ECE 220

Lecture x000C - 10/07

Slides based on material originally by: Yuting Chen & Thomas Moon

Review

Review

- Last time we discussed sorting & searching

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  - Selection sort

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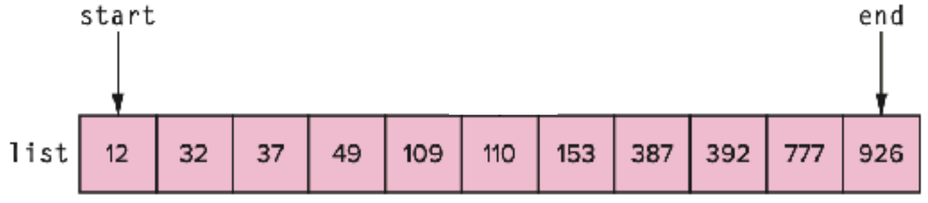
Complete codes for available on Github: https://github.com/iabraham/ece220-fa25/tree/main/lec11_1002/sorts

Lesson objectives

- Understand recursion as a *divide-and-conquer* programming technique.
    - Understand difference between recursive cases and terminating/ base cases.
    - Understand recursion implicitly uses stack data structure.
- Be able to implement recursive functions in C.
- Be (in principle) be able to convert recursive functions to iterative ones.
- Be able to lower recursive C functions to LC3 assembly.

Recap binary search

Key = 109



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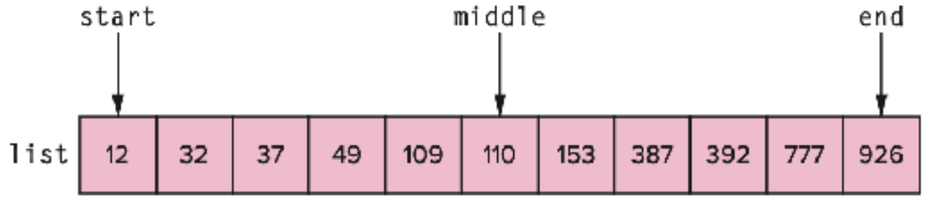

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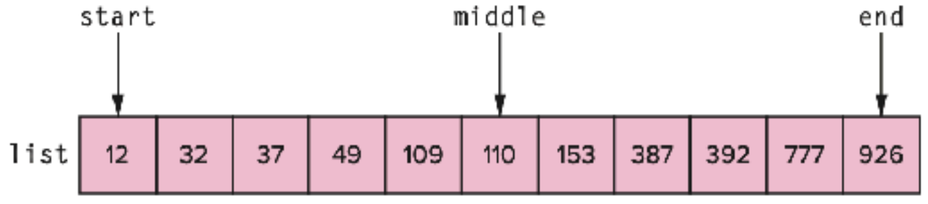
Key = 109 **Pick middle** **Is middle == key?**



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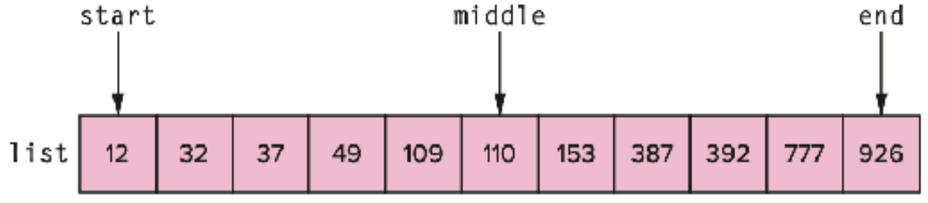

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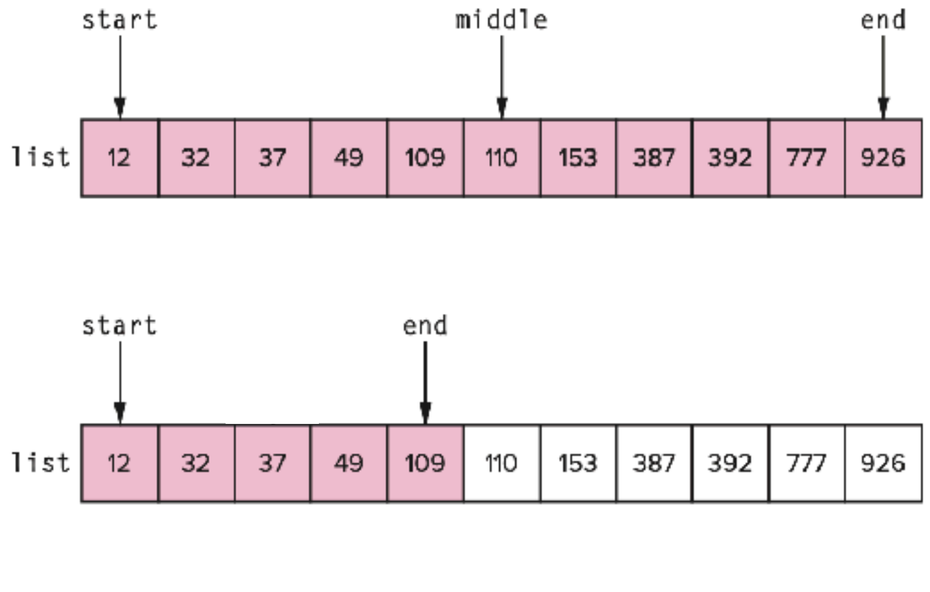

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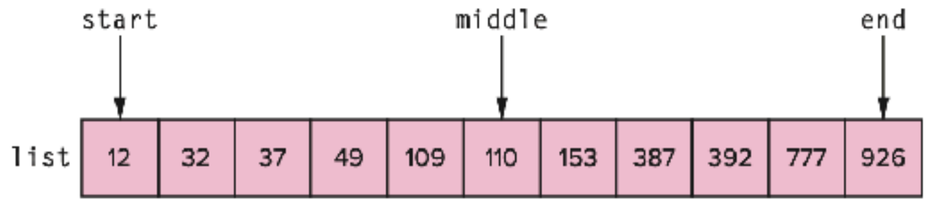

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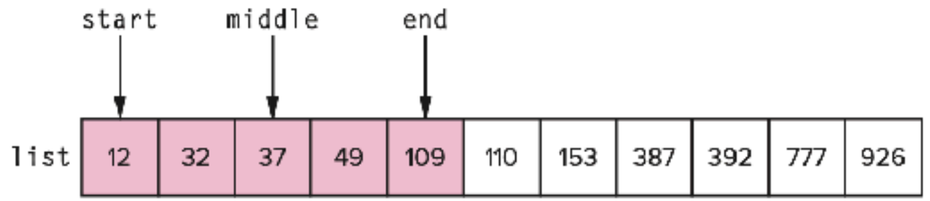

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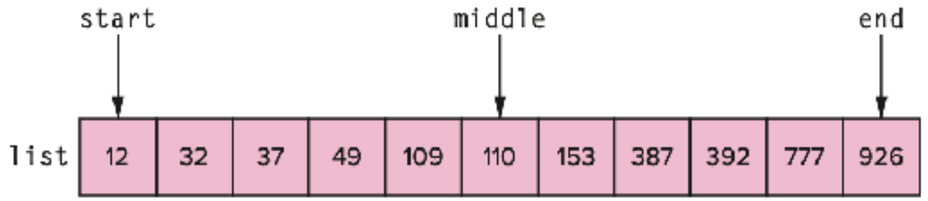

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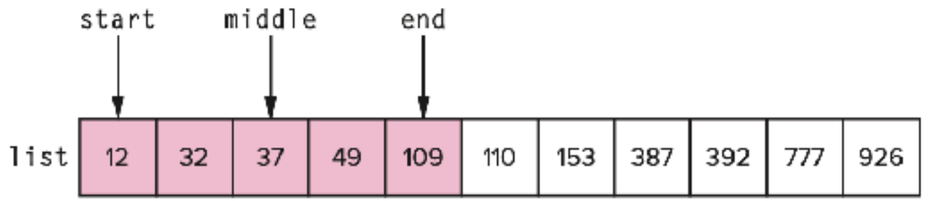

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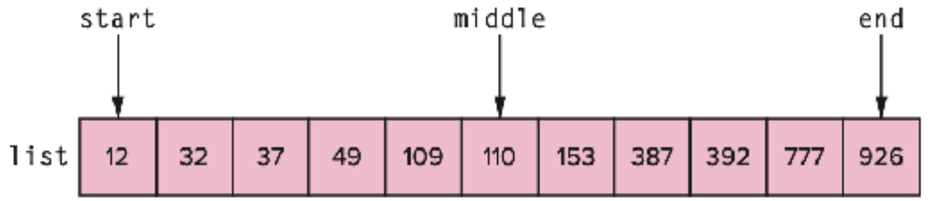

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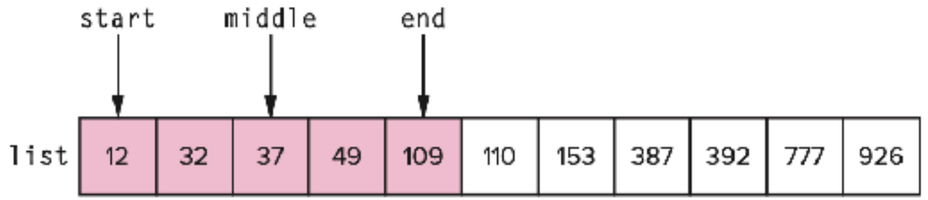

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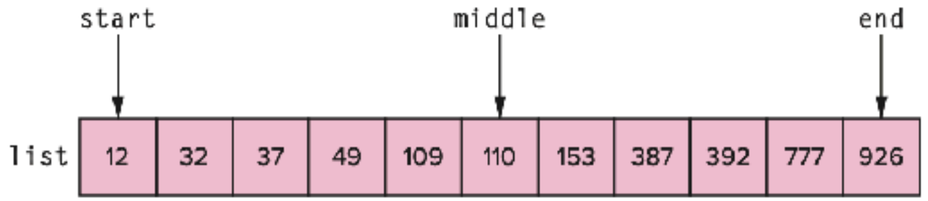

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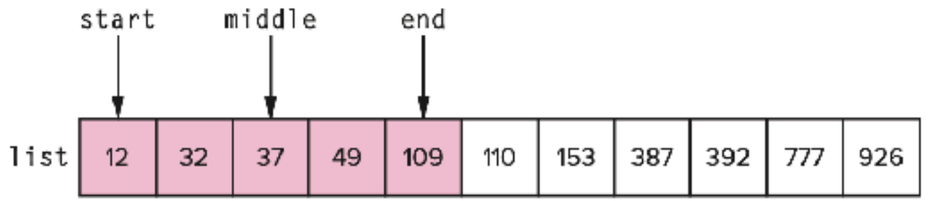

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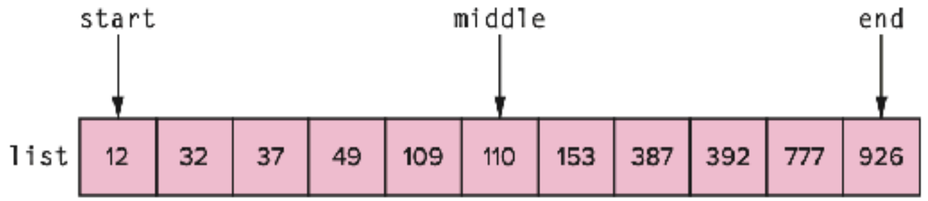

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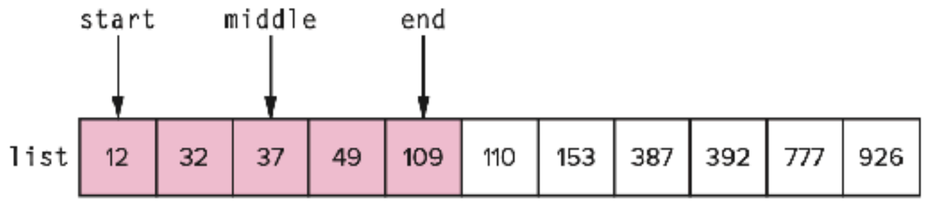

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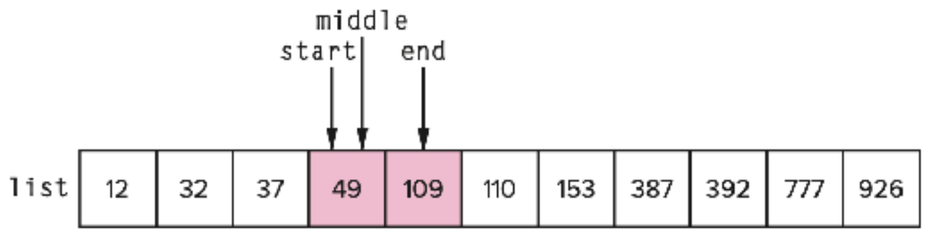

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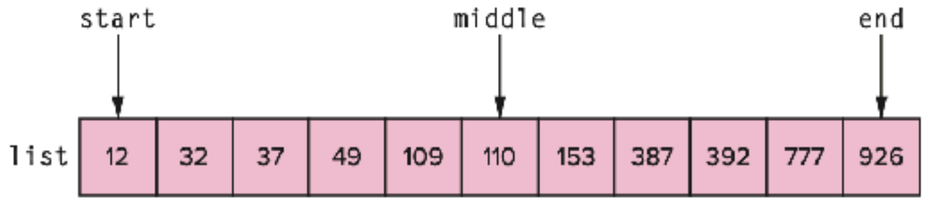

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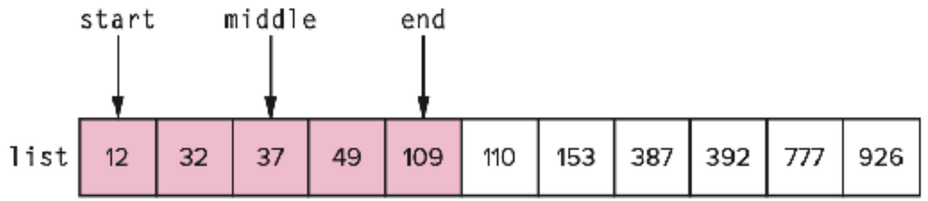

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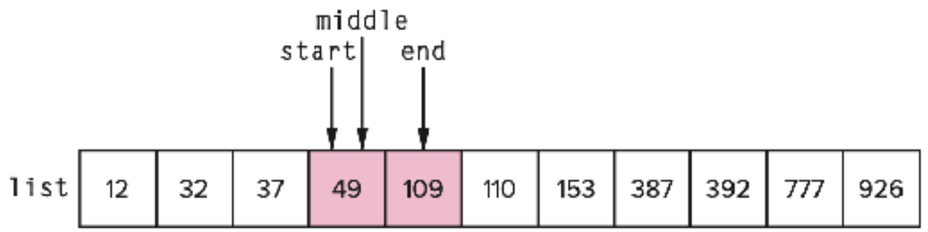

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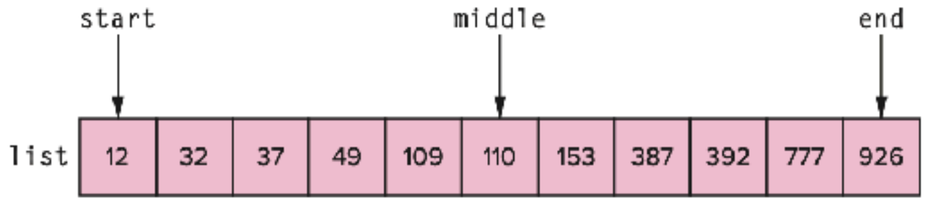

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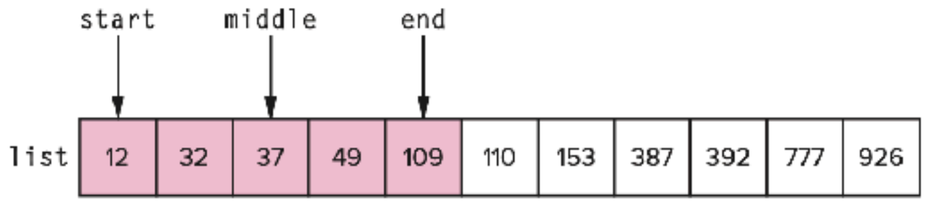

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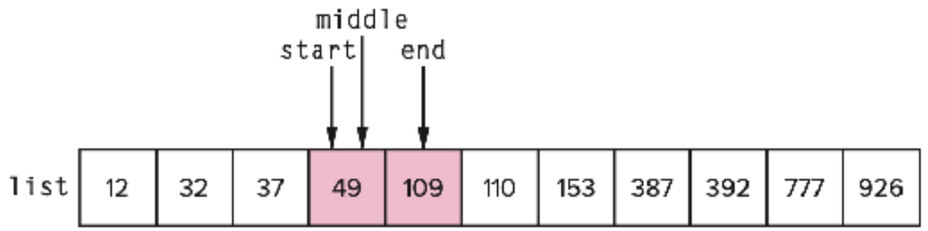

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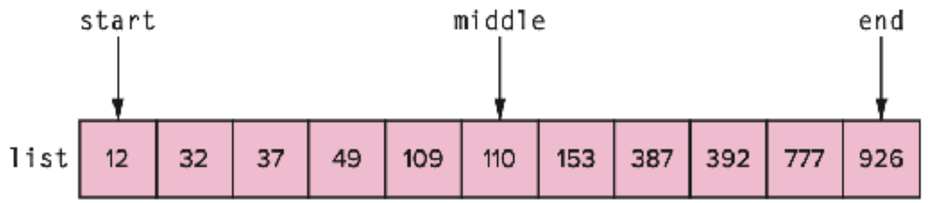

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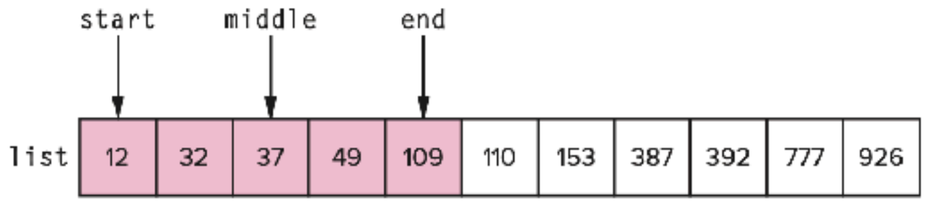

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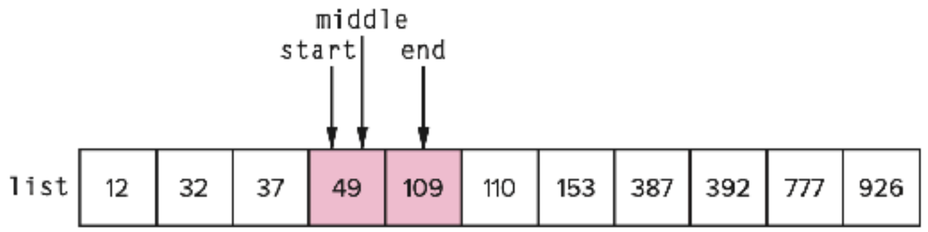

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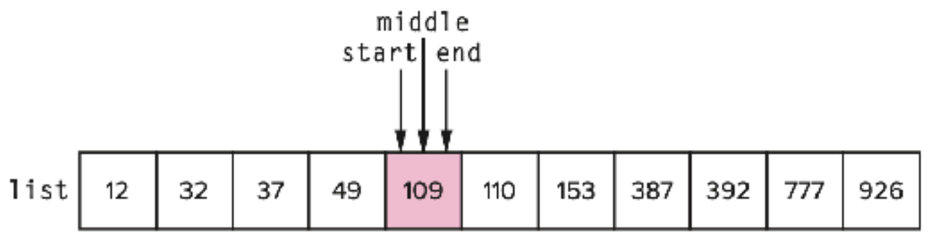

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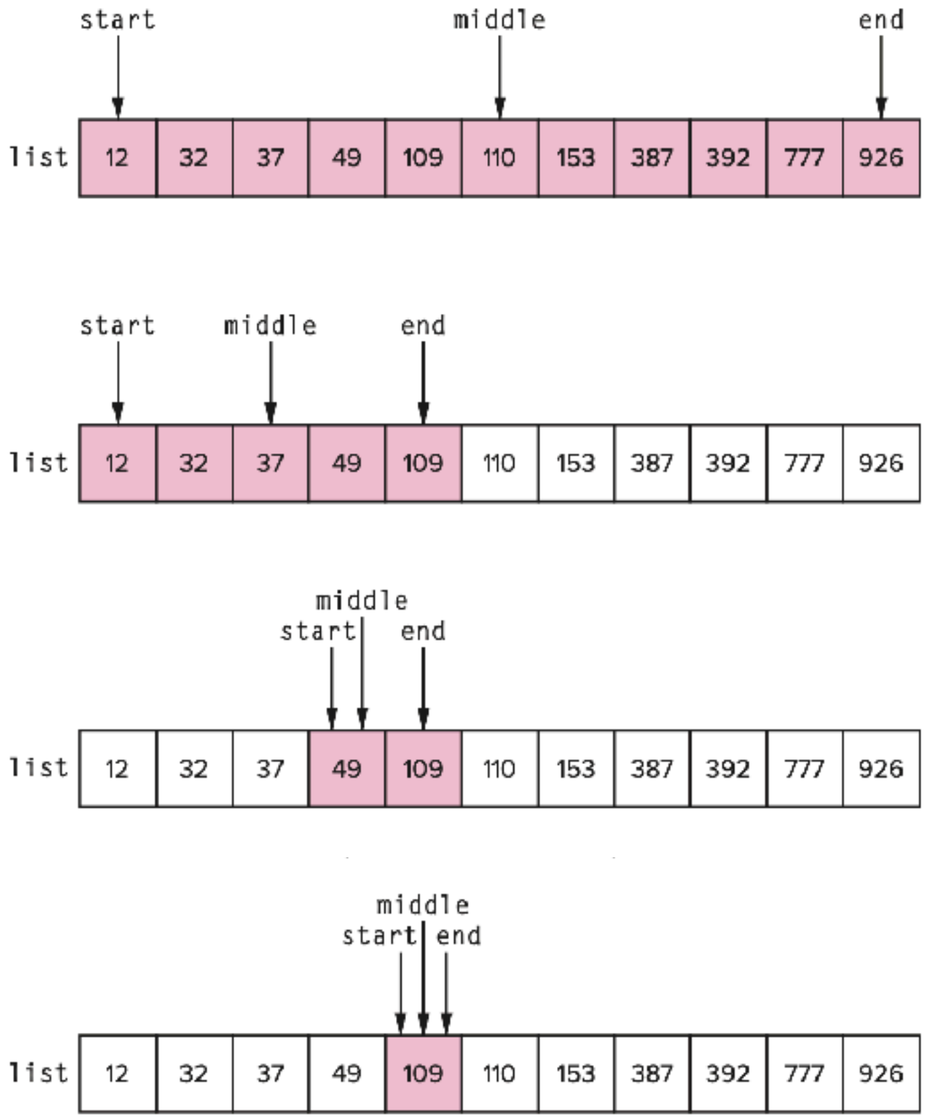

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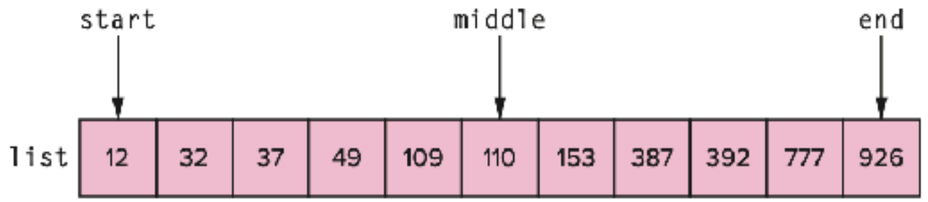
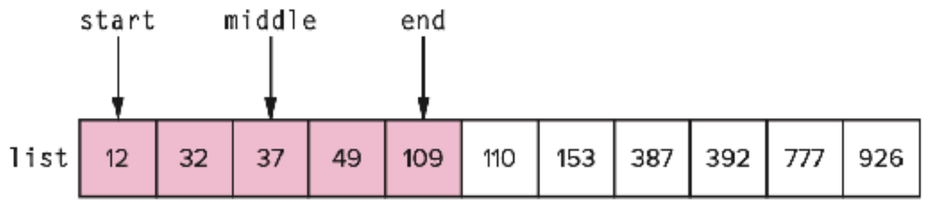
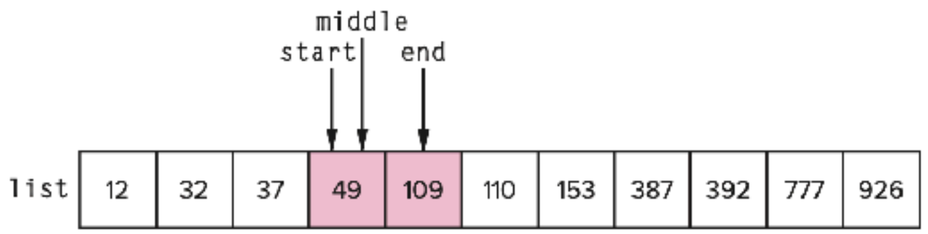
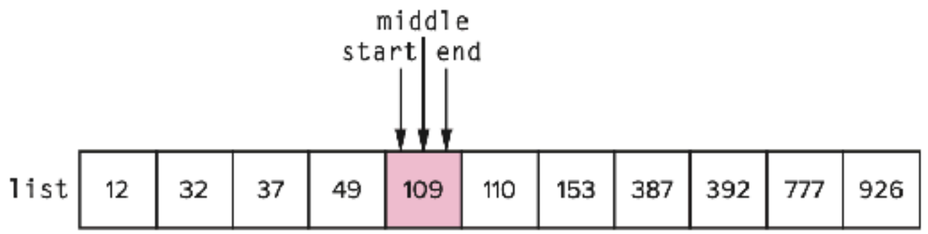
```c
int binary(int arr[], int n, int key){
  int start = 0;       // Left pointer
  int end = _____; // Right pointer

  while (end >= start){
    int mid = (____________) / 2; // Pick middle element

    // Logic to focus search on left or right of mid
    if (key == arr[mid])
      return mid;
    else if (key < arr[mid])
      end = _______;
    else
      start = ________;
  }
  return -1; // Loop exited, element not present.
}
```









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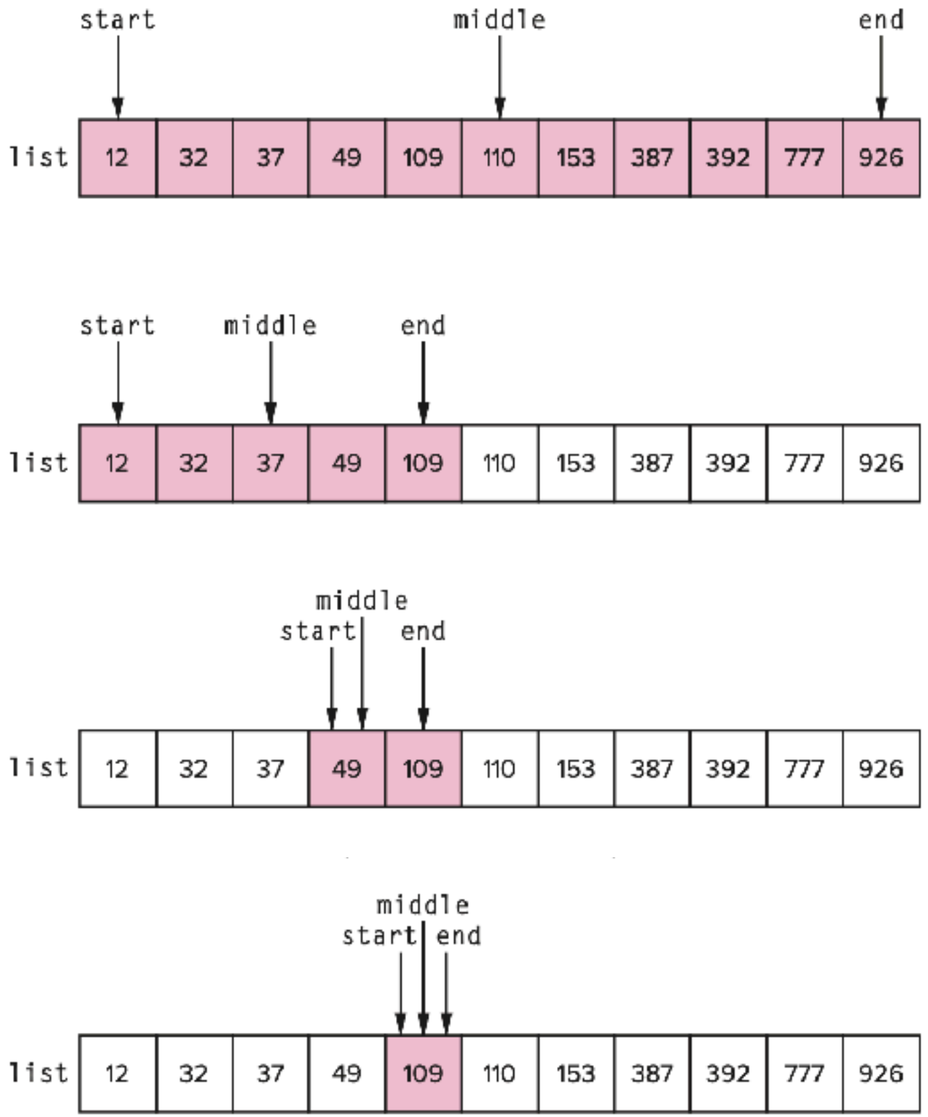

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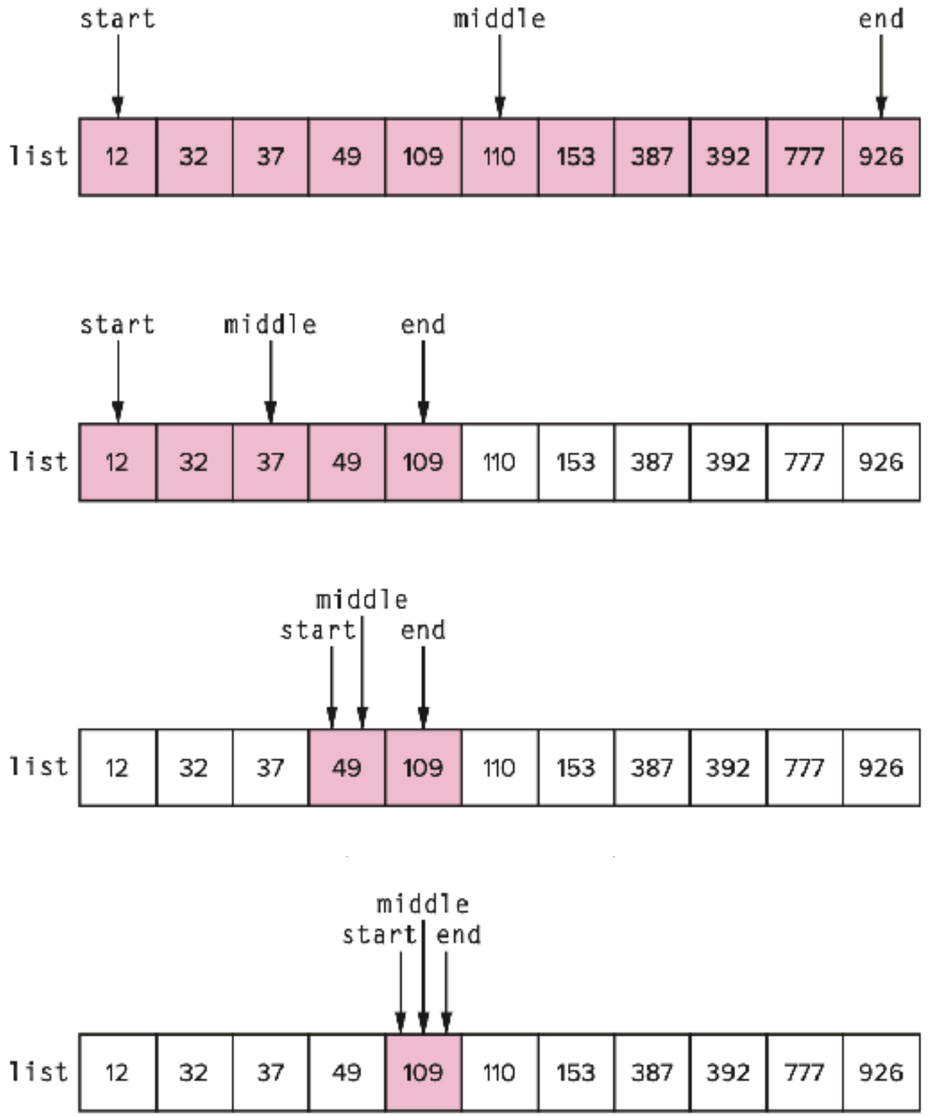

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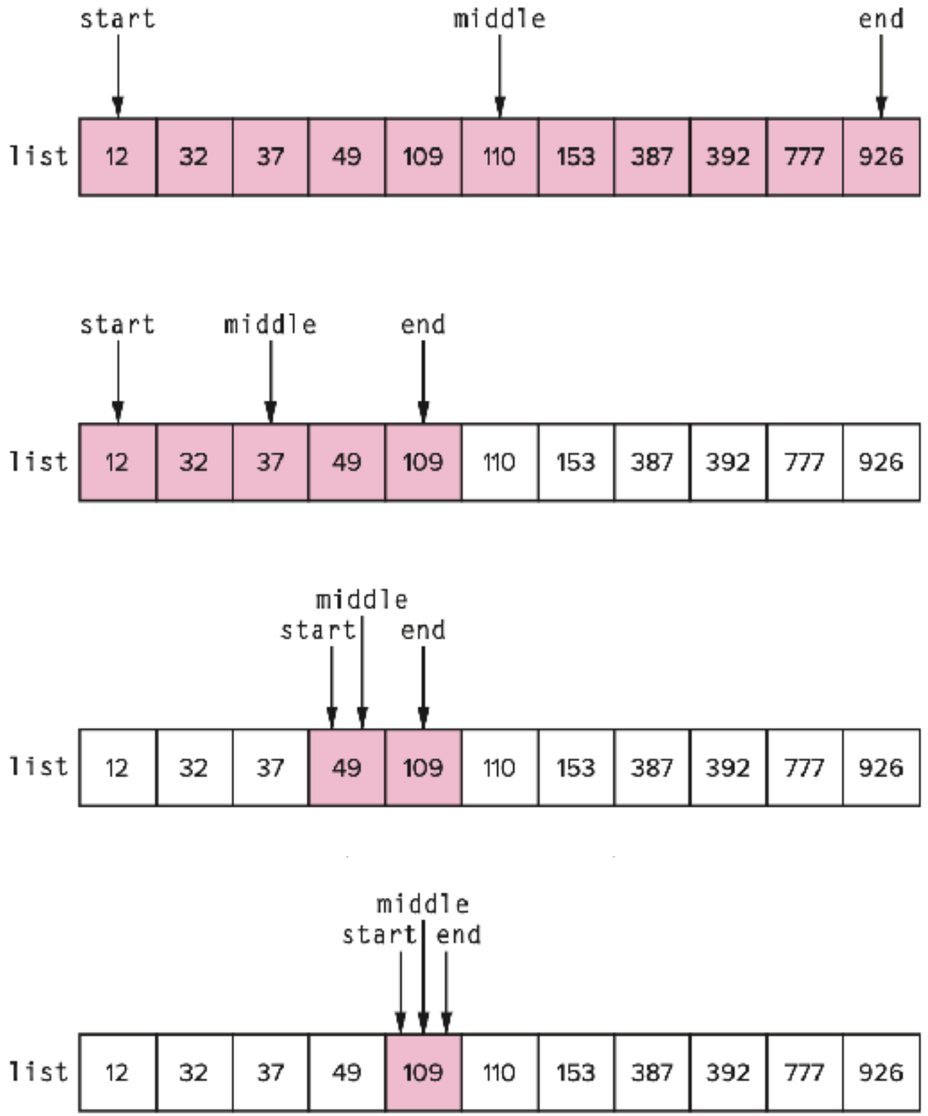

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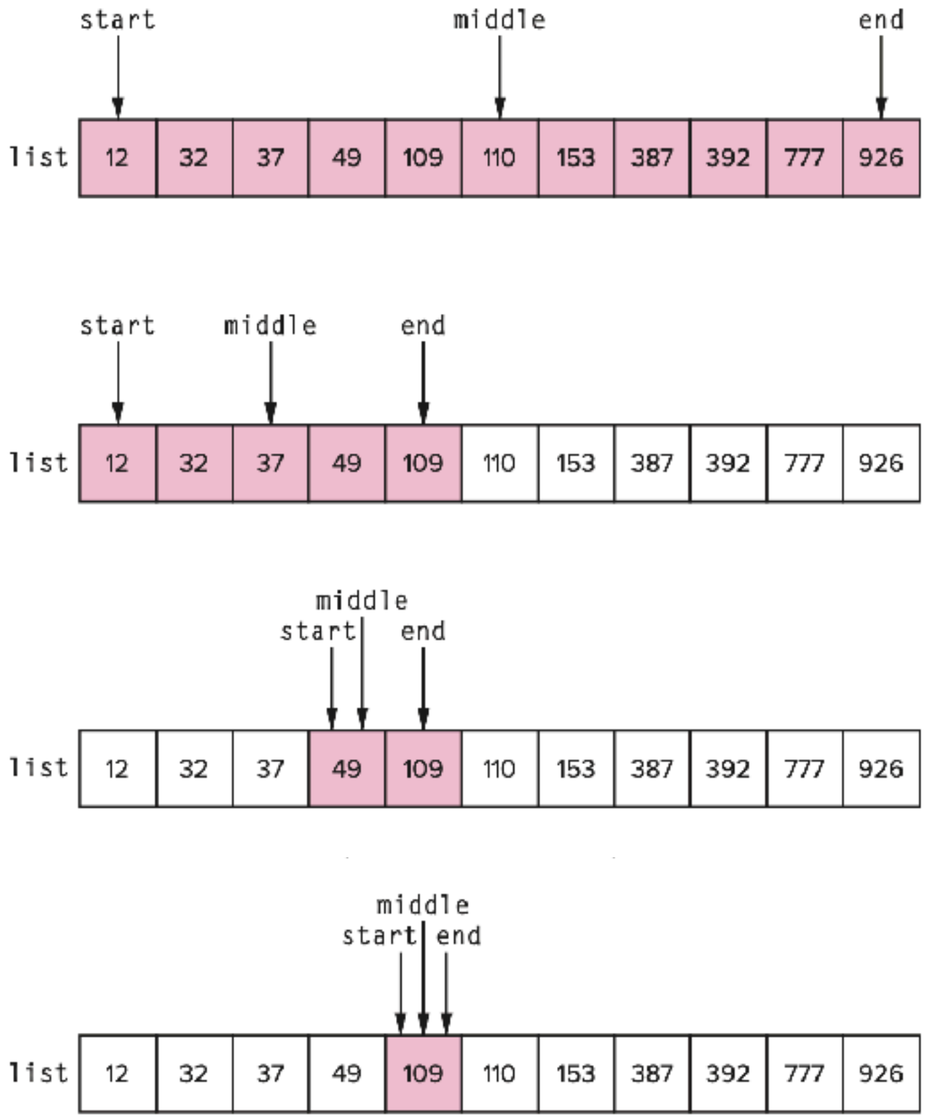

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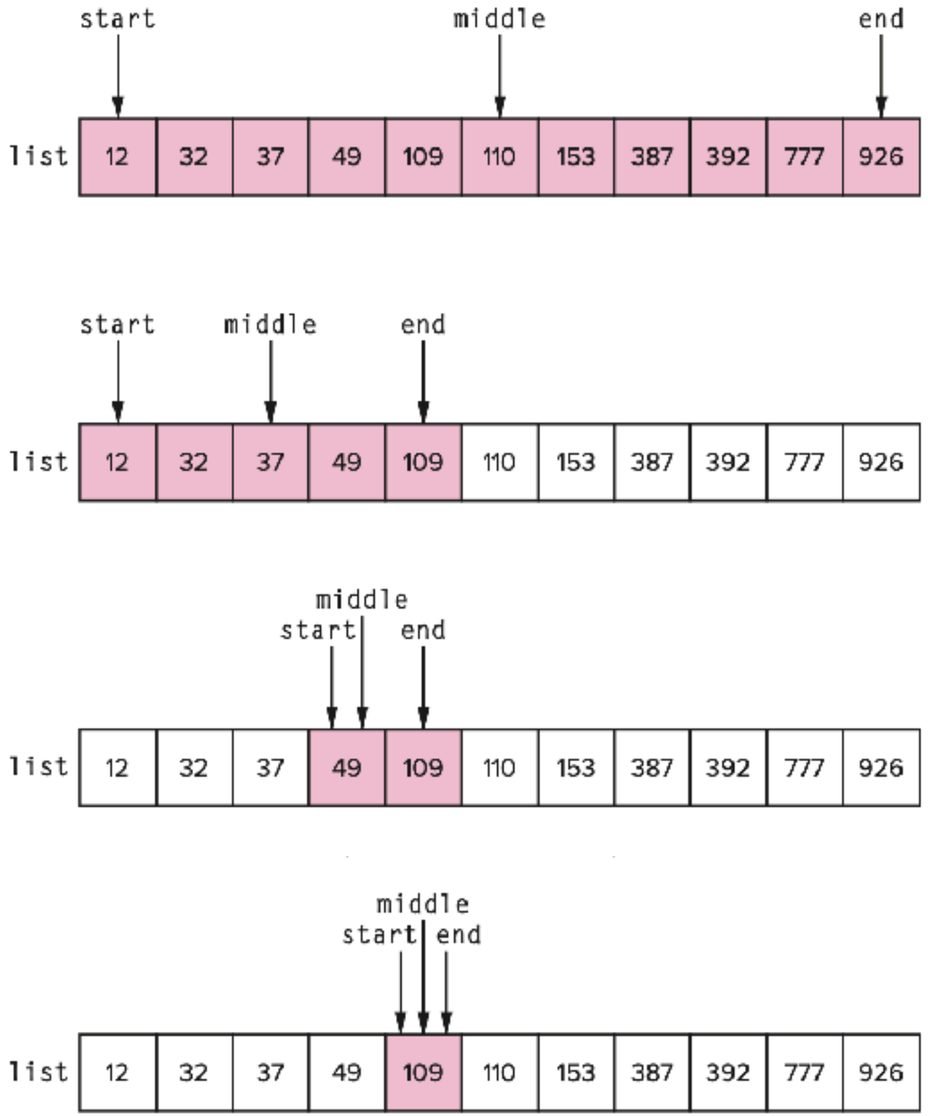

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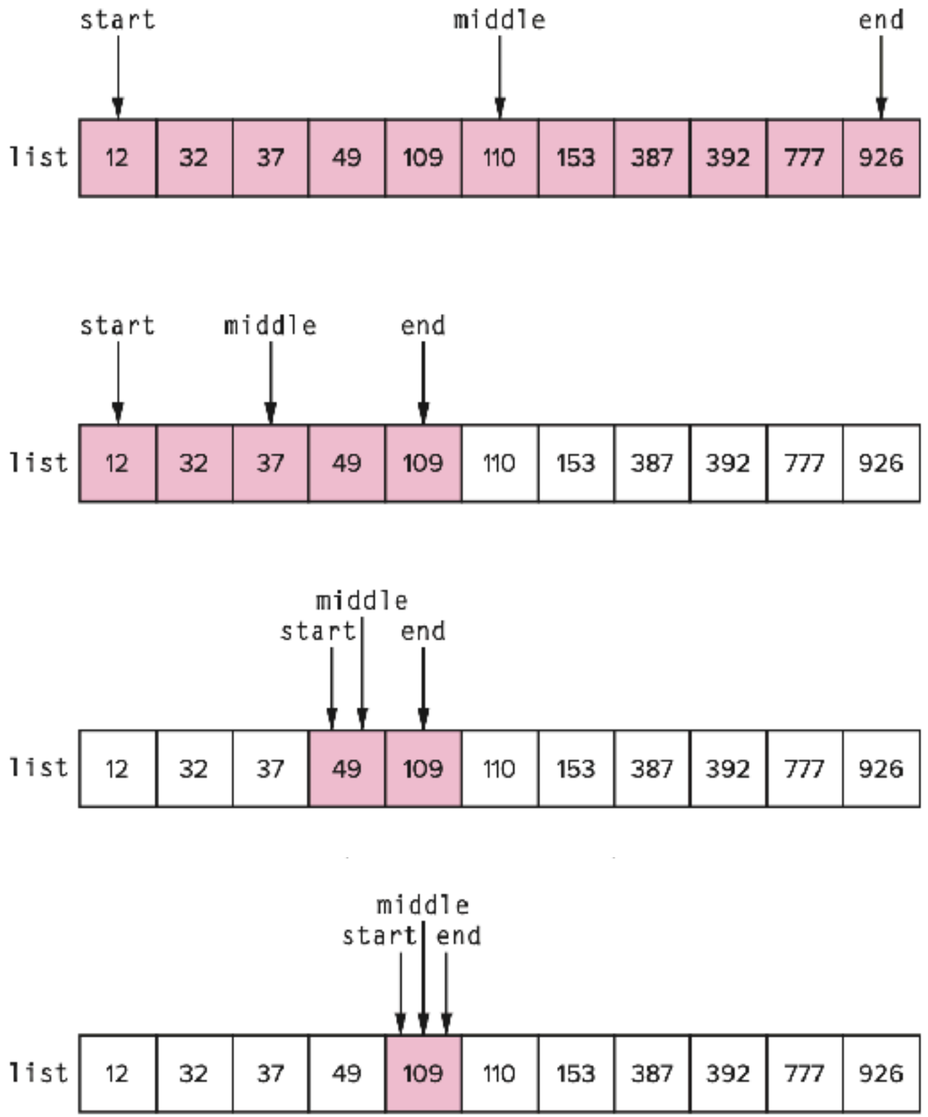

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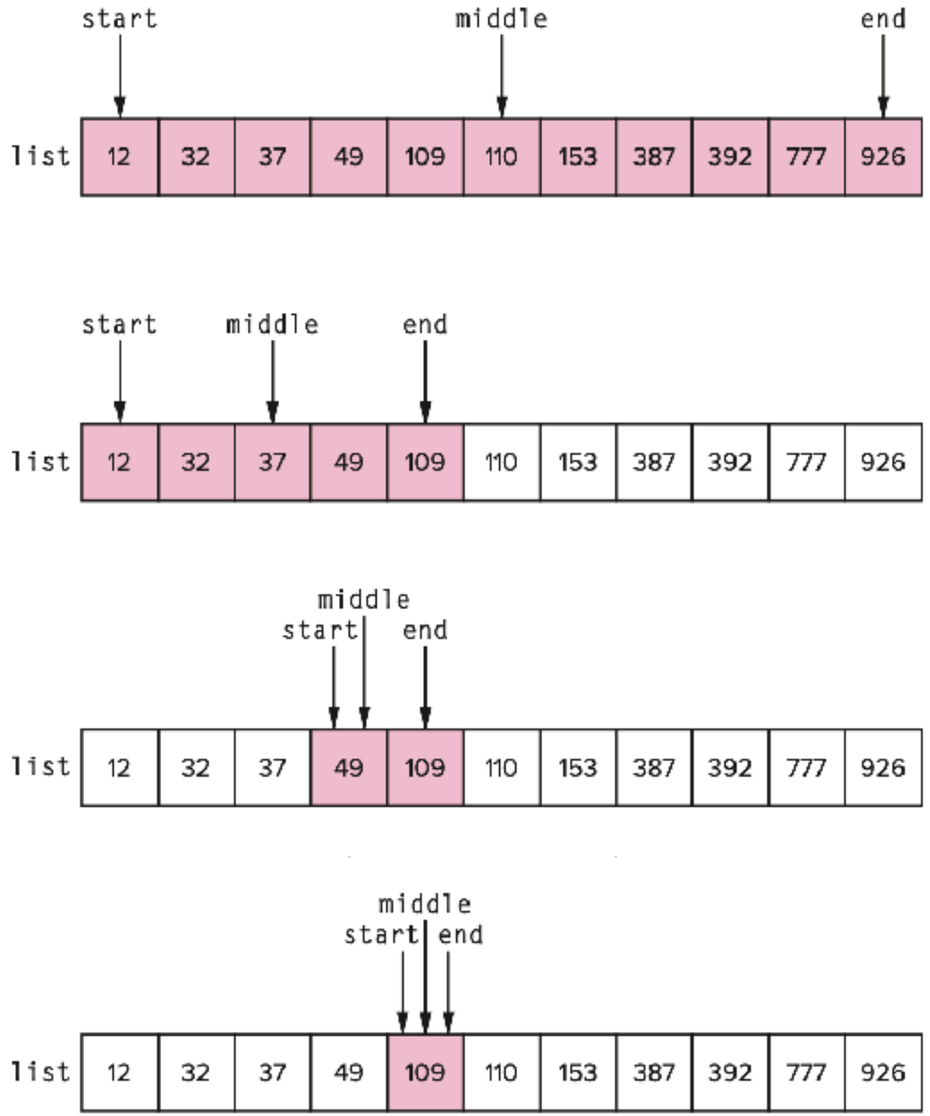
  - We already saw this in QuickSort!



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Examples: Factorial function, Fibonacci series, binary search, etc.

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  - The input to the next call gets reduced gradually to the terminating case.

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- Mathematical definition

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$$n! = n \cdot (n-1) \cdot (n-2) \ldots \cdot 2 \cdot 1 \qquad for \quad n > 0$$

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$$n! = n \cdot (n-1) \cdot (n-2) \ldots \cdot 2 \cdot 1 \qquad for \quad n > 0$$

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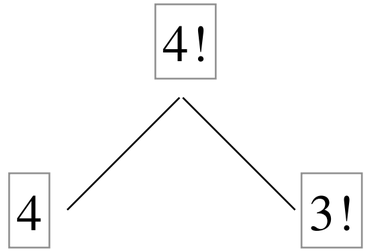

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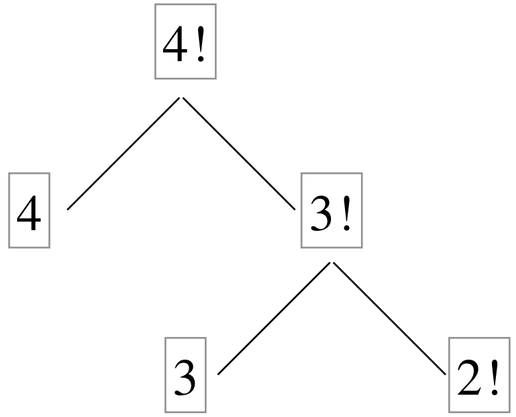

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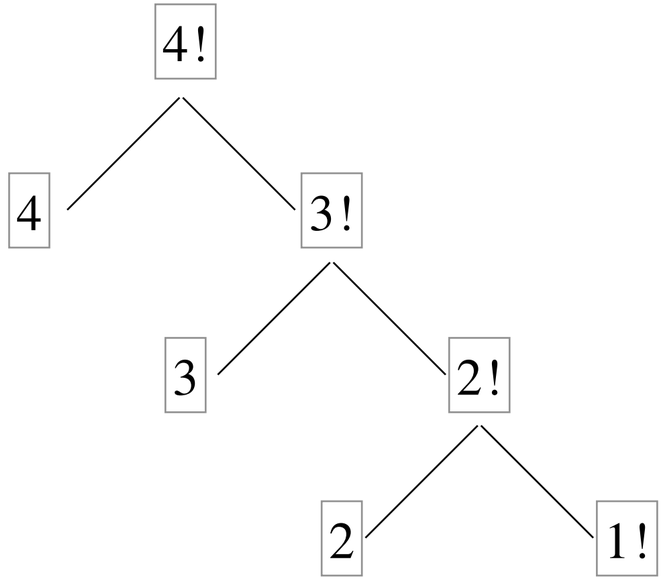

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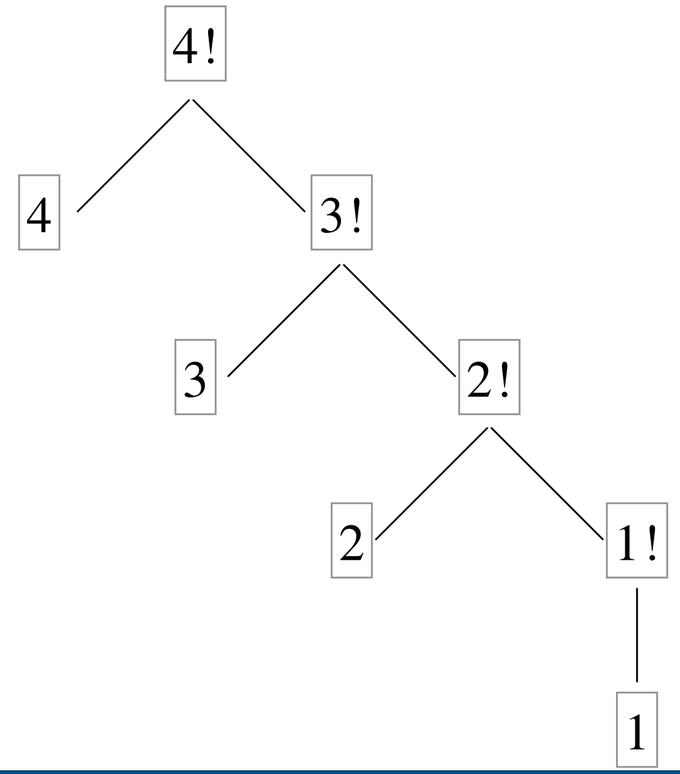

Example: Factorial

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```
int Factorial(int n){
     if (n == 1)
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     else
          return n * Factorial(n - 1);
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Example: Factorial

```
fn = Factorial(4);
```

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```
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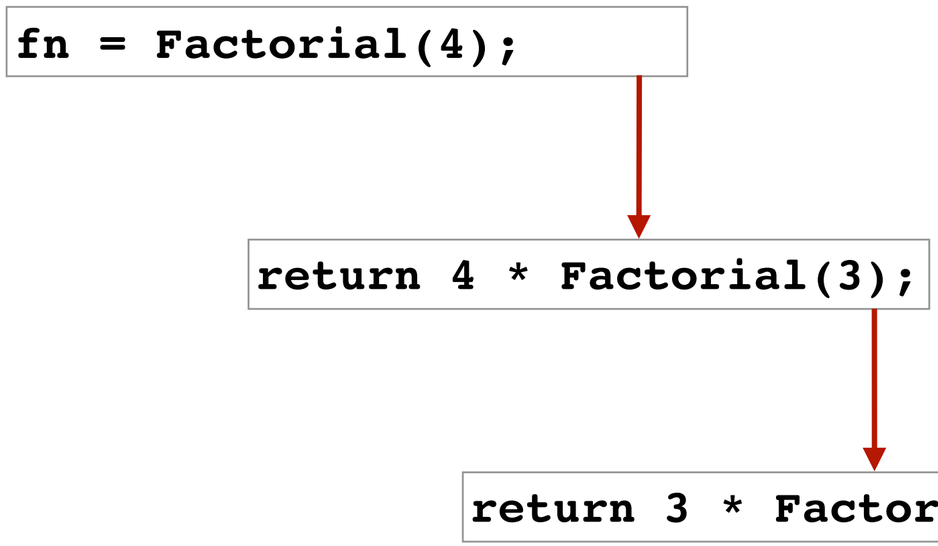
Example: Factorial

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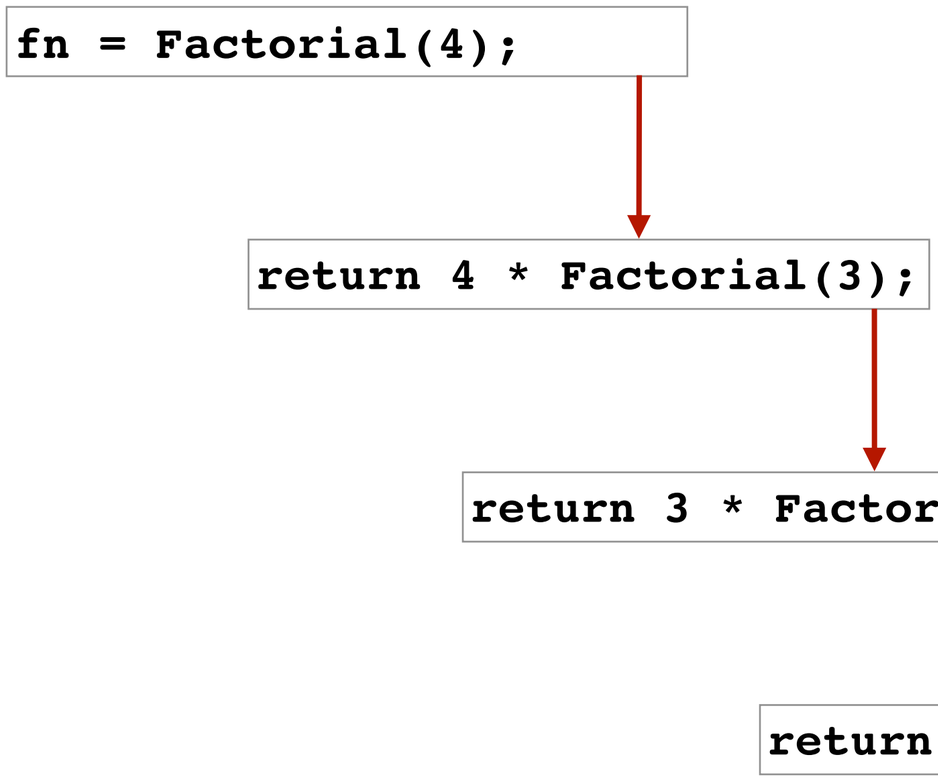


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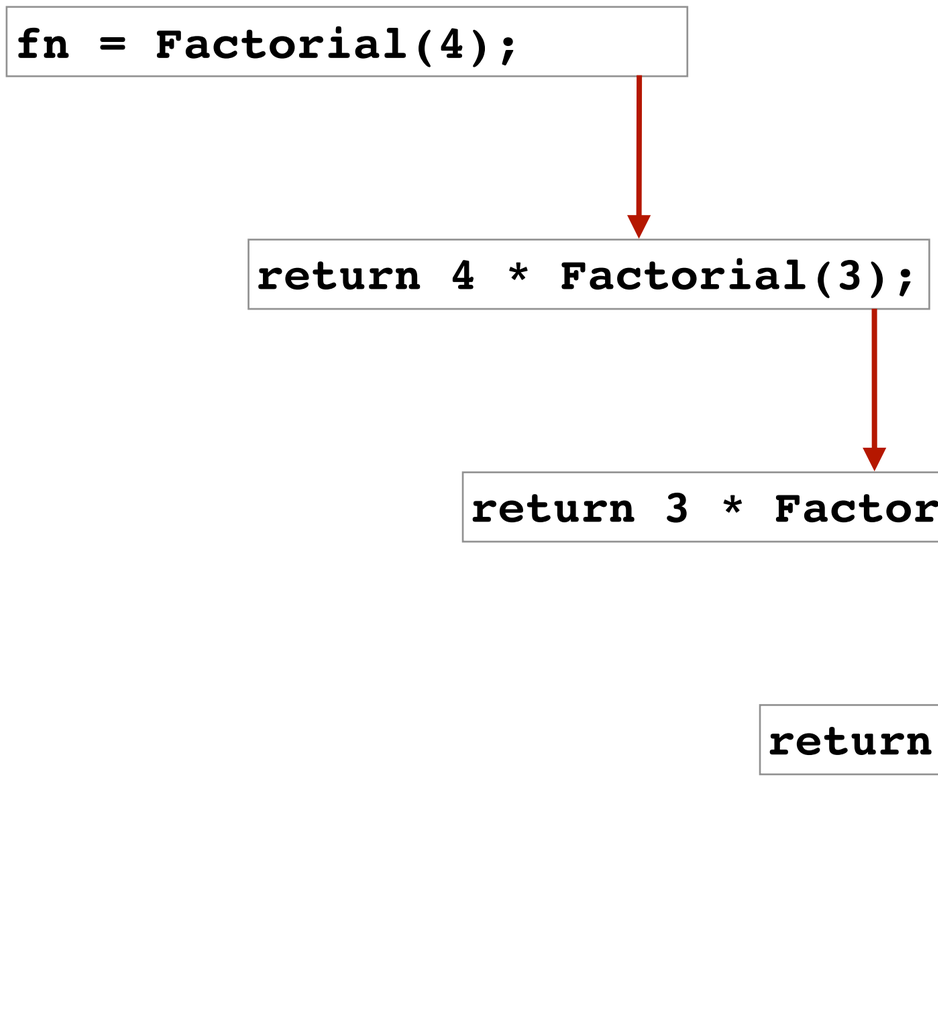


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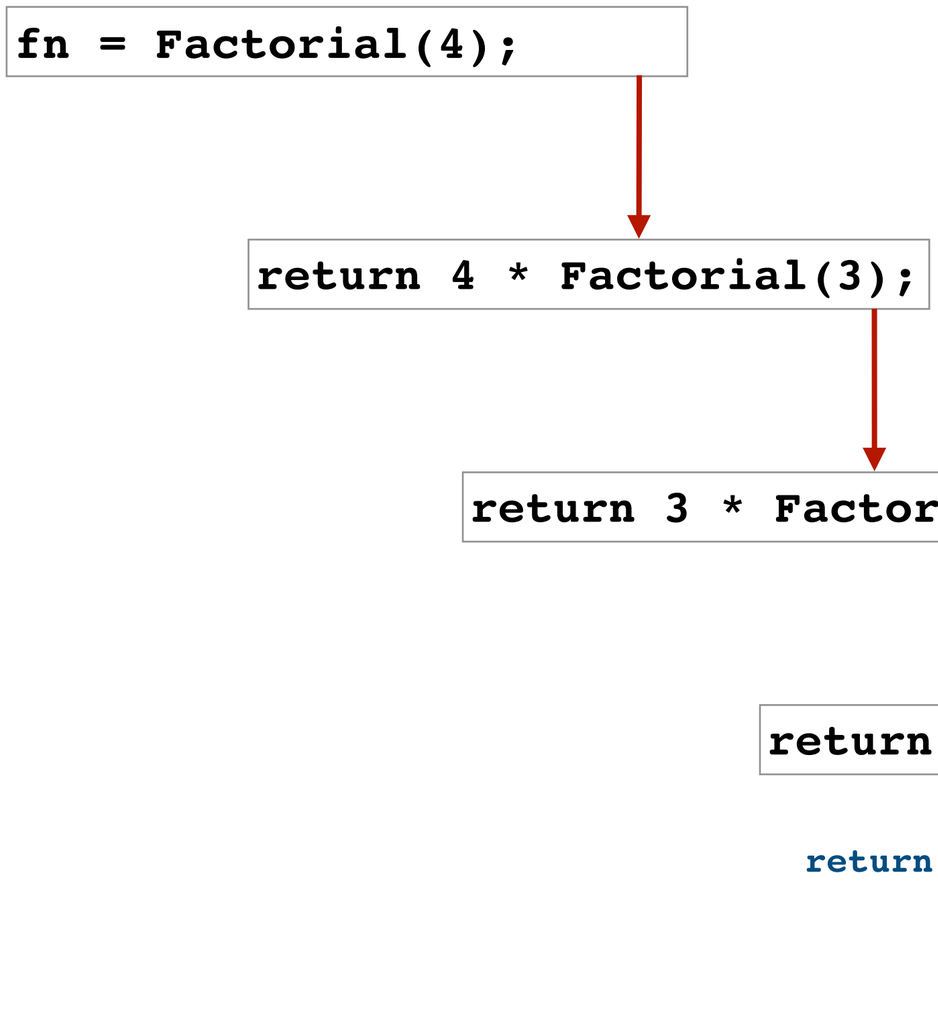


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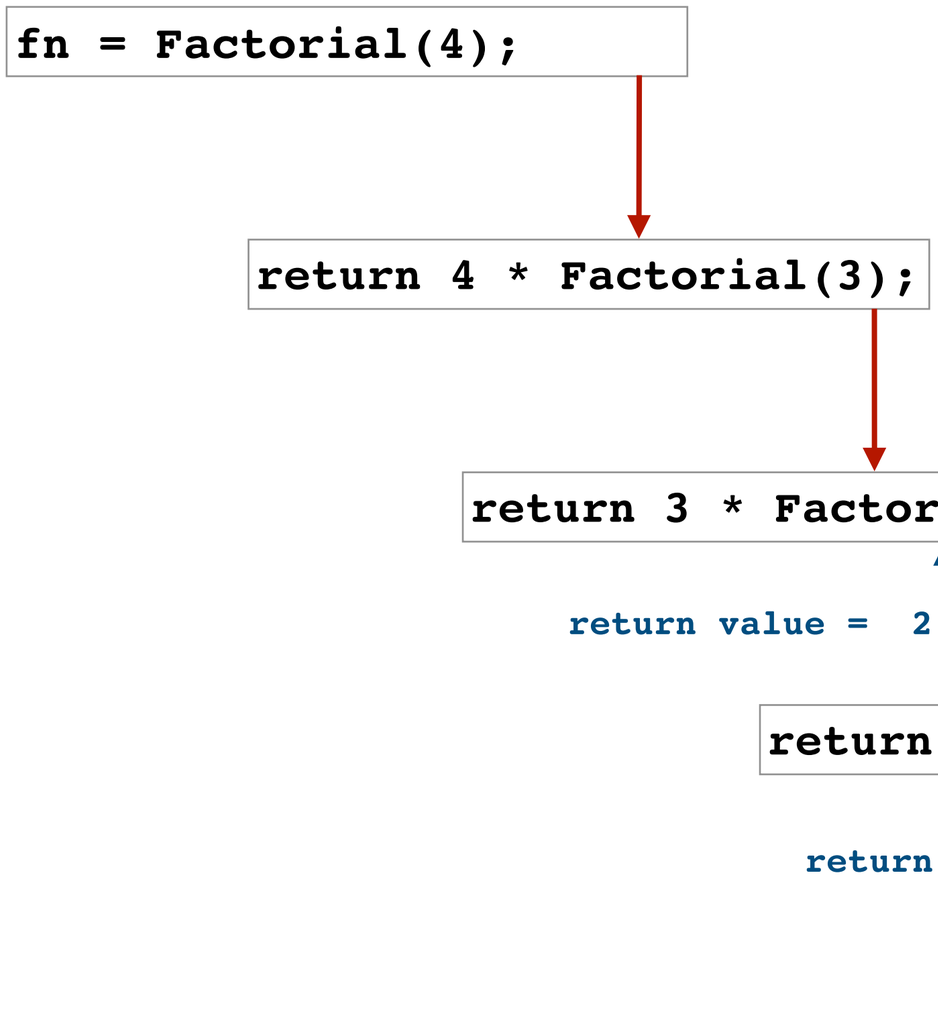


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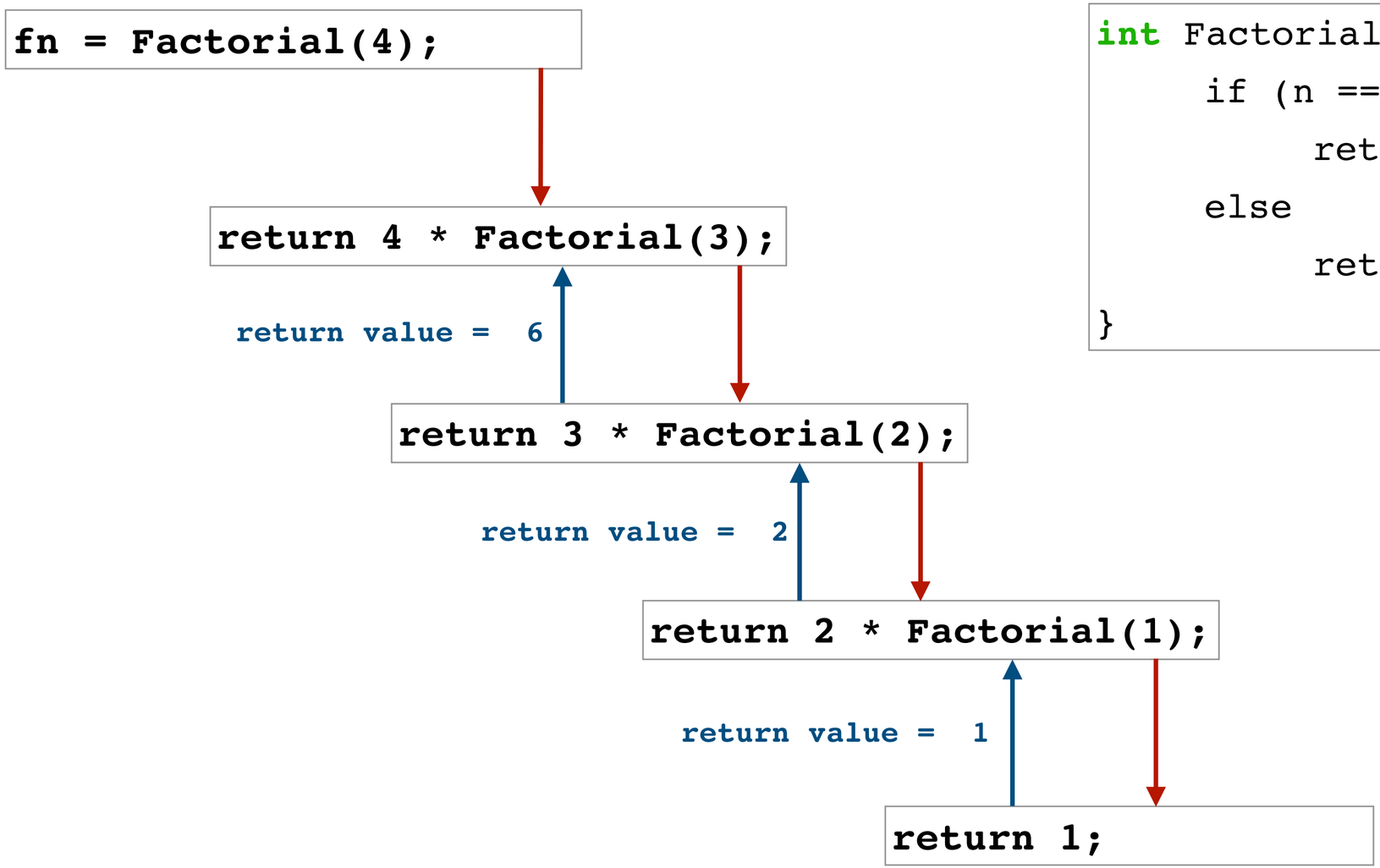


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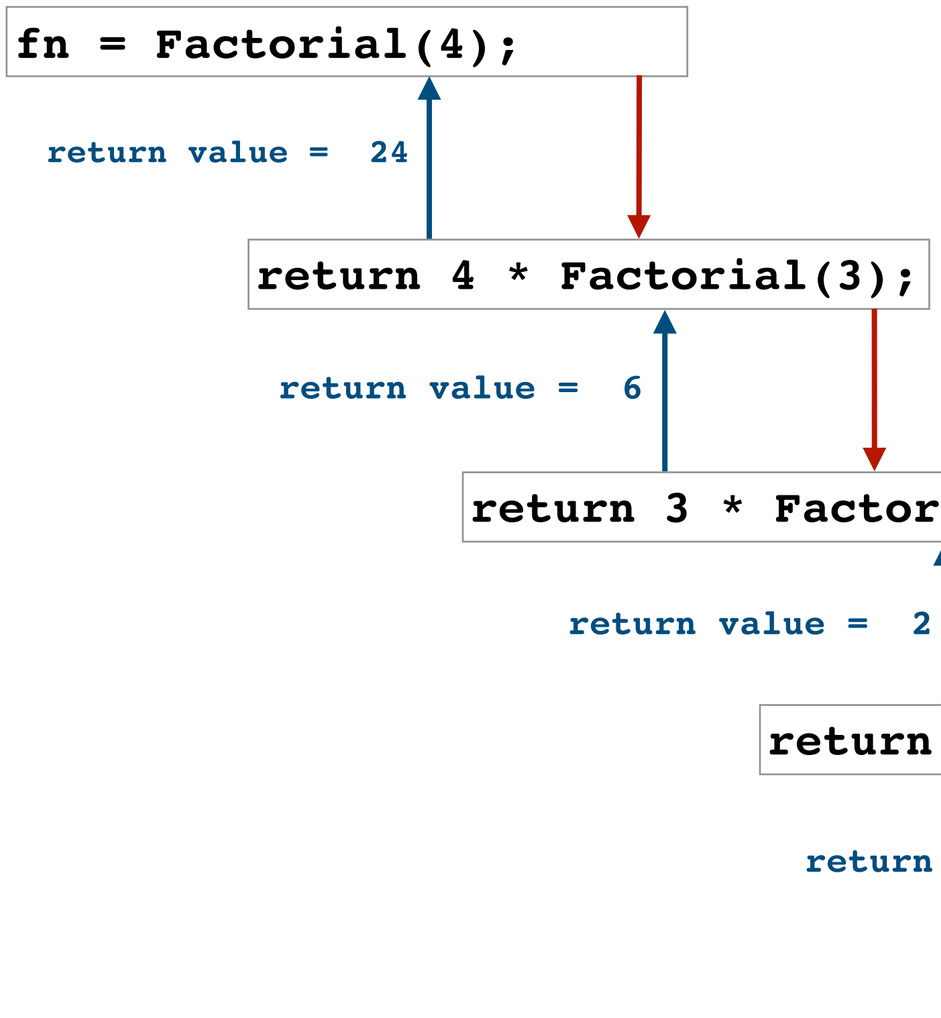


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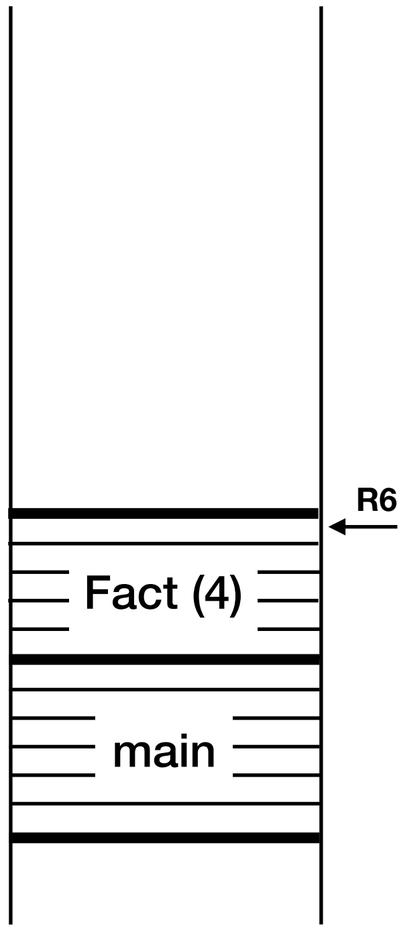
RTS During Execution of Factorial

main calls Factorial(4)



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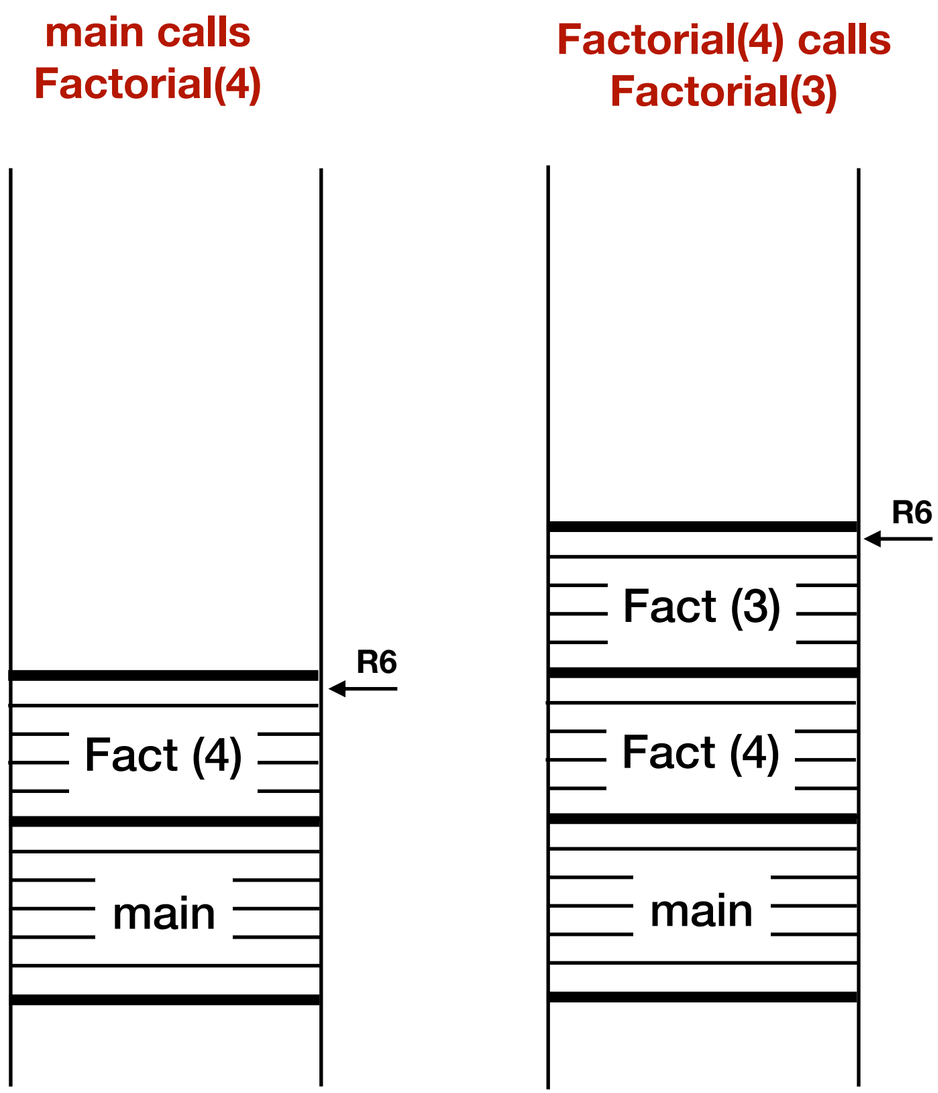

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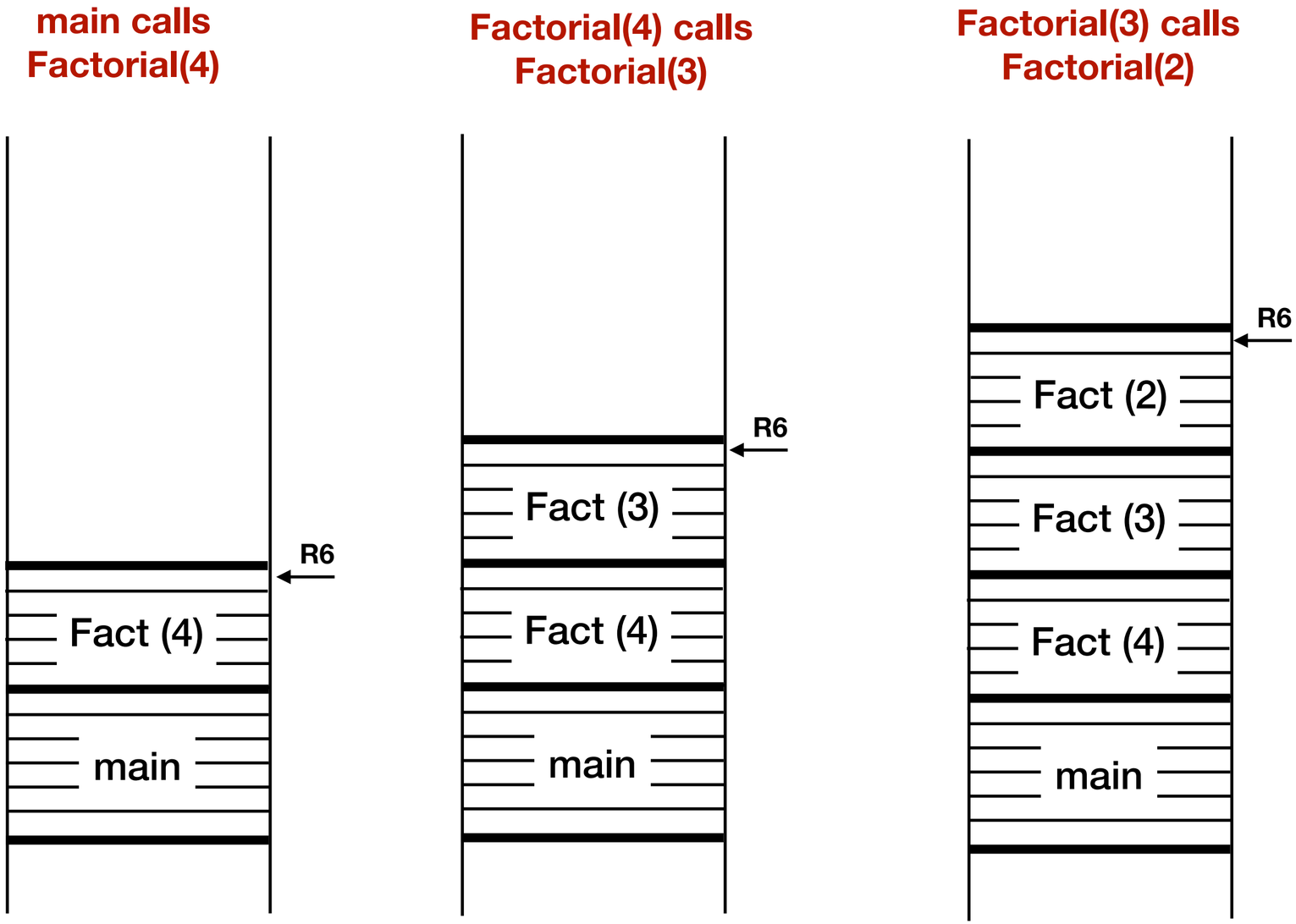

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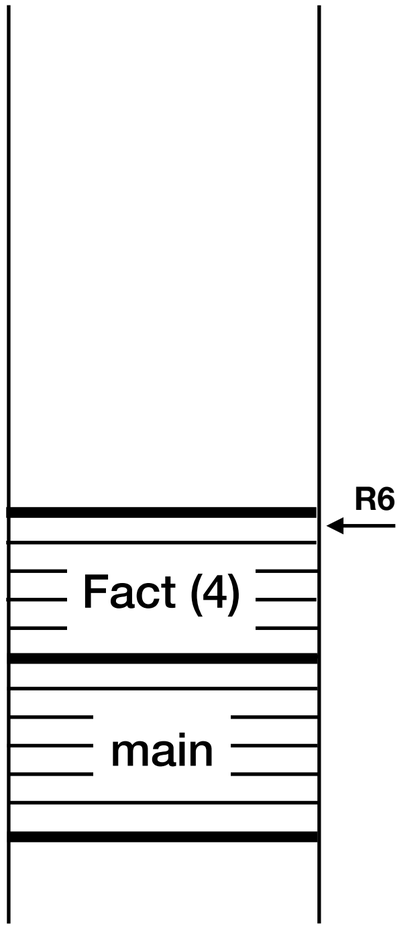
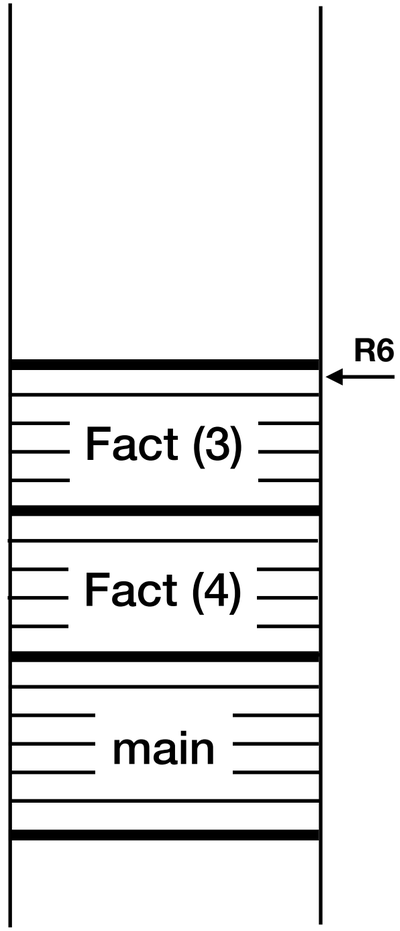
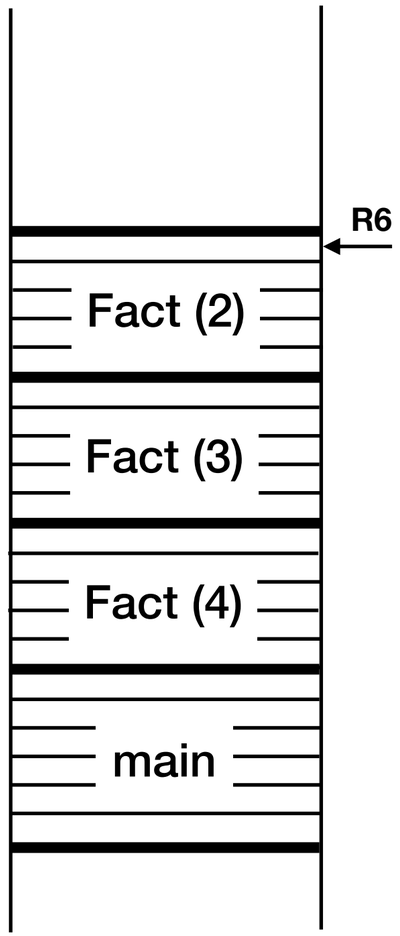


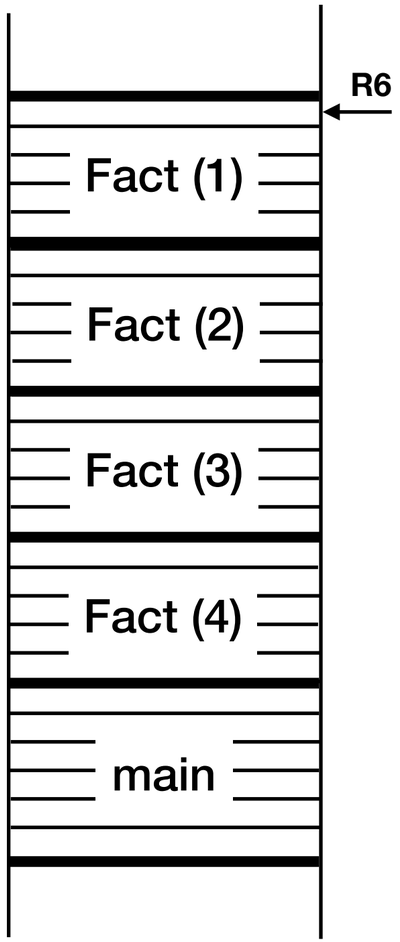
Factorial(4) calls Factorial(3)



Factorial(3) calls Factorial(2)



Factorial(2) calls Factorial(1)



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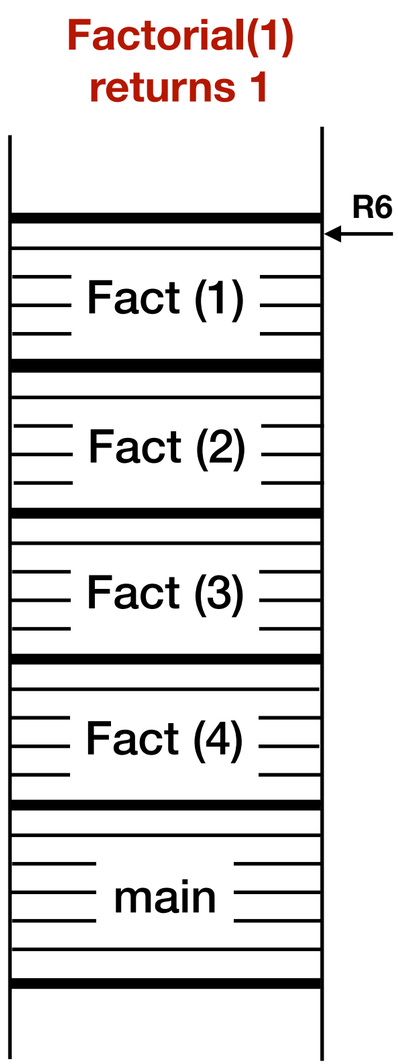

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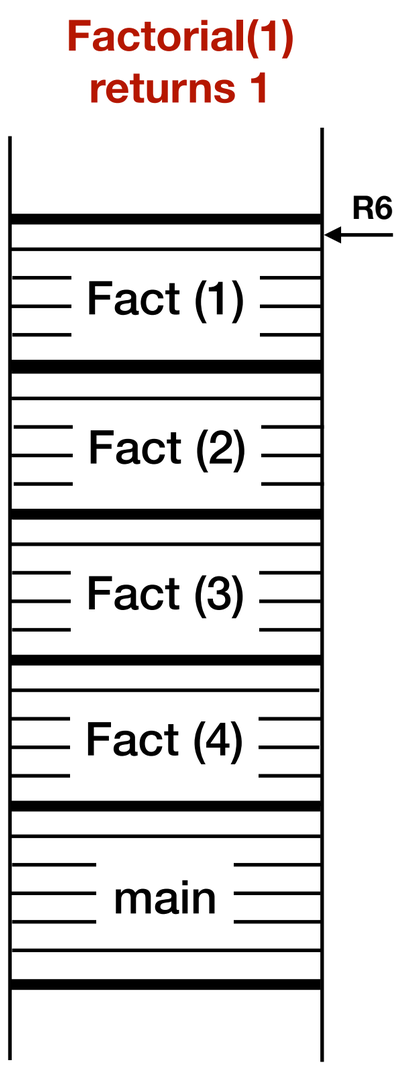
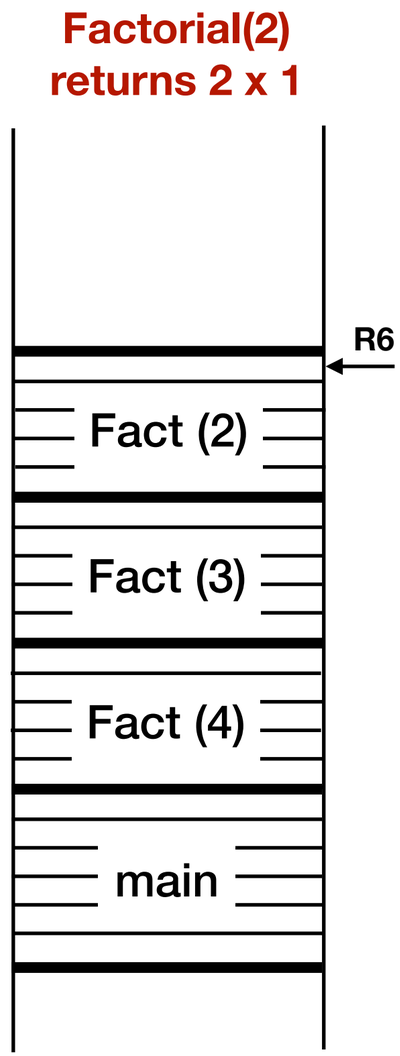
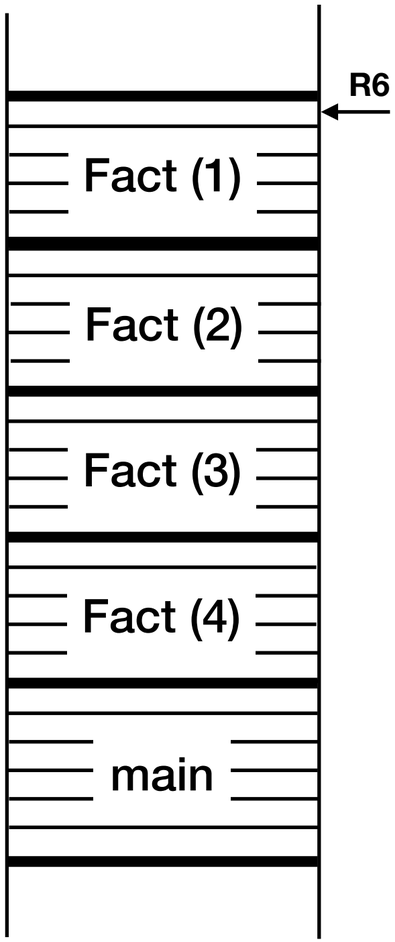


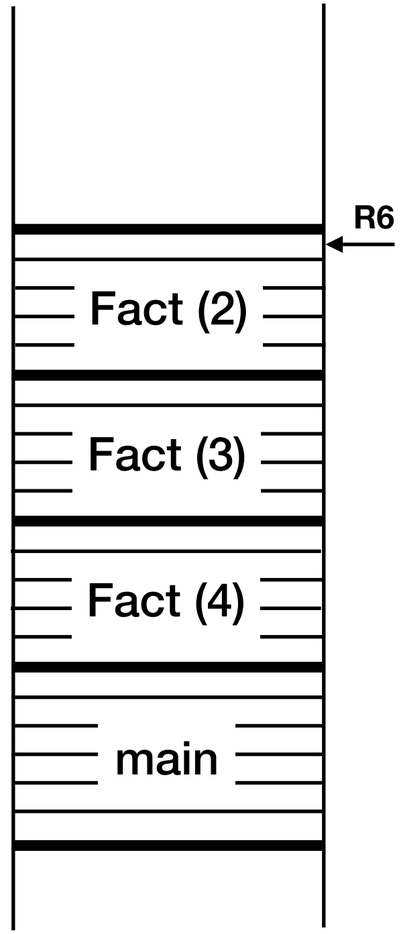


RTS During Execution of Factorial

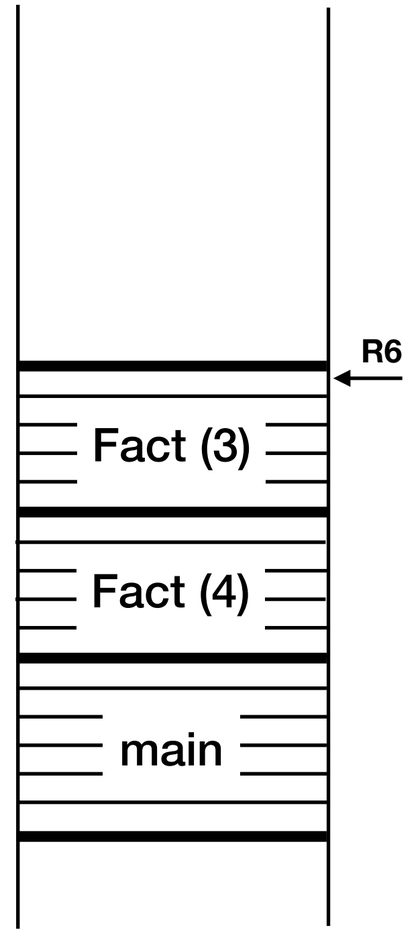
Factorial(1) returns 1



Factorial(2) returns 2 x 1



Factorial(3) returns 3 x 2



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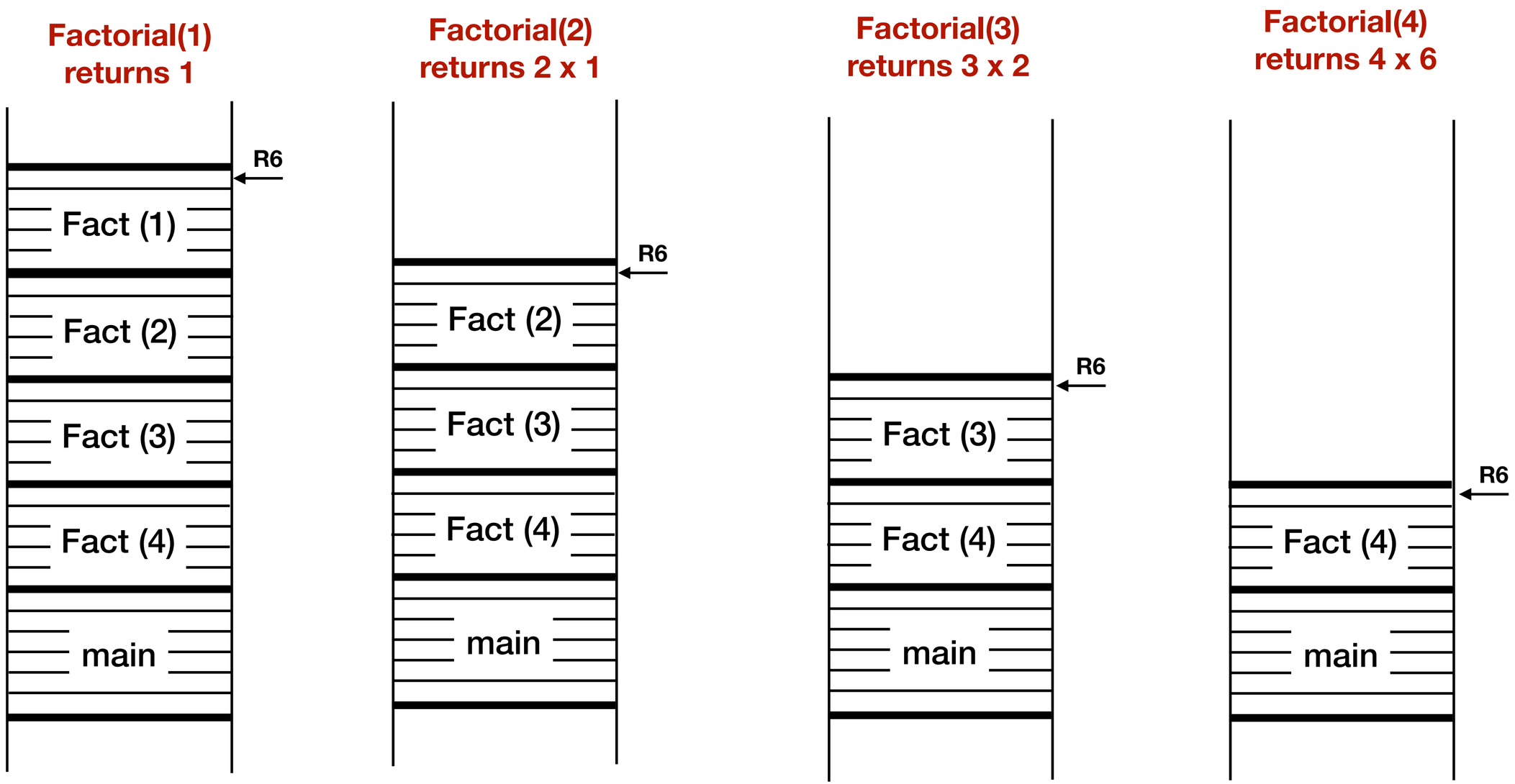

Example: Fibonacci series

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Mathematical definition:

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Fibonacci Series: 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, ……

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= (Fibonacci(1) + Fibonacci(0)) + Fibonacci(1)

= 1 + 1 + 1 = 3

Example: Fibonacci series

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```
int Fibonacci (int n) {
    int sum;
    if (n == 0 || n == 1)
        return 1;
    else {
        sum = (Fibonacci(n-1) + Fibonacci(n-2));
        return sum;
        }
}
```

Example: Binary search

```c
int binary(int arr[], int n, int key){
  int start = 0;       // Left pointer
  int end = n - 1; // Right pointer

  while (end >= start){
    int mid = (start + end) / 2; // Pick middle element

    // Logic to focus search on left or right of mid
    if (key == arr[mid])
      return mid;
    else if (key < arr[mid])
      end = mid - 1;
    else
      start = mid + 1;
  }
  return -1; // Loop exited, element not present.
}
```

Can we implement binary search in *recursive* way?

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Can we implement binary search in *recursive* way?

- **Need**: A mechanism to keep track of the `start` and `end` indices across recursive calls; local variables won't do (why?).

Example: Binary search

Example: Binary search

```
int binary_search(int item, int list[], int start, int end){
    int middle = (end + start)/2;
    if (end < start)
      return -1;                          // Did not find key
    else if (_________________________) // Found item!
        return middle;
    else if (_________________________)  // Search left half
        return binary_search(item, list, start, middle-1);
    else                                  // Search right half
        return binary_search(_____________________________);
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```

Example: Binary search

Mechanism

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Concept check

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- What are the base/terminating cases in binary search?

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- What are the recursive cases in binary search?

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- Recall the steps ….

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    1. If pointers cross/overlap, **split array**, then **repeat** on each subarray.

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We wrote one function to split array

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5. If neither pointers can move swap elements.
6. Repeat 3-5 while left pointer < right pointer.

And a loop to repeat on subarrays after split

We wrote one function to split array

Recursive implementation

```c
void Swap(int* one, int* two){
  int temp = *one;
  *one = *two;
  *two = temp;
}
```

```c
void QuickSort(int arr[], int start, int end){
  if (start < end){
    int pivotVal = partition(arr, start, end);

    // Now sort left half
    QuickSort(arr, start, _________);

    // And right half
    QuickSort(arr, ___________, end);
  }
}
```

```c
int partition(int arr[], int start, int end){

  int pivotVal = ___________;
  int left = _________; // Initialize left
  int right = _______;   // Initialize right

  while(1){
    do left++; // Increment left till …
    while (_____________________);

    do right--; // Decrement right till …
    while (_____________________);

    if (___________) // Split if overlap
      return right;

    Swap(&arr[left], &arr[right]);
  }
}
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Check Gitlab for reference material: https://github.com/iabraham/ece220-fa25/blob/main/lec12_1007/quicksort.c

Recursive implementation

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  int temp = *one;
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    int pivotVal = partition(arr, start, end);

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    QuickSort(arr, ____________, end);
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  int pivotVal = arr[start];
  int left = start - 1; // Initialize left
  int right = _______;   // Initialize right

  while(1){
    do left++; // Increment left till …
    while (_____________________);

    do right--; // Decrement right till …
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    while (_____________________);

    do right--; // Decrement right till …
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    do left++; // Increment left till …
    while (arr[left] < pivotVal);

    do right--; // Decrement right till …
    while (_____________________);

    if (___________) // Split if overlap
      return right;

    Swap(&arr[left], &arr[right]);
  }
}
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    QuickSort(arr, pivotVal + 1, end);
  }
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  int left = start - 1; // Initialize left
  int right = end + 1;   // Initialize right

  while(1){
    do left++; // Increment left till …
    while (arr[left] < pivotVal);

    do right--; // Decrement right till …
    while (arr[right] > pivotVal);

    if (_____) // Split if overlap
      return right;

    Swap(&arr[left], &arr[right]);
  }
}
```

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    QuickSort(arr, start, pivotVal);

    // And right half
    QuickSort(arr, pivotVal + 1, end);
 }
}
```

```c
int partition(int arr[], int start, int end){

  int pivotVal = arr[start];
  int left = start - 1; // Initialize left
  int right = end + 1;   // Initialize right

  while(1){
    do left++; // Increment left till …
    while (arr[left] < pivotVal);

    do right--; // Decrement right till …
    while (arr[right] > pivotVal);

    if (left >= right) // Split if overlap
      return right;

    Swap(&arr[left], &arr[right]);
  }
}
```

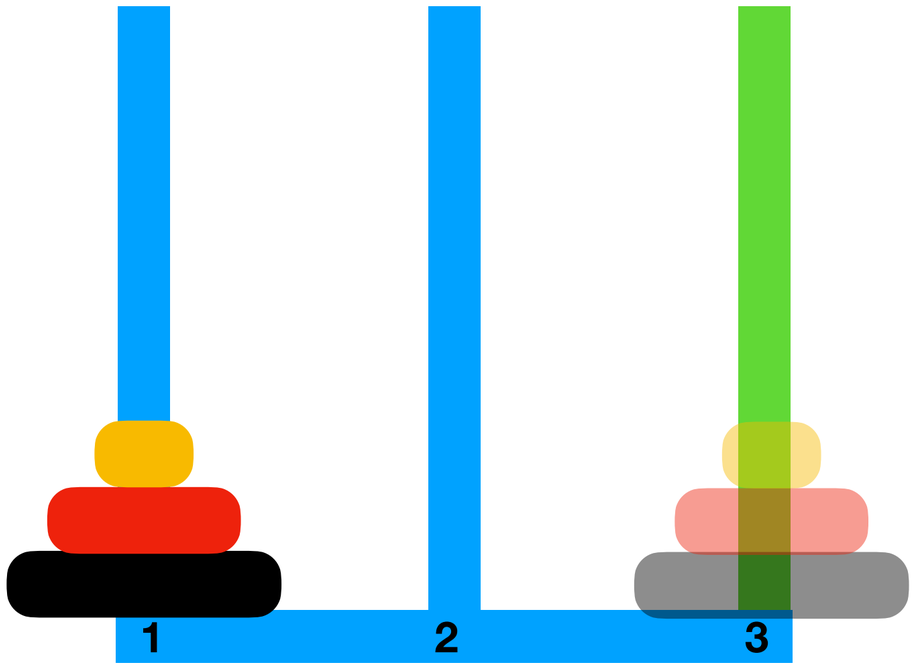
Check Gitlab for reference material: https://github.com/iabraham/ece220-fa25/blob/main/lec12_1007/quicksort.c

Concept check

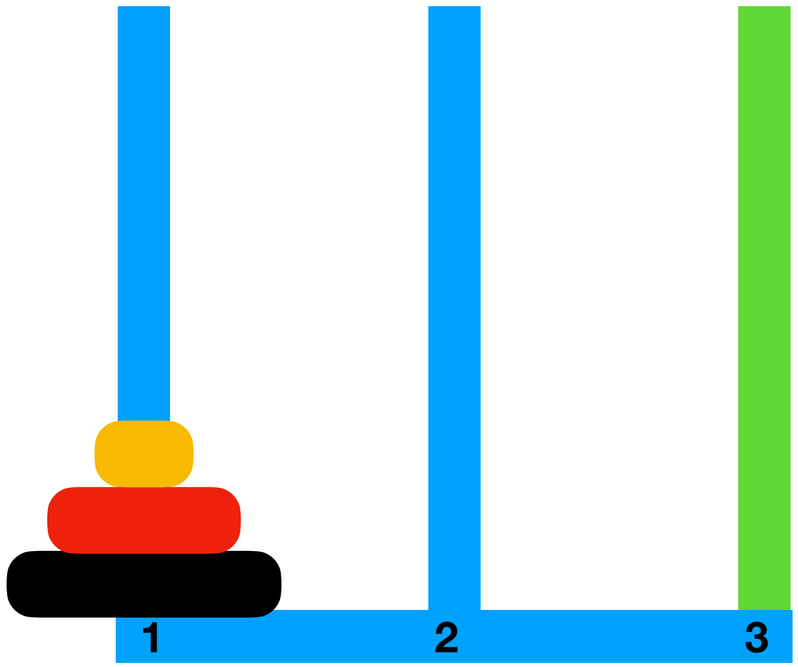
Concept check

- Why did iterative version of Quicksort need a stack to work but iterative binary search did not?

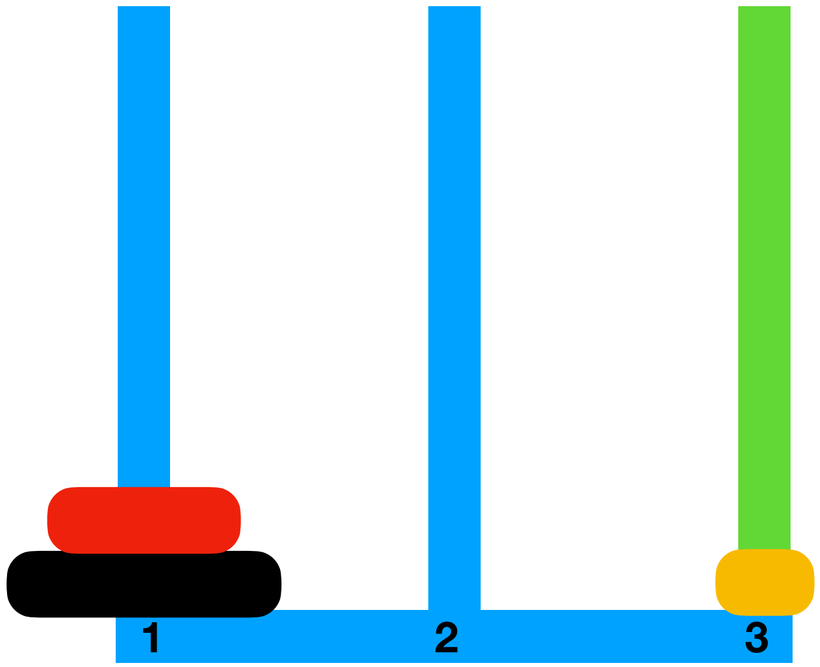
Example: Towers of Hanoi



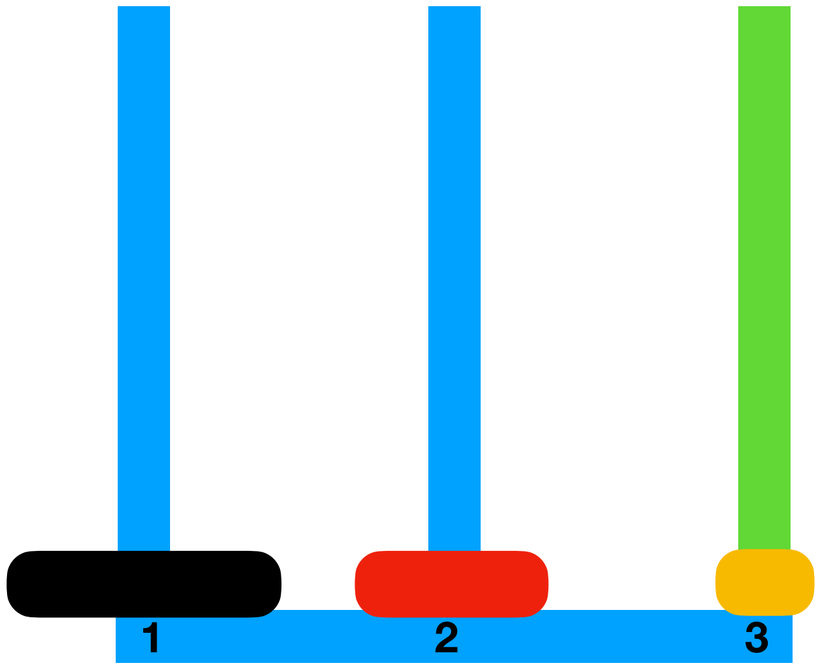
Example: Towers of Hanoi



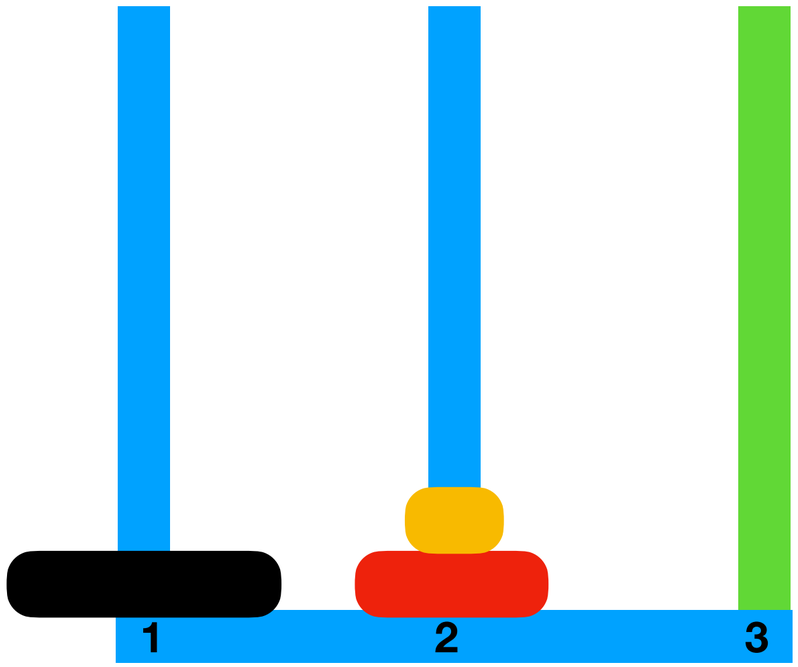
Example: Towers of Hanoi



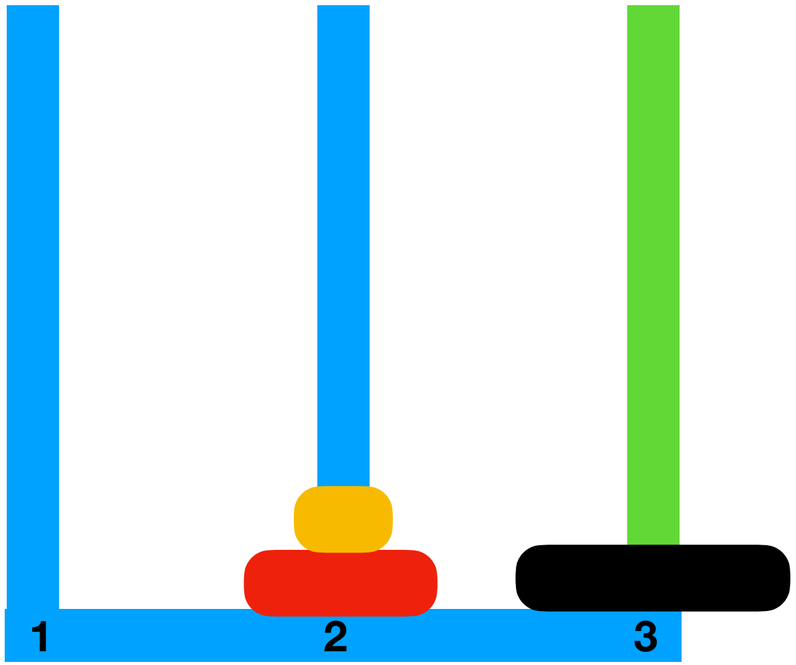
Example: Towers of Hanoi



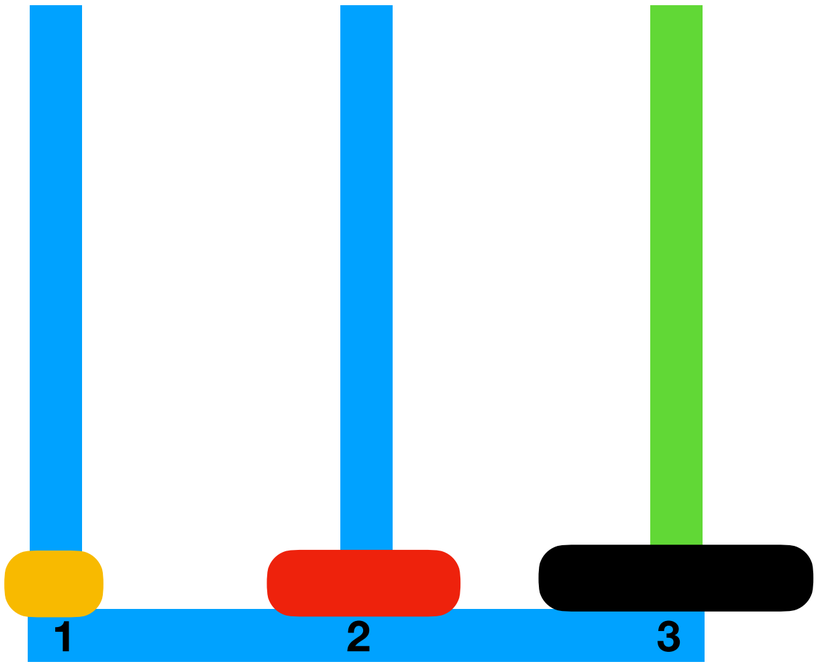
Example: Towers of Hanoi



Example: Towers of Hanoi



Example: Towers of Hanoi



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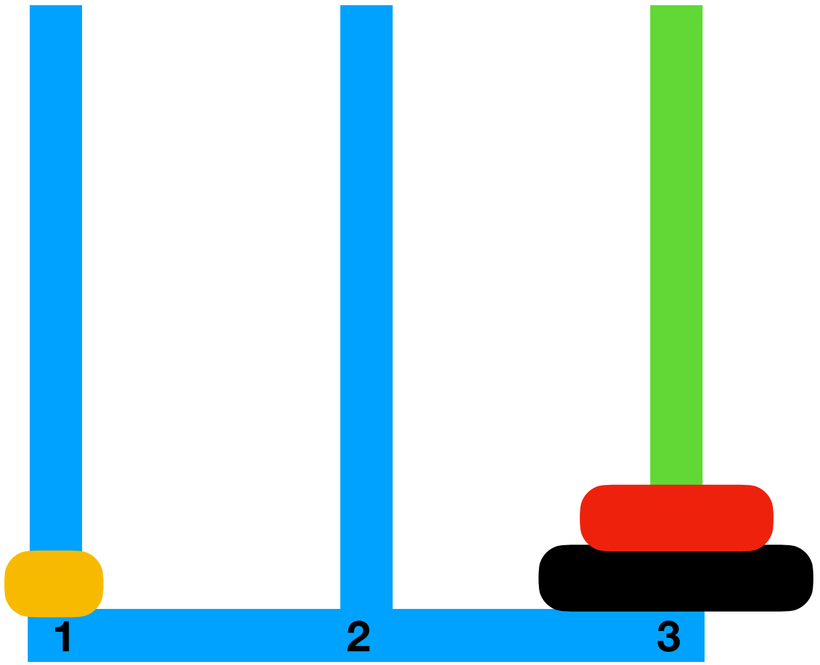

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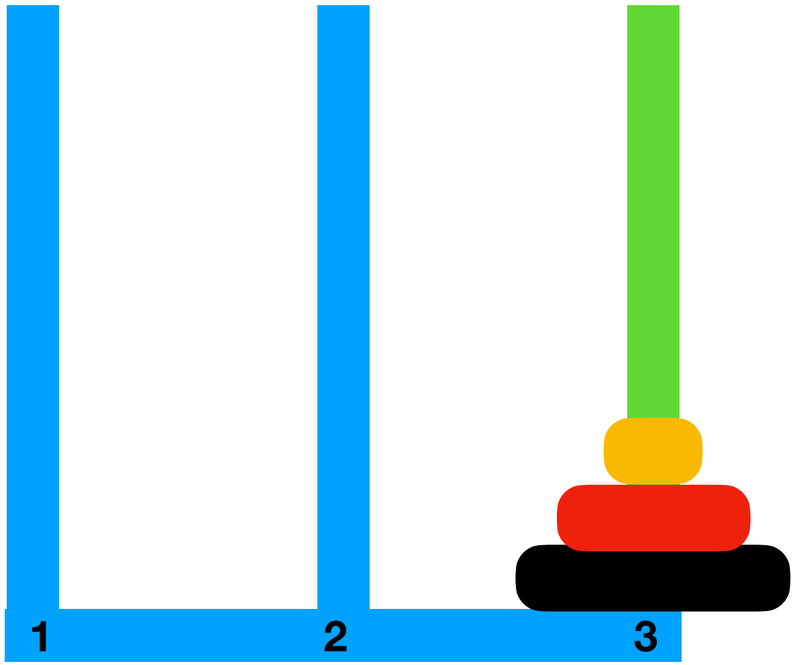

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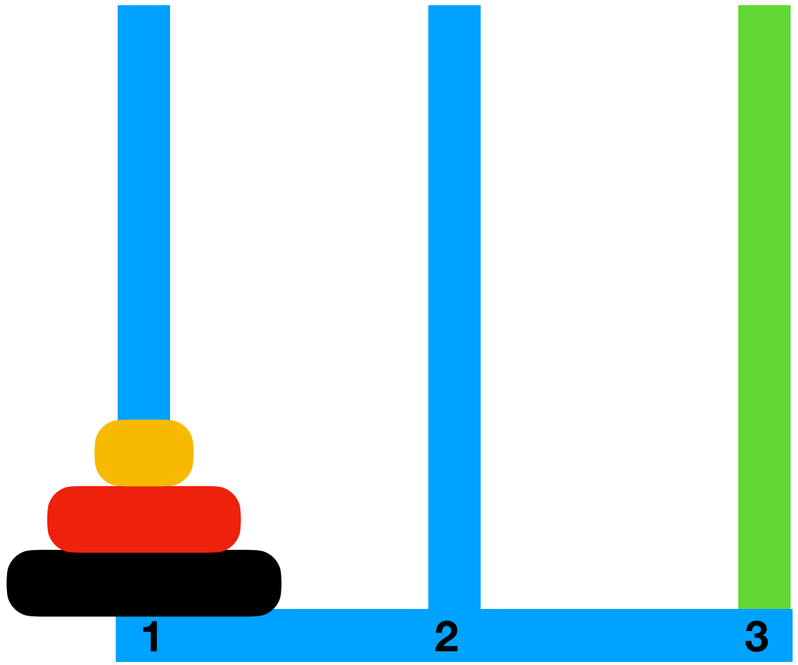
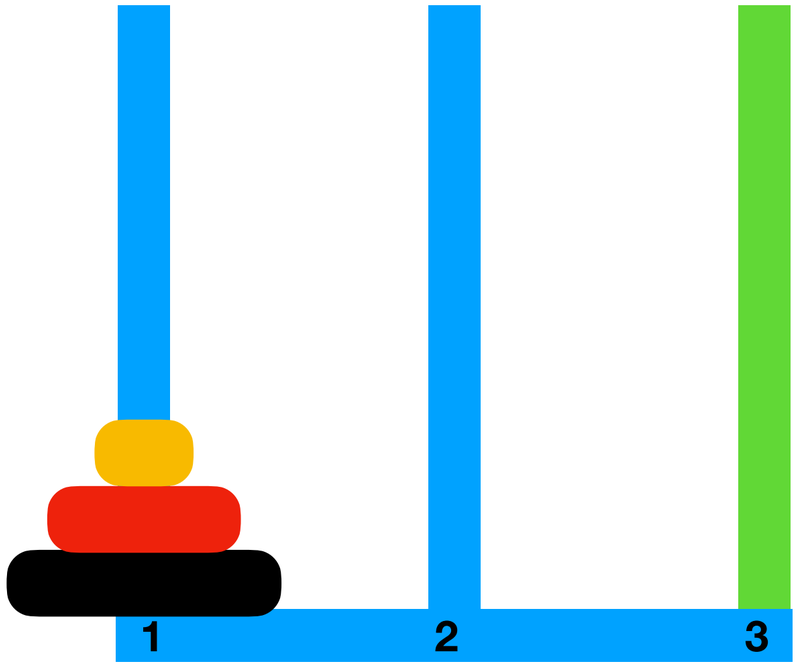


move(diskStacks, src, dest, temp) *// Move 2+ disks*
dmove(disk_num, src, dest) *// Move single disk*

move(disks3, r1, r3, r2)

Example: Towers of Hanoi



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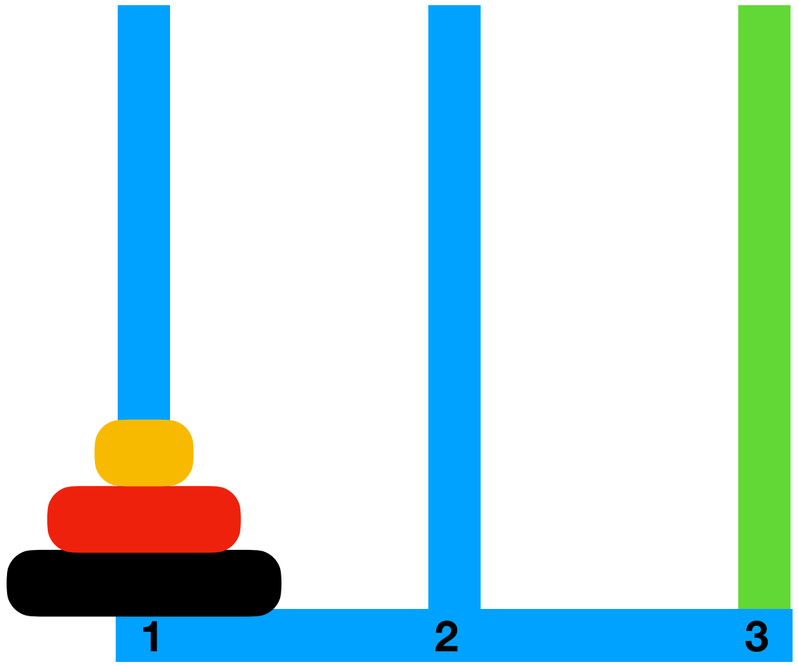

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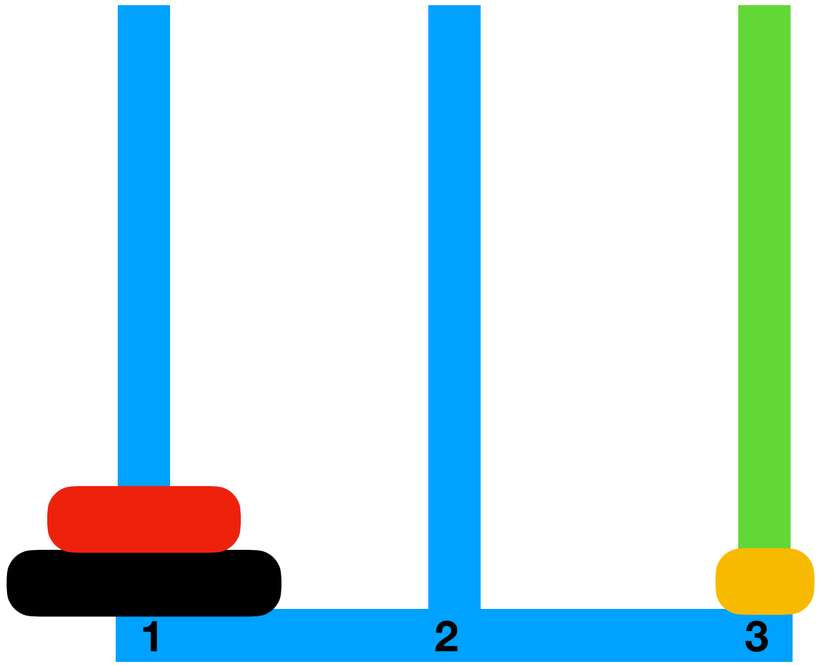

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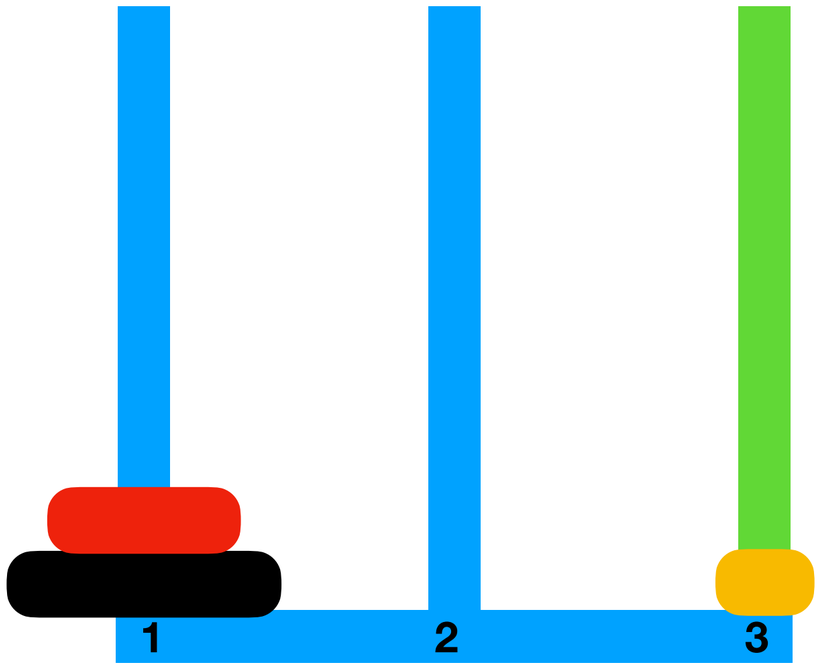

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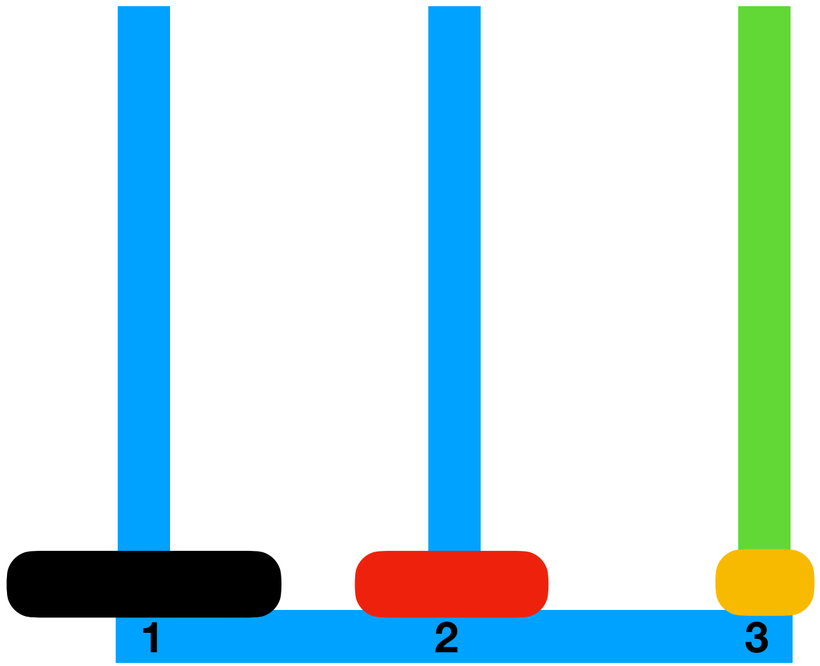

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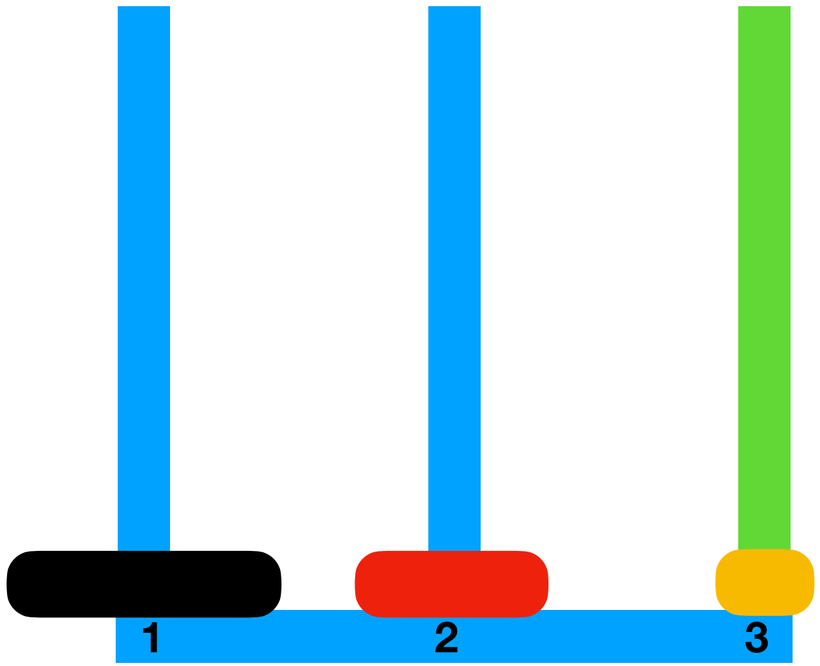

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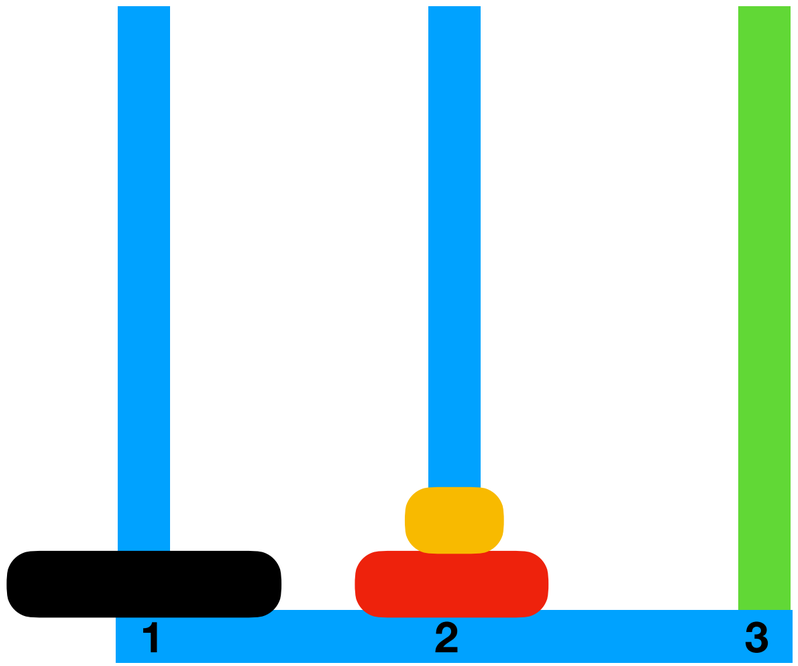

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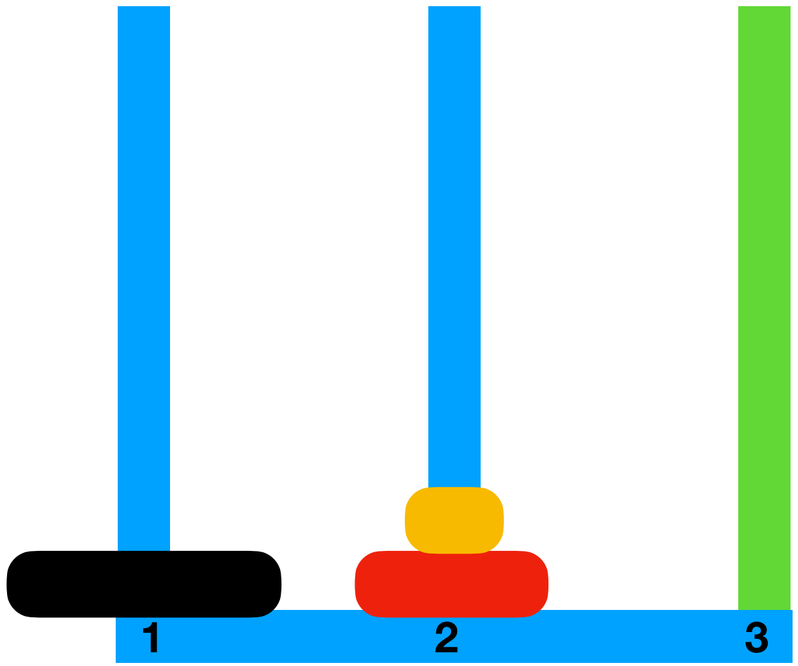

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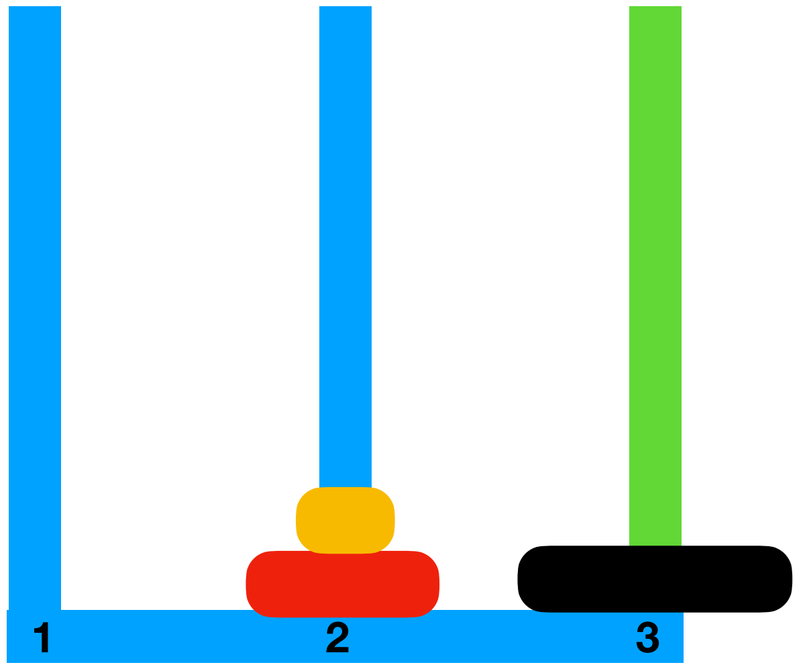

move(disks3, r1, r3, r2)

move(disks2, r1, r2, r3)

+

dmove(disk_3, r1, r3)

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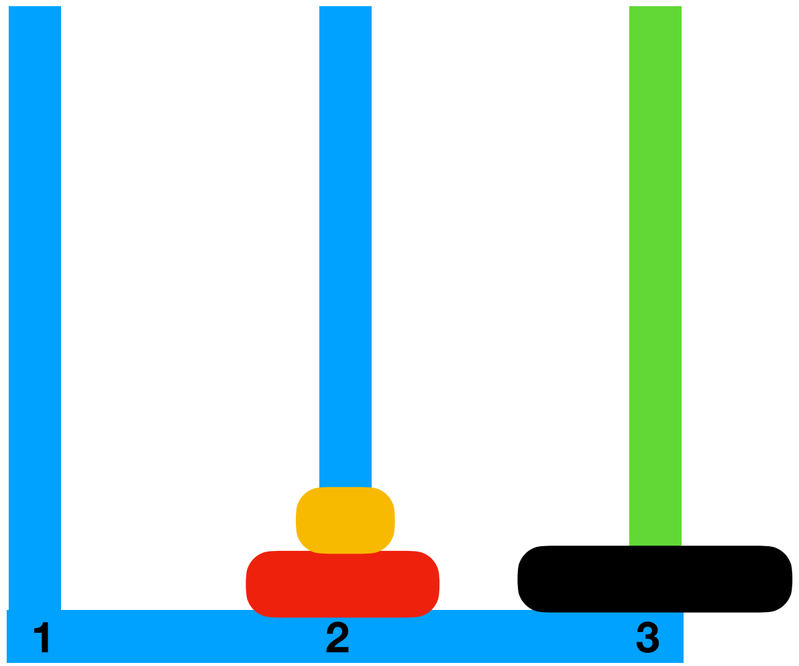

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move(disks2, r1, r2, r3)
+
dmove(disk_3, r1, r3)
+
move(disks2, r2, r3, r1)

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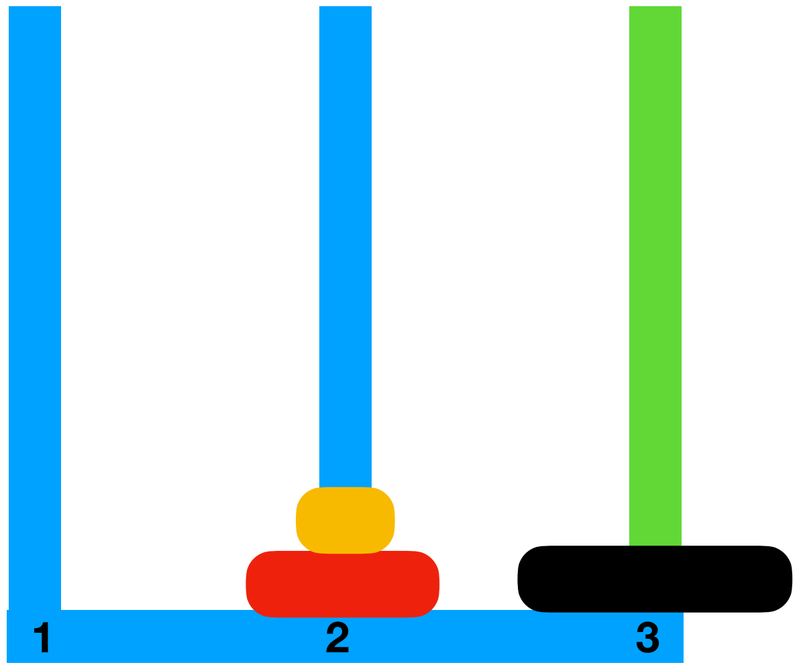

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+
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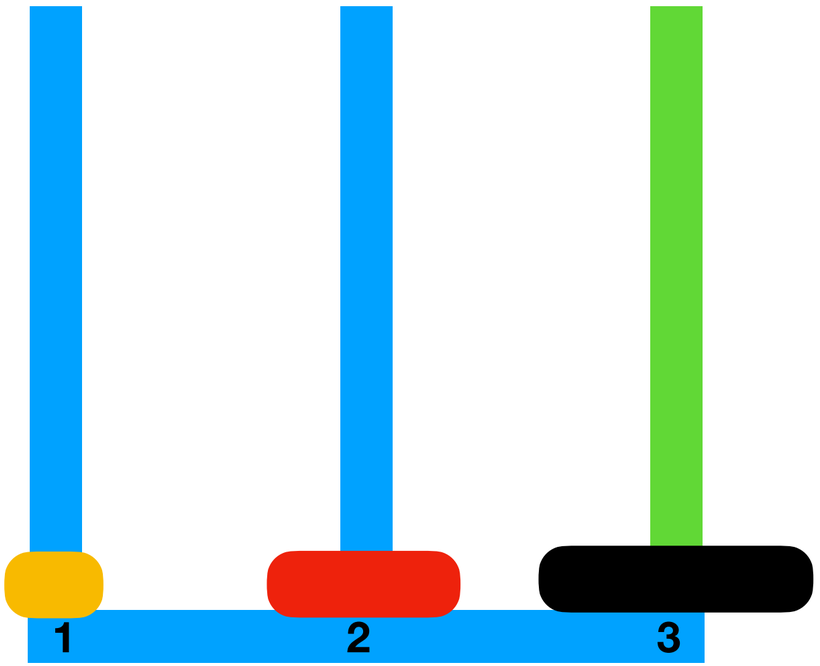
dmove(disk_1, r1, r3)
dmove(disk_2, r1, r2)
dmove(disk_1, r3, r2)
dmove(disk_1, r2, r1)

Example: Towers of Hanoi



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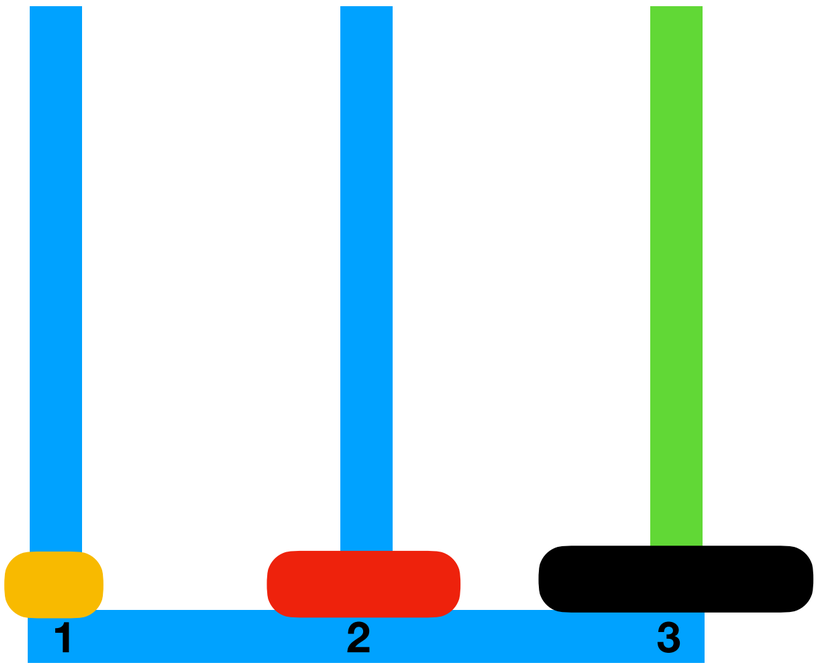
move(disks3, r1, r3, r2)

move(disks2, r1, r2, r3)
+
dmove(disk_3, r1, r3)
+
move(disks2, r2, r3, r1)

dmove(disk_1, r1, r3)
dmove(disk_2, r1, r2)
dmove(disk_1, r3, r2)
dmove(disk_1, r2, r1)

Example: Towers of Hanoi



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move(disks3, r1, r3, r2)

move(disks2, r1, r2, r3)
+
dmove(disk_3, r1, r3)
+
move(disks2, r2, r3, r1)

dmove(disk_1, r1, r3)
dmove(disk_2, r1, r2)
dmove(disk_1, r3, r2)

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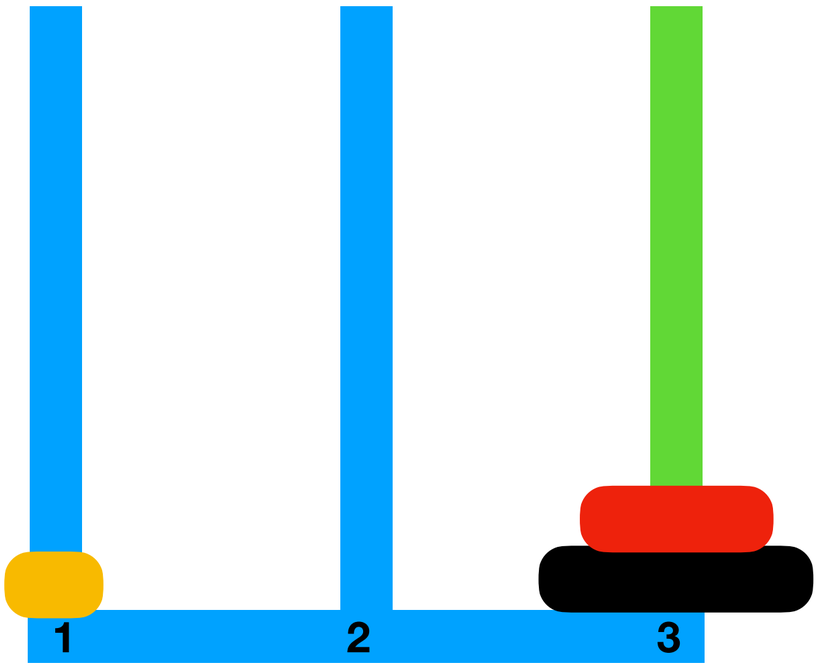
Example: Towers of Hanoi



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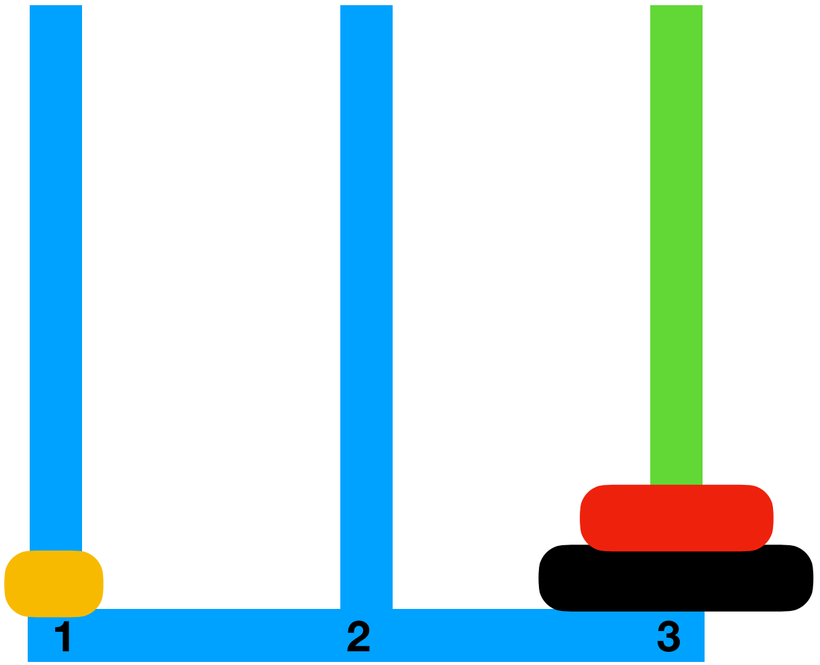
move(disks3, r1, r3, r2)

move(disks2, r1, r2, r3)
+
dmove(disk_3, r1, r3)
+
move(disks2, r2, r3, r1)

dmove(disk_1, r1, r3)
dmove(disk_2, r1, r2)
dmove(disk_1, r3, r2)

dmove(disk_1, r2, r1)
dmove(disk_2, r2, r3)

Example: Towers of Hanoi



move(diskStacks, src, dest, temp) *// Move 2+ disks*
dmove(disk_num, src, dest) *// Move single disk*

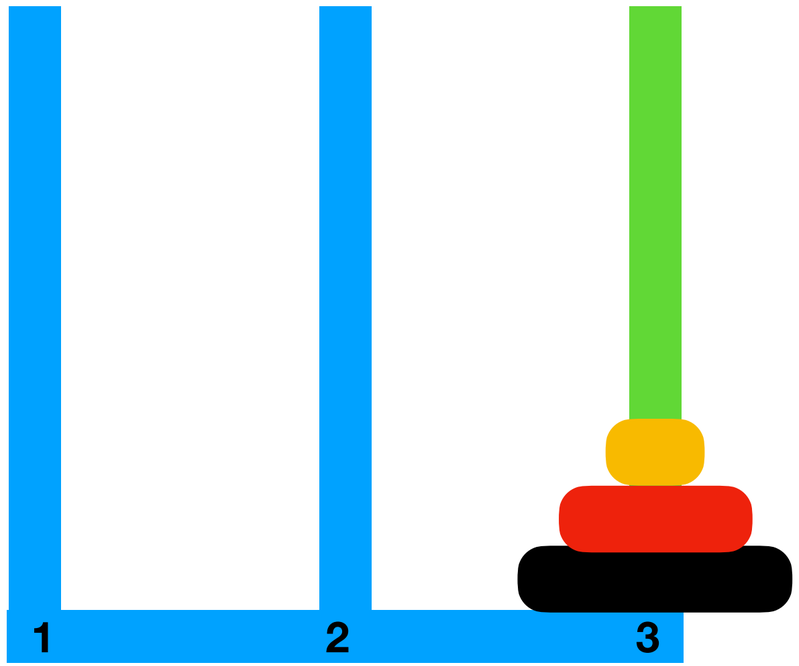
move(disks3, r1, r3, r2)

move(disks2, r1, r2, r3)
+
dmove(disk_3, r1, r3)
+
move(disks2, r2, r3, r1)

dmove(disk_1, r1, r3)
dmove(disk_2, r1, r2)
dmove(disk_1, r3, r2)

dmove(disk_1, r2, r1)
dmove(disk_2, r2, r3)
dmove(disk_1, r1, r3)

Example: Towers of Hanoi



move(diskStacks, src, dest, temp) *// Move 2+ disks*
dmove(disk_num, src, dest) *// Move single disk*

move(disks3, r1, r3, r2)

move(disks2, r1, r2, r3)
+
dmove(disk_3, r1, r3)
+
move(disks2, r2, r3, r1)

dmove(disk_1, r1, r3)
dmove(disk_2, r1, r2)
dmove(disk_1, r3, r2)

dmove(disk_1, r2, r1)
dmove(disk_2, r2, r3)
dmove(disk_1, r1, r3)

Recursion in Towers of Hanoi

Recursion in Towers of Hanoi

move(disks3, r1, r3, r2)

move(disks4, r1, r3, r2)

Recursion in Towers of Hanoi

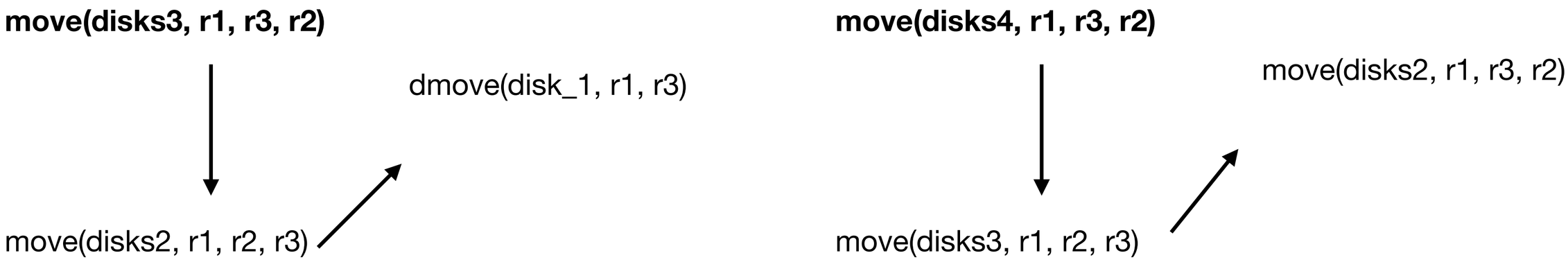
move(disks3, r1, r3, r2)

move(disks2, r1, r2, r3)

move(disks4, r1, r3, r2)

move(disks3, r1, r2, r3)

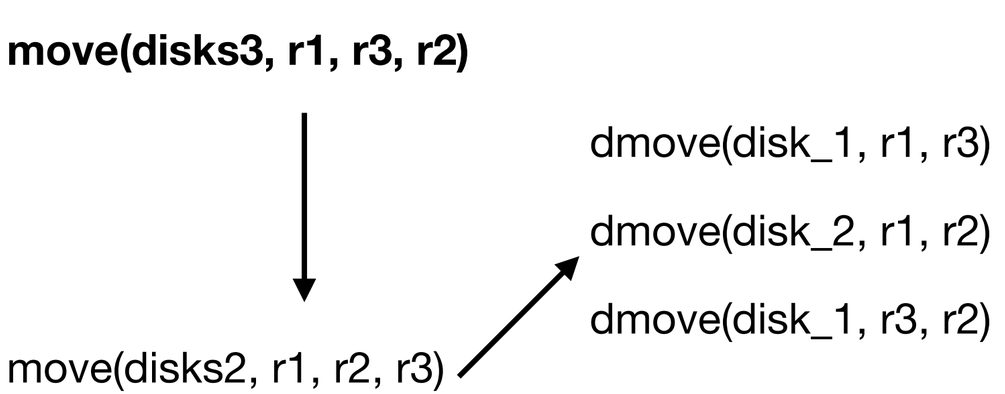
Recursion in Towers of Hanoi

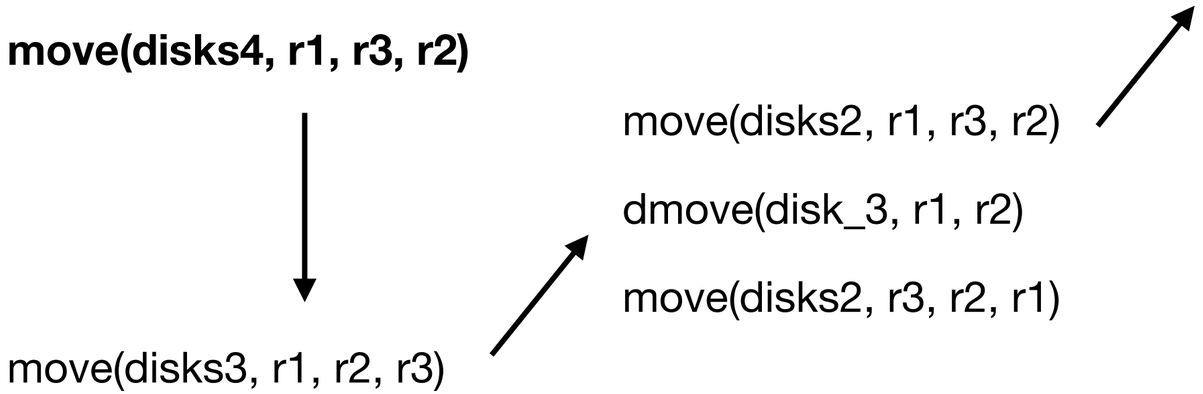


Recursion in Towers of Hanoi

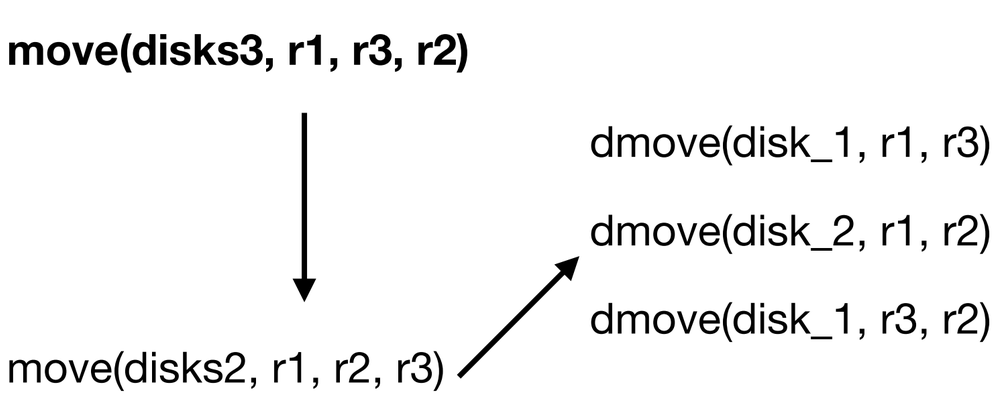
Recursion in Towers of Hanoi

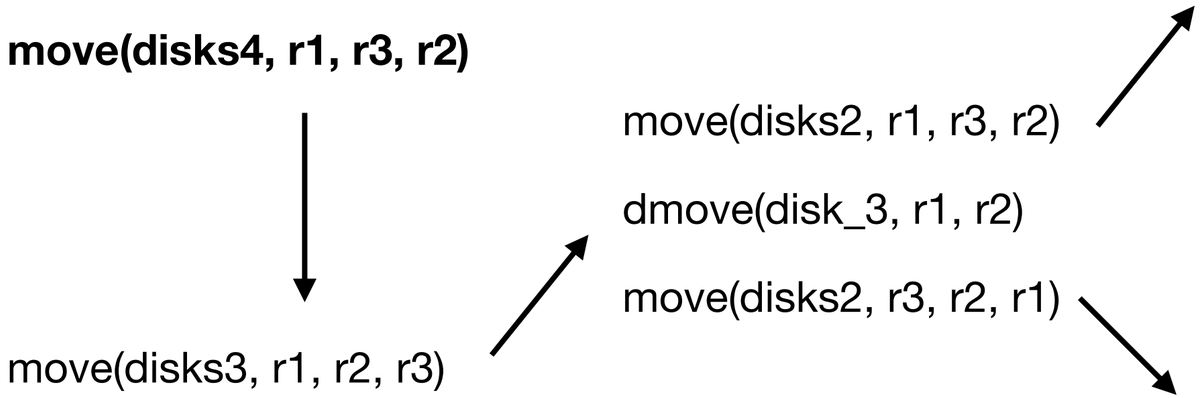
Recursion in Towers of Hanoi



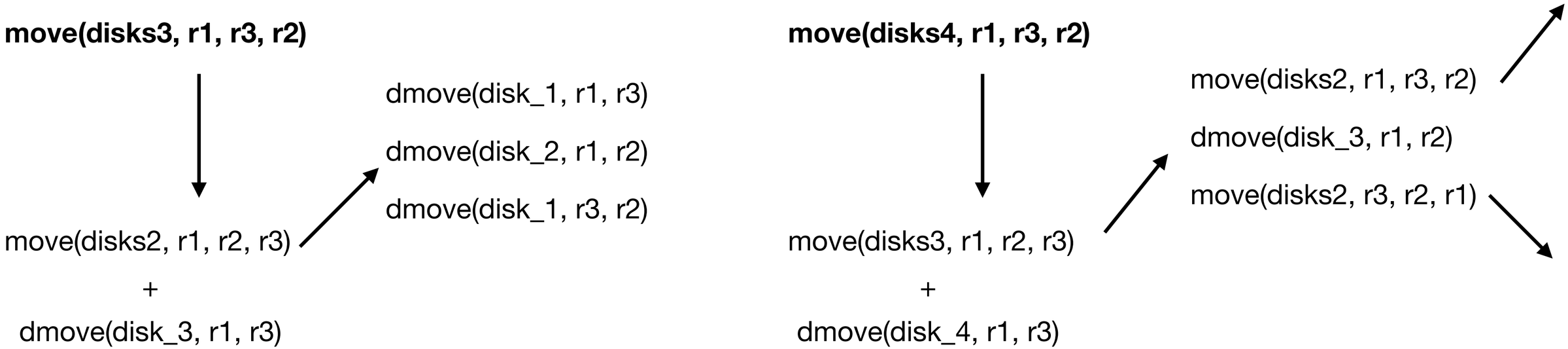


Recursion in Towers of Hanoi

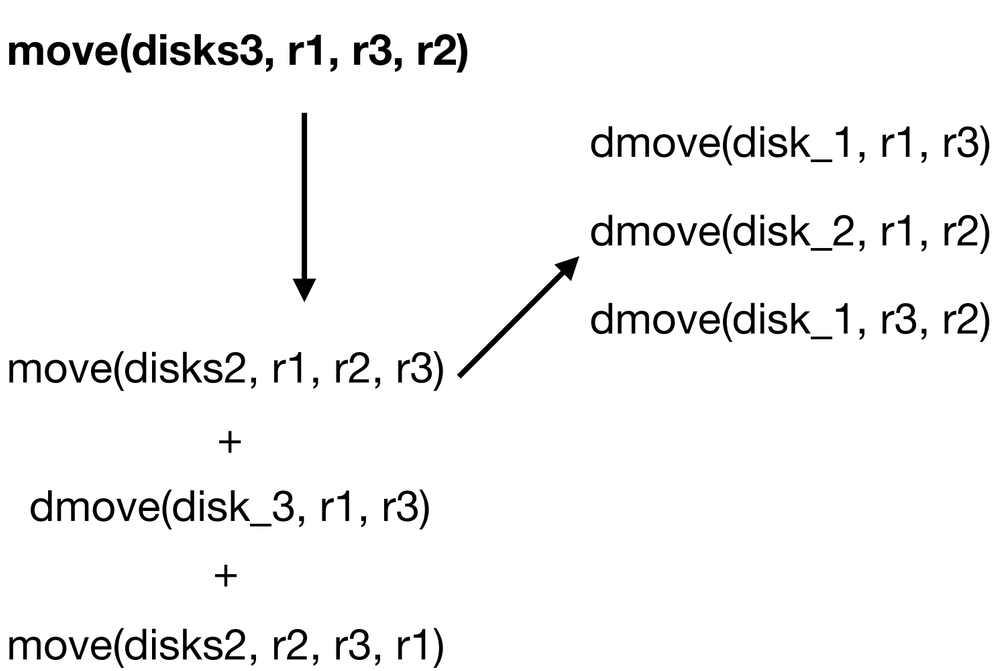


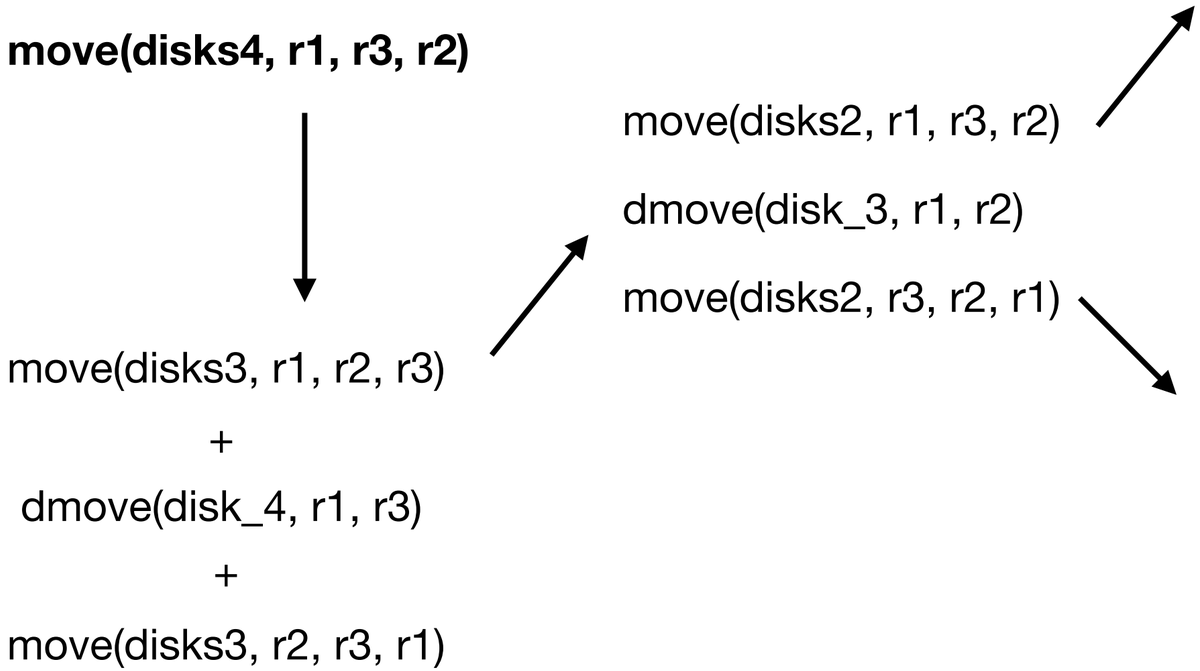


Recursion in Towers of Hanoi

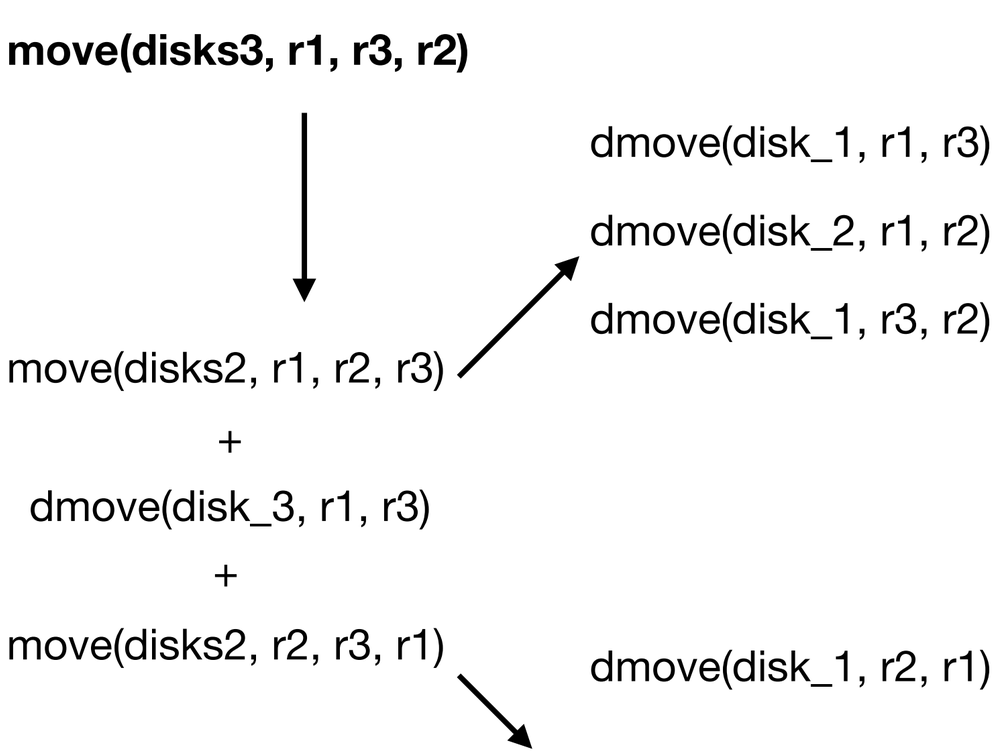


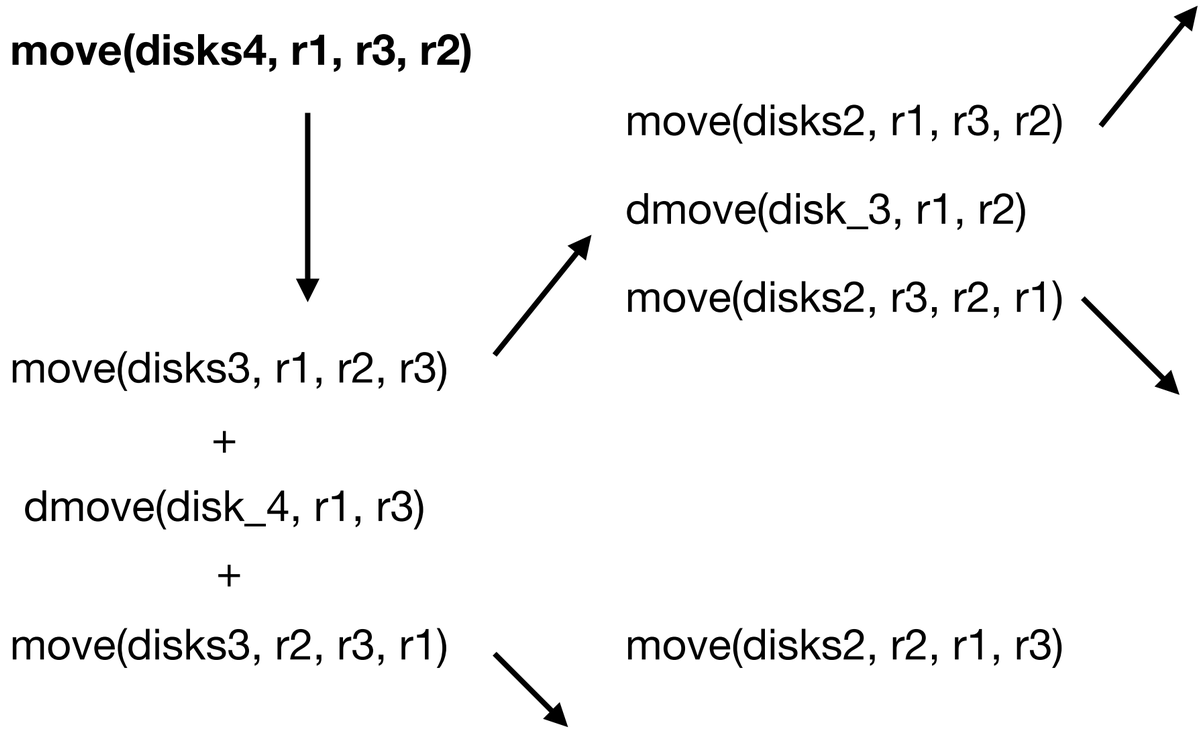
Recursion in Towers of Hanoi



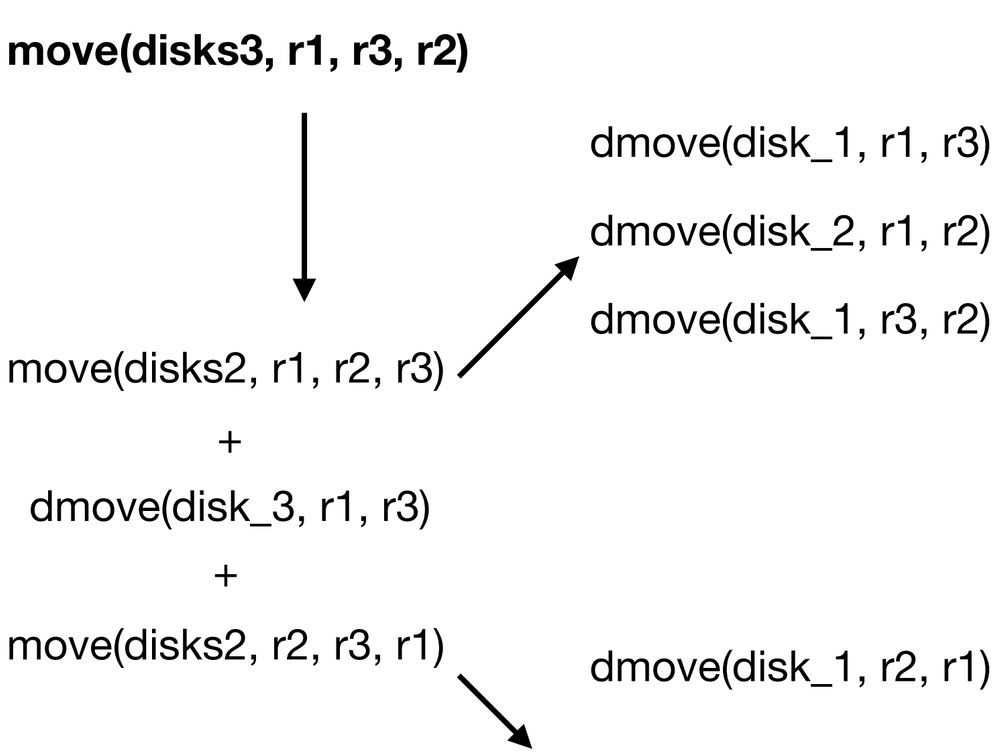


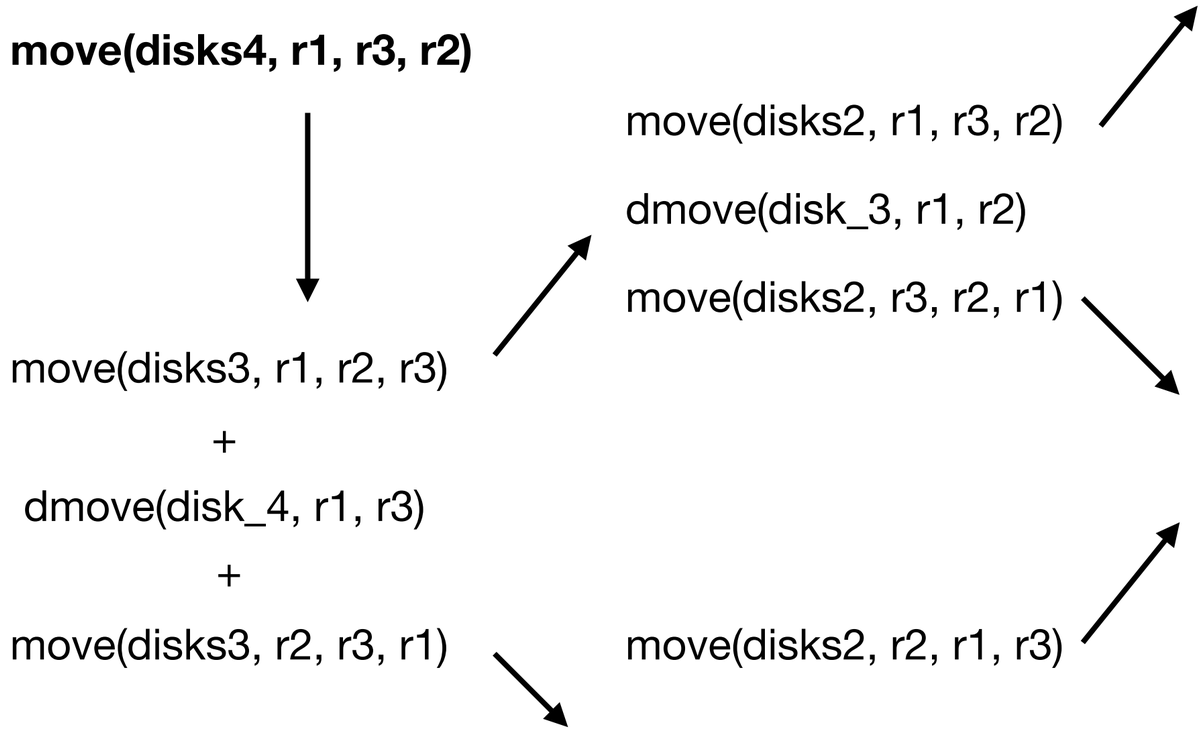
Recursion in Towers of Hanoi



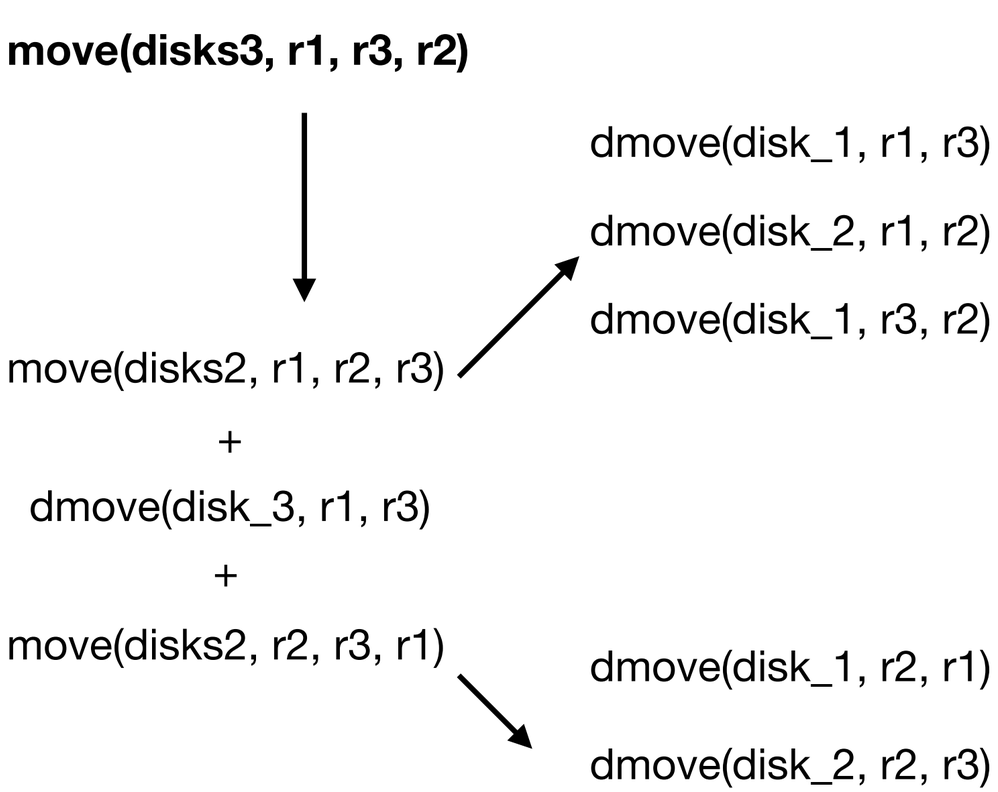


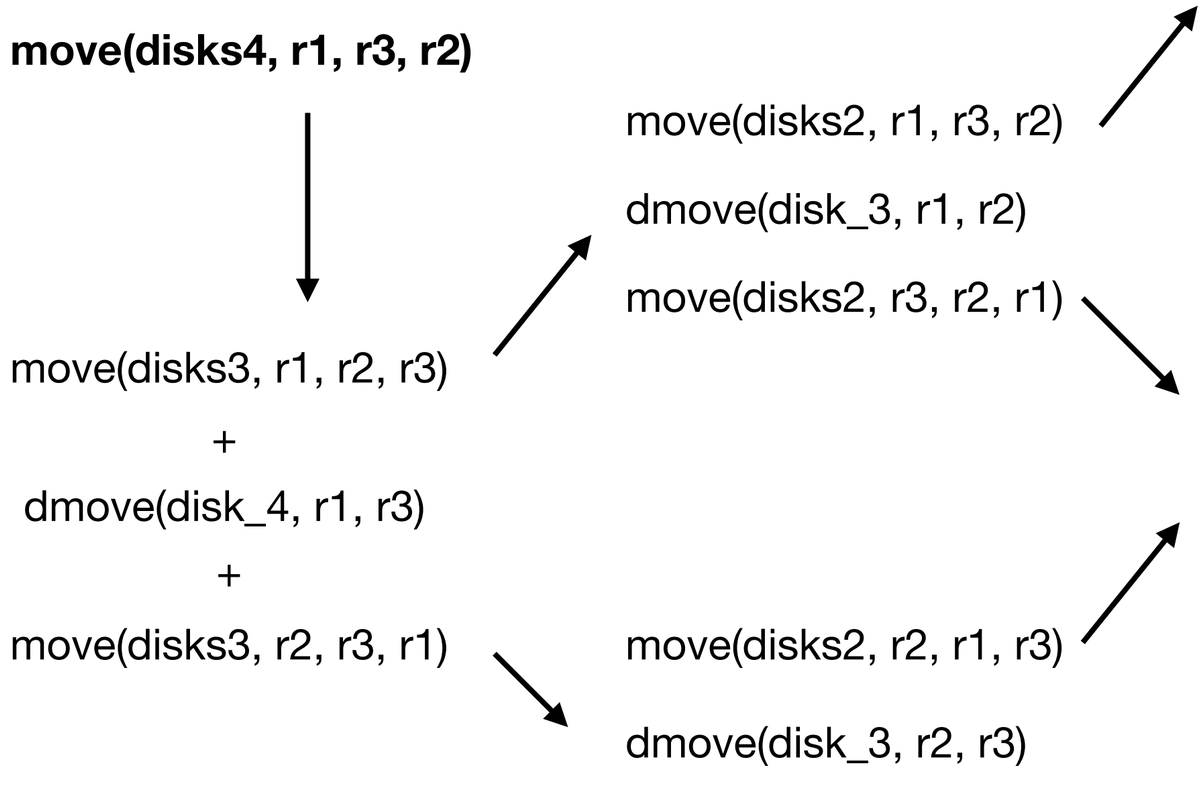
Recursion in Towers of Hanoi



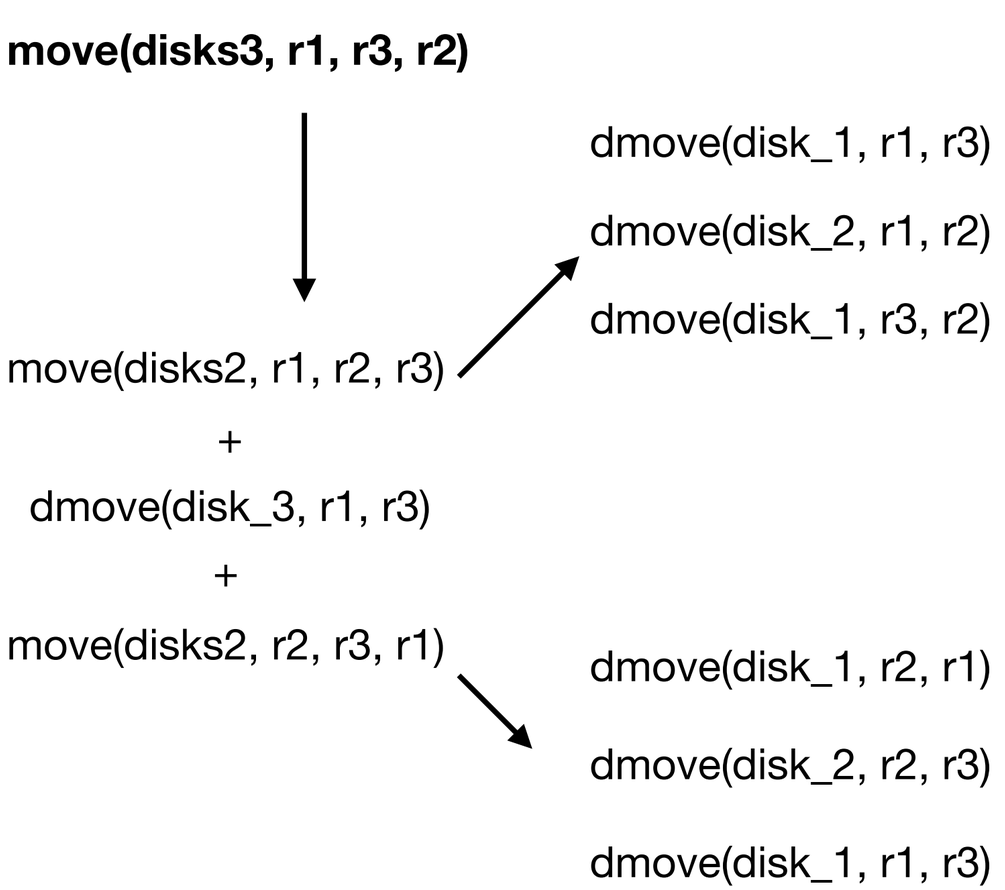


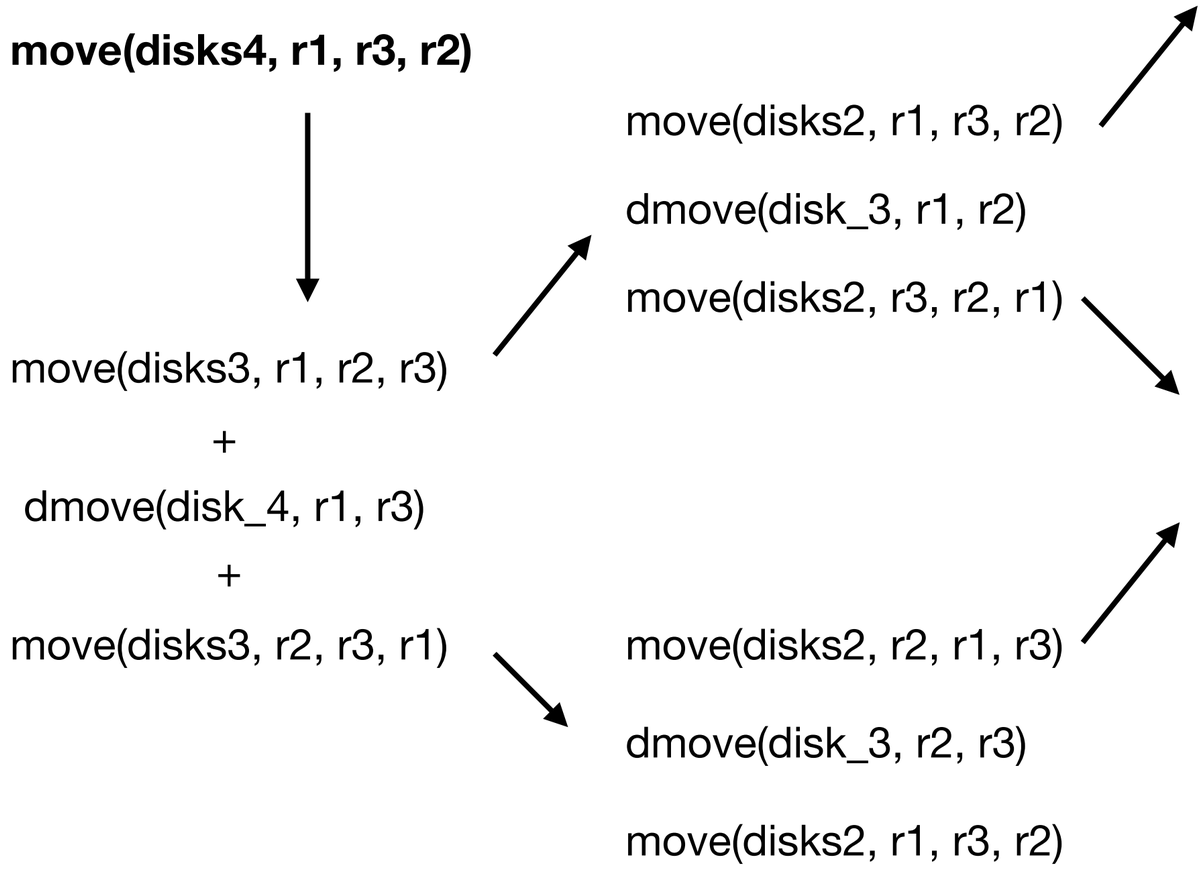
Recursion in Towers of Hanoi



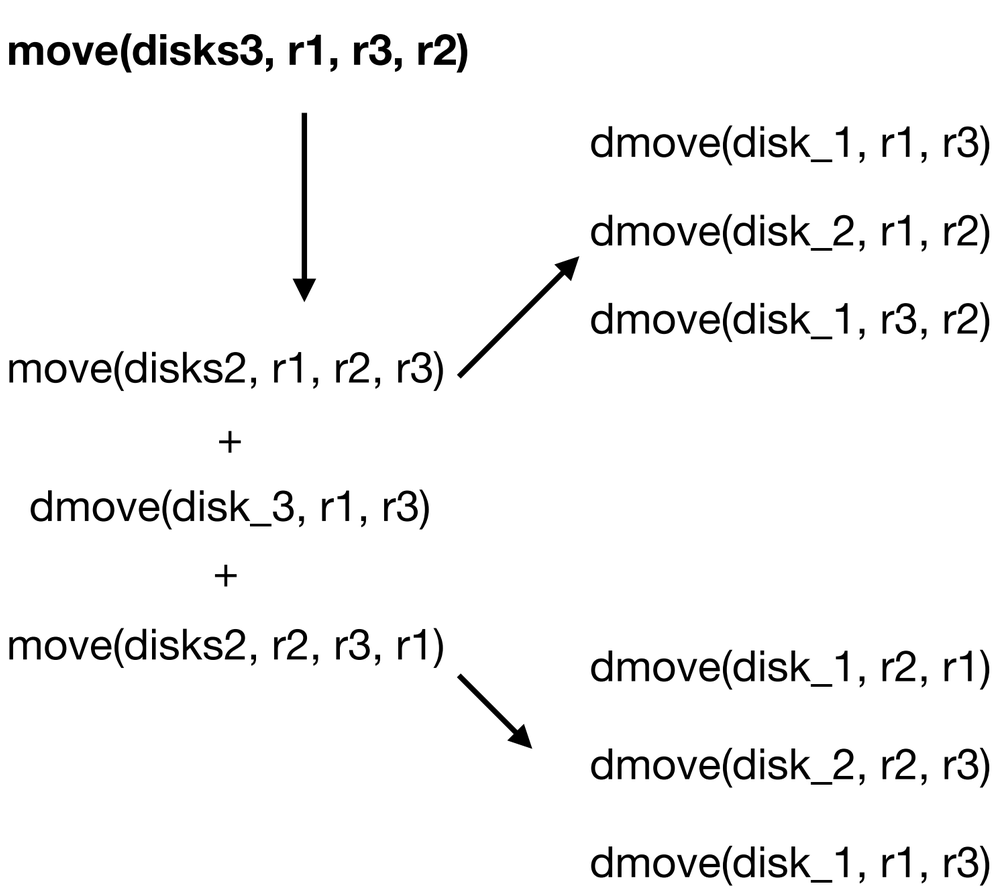


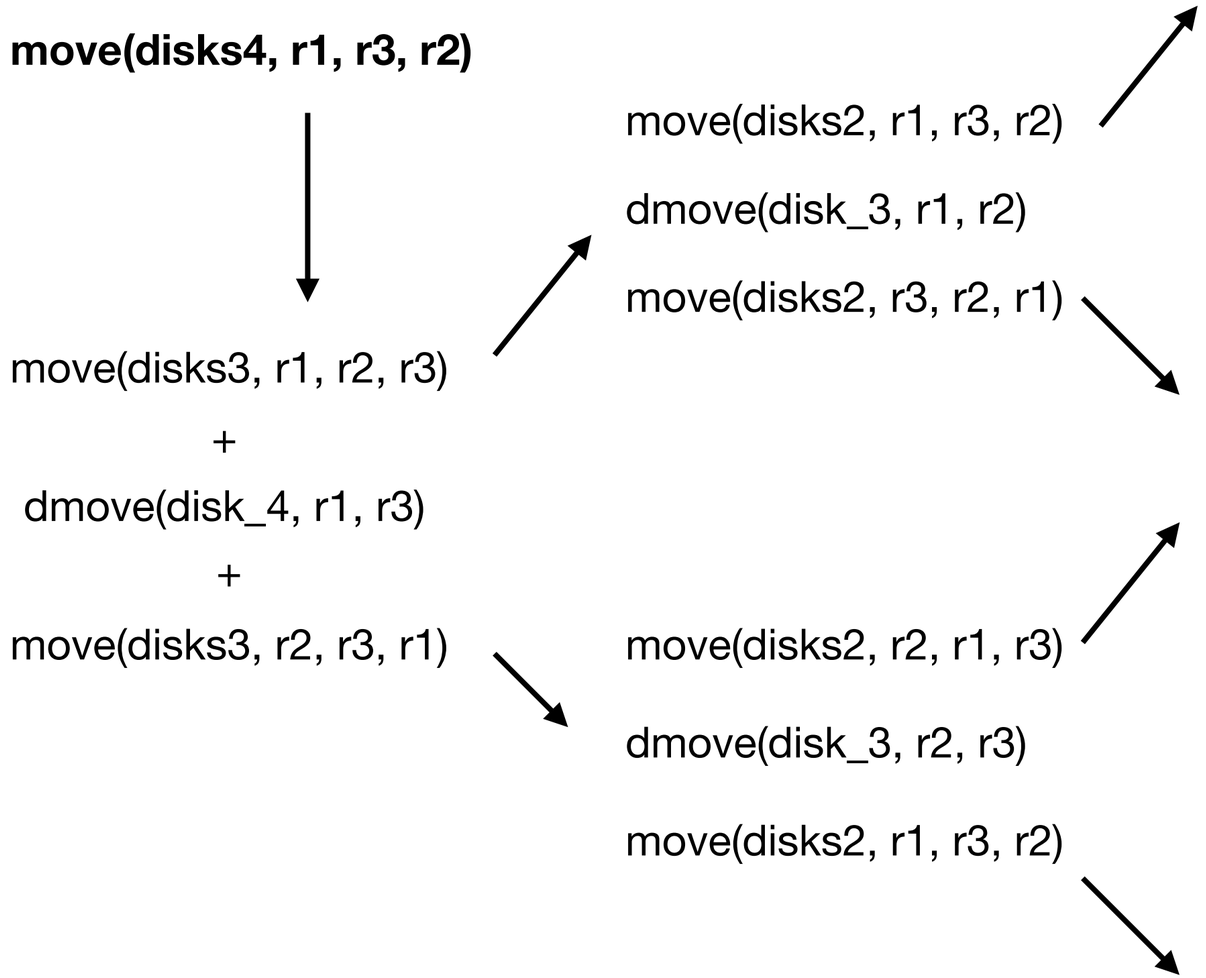
Recursion in Towers of Hanoi





Recursion in Towers of Hanoi





Towers of Hanoi: General formula?

Towers of Hanoi: General formula?

```
void move(disknum, src, dest, temp){
  if (disknum > 1){
    move(disknum - 1, src, temp, dest);

    printf("Moved disk %d from rod %d to %d\n", disknum, src, dest);

    move(disknum - 1, temp, dest, src);
  }
  else
    printf("Moved disk 1 from rod %d to %d\n", src, dest);
}
```

Towers of Hanoi: General formula?

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```

Move a sub-stack to from src to intermediate rod

Towers of Hanoi: General formula?

Move a sub-stack to from src to intermediate rod

Direct move of single disk.

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void move(disknum, src, dest, temp){
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Move a sub-stack to from src to intermediate rod

Direct move of single disk.

Move the sub-stack from intermediate to dest rod

Towers of Hanoi: General formula?

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```

Move a sub-stack to from src to intermediate rod

Direct move of single disk.

Move the sub-stack from intermediate to dest rod

Base case; all moves involving sub-stacks end-up here.

Recursion in LC3

Recursion in LC3

```
int running_sum(int n){
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}

int main(void){
    int n = 5;
    running_sum(5);
}
```

Recursion in LC3

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How can we write equivalent LC3 code?

Recursion in LC3

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    int fn;
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        fn = 1;
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    int n = 5;
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How can we write equivalent LC3 code?

Recall function calls are implemented using the run time stack.

Recursion in LC3

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}

int main(void){
    int n = 5;
    running_sum(5);
}
```

How can we write equivalent LC3 code?

Recall function calls are implemented using the run time stack.

Recursive calls need not be treated any different from normal function calls!

Generating an activation record

Check Github for reference material: https://github.com/iabraham/ece220-sp25/blob/main/lec12/README.md

Generating an activation record

1. *Caller* build-up: Push callee's arguments onto stack

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Generating an activation record

1. *Caller* build-up: Push callee's arguments onto stack
2. Pass control to callee (`JSR/JSRR`)

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Generating an activation record

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Caller

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Generating an activation record

1. *Caller* build-up: Push callee's arguments onto stack
2. Pass control to callee (`JSR/JSRR`)

Caller

3. *Callee* build-up: (push bookkeeping info and local variables onto stack)

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Generating an activation record

1. *Caller* build-up: Push callee's arguments onto stack
2. Pass control to callee (`JSR/JSRR`)

Caller

3. *Callee* build-up: (push bookkeeping info and local variables onto stack)
4. Execute function

Check Github for reference material: https://github.com/iabraham/ece220-sp25/blob/main/lec12/README.md

Generating an activation record

1. *Caller* build-up: Push callee's arguments onto stack
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Caller

3. *Callee* build-up: (push bookkeeping info and local variables onto stack)
4. Execute function
5. *Callee* tear-down (update return value, pop local variables, caller's frame pointer and return address from stack)

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Generating an activation record

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6. Return to caller (`RET`)

Callee

7. *Caller* tear-down (pop callee's return value and arguments from stack)

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Generating an activation record

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Caller

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Recursion in LC3

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int running_sum(int n){
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}

int main(void){
    int n = 4;
    int answer;
    answer = running_sum(n);
}
```

Recursion in LC3

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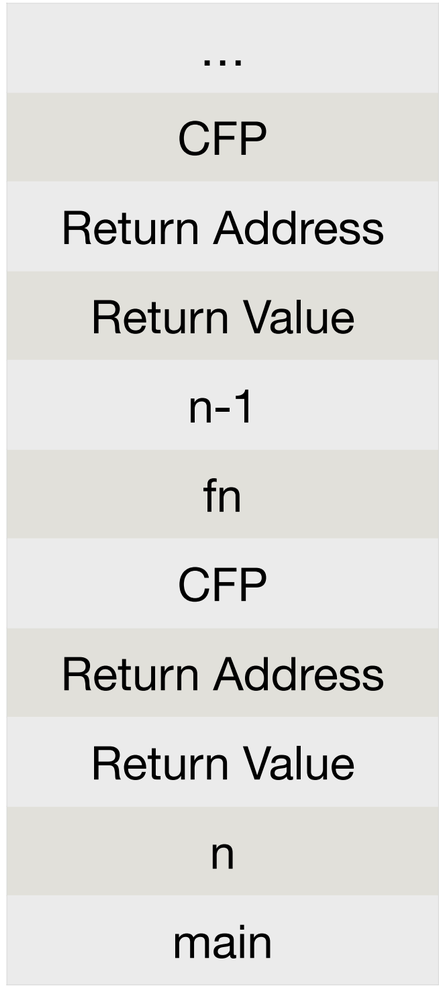

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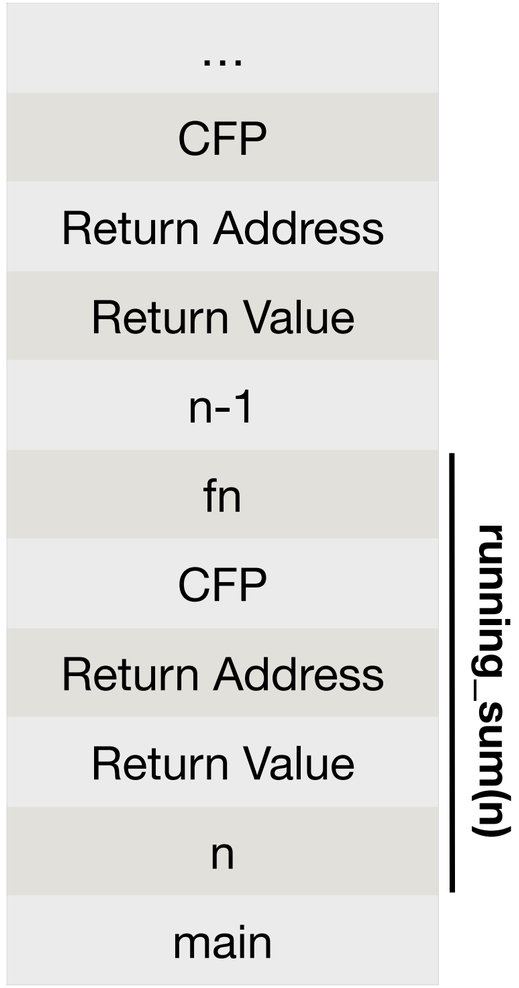

Recursion in LC3

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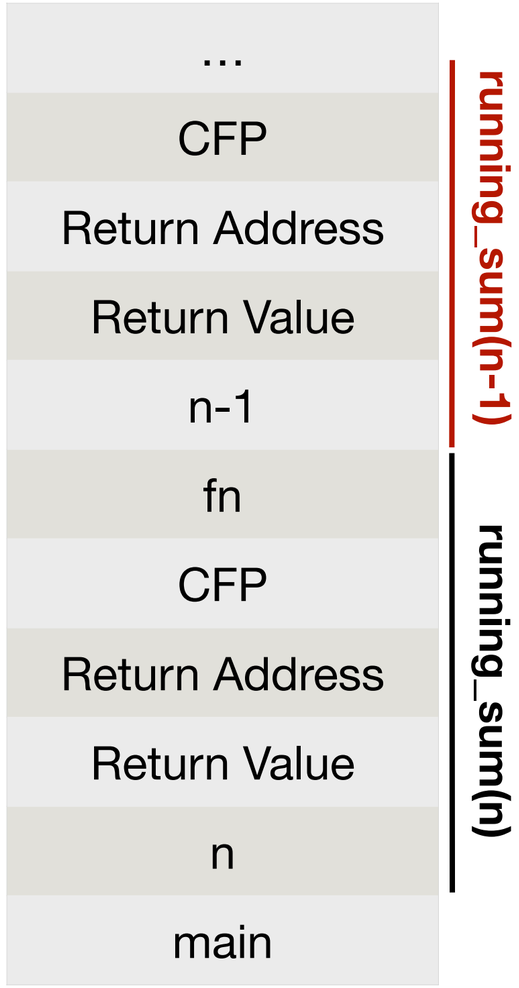

Recursion in LC3

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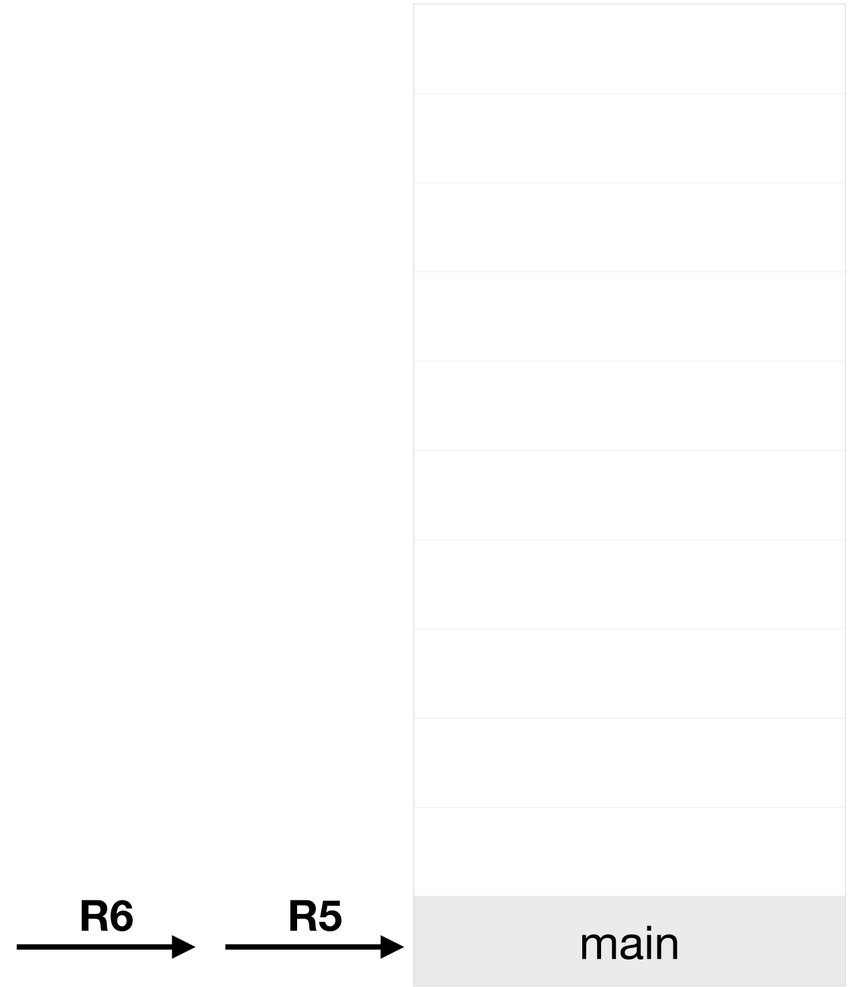
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}
```

```c
int main(void){
    int n = 4;
    int answer;
    answer = running_sum(n);
}
```

```
;Caller set-up
```

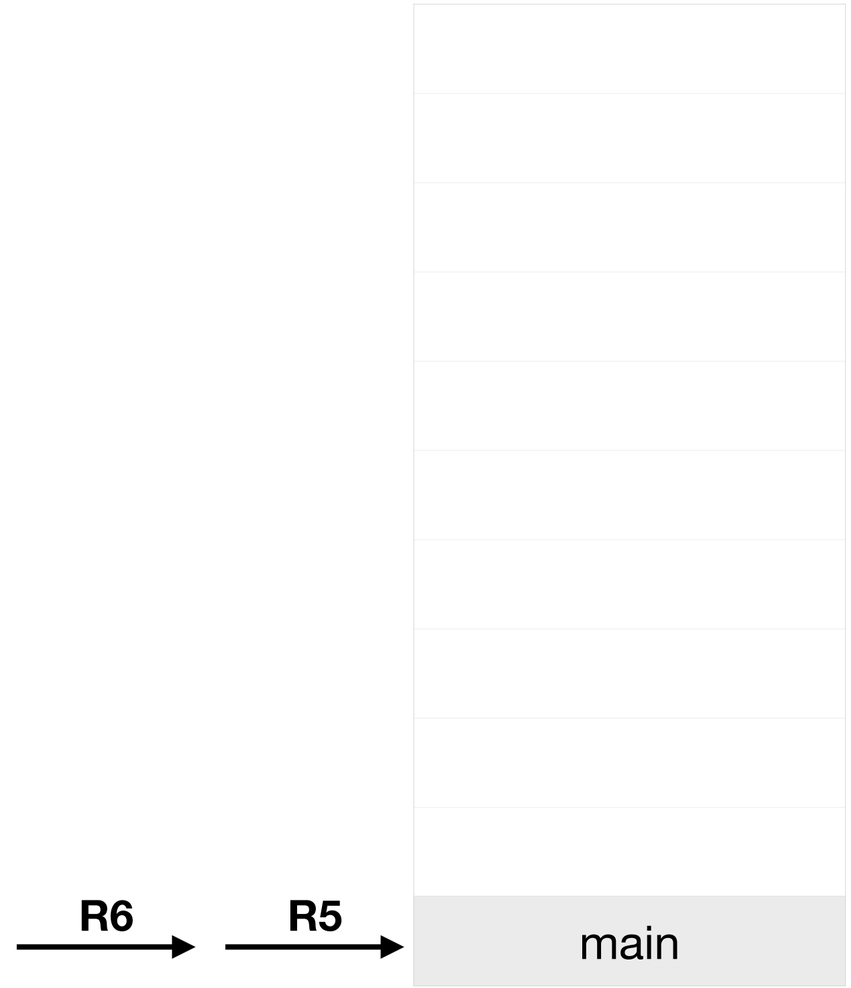


```c
int running_sum(int n){
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}
```

Recursion in LC3

```c
int main(void){
    int n = 4;
    int answer;
    answer = running_sum(n);
}
```

```
;Caller set-up
LDR R0, R5, #0
```



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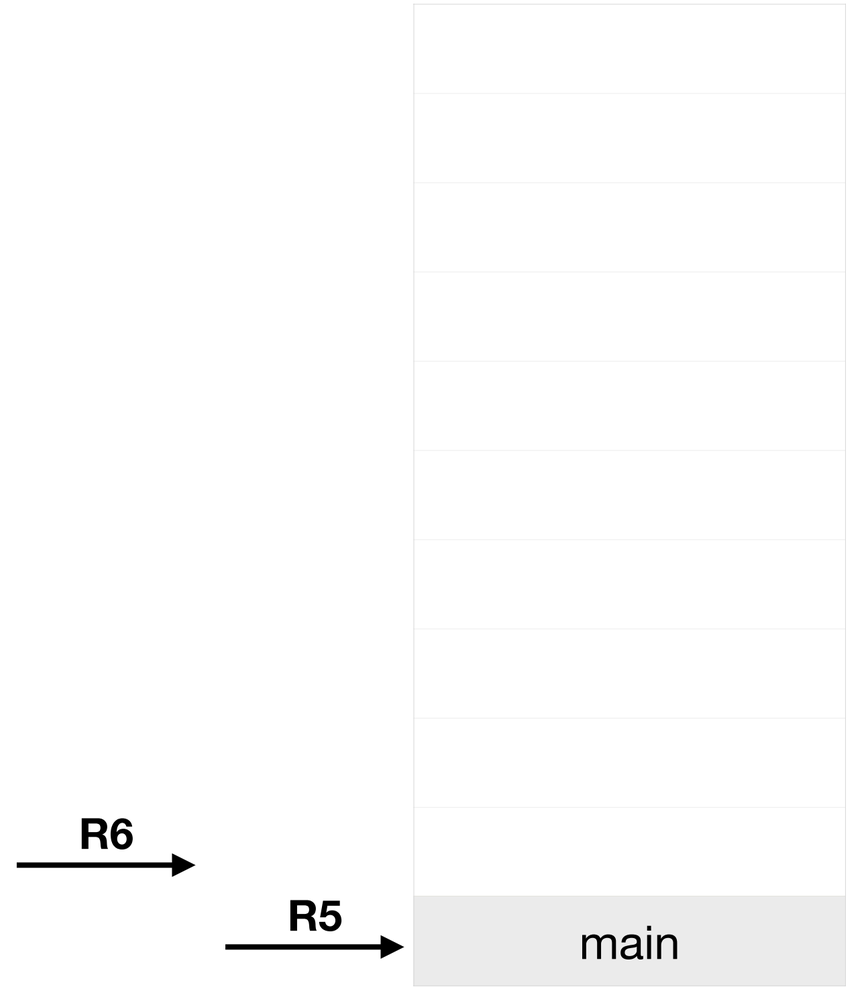
int running_sum(int n){
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}
```

Recursion in LC3

```c
int main(void){
    int n = 4;
    int answer;
    answer = running_sum(n);
}
```

```
;Caller set-up
LDR R0, R5, #0
ADD R6, R6, #-1
```



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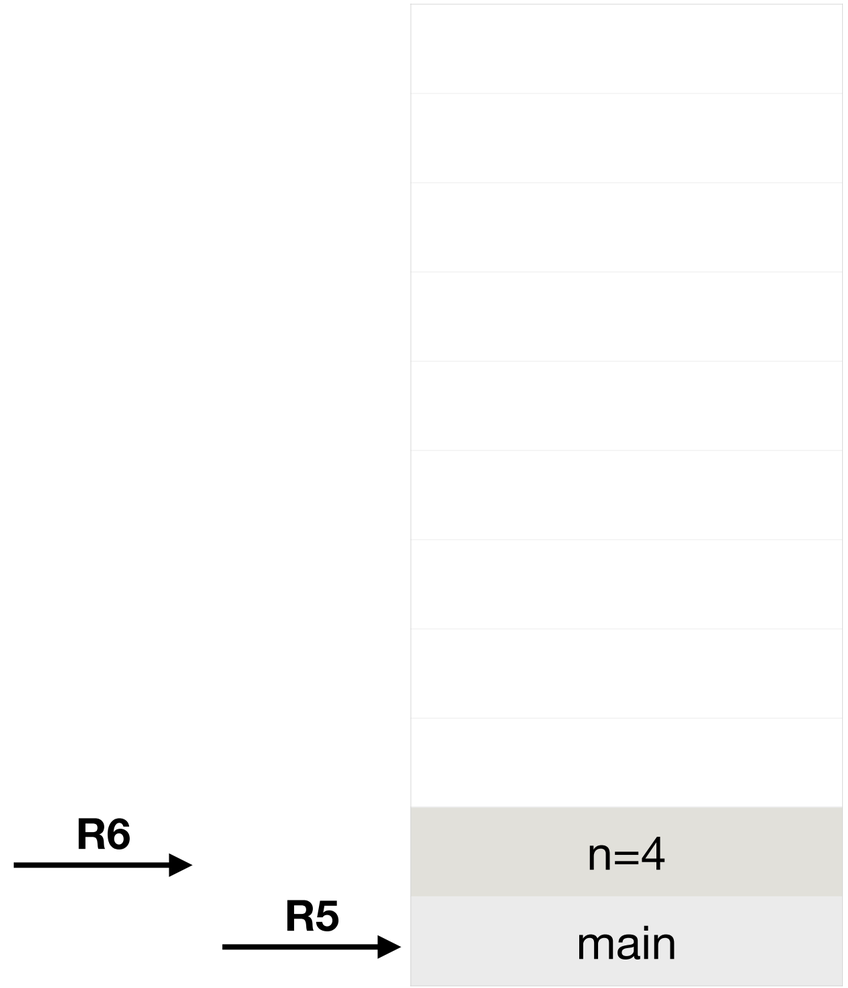
    return fn;
}
```

Recursion in LC3

```c
int main(void){
    int n = 4;
    int answer;
    answer = running_sum(n);
}
```

```
;Caller set-up
LDR R0, R5, #0
ADD R6, R6, #-1
STR R0, R6, #0
```



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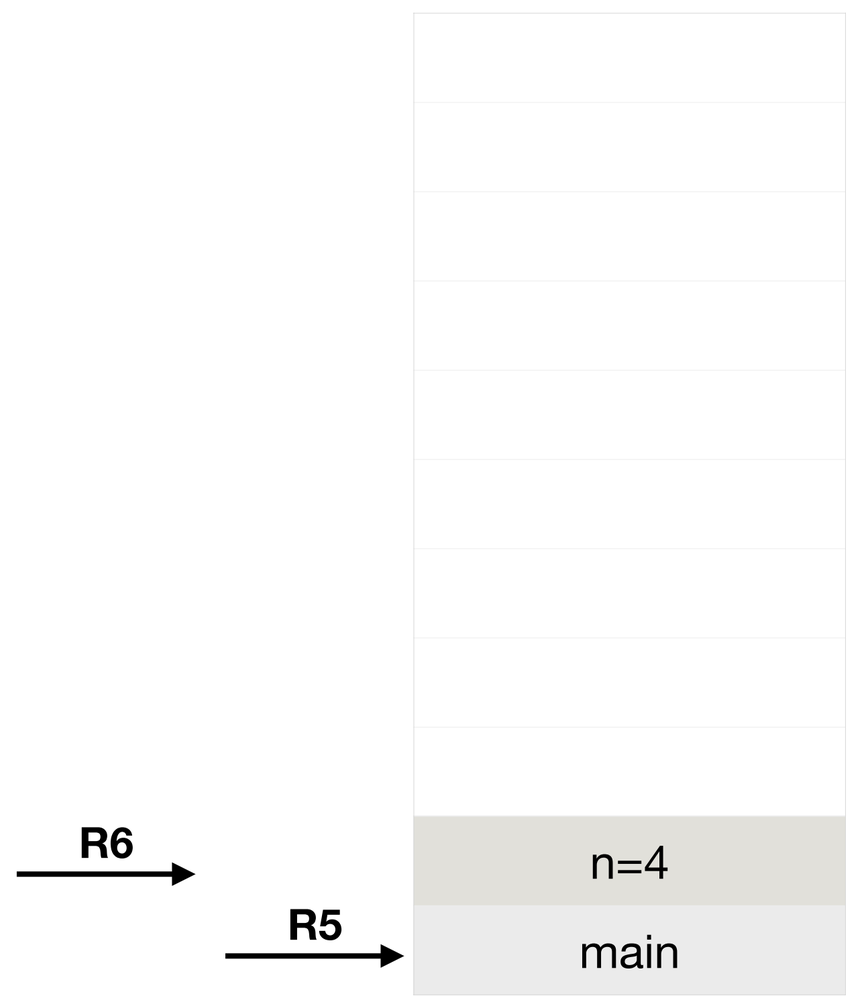
}
```

Recursion in LC3

```c
int main(void){
    int n = 4;
    int answer;
    answer = running_sum(n);
}
```

```
;Caller set-up
LDR R0, R5, #0
ADD R6, R6, #-1
STR R0, R6, #0
JSR RUNNING

RUNNING
;callee set-up of Running(n)'s activation record
```



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    else
        fn = n + running_sum(n-1);
    return fn;
}
```

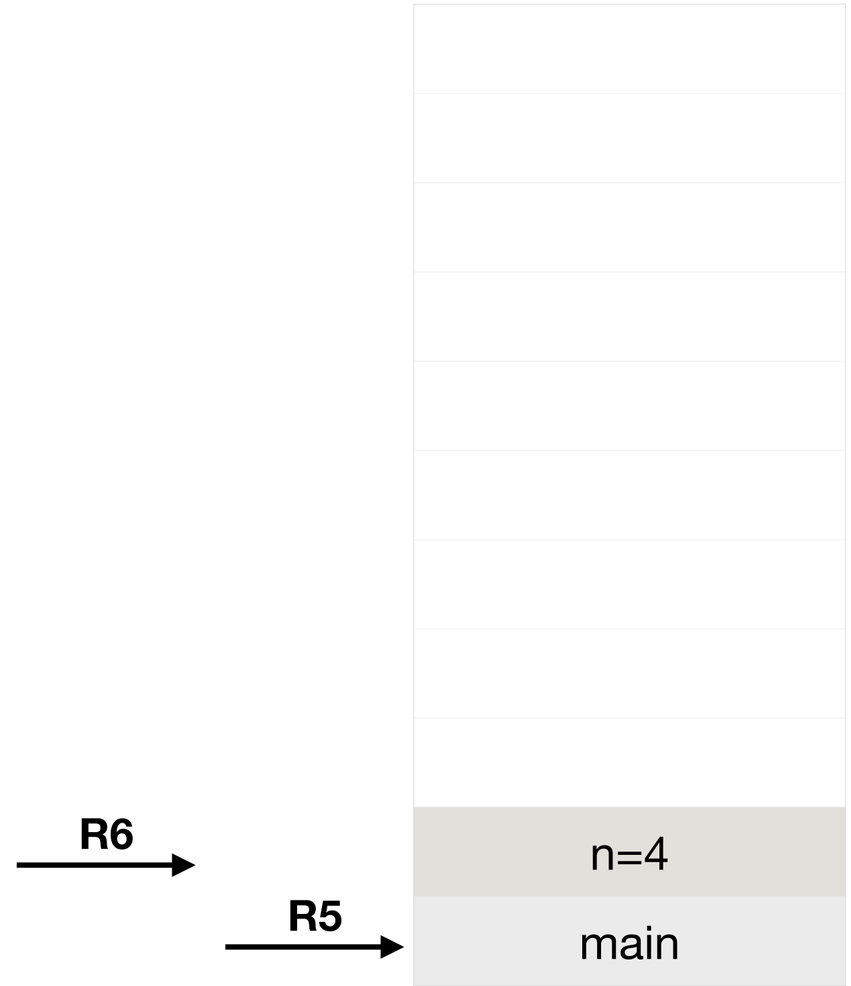
Recursion in LC3

```c
int main(void){
    int n = 4;
    int answer;
    answer = running_sum(n);
}
```

```
;Caller set-up
LDR R0, R5, #0
ADD R6, R6, #-1
STR R0, R6, #0
JSR RUNNING

RUNNING
;callee set-up of Running(n)'s activation record
;push return value, return address & caller's frame pointer
```

R0 | 4



```c
int running_sum(int n){
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}
```

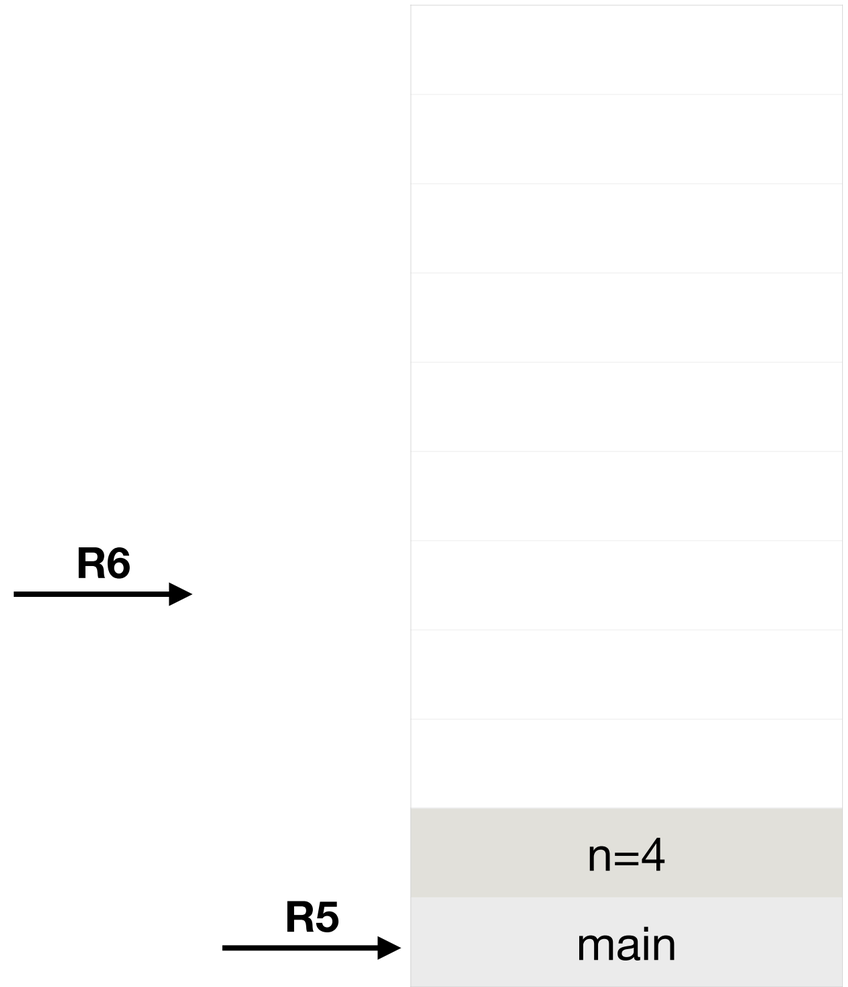
```c
int main(void){
    int n = 4;
    int answer;
    answer = running_sum(n);
}
```

Recursion in LC3

```
;Caller set-up
LDR R0, R5, #0
ADD R6, R6, #-1
STR R0, R6, #0
JSR RUNNING

RUNNING
;callee set-up of Running(n)'s activation record
;push return value, return address & caller's frame pointer
ADD R6, R6, #-3
```

R0	4



```c
int running_sum(int n){
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}
```

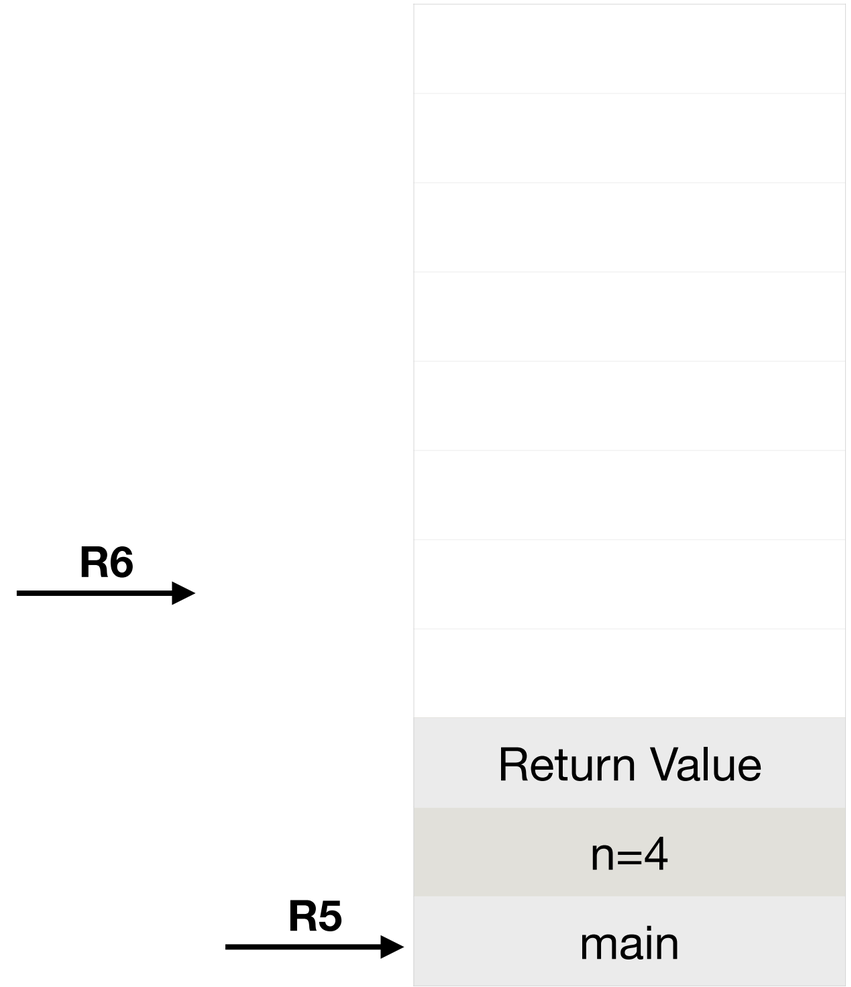
```c
int main(void){
    int n = 4;
    int answer;
    answer = running_sum(n);
}
```

Recursion in LC3

```
;Caller set-up
LDR R0, R5, #0
ADD R6, R6, #-1
STR R0, R6, #0
JSR RUNNING

RUNNING
;callee set-up of Running(n)'s activation record
;push return value, return address & caller's frame pointer
ADD R6, R6, #-3
```

R0	4



```c
int running_sum(int n){
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}
```

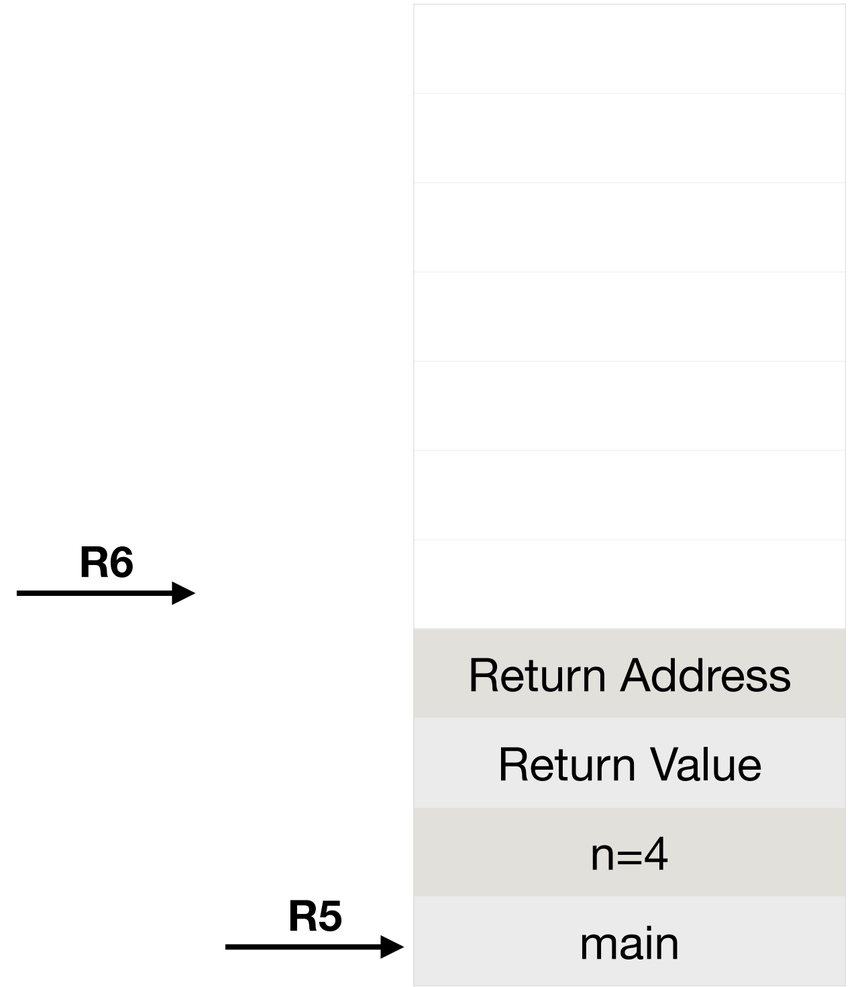
Recursion in LC3

```c
int main(void){
    int n = 4;
    int answer;
    answer = running_sum(n);
}
```

```
;Caller set-up
LDR R0, R5, #0
ADD R6, R6, #-1
STR R0, R6, #0
JSR RUNNING

RUNNING
;callee set-up of Running(n)'s activation record
;push return value, return address & caller's frame pointer
ADD R6, R6, #-3
STR R7, R6, #1
```

R0 | 4



```c
int running_sum(int n){
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}
```

Recursion in LC3

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int main(void){
    int n = 4;
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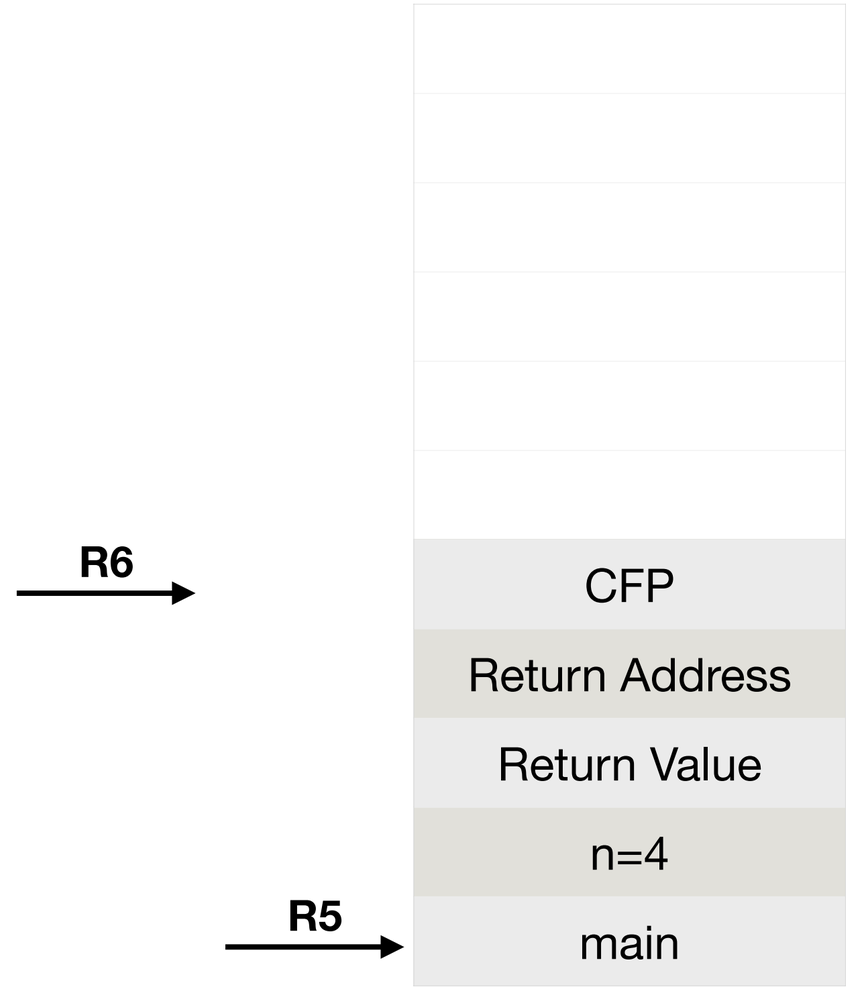
    answer = running_sum(n);
}
```

```
;Caller set-up
LDR R0, R5, #0
ADD R6, R6, #-1
STR R0, R6, #0
JSR RUNNING

RUNNING
;callee set-up of Running(n)'s activation record
;push return value, return address & caller's frame pointer
ADD R6, R6, #-3
STR R7, R6, #1
STR R5, R6, #0
```

R0	4



```c
int running_sum(int n){
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}
```

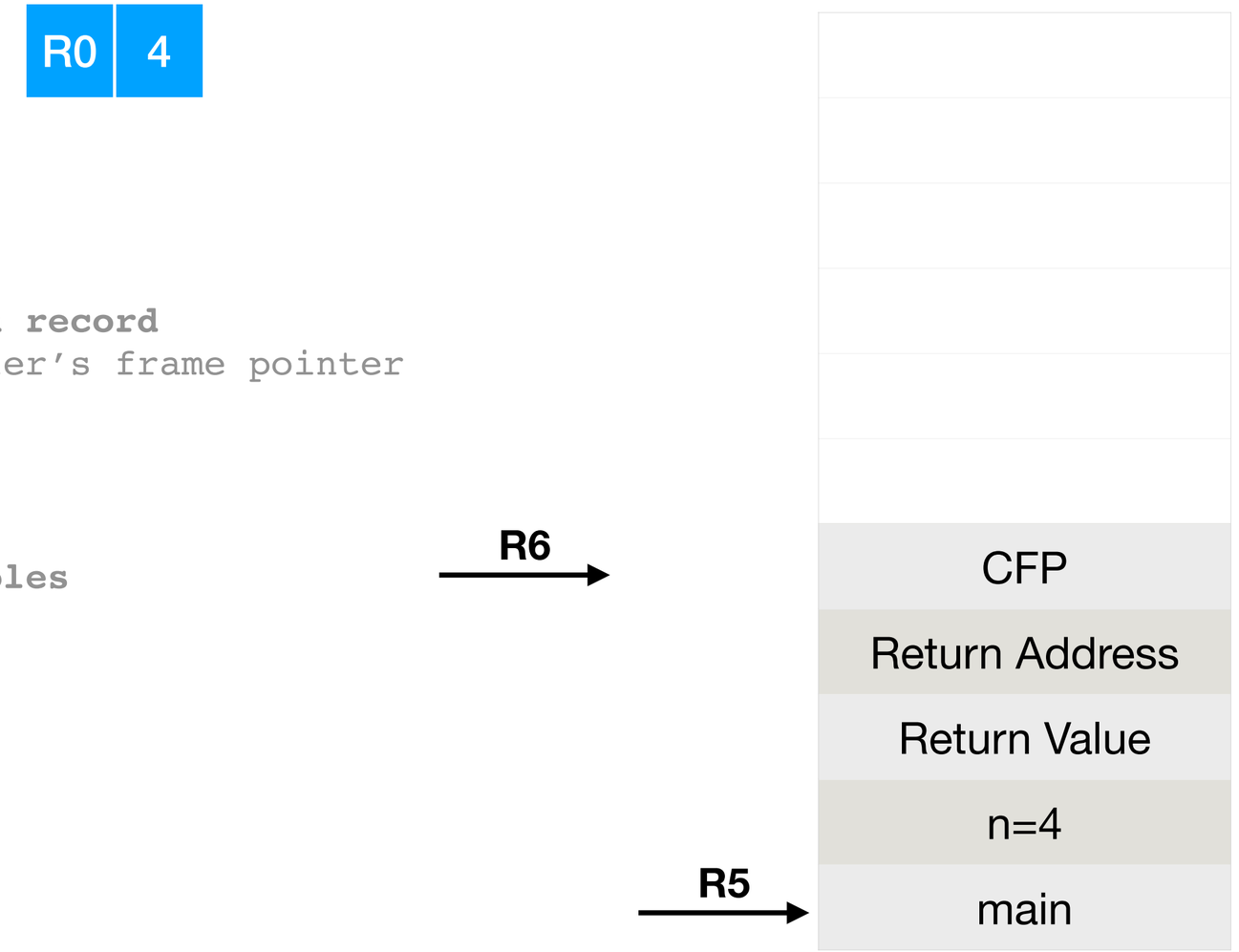
Recursion in LC3

```c
int main(void){
    int n = 4;
    int answer;
    answer = running_sum(n);
}
```

```
;Caller set-up
LDR R0, R5, #0
ADD R6, R6, #-1
STR R0, R6, #0
JSR RUNNING

RUNNING
;callee set-up of Running(n)'s activation record
;push return value, return address & caller's frame pointer
ADD R6, R6, #-3
STR R7, R6, #1
STR R5, R6, #0

;update frame pointer & push local variables
```



```c
int running_sum(int n){
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}
```

Recursion in LC3

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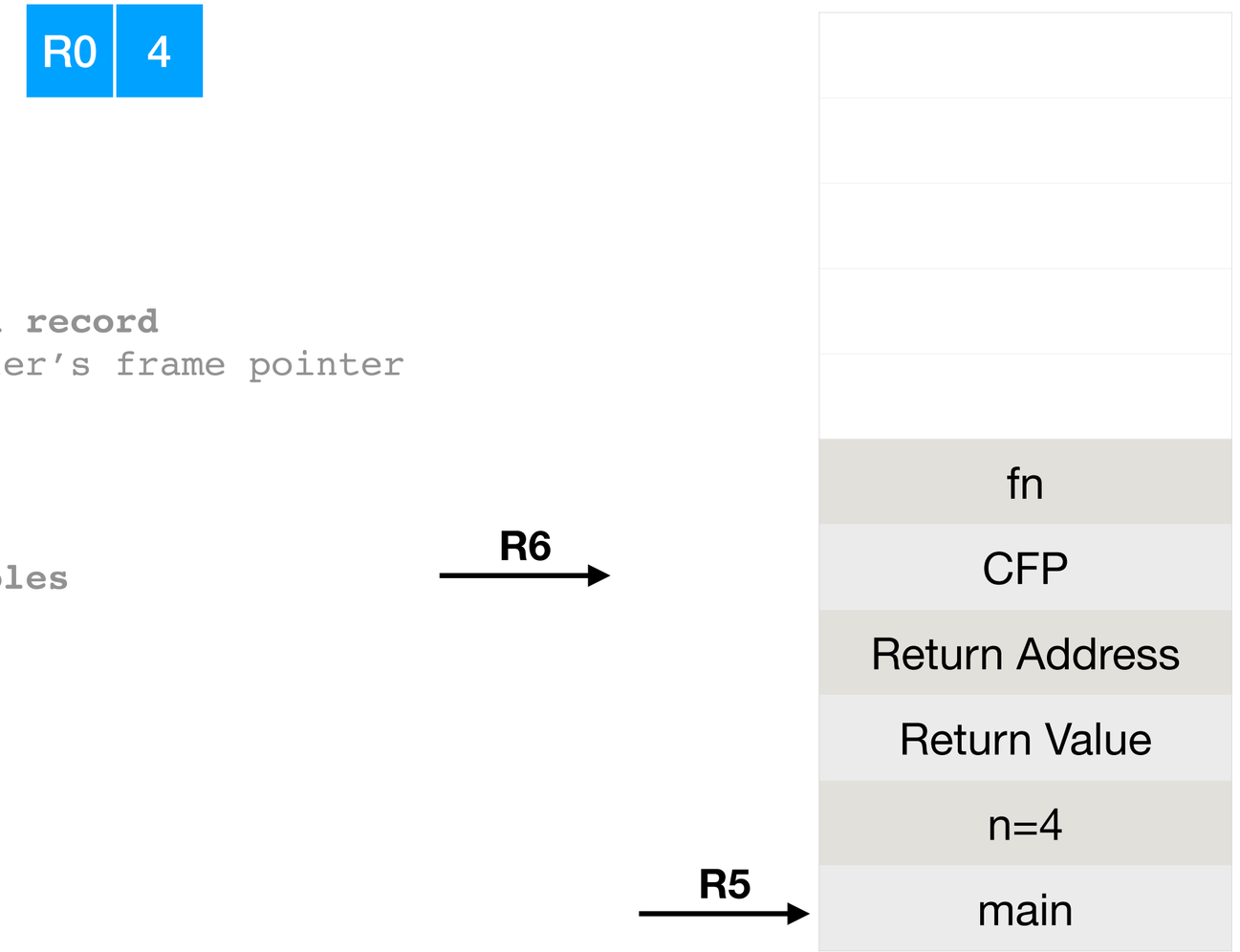
    int n = 4;
    int answer;
    answer = running_sum(n);
}
```

```
;Caller set-up
LDR R0, R5, #0
ADD R6, R6, #-1
STR R0, R6, #0
JSR RUNNING

RUNNING
;callee set-up of Running(n)'s activation record
;push return value, return address & caller's frame pointer
ADD R6, R6, #-3
STR R7, R6, #1
STR R5, R6, #0

;update frame pointer & push local variables
```



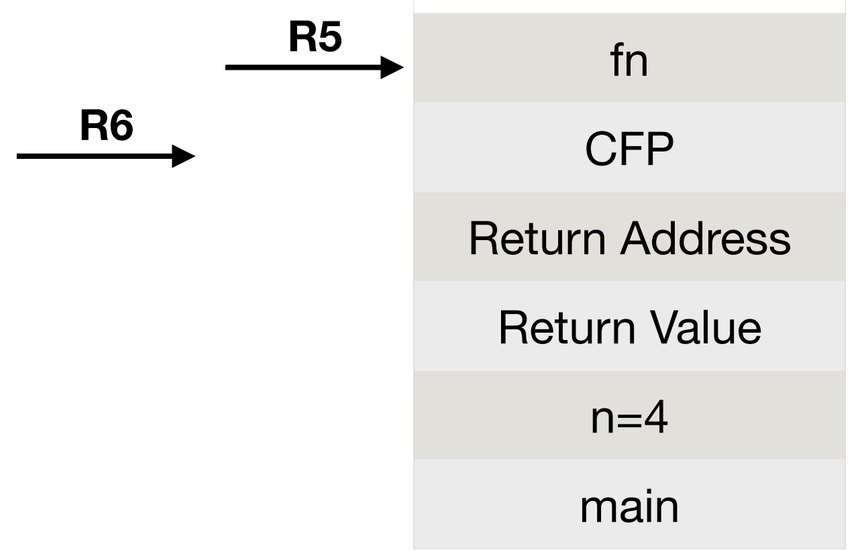
```c
int running_sum(int n){
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}
```

Recursion in LC3

```c
int main(void){
    int n = 4;
    int answer;
    answer = running_sum(n);
}
```

```
;Caller set-up
LDR R0, R5, #0
ADD R6, R6, #-1
STR R0, R6, #0
JSR RUNNING

RUNNING
;callee set-up of Running(n)'s activation record
;push return value, return address & caller's frame pointer
ADD R6, R6, #-3
STR R7, R6, #1
STR R5, R6, #0

;update frame pointer & push local variables
ADD R5, R6, #-1
```

R0	4



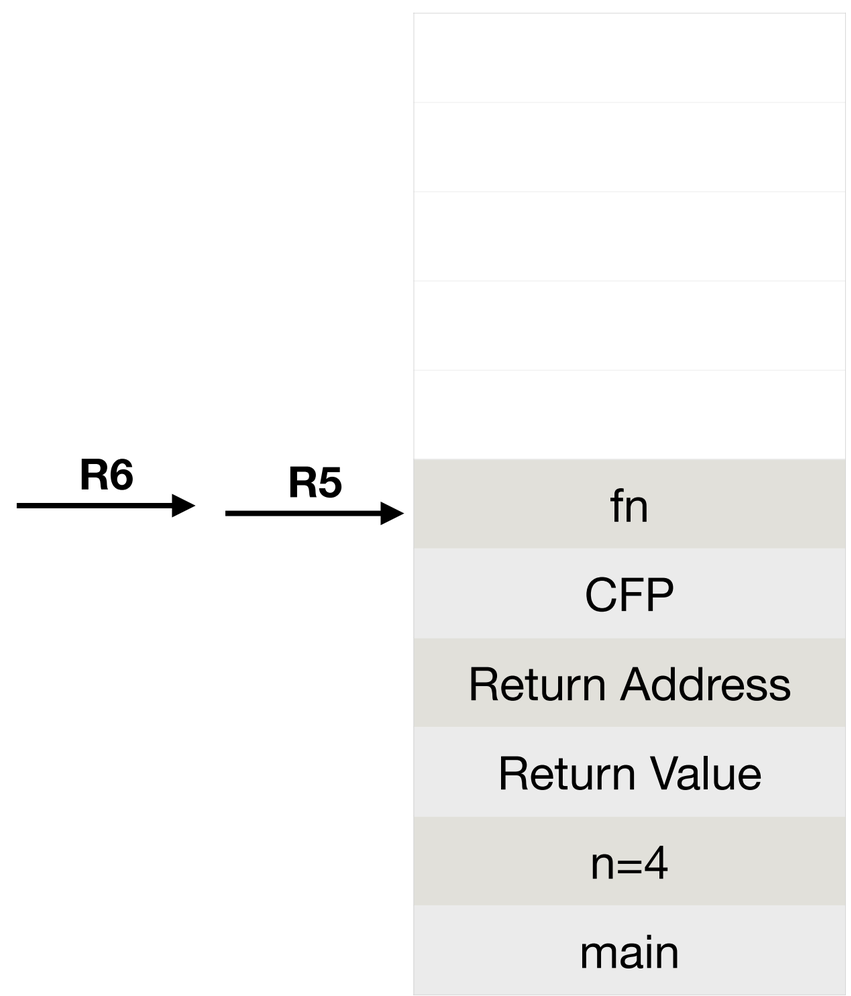
```c
int running_sum(int n){
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}
```

Recursion in LC3

```c
int main(void){
    int n = 4;
    int answer;
    answer = running_sum(n);
}
```

```
;Caller set-up
LDR R0, R5, #0
ADD R6, R6, #-1
STR R0, R6, #0
JSR RUNNING

RUNNING
;callee set-up of Running(n)'s activation record
;push return value, return address & caller's frame pointer
ADD R6, R6, #-3
STR R7, R6, #1
STR R5, R6, #0

;update frame pointer & push local variables
ADD R5, R6, #-1
ADD R6, R6, #-1
```

R0	4



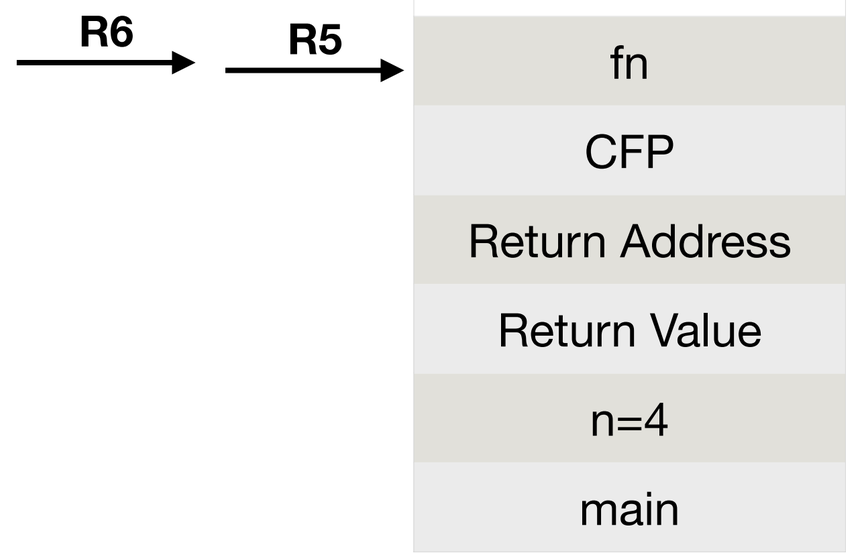
```c
int running_sum(int n){
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}
```

Recursion in LC3

```c
int main(void){
    int n = 4;
    int answer;
    answer = running_sum(n);
}
```

```
;Caller set-up
LDR R0, R5, #0
ADD R6, R6, #-1
STR R0, R6, #0
JSR RUNNING

RUNNING
;callee set-up of Running(n)'s activation record
;push return value, return address & caller's frame pointer
ADD R6, R6, #-3
STR R7, R6, #1
STR R5, R6, #0

;update frame pointer & push local variables
ADD R5, R6, #-1
ADD R6, R6, #-1

;function logic
```

R0	4



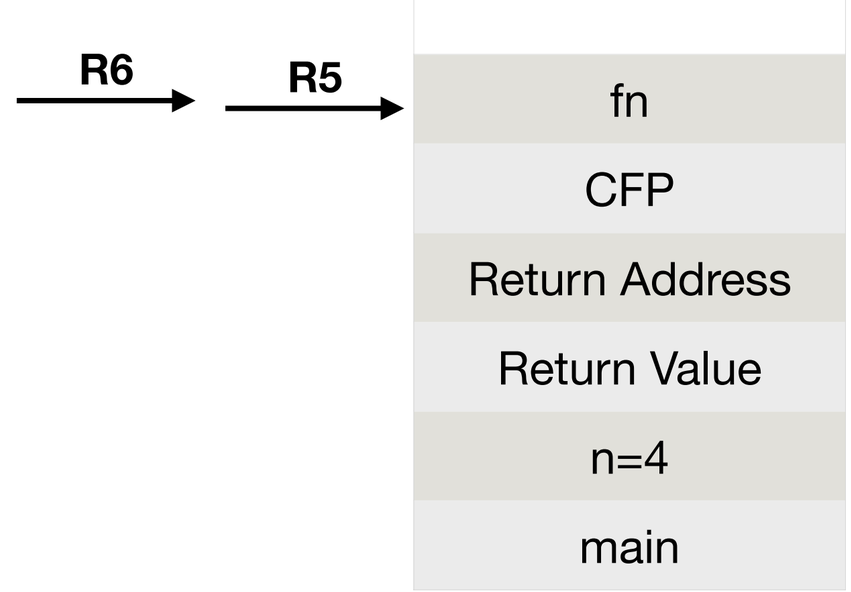
```c
int running_sum(int n){
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}
```

Recursion in LC3

```c
int main(void){
    int n = 4;
    int answer;
    answer = running_sum(n);
}
```

```
;Caller set-up
LDR R0, R5, #0
ADD R6, R6, #-1
STR R0, R6, #0
JSR RUNNING

RUNNING
;callee set-up of Running(n)'s activation record
;push return value, return address & caller's frame pointer
ADD R6, R6, #-3
STR R7, R6, #1
STR R5, R6, #0

;update frame pointer & push local variables
ADD R5, R6, #-1
ADD R6, R6, #-1

;function logic
;base case (n==1)
```

R0	4



```c
int running_sum(int n){
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}
```

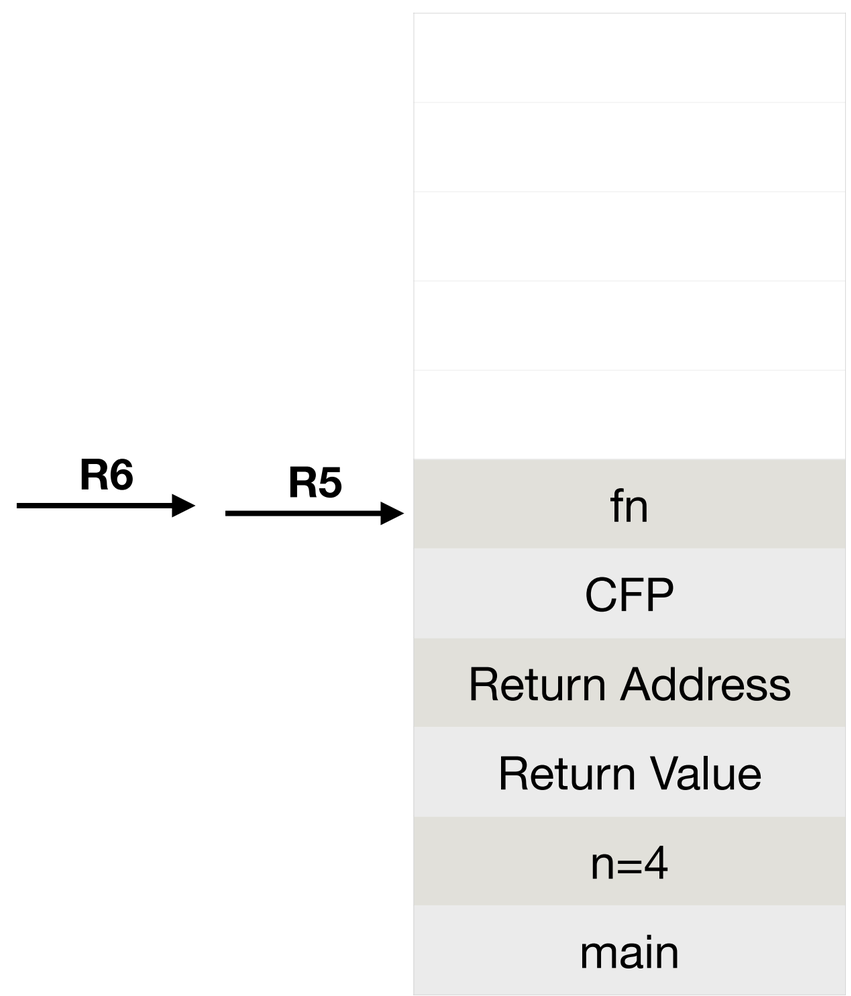
Recursion in LC3

```c
int main(void){
    int n = 4;
    int answer;
    answer = running_sum(n);
}
```

```
;Caller set-up
LDR R0, R5, #0
ADD R6, R6, #-1
STR R0, R6, #0
JSR RUNNING

RUNNING
;callee set-up of Running(n)'s activation record
;push return value, return address & caller's frame pointer
ADD R6, R6, #-3
STR R7, R6, #1
STR R5, R6, #0

;update frame pointer & push local variables
ADD R5, R6, #-1
ADD R6, R6, #-1

;function logic
;base case (n==1)
LDR R1, R5, #4
```

R0	4

R1	4



```c
int running_sum(int n){
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}
```

Recursion in LC3

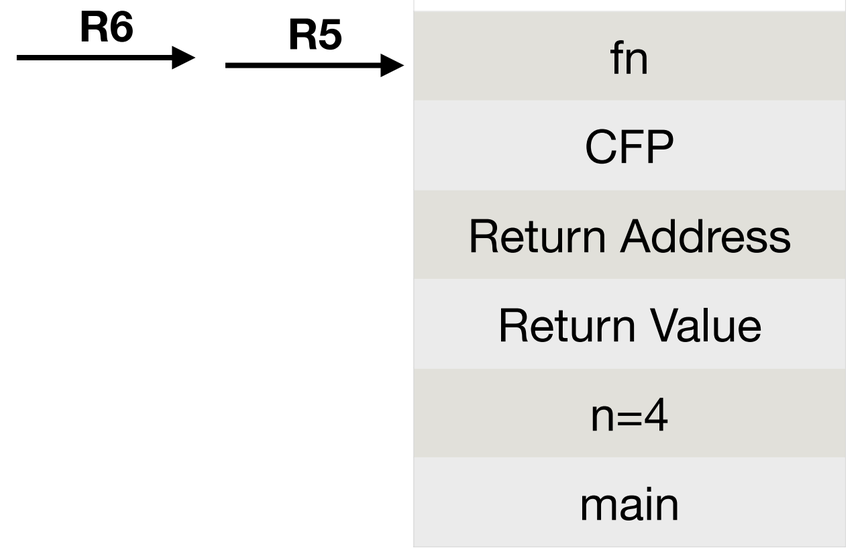
```c
int main(void){
    int n = 4;
    int answer;
    answer = running_sum(n);
}
```

```
;Caller set-up
LDR R0, R5, #0
ADD R6, R6, #-1
STR R0, R6, #0
JSR RUNNING

RUNNING
;callee set-up of Running(n)'s activation record
;push return value, return address & caller's frame pointer
ADD R6, R6, #-3
STR R7, R6, #1
STR R5, R6, #0

;update frame pointer & push local variables
ADD R5, R6, #-1
ADD R6, R6, #-1

;function logic
;base case (n==1)
LDR R1, R5, #4
ADD R2, R1, #-1
```

R0	4

R1	4

R2	3



```c
int running_sum(int n){
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}
```

Recursion in LC3

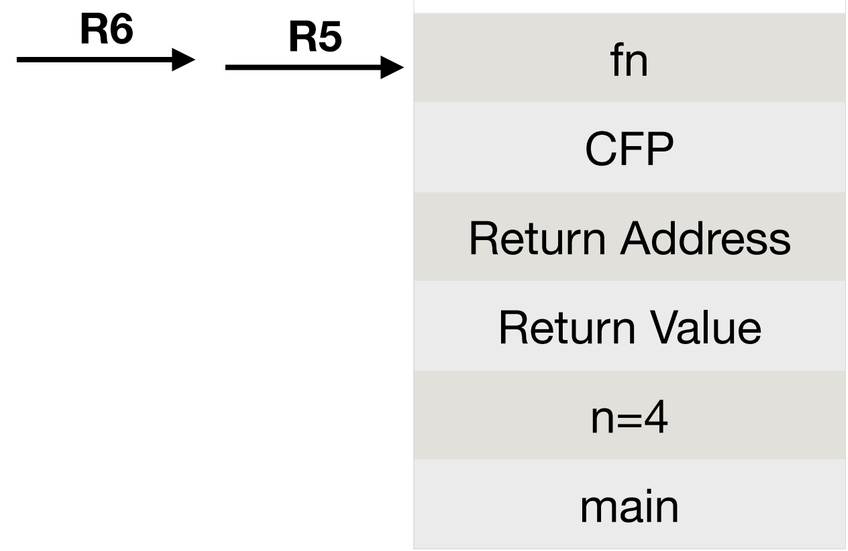
```c
int main(void){
    int n = 4;
    int answer;
    answer = running_sum(n);
}
```

```
;Caller set-up
LDR R0, R5, #0
ADD R6, R6, #-1
STR R0, R6, #0
JSR RUNNING

RUNNING
;callee set-up of Running(n)'s activation record
;push return value, return address & caller's frame pointer
ADD R6, R6, #-3
STR R7, R6, #1
STR R5, R6, #0

;update frame pointer & push local variables
ADD R5, R6, #-1
ADD R6, R6, #-1

;function logic
;base case (n==1)
LDR R1, R5, #4
ADD R2, R1, #-1
BRz BASE_CASE
```

R0	4

R1	4

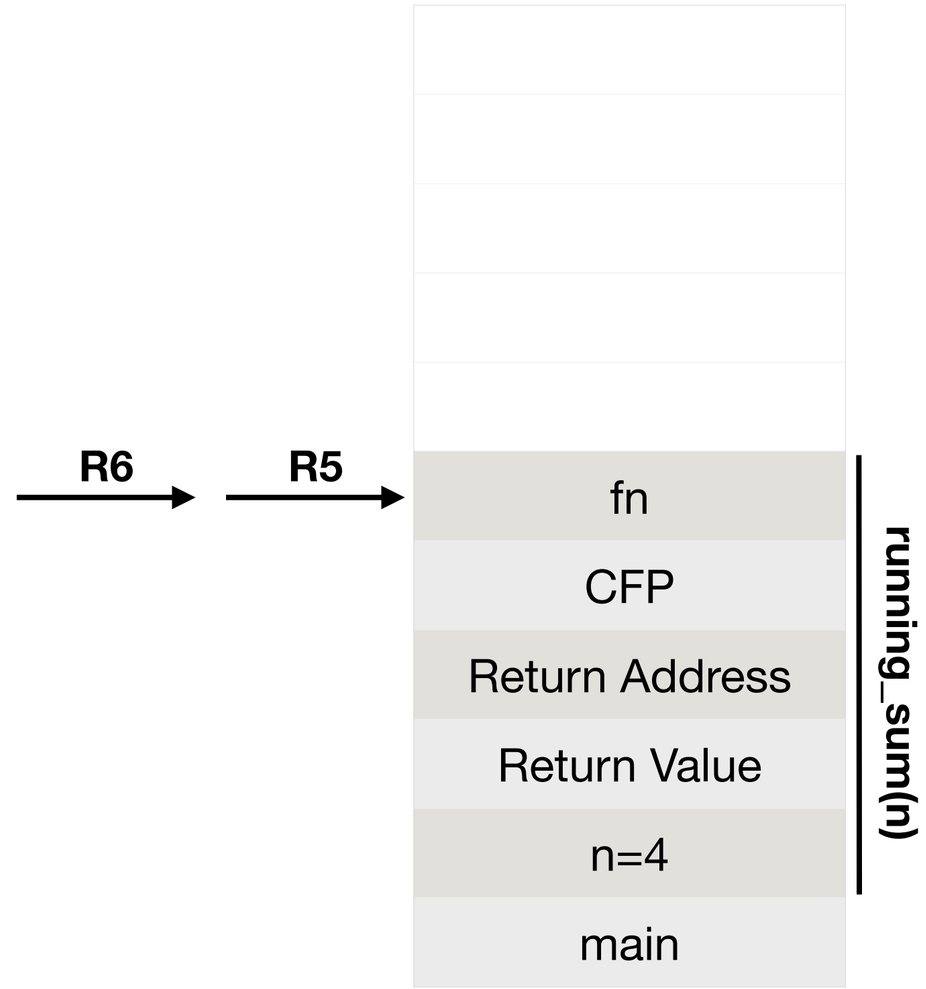
R2	3



Recursion in LC3

```c
int running_sum(int n){
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}
```

```c
int main(void){
    int n = 4;
    int answer;
    answer = running_sum(n);
}
```



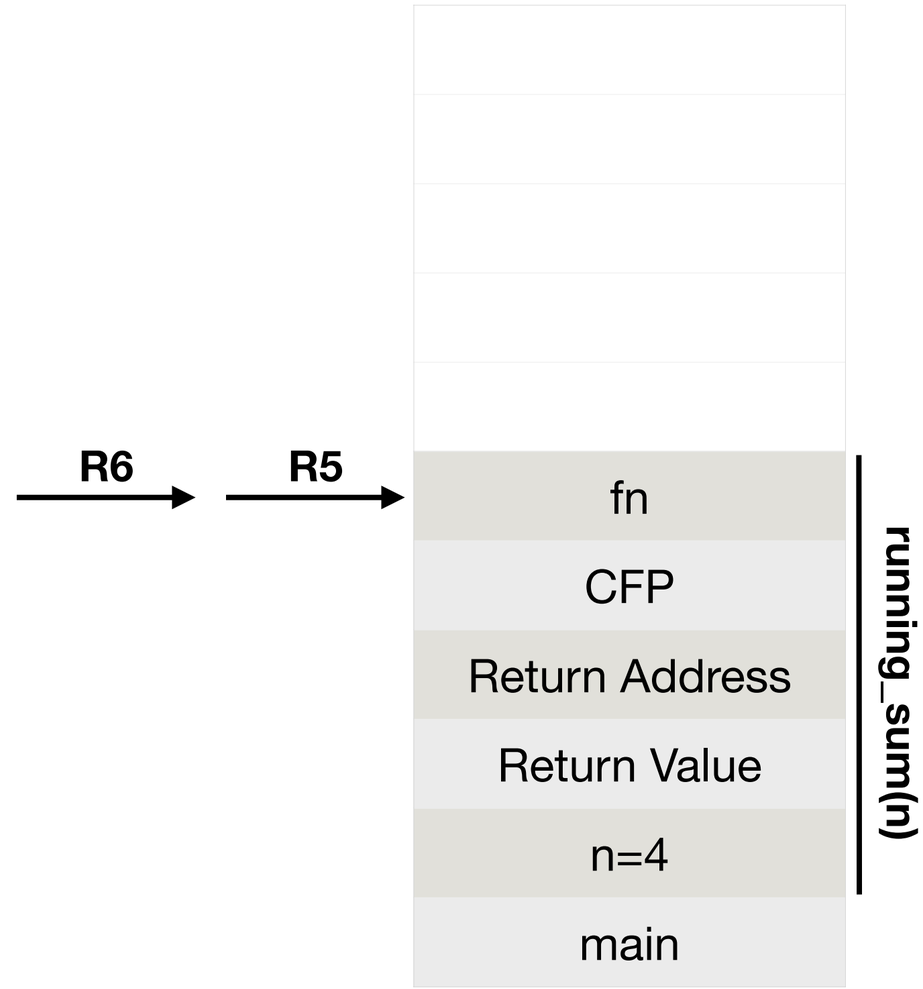
```c
int running_sum(int n){
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}
```

Recursion in LC3

```c
int main(void){
    int n = 4;
    int answer;
    answer = running_sum(n);
}
```

R2 | 3

```
;Recursive case
;Caller setup: push argument n-1 onto RTS
ADD R6, R6, #-1
```



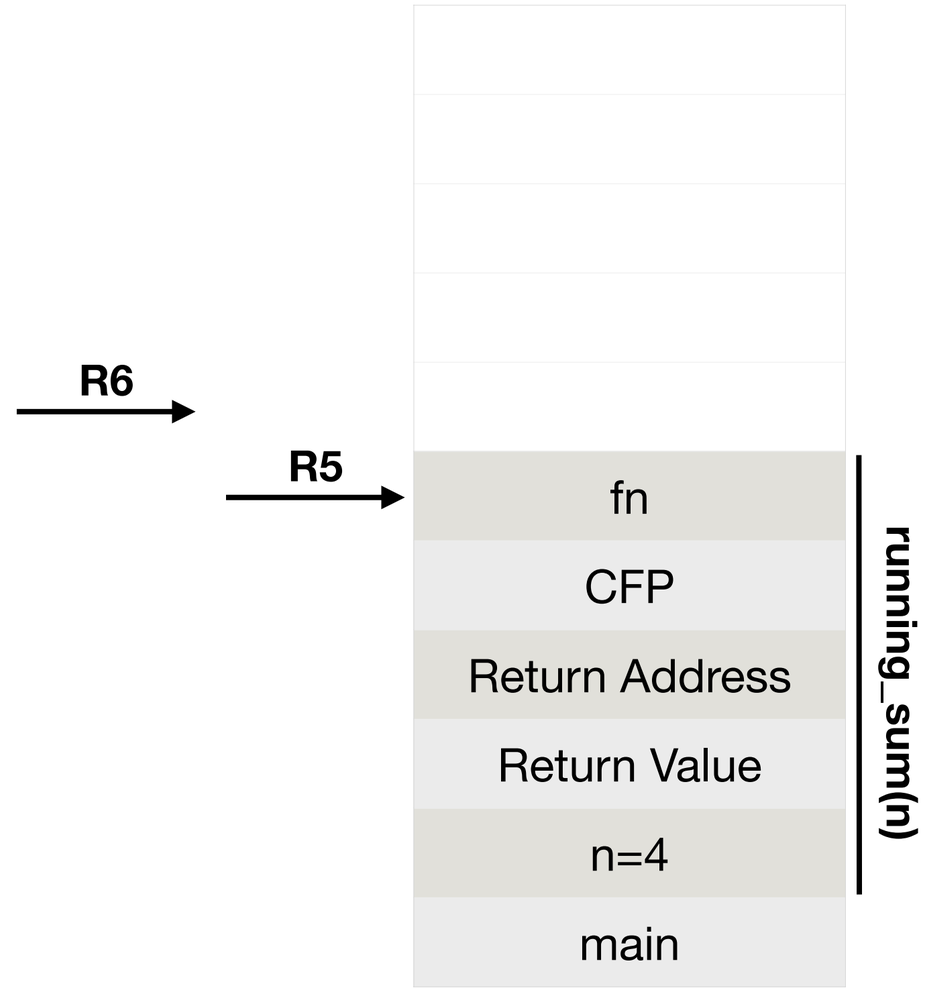
```c
int running_sum(int n){
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}
```

Recursion in LC3

```c
int main(void){
    int n = 4;
    int answer;
    answer = running_sum(n);
}
```

R2 | 3

```
;Recursive case
;Caller setup: push argument n-1 onto RTS
ADD R6, R6, #-1
STR R2, R6, #0 ; R2 = n - 1
```



```c
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    int fn;
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        fn = 1;
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        fn = n + running_sum(n-1);
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Recursion in LC3

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    int n = 4;
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}
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R2 | 3

```
;Recursive case
;Caller setup: push argument n-1 onto RTS
ADD R6, R6, #-1
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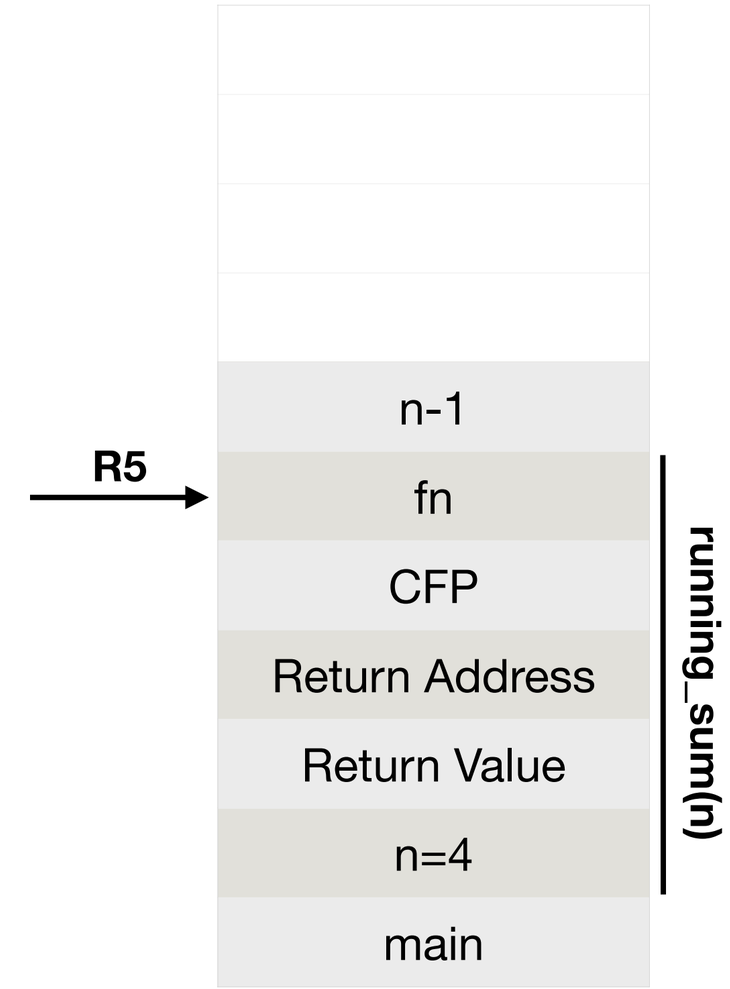


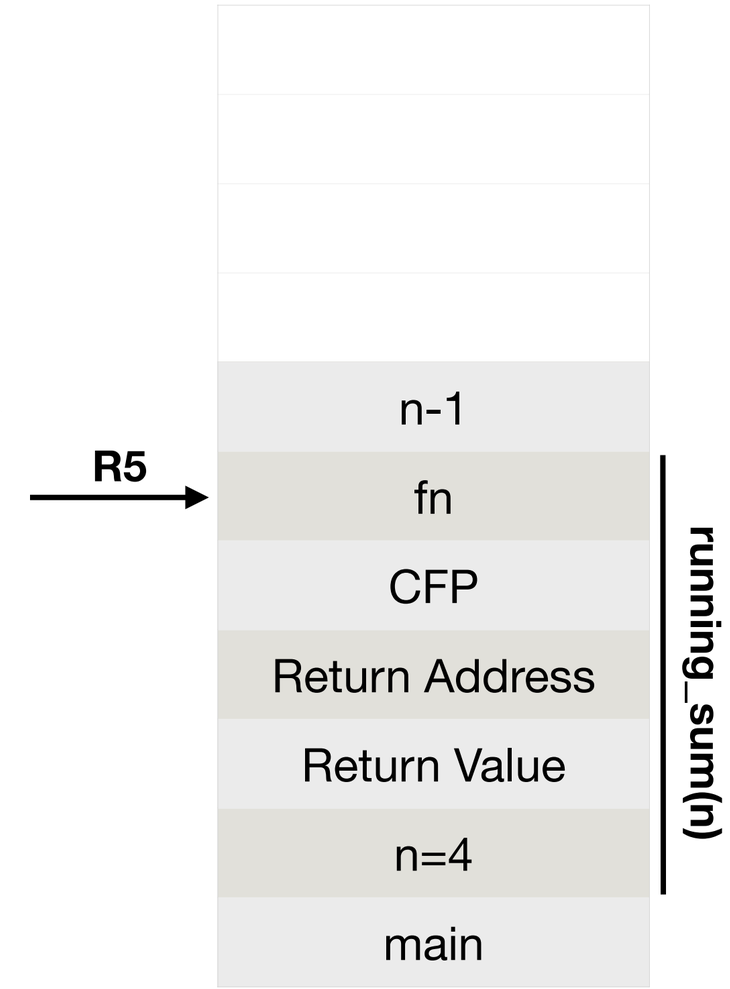
```c
int running_sum(int n){
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}
```

Recursion in LC3

```c
int main(void){
    int n = 4;
    int answer;
    answer = running_sum(n);
}
```

R2 | 3

```
;Recursive case
;Caller setup: push argument n-1 onto RTS
ADD R6, R6, #-1
STR R2, R6, #0 ; R2 = n - 1
JSR RUNNING ; call Running(n-1)
```



```c
int running_sum(int n){
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}
```

Recursion in LC3

```c
int main(void){
    int n = 4;
    int answer;
    answer = running_sum(n);
}
```

R2 | 3

```
;Recursive case
;Caller setup: push argument n-1 onto RTS
ADD R6, R6, #-1
STR R2, R6, #0 ; R2 = n - 1
JSR RUNNING ; call Running(n-1)
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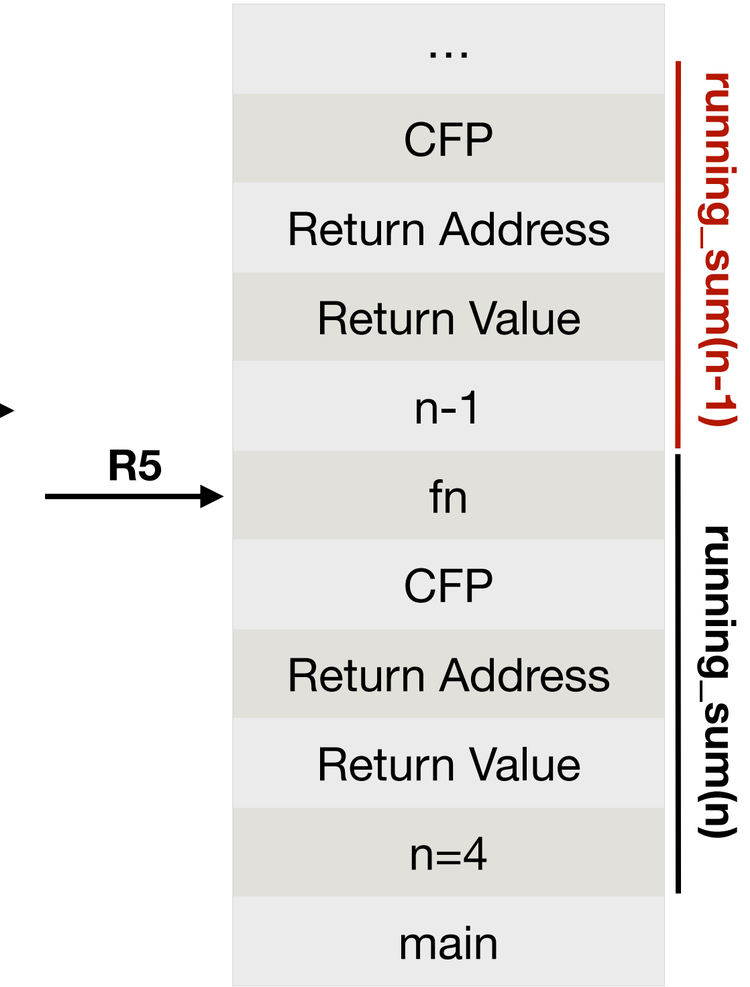


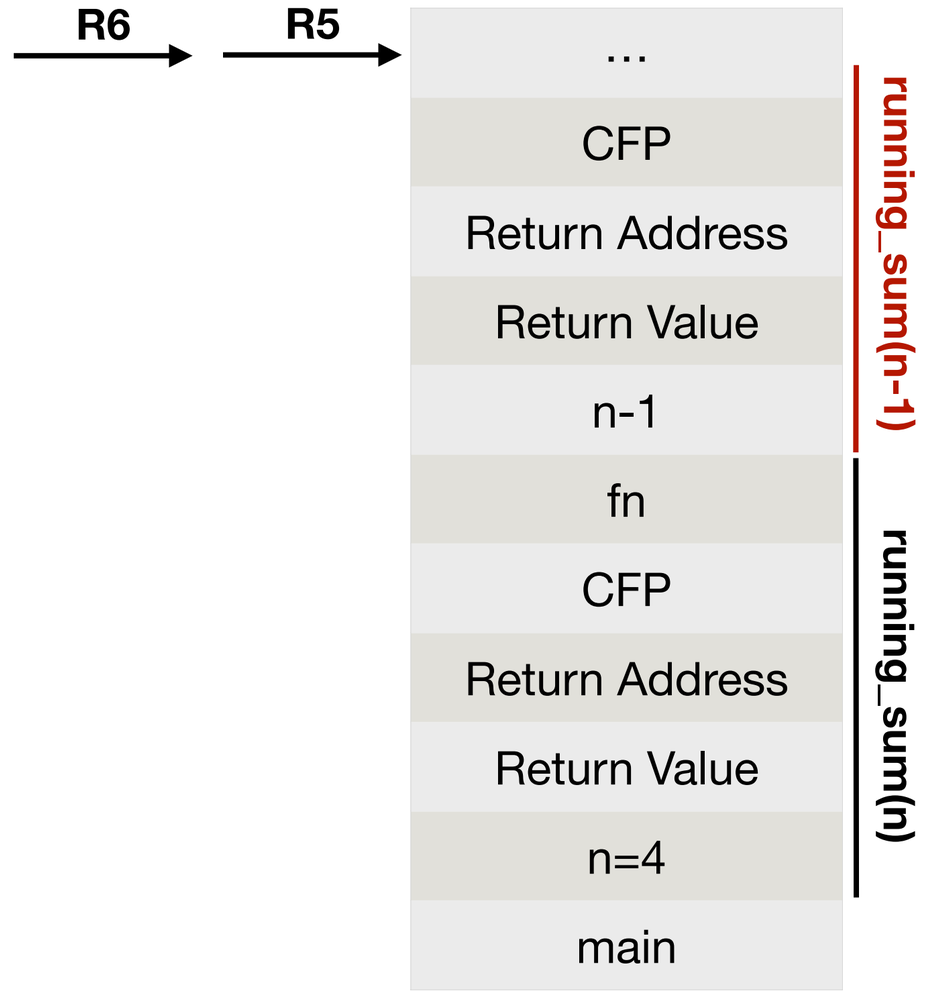
```c
int running_sum(int n){
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}
```

Recursion in LC3

```c
int main(void){
    int n = 4;
    int answer;
    answer = running_sum(n);
}
```

R2 | 3

```
;Recursive case
;Caller setup: push argument n-1 onto RTS
ADD R6, R6, #-1
STR R2, R6, #0 ; R2 = n - 1
JSR RUNNING ; call Running(n-1)
```



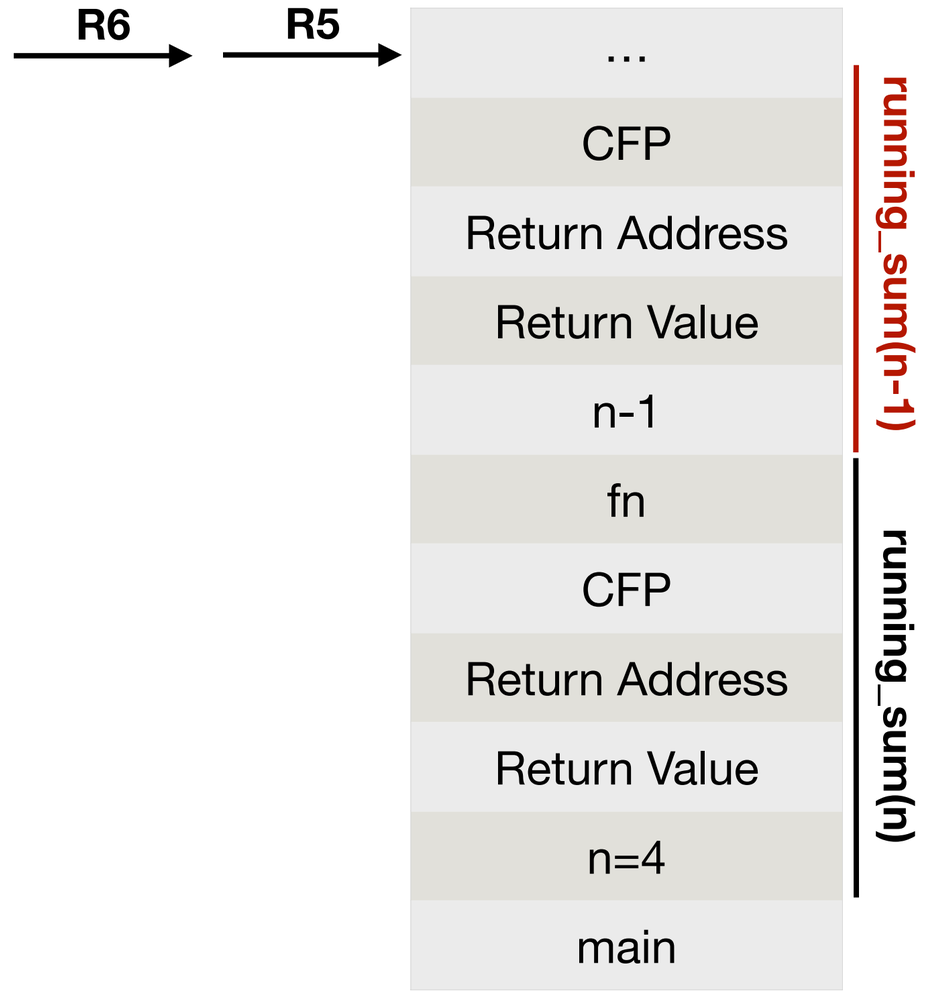
```c
int running_sum(int n){
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}
```

Recursion in LC3

```c
int main(void){
    int n = 4;
    int answer;
    answer = running_sum(n);
}
```

R2 | 3

```
;Recursive case
;Caller setup: push argument n-1 onto RTS
ADD R6, R6, #-1
STR R2, R6, #0 ; R2 = n - 1
JSR RUNNING ; call Running(n-1)
```



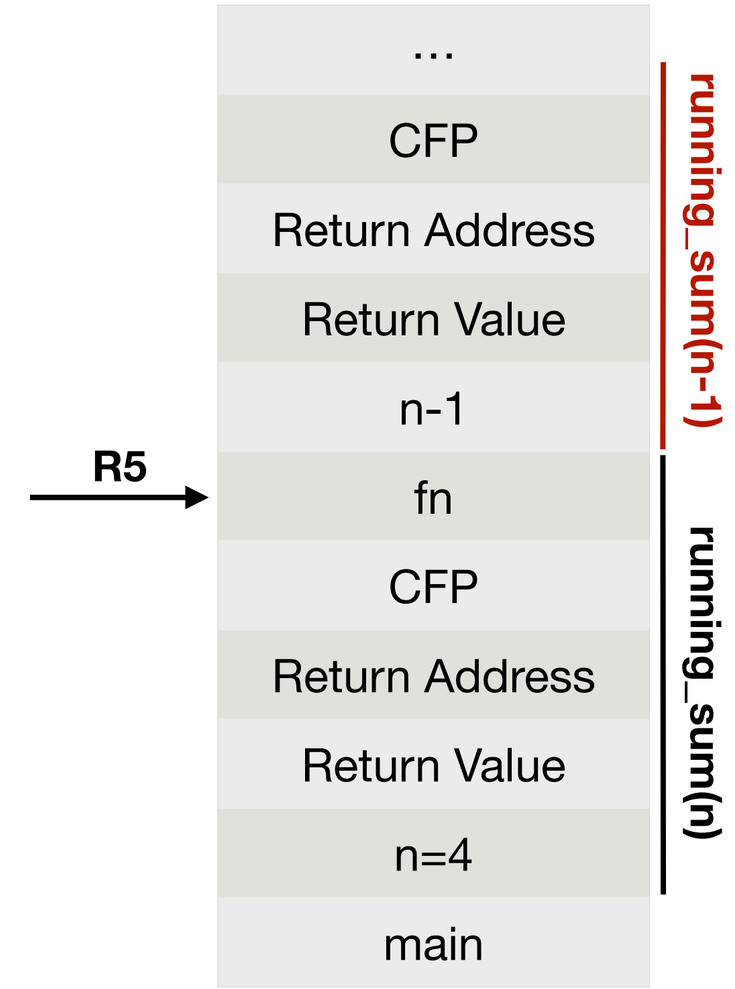
```c
int running_sum(int n){
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}
```

Recursion in LC3

```c
int main(void){
    int n = 4;
    int answer;
    answer = running_sum(n);
}
```

R2 | 3

```
;Recursive case
;Caller setup: push argument n-1 onto RTS
ADD R6, R6, #-1
STR R2, R6, #0 ; R2 = n - 1
JSR RUNNING ; call Running(n-1)
```



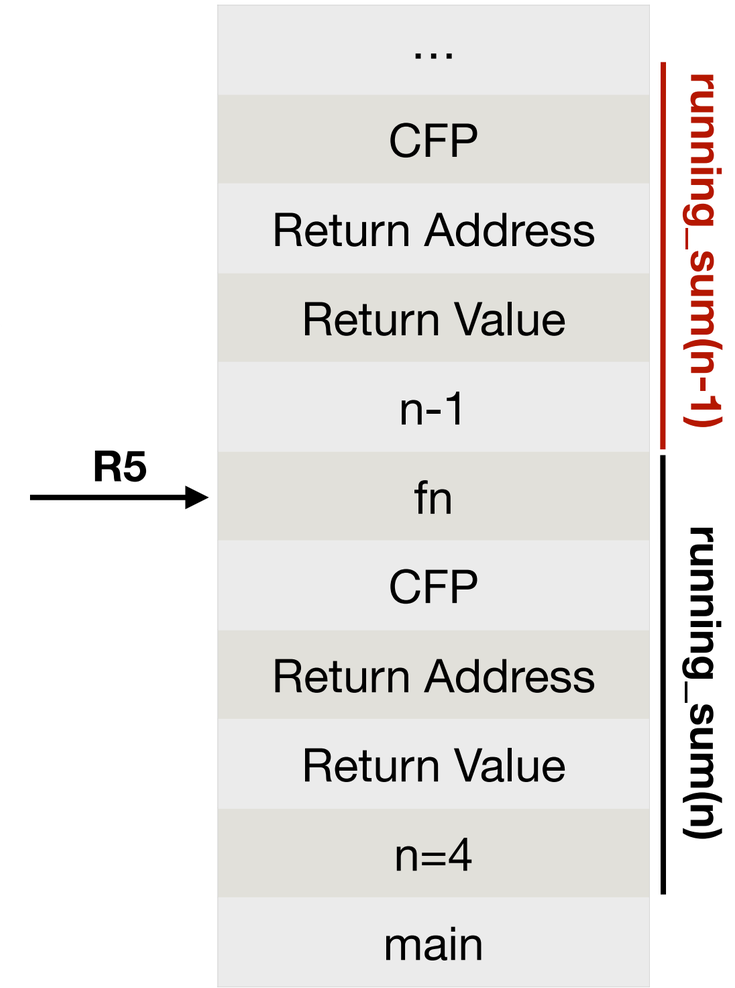
```c
int running_sum(int n){
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}
```

Recursion in LC3

```c
int main(void){
    int n = 4;
    int answer;
    answer = running_sum(n);
}
```

R2 | 3

```
;Recursive case
;Caller setup: push argument n-1 onto RTS
ADD R6, R6, #-1
STR R2, R6, #0 ; R2 = n - 1
JSR RUNNING ; call Running(n-1)



;Caller tear-down for Running(n-1)
```



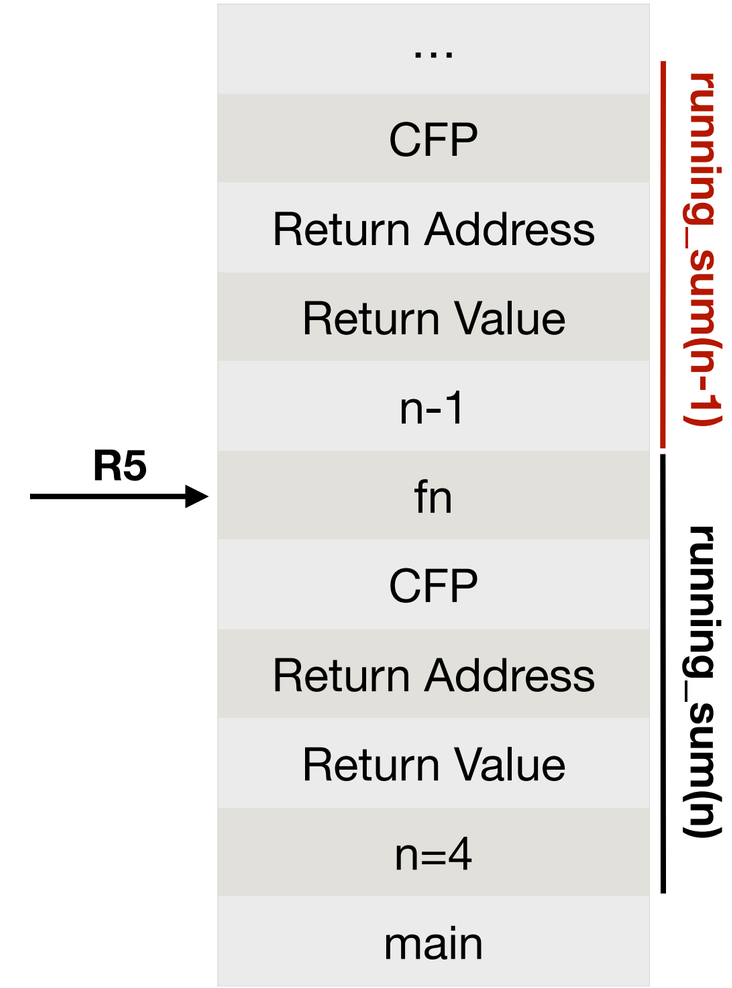
```c
int running_sum(int n){
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}
```

Recursion in LC3

```c
int main(void){
    int n = 4;
    int answer;
    answer = running_sum(n);
}
```

R2 | 3

```
;Recursive case
;Caller setup: push argument n-1 onto RTS
ADD R6, R6, #-1
STR R2, R6, #0 ; R2 = n - 1
JSR RUNNING ; call Running(n-1)



;Caller tear-down for Running(n-1)
;pop Running(n-1)'s return value to R0
```



```c
int running_sum(int n){
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}
```

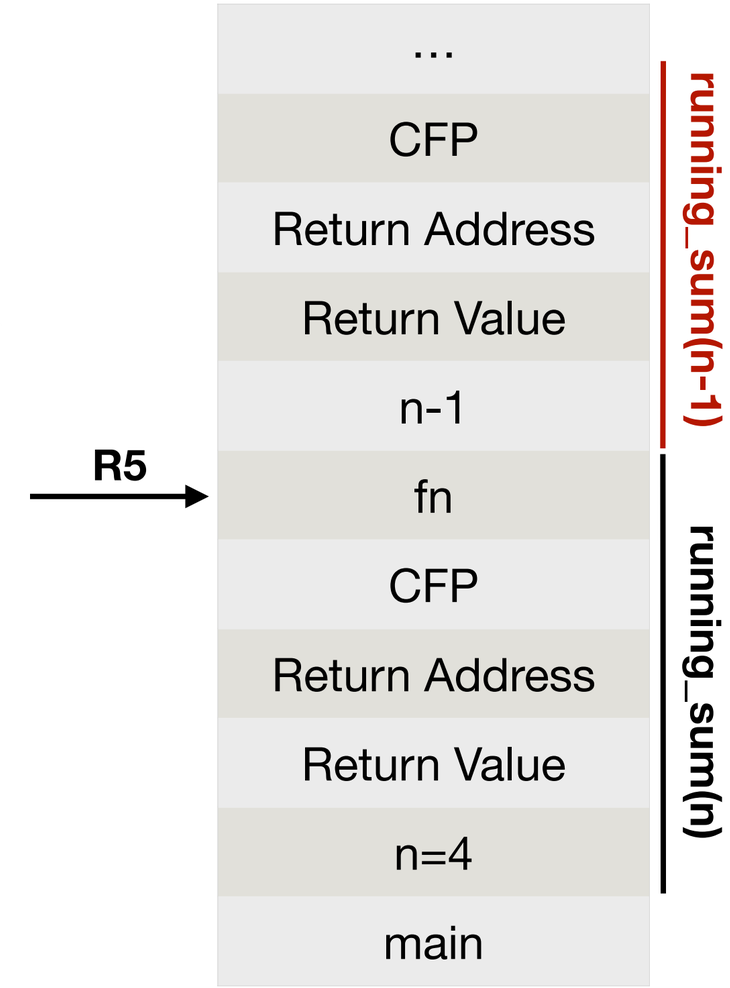
Recursion in LC3

```c
int main(void){
    int n = 4;
    int answer;
    answer = running_sum(n);
}
```

R2 | 3

```
;Recursive case
;Caller setup: push argument n-1 onto RTS
ADD R6, R6, #-1
STR R2, R6, #0 ; R2 = n - 1
JSR RUNNING ; call Running(n-1)



;Caller tear-down for Running(n-1)
;pop Running(n-1)'s return value to R0
LDR R0, R6, #0
```

R0 | 6



Recursion in LC3

```c
int running_sum(int n){
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}
```

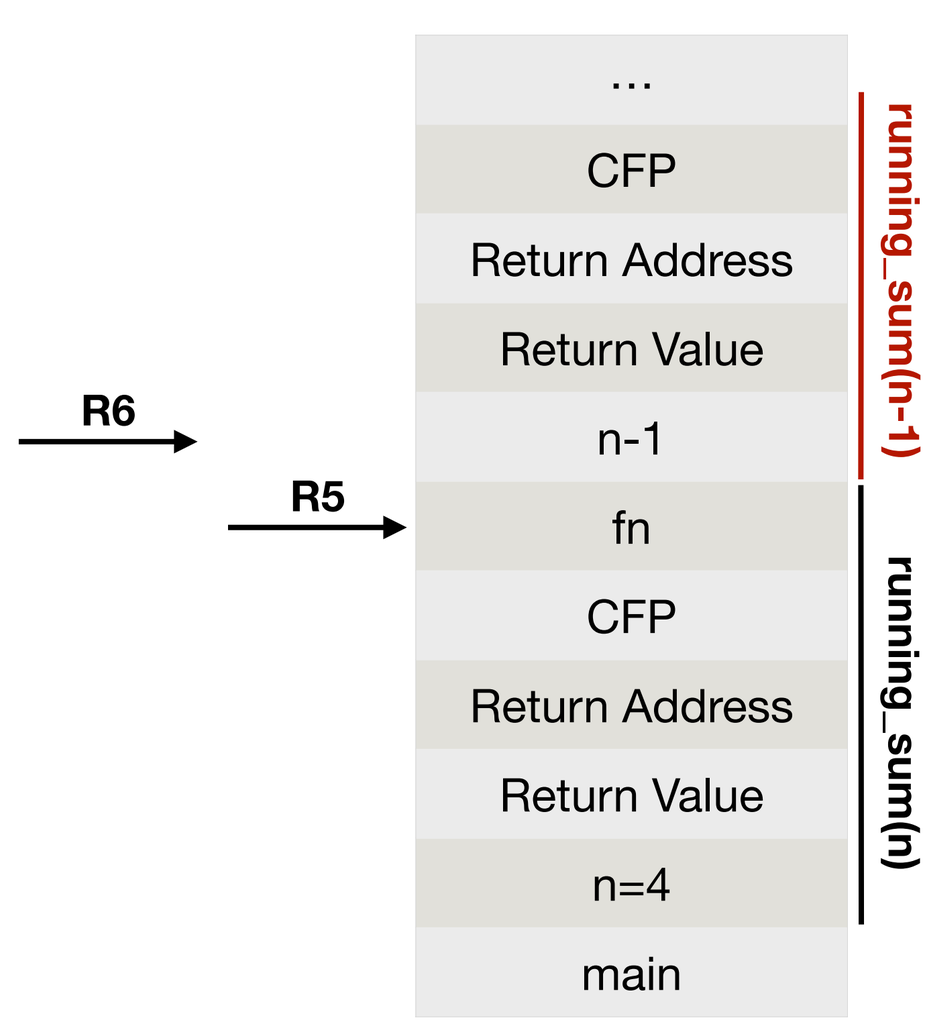
```c
int main(void){
    int n = 4;
    int answer;
    answer = running_sum(n);
}
```

R2 | 3

R0 | 6

```
;Recursive case
;Caller setup: push argument n-1 onto RTS
ADD R6, R6, #-1
STR R2, R6, #0 ; R2 = n - 1
JSR RUNNING ; call Running(n-1)
```

```
;Caller tear-down for Running(n-1)
;pop Running(n-1)'s return value to R0
LDR R0, R6, #0
ADD R6, R6, #1
```



```c
int running_sum(int n){
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}
```

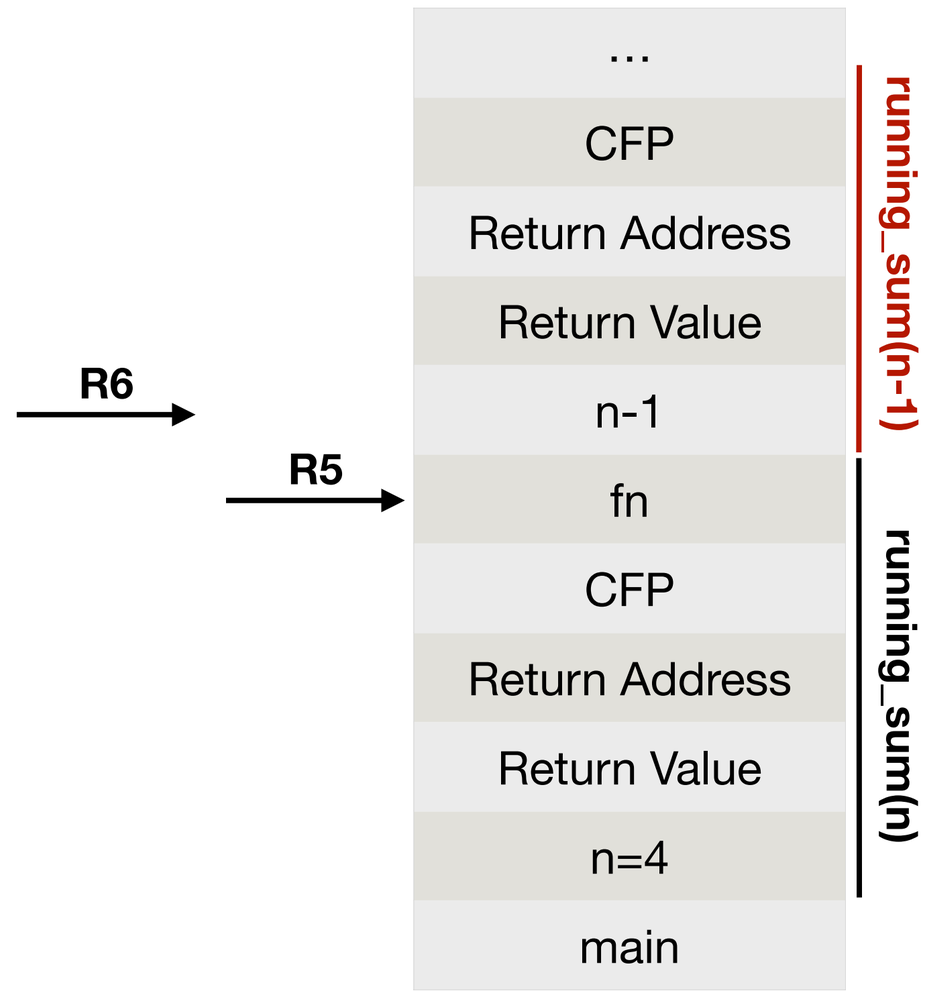
Recursion in LC3

```c
int main(void){
    int n = 4;
    int answer;
    answer = running_sum(n);
}
```

R2 | 3

R0 | 6

```
;Recursive case
;Caller setup: push argument n-1 onto RTS
ADD R6, R6, #-1
STR R2, R6, #0 ; R2 = n - 1
JSR RUNNING ; call Running(n-1)


;Caller tear-down for Running(n-1)
;pop Running(n-1)'s return value to R0
LDR R0, R6, #0
ADD R6, R6, #1

;pop Running(n-1)'s argument
```



```c
int running_sum(int n){
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}
```

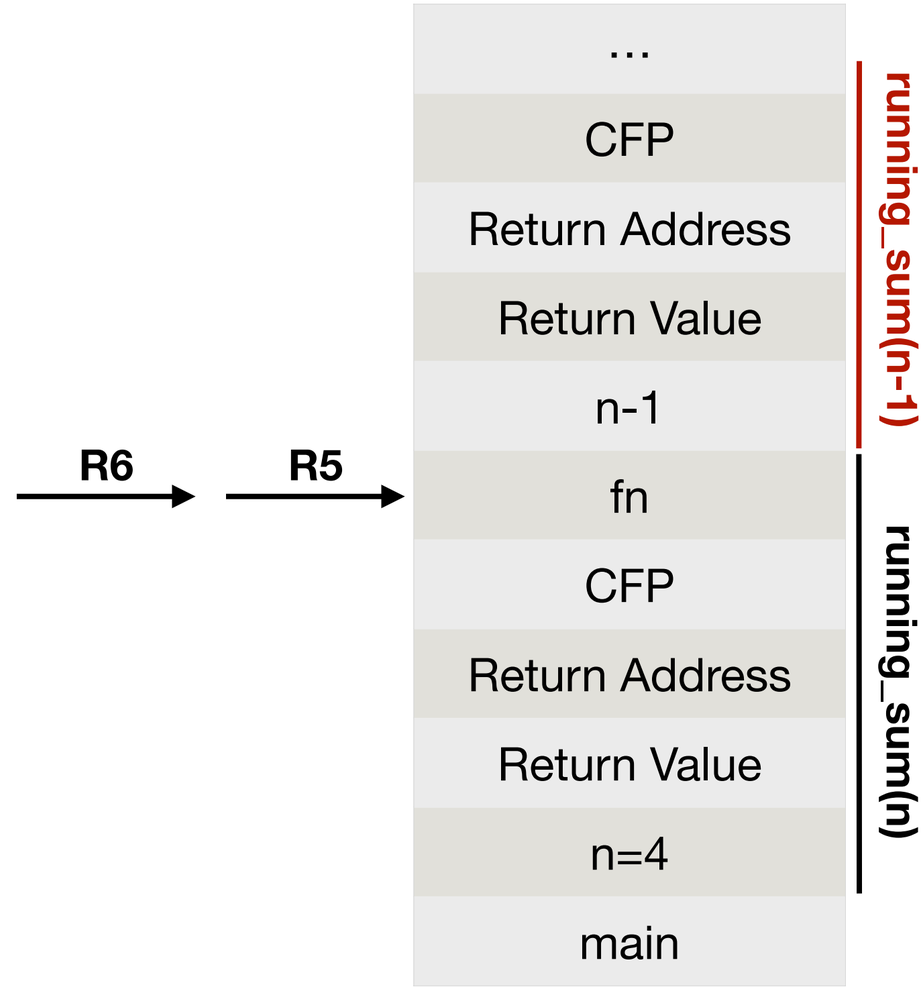
Recursion in LC3

```c
int main(void){
    int n = 4;
    int answer;
    answer = running_sum(n);
}
```

R2 | 3

R0 | 6

```
;Recursive case
;Caller setup: push argument n-1 onto RTS
ADD R6, R6, #-1
STR R2, R6, #0 ; R2 = n - 1
JSR RUNNING ; call Running(n-1)



;Caller tear-down for Running(n-1)
;pop Running(n-1)'s return value to R0
LDR R0, R6, #0
ADD R6, R6, #1

;pop Running(n-1)'s argument
ADD R6, R6, #1
```



```c
int running_sum(int n){
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}
```

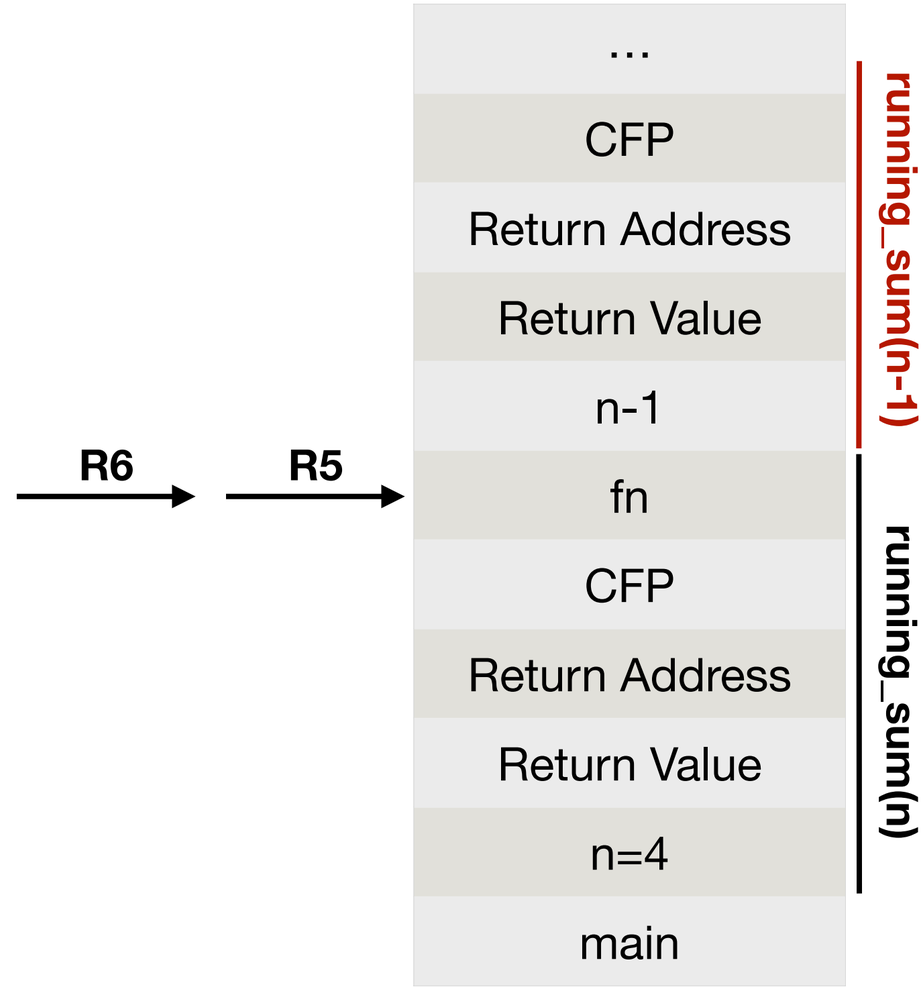
Recursion in LC3

```c
int main(void){
    int n = 4;
    int answer;
    answer = running_sum(n);
}
```

R2 | 3

R0 | 6

```
;Recursive case
;Caller setup: push argument n-1 onto RTS
ADD R6, R6, #-1
STR R2, R6, #0 ; R2 = n - 1
JSR RUNNING ; call Running(n-1)



;Caller tear-down for Running(n-1)
;pop Running(n-1)'s return value to R0
LDR R0, R6, #0
ADD R6, R6, #1

;pop Running(n-1)'s argument
ADD R6, R6, #1

;calculate n + Running(n-1)
```



Recursion in LC3

```c
int running_sum(int n){
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}
```

```c
int main(void){
    int n = 4;
    int answer;
    answer = running_sum(n);
}
```

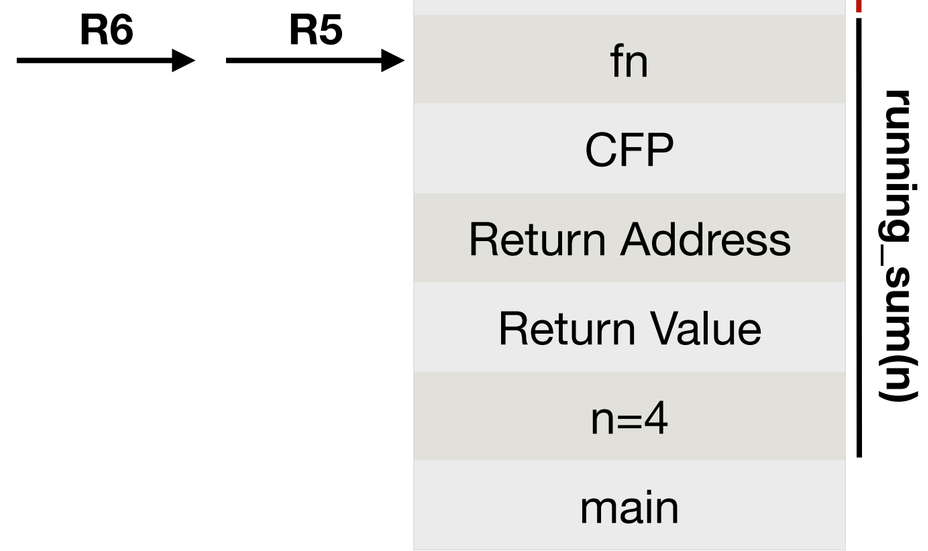
R2 | 3

```
;Recursive case
;Caller setup: push argument n-1 onto RTS
ADD R6, R6, #-1
STR R2, R6, #0 ; R2 = n - 1
JSR RUNNING ; call Running(n-1)
```

R0 | 6

R1 | 4

```
;Caller tear-down for Running(n-1)
;pop Running(n-1)'s return value to R0
LDR R0, R6, #0
ADD R6, R6, #1

;pop Running(n-1)'s argument
ADD R6, R6, #1

;calculate n + Running(n-1)
LDR R1, R5, #4
```



Recursion in LC3

```c
int running_sum(int n){
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}
```

```c
int main(void){
    int n = 4;
    int answer;
    answer = running_sum(n);
}
```

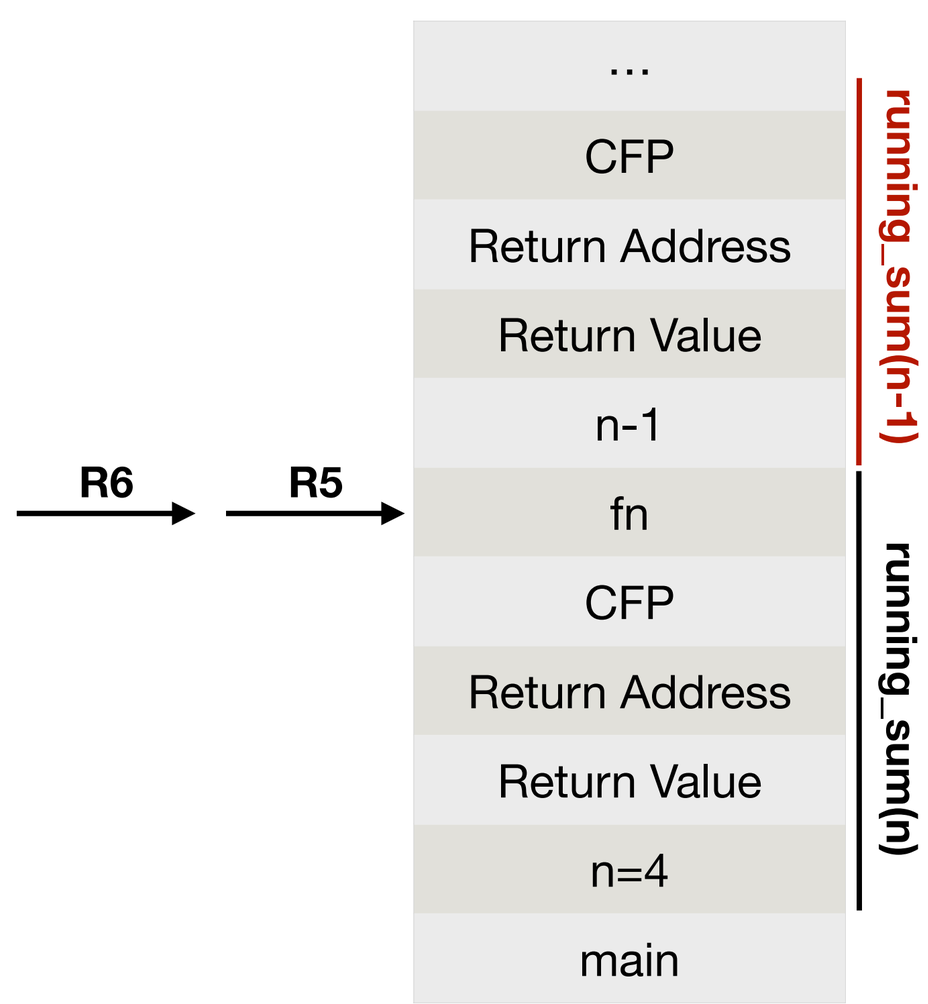
R2 | 3

```
;Recursive case
;Caller setup: push argument n-1 onto RTS
ADD R6, R6, #-1
STR R2, R6, #0 ; R2 = n - 1
JSR RUNNING ; call Running(n-1)
```

R0 | 10

R1 | 4

```
;Caller tear-down for Running(n-1)
;pop Running(n-1)'s return value to R0
LDR R0, R6, #0
ADD R6, R6, #1

;pop Running(n-1)'s argument
ADD R6, R6, #1

;calculate n + Running(n-1)
LDR R1, R5, #4
ADD R0, R1, R0
```



Recursion in LC3

```c
int running_sum(int n){
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}
```

```c
int main(void){
    int n = 4;
    int answer;
    answer = running_sum(n);
}
```

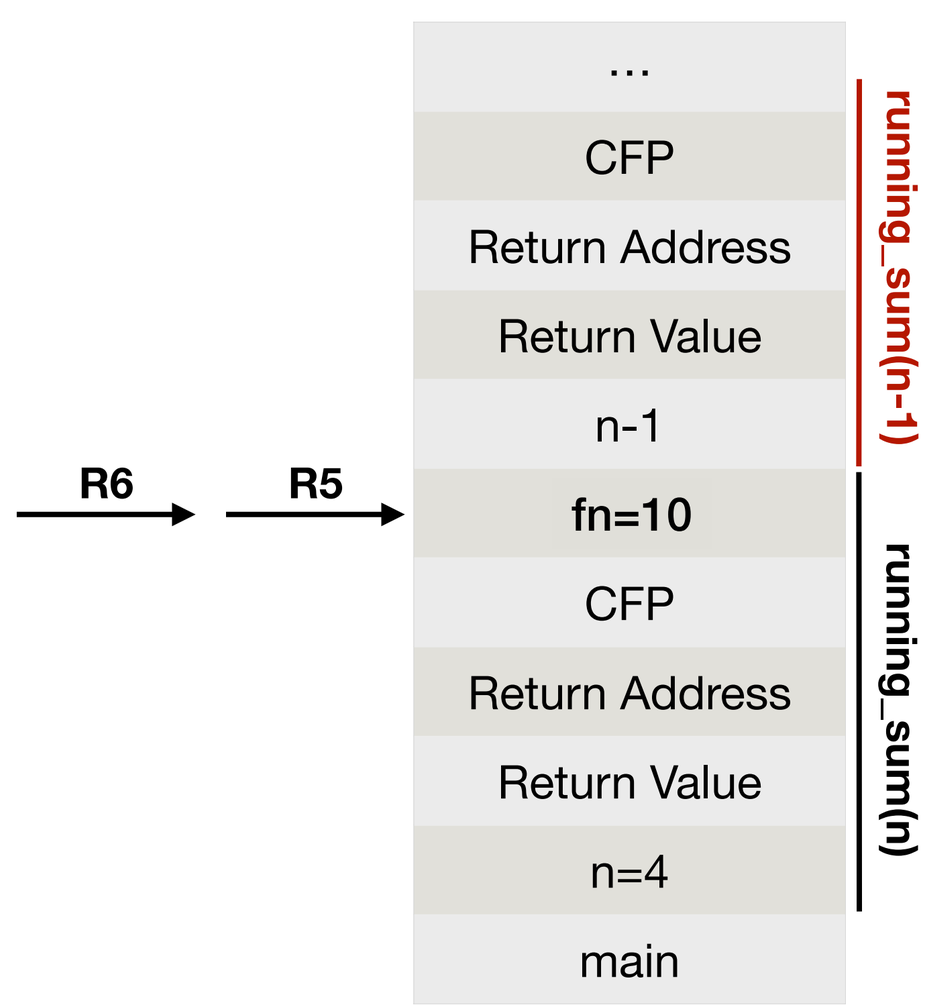
R2 | 3

```
;Recursive case
;Caller setup: push argument n-1 onto RTS
ADD R6, R6, #-1
STR R2, R6, #0 ; R2 = n - 1
JSR RUNNING ; call Running(n-1)
```

R0 | 10

R1 | 4

```
;Caller tear-down for Running(n-1)
;pop Running(n-1)'s return value to R0
LDR R0, R6, #0
ADD R6, R6, #1

;pop Running(n-1)'s argument
ADD R6, R6, #1

;calculate n + Running(n-1)
LDR R1, R5, #4
ADD R0, R1, R0
STR R0, R5, #0 ;store result in fn
```



Recursion in LC3

```c
int running_sum(int n){
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}
```

```c
int main(void){
    int n = 4;
    int answer;
    answer = running_sum(n);
}
```

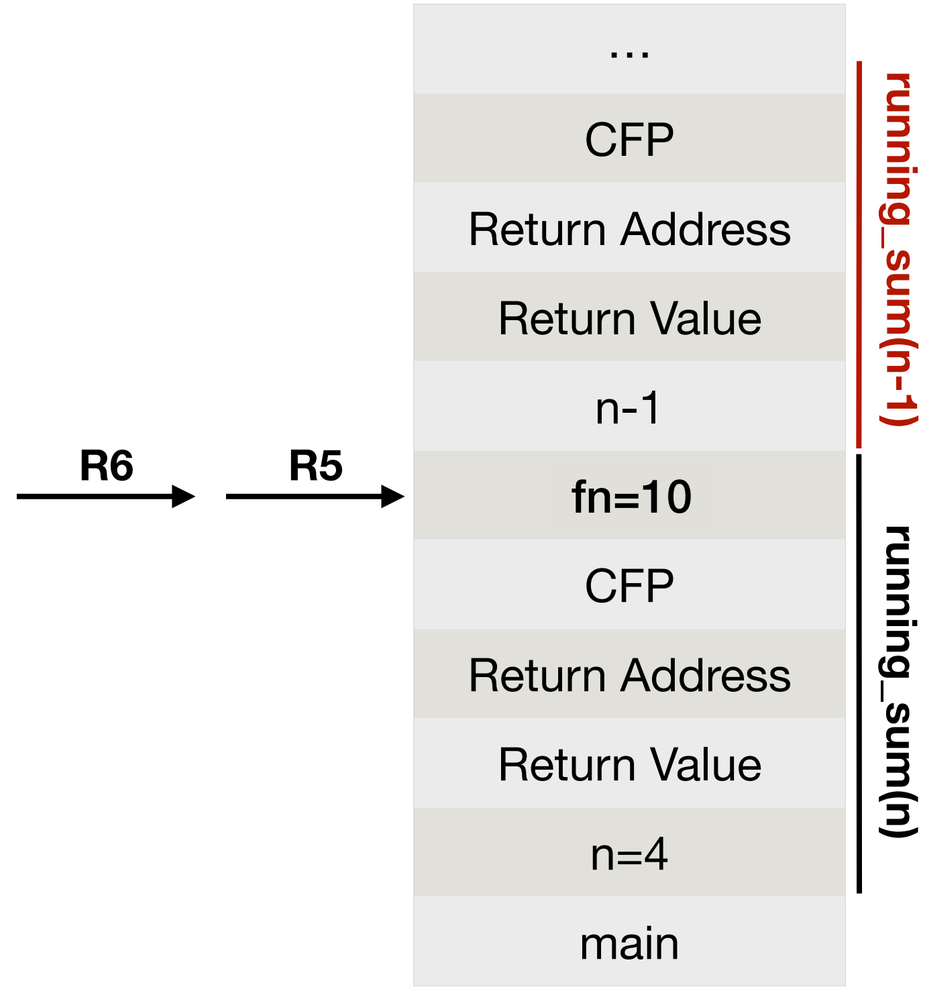
R2 | 3

```
;Recursive case
;Caller setup: push argument n-1 onto RTS
ADD R6, R6, #-1
STR R2, R6, #0 ; R2 = n - 1
JSR RUNNING ; call Running(n-1)
```

R0 | 10

R1 | 4

```
;Caller tear-down for Running(n-1)
;pop Running(n-1)'s return value to R0
LDR R0, R6, #0
ADD R6, R6, #1

;pop Running(n-1)'s argument
ADD R6, R6, #1

;calculate n + Running(n-1)
LDR R1, R5, #4
ADD R0, R1, R0
STR R0, R5, #0 ;store result in fn

;ready to return
```



Recursion in LC3

```c
int running_sum(int n){
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}
```

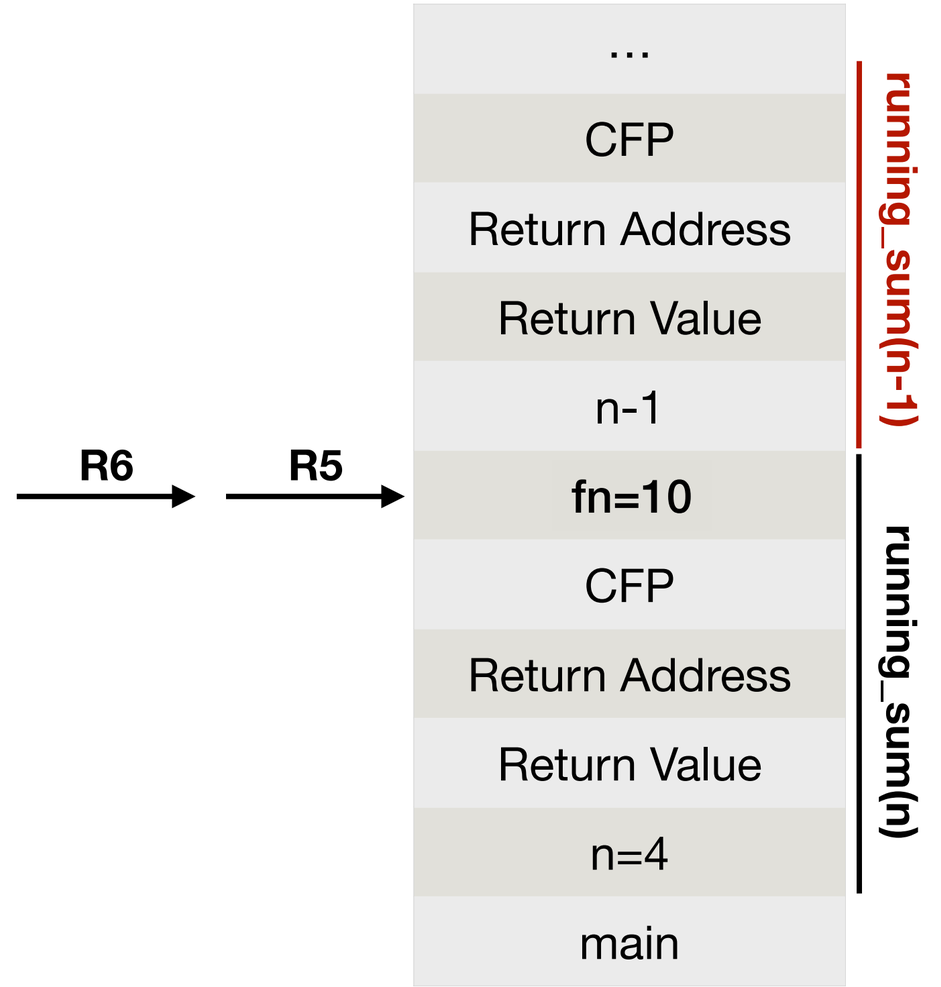
```c
int main(void){
    int n = 4;
    int answer;
    answer = running_sum(n);
}
```

R2 | 3

```
;Recursive case
;Caller setup: push argument n-1 onto RTS
ADD R6, R6, #-1
STR R2, R6, #0 ; R2 = n - 1
JSR RUNNING ; call Running(n-1)



;Caller tear-down for Running(n-1)
;pop Running(n-1)'s return value to R0
LDR R0, R6, #0
ADD R6, R6, #1

;pop Running(n-1)'s argument
ADD R6, R6, #1

;calculate n + Running(n-1)
LDR R1, R5, #4
ADD R0, R1, R0
STR R0, R5, #0 ;store result in fn

;ready to return
BRnzp RETURN
```

R0 | 10

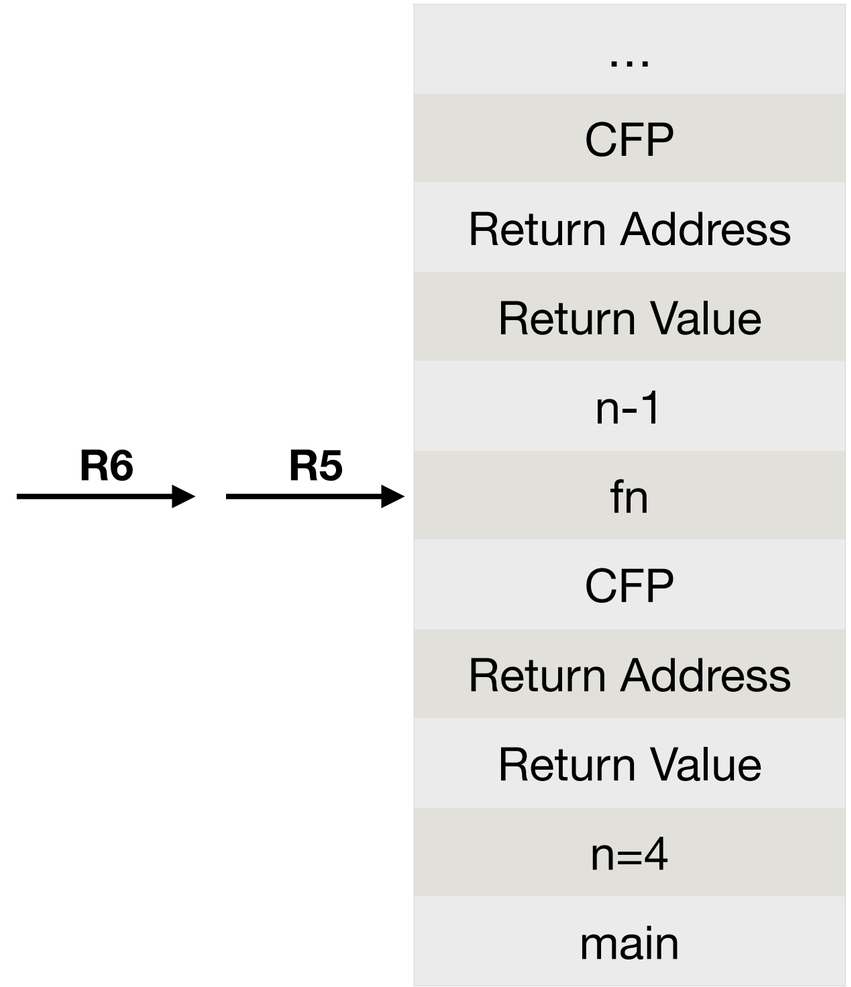
R1 | 4



Recursion in LC3

```c
int running_sum(int n){
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}
```

```c
int main(void){
    int n = 4;
    int answer;
    answer = running_sum(n);
}
```

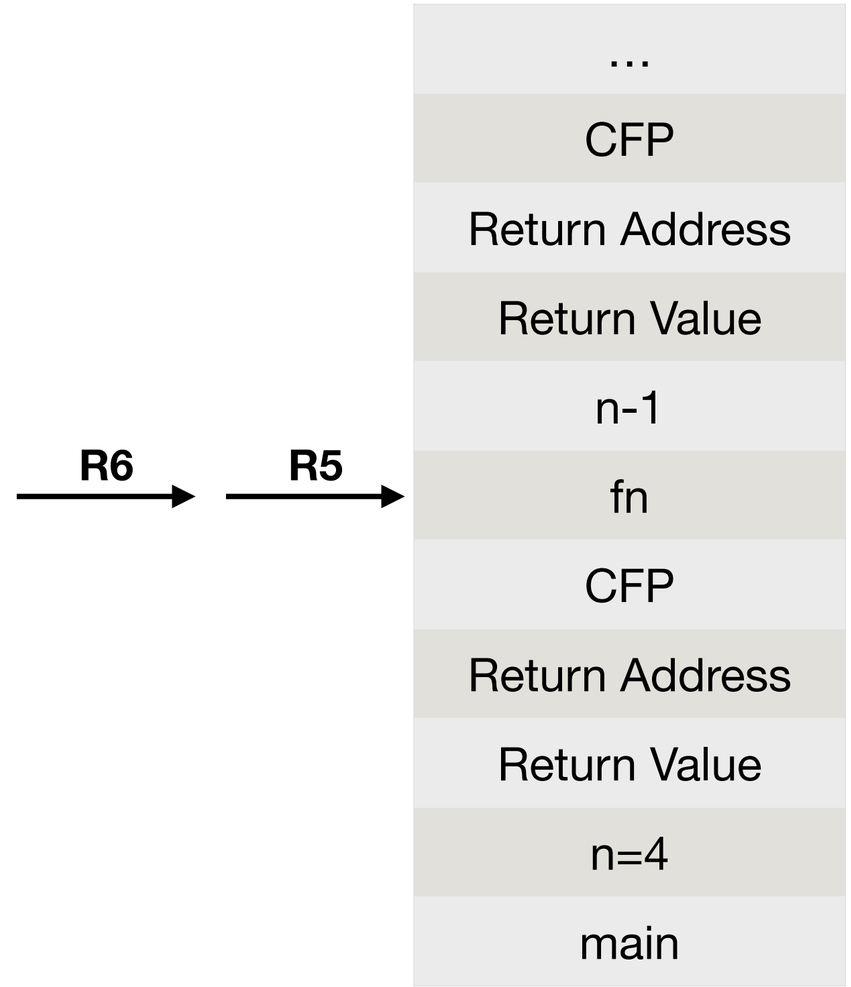


```c
int running_sum(int n){
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}
```

Recursion in LC3

```c
int main(void){
    int n = 4;
    int answer;
    answer = running_sum(n);
}
```

```
BASE_CASE
AND R2, R2, #0
ADD R2, R2, #1
```

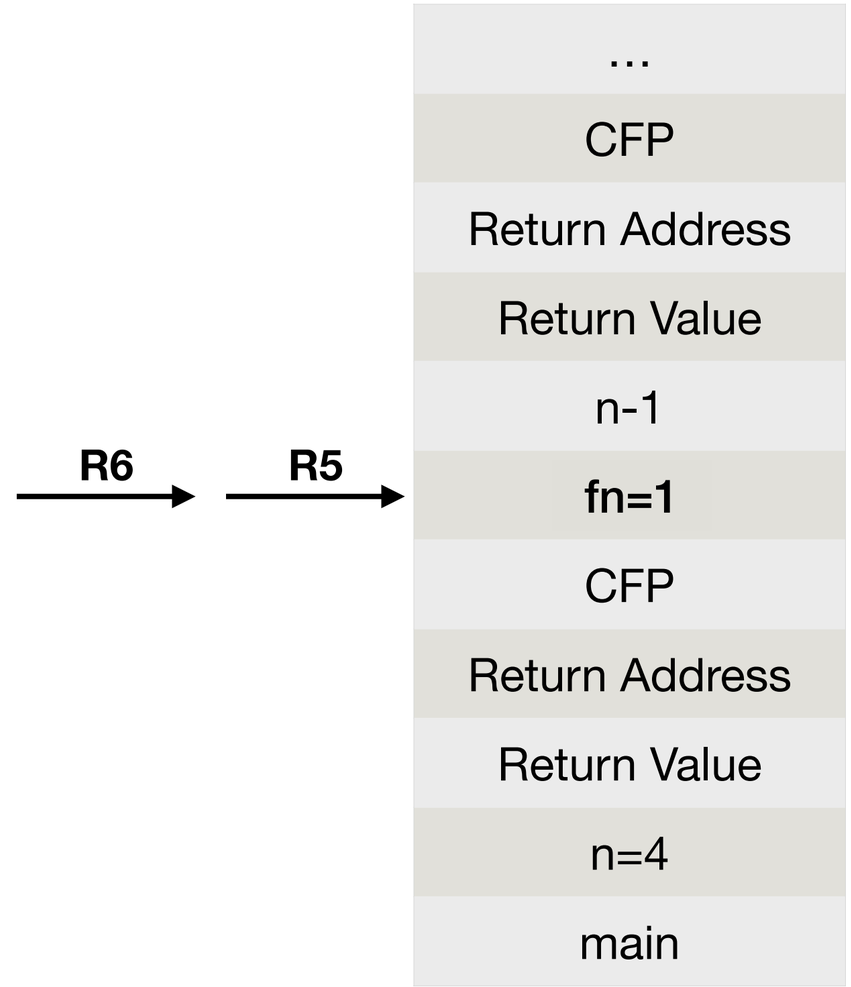


```c
int running_sum(int n){
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}
```

Recursion in LC3

```c
int main(void){
    int n = 4;
    int answer;
    answer = running_sum(n);
}
```

```
BASE_CASE
AND R2, R2, #0
ADD R2, R2, #1
STR R2, R5, #0 ;set fn = 1
```

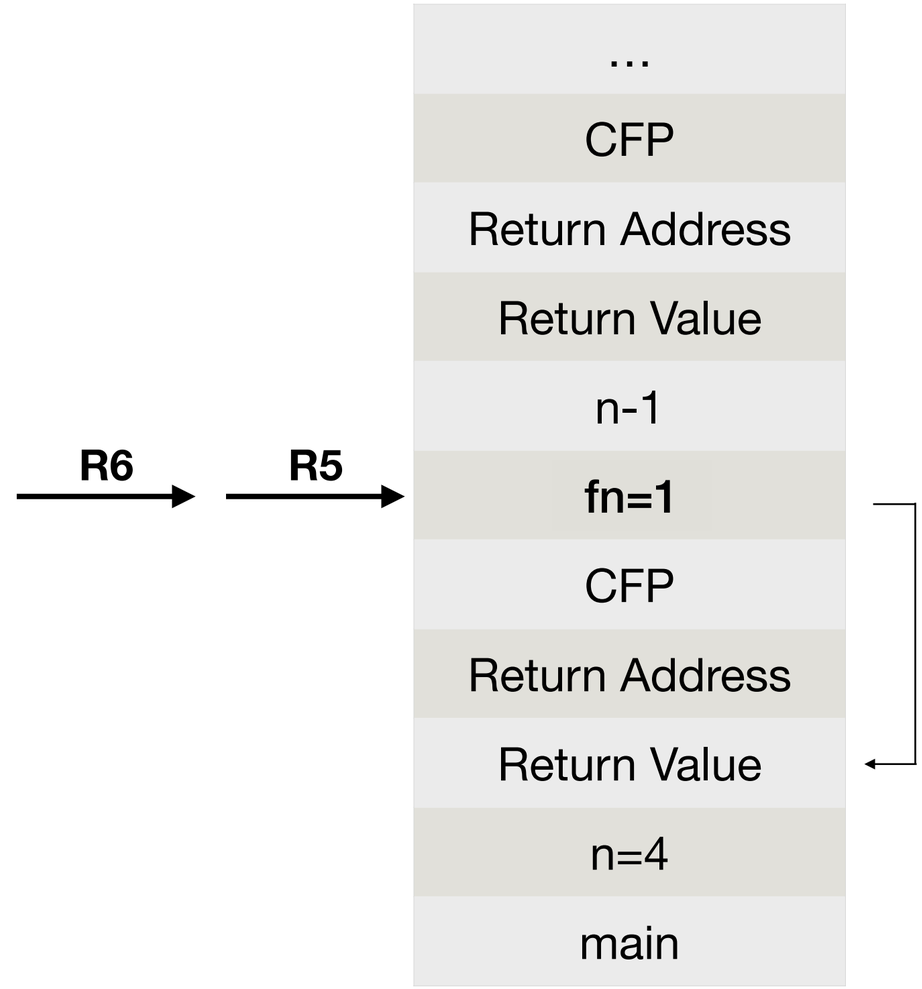


```c
int running_sum(int n){
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}
```

Recursion in LC3

```c
int main(void){
    int n = 4;
    int answer;
    answer = running_sum(n);
}
```

```
BASE_CASE
AND R2, R2, #0
ADD R2, R2, #1
STR R2, R5, #0 ;set fn = 1

RETURN
;set return value
LDR R0, R5, #0
STR R0, R5, #3
```

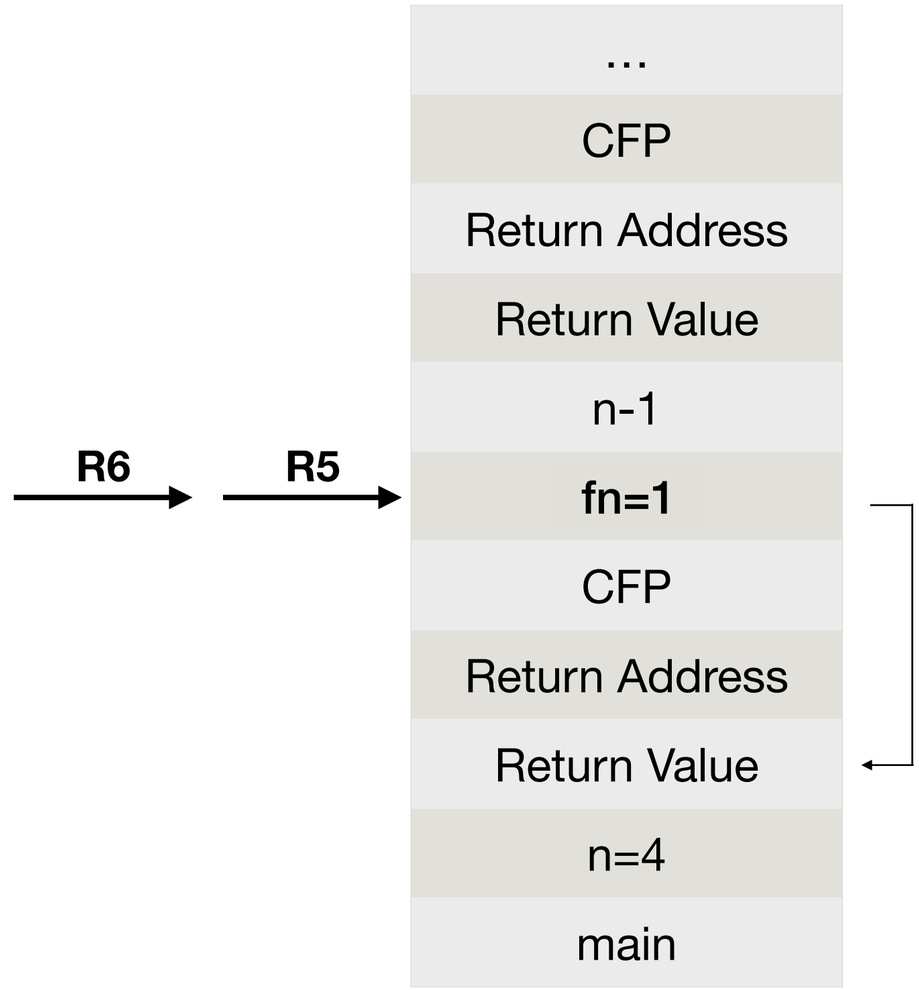


```c
int running_sum(int n){
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}
```

```c
int main(void){
    int n = 4;
    int answer;
    answer = running_sum(n);
}
```

Recursion in LC3

```
BASE_CASE
AND R2, R2, #0
ADD R2, R2, #1
STR R2, R5, #0 ;set fn = 1

RETURN
;set return value
LDR R0, R5, #0
STR R0, R5, #3

;callee tear-down of Running(n)'s activation record
```

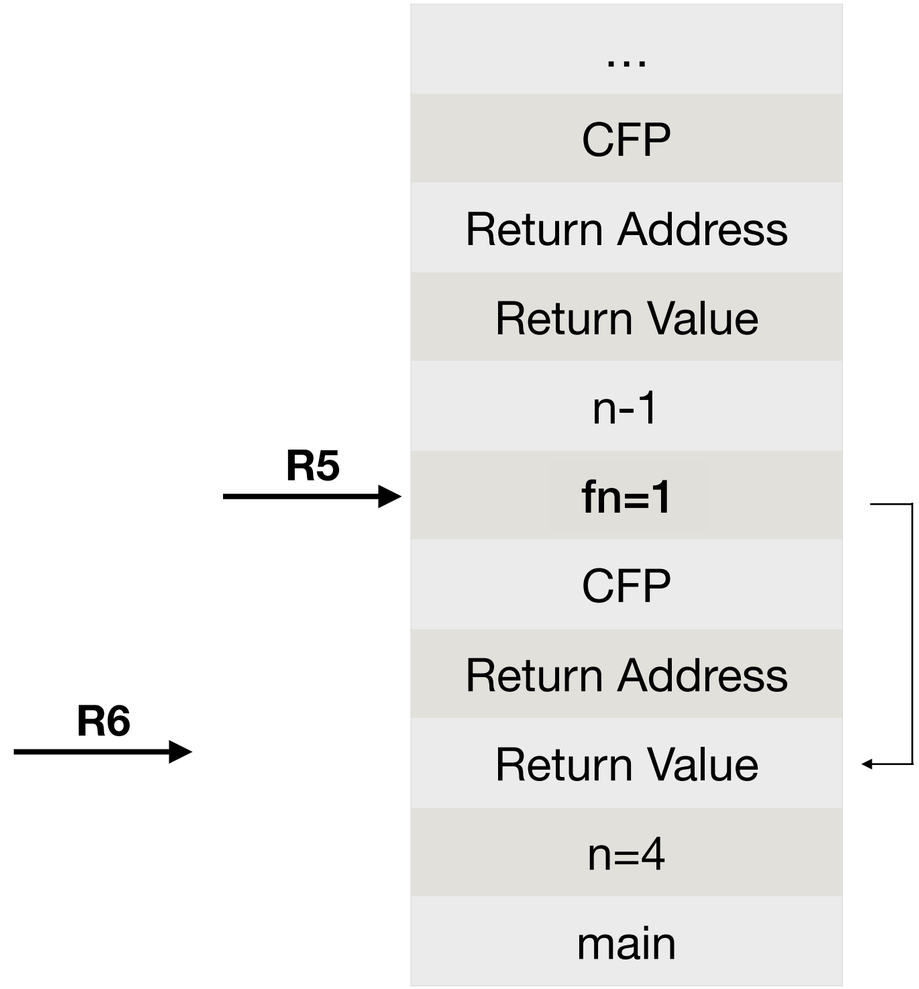


```c
int running_sum(int n){
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}
```

```c
int main(void){
    int n = 4;
    int answer;
    answer = running_sum(n);
}
```

Recursion in LC3

```
BASE_CASE
AND R2, R2, #0
ADD R2, R2, #1
STR R2, R5, #0 ;set fn = 1

RETURN
;set return value
LDR R0, R5, #0
STR R0, R5, #3

;callee tear-down of Running(n)'s activation record
ADD R6, R5, #3 ;pop local variables
```

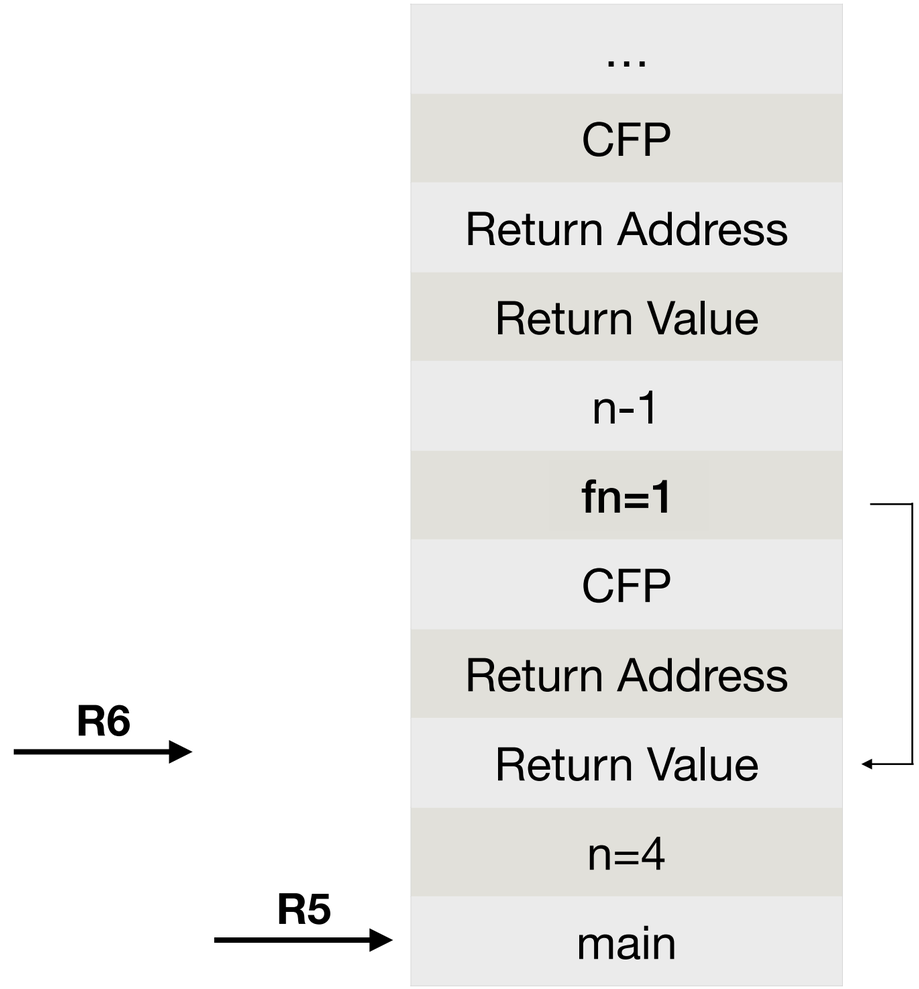


```c
int running_sum(int n){
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}
```

Recursion in LC3

```c
int main(void){
    int n = 4;
    int answer;
    answer = running_sum(n);
}
```

```
BASE_CASE
AND R2, R2, #0
ADD R2, R2, #1
STR R2, R5, #0 ;set fn = 1

RETURN
;set return value
LDR R0, R5, #0
STR R0, R5, #3

;callee tear-down of Running(n)'s activation record
ADD R6, R5, #3 ;pop local variables

;restore caller's frame pointer and return address
LDR R5, R6, #-2
```

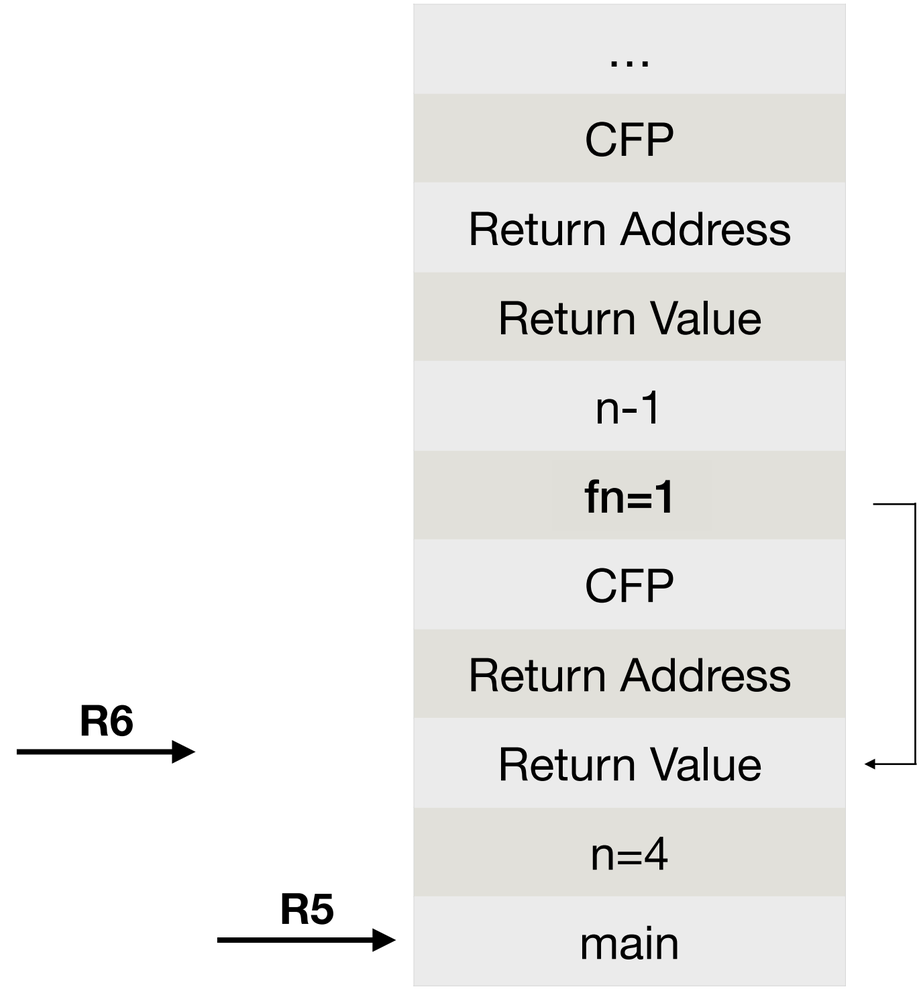


```c
int running_sum(int n){
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}
```

```c
int main(void){
    int n = 4;
    int answer;
    answer = running_sum(n);
}
```

Recursion in LC3

```
BASE_CASE
AND R2, R2, #0
ADD R2, R2, #1
STR R2, R5, #0 ;set fn = 1

RETURN
;set return value
LDR R0, R5, #0
STR R0, R5, #3

;callee tear-down of Running(n)'s activation record
ADD R6, R5, #3 ;pop local variables

;restore caller's frame pointer and return address
LDR R5, R6, #-2
LDR R7, R6, #-1
```

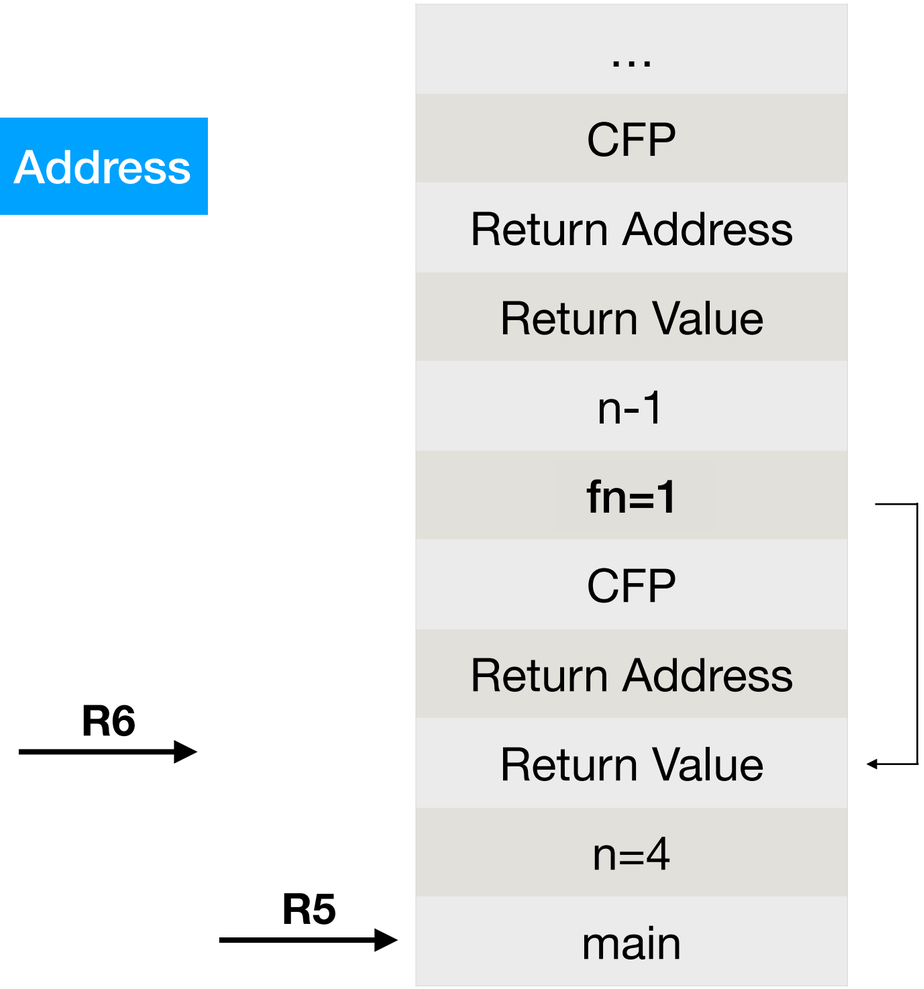


```c
int running_sum(int n){
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}
```

Recursion in LC3

```c
int main(void){
    int n = 4;
    int answer;
    answer = running_sum(n);
}
```

```
BASE_CASE
AND R2, R2, #0
ADD R2, R2, #1
STR R2, R5, #0 ;set fn = 1

RETURN
;set return value
LDR R0, R5, #0
STR R0, R5, #3

;callee tear-down of Running(n)'s activation record
ADD R6, R5, #3 ;pop local variables

;restore caller's frame pointer and return address
LDR R5, R6, #-2
LDR R7, R6, #-1
```

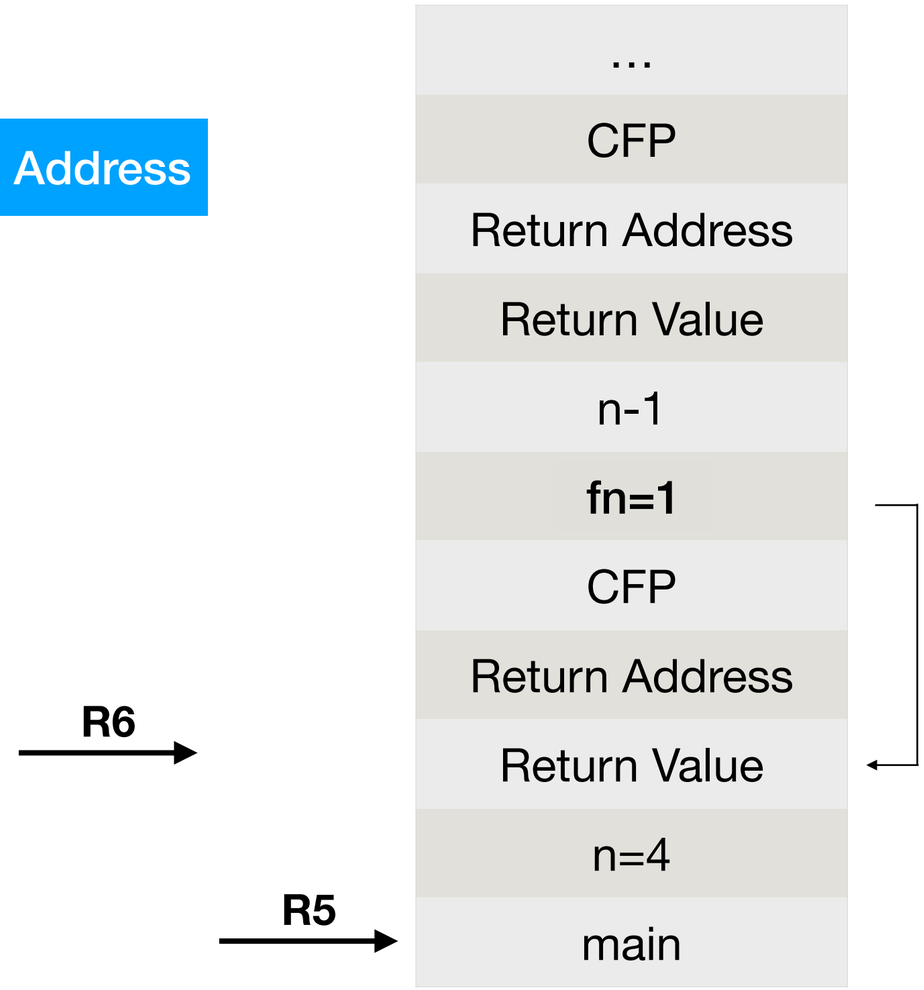


```c
int running_sum(int n){
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}
```

Recursion in LC3

```c
int main(void){
    int n = 4;
    int answer;
    answer = running_sum(n);
}
```

```
BASE_CASE
AND R2, R2, #0
ADD R2, R2, #1
STR R2, R5, #0 ;set fn = 1

RETURN
;set return value
LDR R0, R5, #0
STR R0, R5, #3

;callee tear-down of Running(n)'s activation record
ADD R6, R5, #3 ;pop local variables

;restore caller's frame pointer and return address
LDR R5, R6, #-2
LDR R7, R6, #-1


;return to caller
```



```c
int running_sum(int n){
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    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
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Recursion in LC3

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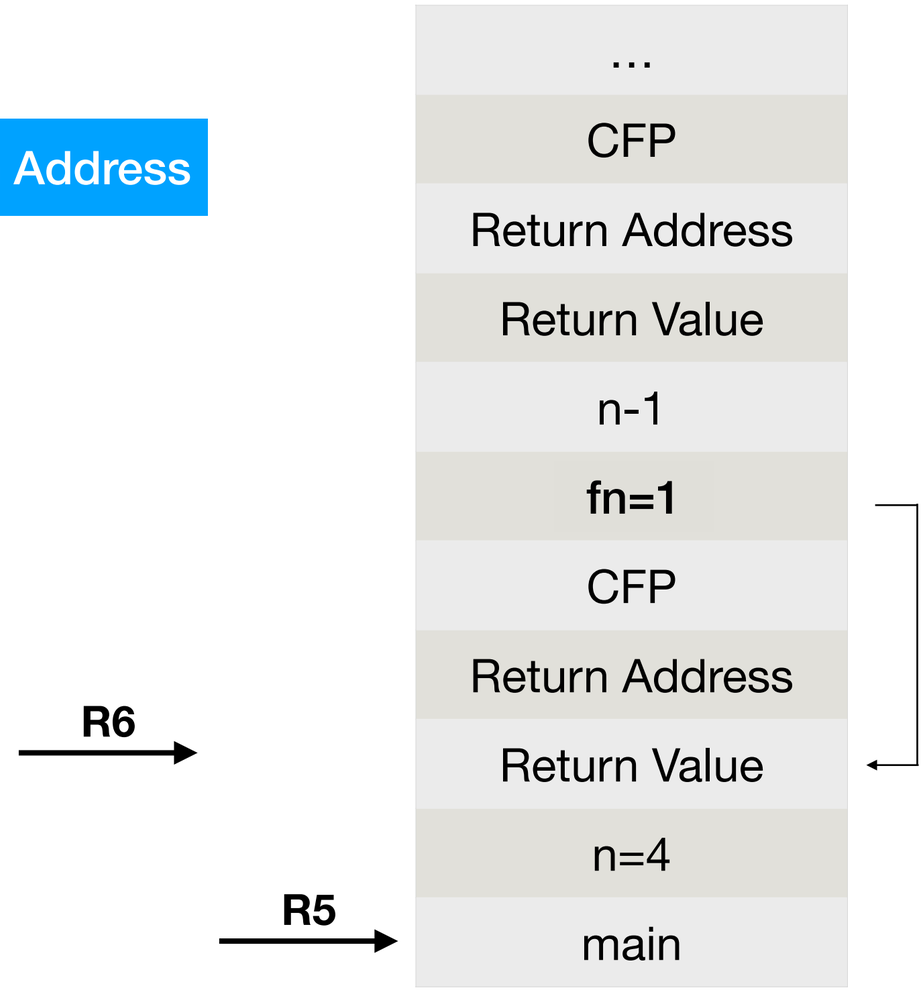
;return to caller
RET
```



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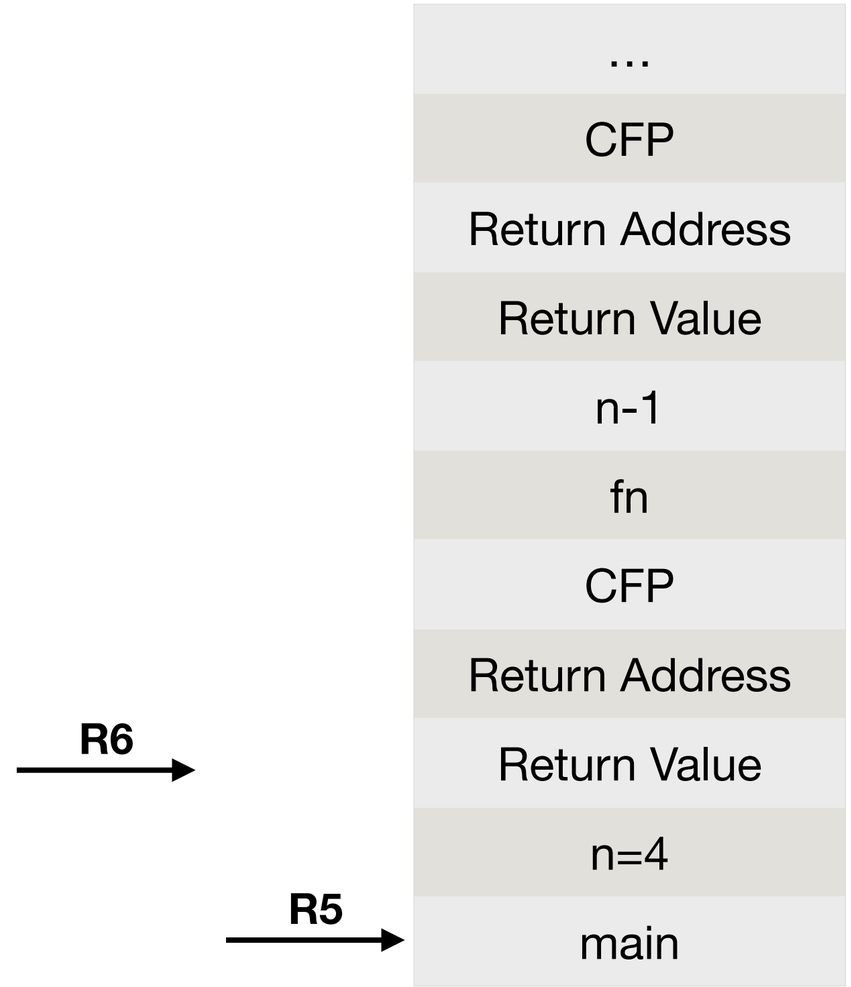
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}
```

Recursion in LC3

```c
int main(void){
    int n = 4;
    int answer;
    answer = running_sum(4);
}
```

```
;Caller stack Tear-down for Running(n)
```



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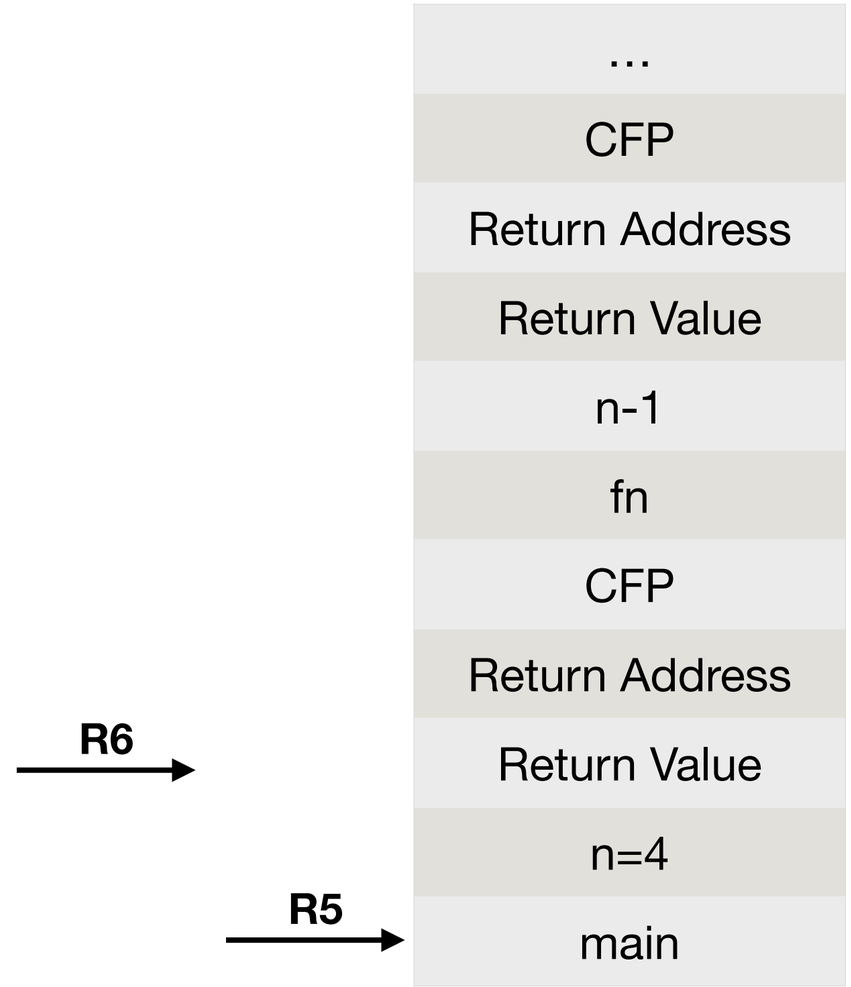
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}
```

Recursion in LC3

```c
int main(void){
    int n = 4;
    int answer;
    answer = running_sum(4);
}
```

```
;Caller stack Tear-down for Running(n)
LDR R0, R6, #0  ;copy return value to R0
```



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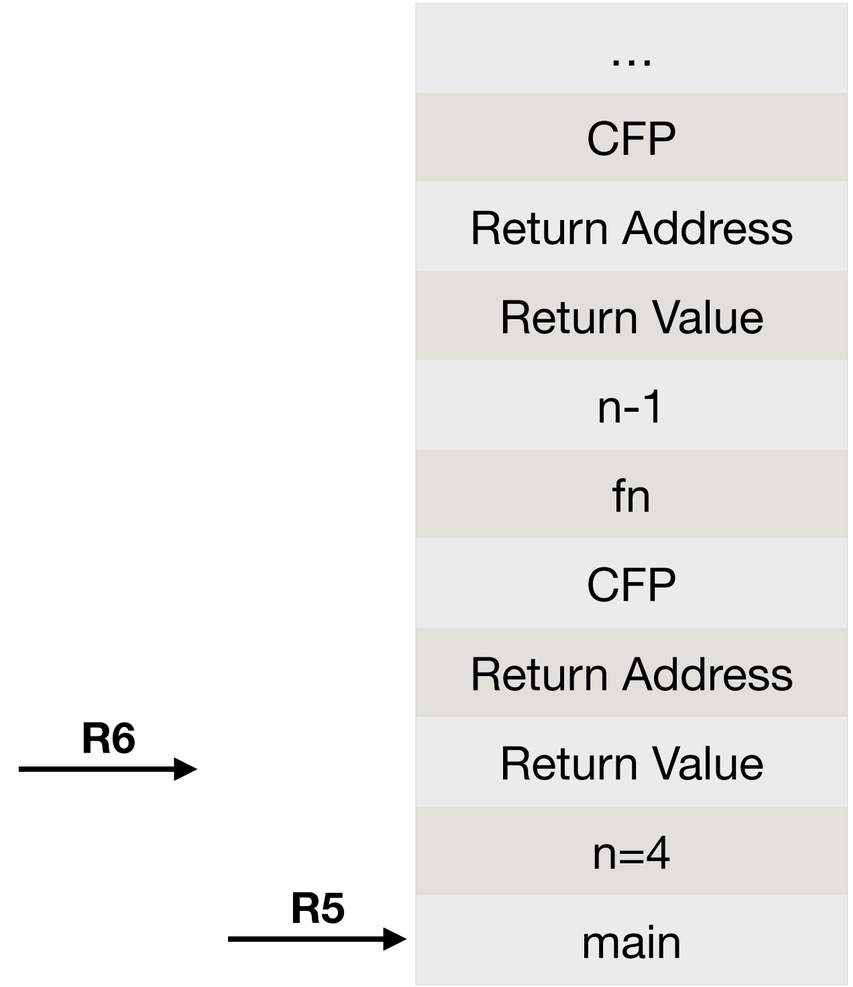
int running_sum(int n){
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}
```

Recursion in LC3

```c
int main(void){
    int n = 4;
    int answer;
    answer = running_sum(4);
}
```

```
;Caller stack Tear-down for Running(n)
LDR R0, R6, #0  ;copy return value to R0
STR R0, R5, #-1 ;save return value to answer
```

Inside main's activation frame, answer is the second local variable



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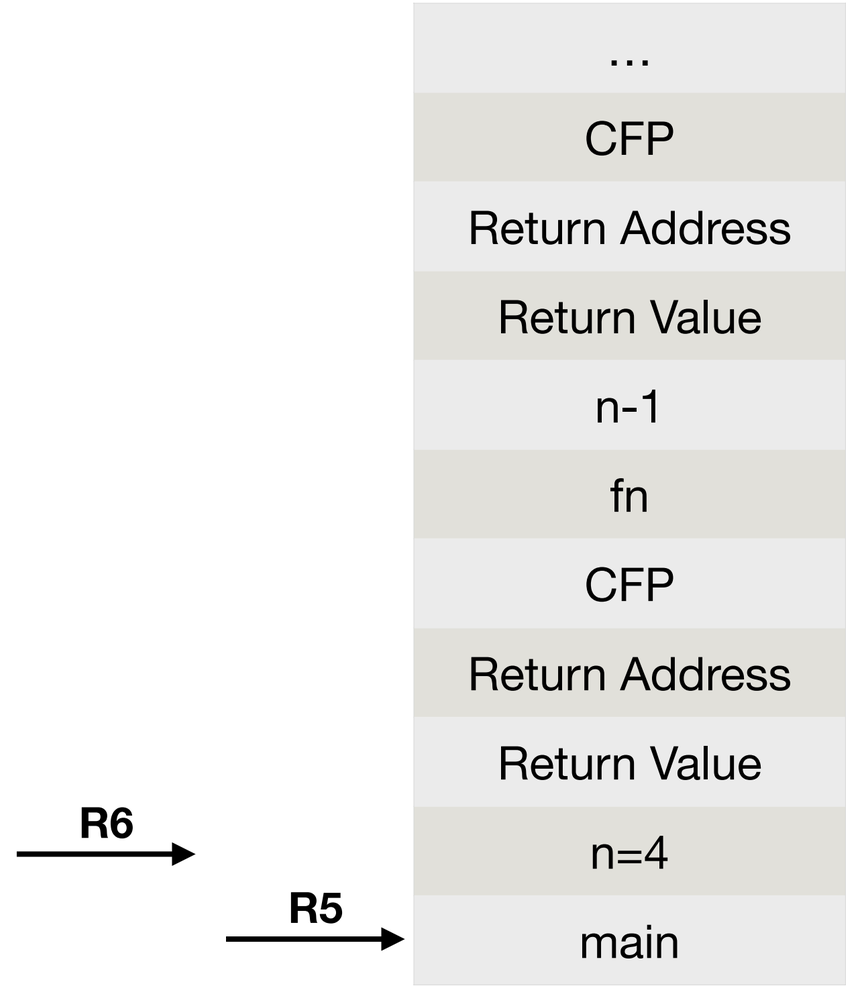
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}
```

Recursion in LC3

```c
int main(void){
    int n = 4;
    int answer;
    answer = running_sum(4);
}
```

```
;Caller stack Tear-down for Running(n)
LDR R0, R6, #0  ;copy return value to R0
STR R0, R5, #-1 ;save return value to answer
ADD R6, R6, #1  ;pop return value from stack
```

Inside main's activation frame, answer is the second local variable



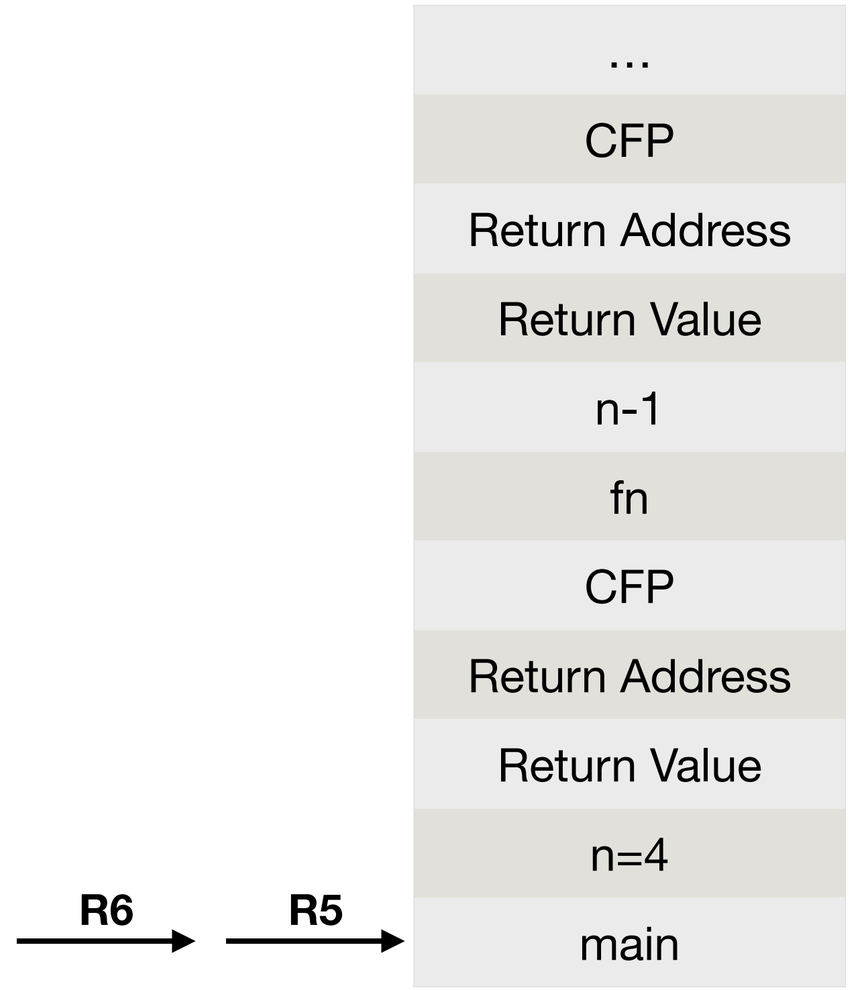
```c
int running_sum(int n){
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}
```

```c
int main(void){
    int n = 4;
    int answer;
    answer = running_sum(4);
}
```

Recursion in LC3

```
;Caller stack Tear-down for Running(n)
LDR R0, R6, #0  ;copy return value to R0
STR R0, R5, #-1 ;save return value to answer
ADD R6, R6, #1  ;pop return value from stack
ADD R6, R6, #1  ;pop argument from stack
```

Inside main's activation frame, answer is the second local variable



```c
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    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}
```
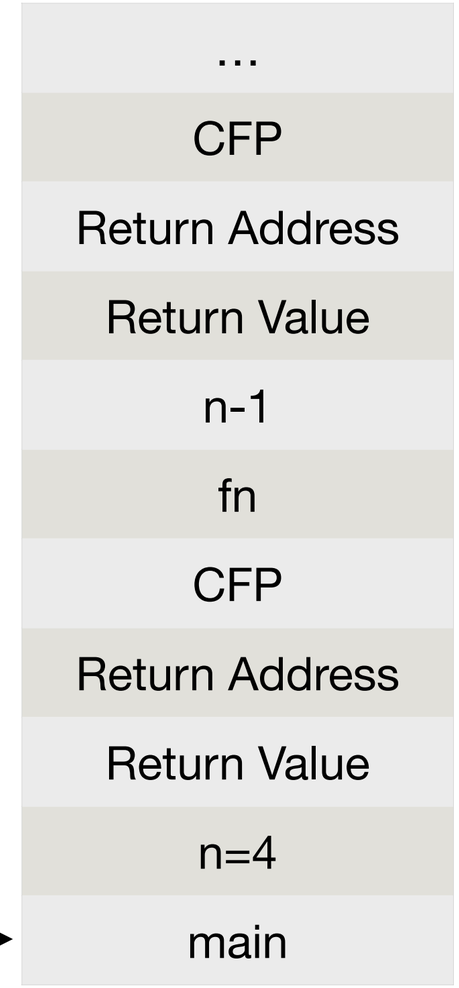

```c
int main(void){
    int n = 4;
    int answer;
    answer = running_sum(4);
}
```

Recursion in LC3

```
;Caller stack Tear-down for Running(n)
LDR R0, R6, #0  ;copy return value to R0
STR R0, R5, #-1 ;save return value to answer
ADD R6, R6, #1  ;pop return value from stack
ADD R6, R6, #1  ;pop argument from stack
```

Inside main's activation frame, answer is the second local variable



Back to where we started!

```c
int running_sum(int n){
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}
```

```c
int main(void){
    int n = 4;
    int answer;
    answer = running_sum(4);
}
```

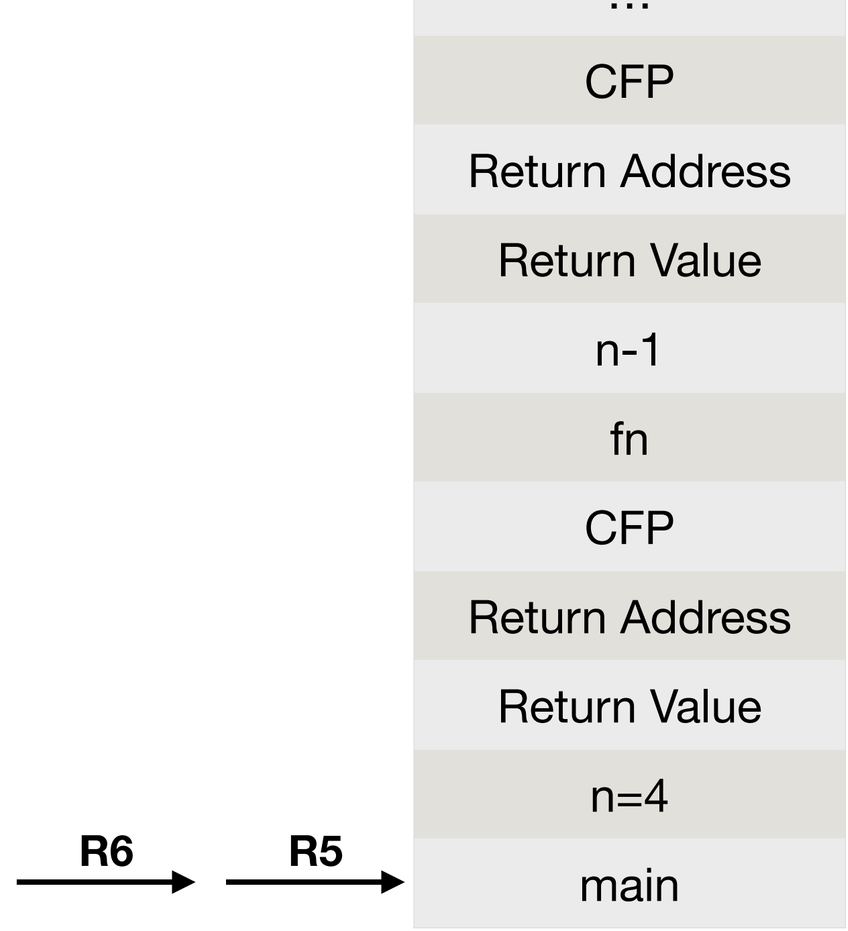
Recursion in LC3

```
;Caller stack Tear-down for Running(n)
LDR R0, R6, #0  ;copy return value to R0
STR R0, R5, #-1 ;save return value to answer
ADD R6, R6, #1  ;pop return value from stack
ADD R6, R6, #1  ;pop argument from stack
```

→ Inside main's activation frame, answer is the second local variable

Practice practice practice!

Back to where we started!



```c
int running_sum(int n){
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}
```

```c
int main(void){
    int n = 4;
    int answer;
    answer = running_sum(4);
}
```

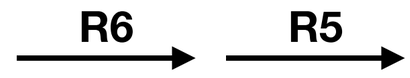
Recursion in LC3

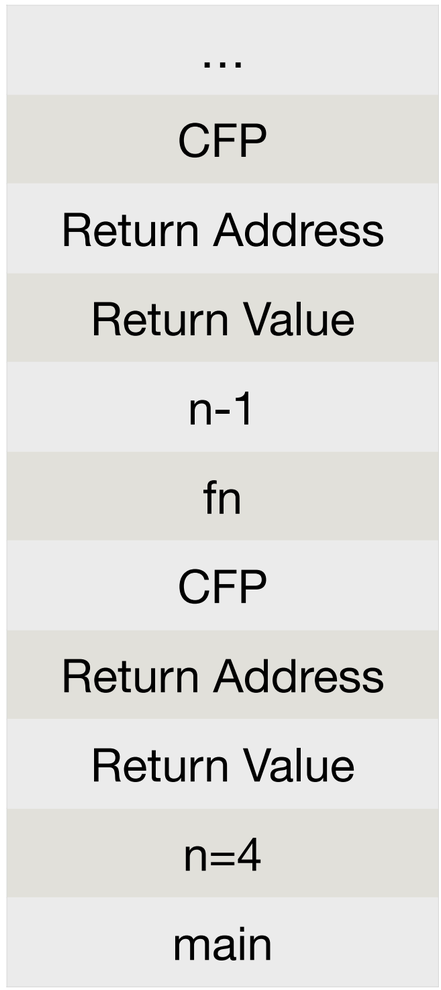
```
;Caller stack Tear-down for Running(n)
LDR R0, R6, #0  ;copy return value to R0
STR R0, R5, #-1 ;save return value to answer
ADD R6, R6, #1  ;pop return value from stack
ADD R6, R6, #1  ;pop argument from stack
```

→ Inside main's activation frame, answer is the second local variable

Practice practice practice!

Back to where we started!





[Full LC3 Code]

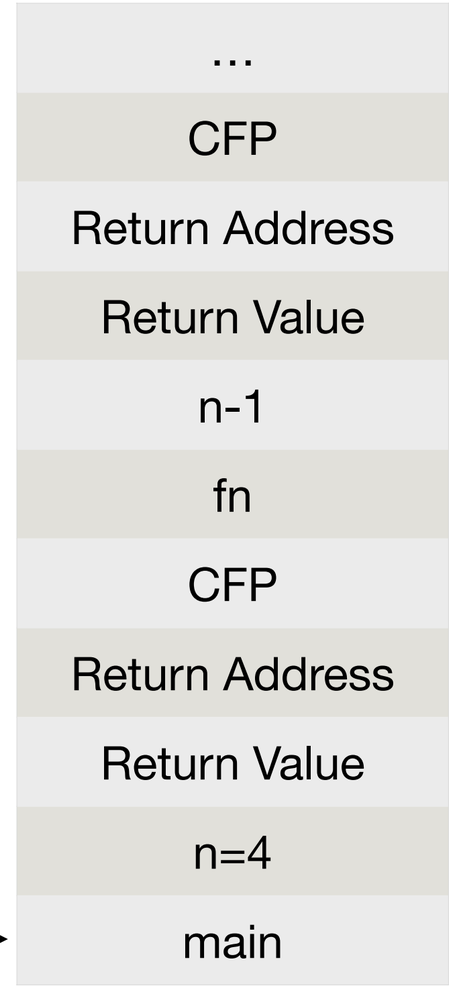
```c
int running_sum(int n){
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}
```

```c
int main(void){
    int n = 4;
    int answer;
    answer = running_sum(4);
}
```

Recursion in LC3

```
;Caller stack Tear-down for Running(n)
LDR R0, R6, #0  ;copy return value to R0
STR R0, R5, #-1 ;save return value to answer
ADD R6, R6, #1  ;pop return value from stack
ADD R6, R6, #1  ;pop argument from stack
```

Inside main's activation frame, answer is the second local variable



Practice practice practice!

Back to where we started!

[Github C to LC3 Steps]() [Full LC3 Code]()

Next time

- More problem solving with recursion.
  - A small chess problem
  - Solving a maze
- When is recursion good vs. bad?