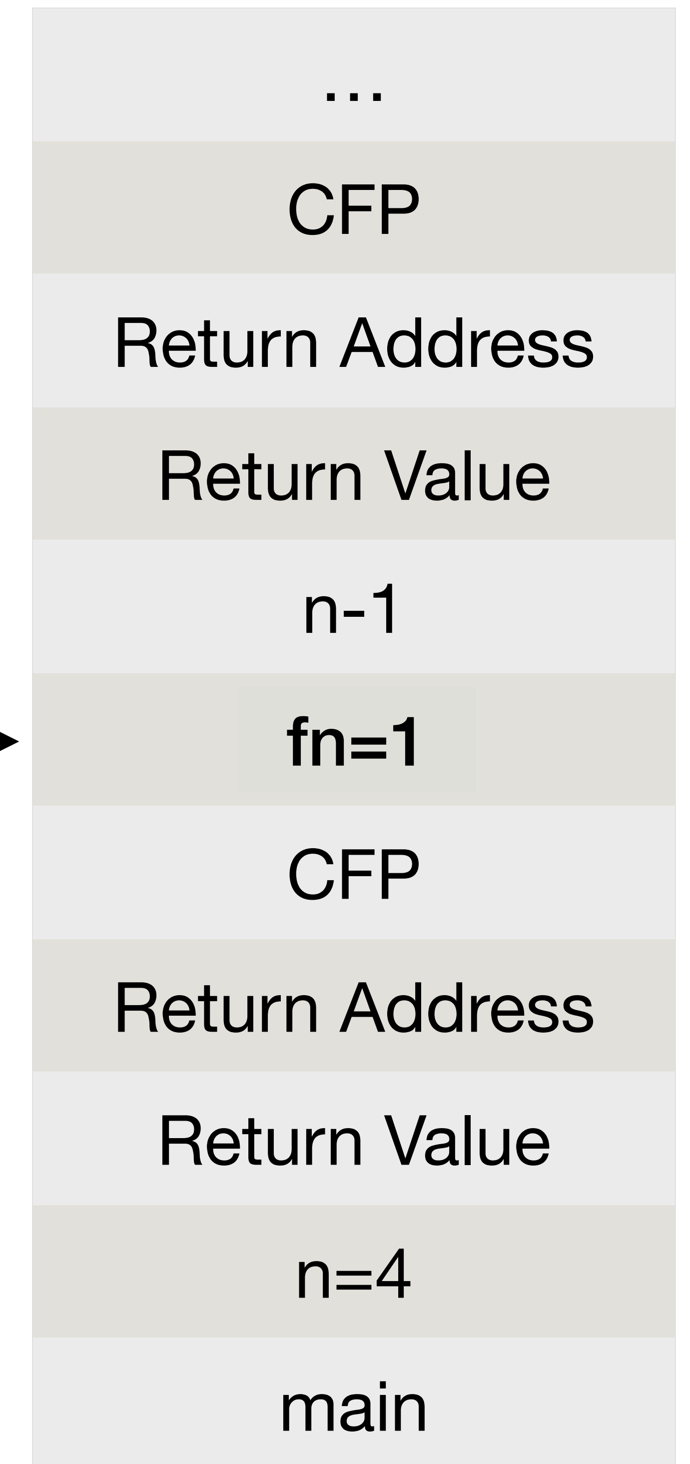
Recursion in LC3

```
BASE_CASE
AND R2, R2, #0
ADD R2, R2, #1
STR R2, R5, #0 ;set fn = 1

RETURN
;set return value
LDR R0, R5, #0
STR R0, R5, #3

;callee tear-down of Running(n)'s activation record
```





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    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}
```
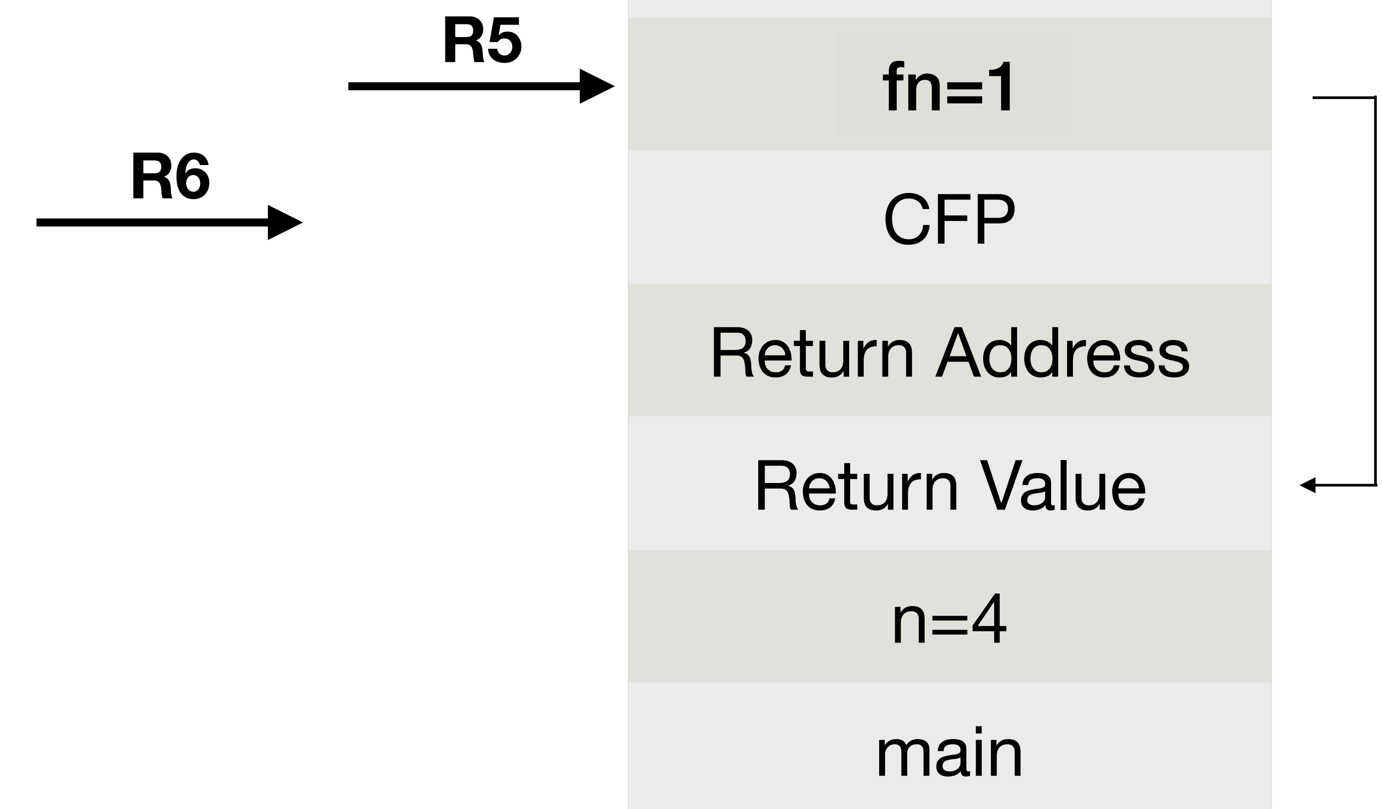

Recursion in LC3

```
BASE_CASE
AND R2, R2, #0
ADD R2, R2, #1
STR R2, R5, #0 ;set fn = 1

RETURN
;set return value
LDR R0, R5, #0
STR R0, R5, #3

;callee tear-down of Running(n)'s activation record
ADD R6, R6, #1 ;pop local variables
```



```c
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    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}
```

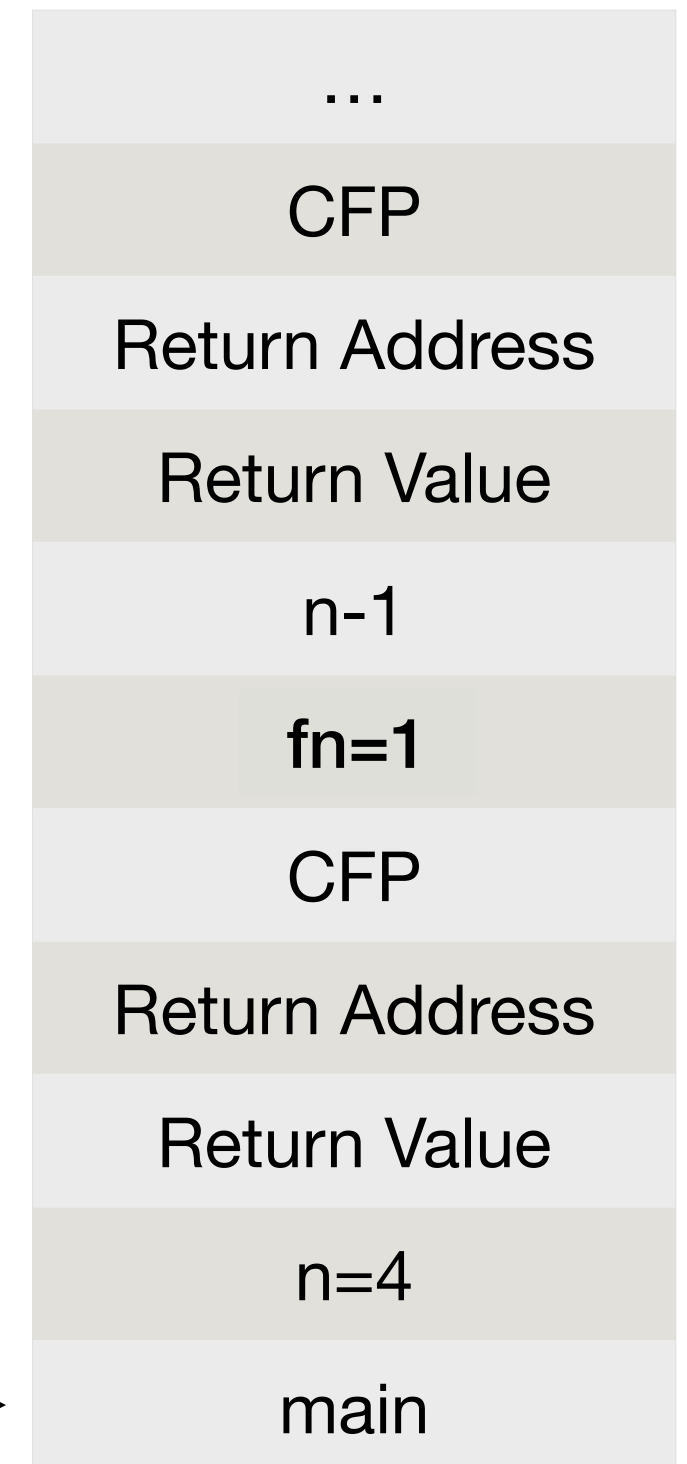
Recursion in LC3

```
BASE_CASE
AND R2, R2, #0
ADD R2, R2, #1
STR R2, R5, #0 ;set fn = 1

RETURN
;set return value
LDR R0, R5, #0
STR R0, R5, #3

;callee tear-down of Running(n)'s activation record
ADD R6, R6, #1 ;pop local variables

;restore caller's frame pointer and return address
LDR R5, R6, #0
```



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    int fn;
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    else
        fn = n + running_sum(n-1);
    return fn;
}
```

Recursion in LC3

```
BASE_CASE
AND R2, R2, #0
ADD R2, R2, #1
STR R2, R5, #0 ;set fn = 1

RETURN
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LDR R0, R5, #0
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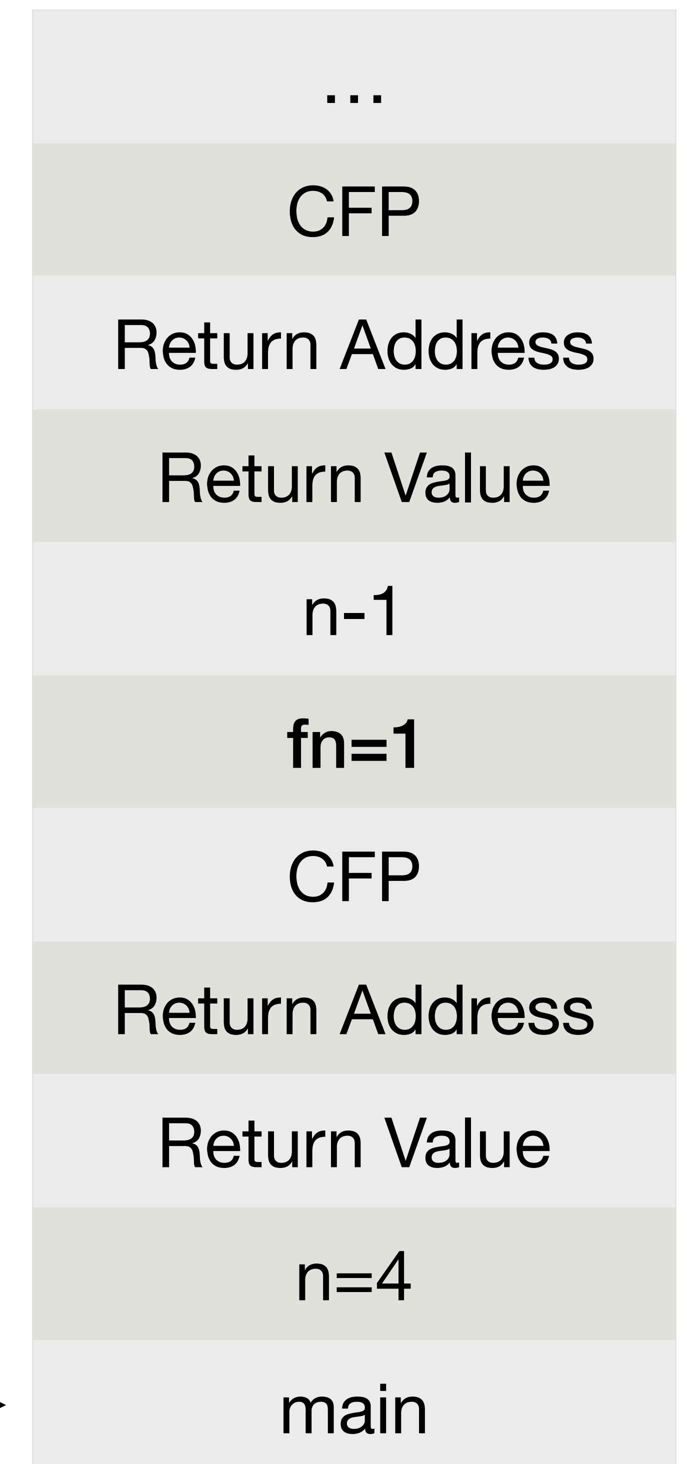
;restore caller's frame pointer and return address
LDR R5, R6, #0
ADD R6, R6, #1
```



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    int fn;
    if (n==1)
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    else
        fn = n + running_sum(n-1);
    return fn;
}
```
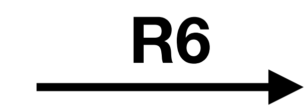
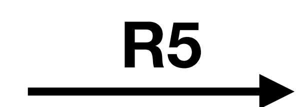
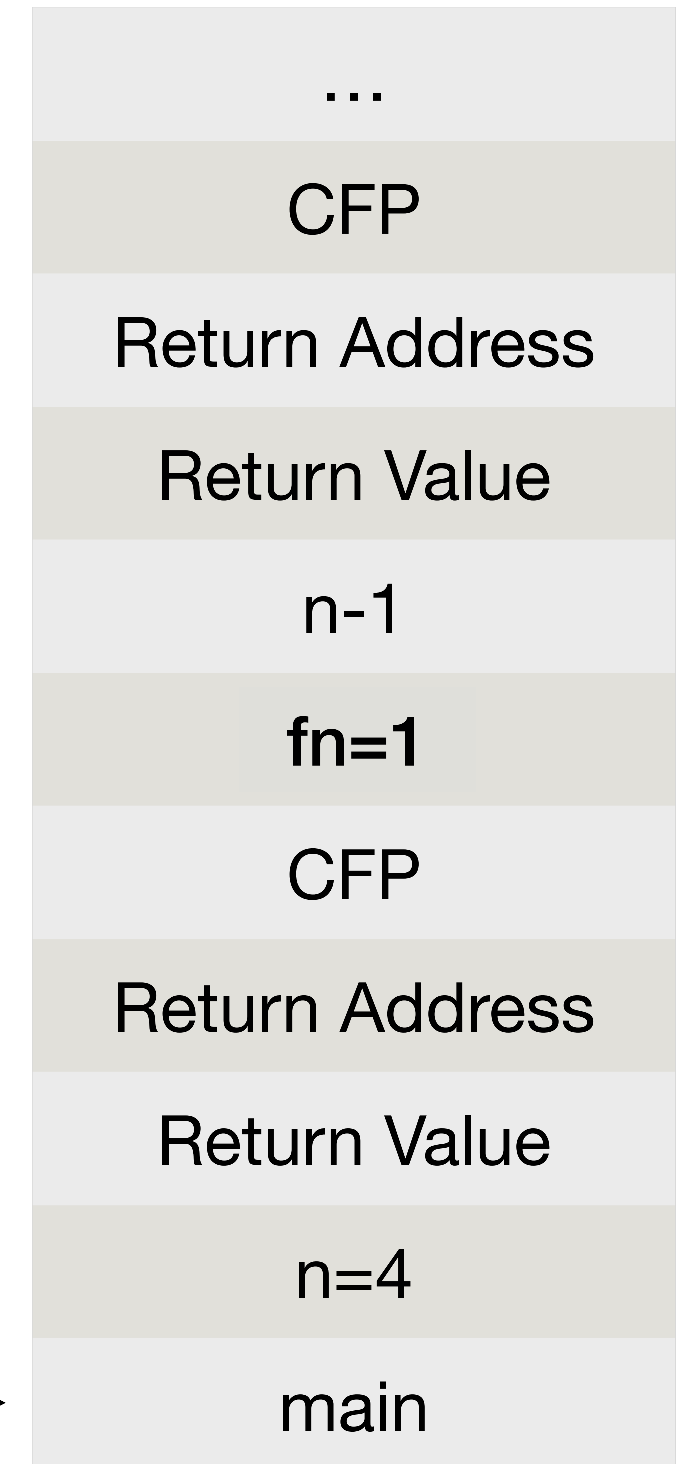

Recursion in LC3

```
BASE_CASE
AND R2, R2, #0
ADD R2, R2, #1
STR R2, R5, #0 ;set fn = 1

RETURN
;set return value
LDR R0, R5, #0
STR R0, R5, #3

;callee tear-down of Running(n)'s activation record
ADD R6, R6, #1 ;pop local variables

;restore caller's frame pointer and return address
LDR R5, R6, #0
ADD R6, R6, #1
LDR R7, R6, #0
```







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}
```

Recursion in LC3

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BASE_CASE
AND R2, R2, #0
ADD R2, R2, #1
STR R2, R5, #0 ;set fn = 1

RETURN
;set return value
LDR R0, R5, #0
STR R0, R5, #3

;callee tear-down of Running(n)'s activation record
ADD R6, R6, #1 ;pop local variables

;restore caller's frame pointer and return address
LDR R5, R6, #0
ADD R6, R6, #1
LDR R7, R6, #0
ADD R6, R6, #1
```

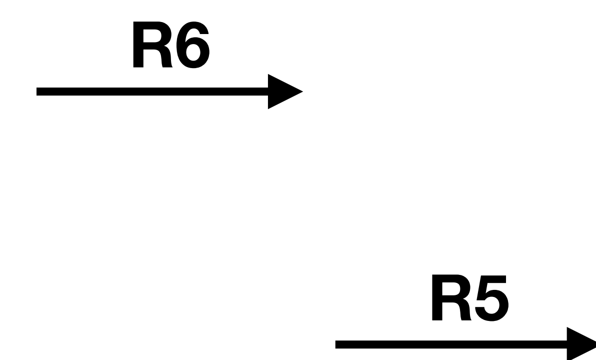

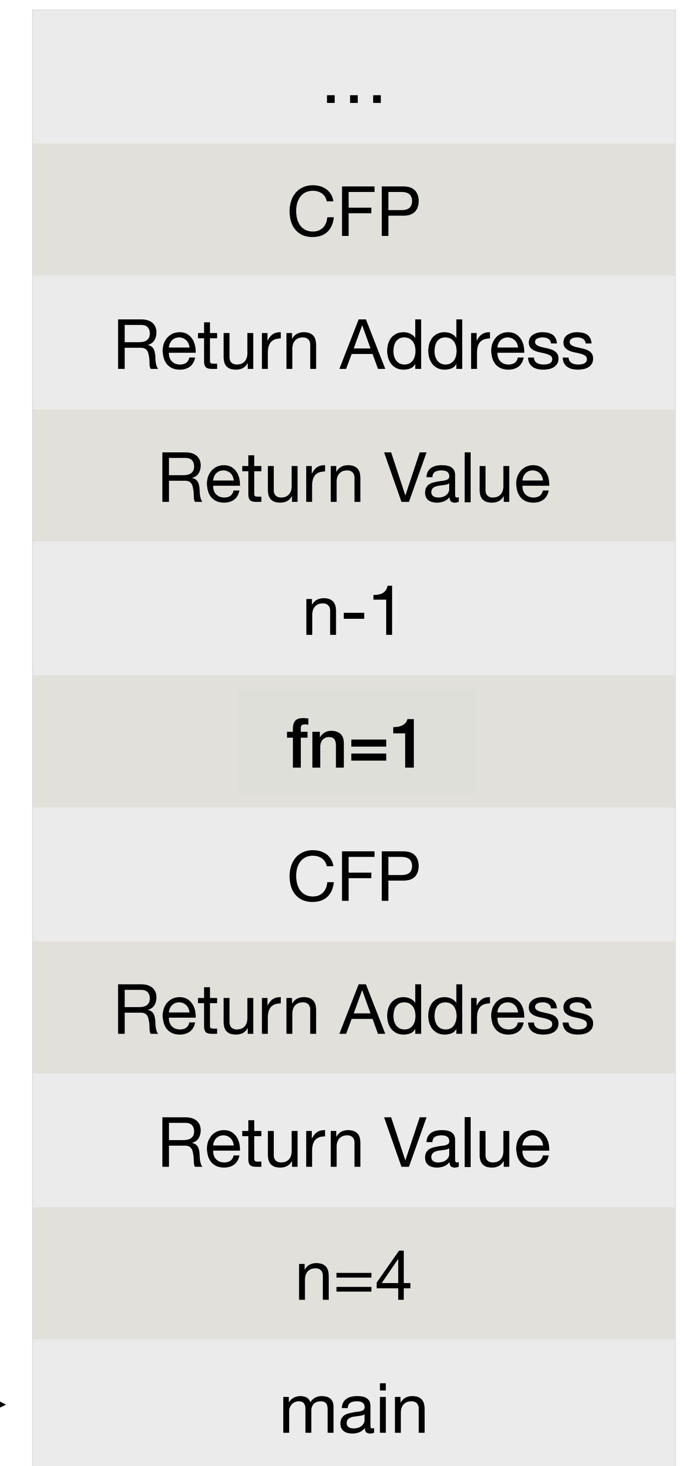

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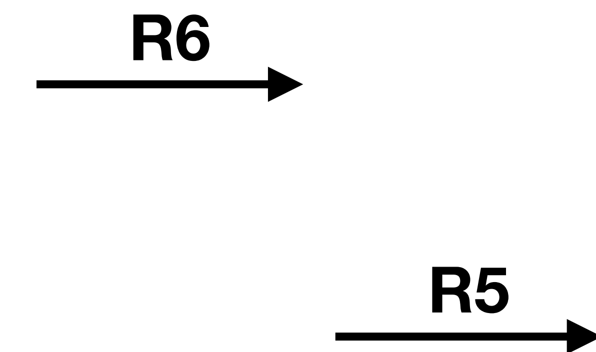
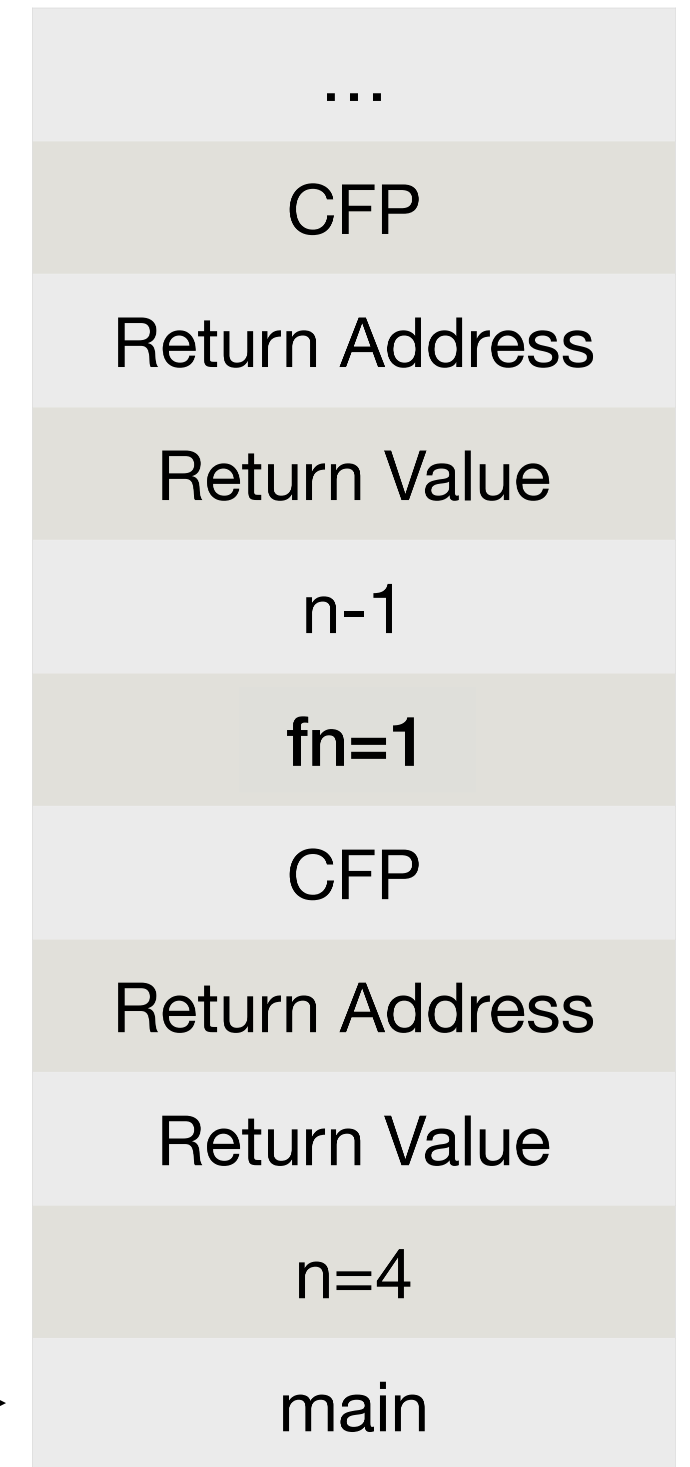
;set return value
LDR R0, R5, #0
STR R0, R5, #3

;callee tear-down of Running(n)'s activation record
ADD R6, R6, #1 ;pop local variables

;restore caller's frame pointer and return address
LDR R5, R6, #0
ADD R6, R6, #1
LDR R7, R6, #0
ADD R6, R6, #1

;return to caller
```





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Recursion in LC3

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;restore caller's frame pointer and return address
LDR R5, R6, #0
ADD R6, R6, #1
LDR R7, R6, #0
ADD R6, R6, #1

;return to caller
RET
```

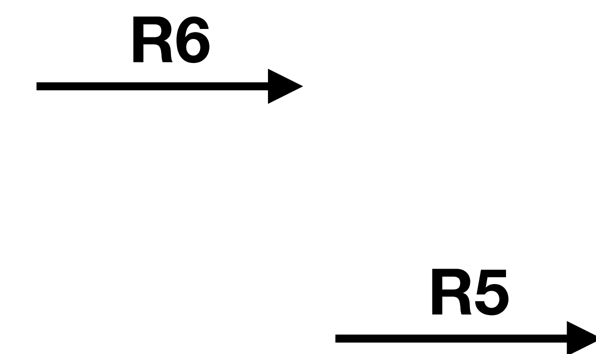

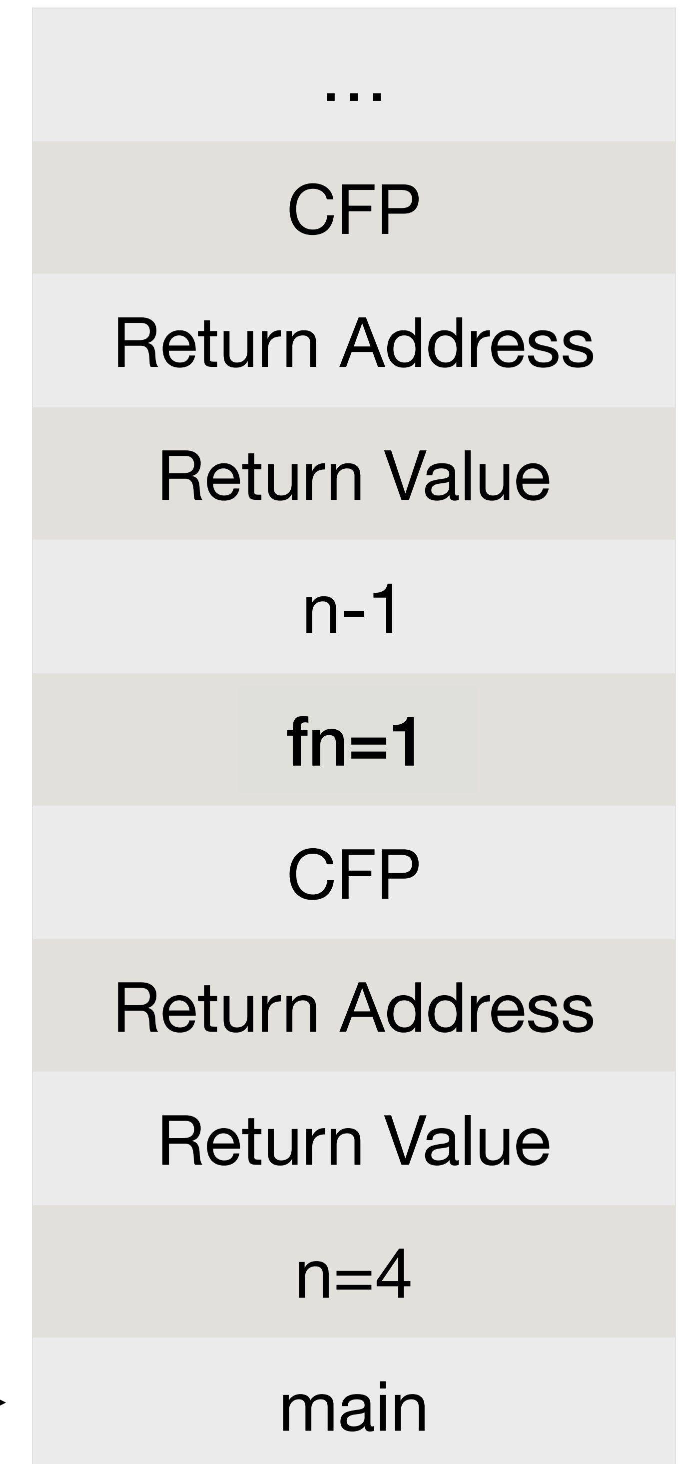

Next time

- More problem solving with recursion.
  - A small chess problem
  - Solving a maze
- When is recursion good vs. bad?