# ECE 220

## Lecture x000C - 10/08

Slides based on material originally by: Yuting Chen & Thomas Moon

# Review

- Last time we discussed sorting & searching

C • Selection sort

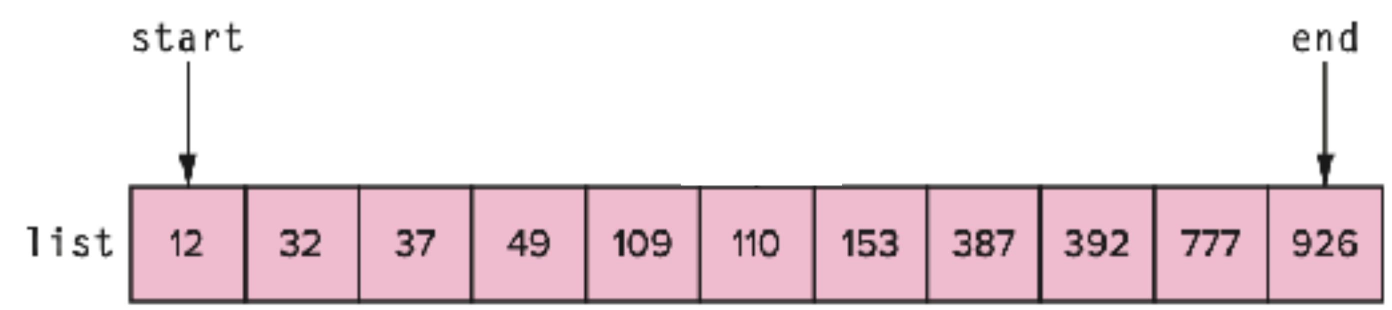
A • Insertion sort

B • Bubble sort

A. Select next element and move things to place it in proper spot
B. Keep comparing pairs and swapping them till no more swaps
C. Find minimum in each pass and bring to appropriate spot
D. Pick pivot & move elements to left (lesser) or right (greater) of pivot

Complete codes available on Gitlab (1003): https://gitlab.engr.illinois.edu/itabrah2/ece220-fa24

# Recap binary search




**Key = 109**

**Pick middle**
**Is middle > key?**

**Is middle == key?**
**Go left**

**Pick middle**
**Is middle > key?**

**Is middle == key?**
**Is middle < key?**
**Go right**

**Pick middle**
**Is middle > key?**

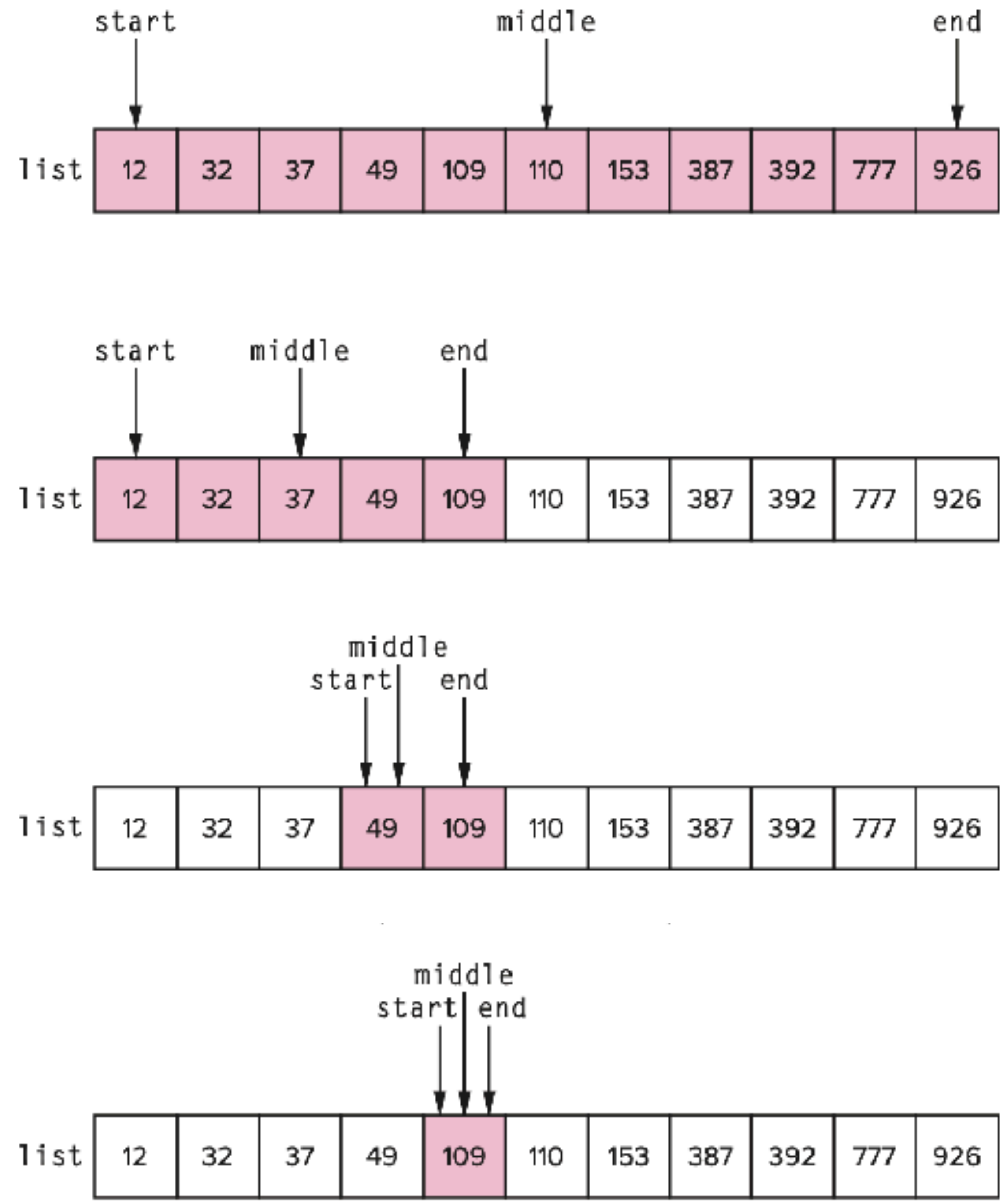
**Is middle == key?**
**Is middle < key?**
**Go right**

**Pick middle**

**Is middle == key?**

# Recap binary search

```c
int binary(int arr[], int n, int key){
  int start = 0;       // Left pointer
  int end = _____; // Right pointer

  while (end >= start){
    int mid = (___________) / 2; // Pick middle element

    // Logic to focus search on left or right of mid
    if (key == arr[mid])
      return mid;
    else if (key < arr[mid])
      end = _______;
    else
      start = ________;
  }
  return -1; // Loop exited, element not present.
}
```



# Binary search

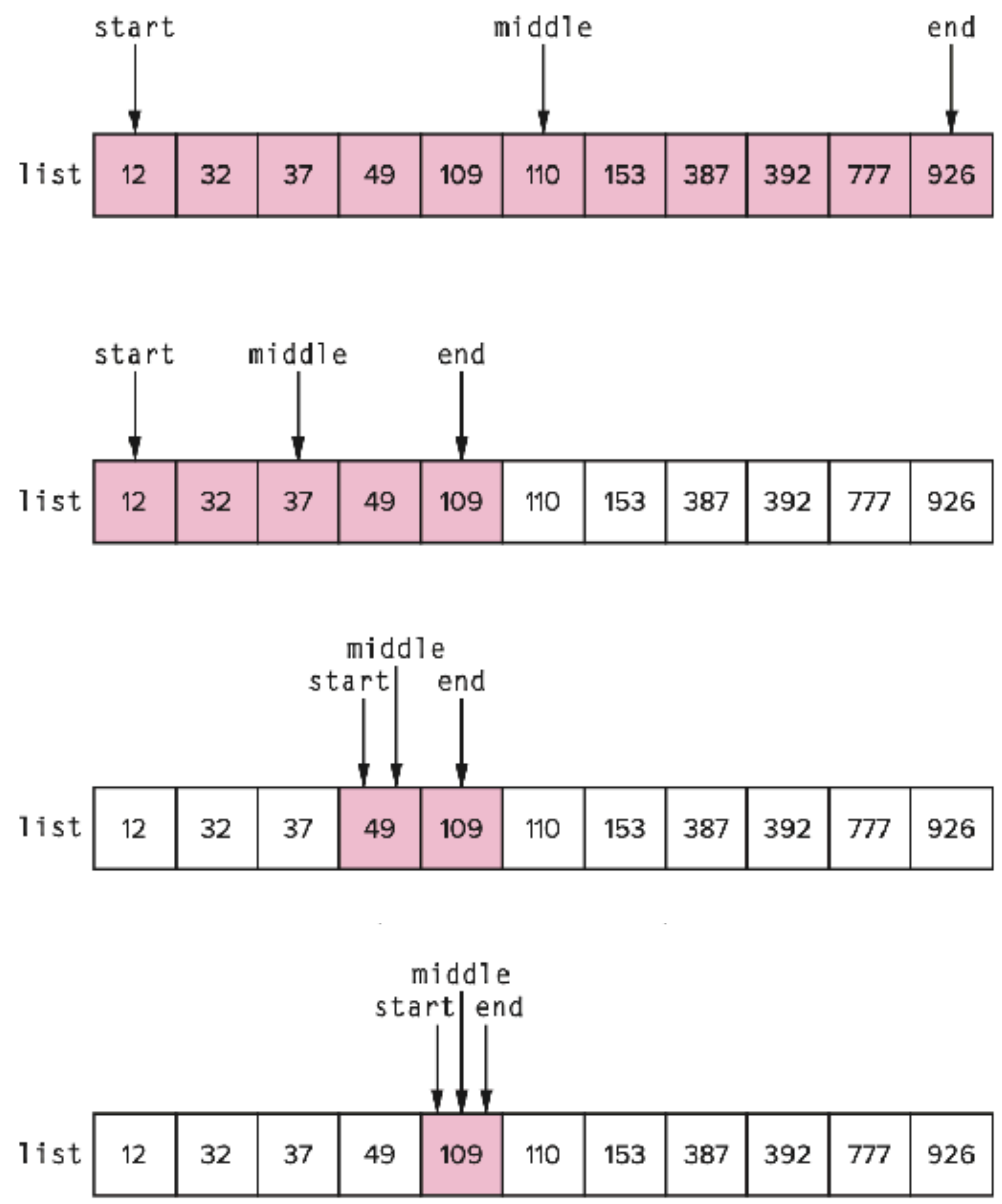
- We are repeating the same process of finding `mid` and going left or right of `mid` on each *subarray.*

- Can we apply

```
binary(arr[], n, key)
```

on each subarray?

- Idea is called *recursion*.



# Recursion

A **recursive function** is one that solves its task by **calling itself** on smaller pieces of data.

- Similar to recurrence function in mathematics
- Like iteration — can be used interchangeably; sometimes recursion results in simpler solution … but not always!
- Must have **at least** *one* base case (terminal case) that ends the recursive process; similar to loop needing condition to exit.

Examples: Factorial function, Fibonacci series, binary search, etc.

# Recursive function

- Base case ( a.k.a terminating case )
  - This case is **required** so the recurrence can terminate.
  - The base case must provide a condition that will eventually become true and returns from the function. **Otherwise, the run-time stack will overflow**.
- Recursive case ( a.k.a inductive case )
  - This case returns a recursive call to the function itself. It breaks down the problem into smaller chunks that can be solved by the *same* function.
  - The input to the next call gets induced gradually.

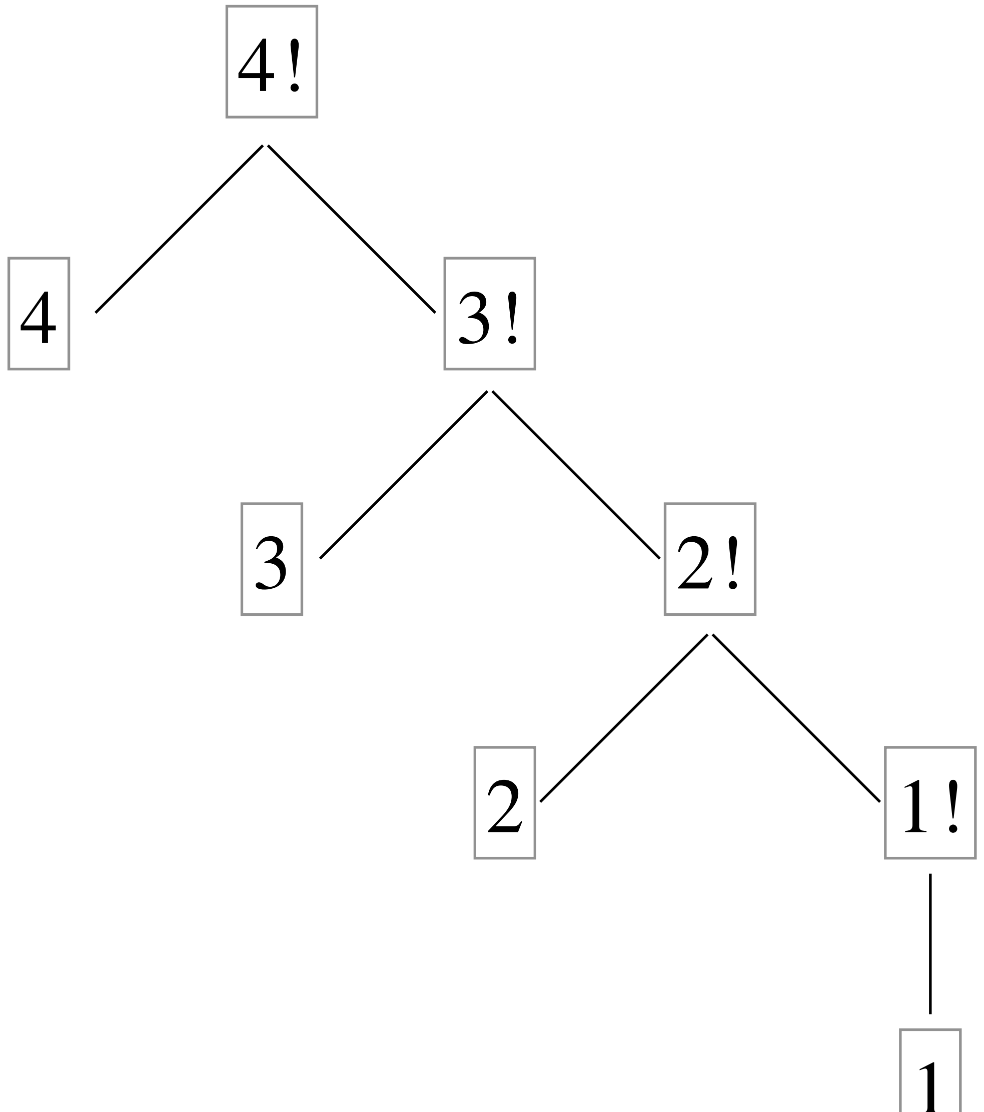
# Example: Factorial

- Mathematical definition

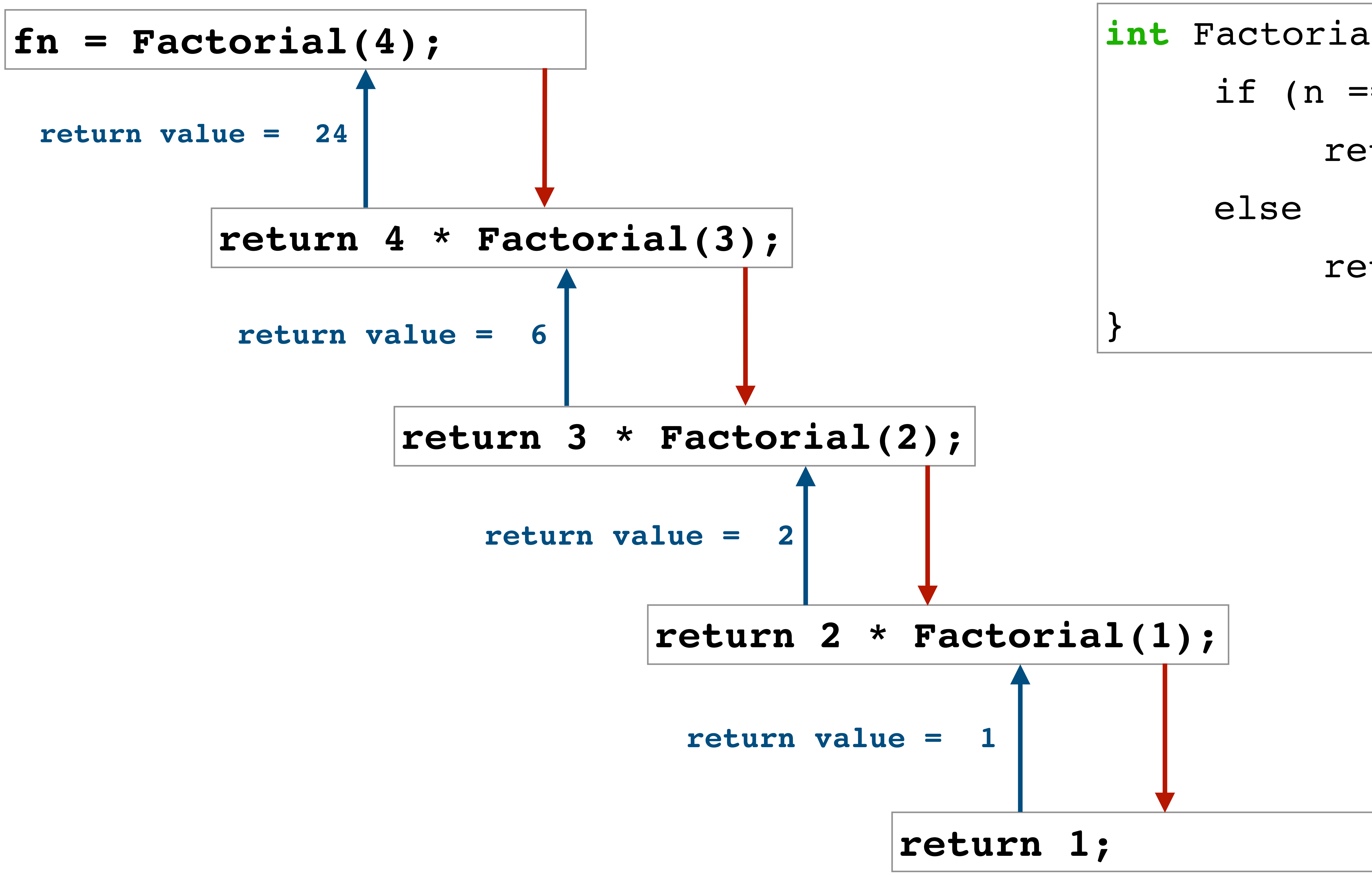
$$n! = n \cdot (n-1) \cdot (n-2) \ldots \cdot 2 \cdot 1 \qquad for \quad n > 0$$

- Recursive form

$$n! = \begin{cases} 1, & \text{if} \quad n = 1 \\ n \cdot (n-1)!, & \text{else} \end{cases}$$

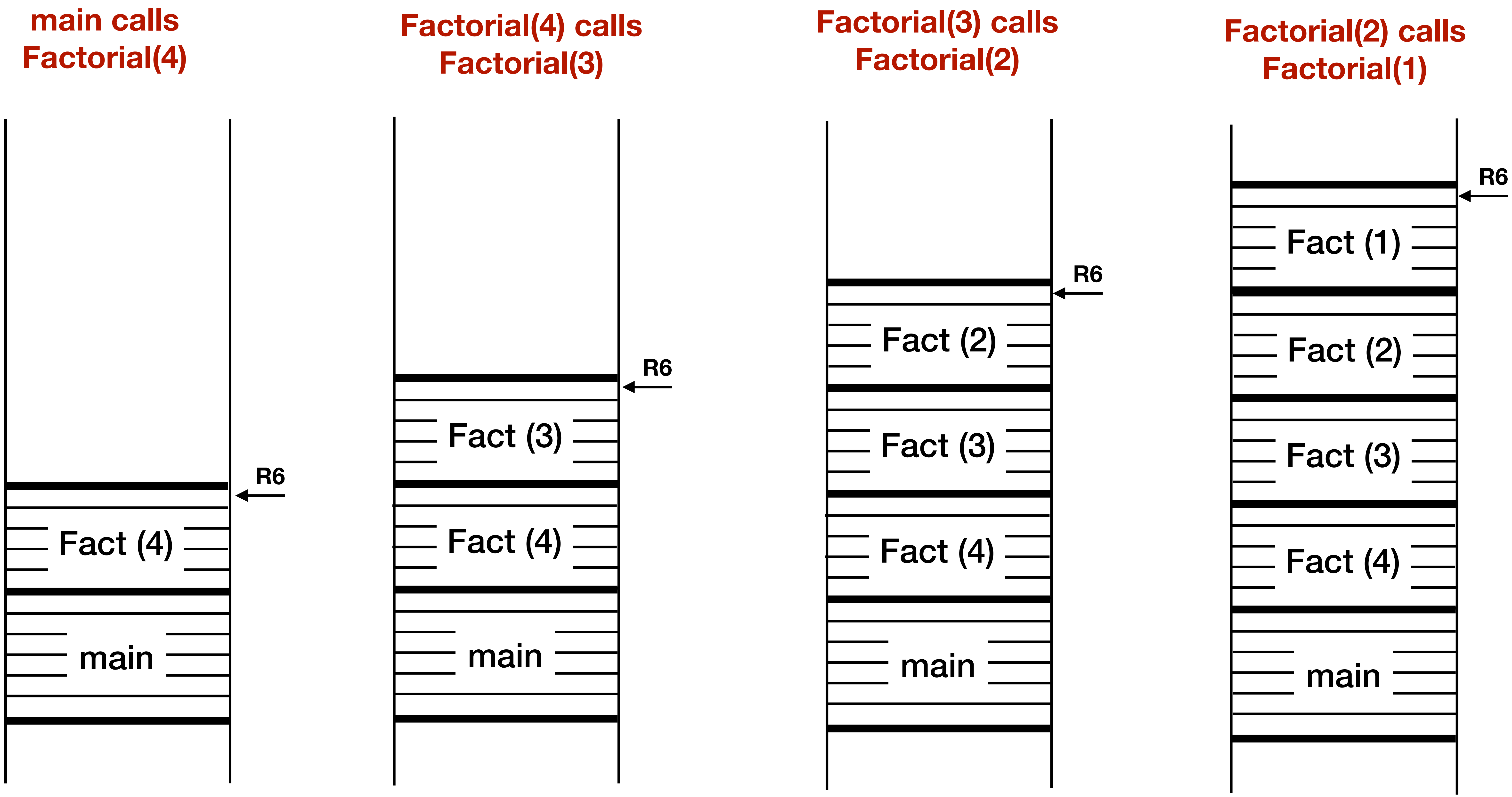


# Example: Factorial

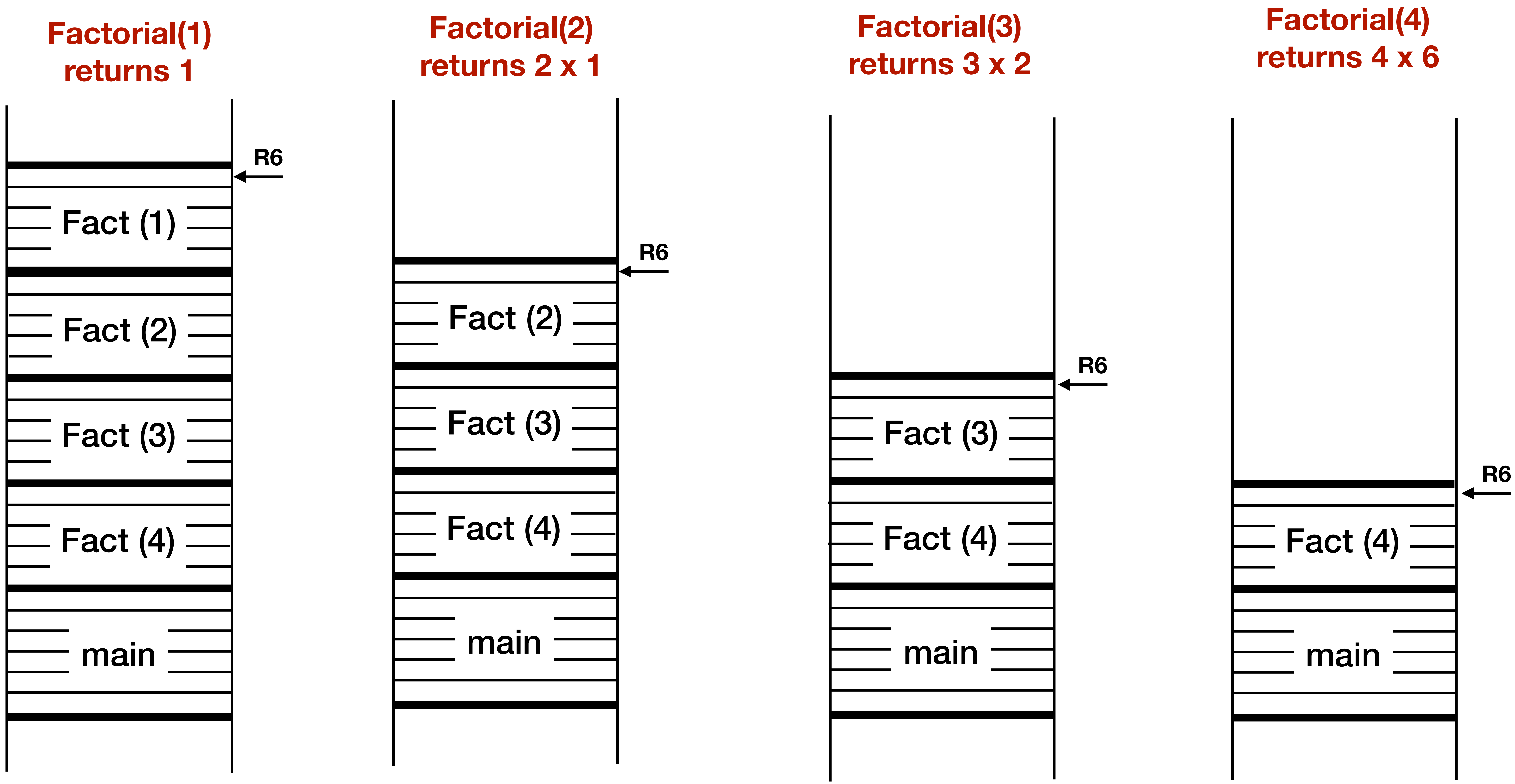



```c
int Factorial(int n){
     if (n == 1)
          return 1;
     else
          return n * Factorial(n - 1);
}
```

# RTS During Execution of Factorial



# RTS During Execution of Factorial



# Example: Fibonacci series

*Mathematical definition:*

Fibonacci Series: 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, ……

$$f(n) = f(n-1) + f(n-2)$$

$$f(1) = 1$$

$$f(0) = 1$$

Fibonacci (3) = Fibonacci(2) + Fibonacci(1)

= (Fibonacci(1) + Fibonacci(0)) + Fibonacci(1)

= 1 + 1 + 1 = 3

# Example: Fibonacci series

```
int Fibonacci (int n) {
    int sum;

    if (n == 0 || n == 1)
        return 1;
    else {
        sum = (Fibonacci(n-1) + Fibonacci(n-2));
        return sum;
        }
}
```

# Example: Binary search

```
int binary(int arr[], int n, int key){
  int start = 0;      // Left pointer
  int end = n - 1; // Right pointer

  while (end >= start){
    int mid = (start + end) / 2; // Pick middle element

    // Logic to focus search on left or right of mid
    if (key == arr[mid])
      return mid;
    else if (key < arr[mid])
      end = mid - 1;
    else
      start = mid + 1;
  }
  return -1; // Loop exited, element not present.
}
```

Can we implement binary search in *recursive* way?

- **Option 1**: Find a mechanism to keep track of the start and end indices across recursive calls; local variables won't do (why?).
- **Option 2**: Pass in subarrays (using pointers) & their lengths to `binary` itself (advanced!).

# Example: Binary search

Option A

```
int binary_opt1(int item, int list[], int start, int end){
    int middle = (end + start)/2;
    if (end < start)
      return -1;                        // Did not find key
    else if (____________________) // Found item!
        return middle;
    else if (___________________)  // Search left half
        return binary_opt1(item, list, start, middle-1);
    else                                // Search right half
        return binary_opt1(_________________________);
}
```

# Example: Binary search

```
int binary_opt1(int item, int list[], int start, int end){
    int middle = (end + start)/2;
    if (end < start)
      return -1;                        // Did not find key
    else if (_____________________) // Found item!
        return middle;
    else if (____________________)  // Search left half
        return binary_opt1(item, list, start, middle-1);
    else                            // Search right half
        return binary_opt1(__________________________);
}
```

# Example: Quicksort

- We already saw Quicksort last time and remarked it was recursive …
- Recall the steps ….

1. Choose first element of given array as pivot.
2. Maintain pointers from the left and right.
3. Increment left pointer while the element it points to is less than pivot
4. Decrement right pointer while the element it points to is greater than pivot.
    1. If pointers cross/overlap, **split array** & **recurse** on each subarray.
5. If neither pointers can move swap elements.
6. Repeat 3-5 while left pointer < right pointer.

And another which will perform recursion after split

We will write one function to split array

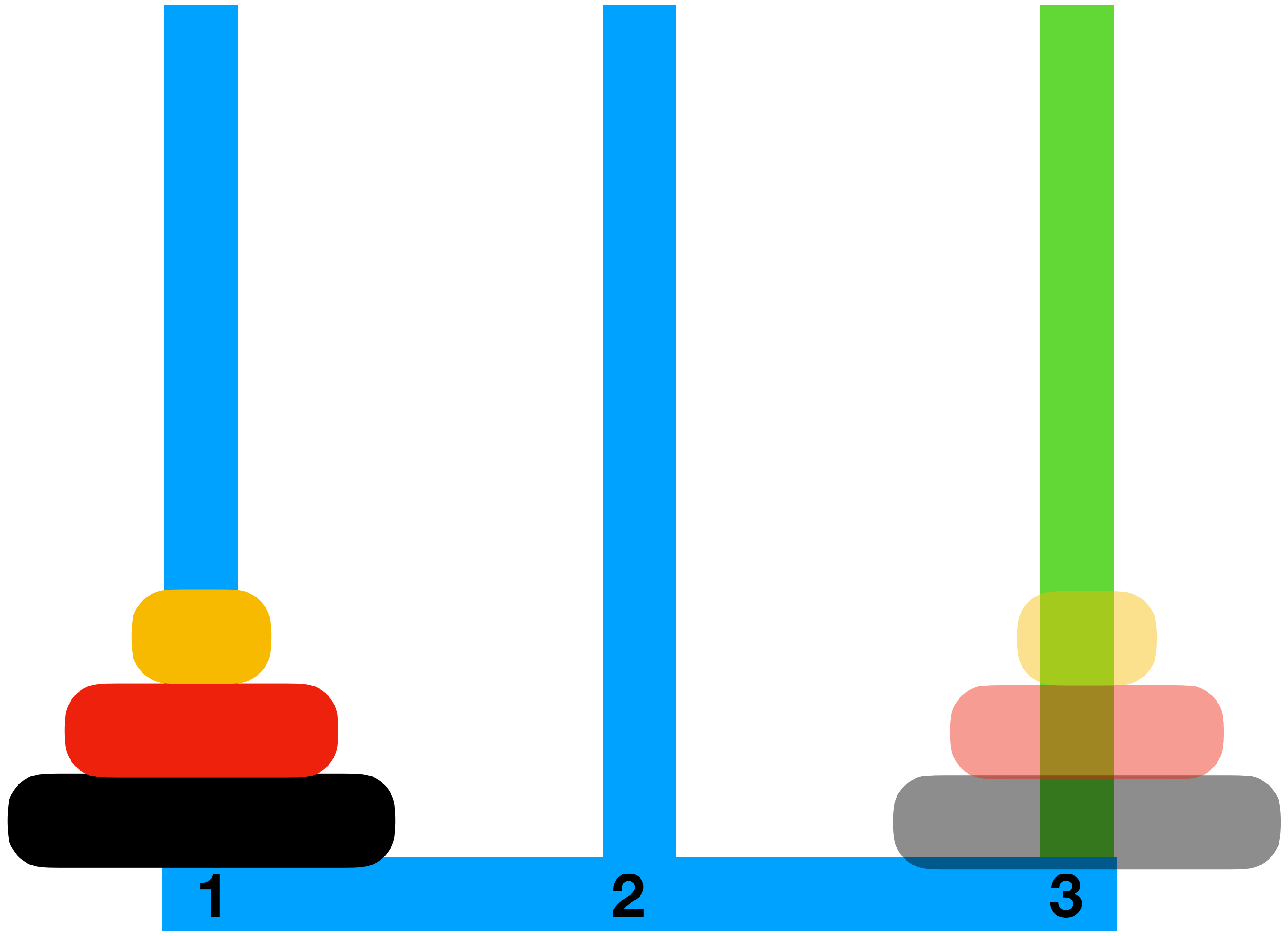
# Implementation

```
void Swap(int* one, int* two){
  int temp = *one;
  *one = *two;
  *two = temp;
}

void QuickSort(int arr[], int start, int end){
  if (start < end){
    int pivotVal = partition(arr, start, end);

    // Now sort left half
    QuickSort(arr, start, ________);

    // And right half
    QuickSort(arr, ____________, end);
  }
}
```

```
int partition(int arr[], int start, int end){

  int pivotVal = __________;
  int i = _________; // Initialize left
  int j = _______;   // Initialize right

  while(1){
    do i++; // Increment left till …
    while (_________________);

    do j--; // Decrement right till …
    while (_________________);

    if (______) // If overlap need to split
      return j;

    Swap(&arr[i], &arr[j]);
  }
}
```

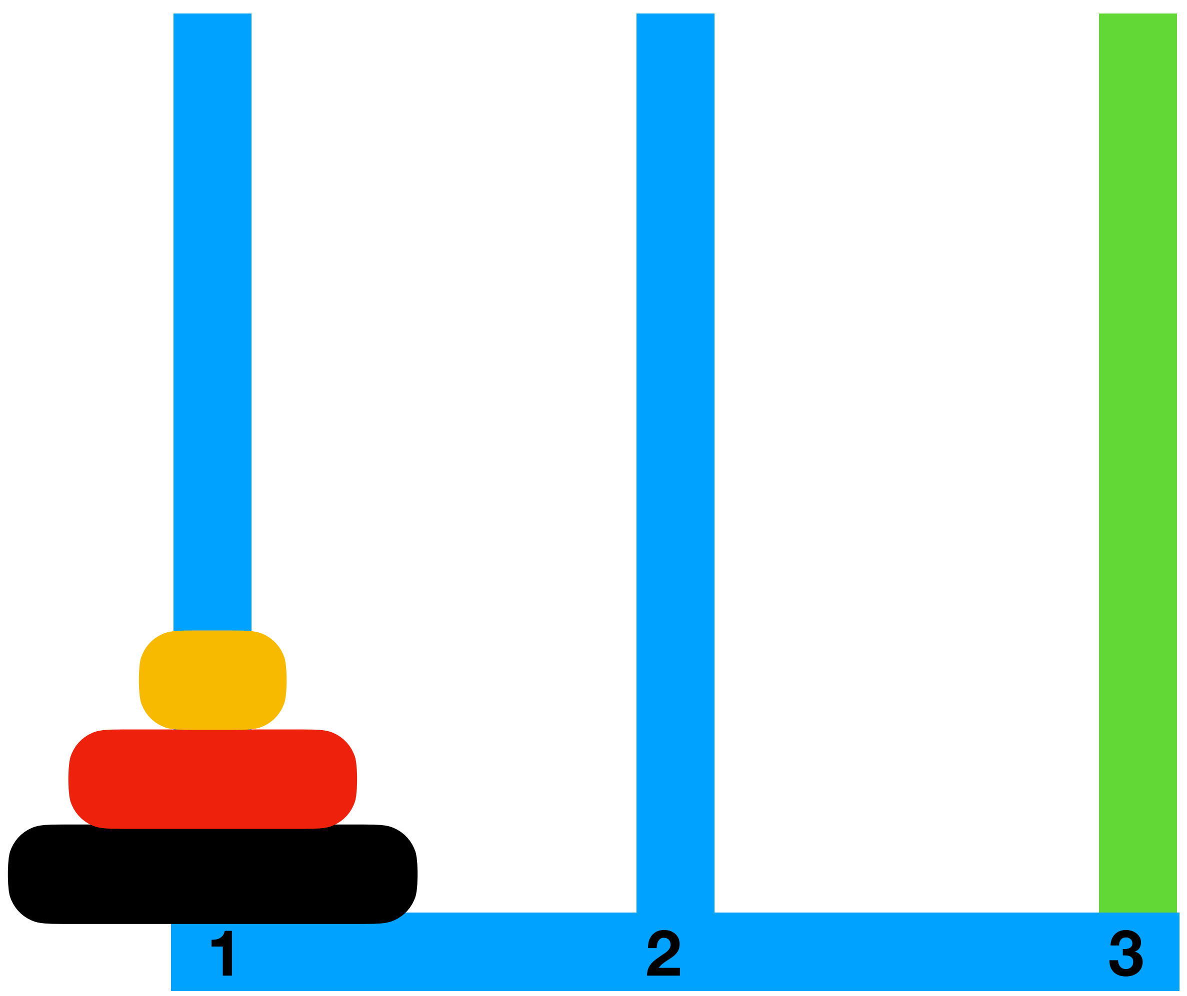
Check Gitlab for reference material: https://gitlab.engr.illinois.edu/itabrah2/ece220-fa24

# Example: Towers of Hanoi



# Example: Towers of Hanoi

*move(diskStacks, src, dest, temp)* *// Move 2+ disks*
*dmove(disk_num, src, dest)* *// Move single disk*




**move(disks3, r1, r3, r2)**

↓

move(disks2, r1, r2, r3)
+
dmove(disk_3, r1, r3)
+
move(disks2, r2, r3, r1)

move(disks2, r1, r2, r3) → dmove(disk_1, r1, r3), dmove(disk_2, r1, r2), dmove(disk_1, r3, r2)

move(disks2, r2, r3, r1) → dmove(disk_1, r2, r1), dmove(disk_2, r2, r3), dmove(disk_1, r1, r3)

# Recursion in Towers of Hanoi

**move(disks3, r1, r3, r2)**

move(disks2, r1, r2, r3)
+
dmove(disk_3, r1, r3)
+
move(disks2, r2, r3, r1)

dmove(disk_1, r1, r3)
dmove(disk_2, r1, r2)
dmove(disk_1, r3, r2)

dmove(disk_1, r2, r1)
dmove(disk_2, r2, r3)
dmove(disk_1, r1, r3)

**move(disks4, r1, r3, r2)**

move(disks3, r1, r2, r3)
+
dmove(disk_4, r1, r3)
+
move(disks3, r2, r3, r1)

move(disks2, r1, r3, r2)
dmove(disk_3, r1, r2)
move(disks2, r3, r2, r1)

move(disks2, r2, r1, r3)
dmove(disk_3, r2, r3)
move(disks2, r1, r3, r2)

# Towers of Hanoi: General formula?

```
void move(disknum, src, dest, temp){
  if (disknum > 1){
    move(disknum - 1, src, temp, dest);

    printf("Moved disk %d from rod %d to %d\n", disknum, src, dest);

    move(disknum - 1, temp, dest, src);
  }
  else
    printf("Moved disk 1 from rod %d to %d\n", src, dest);
}
```

*Move a sub-stack to from src to intermediate rod*

*Direct move of single disk.*

*Move the sub-stack from intermediate to dest rod*

*Base case; all moves involving sub-stacks end-up here.*

# Recursion in LC3

```
int running_sum(int n){
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}

int main(void){
    int n = 5;
    running_sum(5);
}
```

How can we write equivalent LC3 code?

Recall function calls are implemented using the run time stack.

Recursive calls need not be treated any different from normal function calls!

# Generating an activation record

1. *Caller* build-up: Push callee's arguments onto stack
2. Pass control to callee (`JSR/JSRR`)

Caller

3. *Callee* build-up: (push bookkeeping info and local variables onto stack)
4. Execute function
5. *Callee* tear-down (update return value, pop local variables, caller's frame pointer and return address from stack)
6. Return to caller (`RET`)

Callee

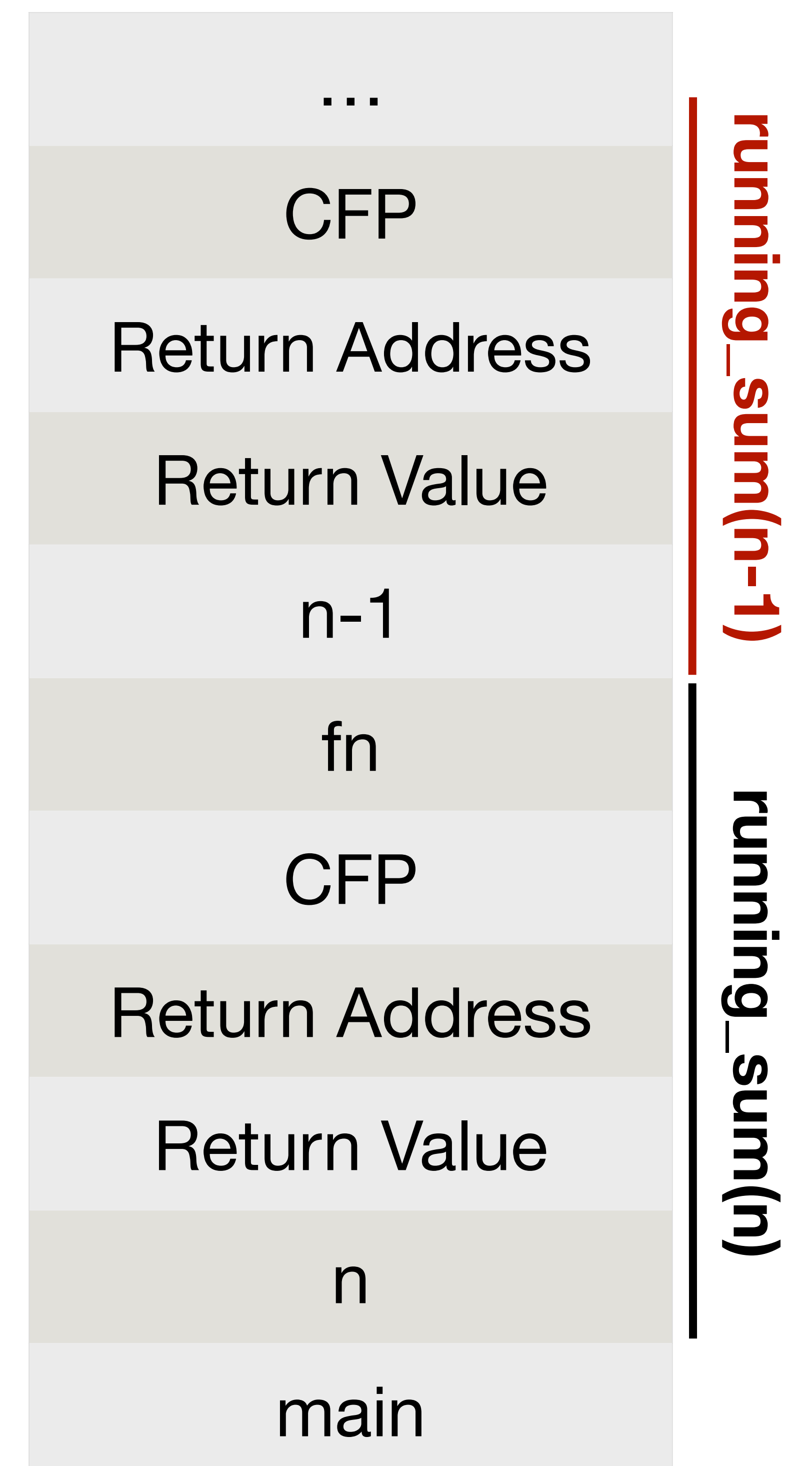
7. *Caller* tear-down (pop callee's return value and arguments from stack)

Caller

Check Gitlab for reference material: https://gitlab.engr.illinois.edu/itabrah2/ece220-sp24

# Recursion in LC3

```c
int running_sum(int n){
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}

int main(void){
    int n = 4;
    running_sum(n);
}
```



```c
int running_sum(int n){
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}
```

```c
int main(void){
    int n = 4;
    int answer;
    answer = running_sum(n);
}
```

# Recursion in LC3

```
;Caller set-up
STR R0, R5, #0
ADD R6, R6, #-1
STR R0, R6, #0
JSR RUNNING

RUNNING
;callee set-up of Running(n)'s activation record
;push return value, return address & caller's frame pointer
ADD R6, R6, #-3
STR R7, R6, #1
STR R5, R6, #0

;push local variables & update frame pointer
ADD R5, R6, #-1
ADD R6, R6, #-1

;function logic
;base case (n==1)
LDR R1, R5, #4
ADD R2, R1, #-1
BRz BASE_CASE
```

R0: 4

R1: 4

R2: 3

| |
|---|
| ... |
| CFP |
| Return Address |
| Return Value |
| n-1 |
| fn |
| CFP |
| Return Address |
| Return Value |
| n=4 |
| main |

R6 → R5 → main

```
int running_sum(int n){
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}
```

```
int main(void){
    int n = 4;
    int answer;
    answer = running_sum(n);
}
```

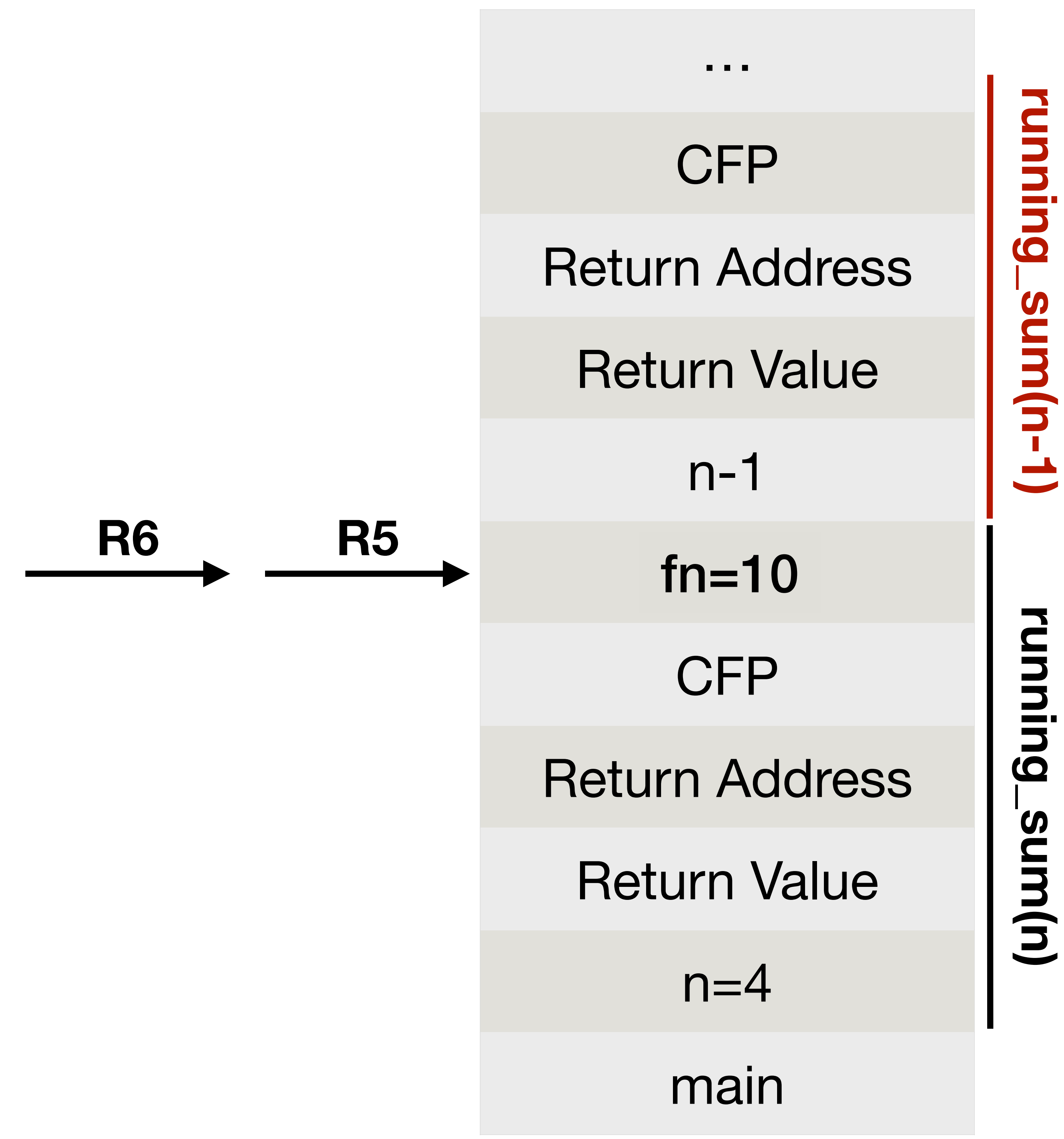
R2 3

# Recursion in LC3

```
;Recursive case
;Caller setup: push argument n-1 onto RTS
ADD R6, R6, #-1
STR R2, R6, #0 ; R2 = n - 1
JSR RUNNING ; call Running(n-1)

;Callee tear-down for Running(n-1) not shown

;Caller tear-down for Running(n-1)
;pop Running(n-1)'s return value to R0
LDR R0, R6, #0
ADD R6, R6, #1

;pop Running(n-1)'s argument
ADD R6, R6, #1

;calculate n + Running(n-1)
LDR R1, R5, #4
ADD R0, R1, R0
STR R0, R5, #0 ;store result in fn

;ready to return
BRnzp RETURN
```

R0 10

R1 4

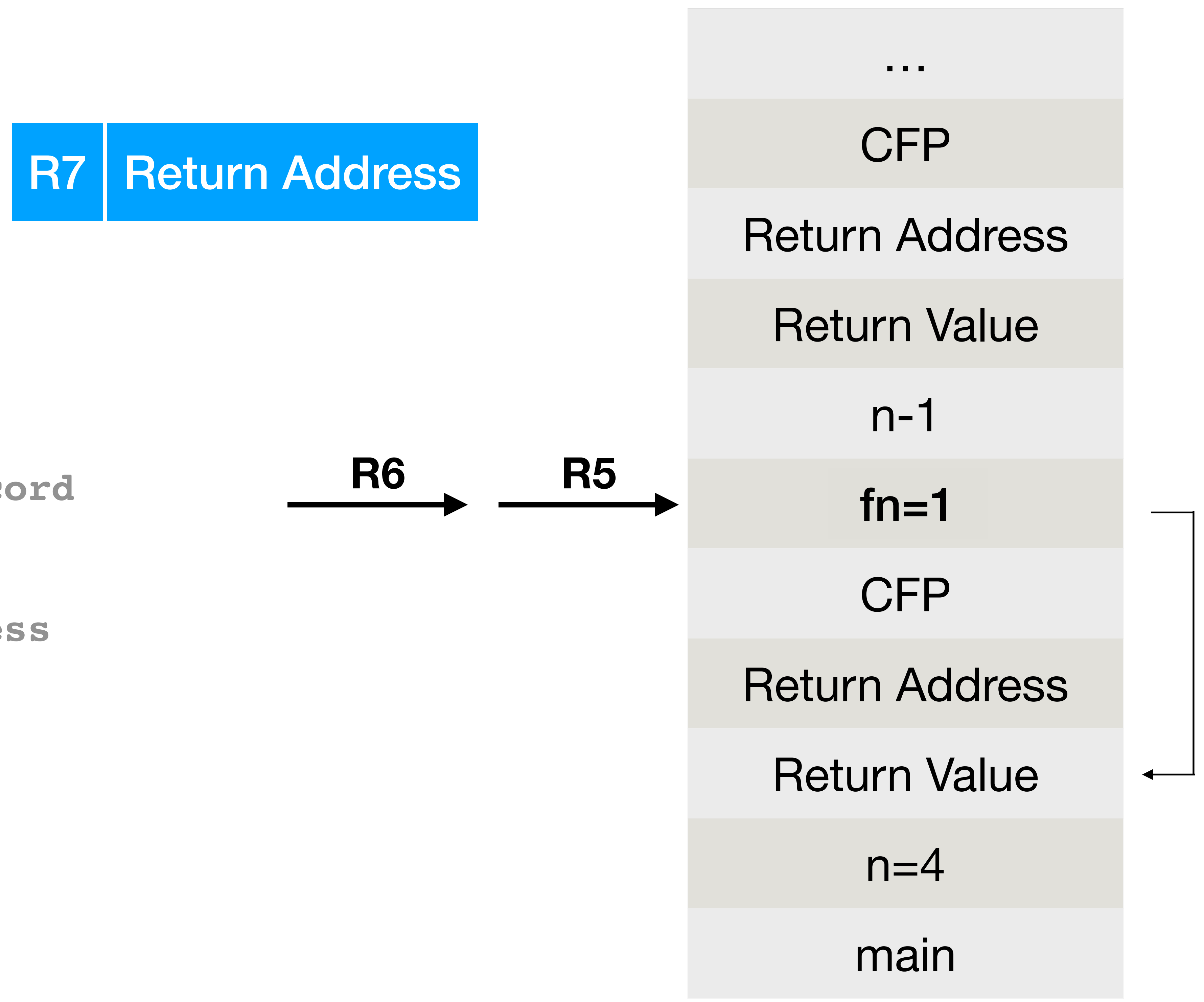


```c
int running_sum(int n){
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}
```

```c
int main(void){
    int n = 4;
    int answer;
    answer = running_sum(n);
}
```

# Recursion in LC3

```
BASE_CASE
AND R2, R2, #0
ADD R2, R2, #1
STR R2, R5, #0 ;set fn = 1

RETURN
;set return value
LDR R0, R5, #0
STR R0, R5, #3

;callee tear-down of Running(n)'s activation record
ADD R6, R6, #1 ;pop local variables

;restore caller's frame pointer and return address
LDR R5, R6, #0
ADD R6, R6, #1
LDR R7, R6, #0
ADD R6, R6, #1

;return to caller
RET
```




**Gitlab C2L3 steps**

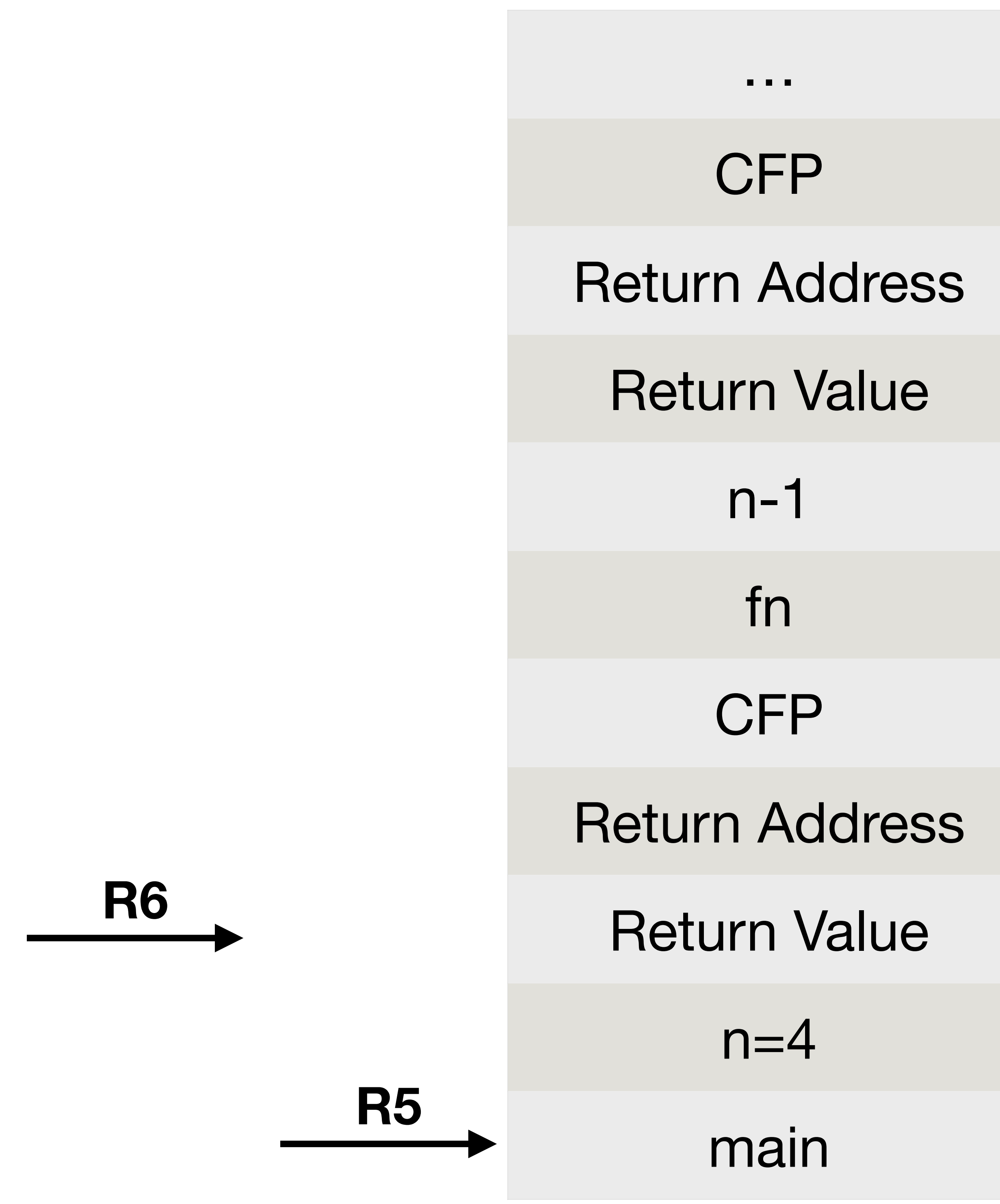
```c
int running_sum(int n){
    int fn;
    if (n==1)
        fn = 1;
    else
        fn = n + running_sum(n-1);
    return fn;
}
```

```c
int main(void){
    int n = 4;
    int answer;
    answer = running_sum(4);
}
```

# Recursion in LC3

```
;Caller stack Tear-down for Running(n)
LDR R0, R6, #0   ;copy return value to R0
STR R0, R5, #-1  ;save return value to answer
ADD R6, R6, #1   ;pop return value from stack
ADD R6, R6, #1   ;pop argument from stack
```

Inside main's activation frame, answer is the second local variable




Back to where we started!

Practice practice practice!

# Next time

- More problem solving with recursion.
  - A small chess problem
  - Solving a maze
- When is recursion good vs. bad?