ECE 220

Lecture x000D - 10/10

Recap

Recap

- Formal introduction to recursion

Recap

- Formal introduction to recursion
    - Factorial

Recap

- Formal introduction to recursion
    - Factorial
    - Binary search

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    - Towers of Hanoi

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- Today: More recursion & problem solving

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  - N - Queens problem
  - Maze solving

Recap

- Formal introduction to recursion
  - Factorial
  - Binary search
  - Towers of Hanoi
  - LC3 implementation
- Today: More recursion & problem solving
  - N - Queens problem
  - Maze solving
  - Exercise(s)

Quick review

```
int running_sum(int n){
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}

int main(void){
    int n = 4;
    running_sum(4);
}
```

Gitlab C2L3 steps

Quick review

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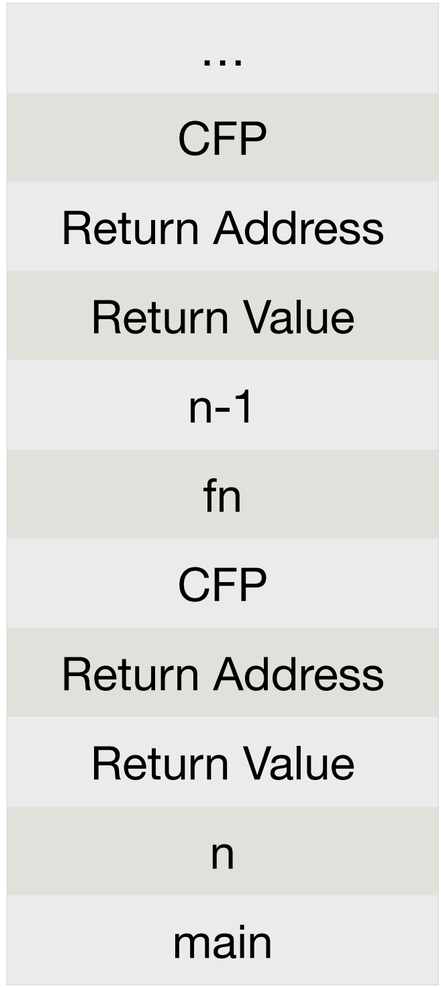

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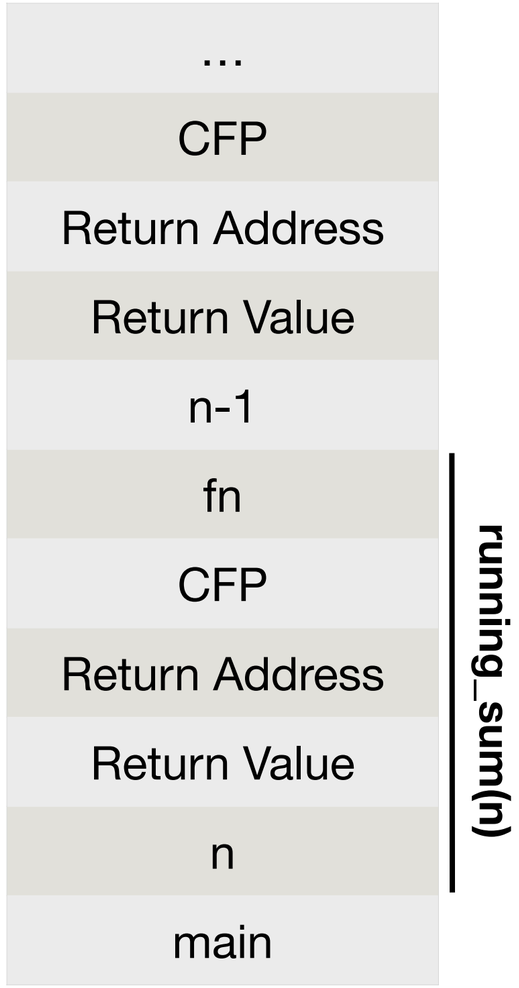

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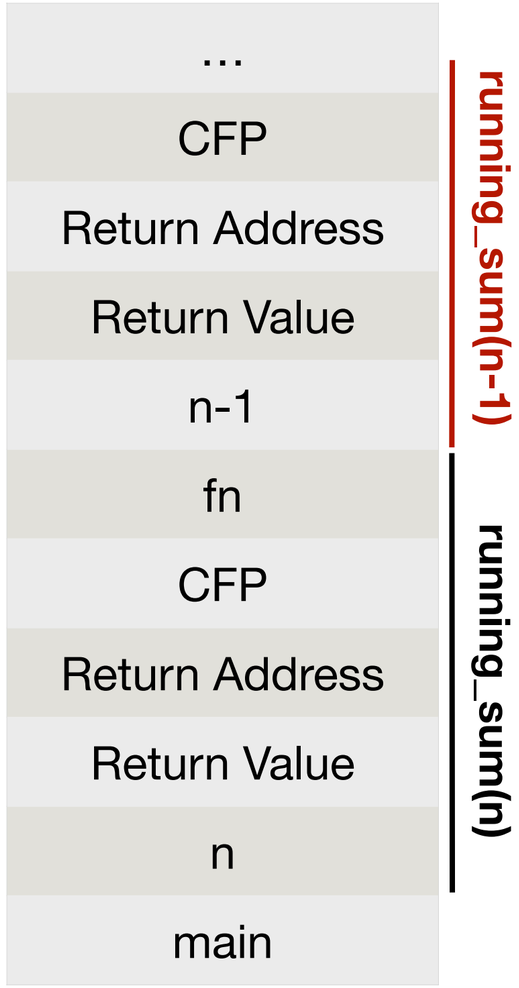

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    answer = running_sum(4);
}
```

```
;Caller set-up for Running(n)
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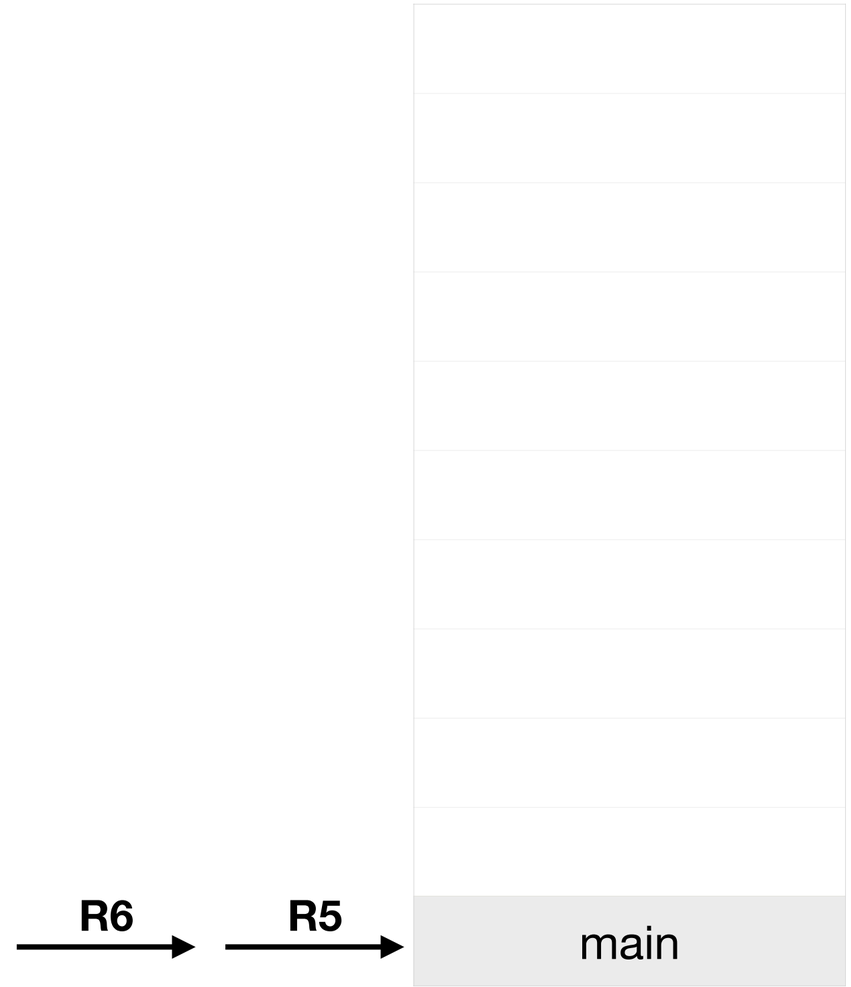

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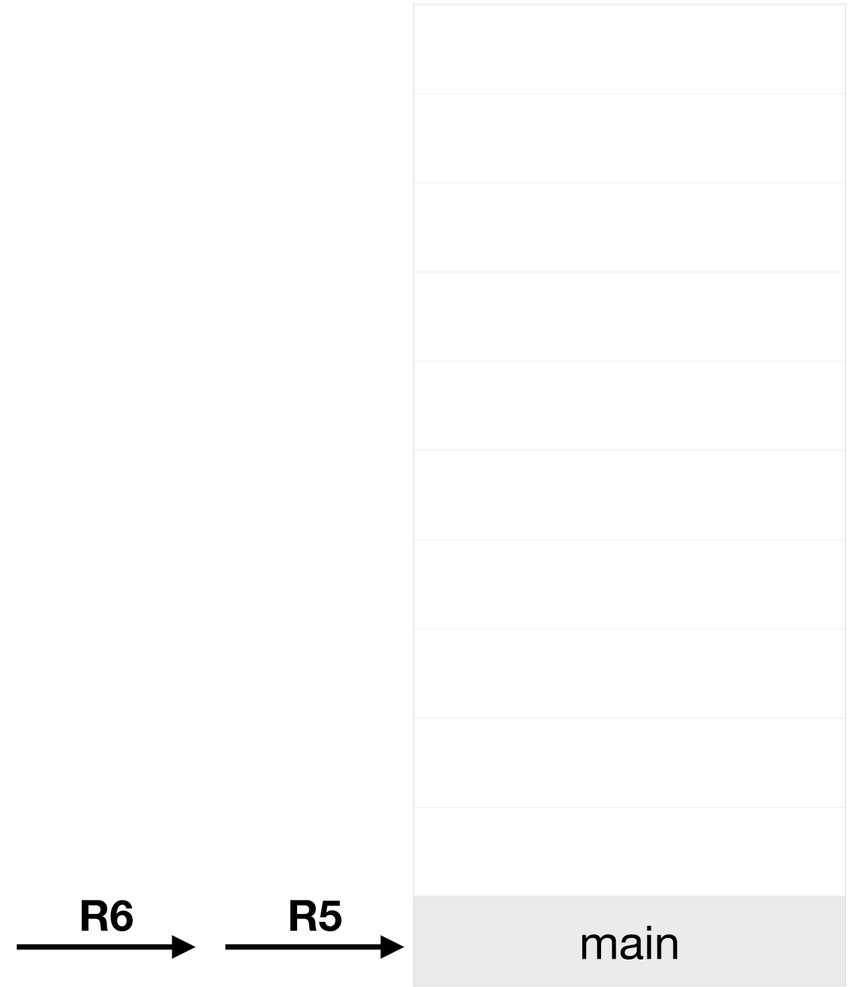

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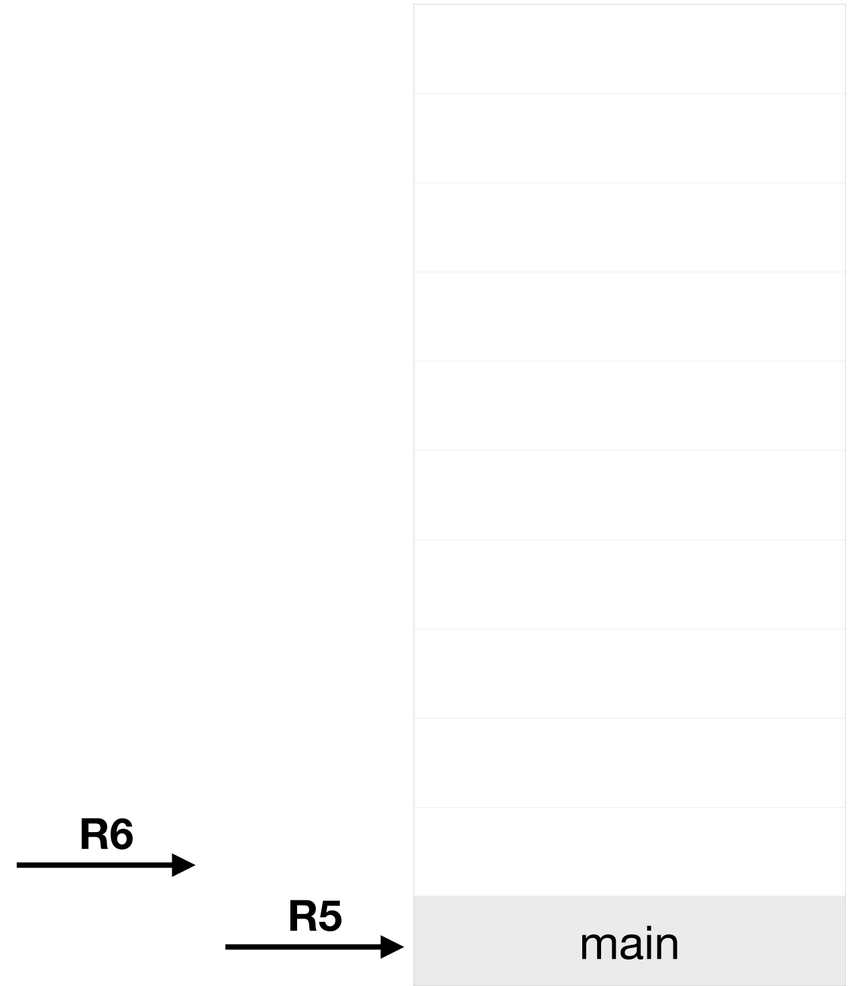

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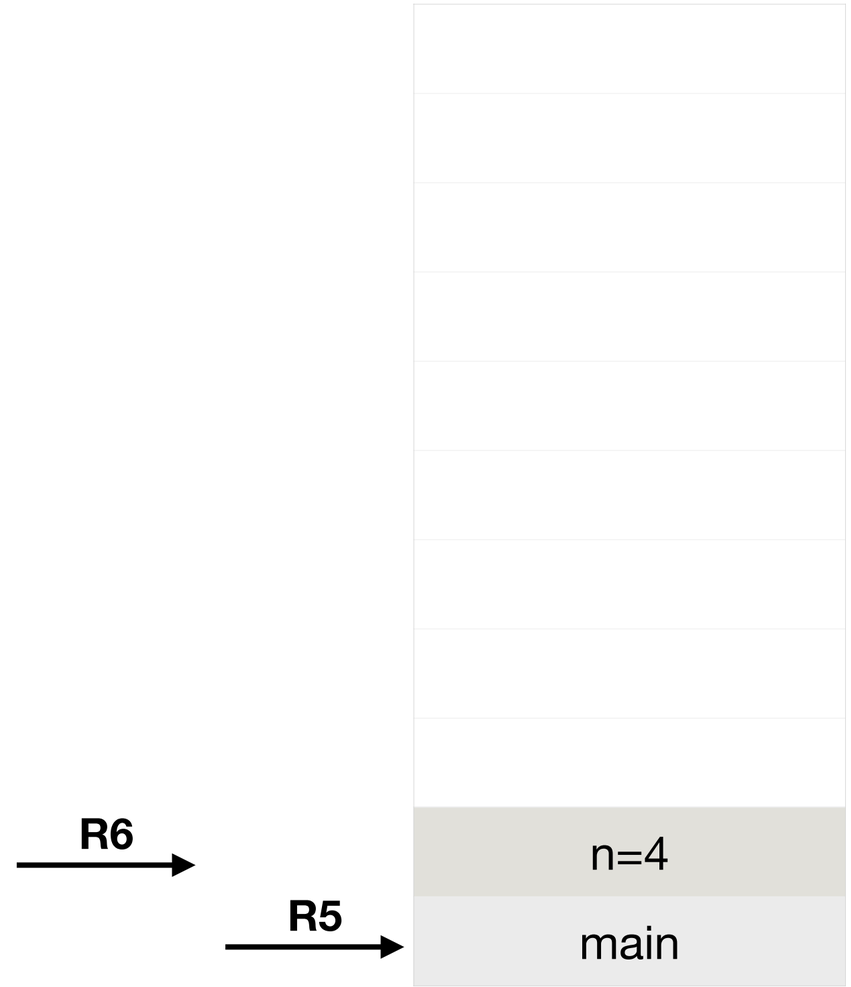

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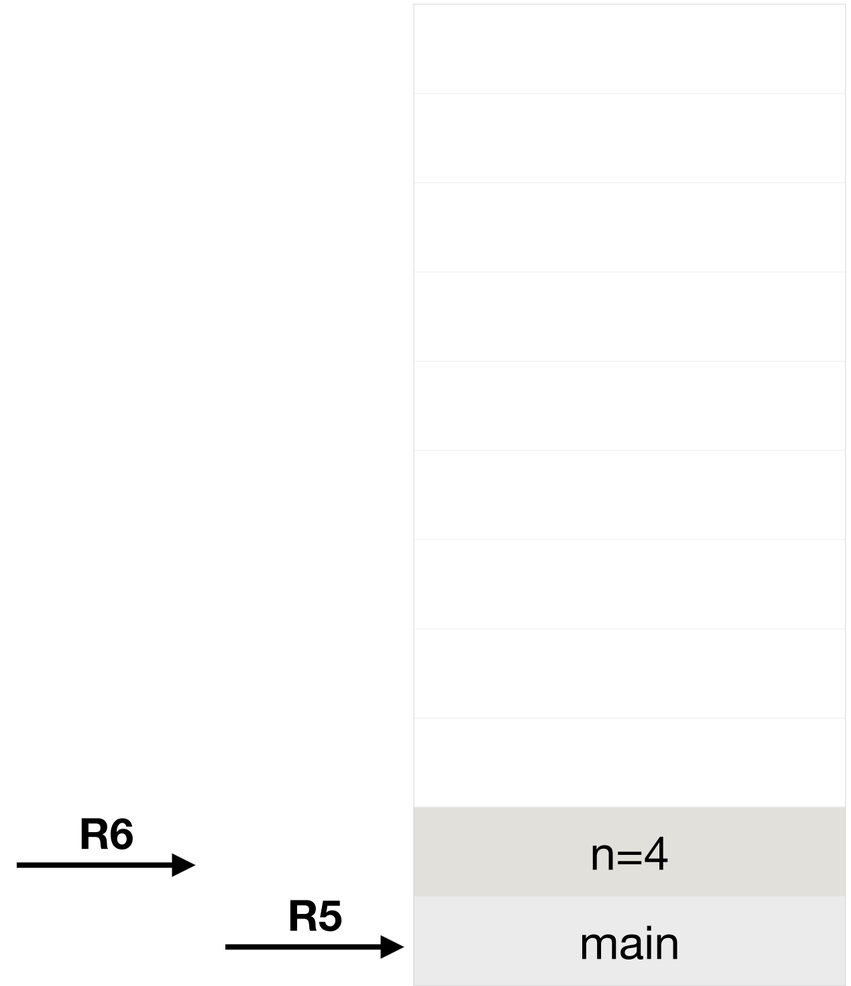

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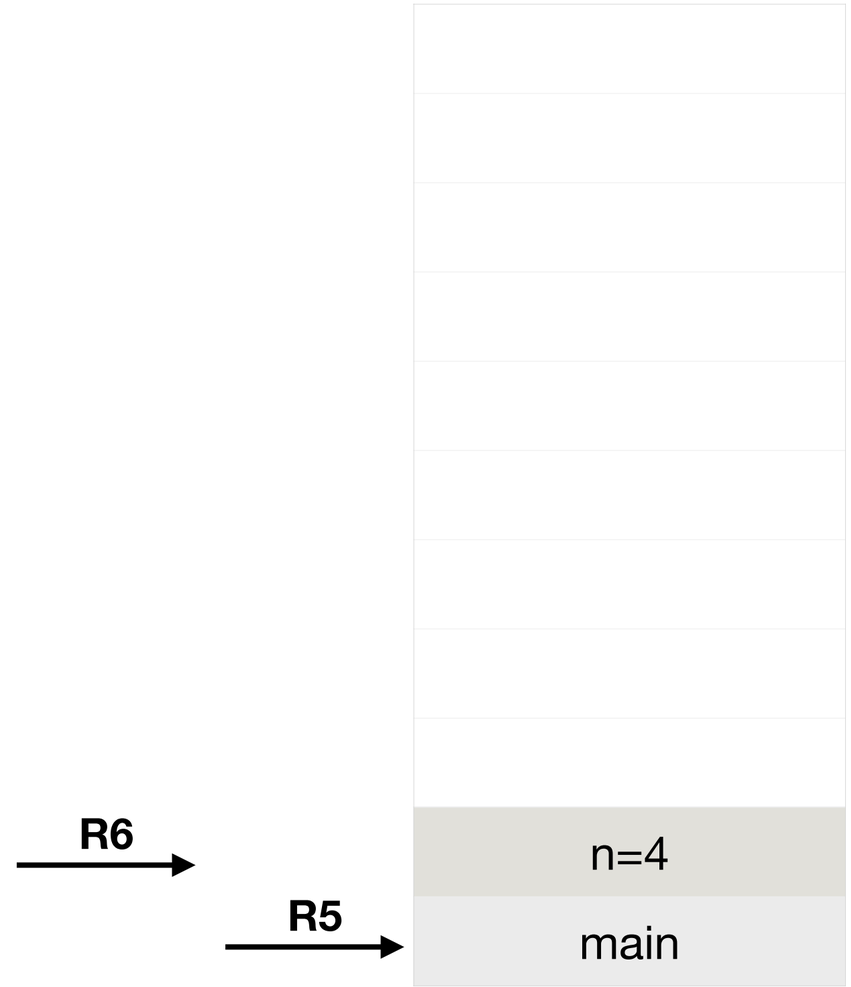

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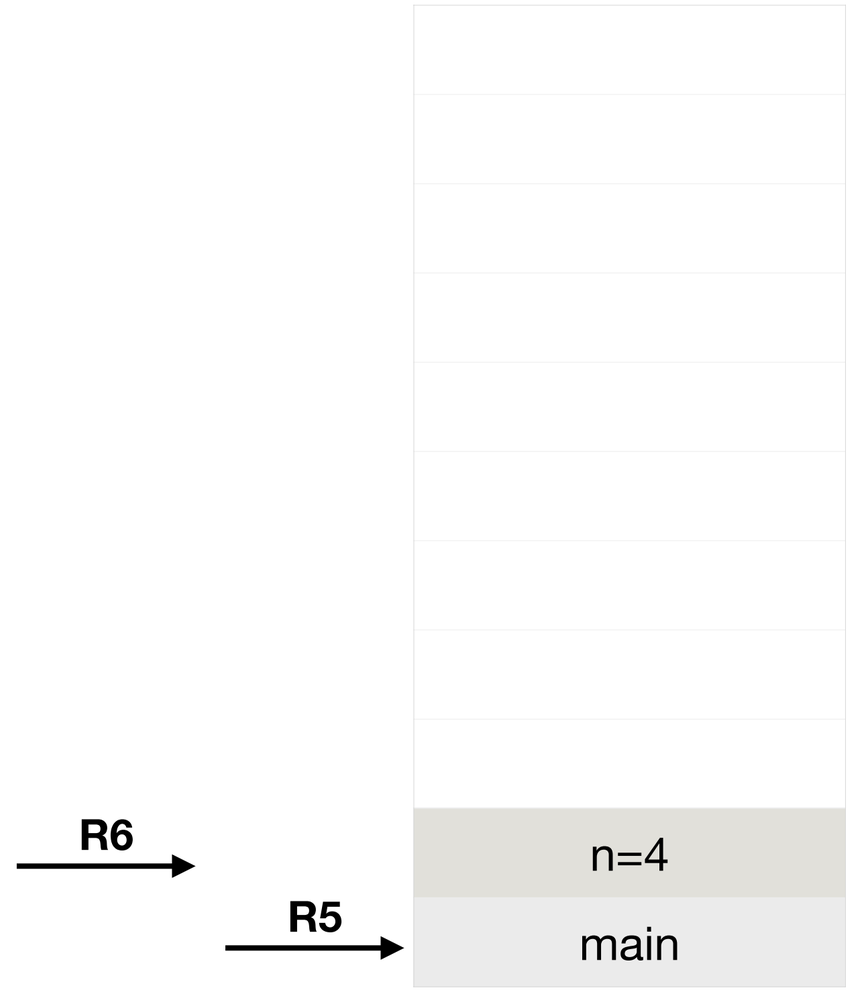

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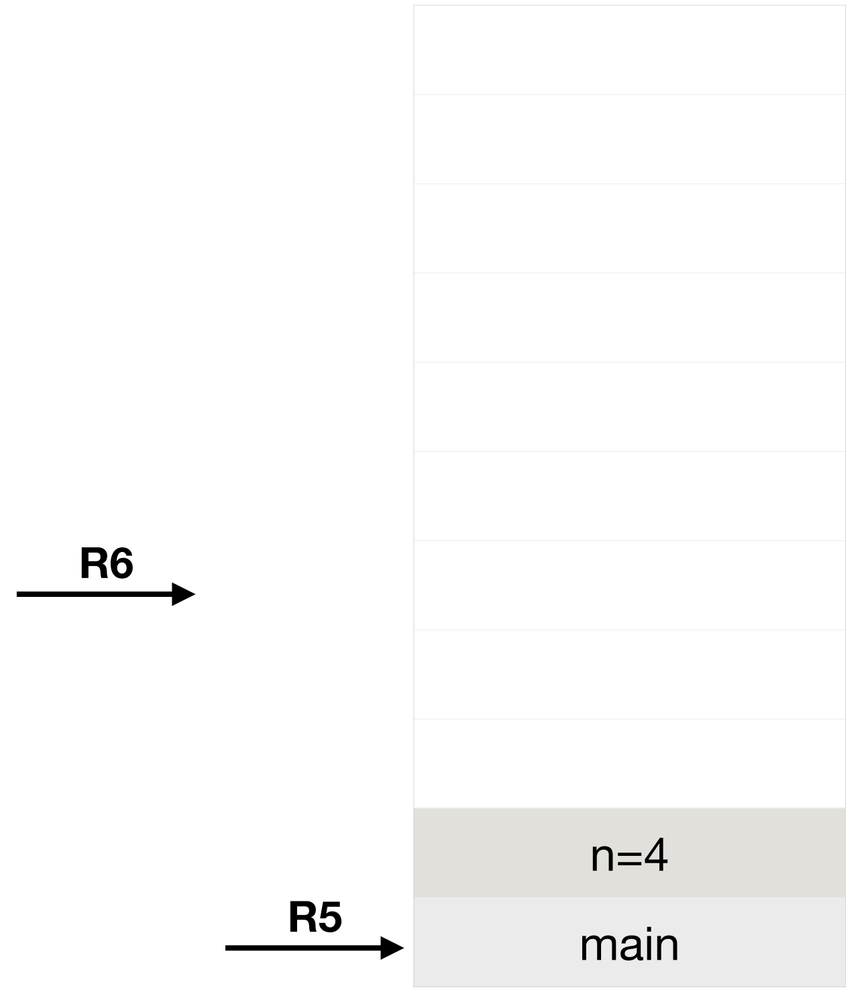

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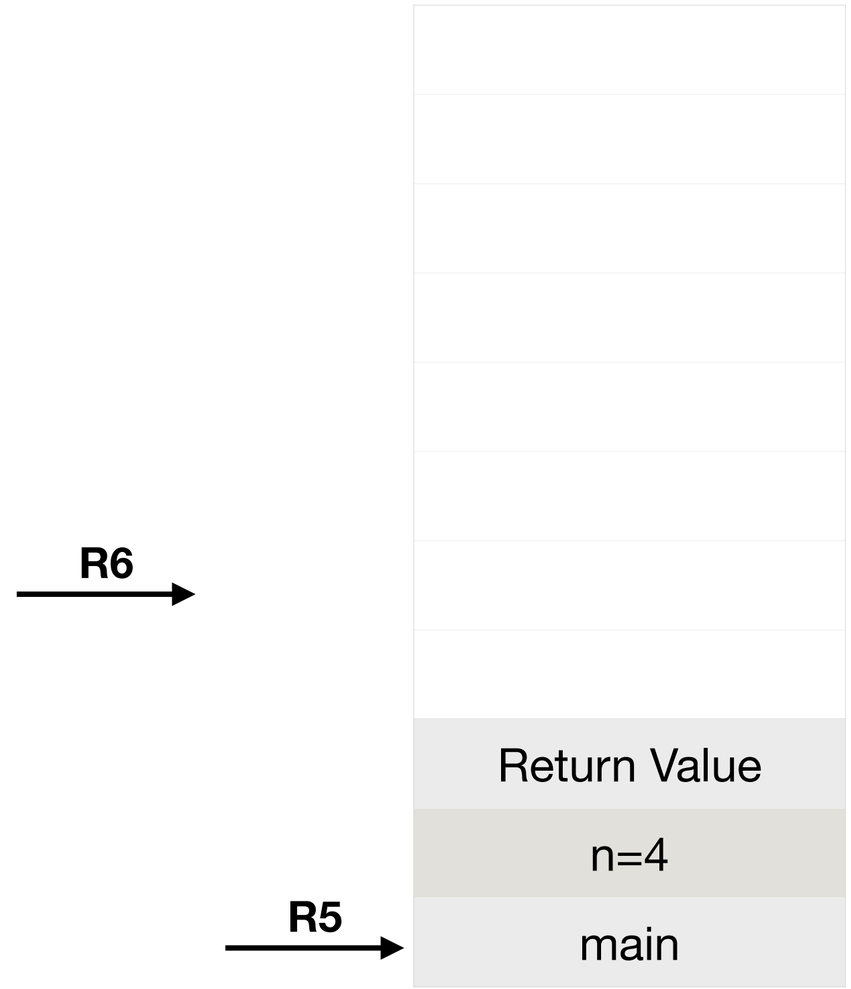

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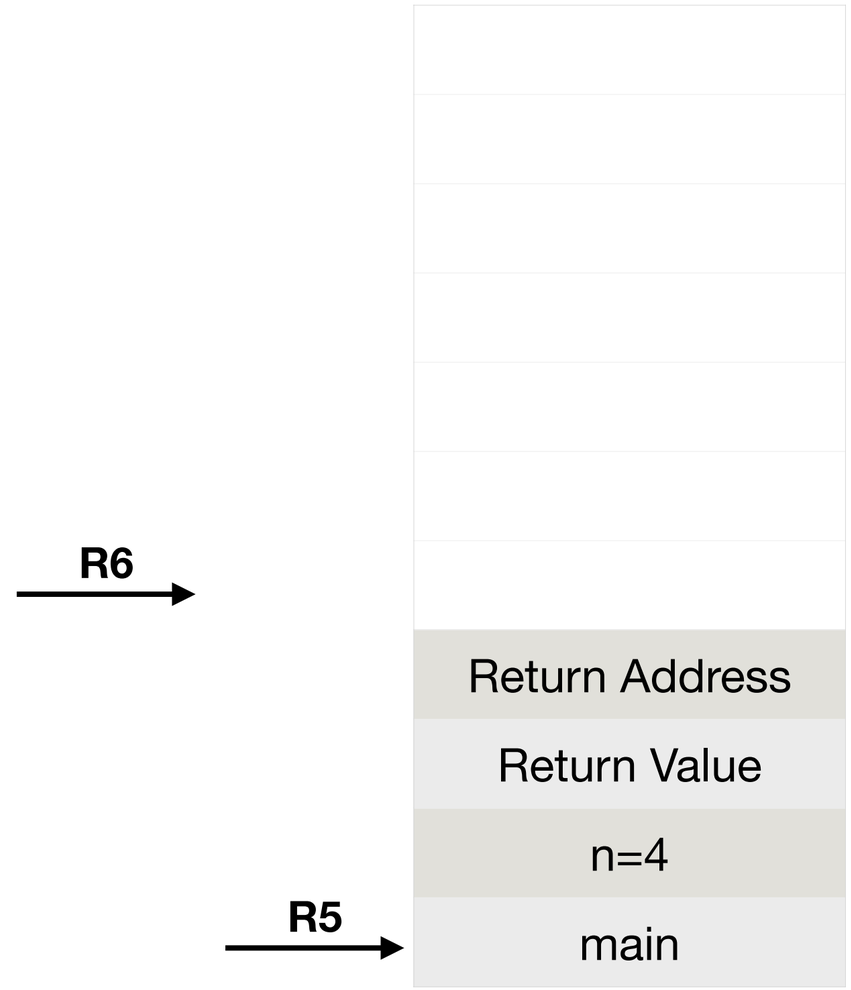

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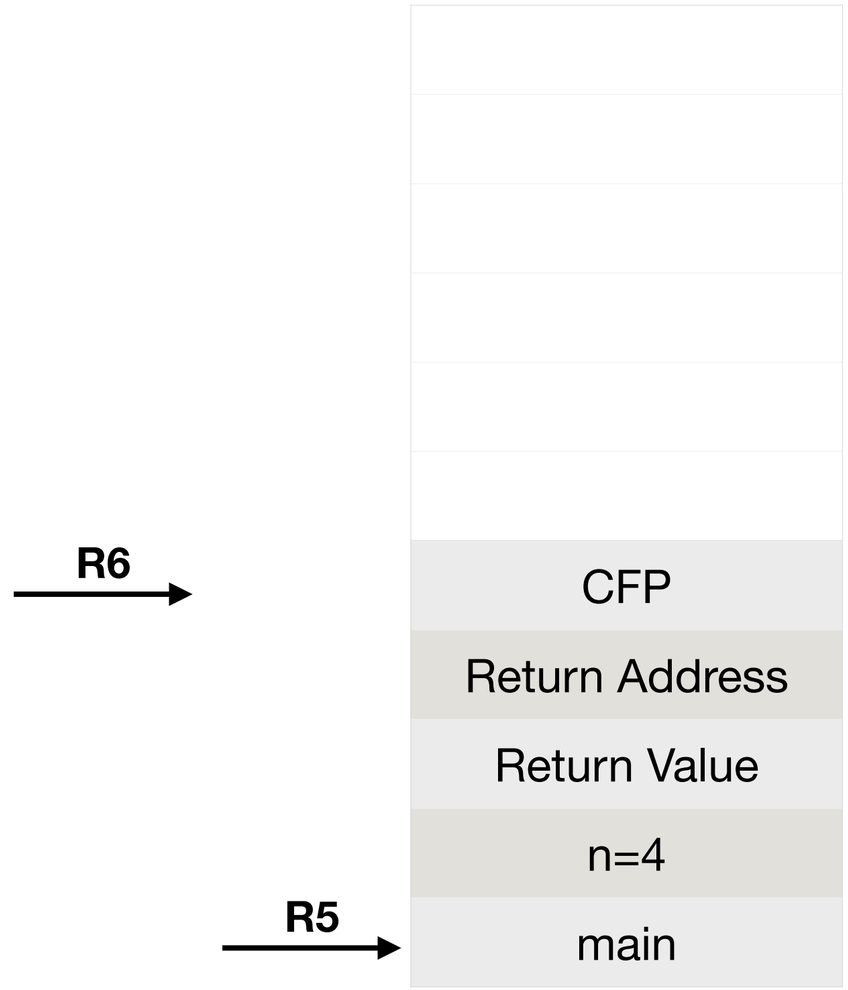

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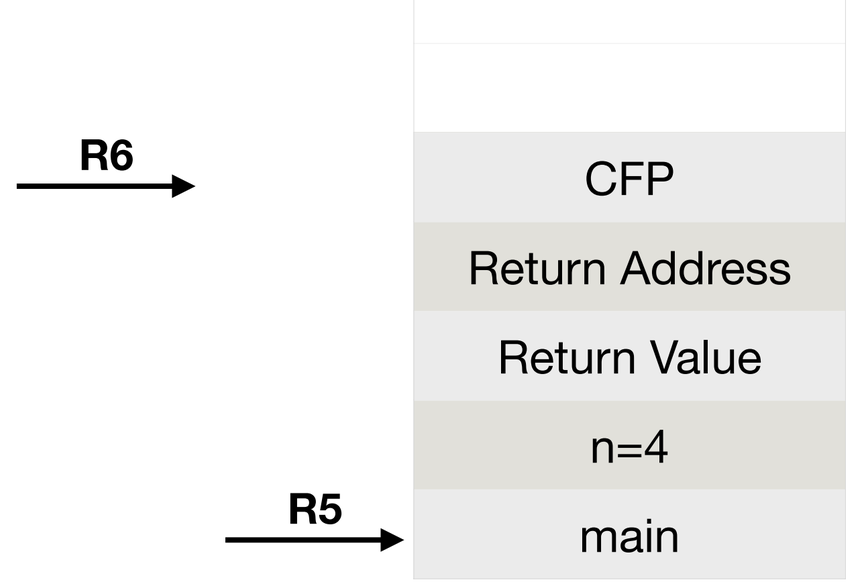

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ADD R5, R6, #-1 ;step 6 on Gitlab
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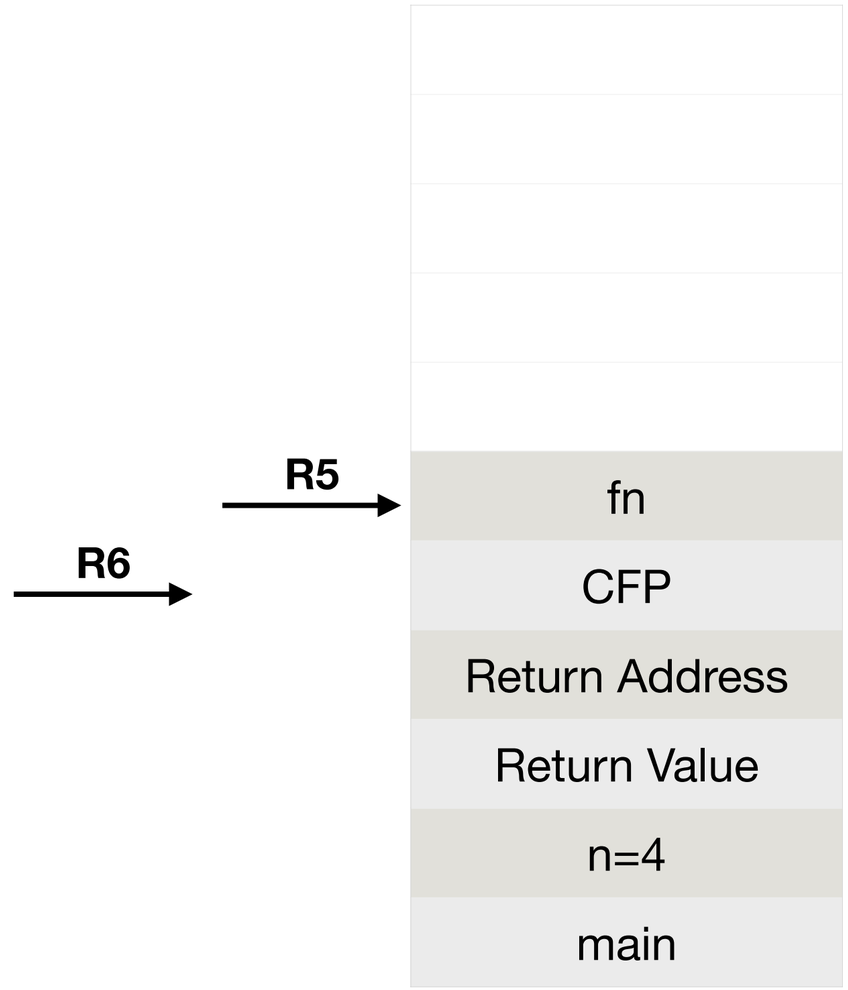

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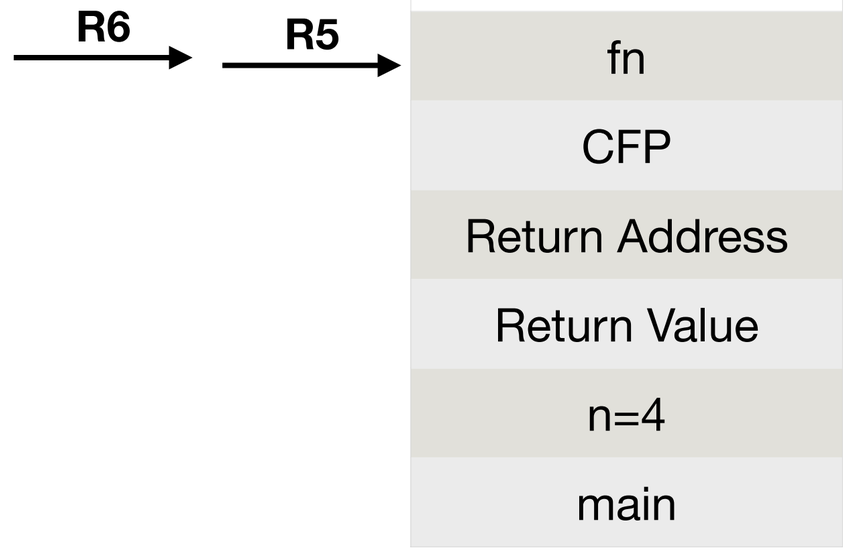

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;function logic
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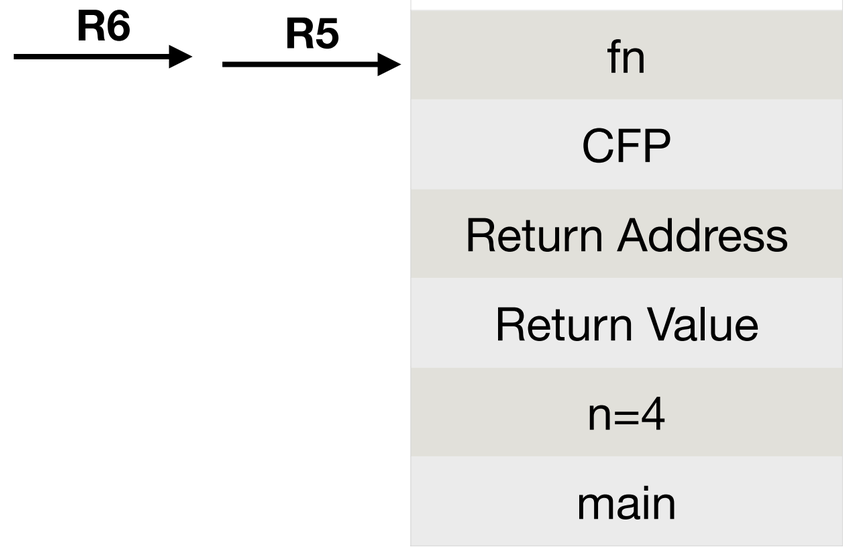

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;function logic
;base case (n==1)
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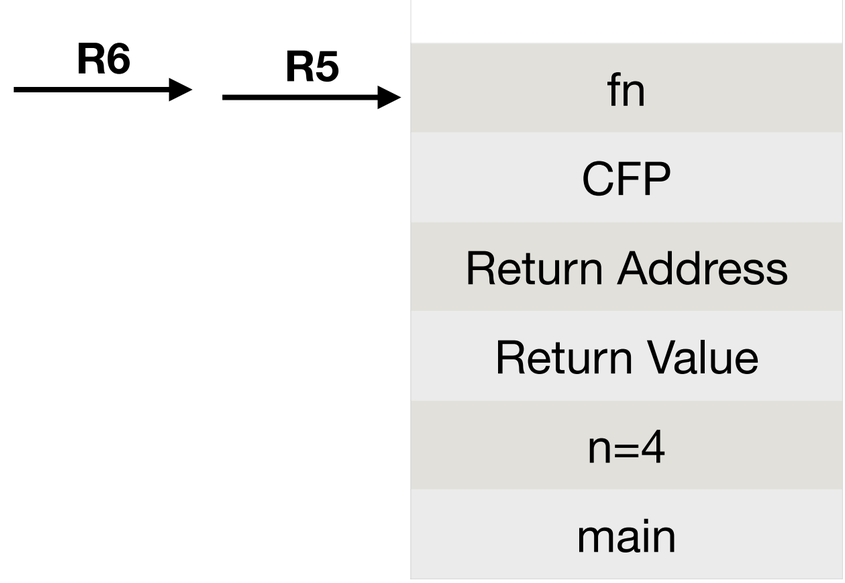

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;function logic
;base case (n==1)
LDR R1, R5, #4
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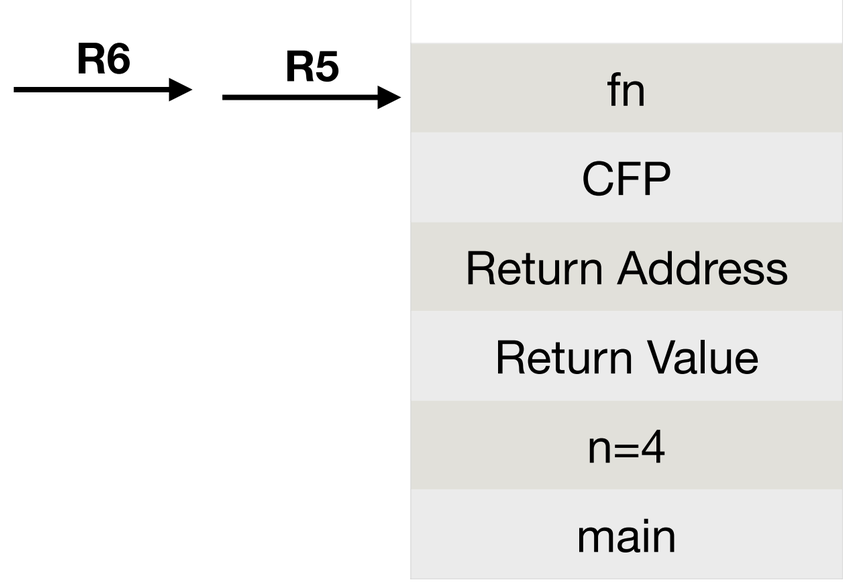

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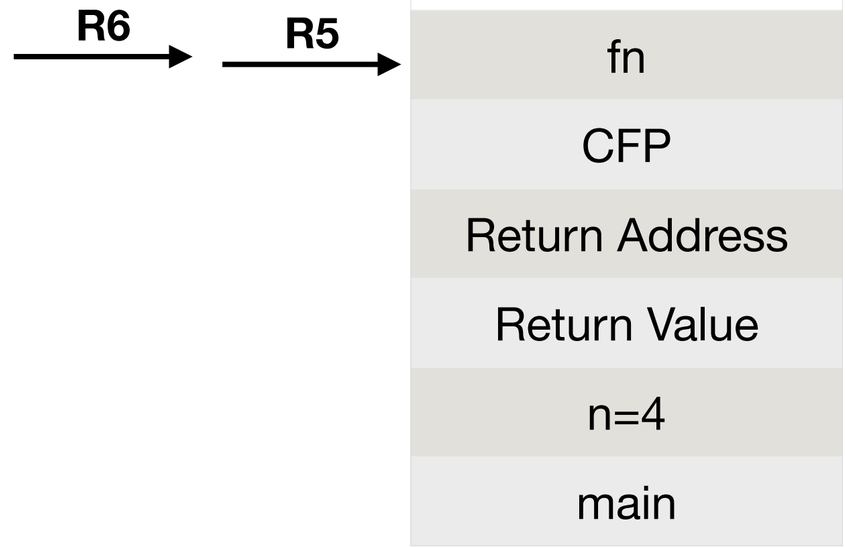
STR R7, R6, #1  ;return address - Step 4 on Gitlab
STR R5, R6, #0  ;CFP - Step 5 on Gitlab

;update frame pointer & make space for local variable
ADD R5, R6, #-1 ;step 6 on Gitlab
ADD R6, R6, #-1 ;step 7 on Gitlab

;function logic
;base case (n==1)
LDR R1, R5, #4
ADD R2, R1, #-1
```



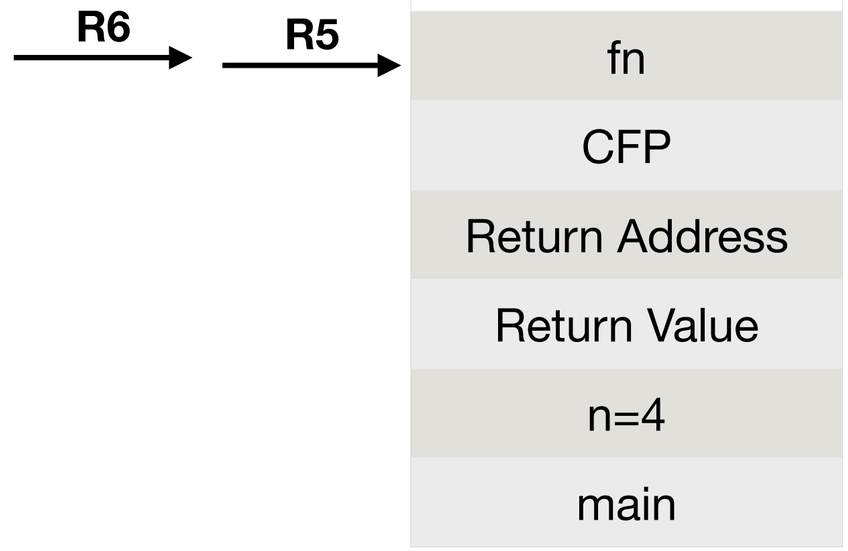
Gitlab C2L3 steps

```c
int running_sum(int n){
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}
```

Review

```c
int main(void){
    int n = 4;
    int answer;
    answer = running_sum(4);
}
```

```
;Caller set-up for Running(n)
STR R0, R5, #0 ; R5 points to main's first local
ADD R6, R6, #-1
STR R0, R6, #0 ; Step 1 on Gitlab
JSR RUNNING ;    Step 2 on Gitlab

RUNNING
;callee set-up of Running(n)'s activation record
;push return value, return address & caller's frame pointer
ADD R6, R6, #-3 ;Step 3 on Gitlab
STR R7, R6, #1  ;return address - Step 4 on Gitlab
STR R5, R6, #0  ;CFP - Step 5 on Gitlab

;update frame pointer & make space for local variable
ADD R5, R6, #-1 ;step 6 on Gitlab
ADD R6, R6, #-1 ;step 7 on Gitlab

;function logic
;base case (n==1)
LDR R1, R5, #4
ADD R2, R1, #-1
BRz BASE_CASE
```

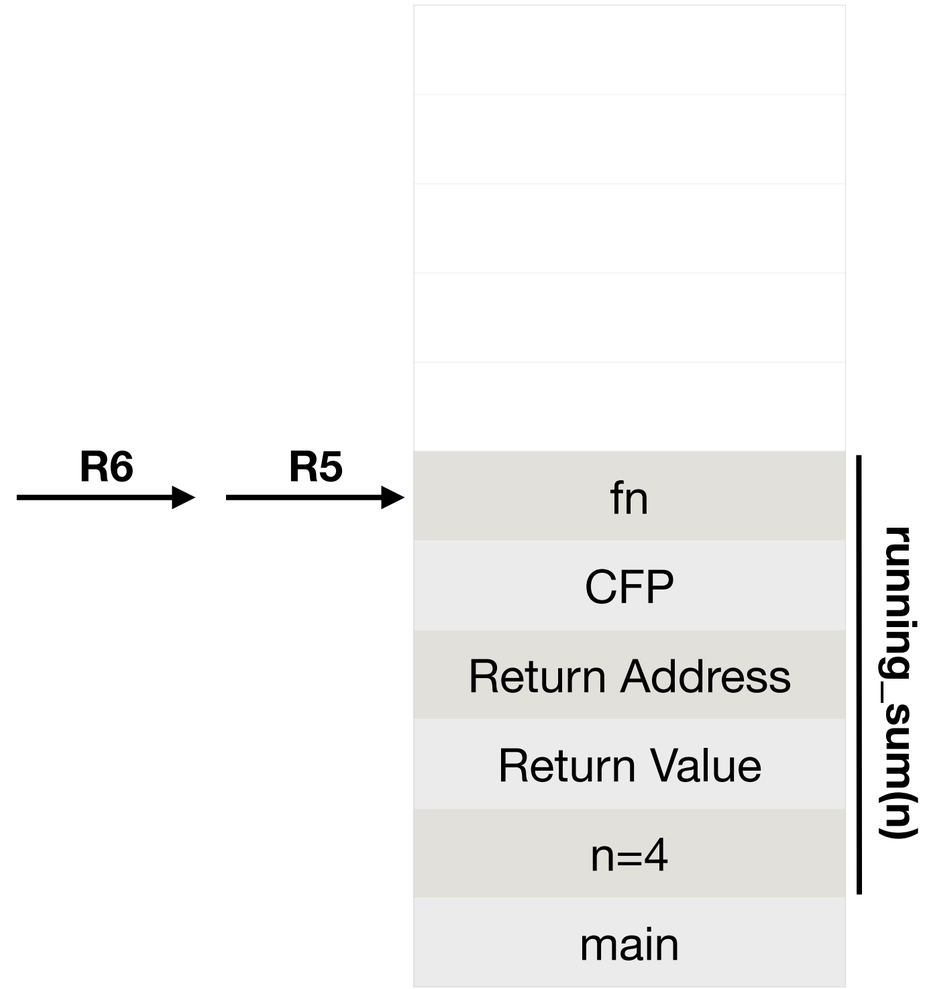


Gitlab C2L3 steps

```c
int running_sum(int n){
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}
```

Review

```c
int main(void){
    int n = 4;
    int answer;
    answer = running_sum(4);
}
```



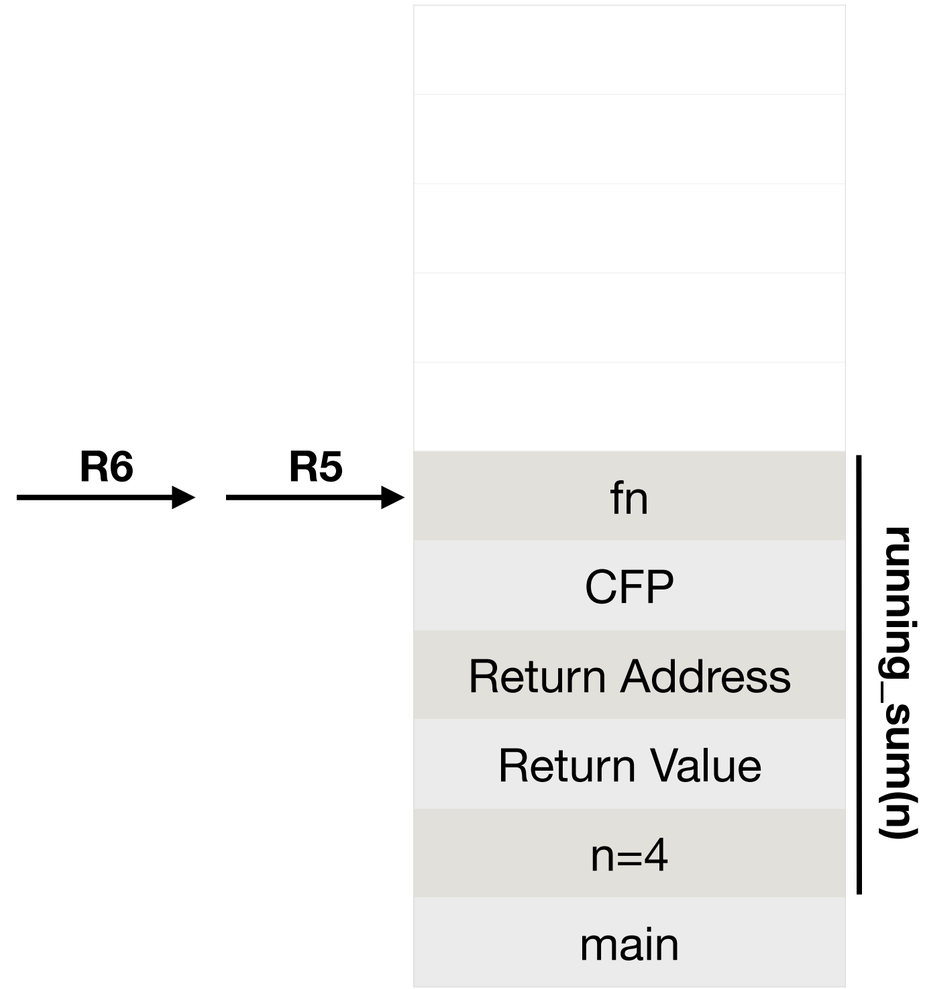
Gitlab C2L3 steps

```c
int running_sum(int n){
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}
```

Review

```c
int main(void){
    int n = 4;
    int answer;
    answer = running_sum(4);
}
```

```
;Recursive case
;Caller setup for Running(n-1): push argument n-1 onto RST
ADD R6, R6, #-1
```



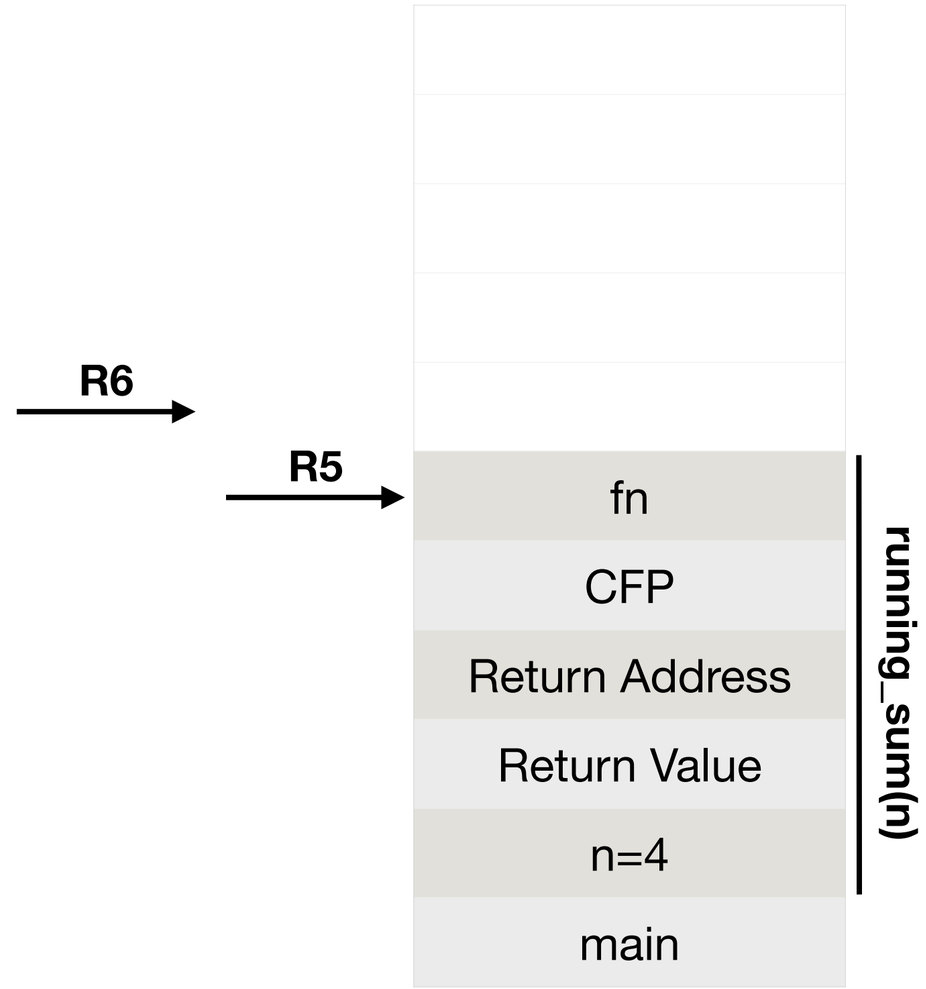
Gitlab C2L3 steps

```c
int running_sum(int n){
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}
```

Review

```c
int main(void){
    int n = 4;
    int answer;
    answer = running_sum(4);
}
```

```
;Recursive case
;Caller setup for Running(n-1): push argument n-1 onto RST
ADD R6, R6, #-1
STR R2, R6, #0 ; R2 = n - 1
```



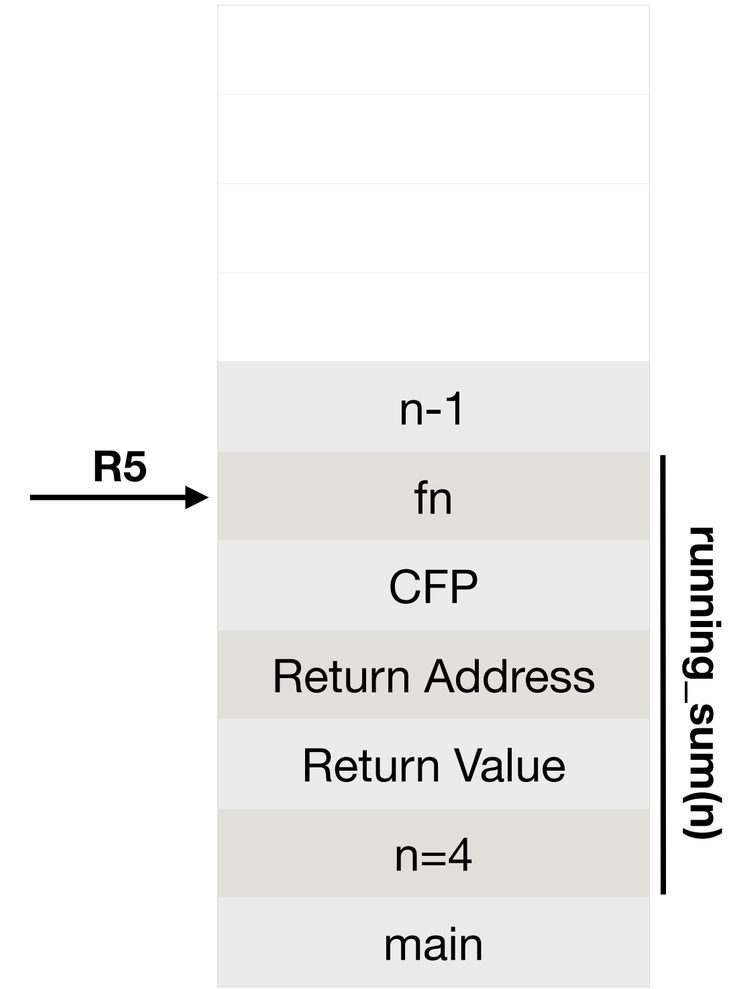
Gitlab C2L3 steps

```c
int running_sum(int n){
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}
```

Review

```c
int main(void){
    int n = 4;
    int answer;
    answer = running_sum(4);
}
```

```
;Recursive case
;Caller setup for Running(n-1): push argument n-1 onto RST
ADD R6, R6, #-1
STR R2, R6, #0 ; R2 = n - 1
```



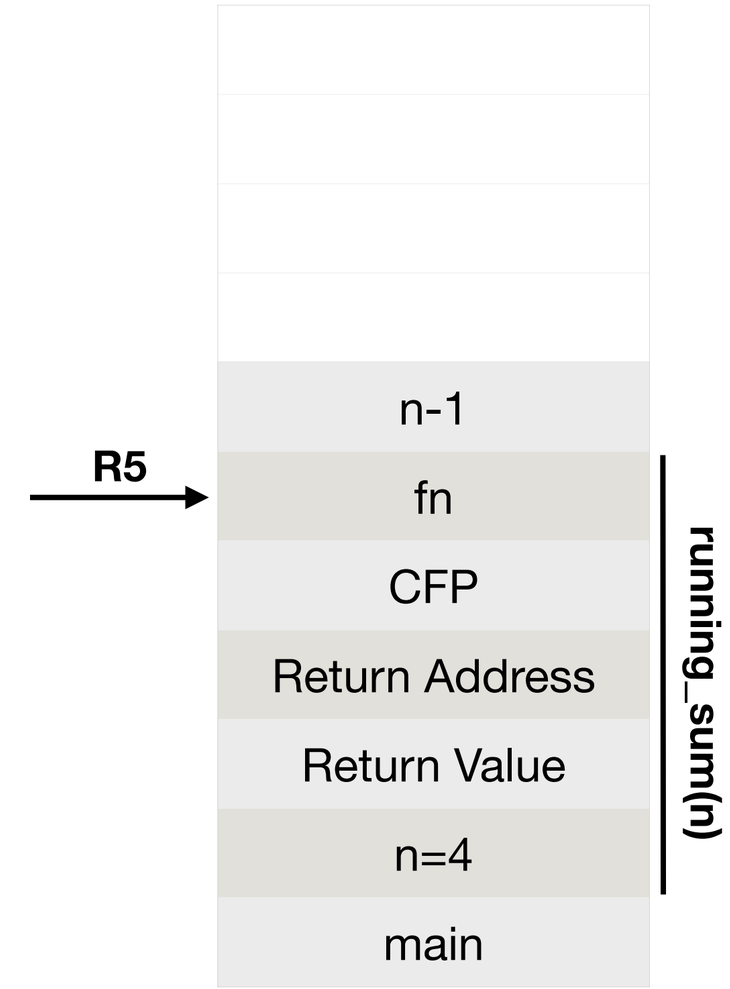
Gitlab C2L3 steps

```c
int running_sum(int n){
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}
```

Review

```c
int main(void){
    int n = 4;
    int answer;
    answer = running_sum(4);
}
```

```
;Recursive case
;Caller setup for Running(n-1): push argument n-1 onto RST
ADD R6, R6, #-1
STR R2, R6, #0 ; R2 = n - 1
JSR RUNNING ; call Running(n-1)
```



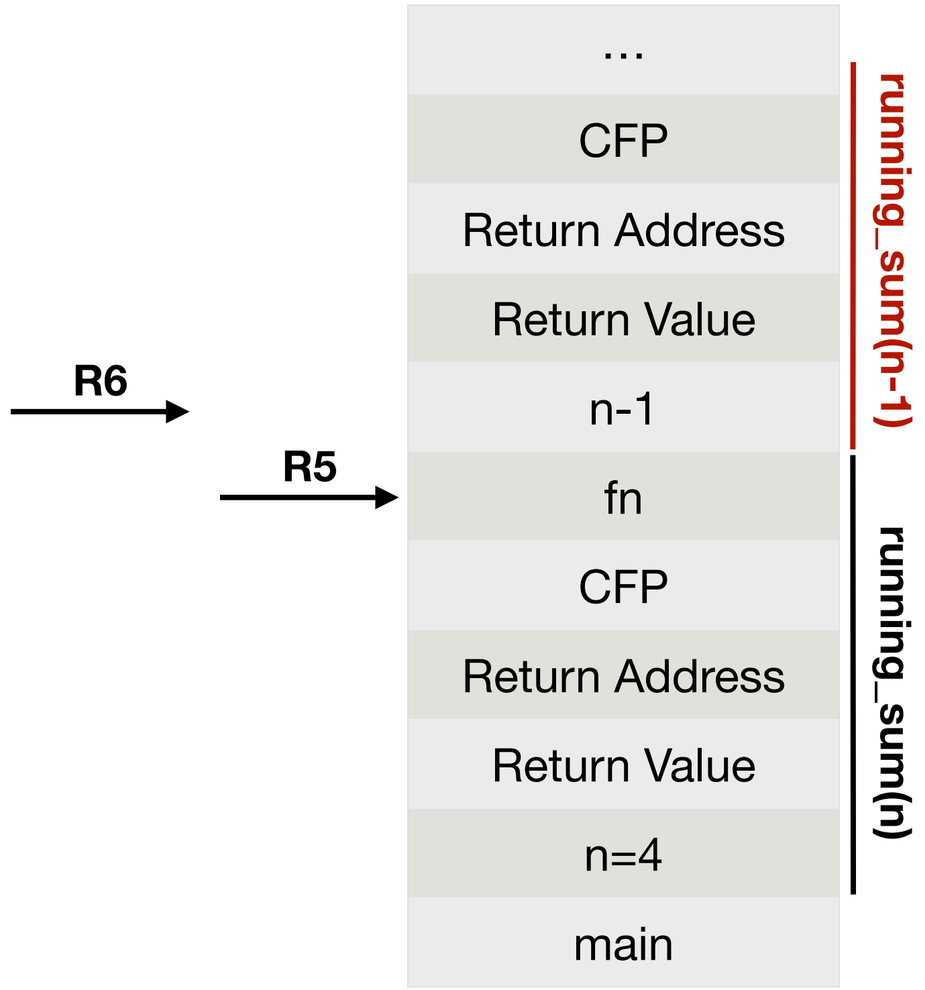
Gitlab C2L3 steps

```c
int running_sum(int n){
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}
```

Review

```c
int main(void){
    int n = 4;
    int answer;
    answer = running_sum(4);
}
```

```
;Recursive case
;Caller setup for Running(n-1): push argument n-1 onto RST
ADD R6, R6, #-1
STR R2, R6, #0 ; R2 = n - 1
JSR RUNNING ; call Running(n-1)
```



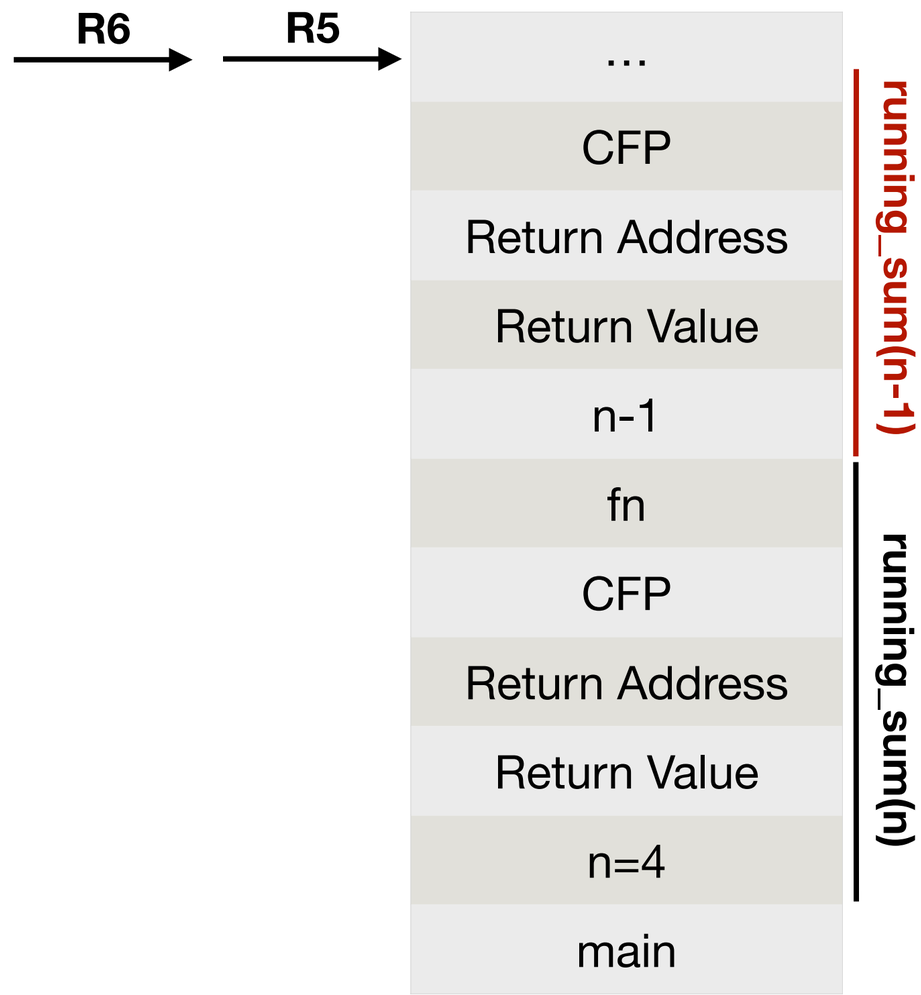
Gitlab C2L3 steps

```c
int running_sum(int n){
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}
```

Review

```c
int main(void){
    int n = 4;
    int answer;
    answer = running_sum(4);
}
```

```
;Recursive case
;Caller setup for Running(n-1): push argument n-1 onto RST
ADD R6, R6, #-1
STR R2, R6, #0 ; R2 = n - 1
JSR RUNNING ; call Running(n-1)
```



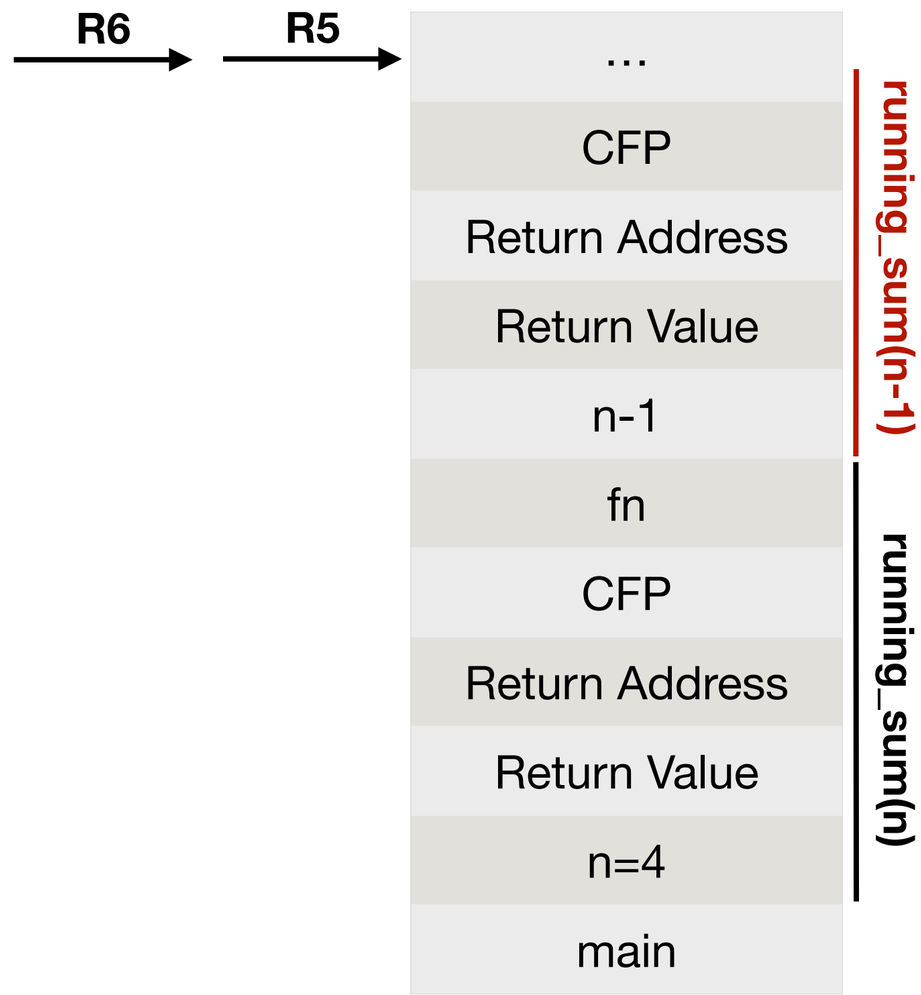
Gitlab C2L3 steps

```c
int running_sum(int n){
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}
```

Review

```c
int main(void){
    int n = 4;
    int answer;
    answer = running_sum(4);
}
```

```
;Recursive case
;Caller setup for Running(n-1): push argument n-1 onto RST
ADD R6, R6, #-1
STR R2, R6, #0 ; R2 = n - 1
JSR RUNNING ; call Running(n-1)

;Callee tear-down for Running(n-1) not shown
```



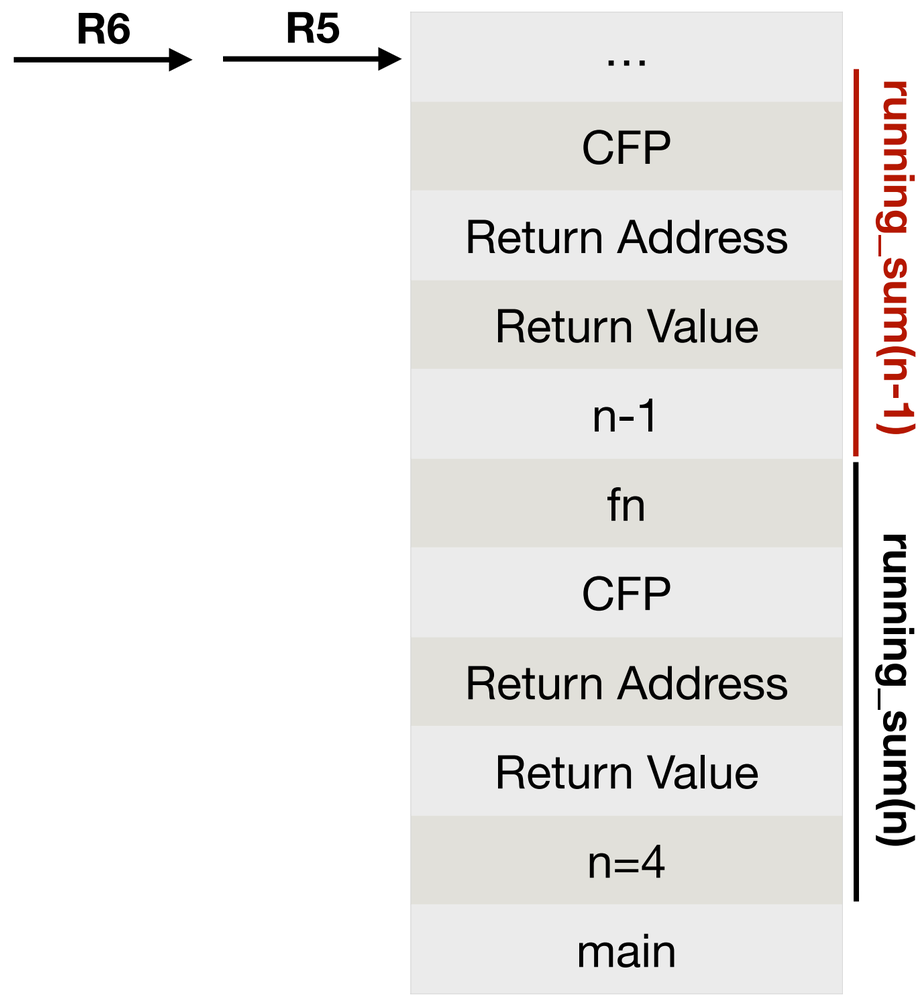
Gitlab C2L3 steps

```c
int running_sum(int n){
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}
```

Review

```c
int main(void){
    int n = 4;
    int answer;
    answer = running_sum(4);
}
```

```
;Recursive case
;Caller setup for Running(n-1): push argument n-1 onto RST
ADD R6, R6, #-1
STR R2, R6, #0 ; R2 = n - 1
JSR RUNNING ; call Running(n-1)

;Callee tear-down for Running(n-1) not shown

;Caller tear-down for Running(n-1)
```



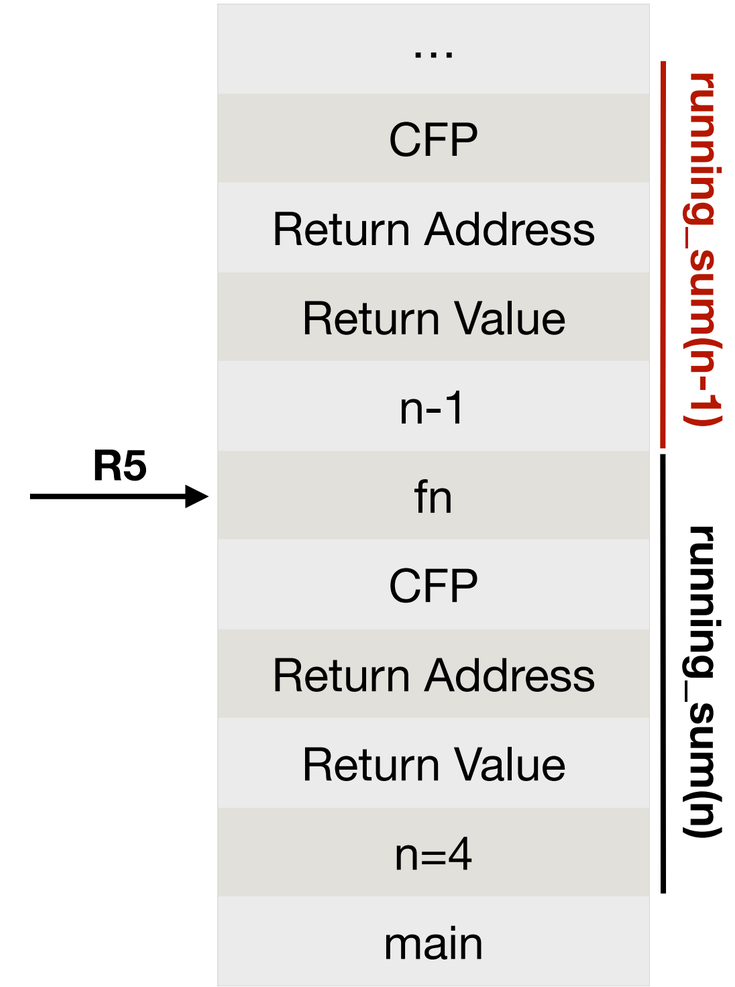
Gitlab C2L3 steps

```c
int running_sum(int n){
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}
```

Review

```c
int main(void){
    int n = 4;
    int answer;
    answer = running_sum(4);
}
```

```
;Recursive case
;Caller setup for Running(n-1): push argument n-1 onto RST
ADD R6, R6, #-1
STR R2, R6, #0 ; R2 = n - 1
JSR RUNNING ; call Running(n-1)

;Callee tear-down for Running(n-1) not shown

;Caller tear-down for Running(n-1)
```



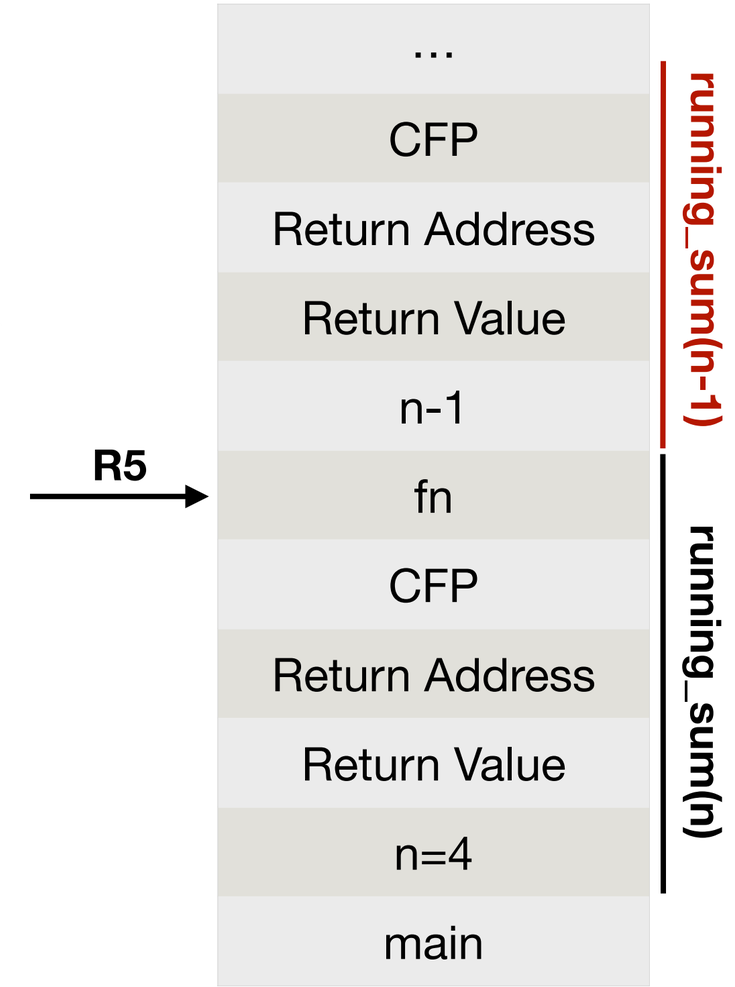
Gitlab C2L3 steps

```c
int running_sum(int n){
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}
```

Review

```c
int main(void){
    int n = 4;
    int answer;
    answer = running_sum(4);
}
```

```
;Recursive case
;Caller setup for Running(n-1): push argument n-1 onto RST
ADD R6, R6, #-1
STR R2, R6, #0 ; R2 = n - 1
JSR RUNNING ; call Running(n-1)

;Callee tear-down for Running(n-1) not shown

;Caller tear-down for Running(n-1)
;pop Running(n-1)'s return value to R0
```



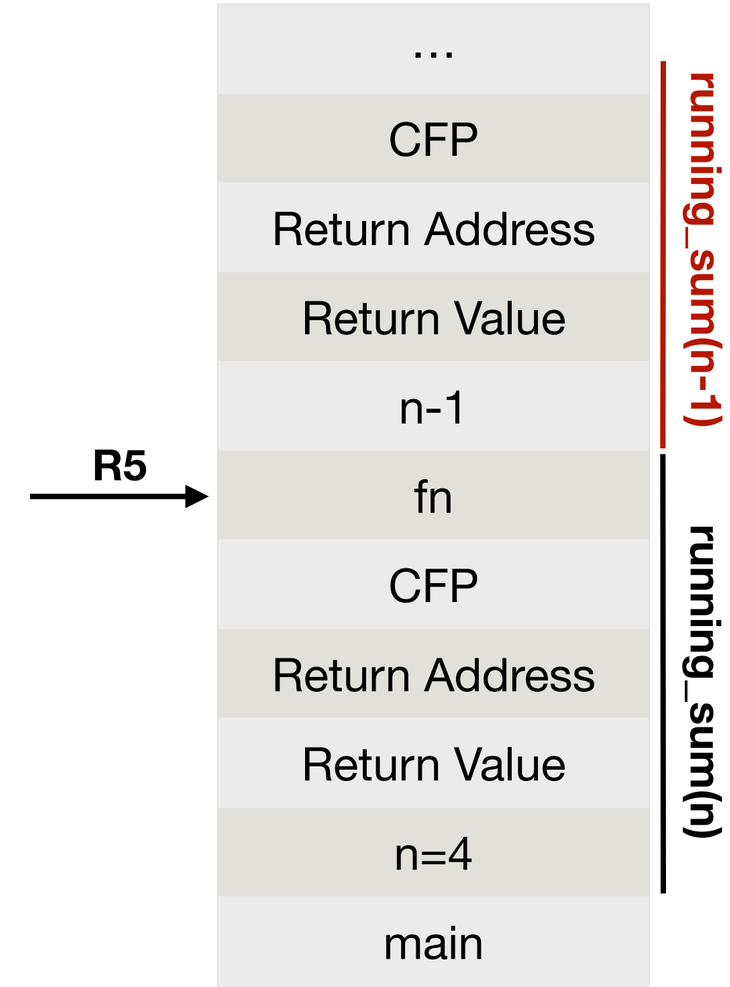
Gitlab C2L3 steps

```c
int running_sum(int n){
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}
```

Review

```c
int main(void){
    int n = 4;
    int answer;
    answer = running_sum(4);
}
```

```
;Recursive case
;Caller setup for Running(n-1): push argument n-1 onto RST
ADD R6, R6, #-1
STR R2, R6, #0 ; R2 = n - 1
JSR RUNNING ; call Running(n-1)

;Callee tear-down for Running(n-1) not shown

;Caller tear-down for Running(n-1)
;pop Running(n-1)'s return value to R0
LDR R0, R6, #0
```



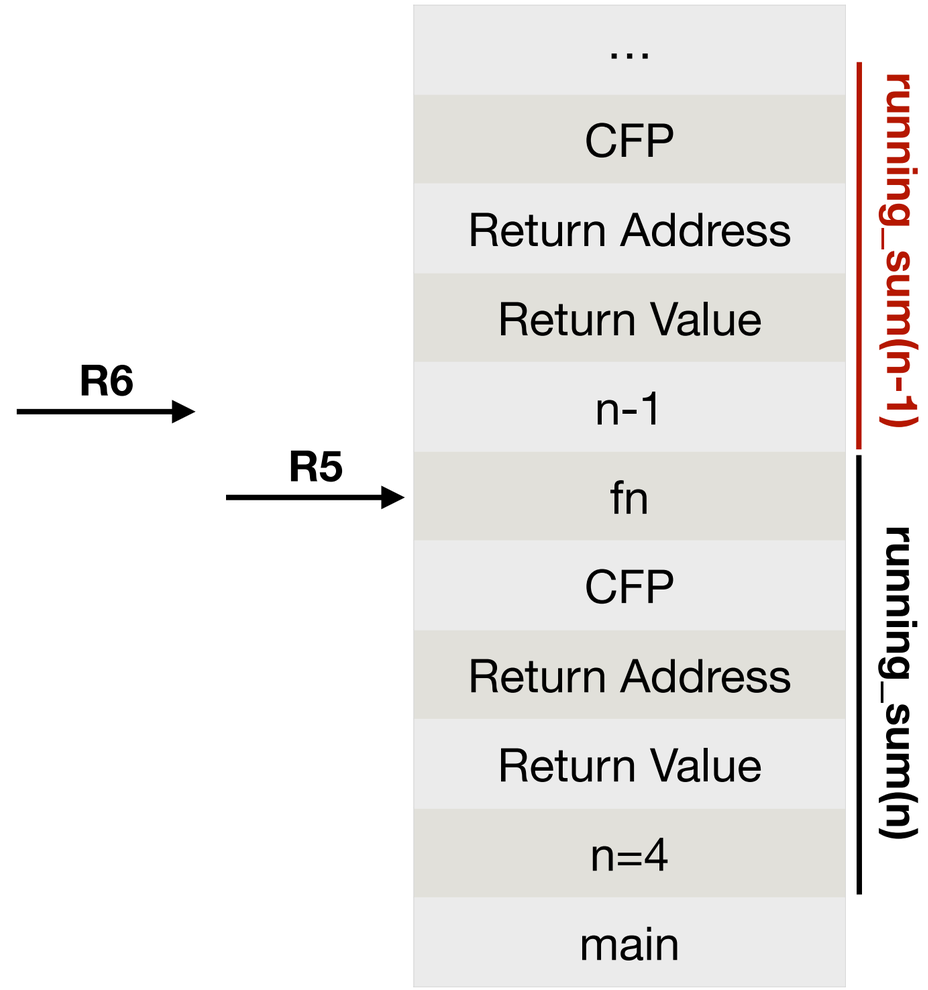
Gitlab C2L3 steps

Review

```c
int running_sum(int n){
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}
```

```c
int main(void){
    int n = 4;
    int answer;
    answer = running_sum(4);
}
```

```
;Recursive case
;Caller setup for Running(n-1): push argument n-1 onto RST
ADD R6, R6, #-1
STR R2, R6, #0 ; R2 = n - 1
JSR RUNNING ; call Running(n-1)

;Callee tear-down for Running(n-1) not shown


;Caller tear-down for Running(n-1)
;pop Running(n-1)'s return value to R0
LDR R0, R6, #0
ADD R6, R6, #1 ; Step 13 on Gitlab
```



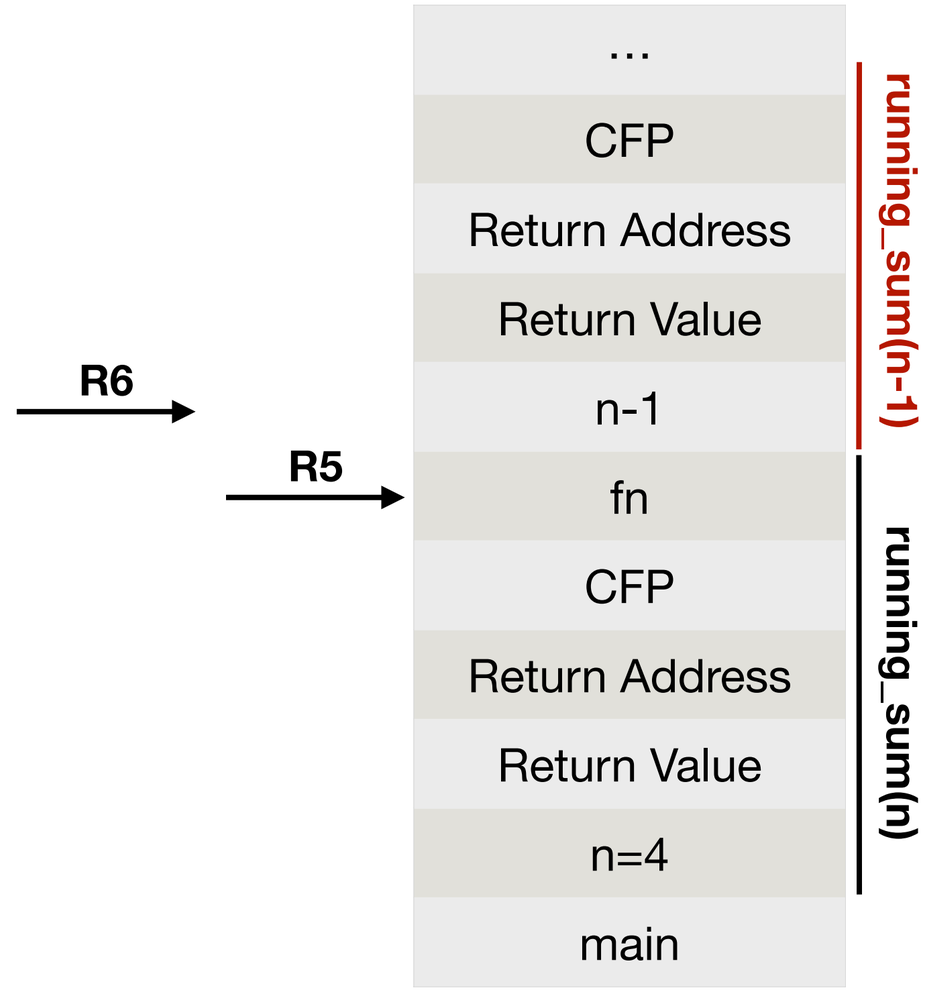
Gitlab C2L3 steps

Review

```c
int running_sum(int n){
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}
```

```c
int main(void){
    int n = 4;
    int answer;
    answer = running_sum(4);
}
```

```
;Recursive case
;Caller setup for Running(n-1): push argument n-1 onto RST
ADD R6, R6, #-1
STR R2, R6, #0 ; R2 = n - 1
JSR RUNNING ; call Running(n-1)

;Callee tear-down for Running(n-1) not shown


;Caller tear-down for Running(n-1)
;pop Running(n-1)'s return value to R0
LDR R0, R6, #0
ADD R6, R6, #1 ; Step 13 on Gitlab

;pop Running(n-1)'s argument
```



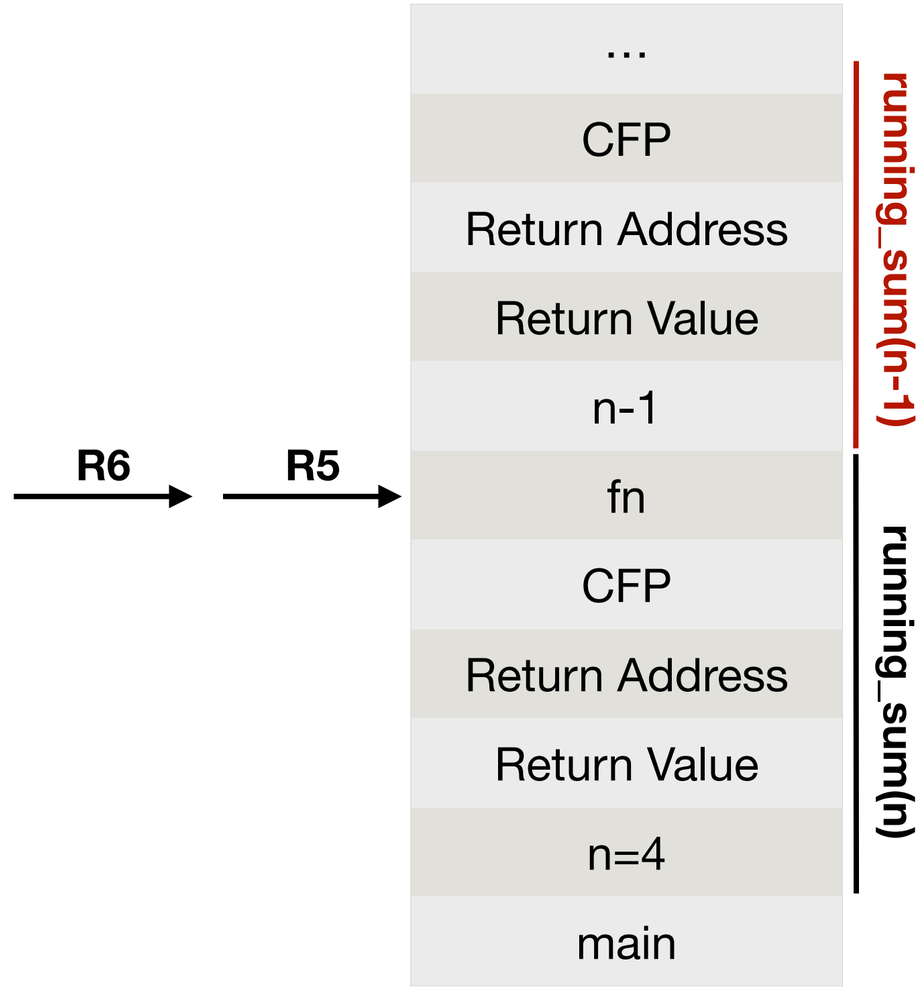
Gitlab C2L3 steps

```c
int running_sum(int n){
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}
```

Review

```c
int main(void){
    int n = 4;
    int answer;
    answer = running_sum(4);
}
```

```
;Recursive case
;Caller setup for Running(n-1): push argument n-1 onto RST
ADD R6, R6, #-1
STR R2, R6, #0 ; R2 = n - 1
JSR RUNNING ; call Running(n-1)

;Callee tear-down for Running(n-1) not shown

;Caller tear-down for Running(n-1)
;pop Running(n-1)'s return value to R0
LDR R0, R6, #0
ADD R6, R6, #1 ; Step 13 on Gitlab

;pop Running(n-1)'s argument
ADD R6, R6, #1 ; Step 14 on Gitlab
```



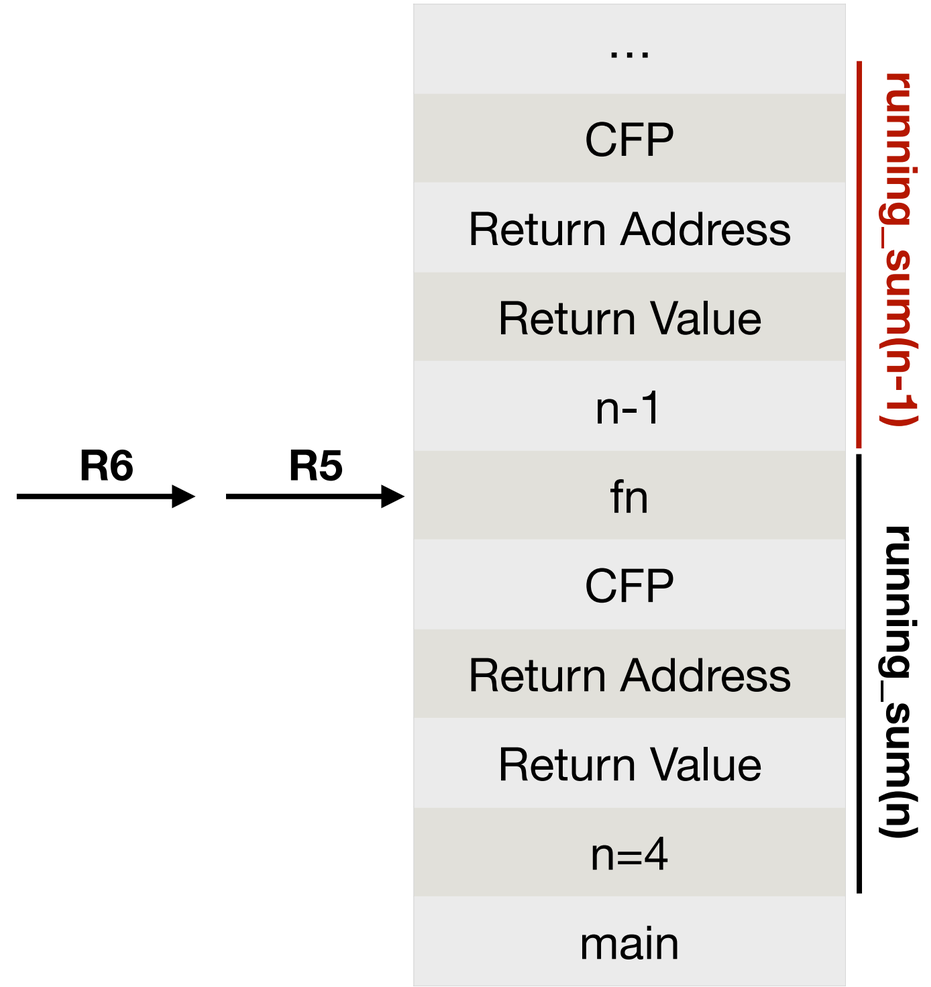
Gitlab C2L3 steps

```c
int running_sum(int n){
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}
```

Review

```c
int main(void){
    int n = 4;
    int answer;
    answer = running_sum(4);
}
```

```
;Recursive case
;Caller setup for Running(n-1): push argument n-1 onto RST
ADD R6, R6, #-1
STR R2, R6, #0 ; R2 = n - 1
JSR RUNNING ; call Running(n-1)

;Callee tear-down for Running(n-1) not shown

;Caller tear-down for Running(n-1)
;pop Running(n-1)'s return value to R0
LDR R0, R6, #0
ADD R6, R6, #1 ; Step 13 on Gitlab

;pop Running(n-1)'s argument
ADD R6, R6, #1 ; Step 14 on Gitlab

;calculate n + Running(n-1)
LDR R1, R5, #4
ADD R0, R1, R0
STR R0, R5, #0 ;store result in fn
```



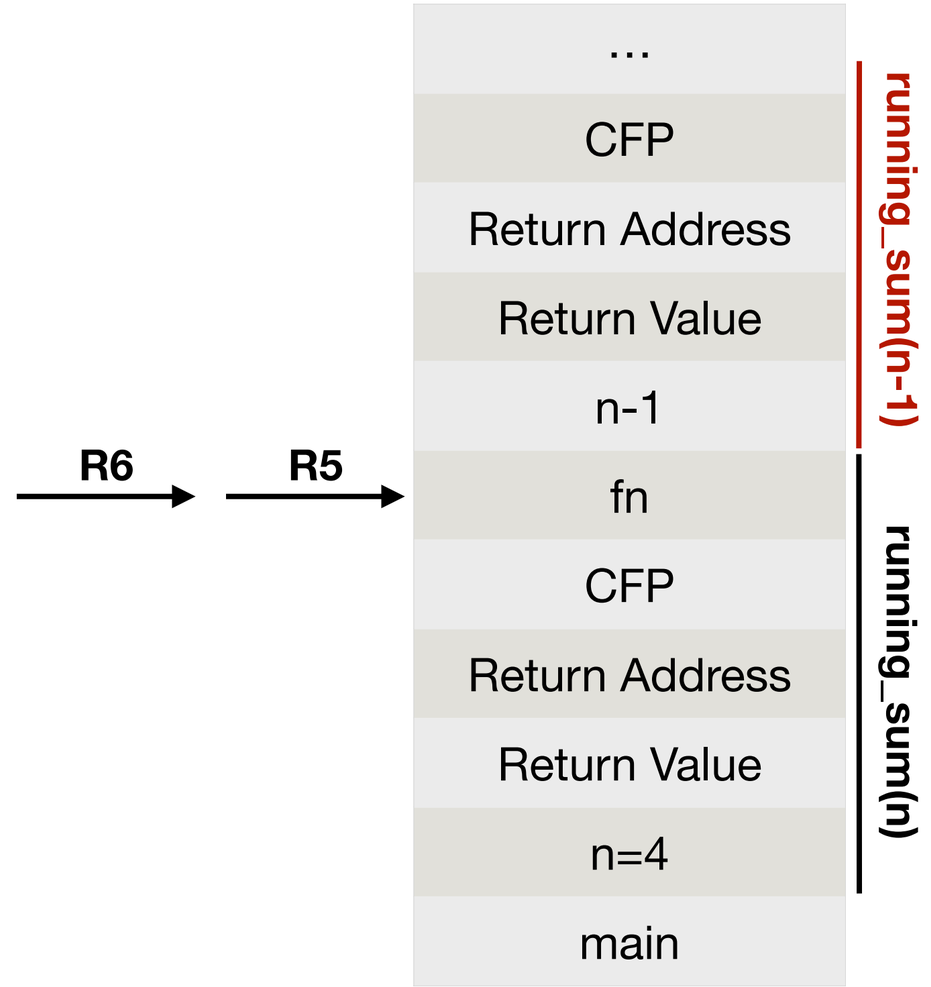
Gitlab C2L3 steps

```c
int running_sum(int n){
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}
```

Review

```c
int main(void){
    int n = 4;
    int answer;
    answer = running_sum(4);
}
```

```
;Recursive case
;Caller setup for Running(n-1): push argument n-1 onto RST
ADD R6, R6, #-1
STR R2, R6, #0 ; R2 = n - 1
JSR RUNNING ; call Running(n-1)

;Callee tear-down for Running(n-1) not shown

;Caller tear-down for Running(n-1)
;pop Running(n-1)'s return value to R0
LDR R0, R6, #0
ADD R6, R6, #1 ; Step 13 on Gitlab

;pop Running(n-1)'s argument
ADD R6, R6, #1 ; Step 14 on Gitlab

;calculate n + Running(n-1)
LDR R1, R5, #4
ADD R0, R1, R0
STR R0, R5, #0 ;store result in fn

;ready to return
```



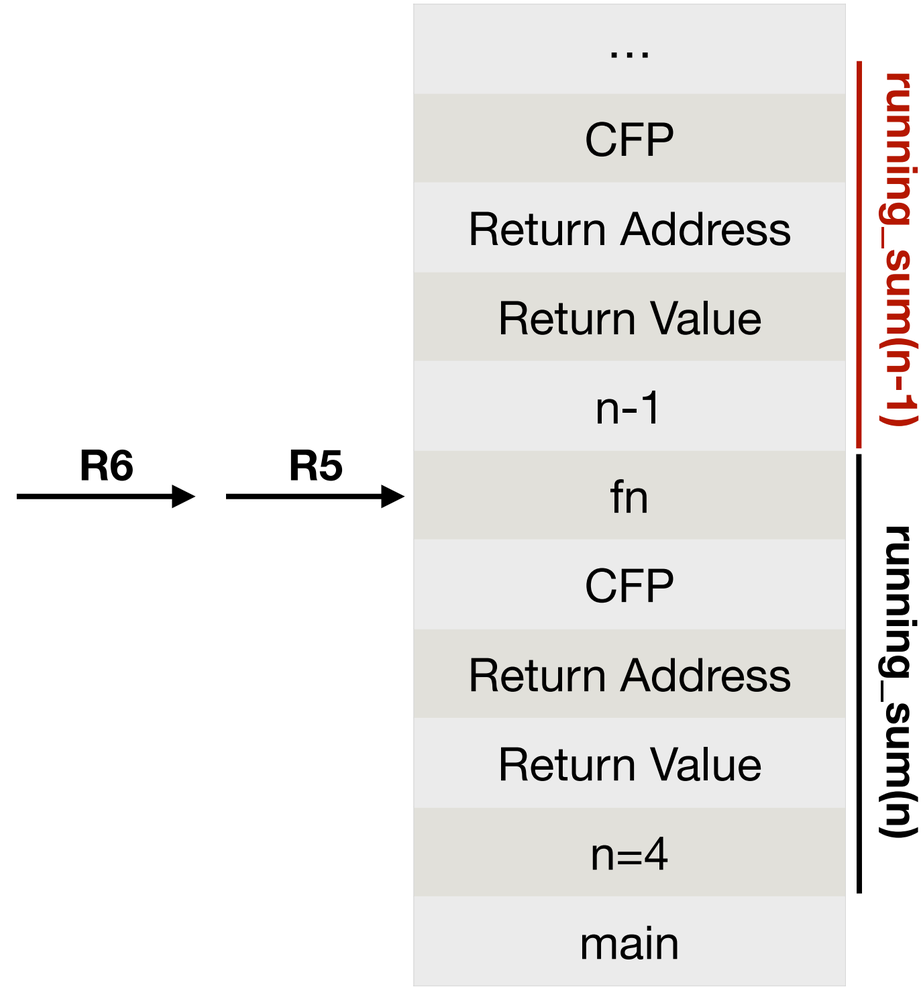
Gitlab C2L3 steps

Review

```c
int running_sum(int n){
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}
```

```c
int main(void){
    int n = 4;
    int answer;
    answer = running_sum(4);
}
```

```
;Recursive case
;Caller setup for Running(n-1): push argument n-1 onto RST
ADD R6, R6, #-1
STR R2, R6, #0 ; R2 = n - 1
JSR RUNNING ; call Running(n-1)

;Callee tear-down for Running(n-1) not shown

;Caller tear-down for Running(n-1)
;pop Running(n-1)'s return value to R0
LDR R0, R6, #0
ADD R6, R6, #1 ; Step 13 on Gitlab

;pop Running(n-1)'s argument
ADD R6, R6, #1 ; Step 14 on Gitlab

;calculate n + Running(n-1)
LDR R1, R5, #4
ADD R0, R1, R0
STR R0, R5, #0 ;store result in fn

;ready to return
BRnzp RETURN
```

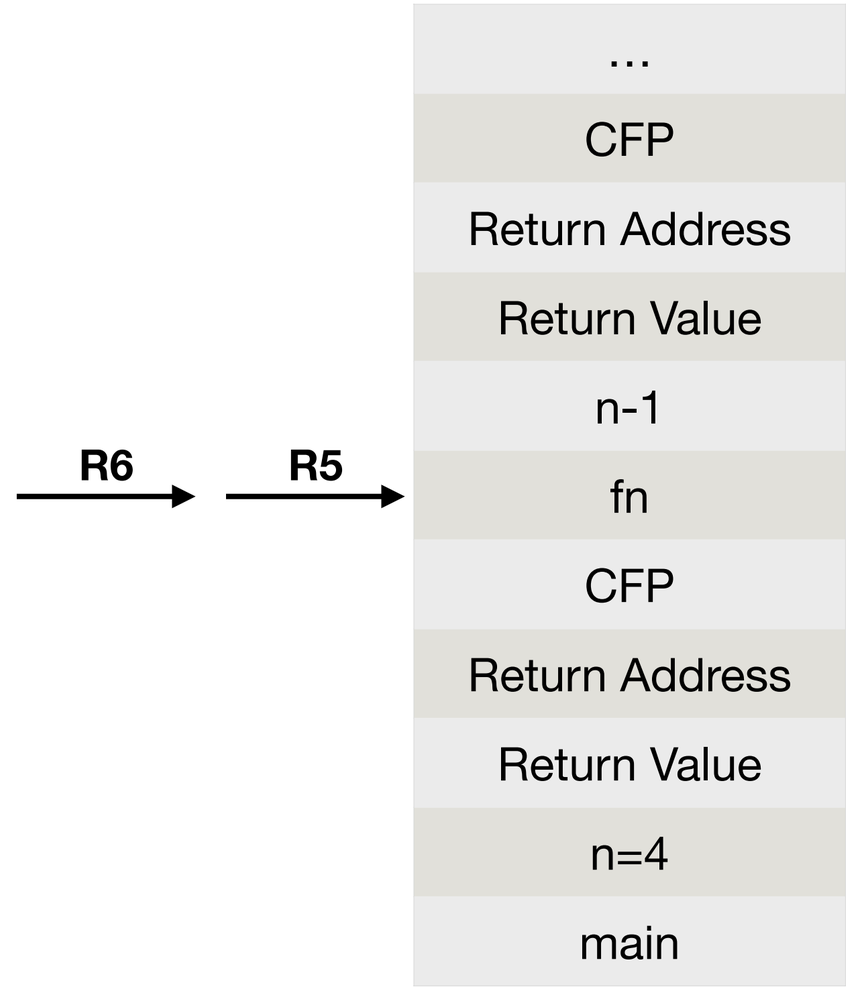


Gitlab C2L3 steps

```c
int running_sum(int n){
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}
```

Review

```c
int main(void){
    int n = 4;
    int answer;
    answer = running_sum(4);
}
```



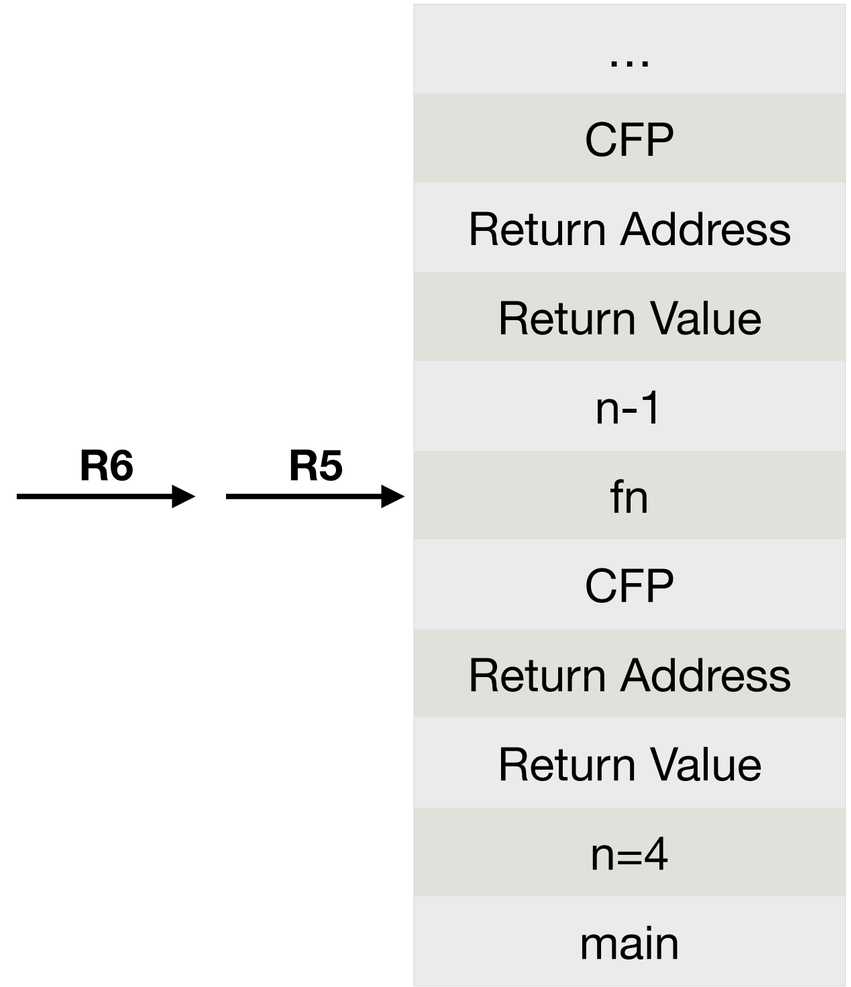
Gitlab C2L3 steps

Review

```c
int running_sum(int n){
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}
```

```c
int main(void){
    int n = 4;
    int answer;
    answer = running_sum(4);
}
```

```
BASE_CASE
AND R2, R2, #0
ADD R2, R2, #1 ;set fn = 1
STR R2, R5, #0
```



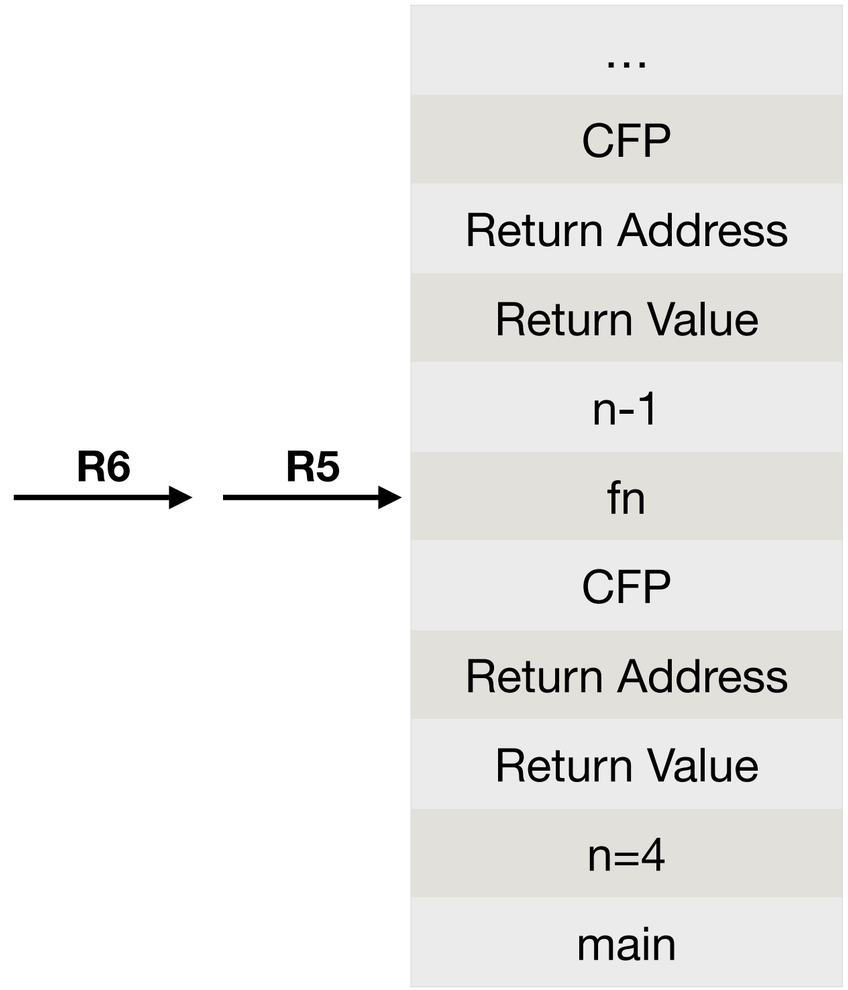
Gitlab C2L3 steps

```c
int running_sum(int n){
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}
```

Review

```c
int main(void){
    int n = 4;
    int answer;
    answer = running_sum(4);
}
```

```
BASE_CASE
AND R2, R2, #0
ADD R2, R2, #1 ;set fn = 1
STR R2, R5, #0

RETURN
;set return value
LDR R0, R5, #0
STR R0, R5, #3 ; END of Step 8 on Gitlab
```



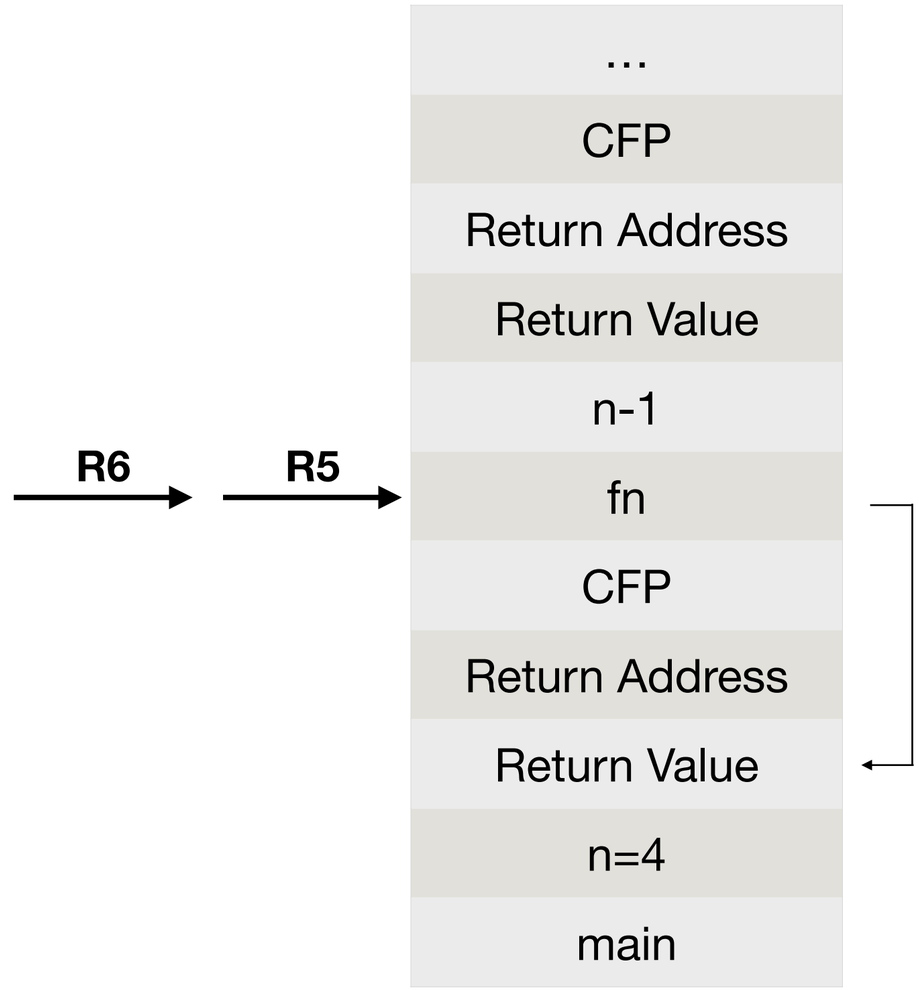
Gitlab C2L3 steps

Review

```c
int running_sum(int n){
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}
```

```c
int main(void){
    int n = 4;
    int answer;
    answer = running_sum(4);
}
```

```
BASE_CASE
AND R2, R2, #0
ADD R2, R2, #1 ;set fn = 1
STR R2, R5, #0

RETURN
;set return value
LDR R0, R5, #0
STR R0, R5, #3 ; END of Step 8 on Gitlab

;callee tear-down of Running(n)'s activation record
```



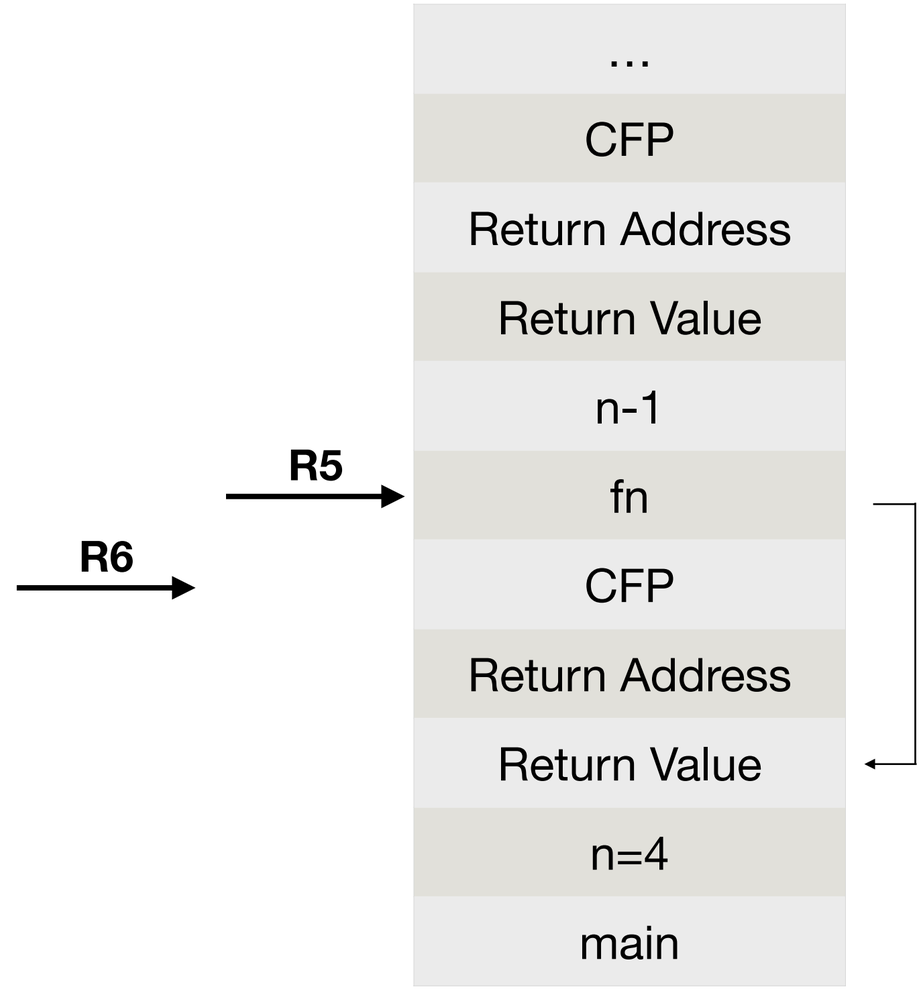
Gitlab C2L3 steps

Review

```
int running_sum(int n){
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}
```

```
int main(void){
    int n = 4;
    int answer;
    answer = running_sum(4);
}
```

```
BASE_CASE
AND R2, R2, #0
ADD R2, R2, #1 ;set fn = 1
STR R2, R5, #0

RETURN
;set return value
LDR R0, R5, #0
STR R0, R5, #3 ; END of Step 8 on Gitlab

;callee tear-down of Running(n)'s activation record
ADD R6, R6, #1 ;pop local variables – Step 9 on Gitlab
```



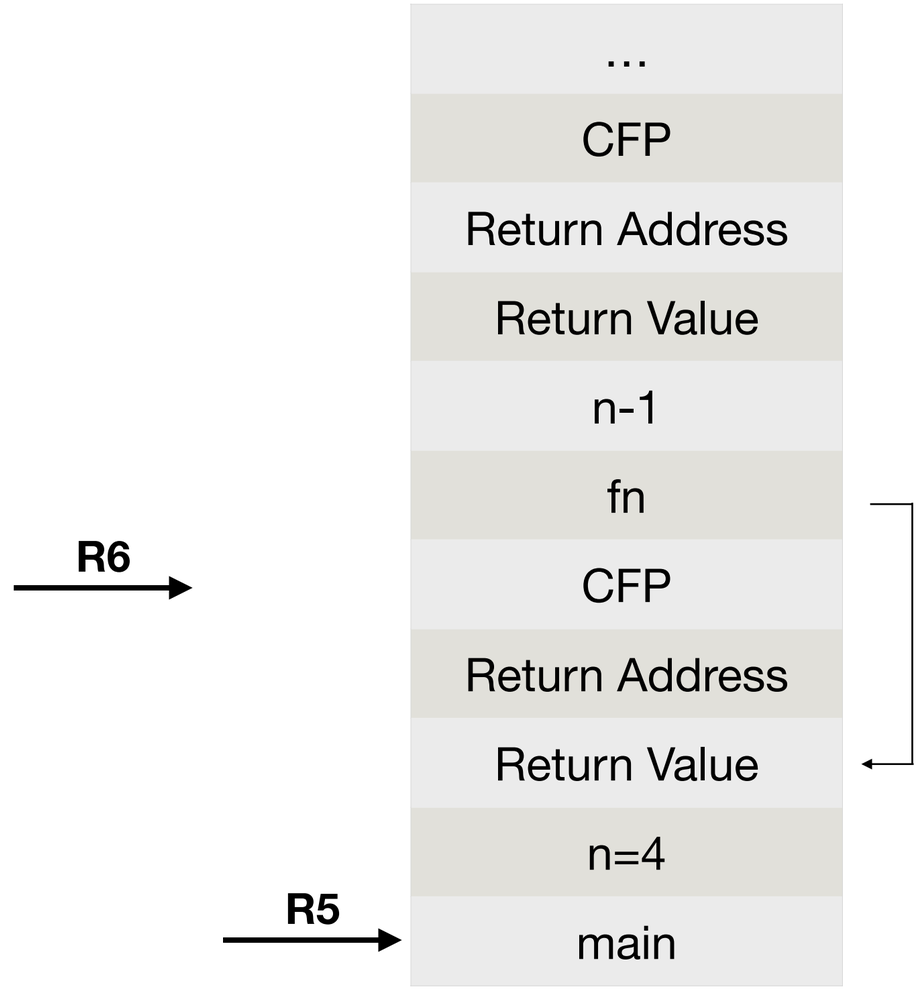
Gitlab C2L3 steps

Review

```c
int running_sum(int n){
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}
```

```c
int main(void){
    int n = 4;
    int answer;
    answer = running_sum(4);
}
```

```
BASE_CASE
AND R2, R2, #0
ADD R2, R2, #1 ;set fn = 1
STR R2, R5, #0

RETURN
;set return value
LDR R0, R5, #3
STR R0, R5, #3 ; END of Step 8 on Gitlab

;callee tear-down of Running(n)'s activation record
ADD R6, R6, #1 ;pop local variables – Step 9 on Gitlab

;restore caller's frame pointer and return address
LDR R5, R6, #0 ; restore CFP – Step 10 on Gitlab
```



Gitlab C2L3 steps

Review

```c
int running_sum(int n){
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}
```

```c
int main(void){
    int n = 4;
    int answer;
    answer = running_sum(4);
}
```

```
BASE_CASE
AND R2, R2, #0
ADD R2, R2, #1 ;set fn = 1
STR R2, R5, #0

RETURN
;set return value
LDR R0, R5, #3
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;restore caller's frame pointer and return address
LDR R5, R6, #0 ; restore CFP – Step 10 on Gitlab
ADD R6, R6, #1
```

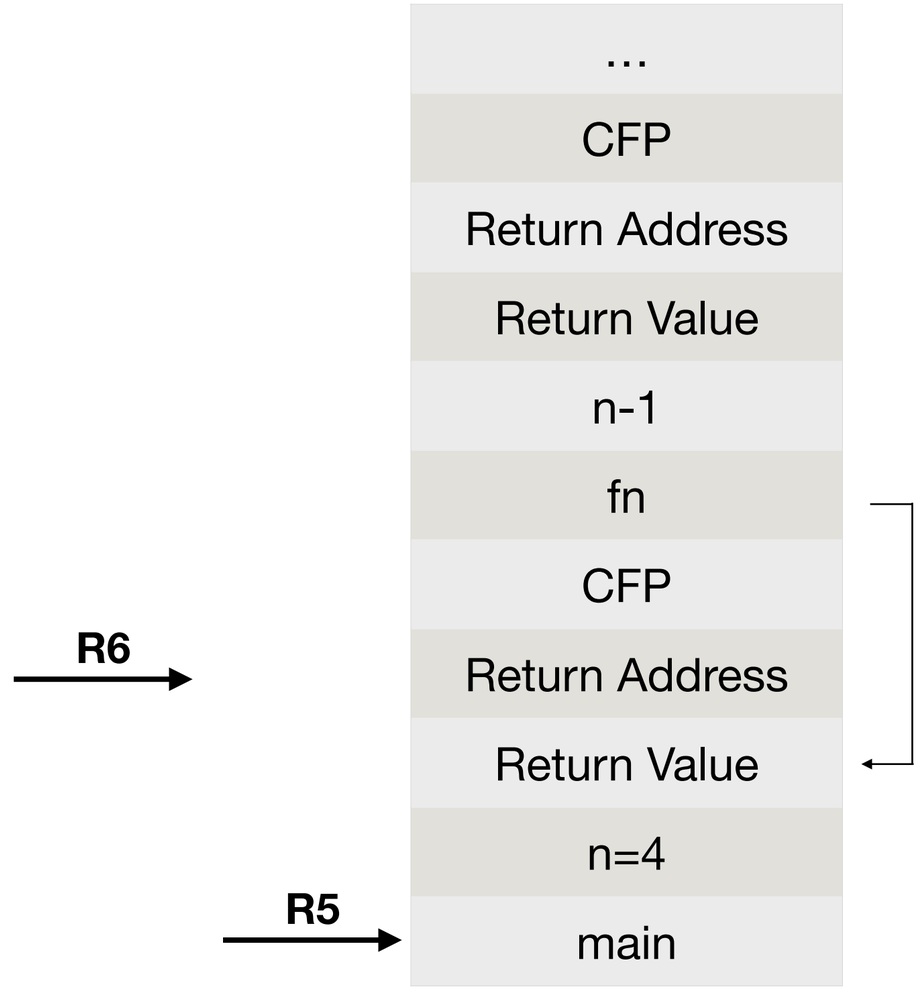

Gitlab C2L3 steps

Review

```c
int running_sum(int n){
    int fn;
    if (n==1)
        fn = 1;
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}
```

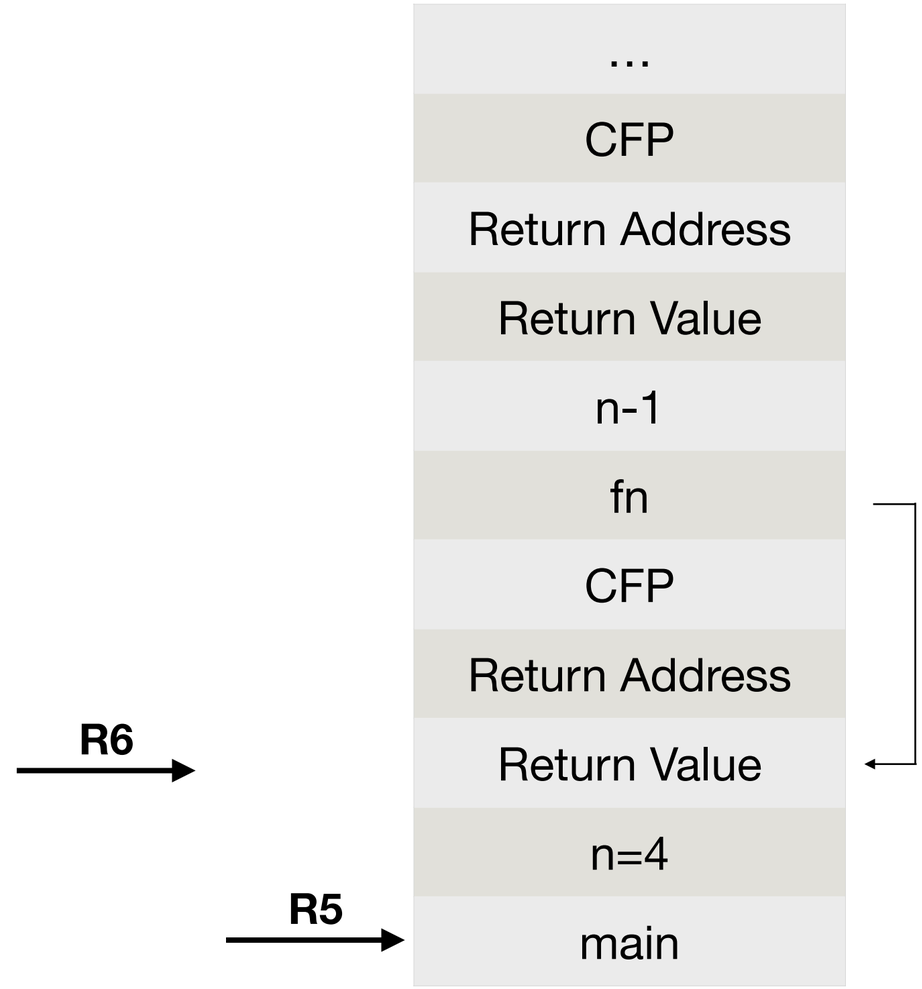
```c
int main(void){
    int n = 4;
    int answer;
    answer = running_sum(4);
}
```

```
BASE_CASE
AND R2, R2, #0
ADD R2, R2, #1 ;set fn = 1
STR R2, R5, #0

RETURN
;set return value
LDR R0, R5, #0
STR R0, R5, #3 ; END of Step 8 on Gitlab

;callee tear-down of Running(n)'s activation record
ADD R6, R6, #1 ;pop local variables – Step 9 on Gitlab

;restore caller's frame pointer and return address
LDR R5, R6, #0 ; restore CFP – Step 10 on Gitlab
ADD R6, R6, #1
LDR R7, R6, #0 ; prime R7 for RET
ADD R6, R6, #1 ; Step 11 on Gitlab
```



Gitlab C2L3 steps

Review

```c
int running_sum(int n){
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}
```

```c
int main(void){
    int n = 4;
    int answer;
    answer = running_sum(4);
}
```

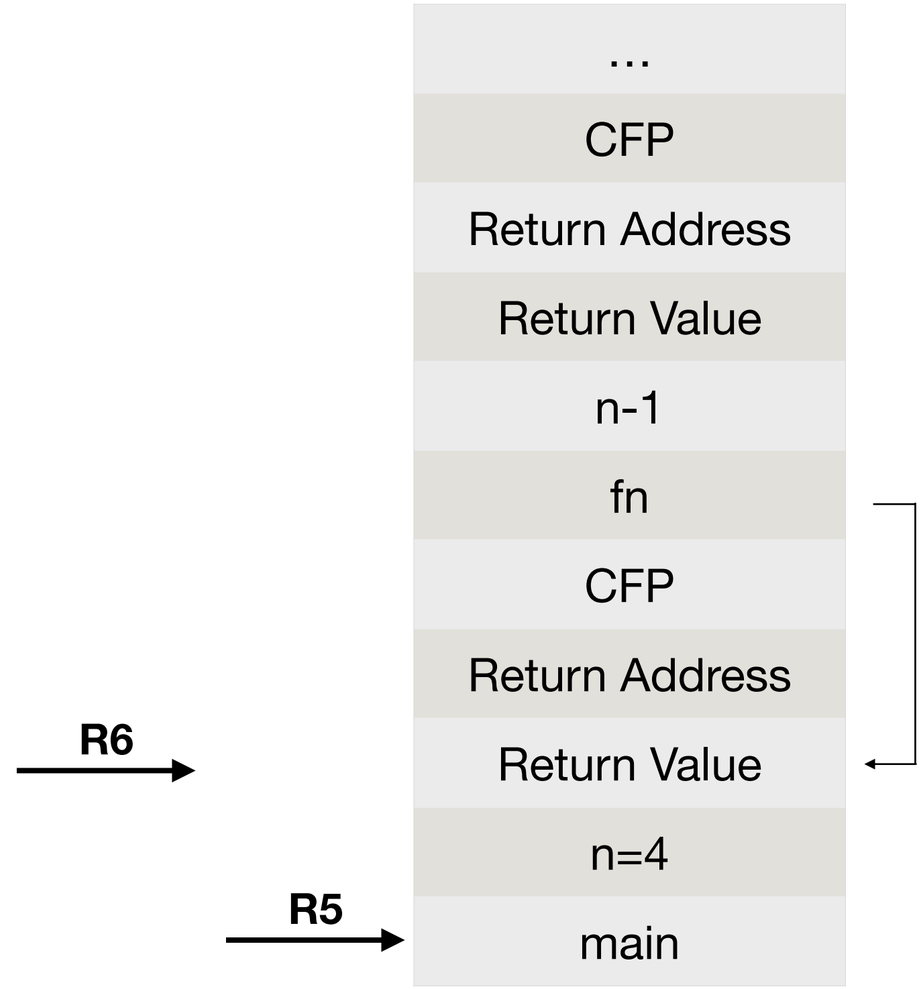
```
BASE_CASE
AND R2, R2, #0
ADD R2, R2, #1 ;set fn = 1
STR R2, R5, #0

RETURN
;set return value
LDR R0, R5, #0
STR R0, R5, #3 ; END of Step 8 on Gitlab

;callee tear-down of Running(n)'s activation record
ADD R6, R6, #1 ;pop local variables – Step 9 on Gitlab

;restore caller's frame pointer and return address
LDR R5, R6, #0 ; restore CFP – Step 10 on Gitlab
ADD R6, R6, #1
LDR R7, R6, #0 ; prime R7 for RET
ADD R6, R6, #1 ; Step 11 on Gitlab

;return to caller
```



Gitlab C2L3 steps

Review

```c
int running_sum(int n){
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}
```

```c
int main(void){
    int n = 4;
    int answer;
    answer = running_sum(4);
}
```

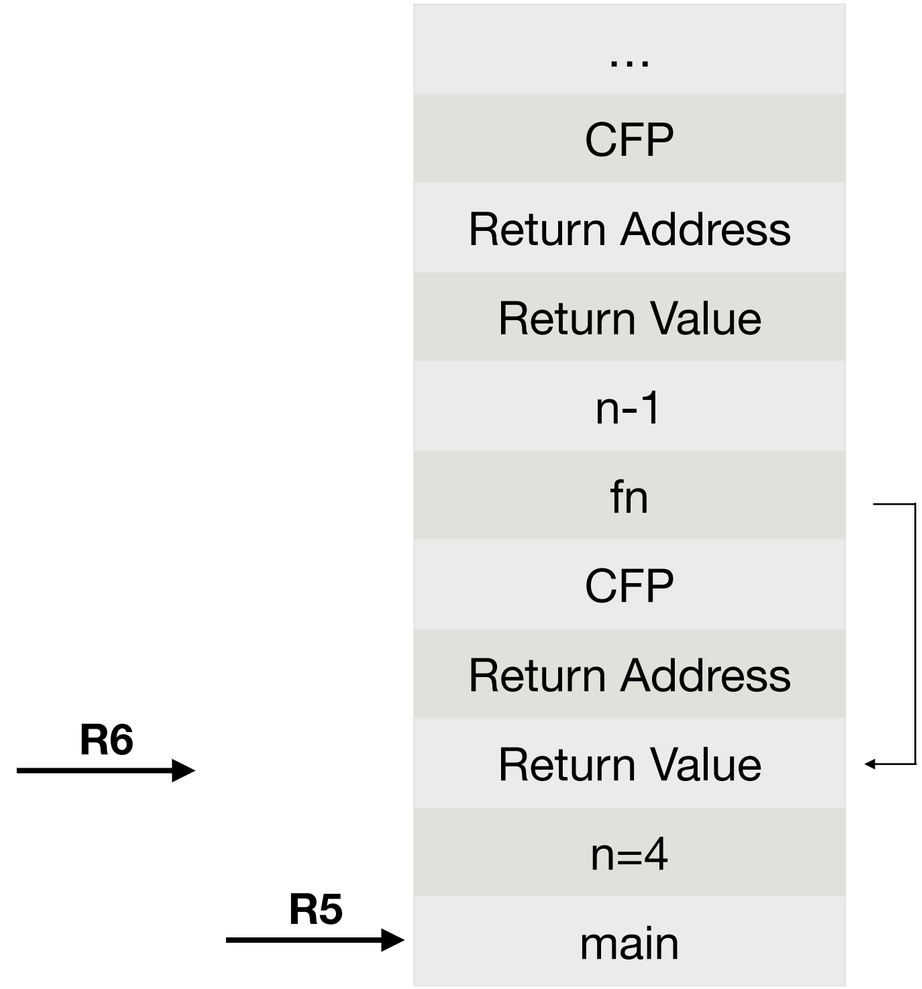
```
BASE_CASE
AND R2, R2, #0
ADD R2, R2, #1 ;set fn = 1
STR R2, R5, #0

RETURN
;set return value
LDR R0, R5, #0
STR R0, R5, #3 ; END of Step 8 on Gitlab

;callee tear-down of Running(n)'s activation record
ADD R6, R6, #1 ;pop local variables – Step 9 on Gitlab

;restore caller's frame pointer and return address
LDR R5, R6, #0 ; restore CFP – Step 10 on Gitlab
ADD R6, R6, #1
LDR R7, R6, #0 ; prime R7 for RET
ADD R6, R6, #1 ; Step 11 on Gitlab

;return to caller
RET ; Step 12 on Gitlab
```



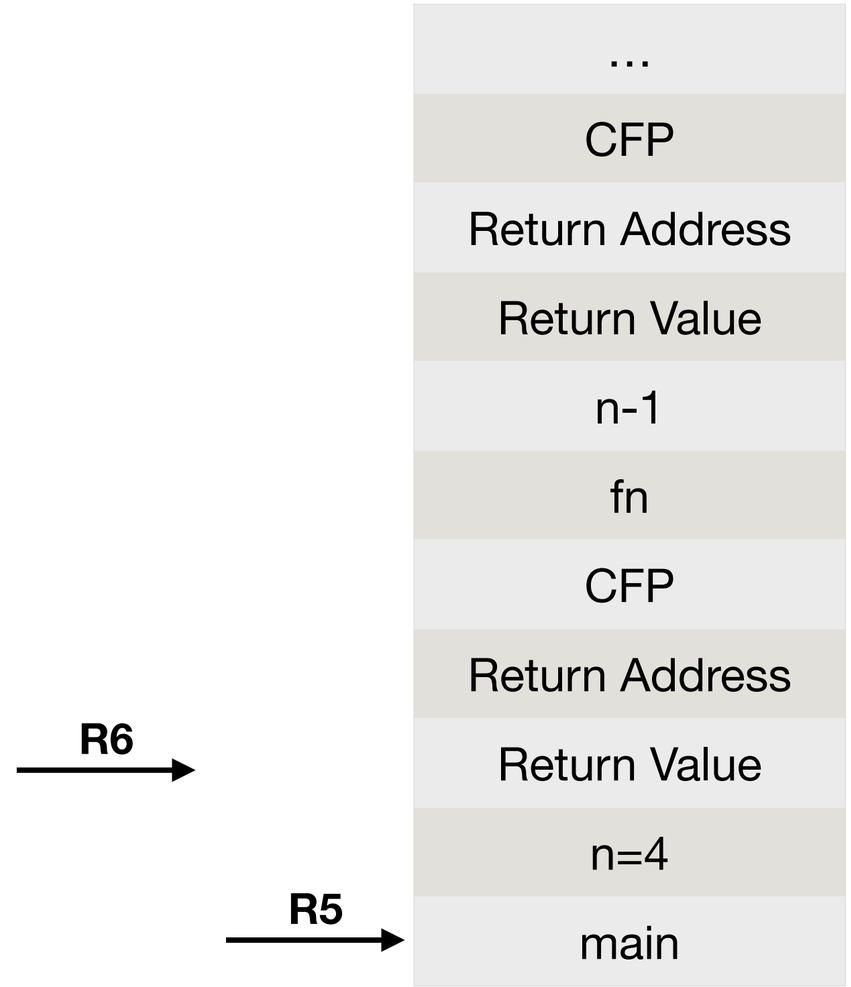
Gitlab C2L3 steps

```c
int running_sum(int n){
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}
```

Review

```c
int main(void){
    int n = 4;
    int answer;
    answer = running_sum(4);
}
```

```
;Caller stack Tear-down for Running(n)
```



Gitlab C2L3 steps

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    int fn;
    if (n==1)
        fn = 1;
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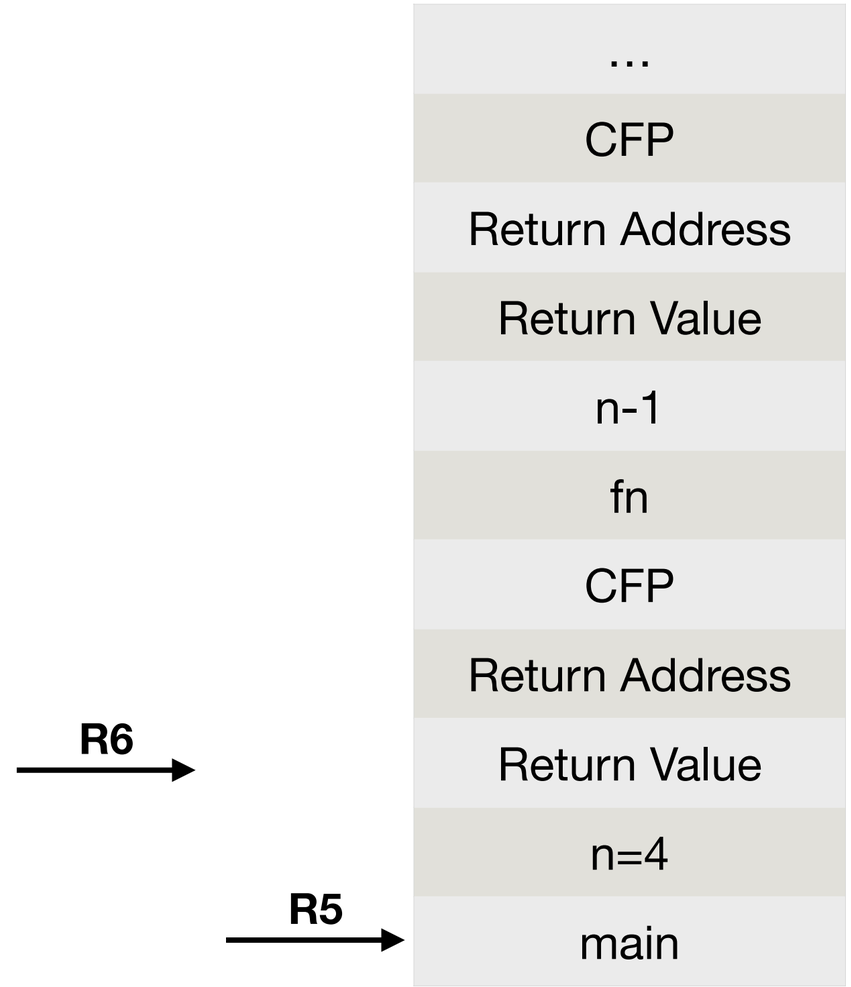
```

Review

```c
int main(void){
    int n = 4;
    int answer;
    answer = running_sum(4);
}
```

```
;Caller stack Tear-down for Running(n)
LDR R0, R6, #0  ;copy return value to R0
```



Gitlab C2L3 steps

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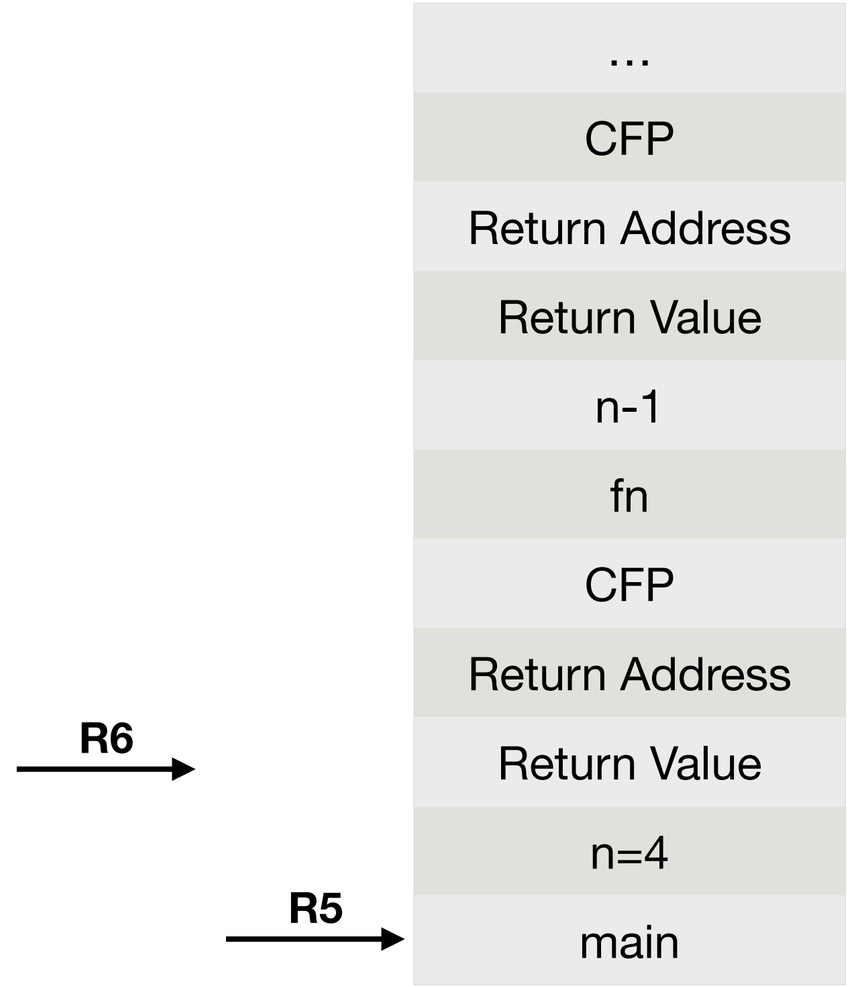
Review

```c
int main(void){
    int n = 4;
    int answer;
    answer = running_sum(4);
}
```

```
;Caller stack Tear-down for Running(n)
LDR R0, R6, #0  ;copy return value to R0
STR R0, R5, #-1 ;save return value to answer
```

Inside main's activation frame, answer is the second local variable



Gitlab C2L3 steps

```c
int running_sum(int n){
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```

Review

```c
int main(void){
    int n = 4;
    int answer;
    answer = running_sum(4);
}
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```
;Caller stack Tear-down for Running(n)
LDR R0, R6, #0  ;copy return value to R0
STR R0, R5, #-1 ;save return value to answer
ADD R6, R6, #1  ;pop return value from stack
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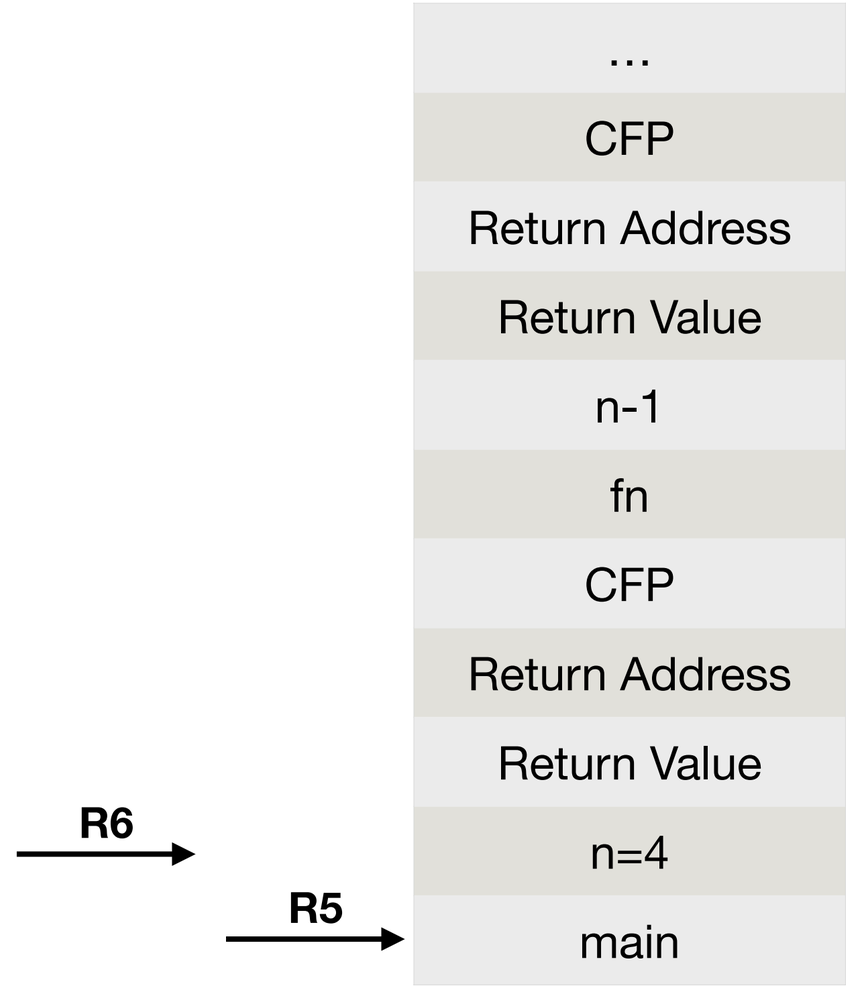

Gitlab C2L3 steps

```c
int running_sum(int n){
    int fn;
    if (n==1)
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        fn = n + running_sum(n-1);
    return fn;
}
```
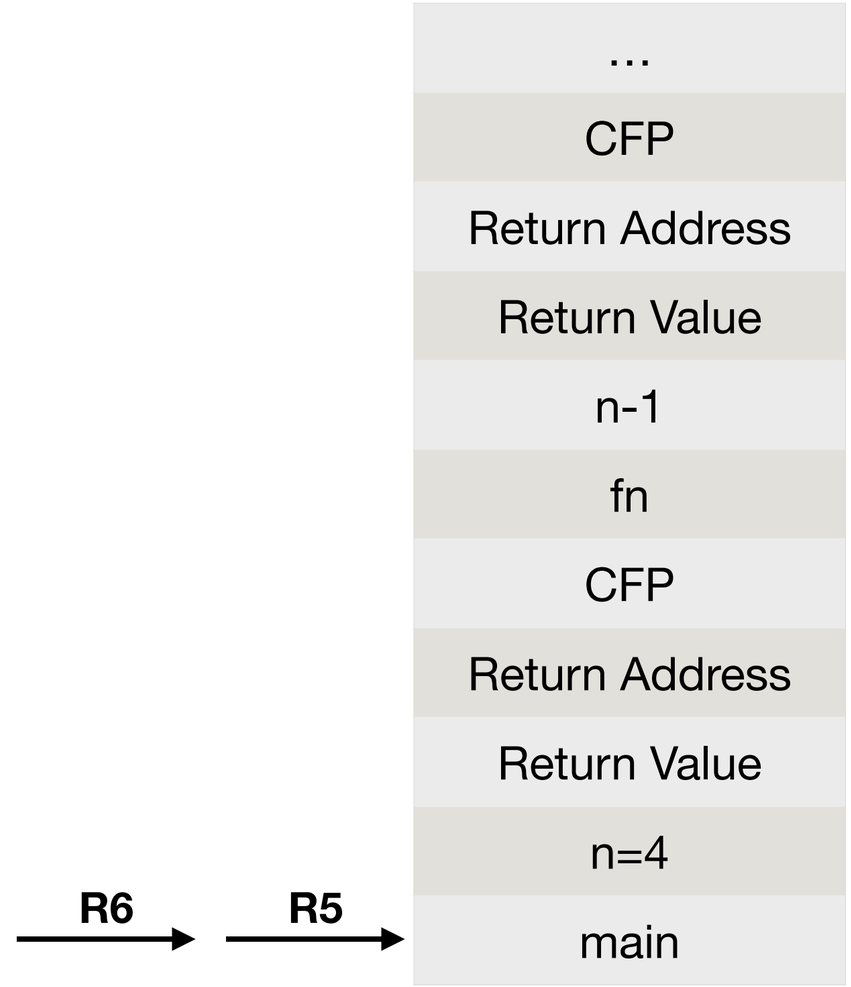

Review

```c
int main(void){
    int n = 4;
    int answer;
    answer = running_sum(4);
}
```

```
;Caller stack Tear-down for Running(n)
LDR R0, R6, #0  ;copy return value to R0
STR R0, R5, #-1 ;save return value to answer
ADD R6, R6, #1  ;pop return value from stack
ADD R6, R6, #1  ;pop argument from stack
```

→ Inside main's activation frame, answer is the second local variable



Gitlab C2L3 steps

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Review

```c
int main(void){
    int n = 4;
    int answer;
    answer = running_sum(4);
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```

```
;Caller stack Tear-down for Running(n)
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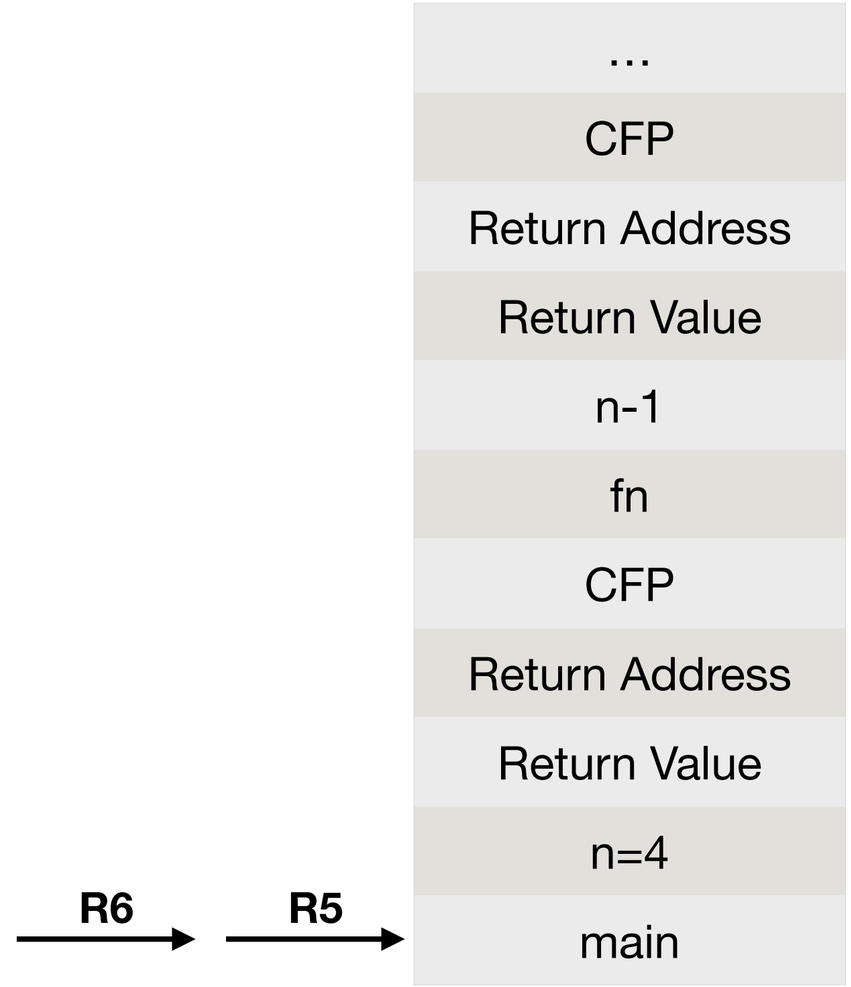

Back to where we started!

Gitlab C2L3 steps

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int running_sum(int n){
    int fn;
    if (n==1)
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}
```

Review

```c
int main(void){
    int n = 4;
    int answer;
    answer = running_sum(4);
}
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```
;Caller stack Tear-down for Running(n)
LDR R0, R6, #0  ;copy return value to R0
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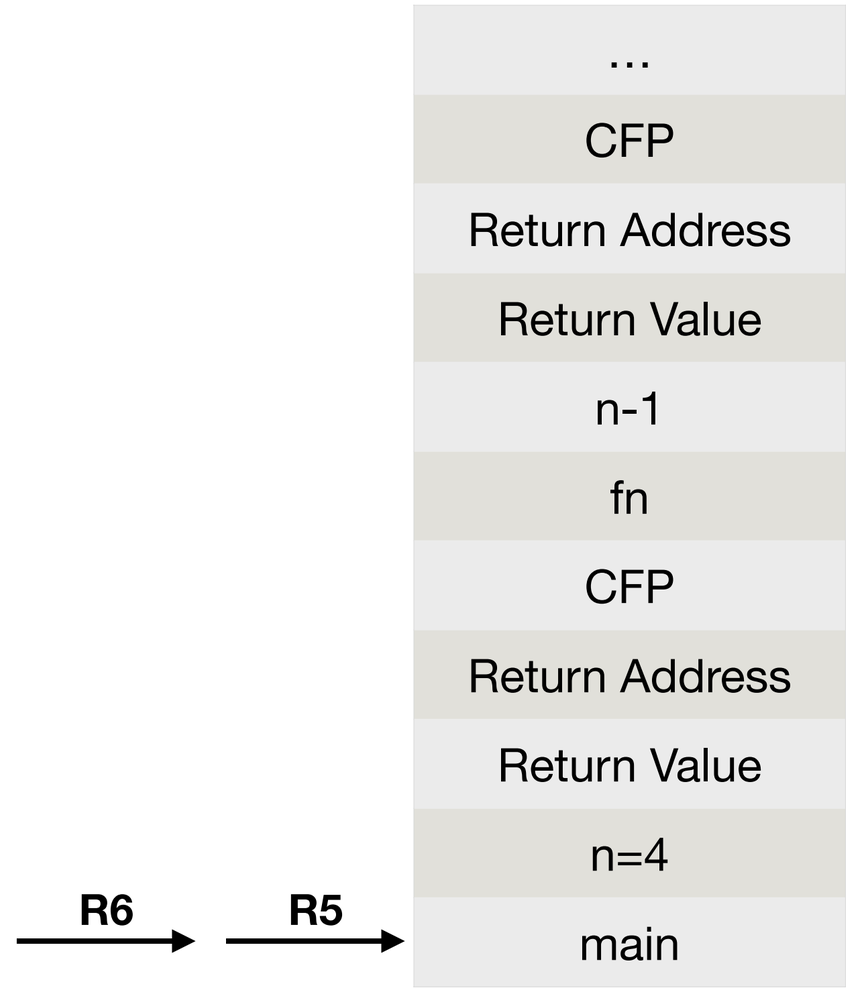
ADD R6, R6, #1  ;pop return value from stack
ADD R6, R6, #1  ;pop argument from stack
```

Inside main's activation frame, answer is the second local variable

Practice practice practice!

Back to where we started!



Gitlab C2L3 steps

Solving a maze

Solving a maze

- We represent a maze by a 2D grid of size $N \times M$

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    - Do not go outside grid

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		X	X	
X			@	X
			X	
E	X			

Solving a maze

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Strategy: Mark current cell as visited and explore solution space. Exploration defined by four possible moves (U, D, L, R).

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	X	X	
X		@	X
		X	
E	X		

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	X	X	
X		@	X
		X	
E	X		

	X	X	
X		V	X
		X	
E	X		

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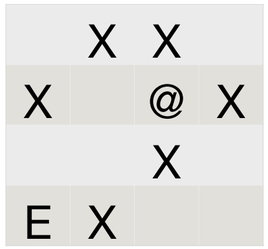

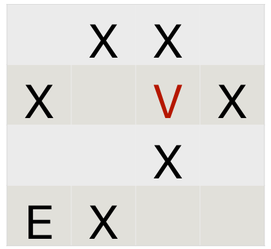

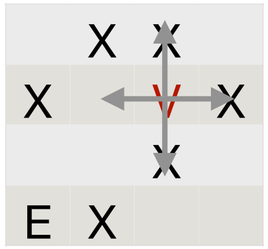

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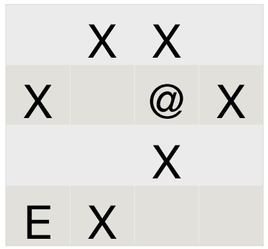

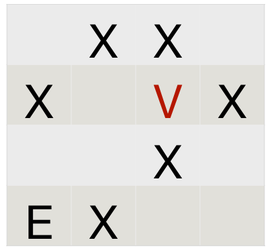

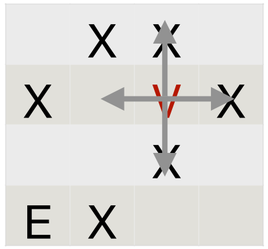

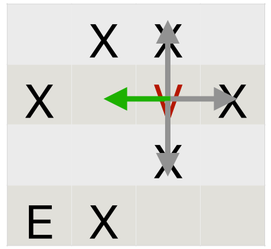

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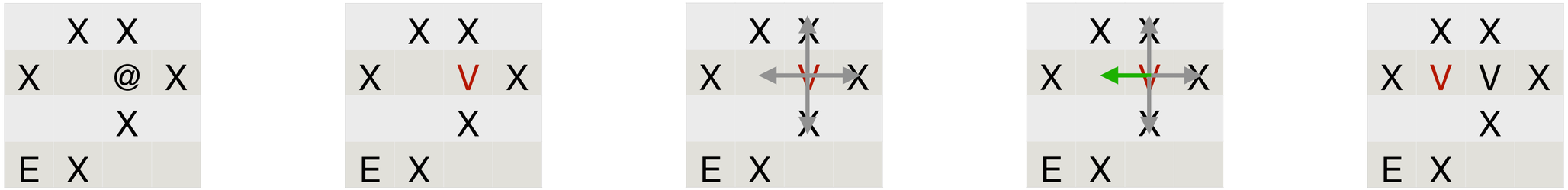

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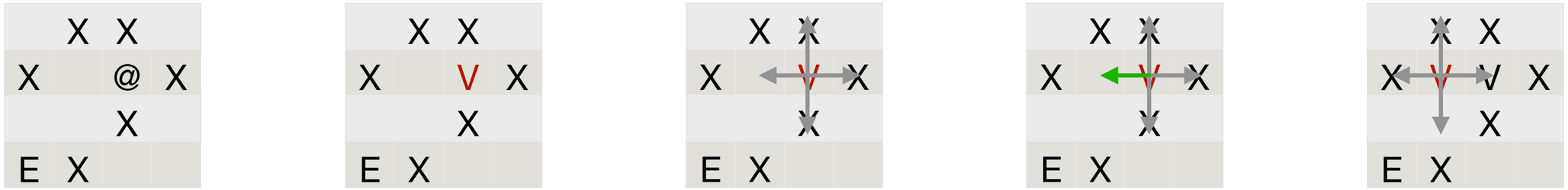

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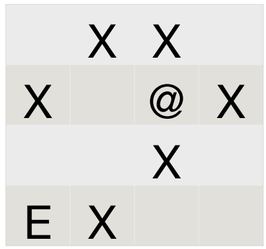

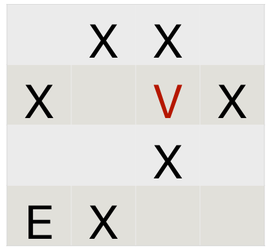

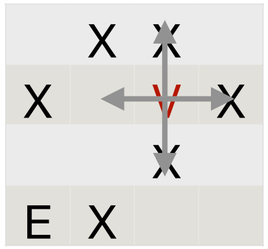

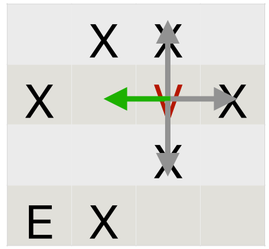

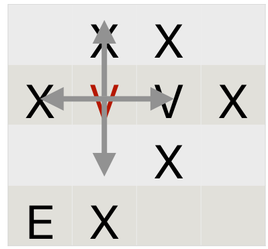

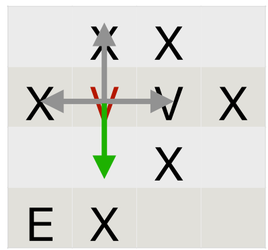

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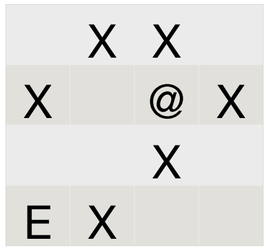

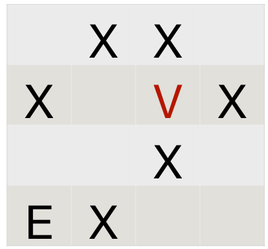

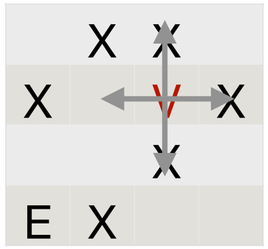

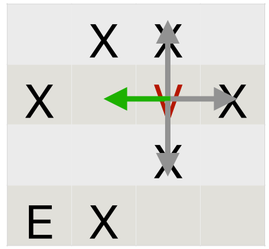

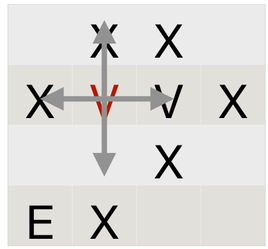

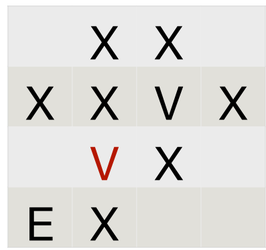

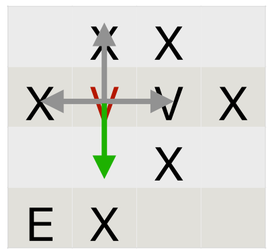

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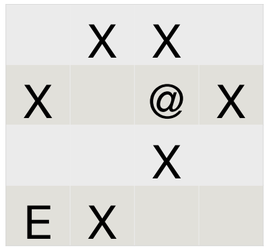

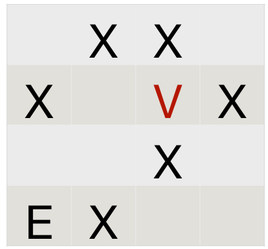

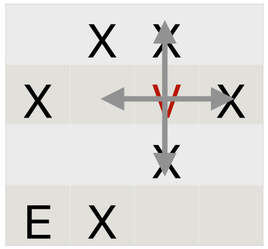

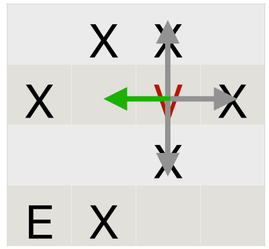

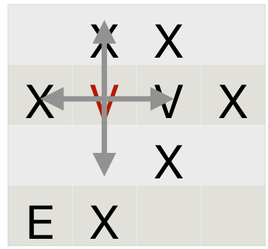

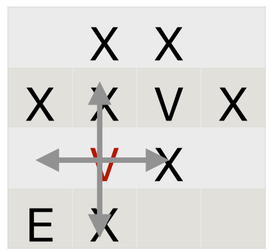

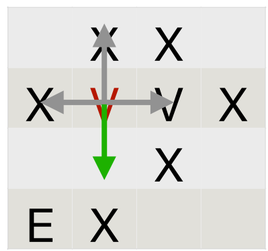

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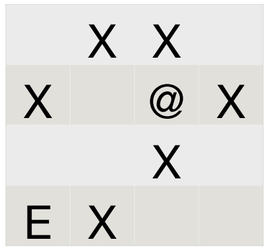

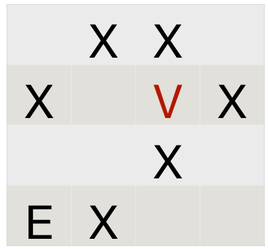

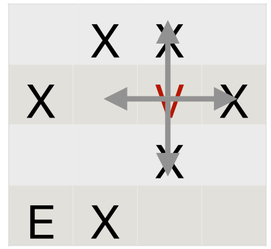

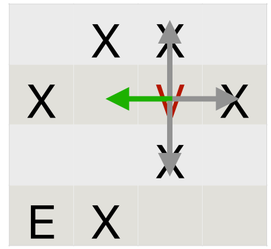

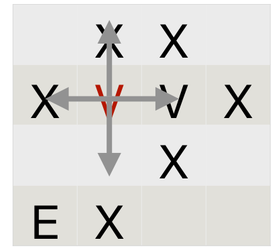

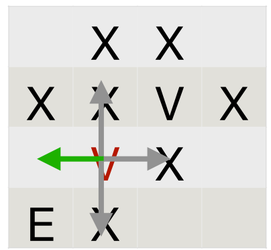

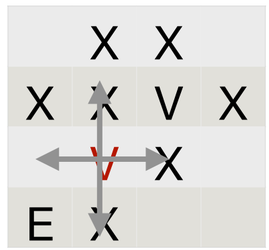

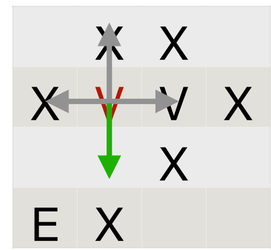

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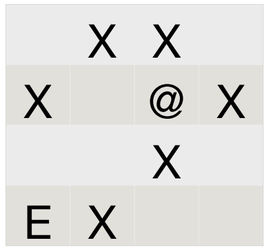

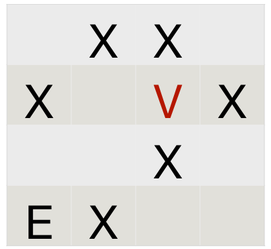

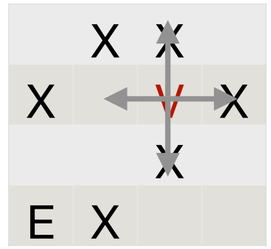

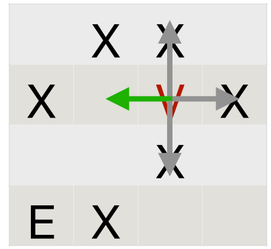

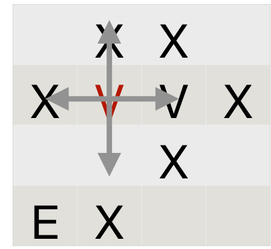

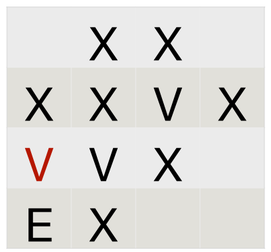

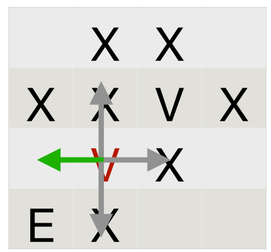

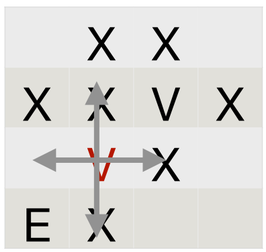

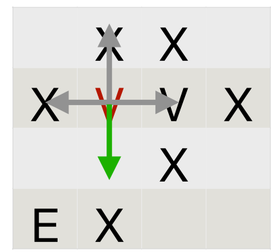

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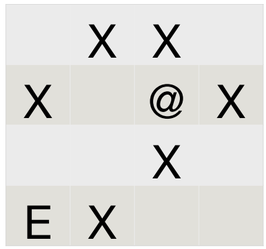

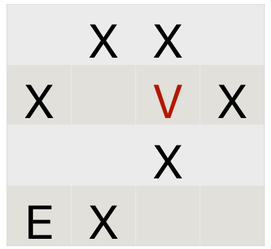

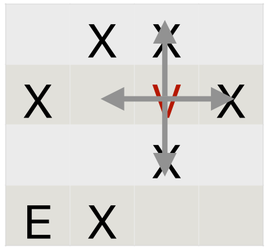

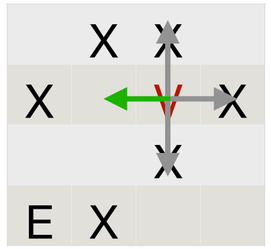

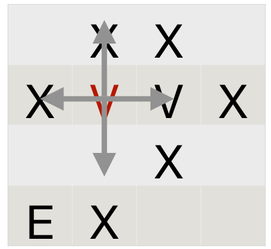

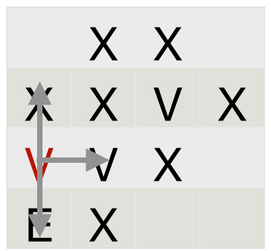

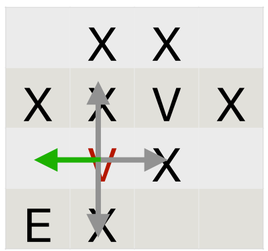

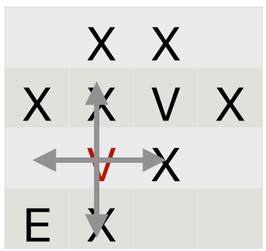

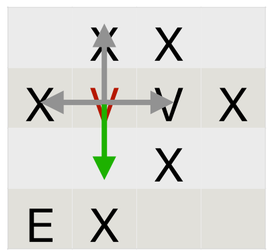

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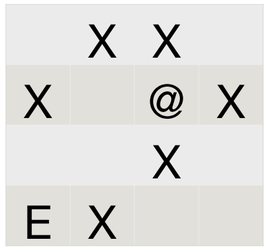

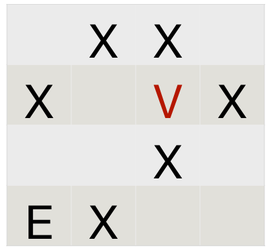

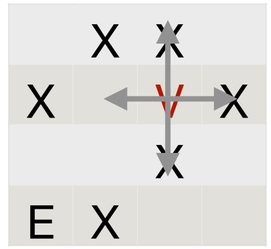

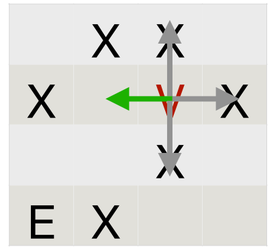

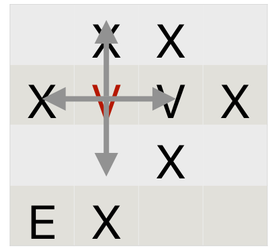

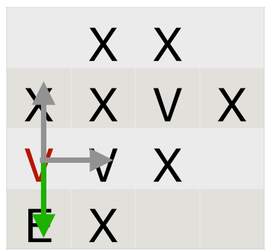

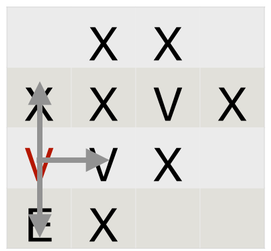

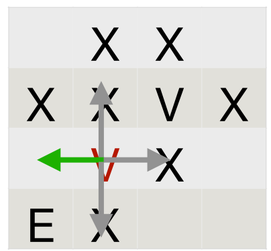

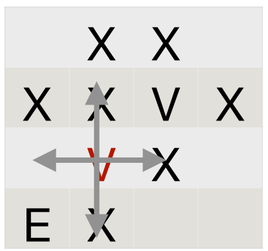

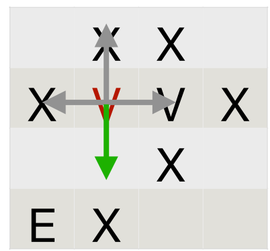

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```
if (xpos < 0 || xpos >= MAZE_WIDTH || ypos < 0 || ypos >= MAZE_HEIGHT)
  return 0;

if (maze[xpos][ypos] == 'E')          // Found the Exit!
  return 1;

if (maze[xpos][ypos] != ' ')          // Space is not empty (possibly X or V)
  return 0;
```

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  - `ExitMaze` is the function exploring the solution space.

```
// Go Down
if (ExitMaze(maze, xpos + 1, ypos)) {
  maze[xpos][ypos]='P';
  return 1;
}

// Go Right
if (ExitMaze(maze, xpos, ypos + 1)) {
  maze[xpos][ypos]='P';
  return 1;
}
```

```
// Go Up
if (ExitMaze(maze, xpos - 1, ypos)) {
  maze[xpos][ypos]='P';
  return 1;
}

// Go Left
if (ExitMaze(maze, xpos, ypos - 1)) {
  maze[xpos][ypos]='P';
  return 1;
}
```

Exercise

- There is an `ExitMaze` function on Gitlab which I tested to work.
- Modify it by adding a `main` function, board definition and try it on this maze.

	X					X
	X			X		
			@	X	X	X
	X					
	X	X	X	X		
X			E	X		
X	X		X	X		X
						X

N - Queens Problem

- In chess, a Queen can attack another piece within its line of sight as long as that piece is in the same: **row**, **column** or **diagonal**.

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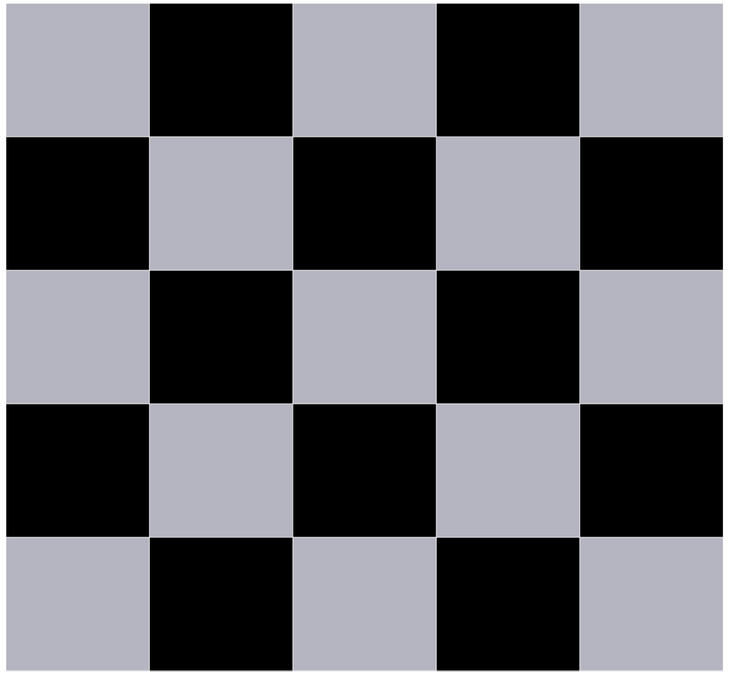

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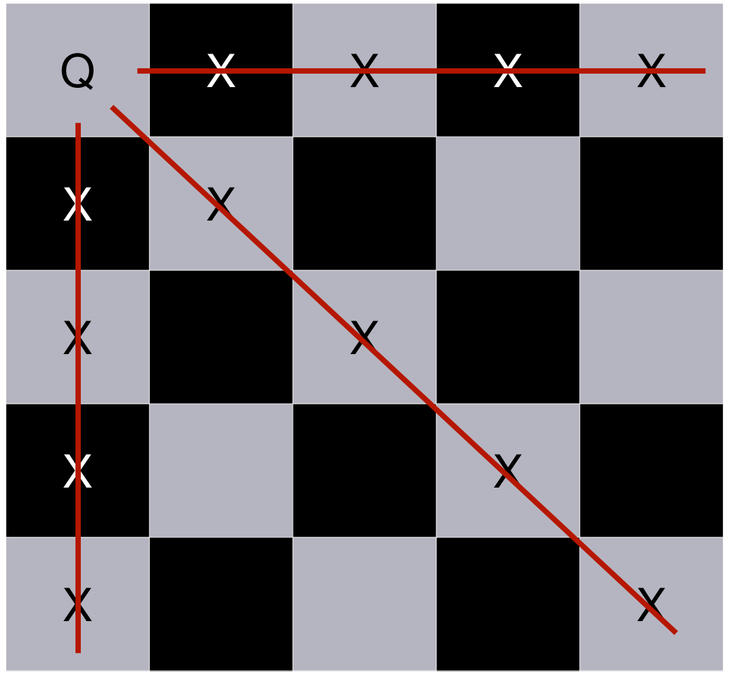

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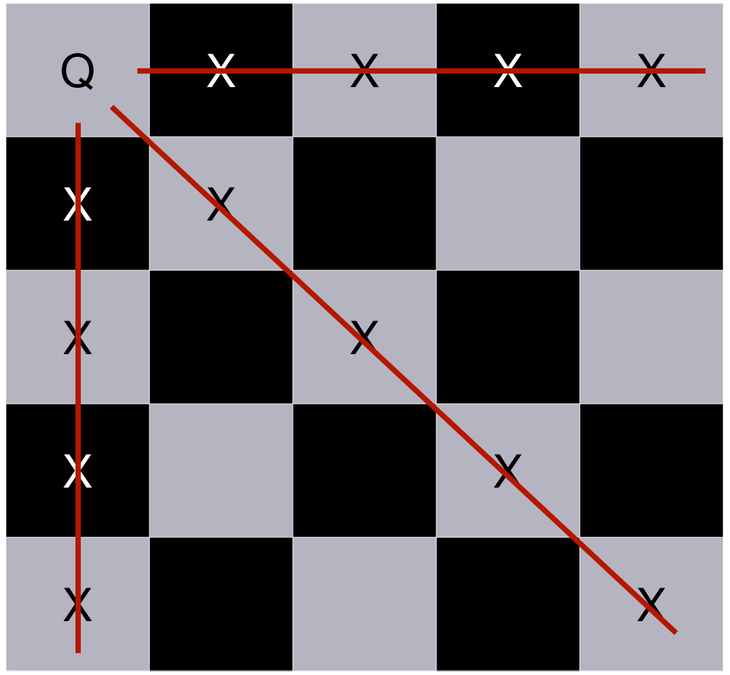

- **Question**: Given an N x N grid, is it possible to place N Queens in the grid so that no two Queens can attack each other ?

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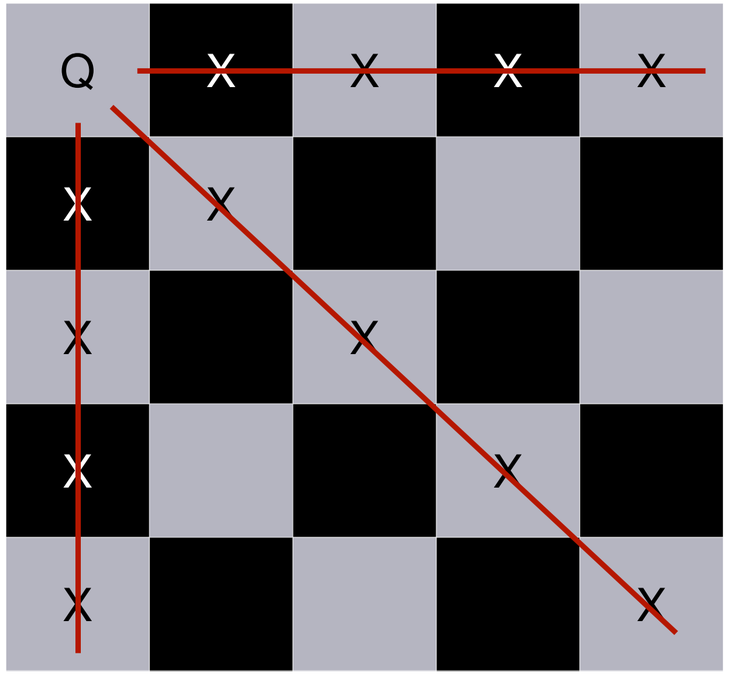

- **Question**: Given an N x N grid, is it possible to place N Queens in the grid so that no two Queens can attack each other ?

- **Answer:** Yes.

N - Queens Problem

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- Here is a possible solution for the 5 x 5 grid.

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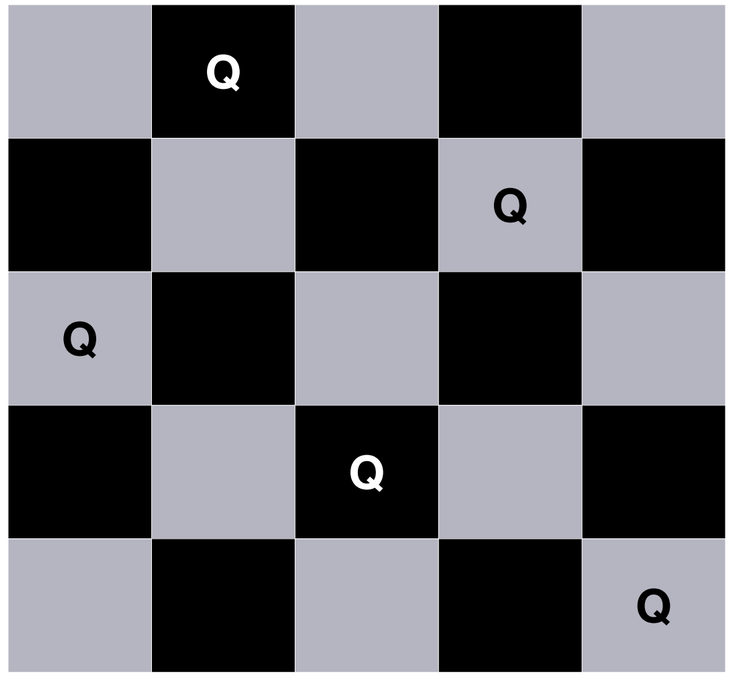

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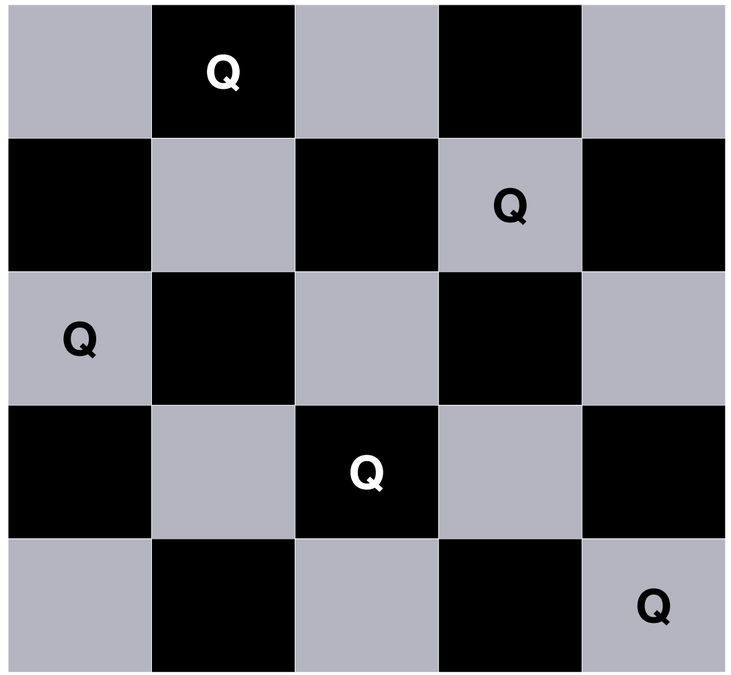
  - Not unique



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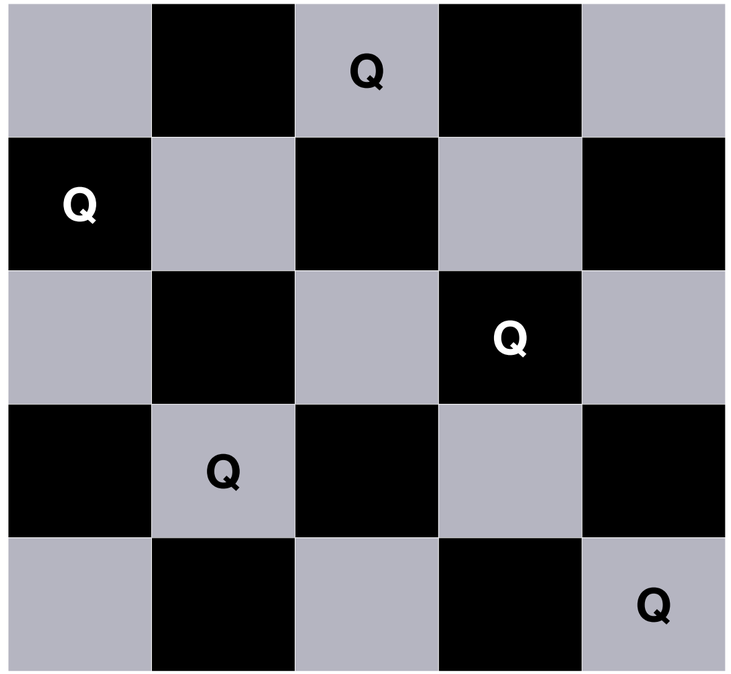

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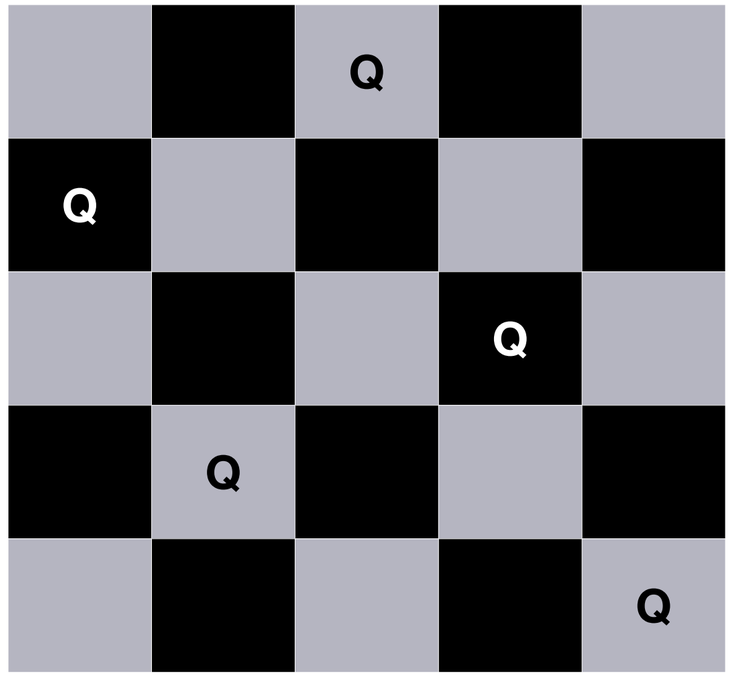

N - Queens Problem

- Here is a possible solution for the 5 x 5 grid.
  - Not unique
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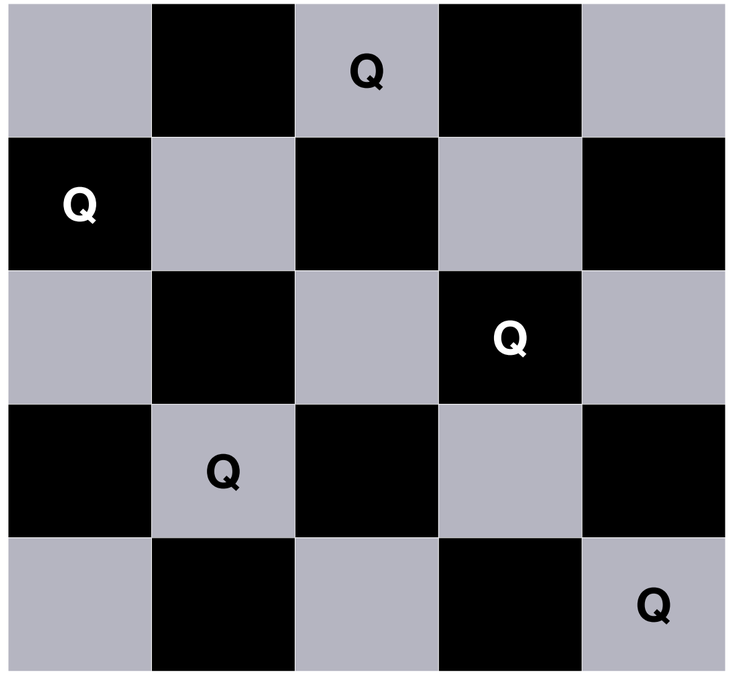
  - Solution: Recursion with *backtracking.*



N - Queens Problem

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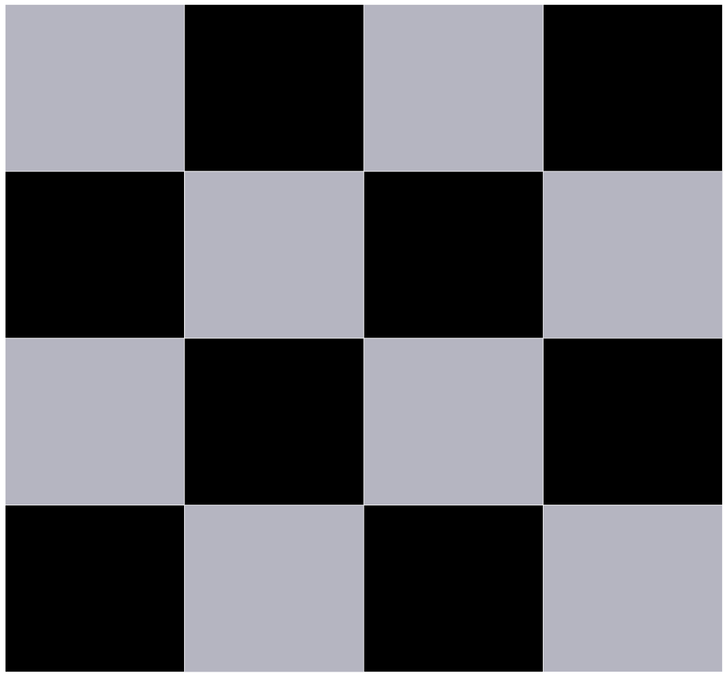

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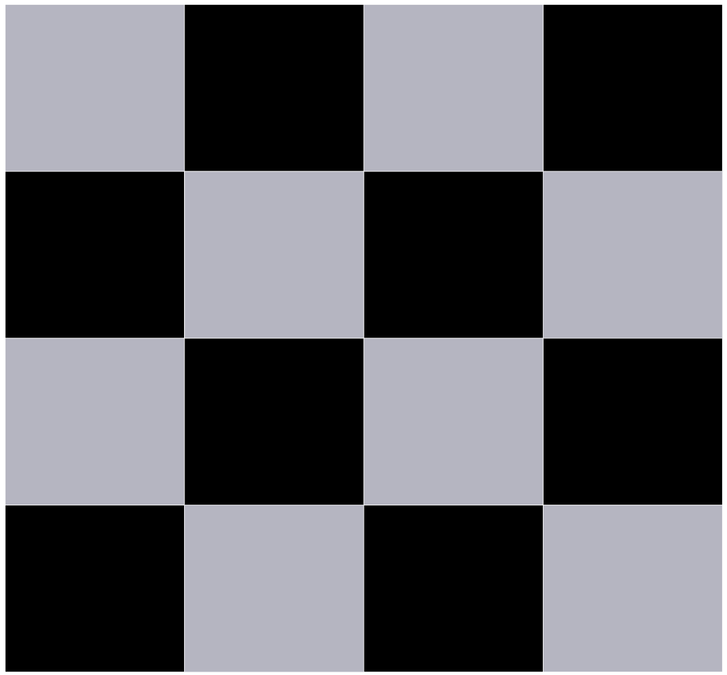

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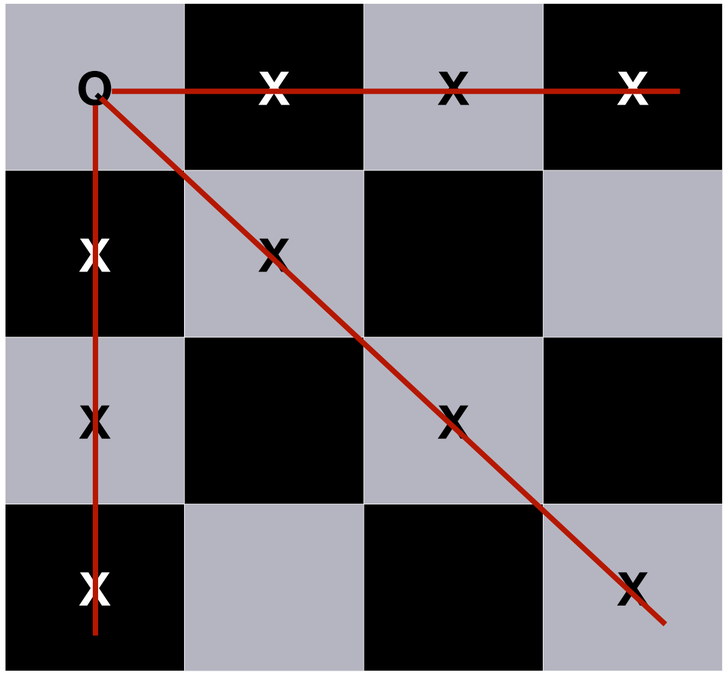

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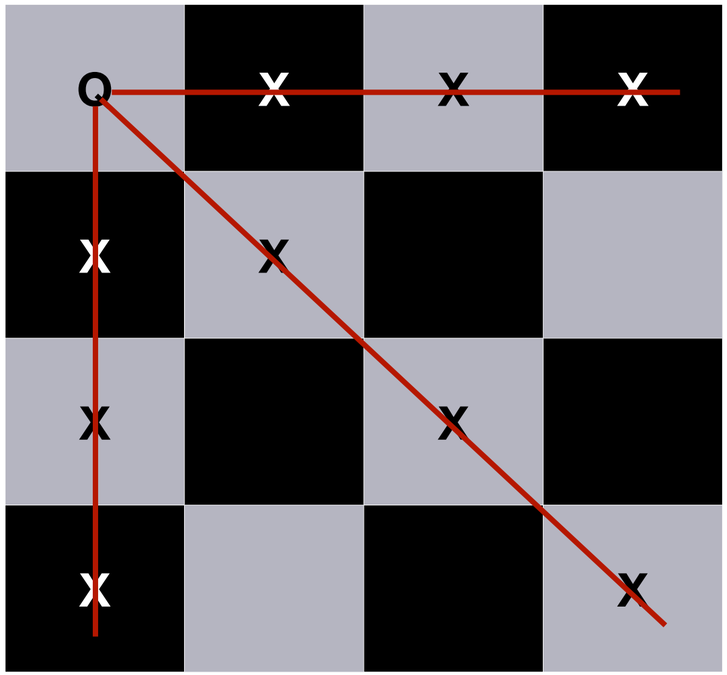

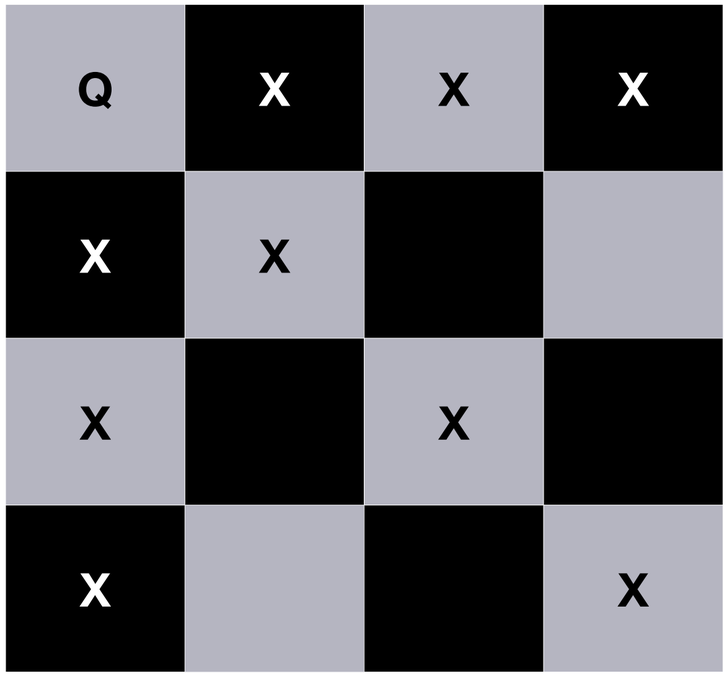

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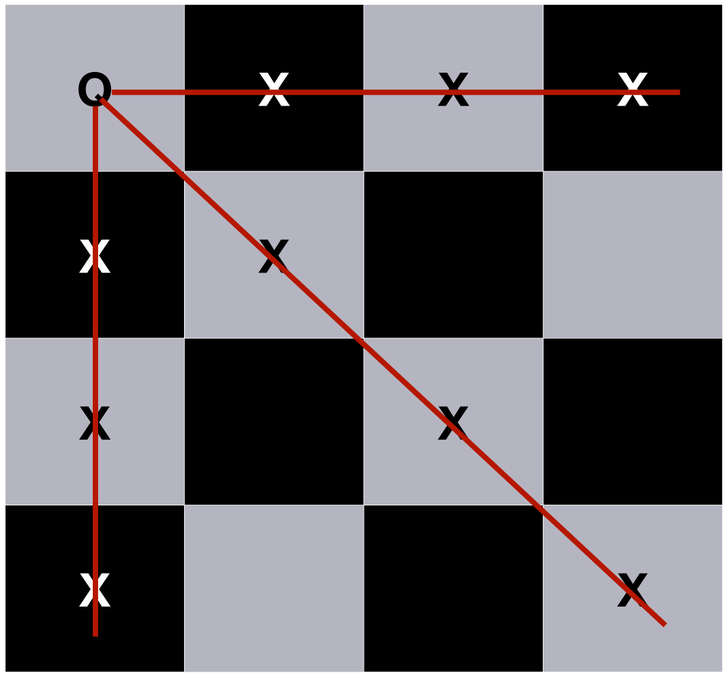

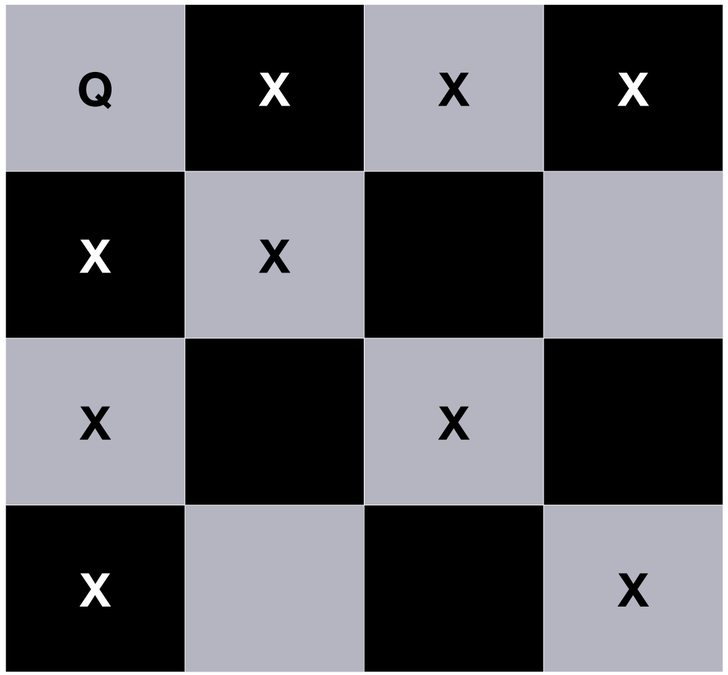

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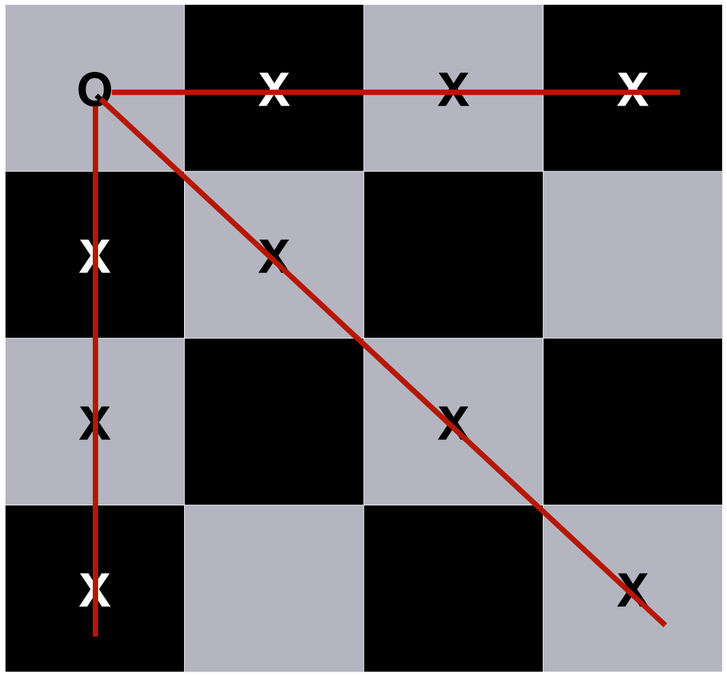

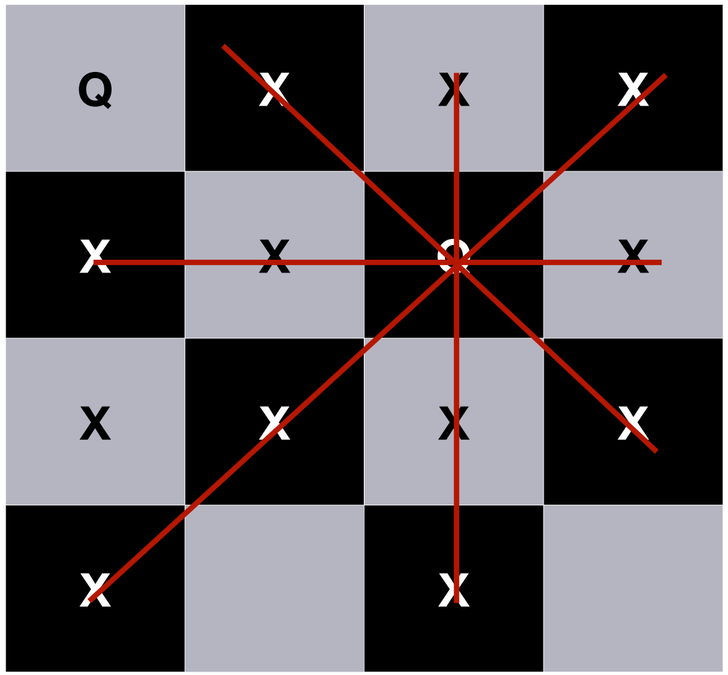

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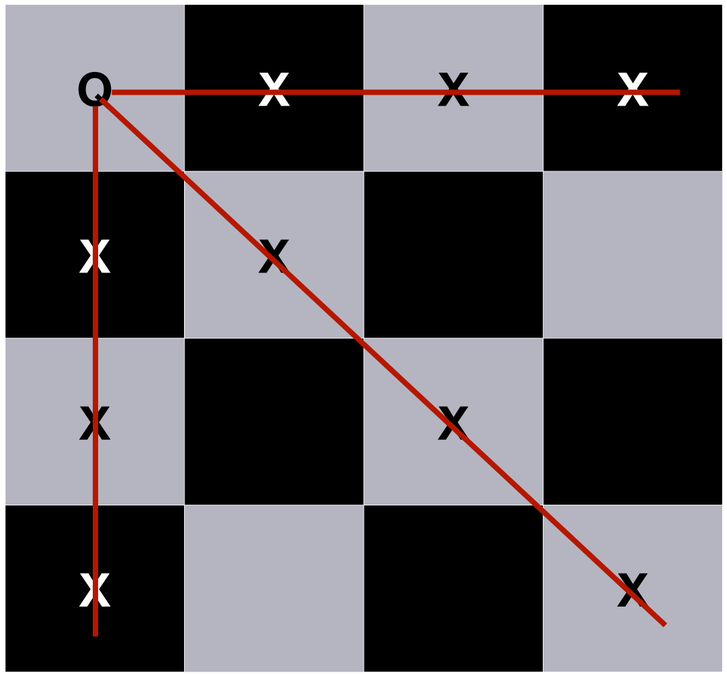

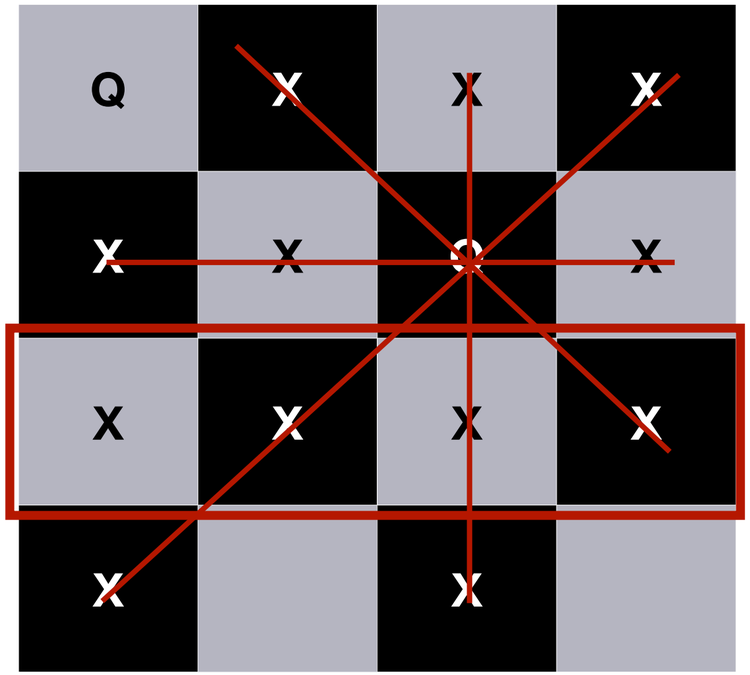

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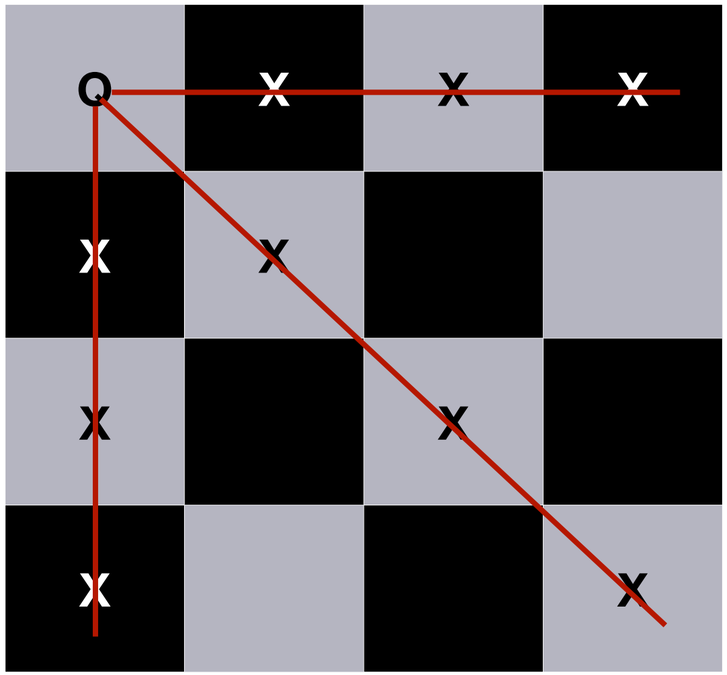

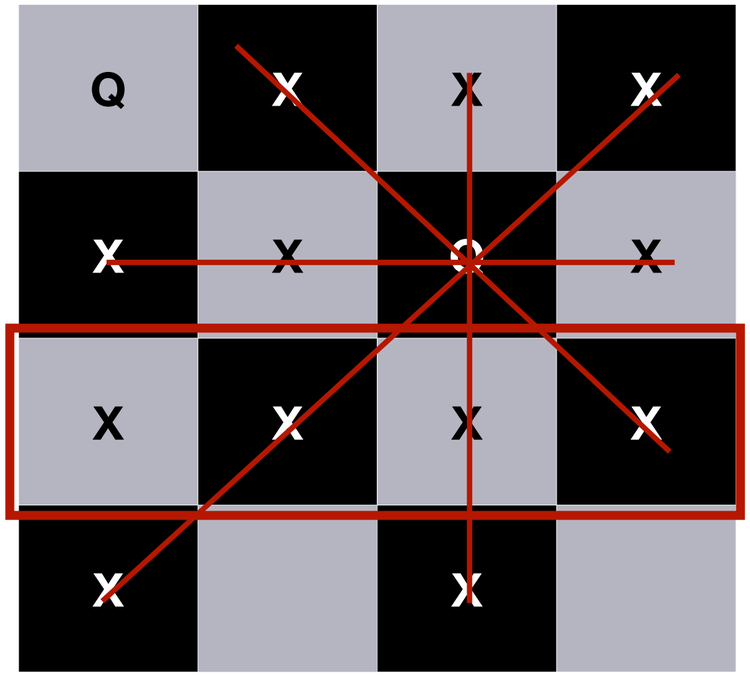

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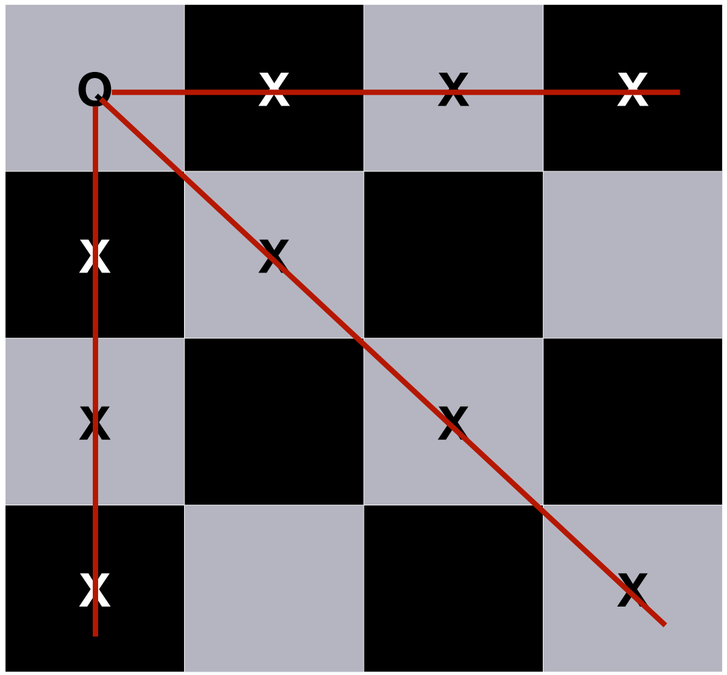

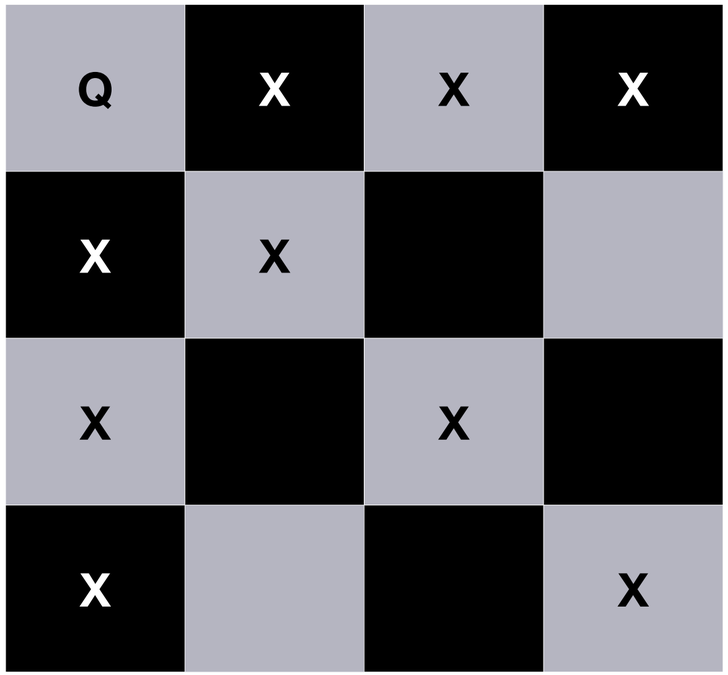

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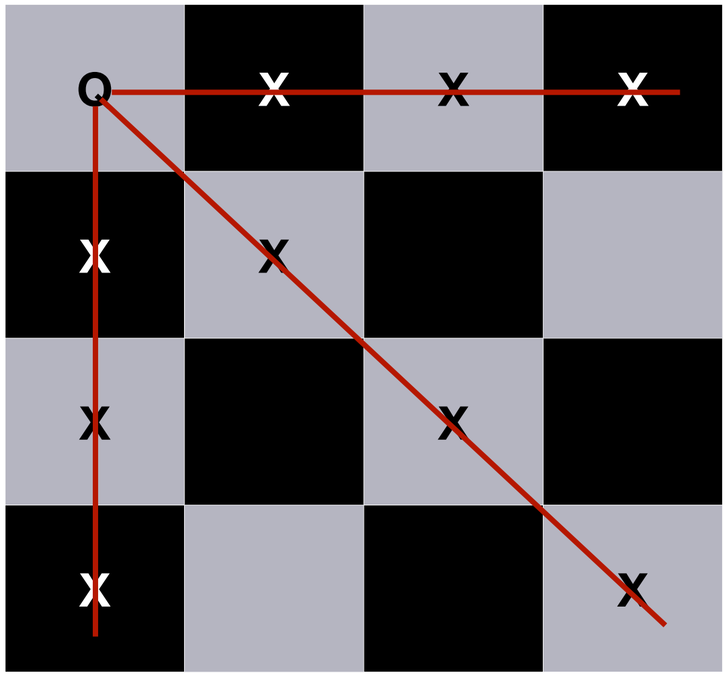

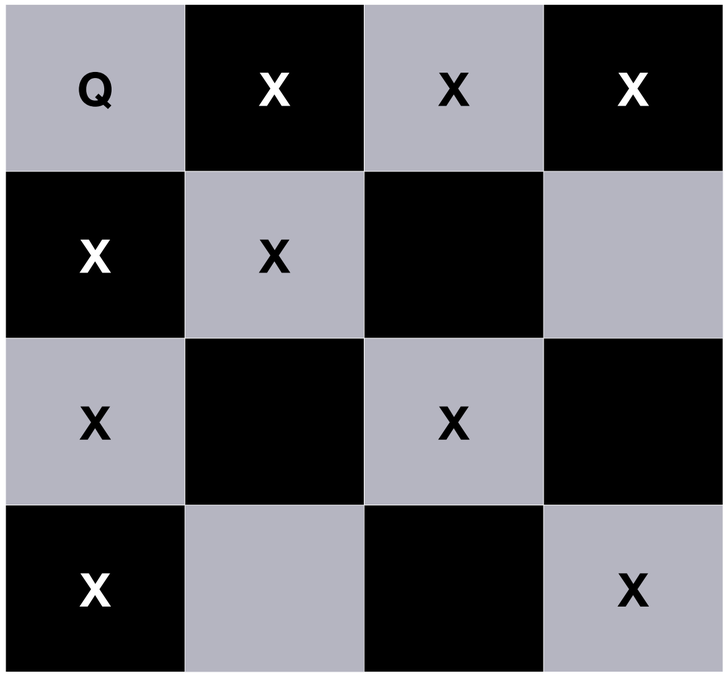

Choice #1.2

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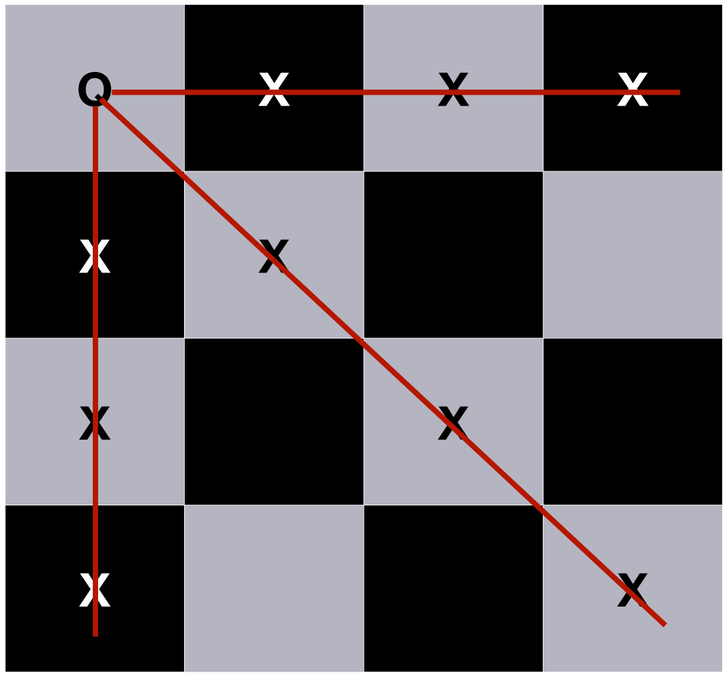

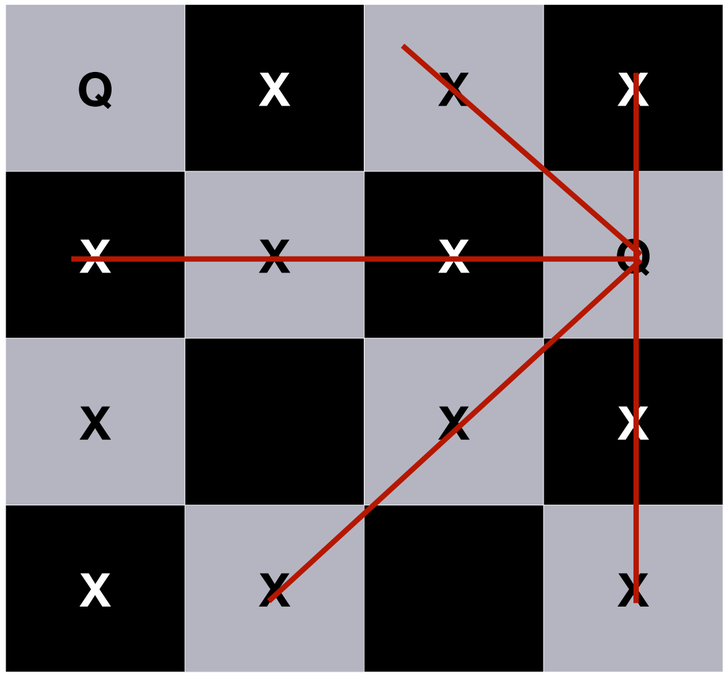

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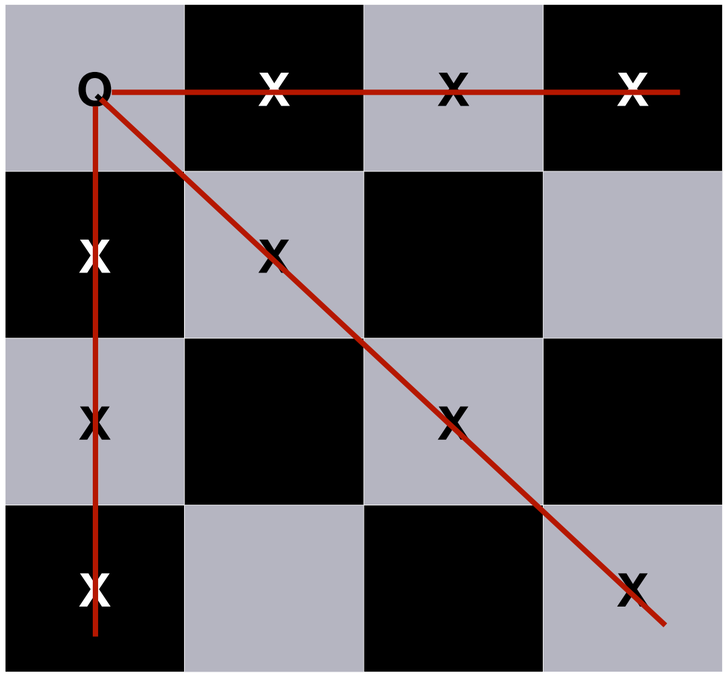

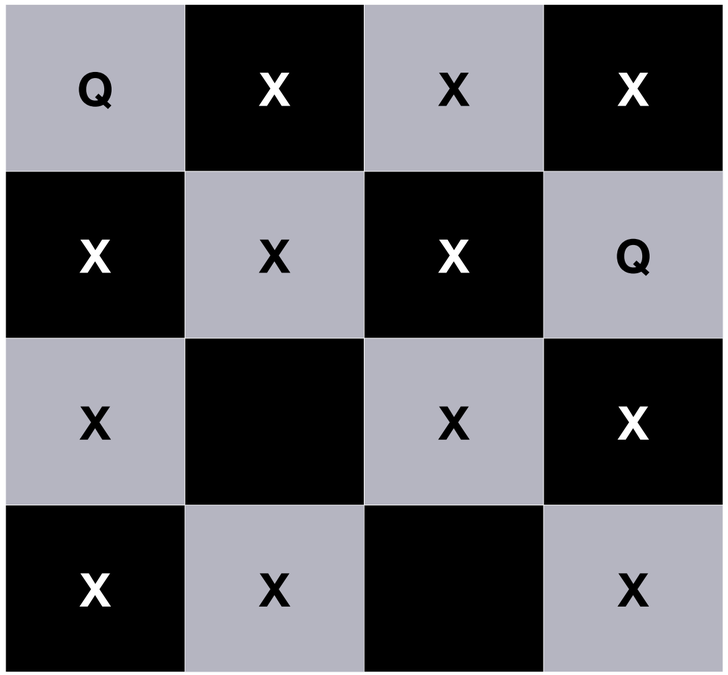

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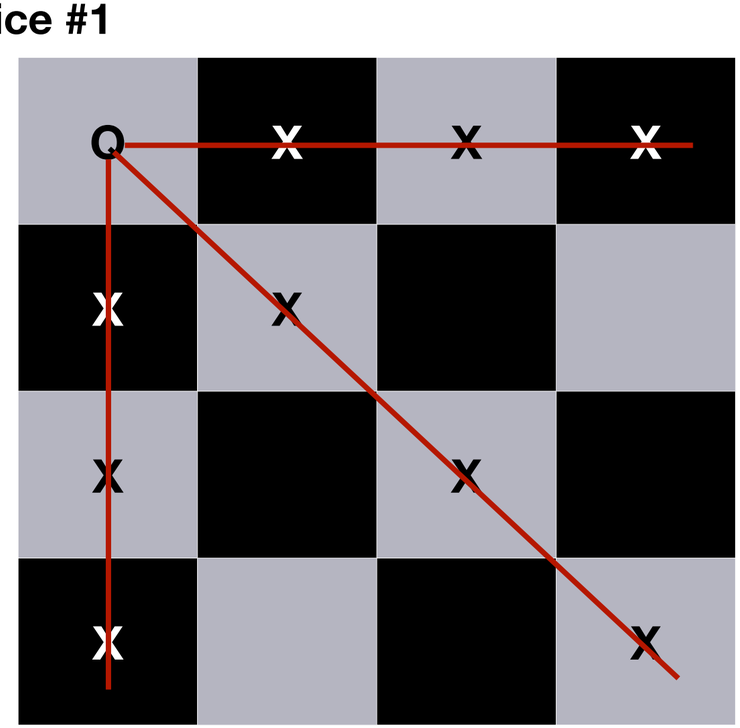

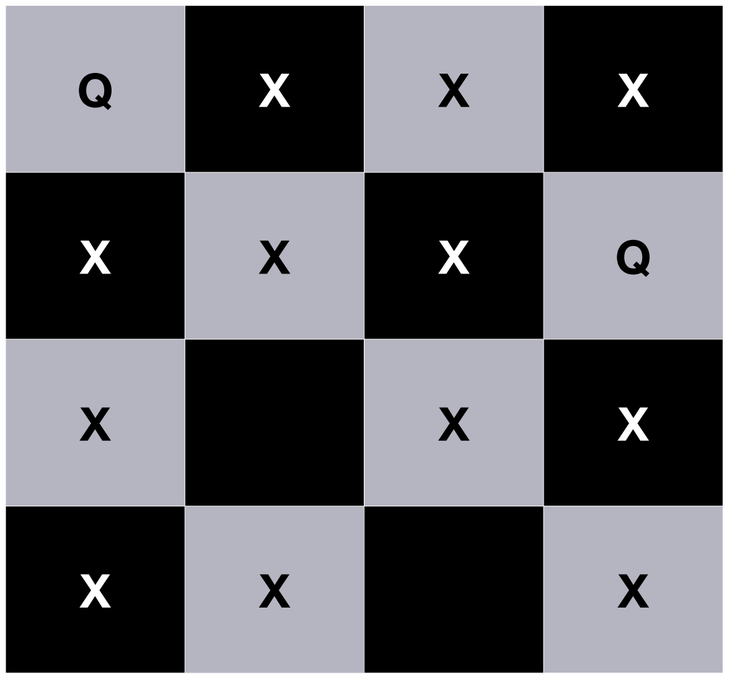

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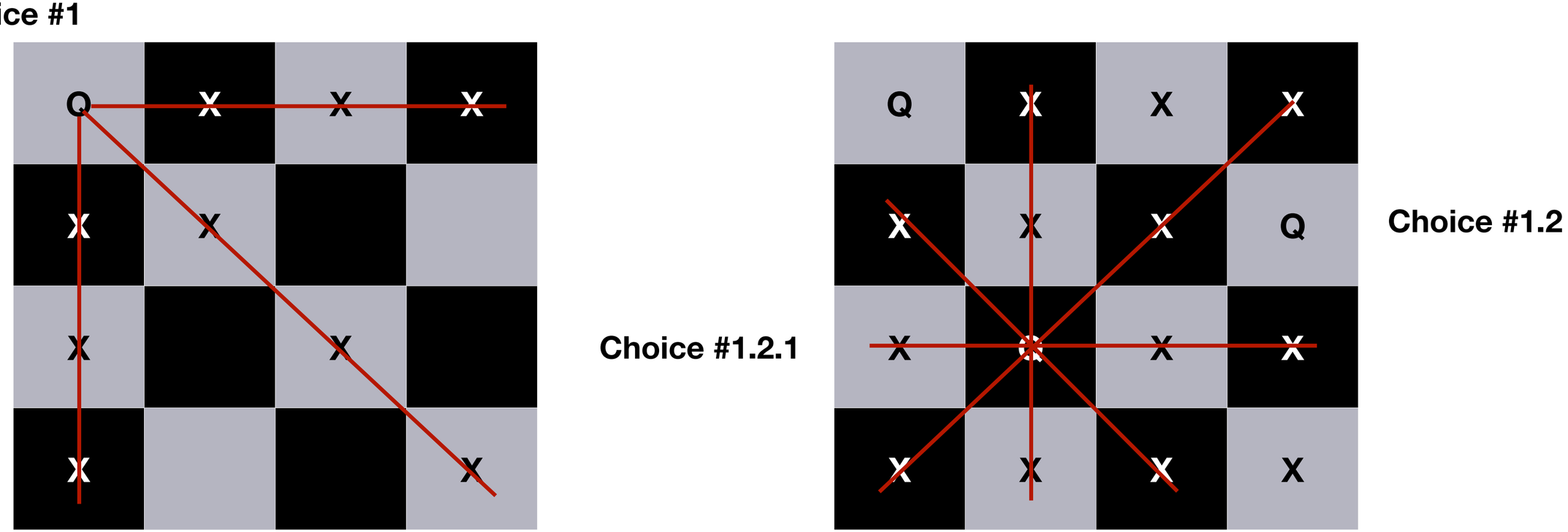

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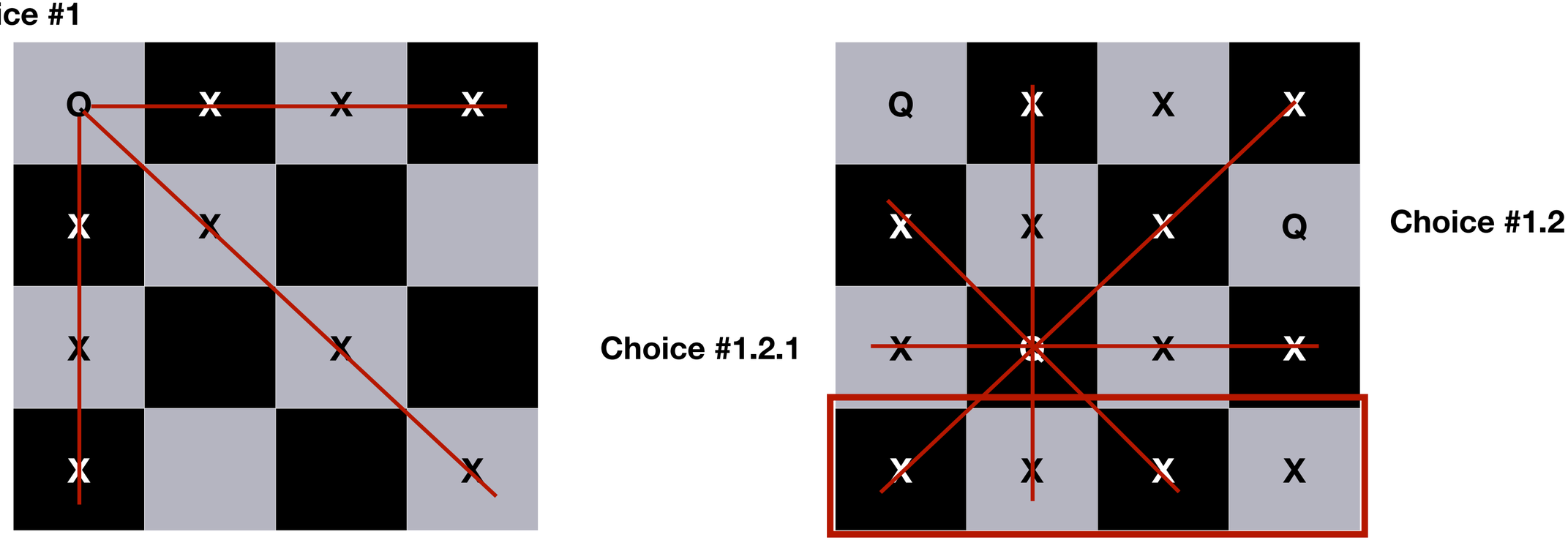

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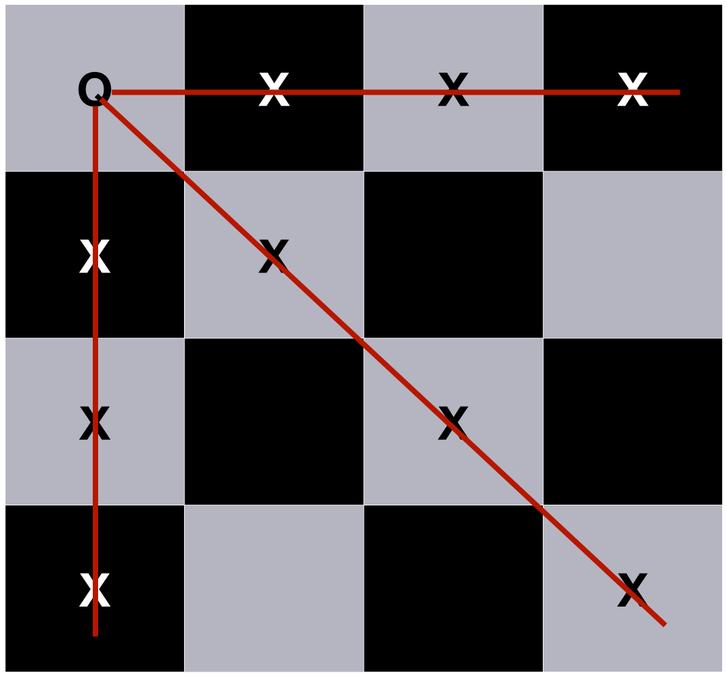
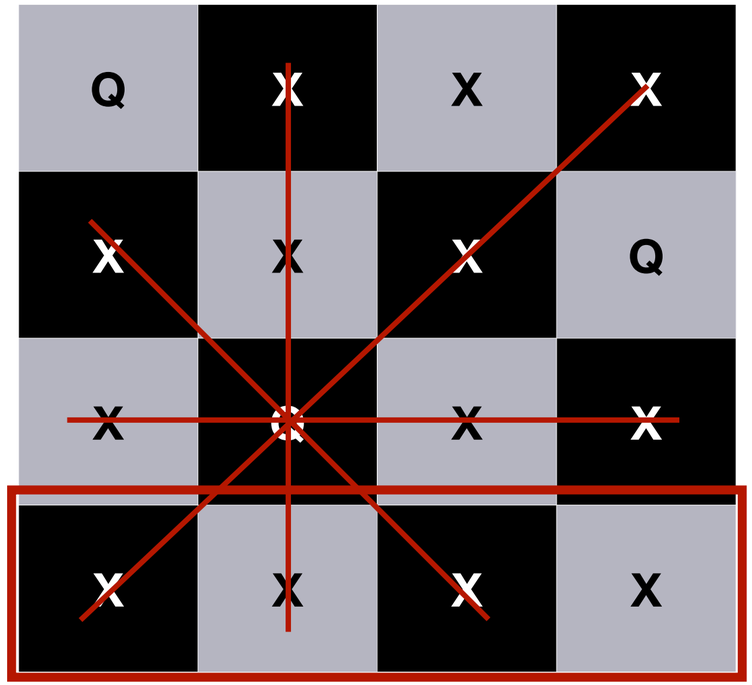


Choice #1.2.1



Choice #1.2

Not a solution!

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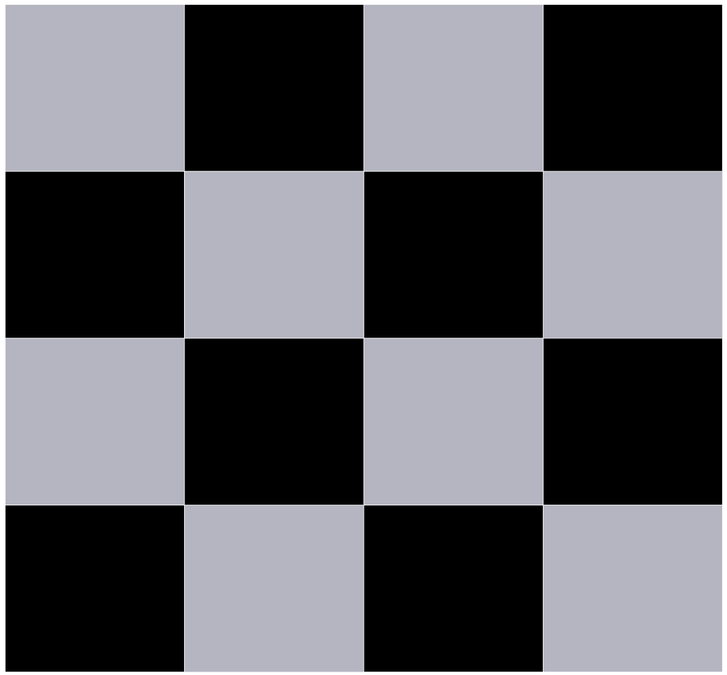

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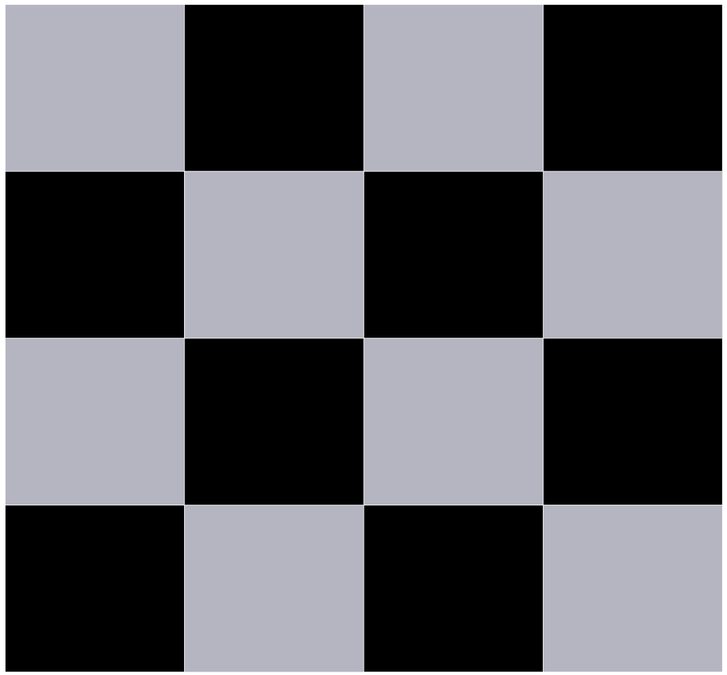

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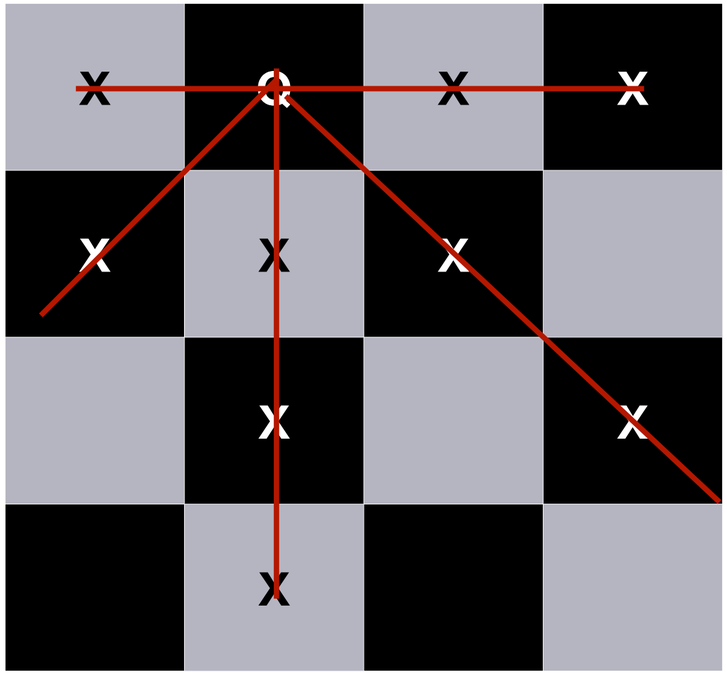

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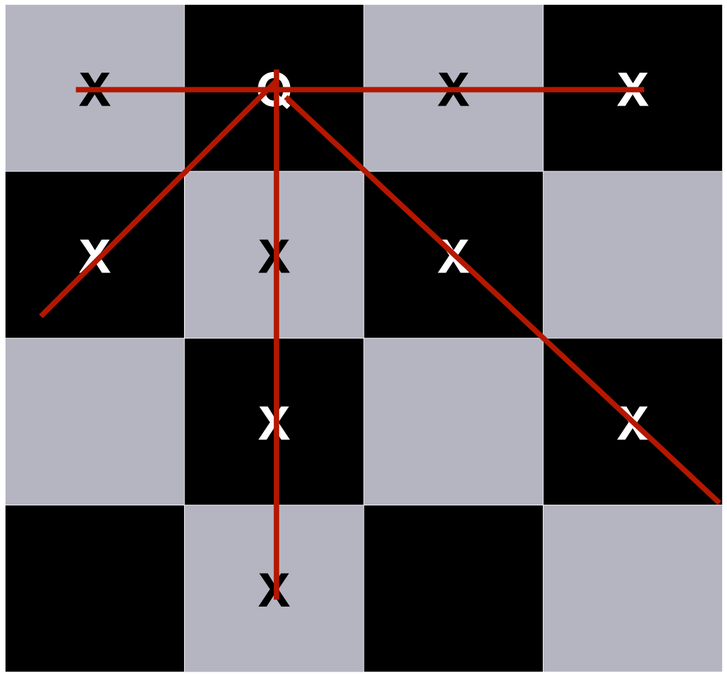

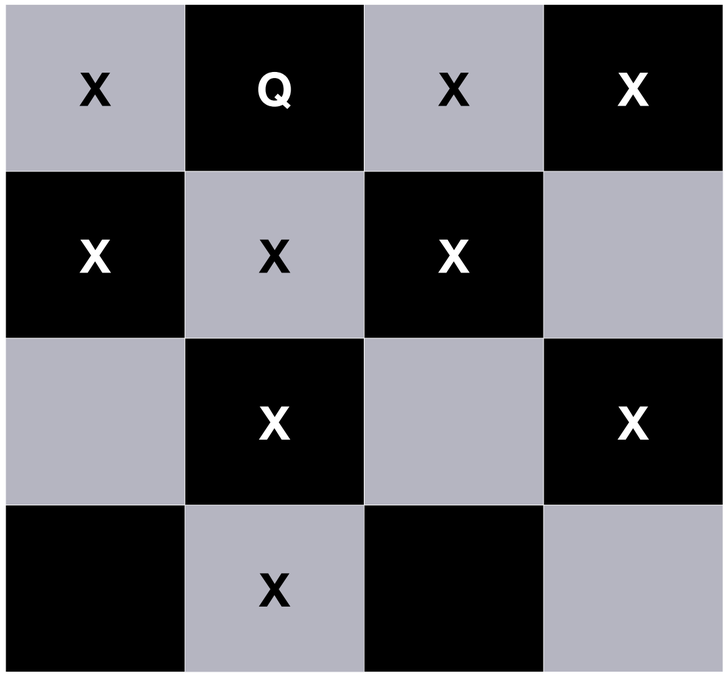

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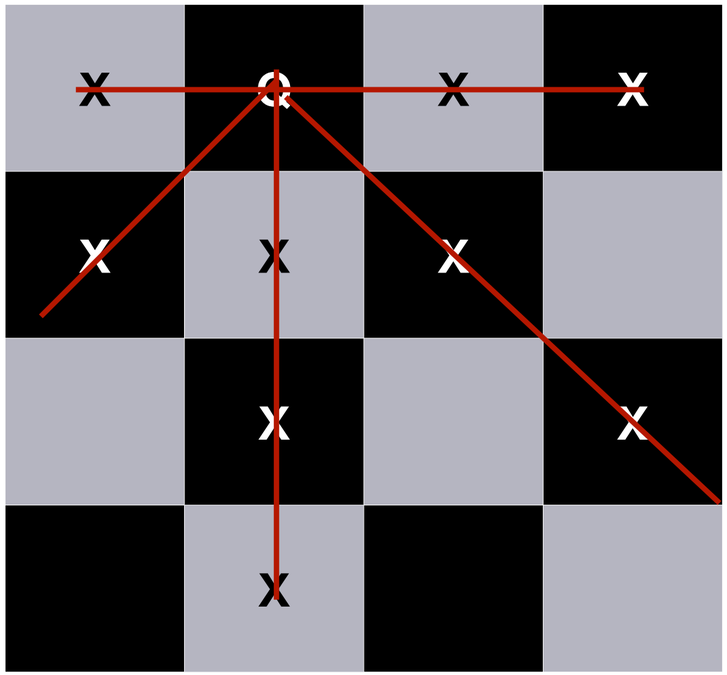

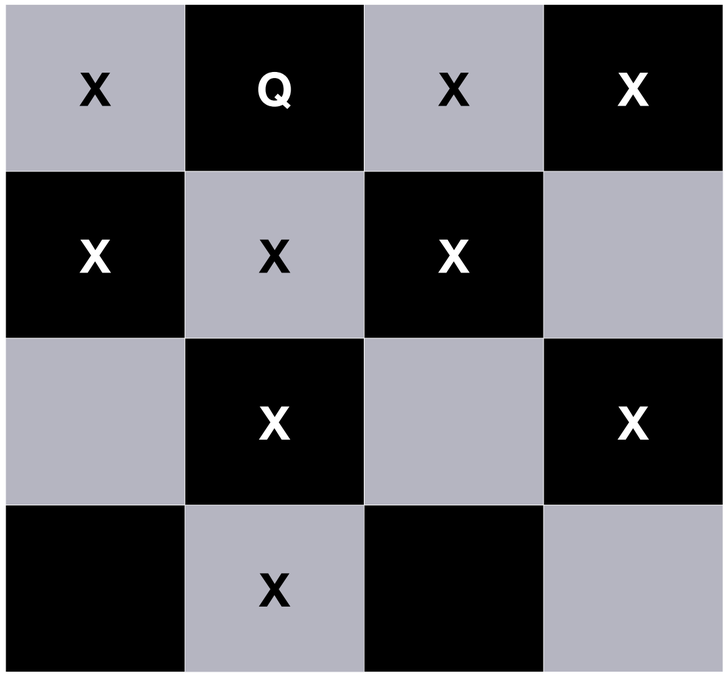

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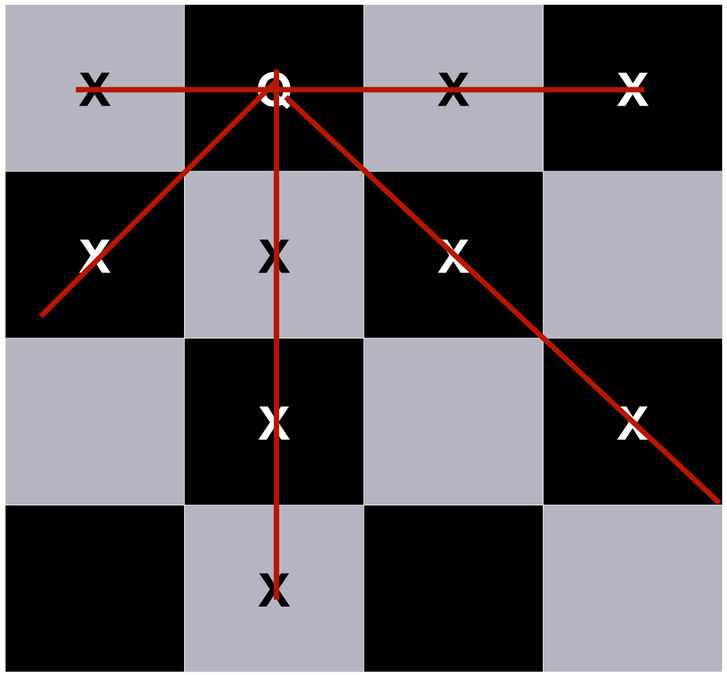

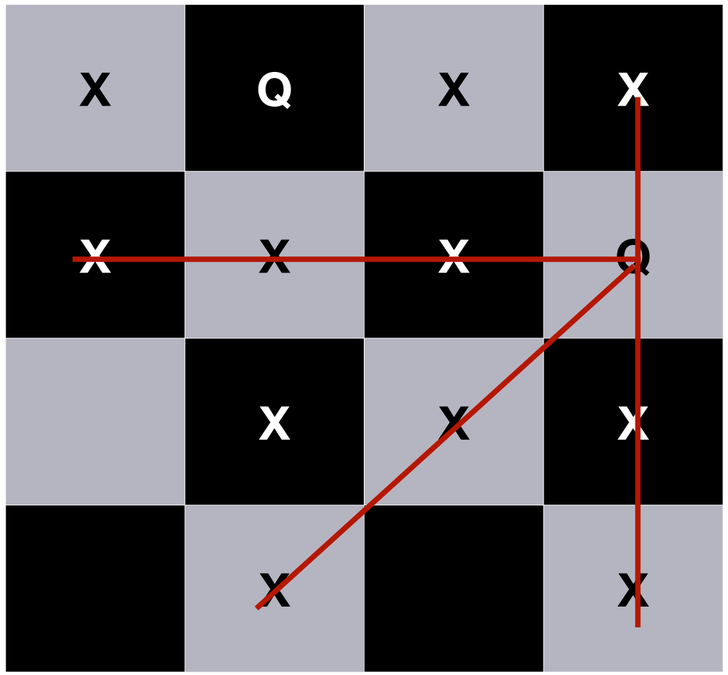

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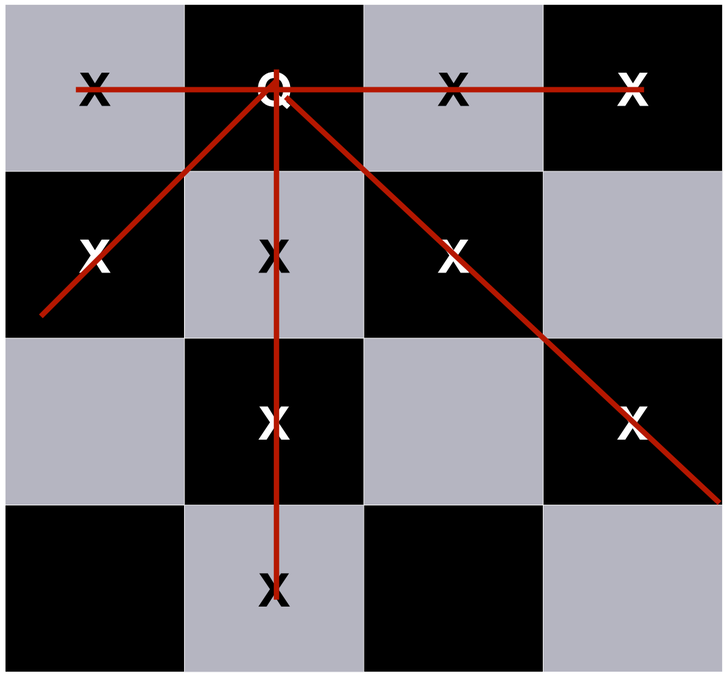

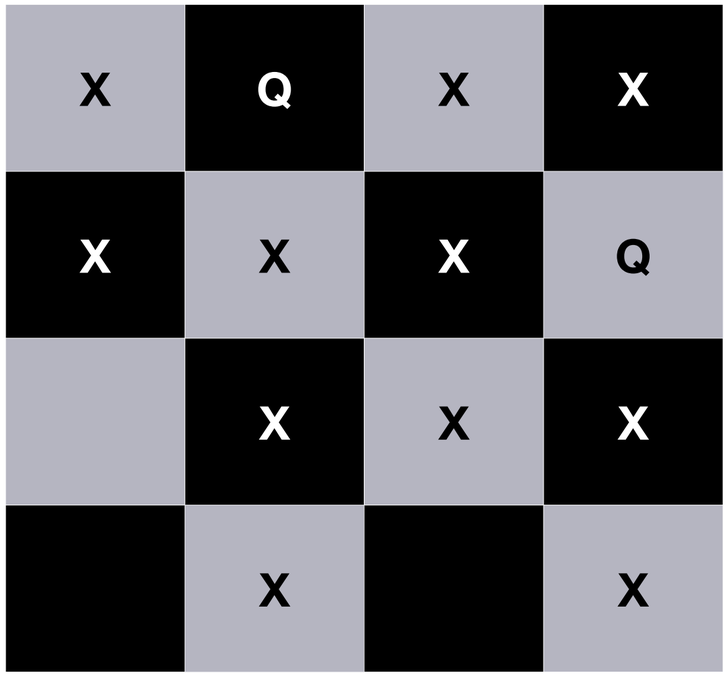

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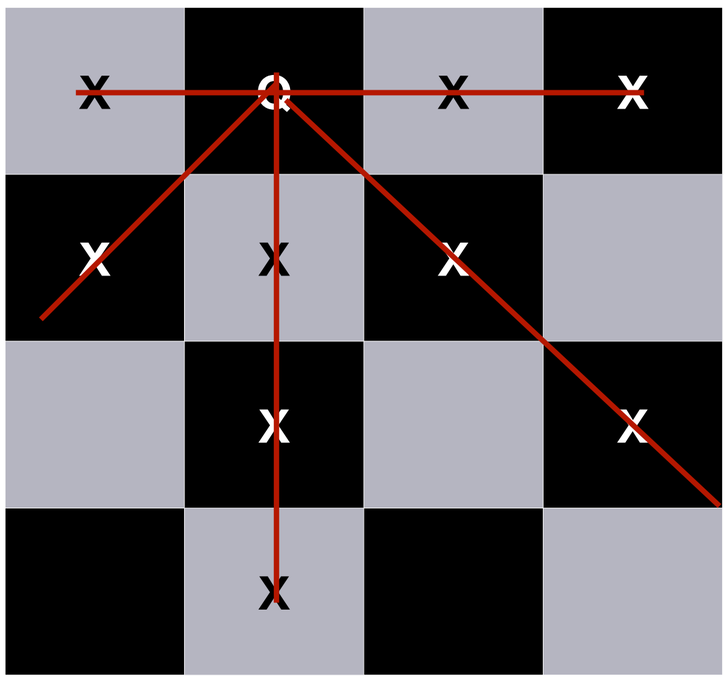

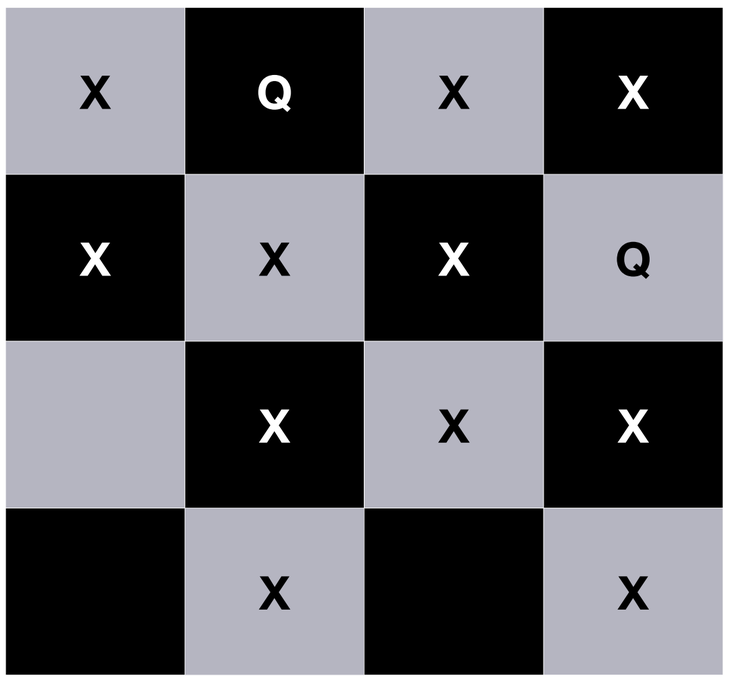

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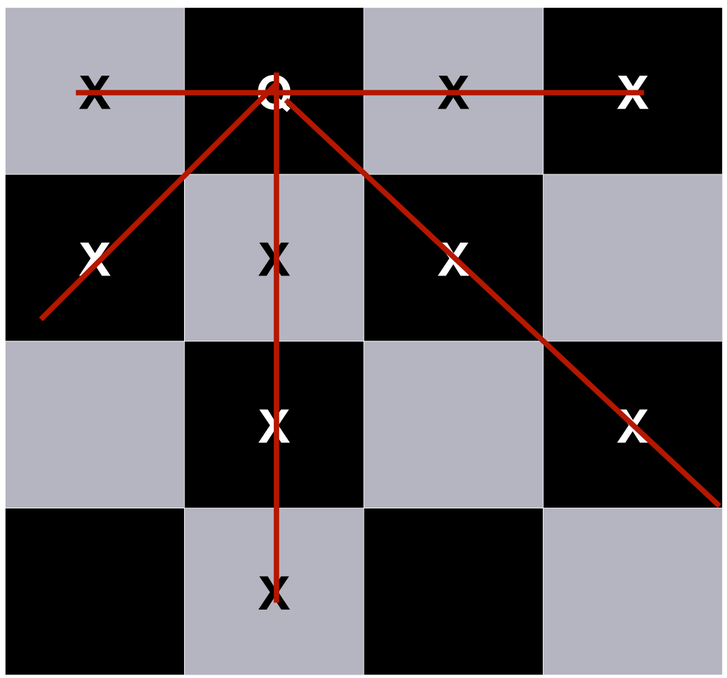

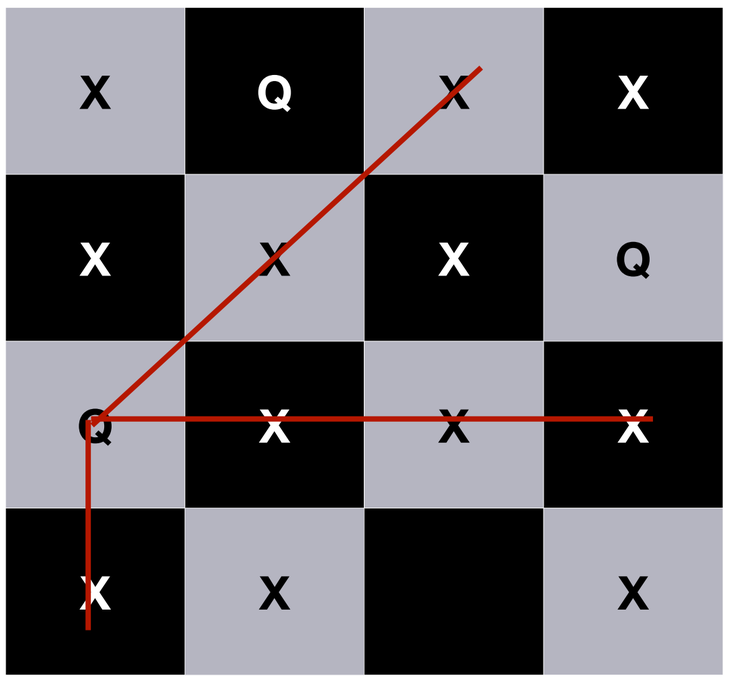

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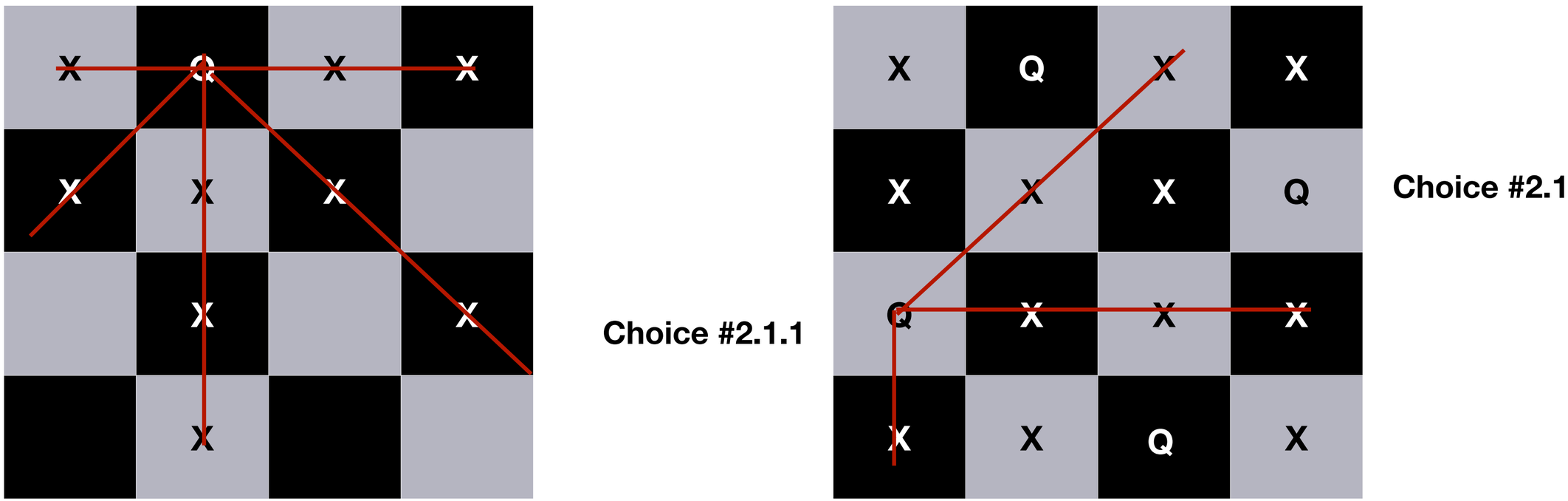

N - Queens Problem

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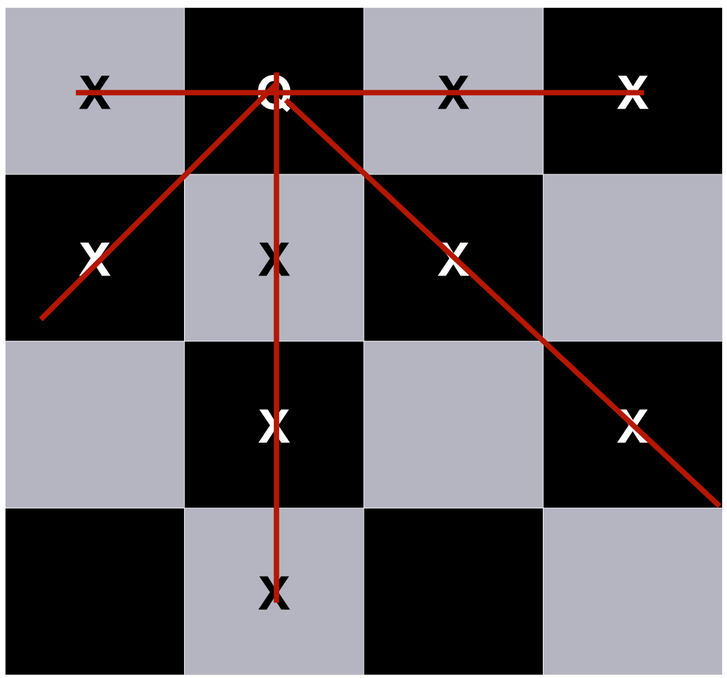

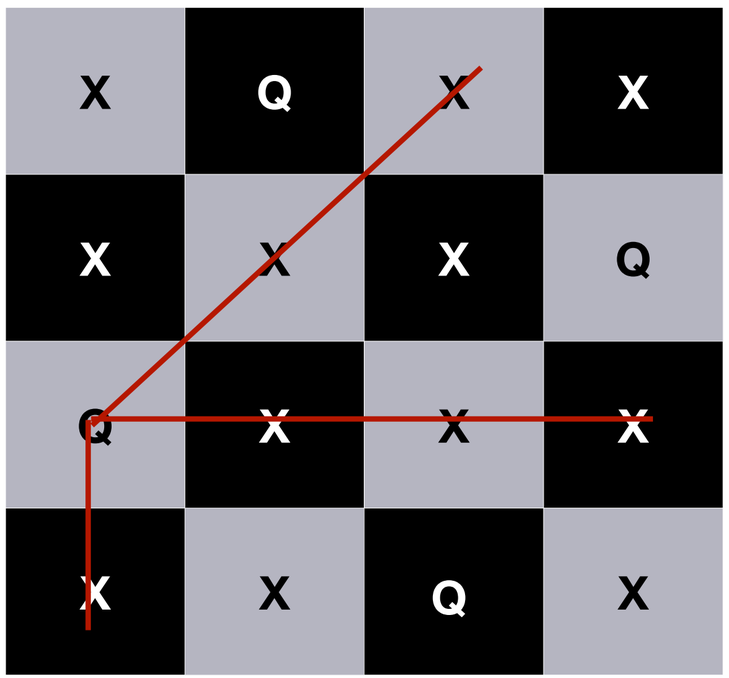

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  → No more rows to fill.

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- We represent the configuration space with a grid.
  - We will denote with digit **zero** an empty spot (maybe safe or unsafe, but its unoccupied).
  - We will denote with the digit **one** a space occupied by a queen.
  - We will fill in rows starting with the first row and proceeding downward.

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```
int is_safe(int board[N][N], int rnum, int cnum);

/*Function places a queen in row rnum */
int place_queen(int board[N][N], int rnum){
  if (          )  // Finished all rows
    return 1;      // Found a solution
  else{
    // Iterate over possible columns
    for(int cnum=0;           ; cnum++)
      if (is_safe(                    )==1){
        board[rnum][cnum] = 1; // Place a queen there
        // Update row number and recurse
        if (                            ==1)
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    - What about diagonals to the bottom left or bottom right?

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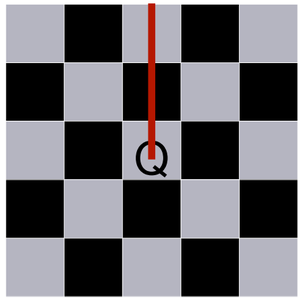


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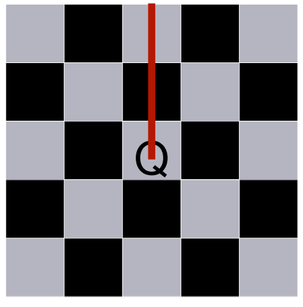


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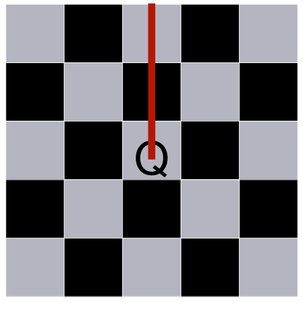

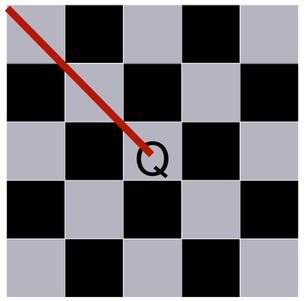


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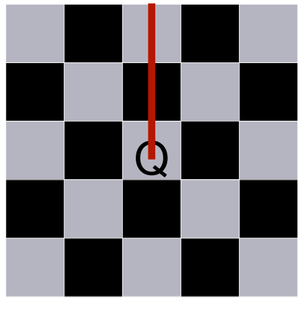

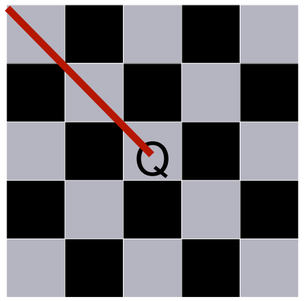


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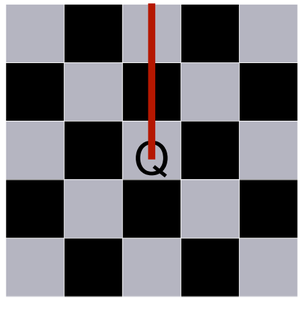

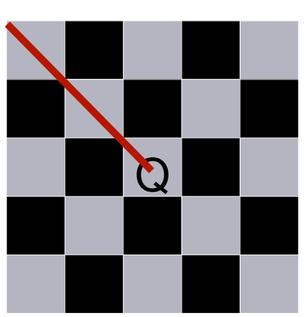

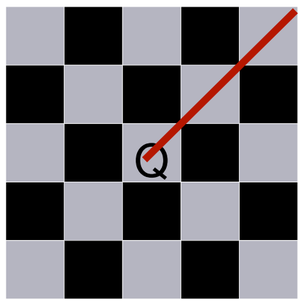


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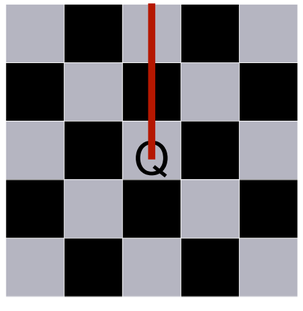

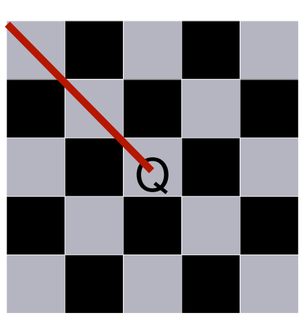

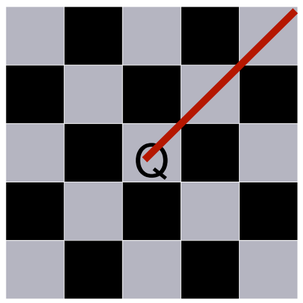


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  for (i=row-1, j=col+1; i>=0 && j<N; i--, j++){
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- Use recursion with backtracking to find a solution

Exercise

```c
#define N 10 // Number allowed
#define M 3  // Types of lengths

// Implement this function
// solution[N]: stores the solution
// idx: index for the solution matrix
// total: remaining length
int solve(int solution[N], int idx, int total);

const int set[M] = {3,7,10};

int main(){
  int solution[N] = {0, 0, 0, 0, 0, 0, 0, 0, 0, 0};
  int total;
  printf("Enter total length: ");
  scanf("%d", &total);
   // Write your code here
}
```

Good recursion vs. bad recursion

- Consider the recursive Fibonacci function from last time.

- Let’s do an activity
- Convert this function to an iterative version.
- Compare run times.

Good recursion vs. bad recursion

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```
long long fib(long long n){
     long long sum;

     if (n == 0 || n == 1)
          return 1;
     else {
          sum = (fib(n-1) + fib(n-2));
          return sum;
          }
}
```

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Exercise for fun outside lecture

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- There is a file on Gitlab which solves the problem for N=5 queens. Make the function work for N=6 and N=7 queens.

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- Exercise for the *curious/mighty/brave*:
  - Modify the source so that it keeps a static variable to keep track of the recursive calls.
  - Varying N, generate a plot (plain old Excel is fine) of N vs number of recursive calls. Try N=4, 5, …, 15. What kind of growth is it?