ECE 220

Lecture x0004 - 09/05

Slides based on material by: Yuting Chen, Yih-Chun Hu & Ujjal Bhowmik

Recap

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- Last time we discussed:
    - Stacks
    - Quarters vs. pancakes
    - Implementing PUSH/POP
    - Examples of use cases for stacks

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Implementation differences in TOS convention

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Implementation differences in TOS convention

- Current top-most element
- Next available spot

A. Balanced parentheses
B. Palindrome check
C. Stack arithmetic

3 4 ×7 2 −3×+

Example from last time

+
x
3
-
2
7
x
4
3

3 4 ×7 2 −3×+

Example from last time

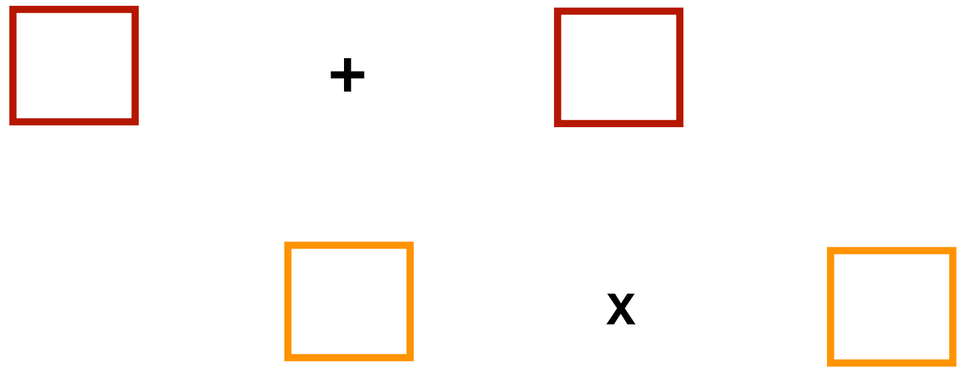
+
x
3
-
2
7
x
4
3

Example from last time

X
3
-
2
7
x
4
3

Example from last time

3
-
2
7
x
4
3



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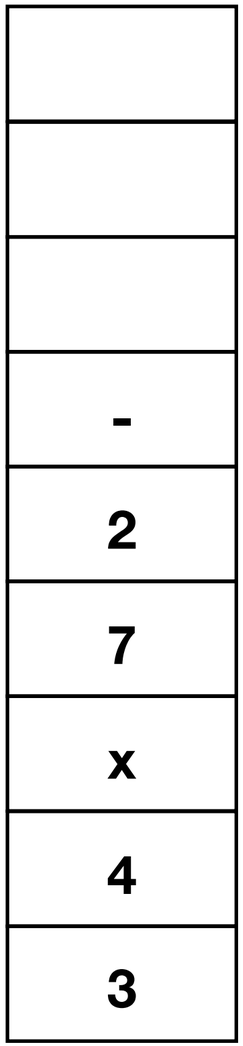

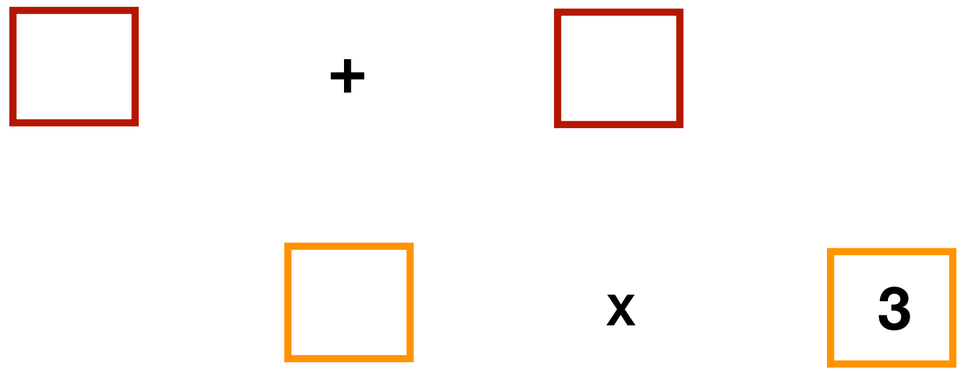

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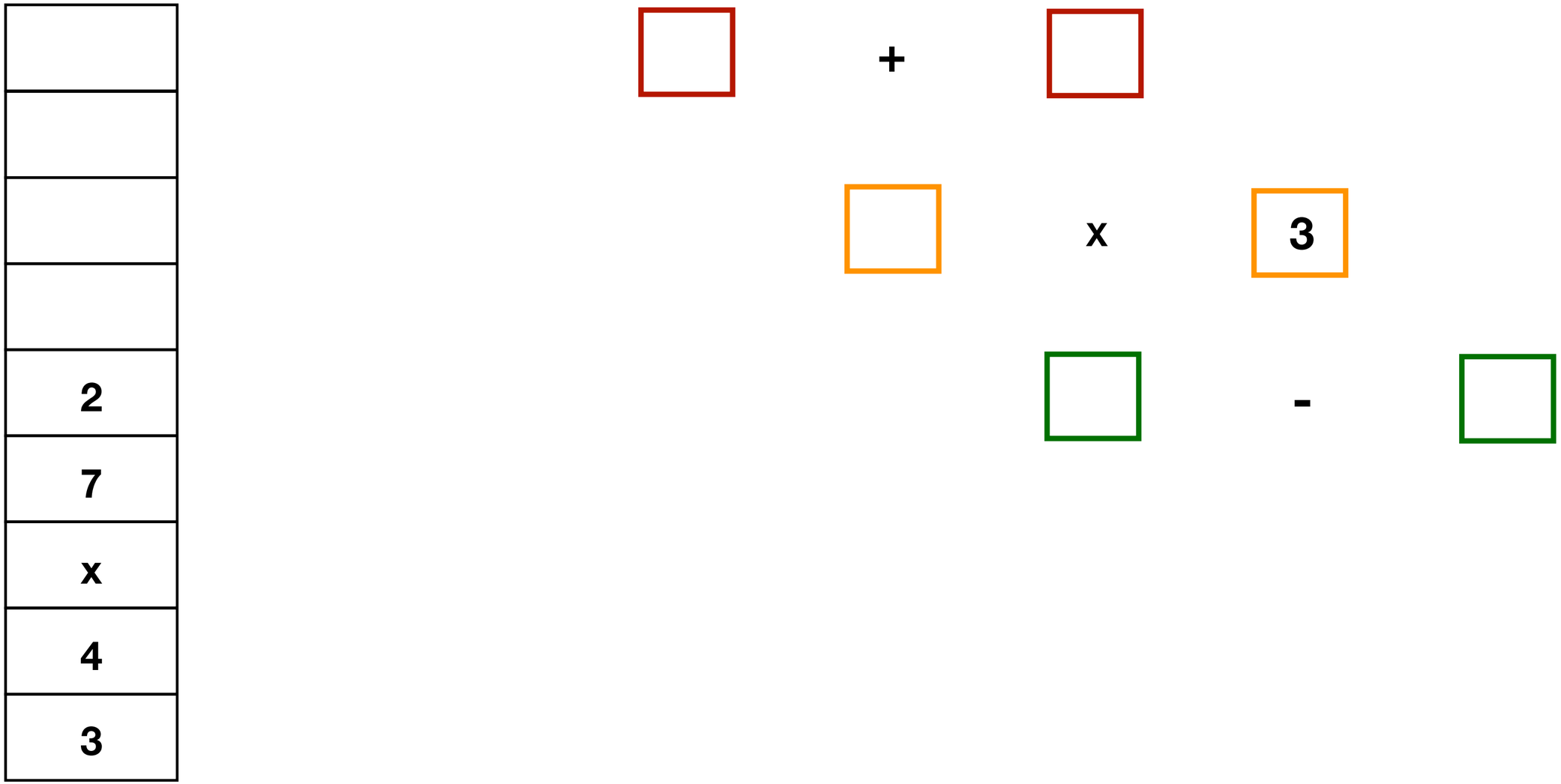

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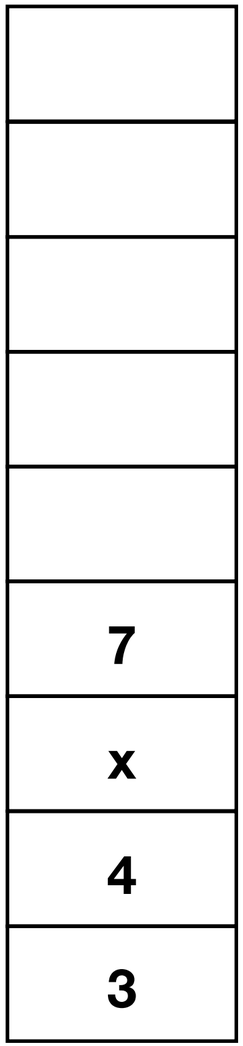

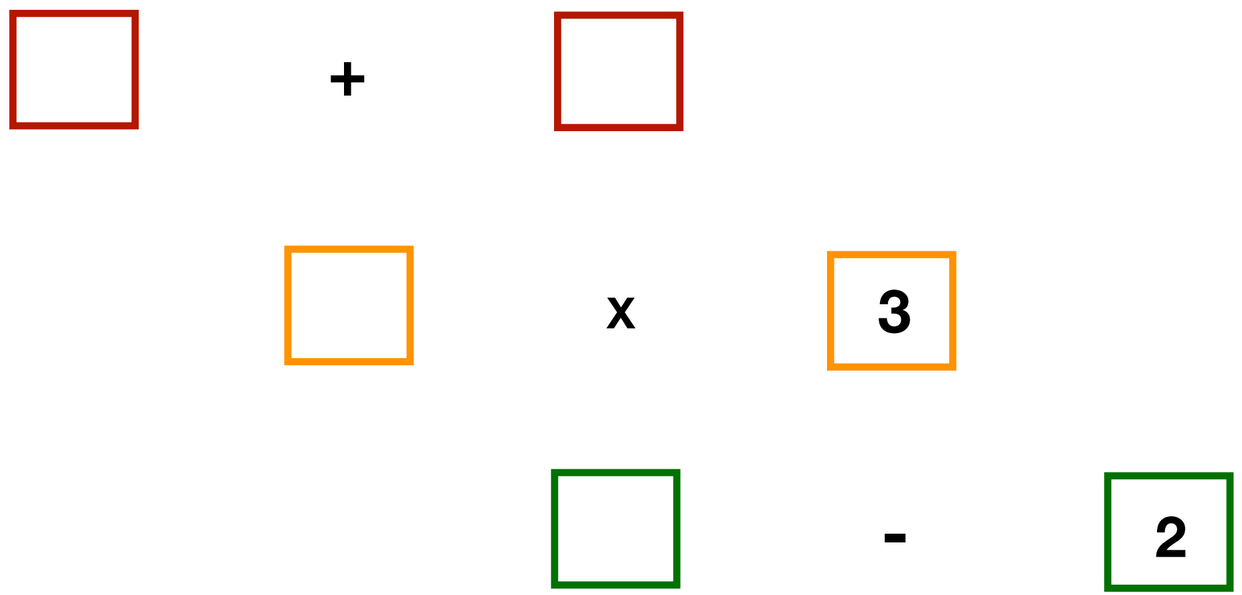

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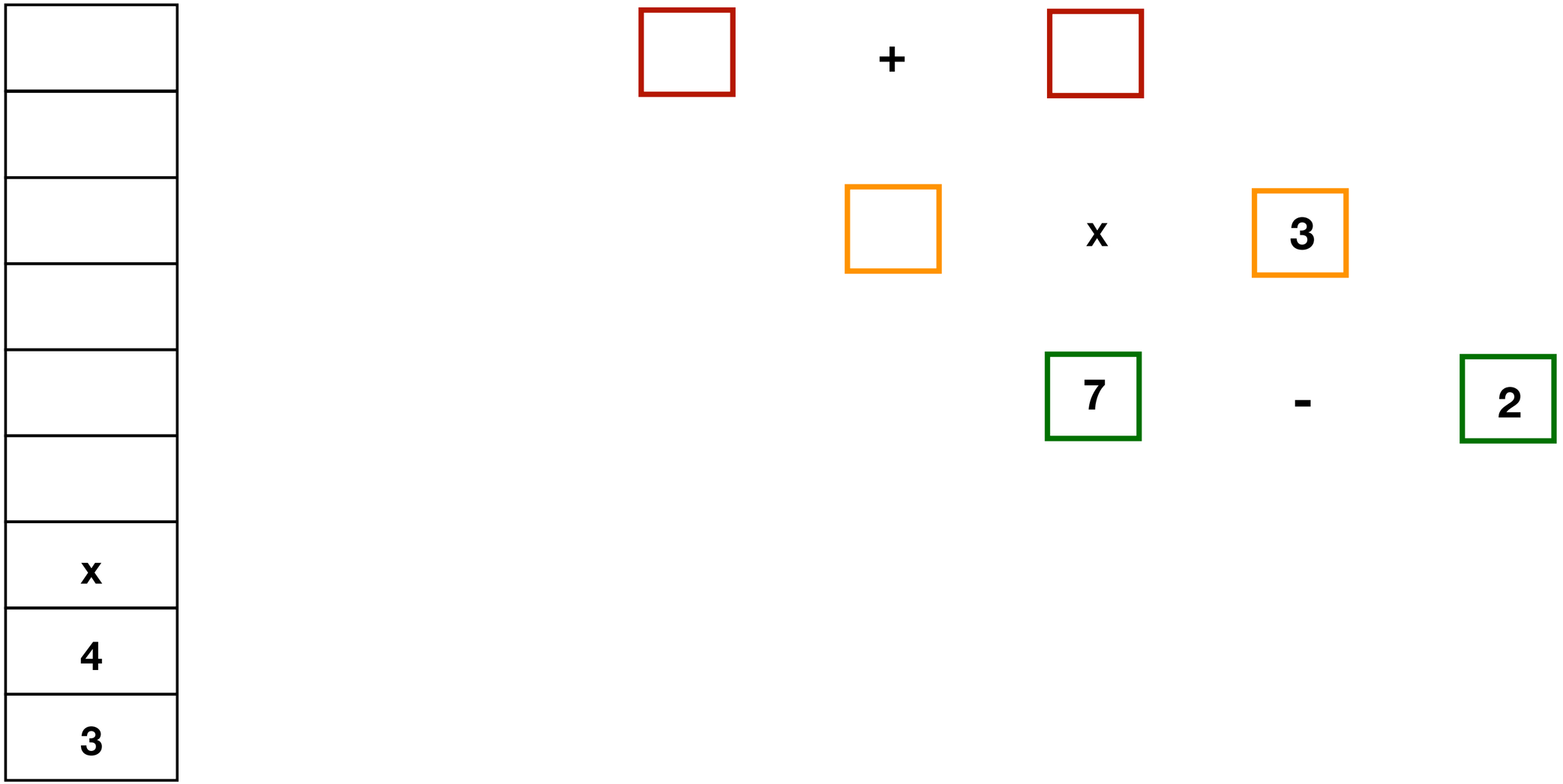

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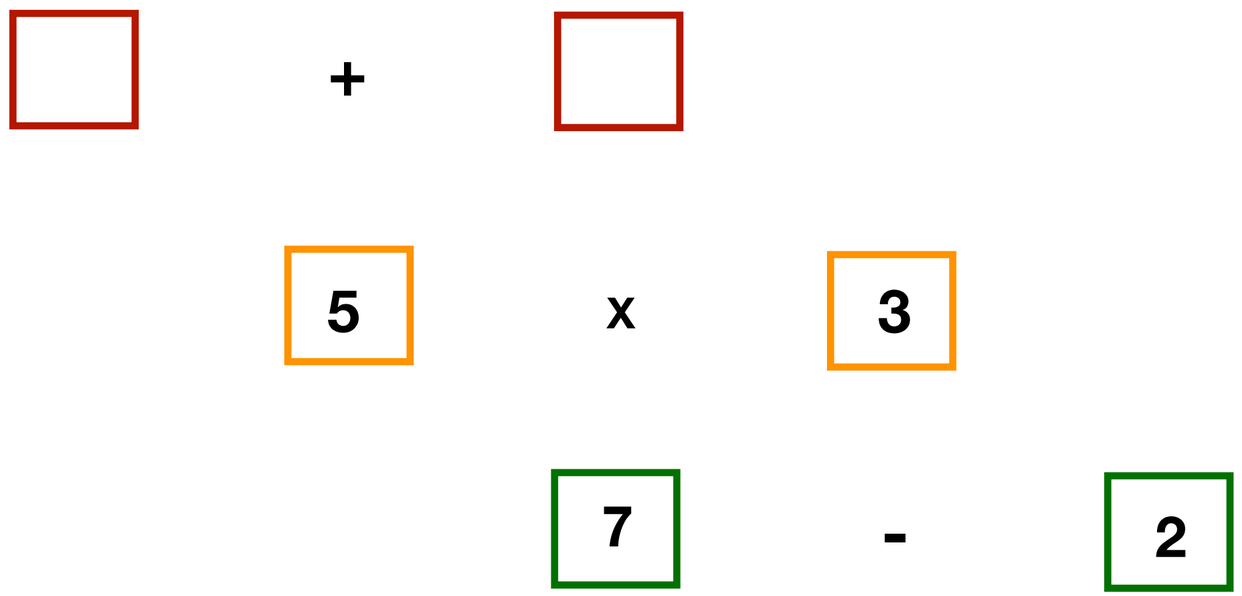

Example from last time

x
4
3

□ + □

5 x 3

7 - 2

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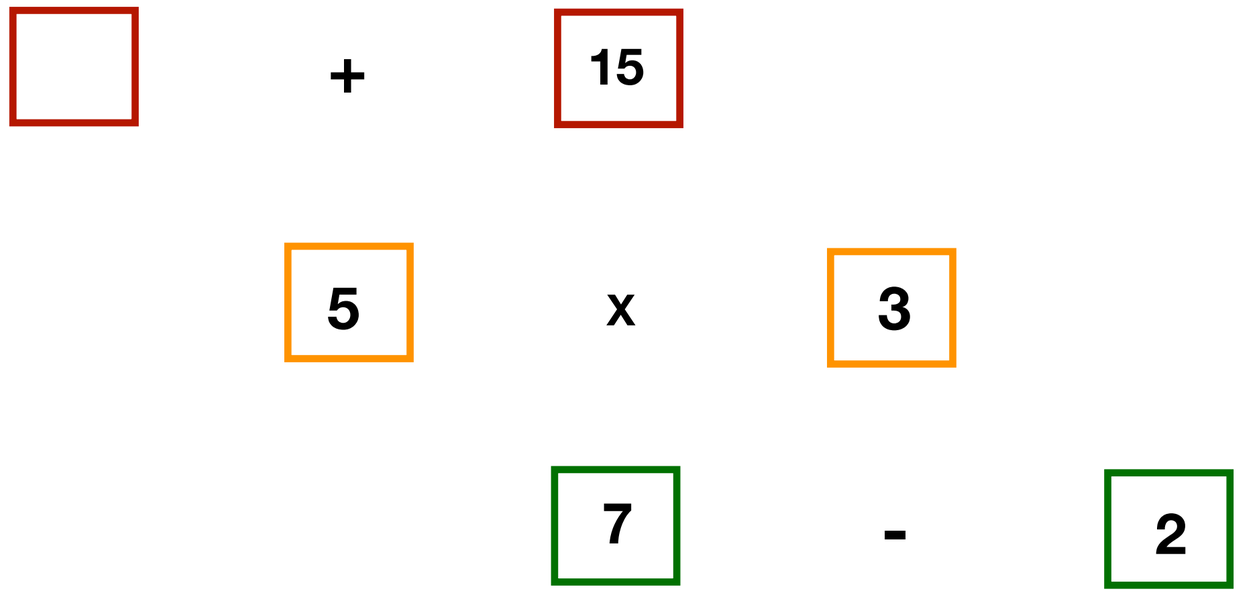

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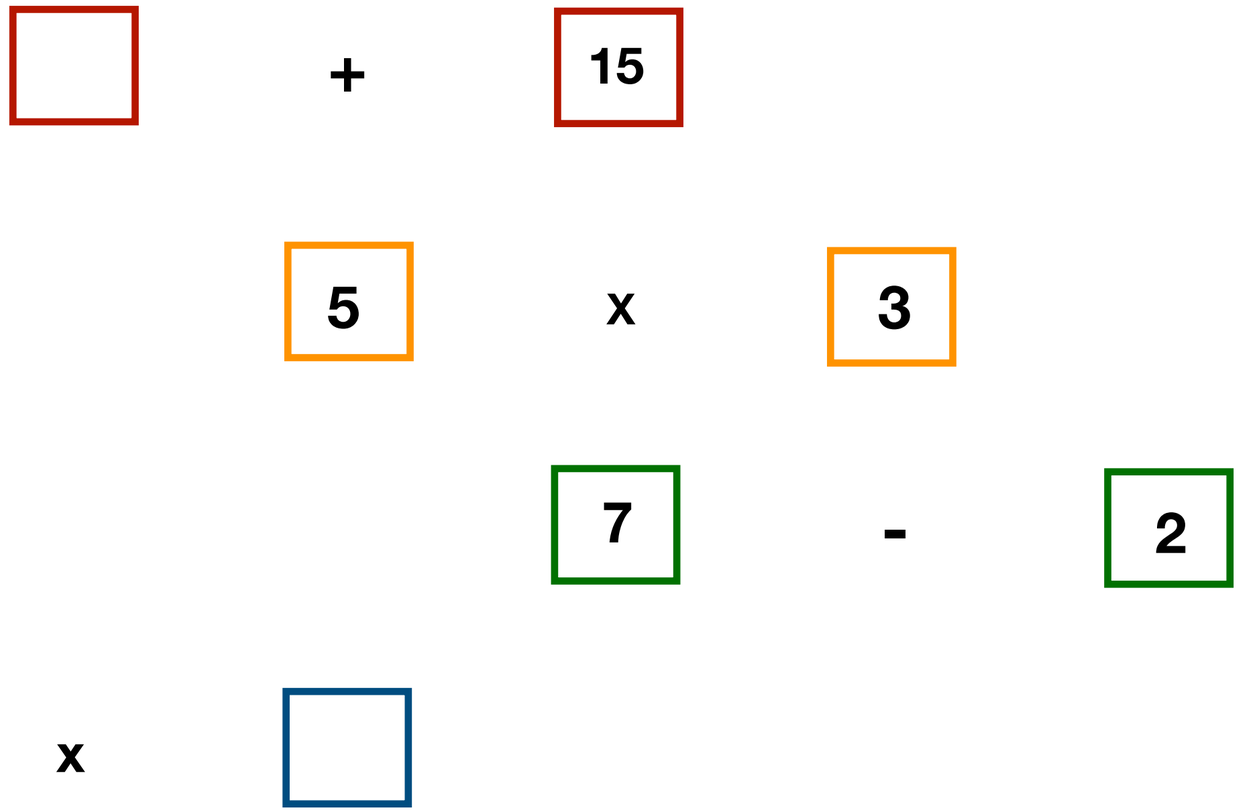

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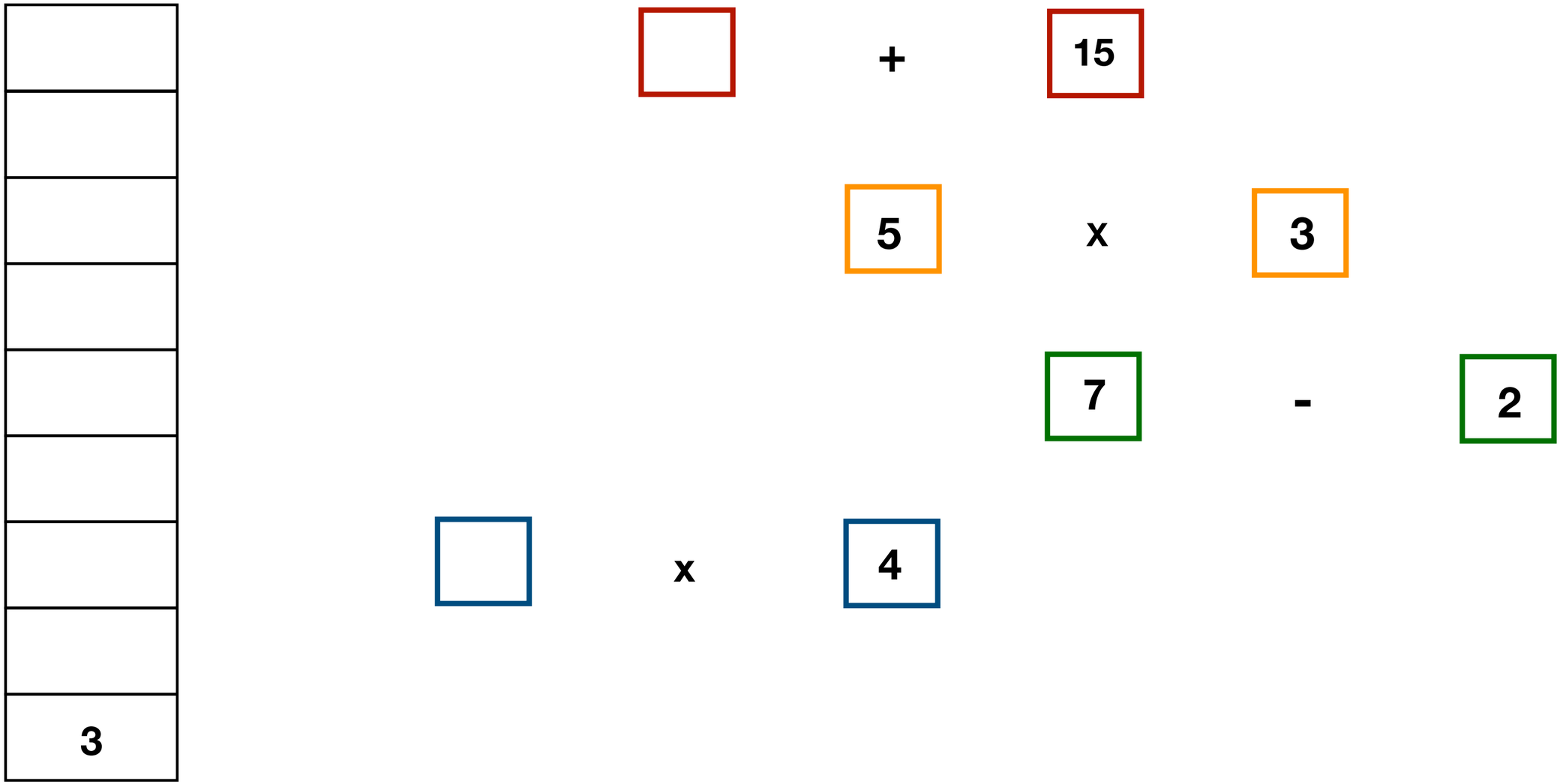

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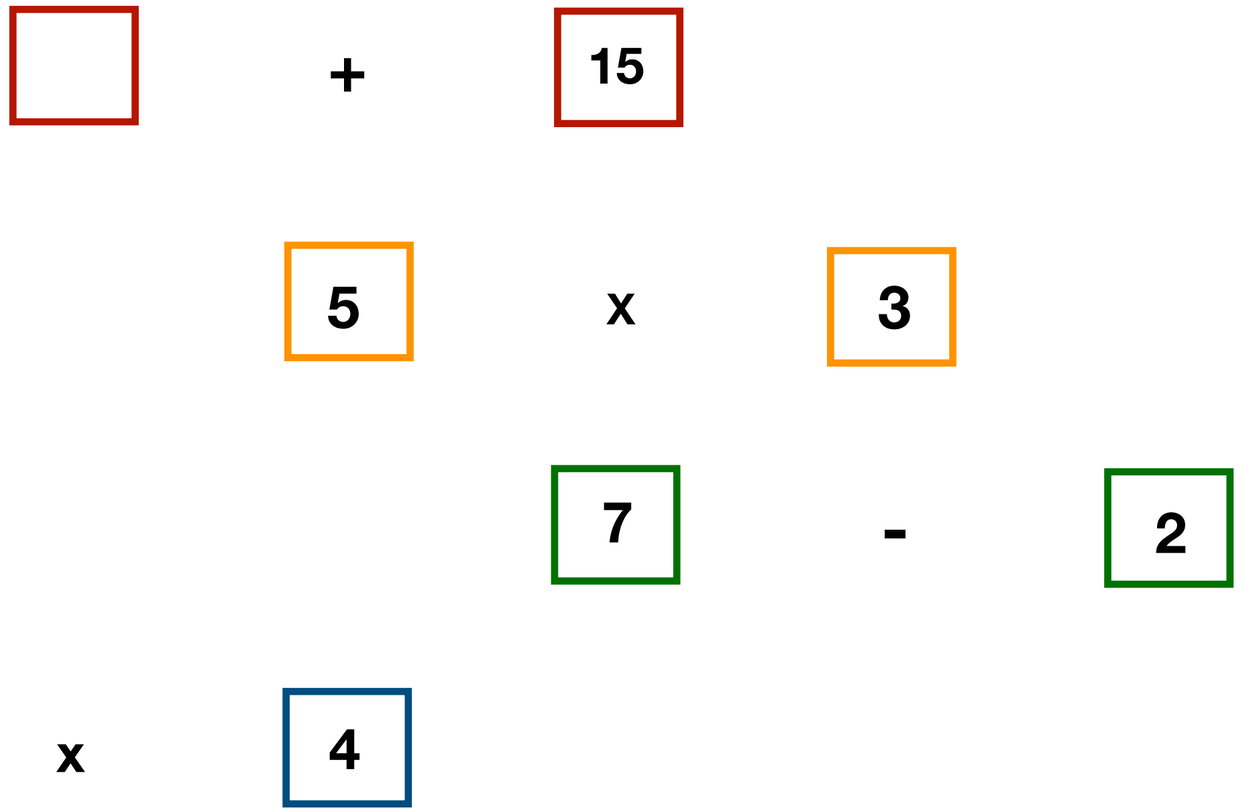

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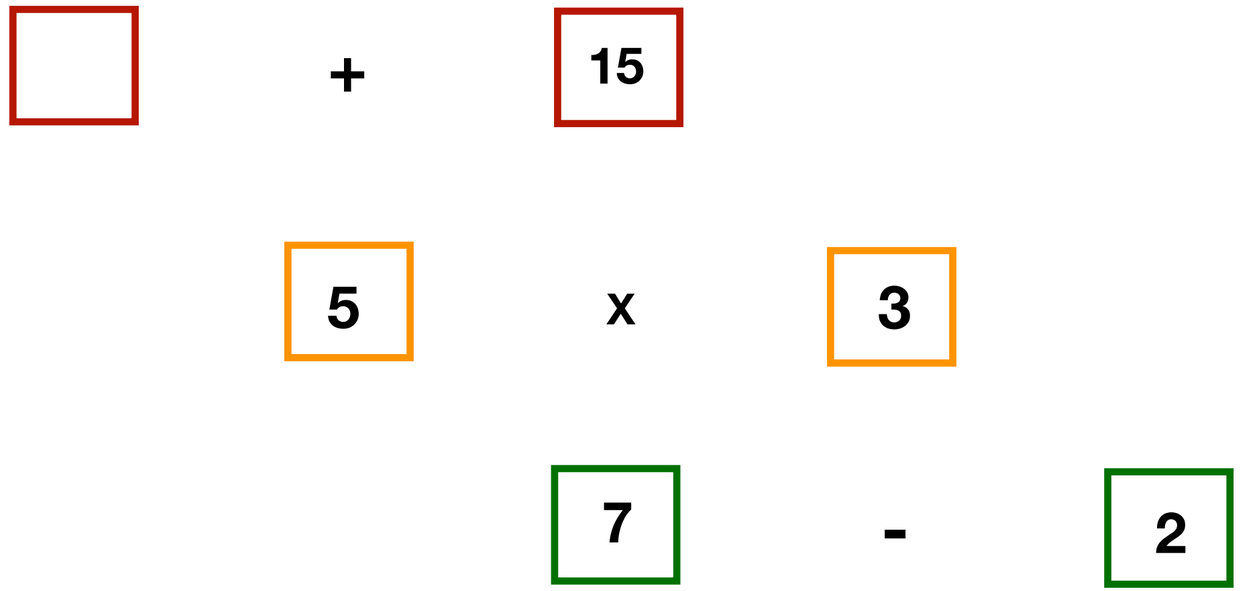

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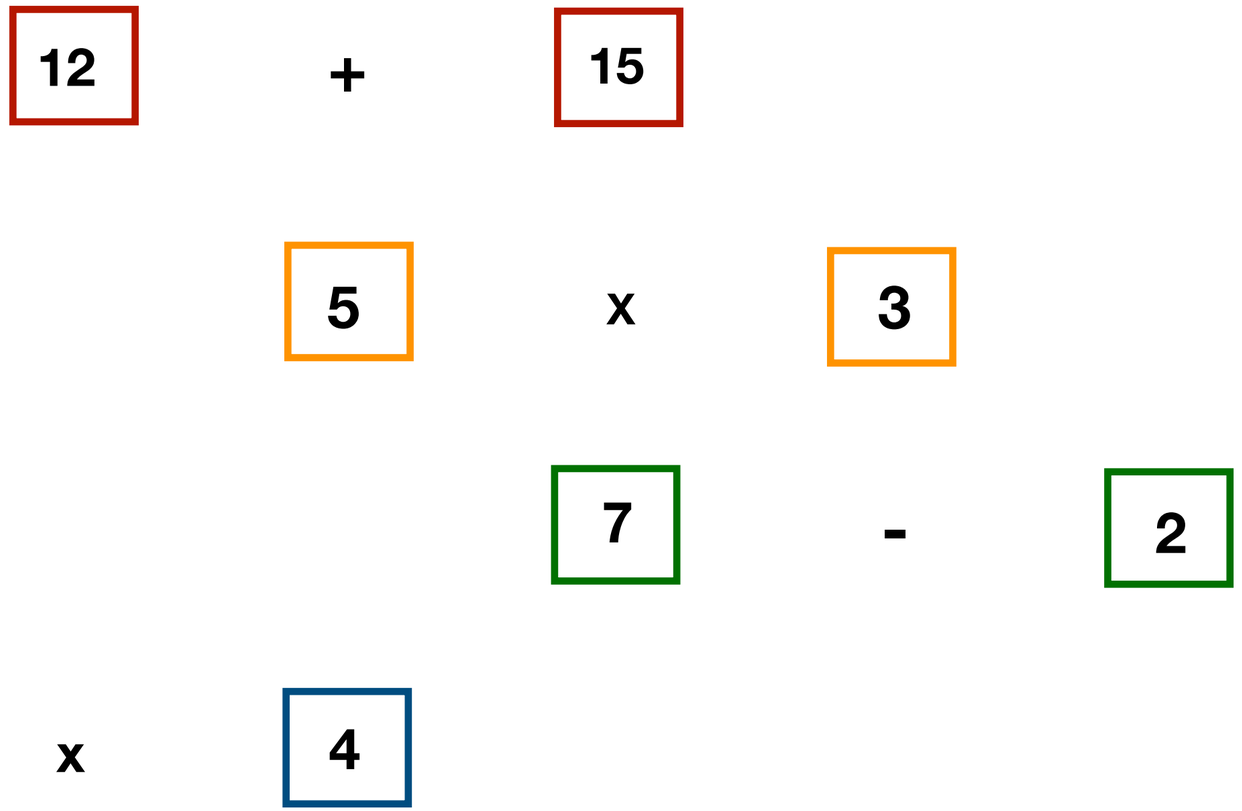

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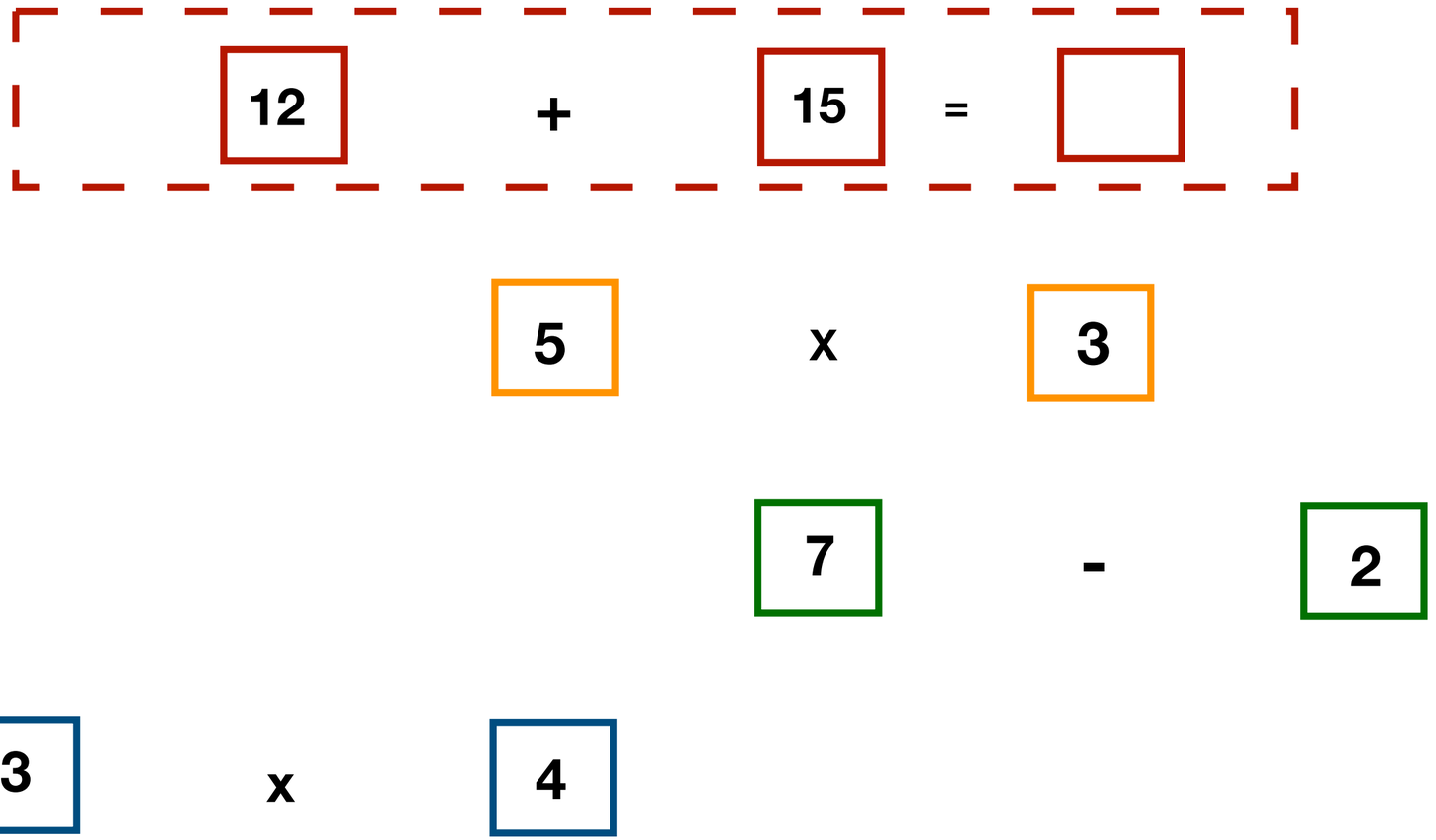

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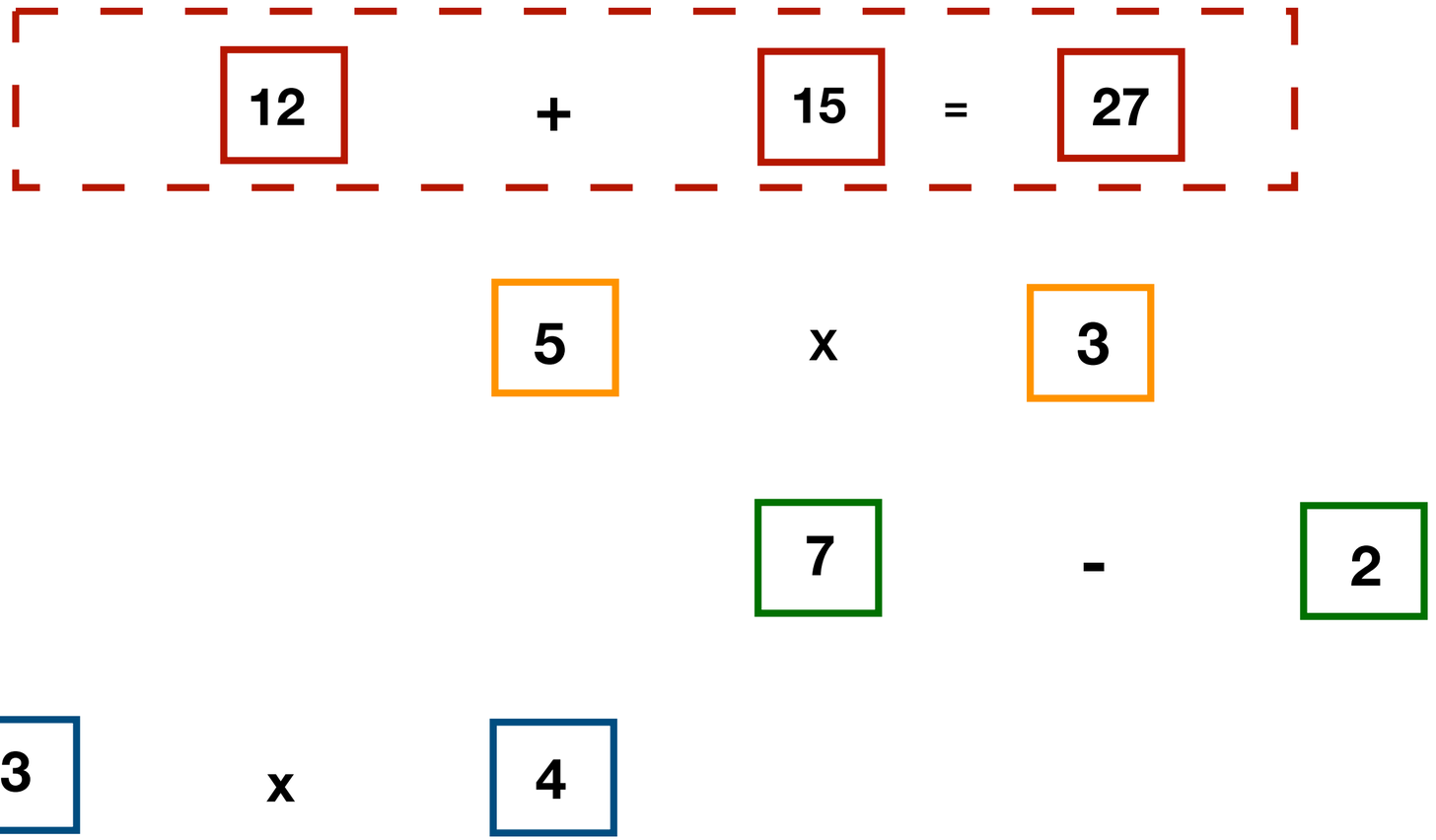

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$$2\ 5 + 4 \times$$

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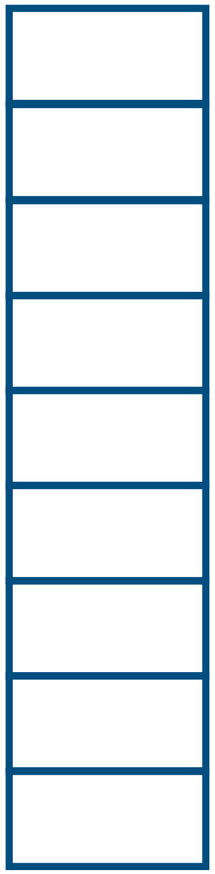

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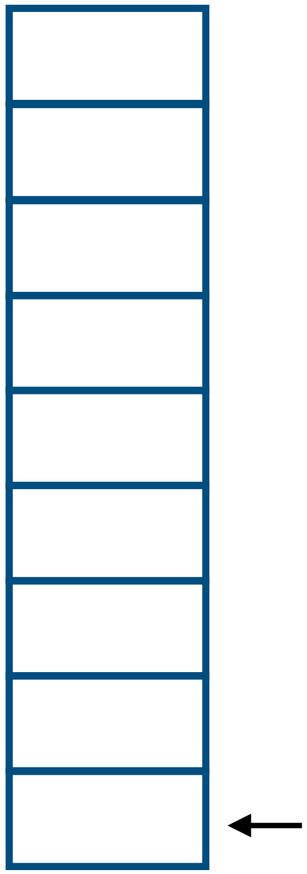

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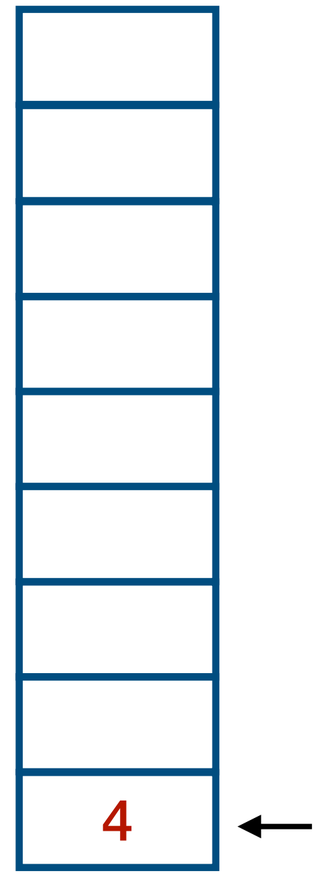

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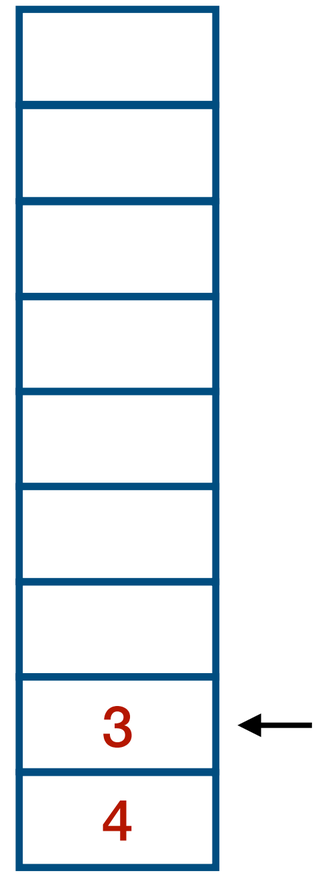

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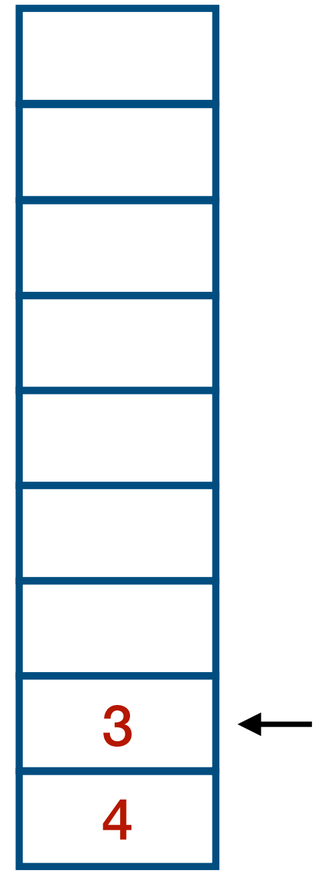

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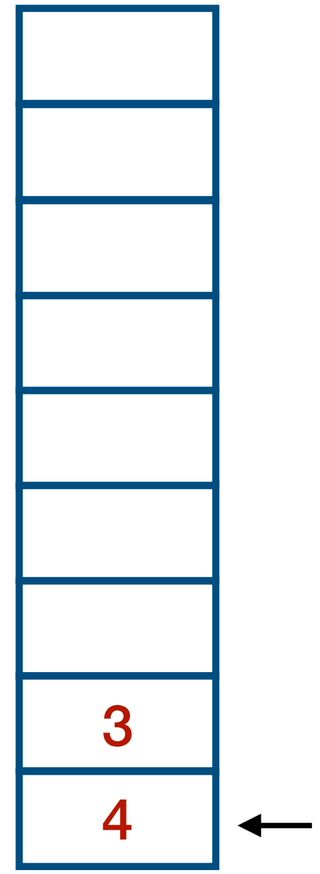

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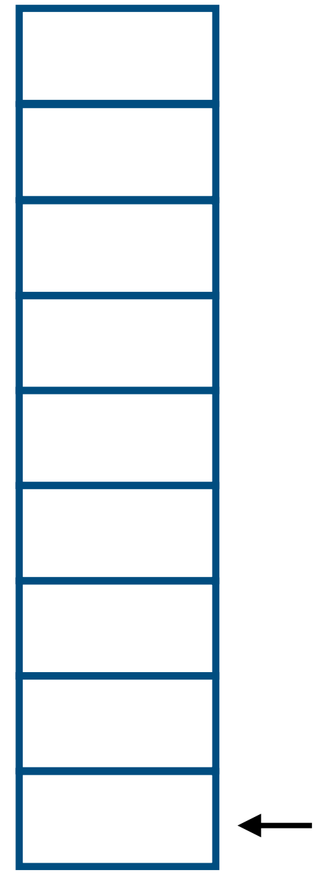

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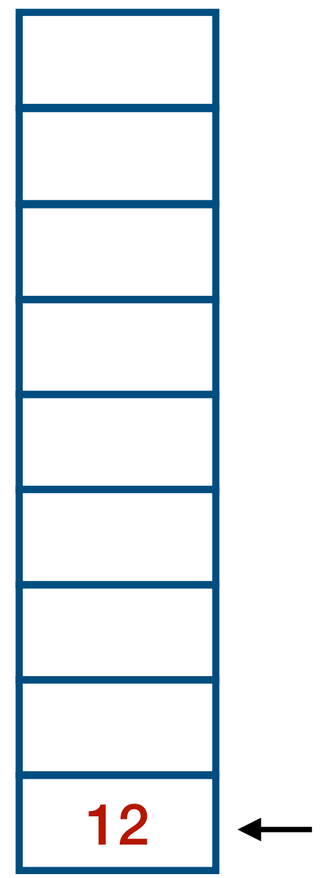

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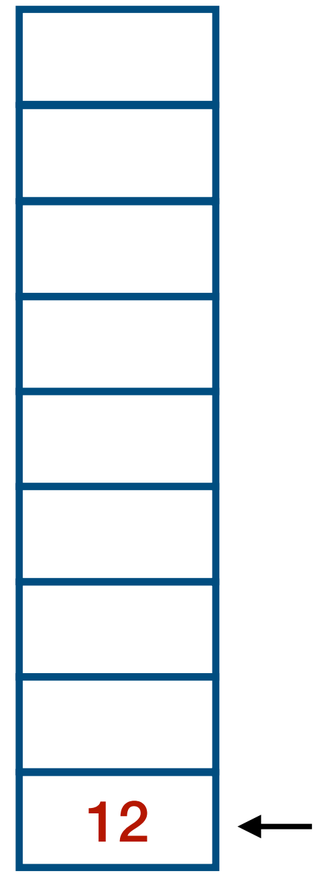

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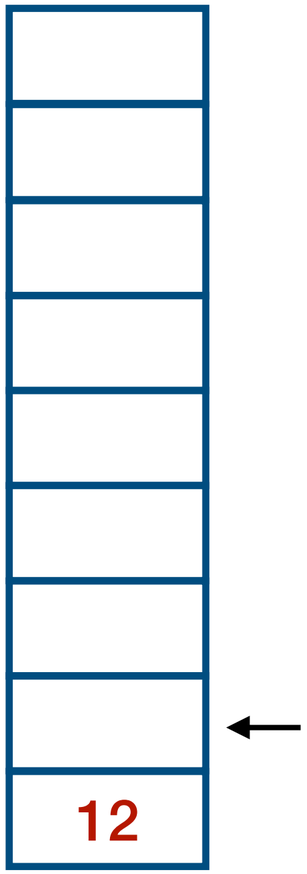

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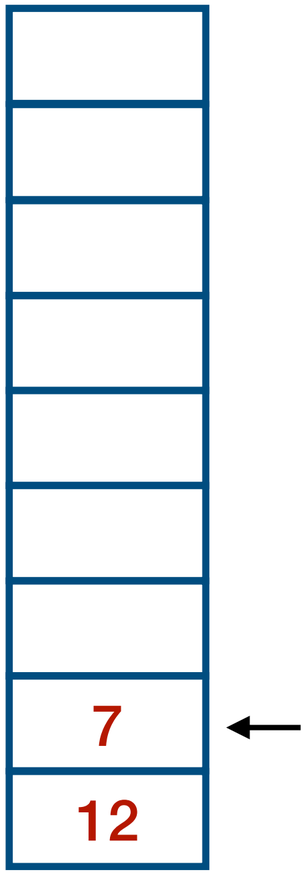

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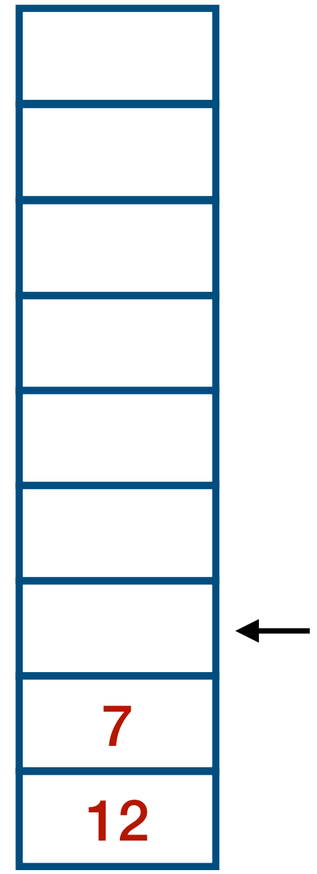

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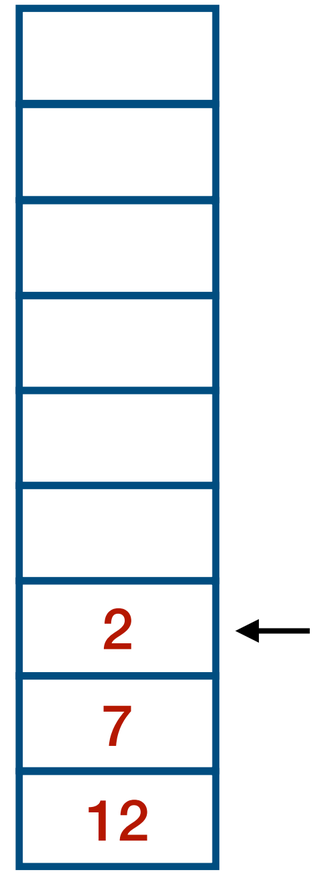

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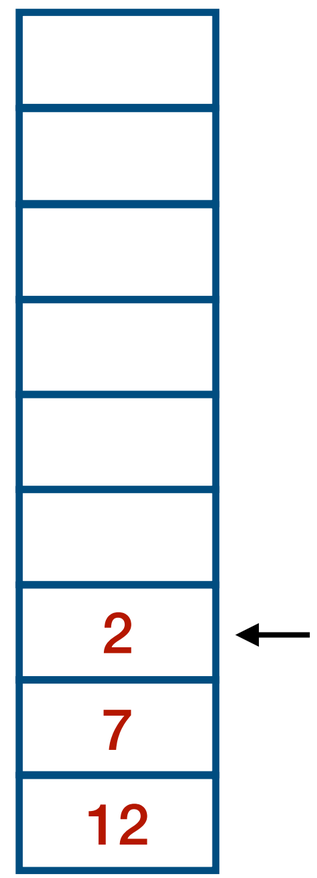

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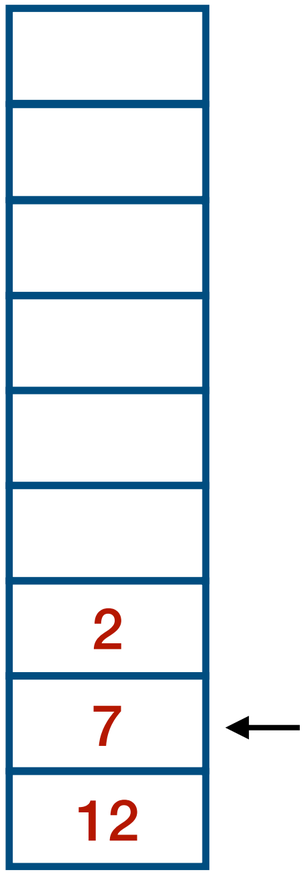

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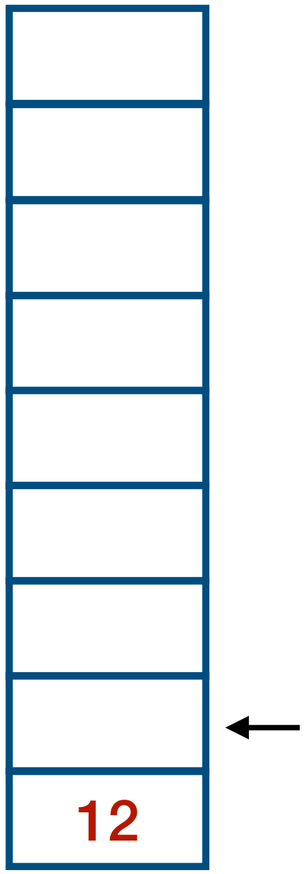

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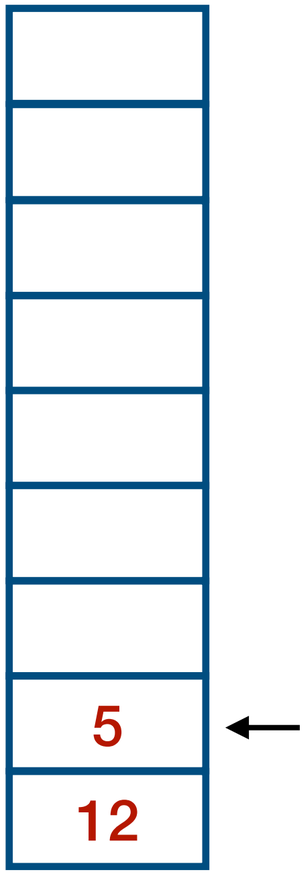

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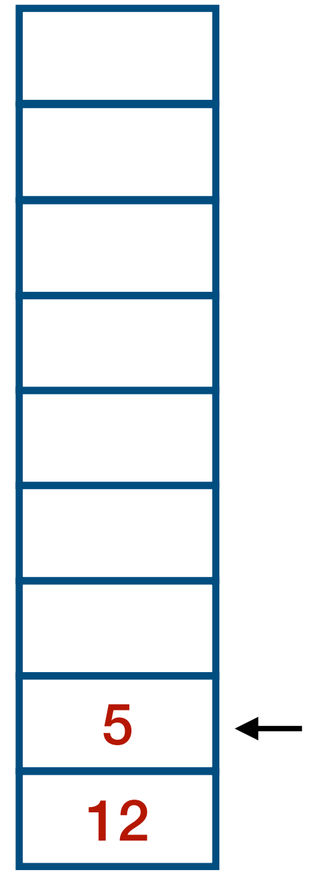

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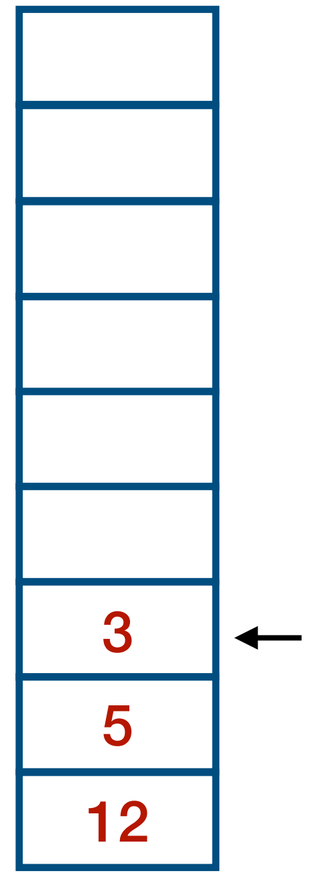

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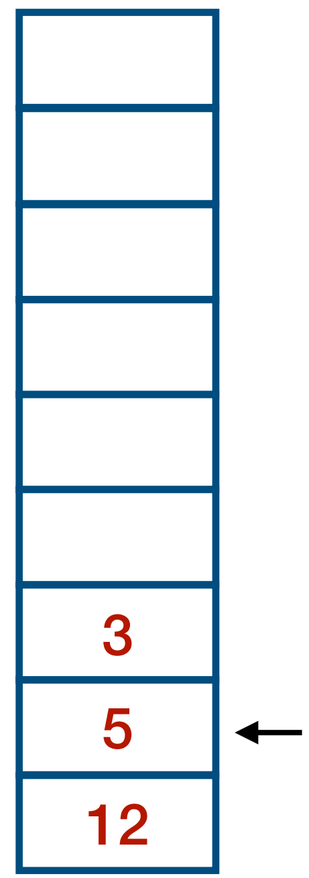
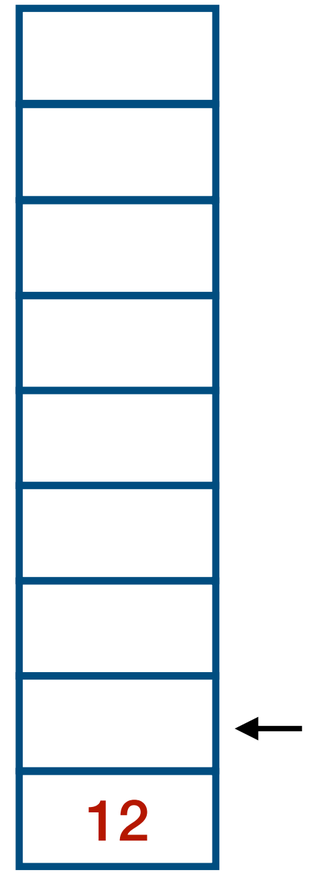


Redo example - single pass?

- Expression: $4\ 3\ \times 7\ 2\ -\ 3\ \times +$
  ↓
- Strategy:
  - Numbers → Push
  - Operator:
    - Pop two elements
    - Perform operation
    - Push on stack



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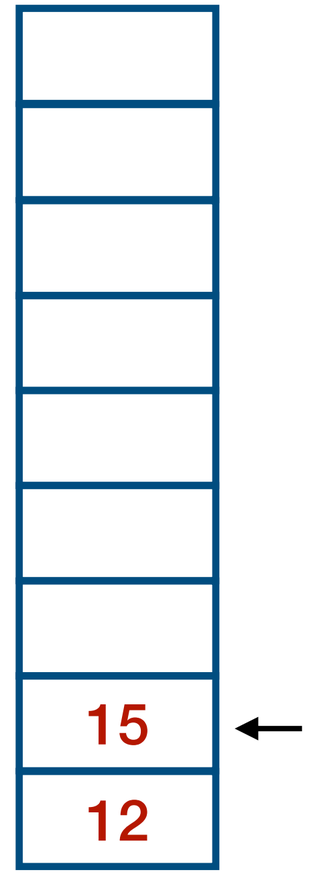

Redo example - single pass?

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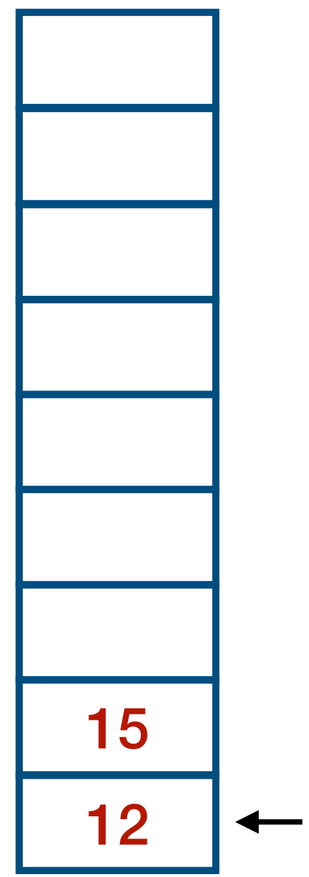

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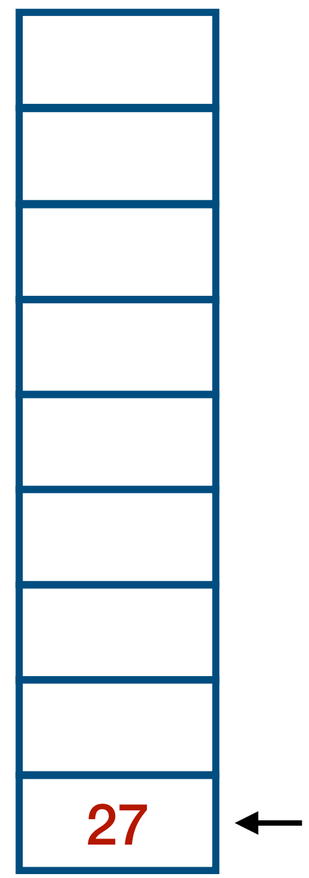

Arithmetic using stack - LC3

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- Compute $(A + B) \times (C + D)$
  and store result in R0

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  and store result in R0

```
;Implementation using registers
LD R0, A
LD R1, B
ADD R1, R0, R1
LD R2, C
LD R3, D
ADD R3, R2, R3
JSR MULT
HALT
```

MULT: subroutine such that
Input: R1, R3 and Output: R0

Arithmetic using stack - LC3

- Compute $(A + B) \times (C + D)$ and store result in R0

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MULT: subroutine such that
Input: R1, R3 and Output: R0

- Compute $A\ B + C\ D + \times$ and store result in R0

Arithmetic using stack - LC3

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HALT
```

MULT: subroutine such that
Input: R1, R3 and Output: R0

- Compute $A\ B + C\ D + \times$ and store result in R0

```
;Implementation using stack
LD RO, A
PUSH
LD RO, B
PUSH
JSR ADD       ;Assuming ADD exists
LD RO, C
PUSH
LD RO, D
PUSH
JSR ADD
JSR MULTIPLY ;Assuming MULTIPLY exists
POP      ;RESULT in R0
```

Arithmetic using stack - LC3

- Compute $(A + B) \times (C + D)$ and store result in R0

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;Implementation using registers
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ADD: POP two numbers, compute and then PUSH result back

Arithmetic using stack - LC3

- Compute $(A + B) \times (C + D)$ and store result in R0

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;Implementation using registers
LD R0, A
LD R1, B
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MULT: subroutine such that Input: R1, R3 and Output: R0

- Compute $A\ B + C\ D + \times$ and store result in R0

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;Implementation using stack
LD RO, A
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LD RO, D
PUSH
JSR ADD
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```

ADD: POP two numbers, compute and then PUSH result back

MULTIPLY: POP two numbers, compute and then PUSH result back

Given that below is an evaluation of an RPN expression: what expression is being evaluated?

Stack usage - memory

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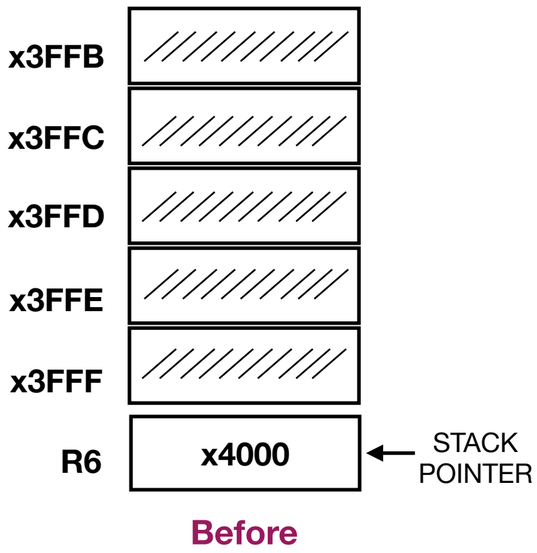
Stack usage - memory



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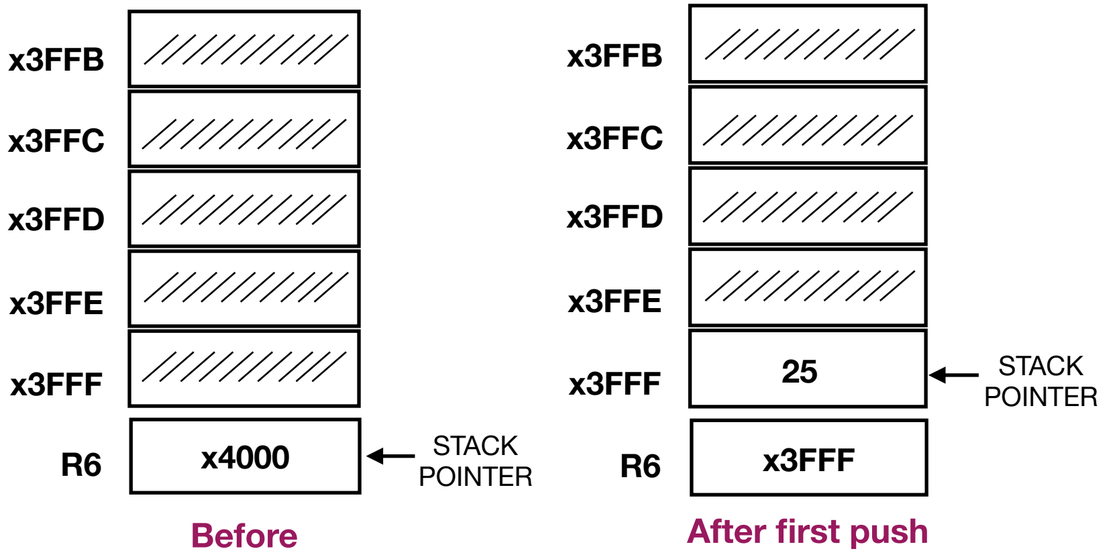

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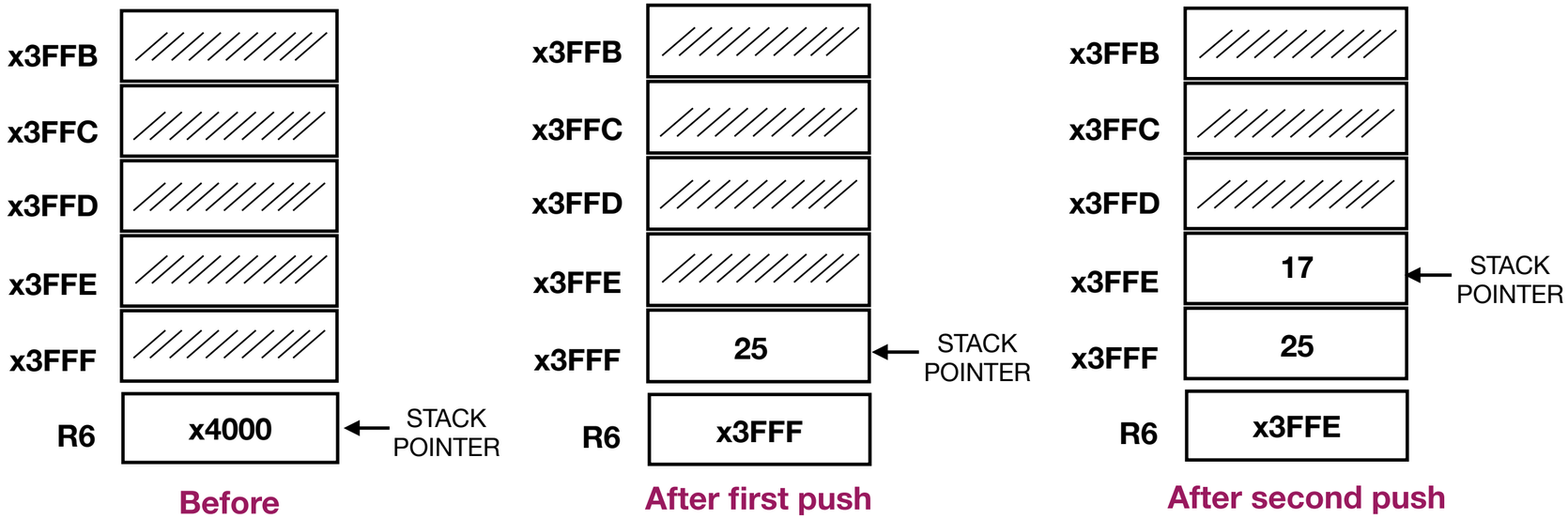

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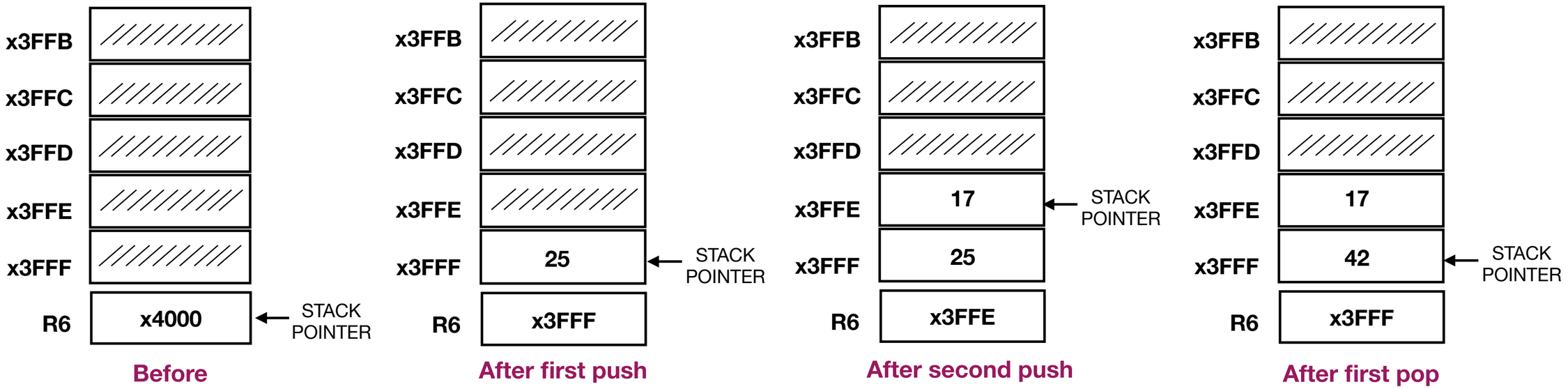

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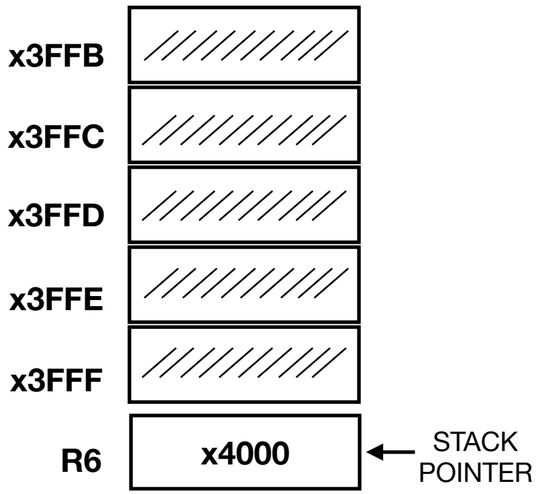
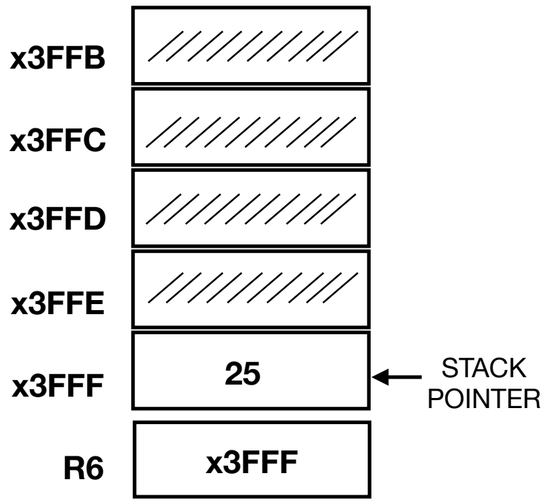
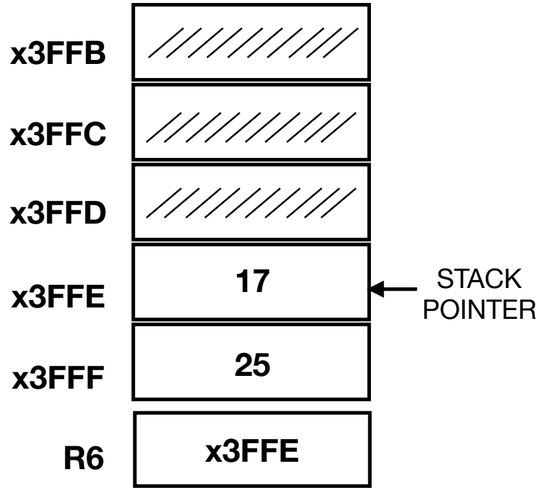


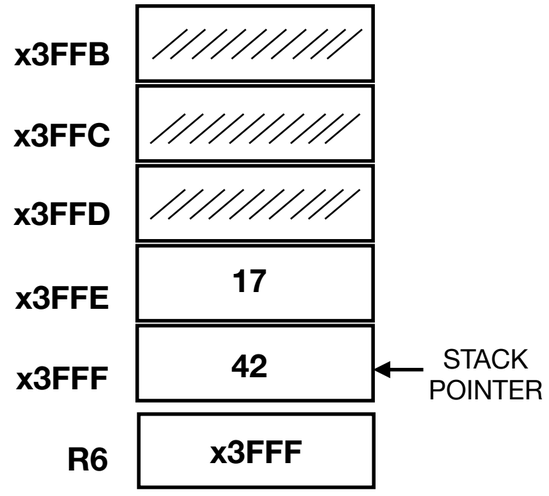
Before



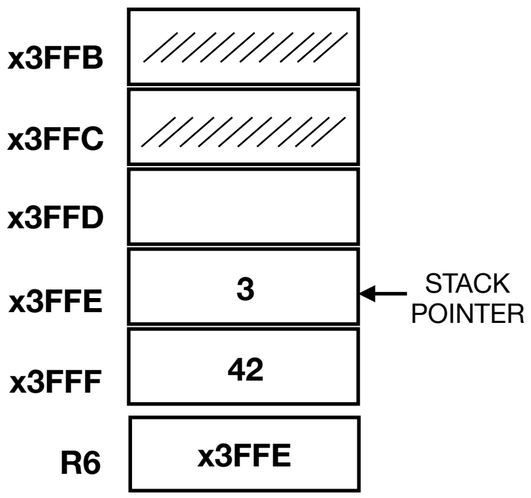
After first push



After second push



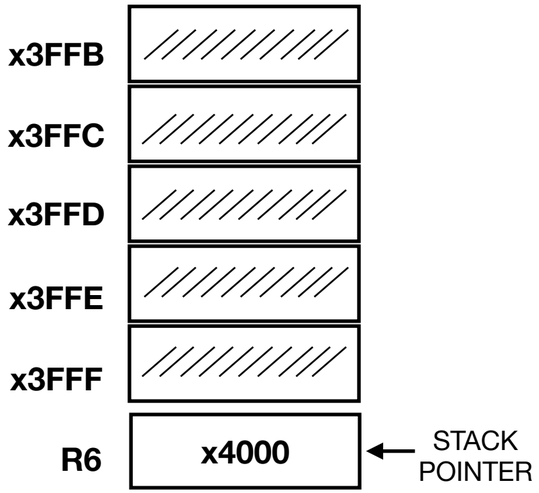
After first pop



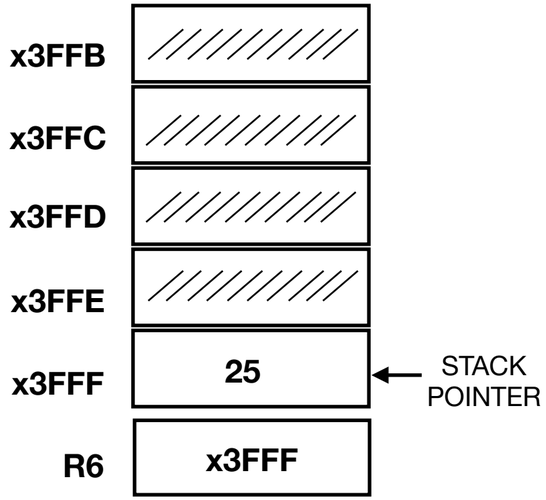
After third push

Given that below is an evaluation of an RPN expression: what expression is being evaluated?

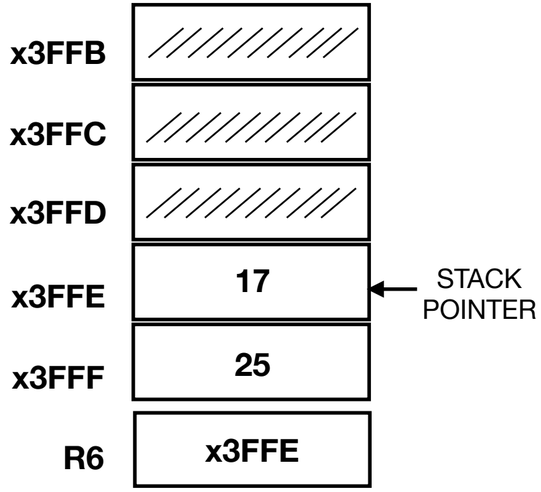
Stack usage - memory



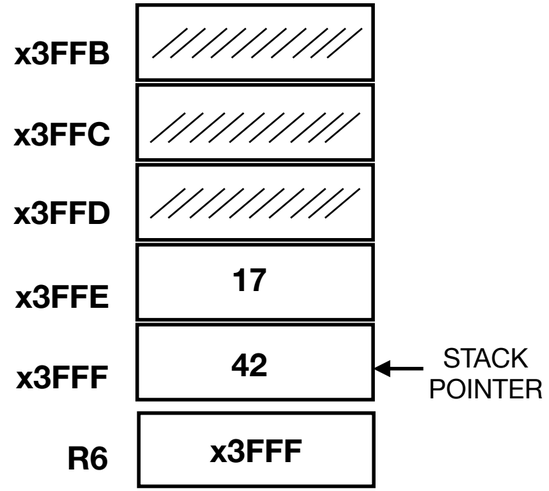
Before



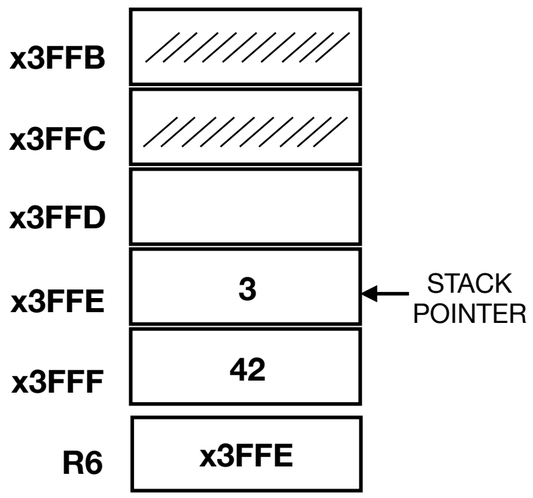
After first push



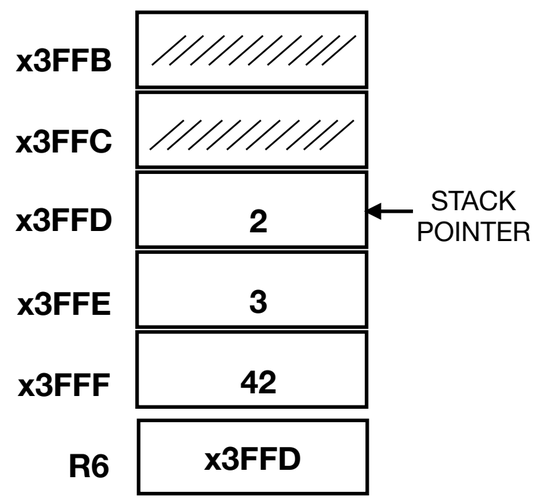
After second push



After first pop



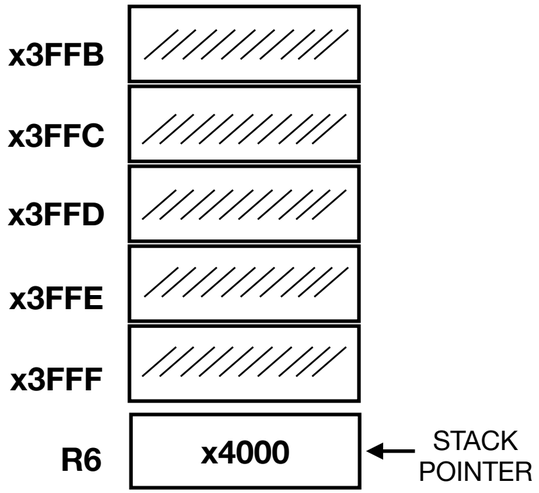
After third push



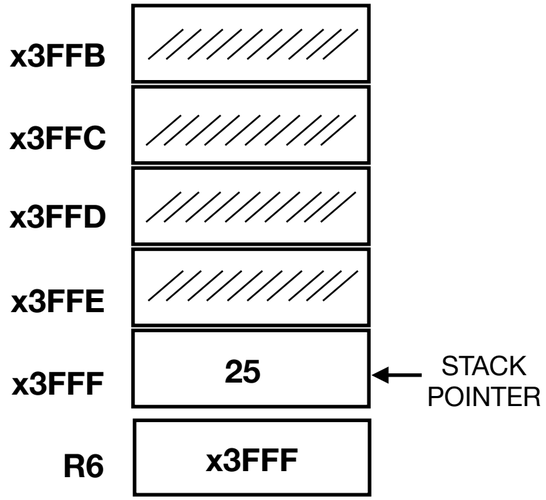
After fourth push

Given that below is an evaluation of an RPN expression: what expression is being evaluated?

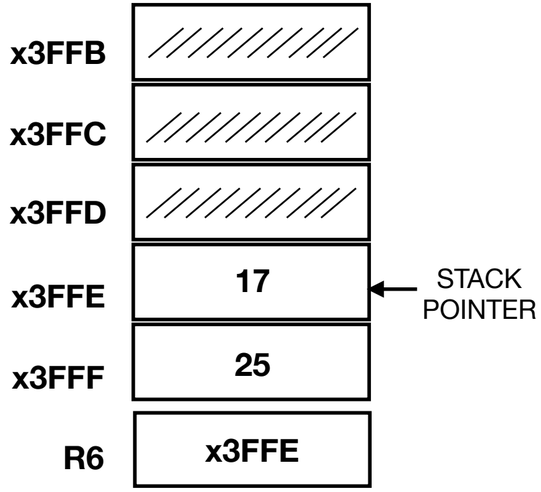
Stack usage - memory



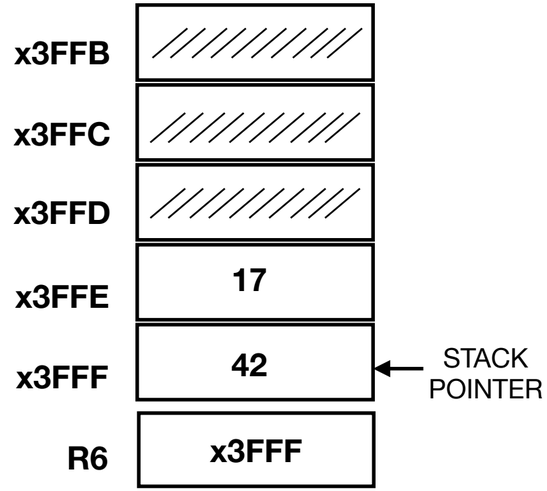
Before



After first push



After second push



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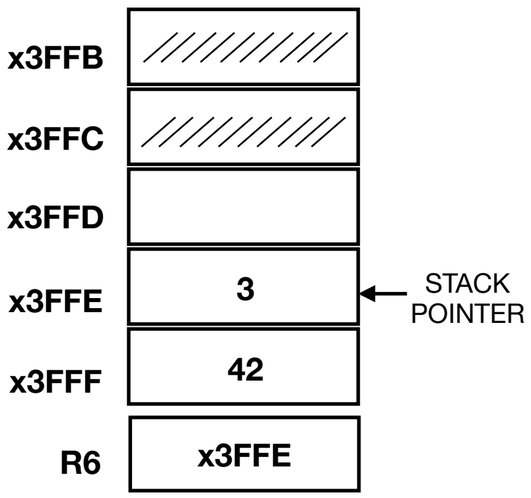

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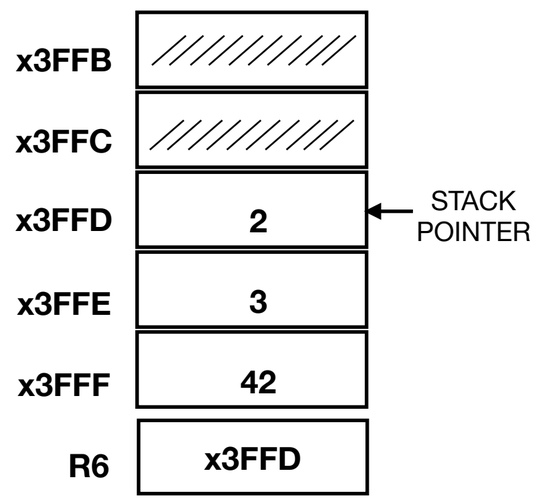

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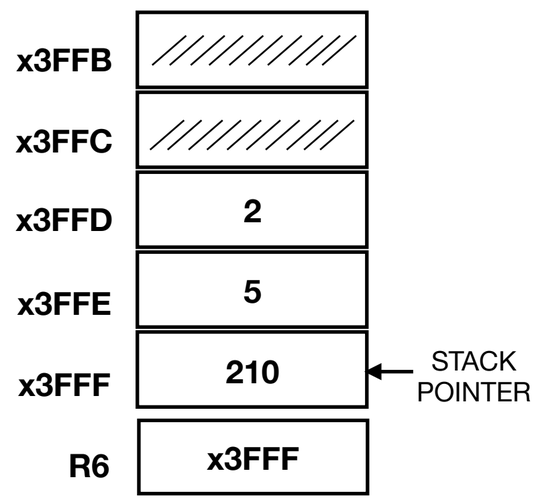
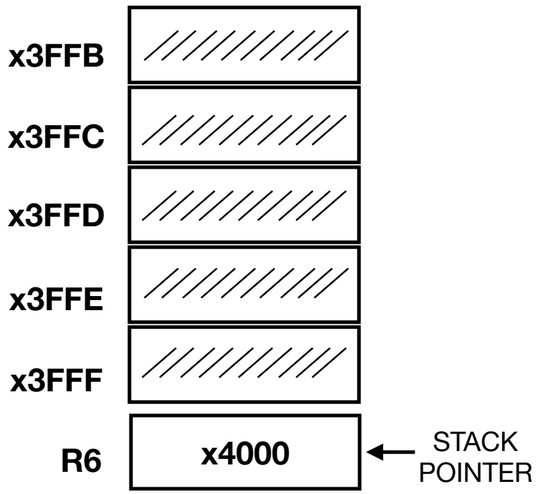


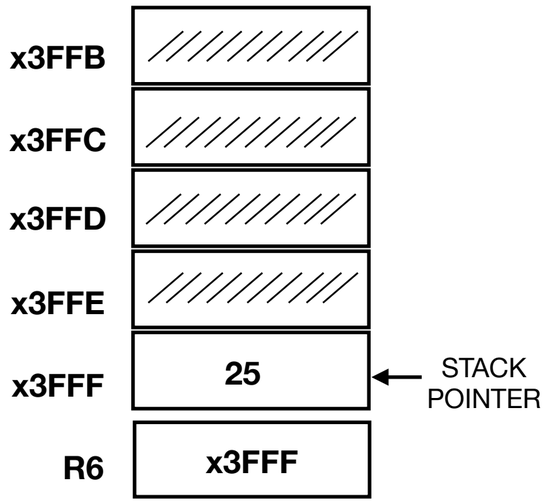
After 2nd & 3rd Ops

Given that below is an evaluation of an RPN expression: what expression is being evaluated?

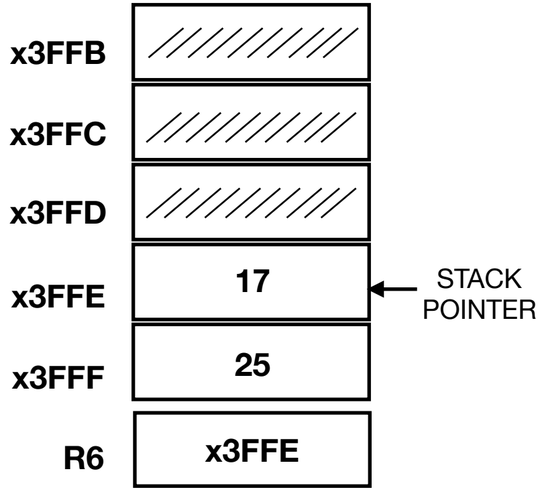
Stack usage - memory



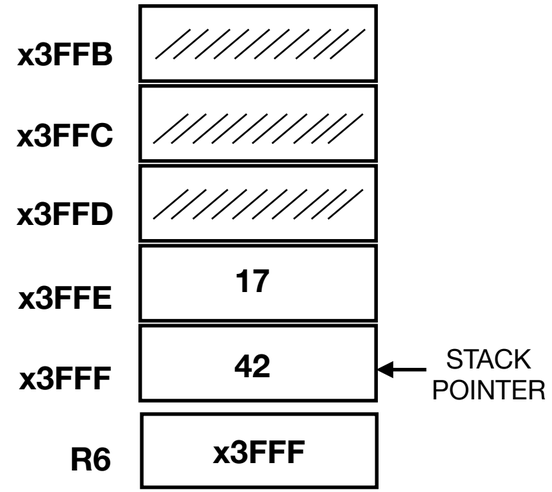
Before



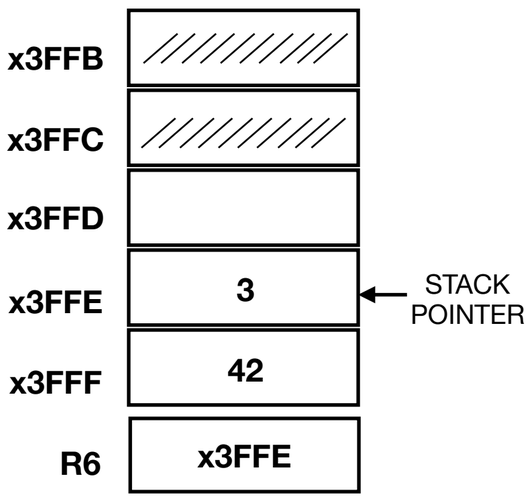
After first push



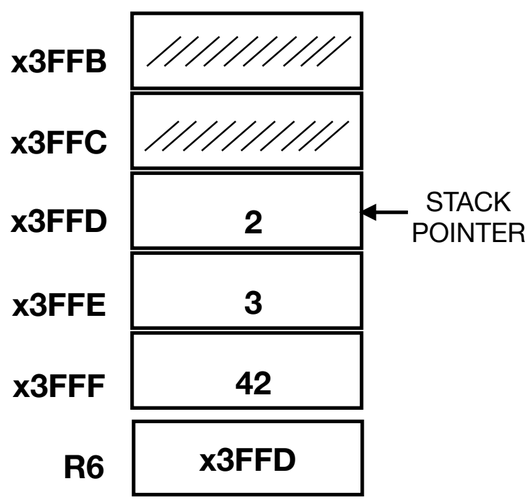
After second push



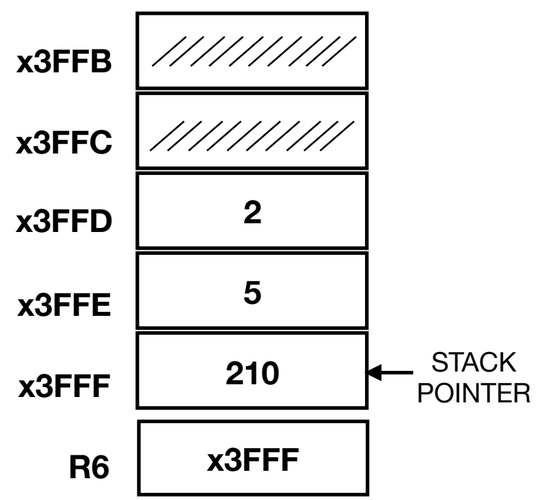
After first pop



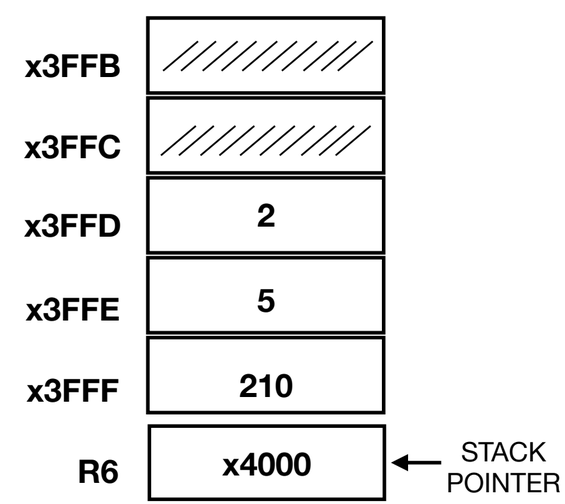
After third push



After fourth push



After 2nd & 3rd Ops

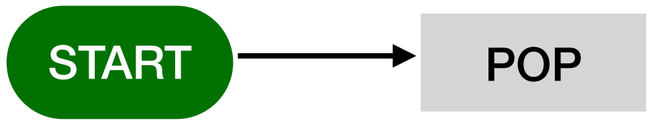


Pop

Flowchart - ADD subroutine

Required reading: Section 10.2 and 10. 3 of Patt & Patel

Flowchart - ADD subroutine



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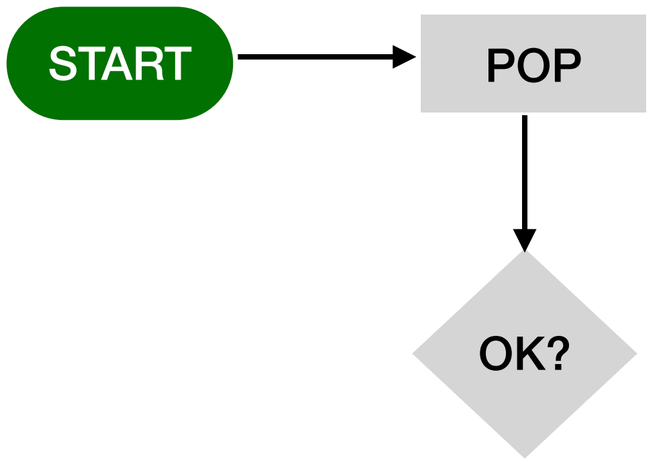
Flowchart - ADD subroutine



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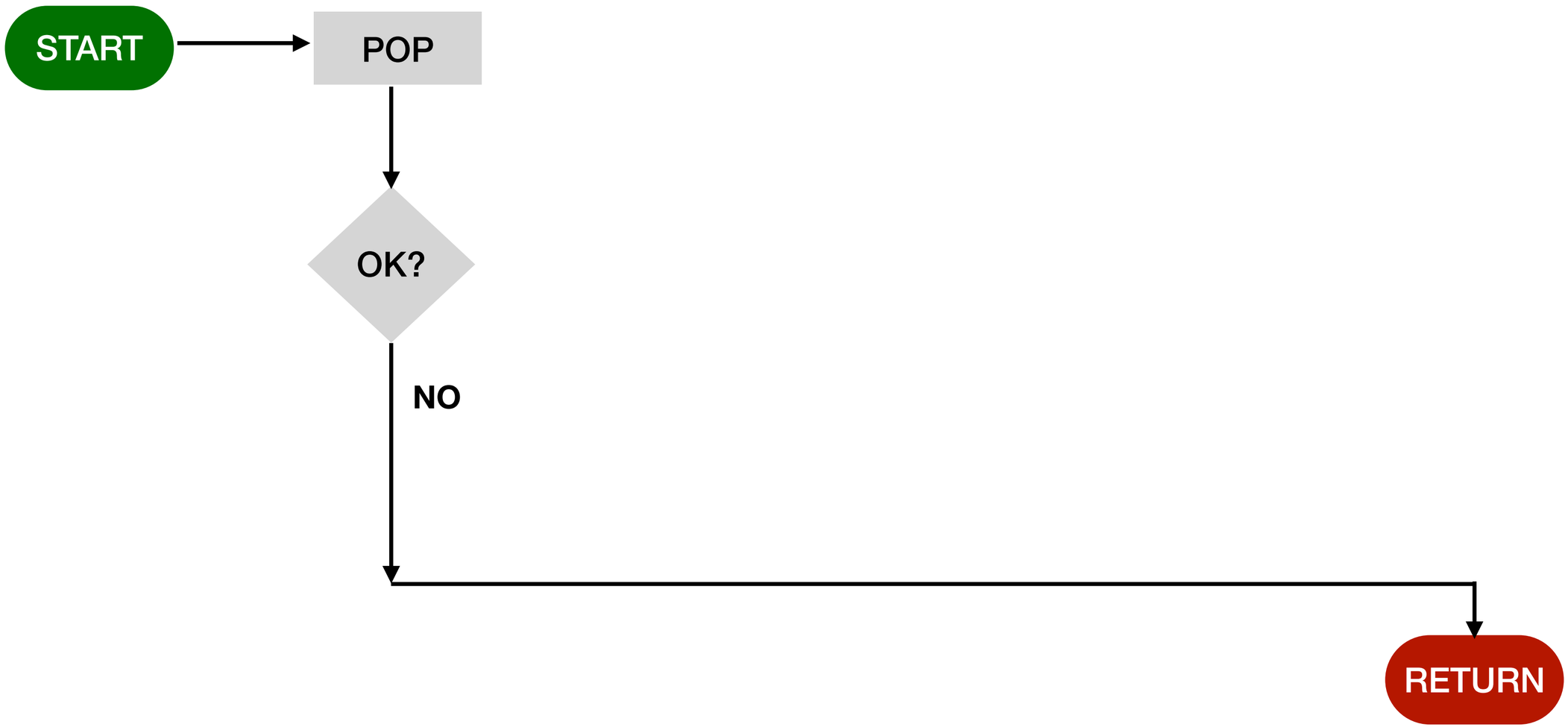

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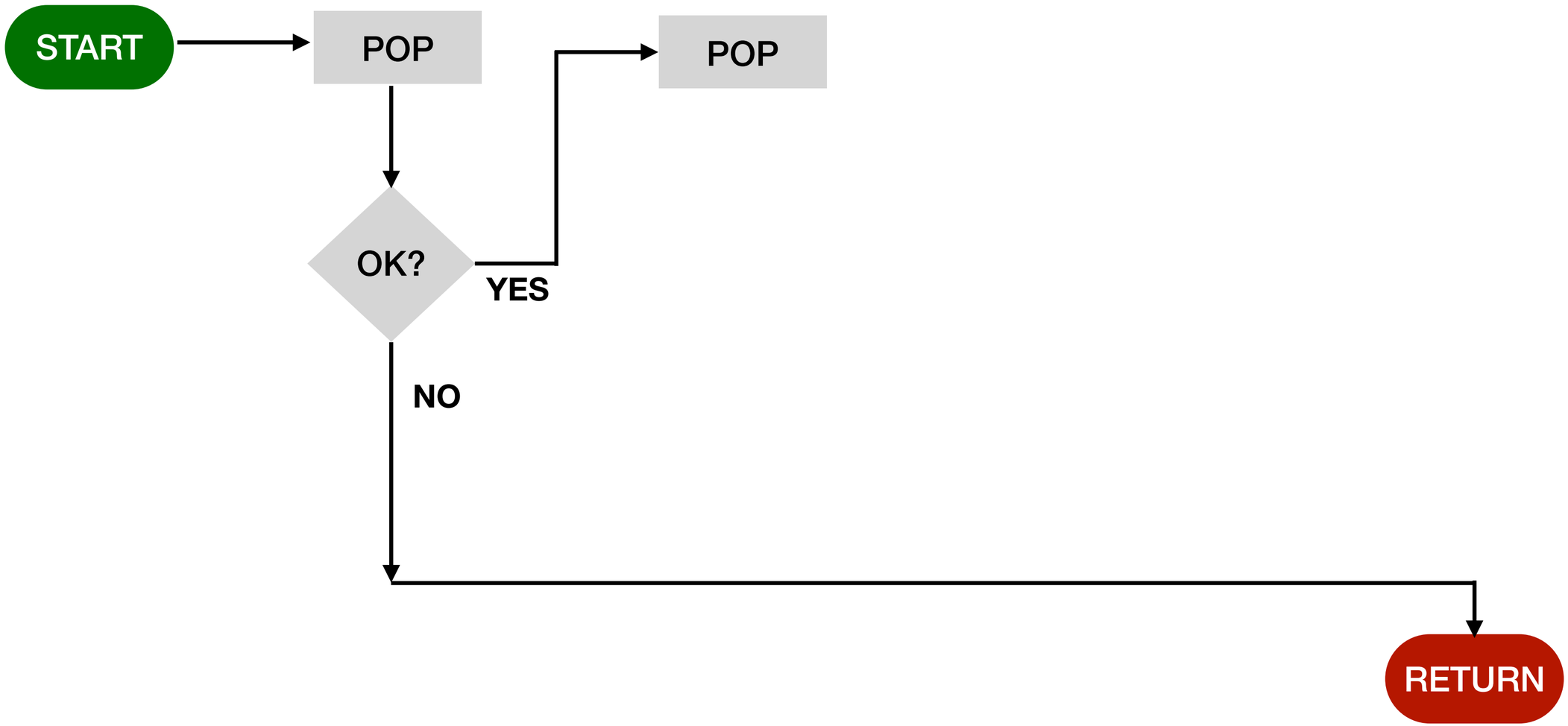

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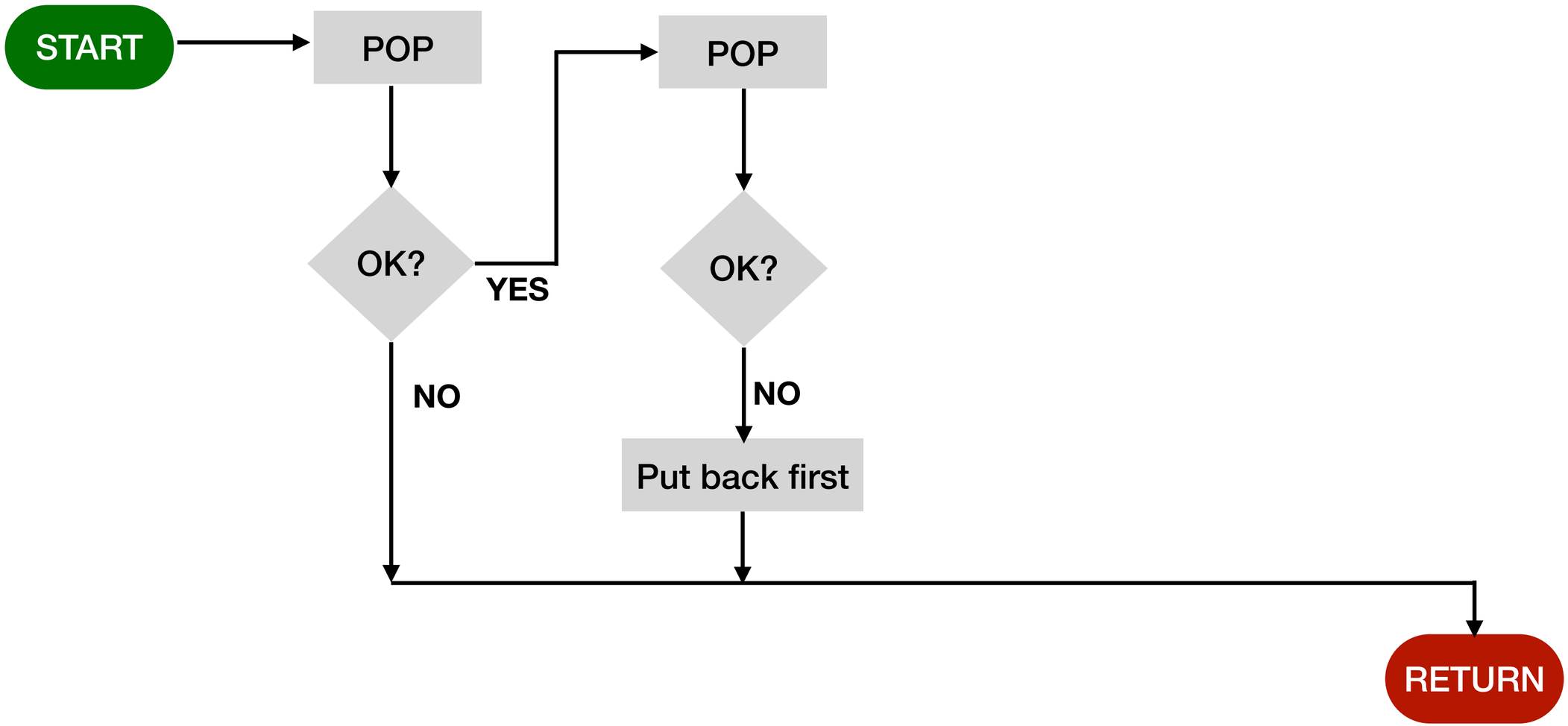

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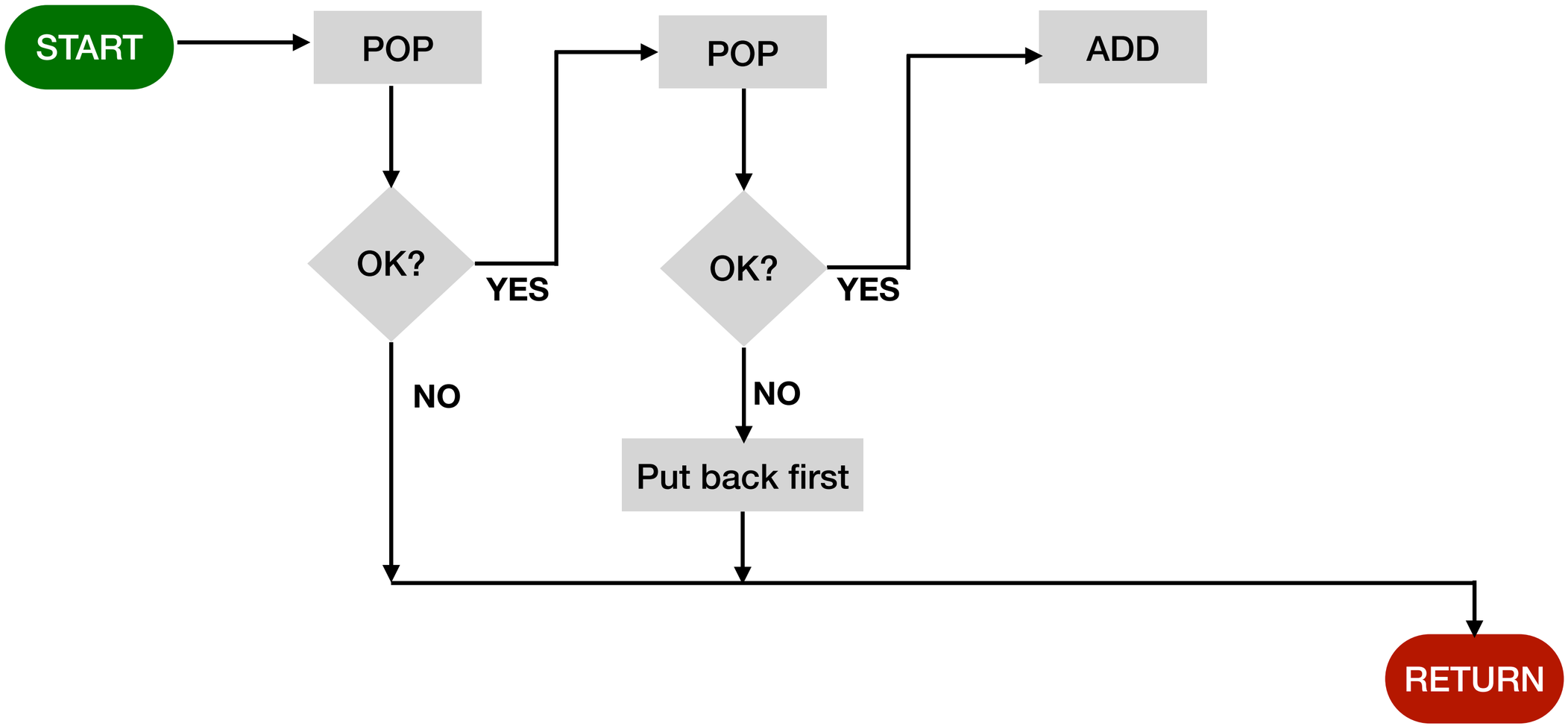
Flowchart - ADD subroutine



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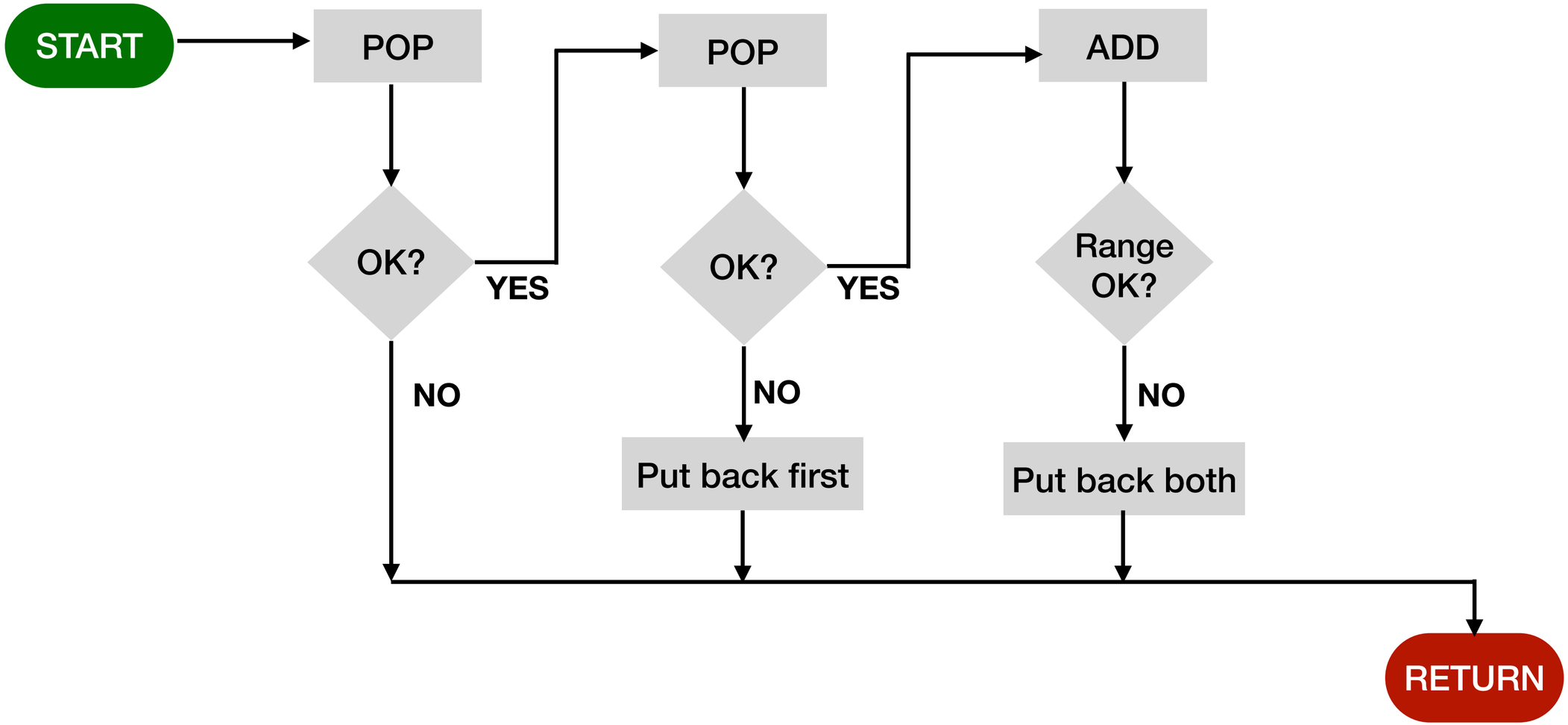
Flowchart - ADD subroutine



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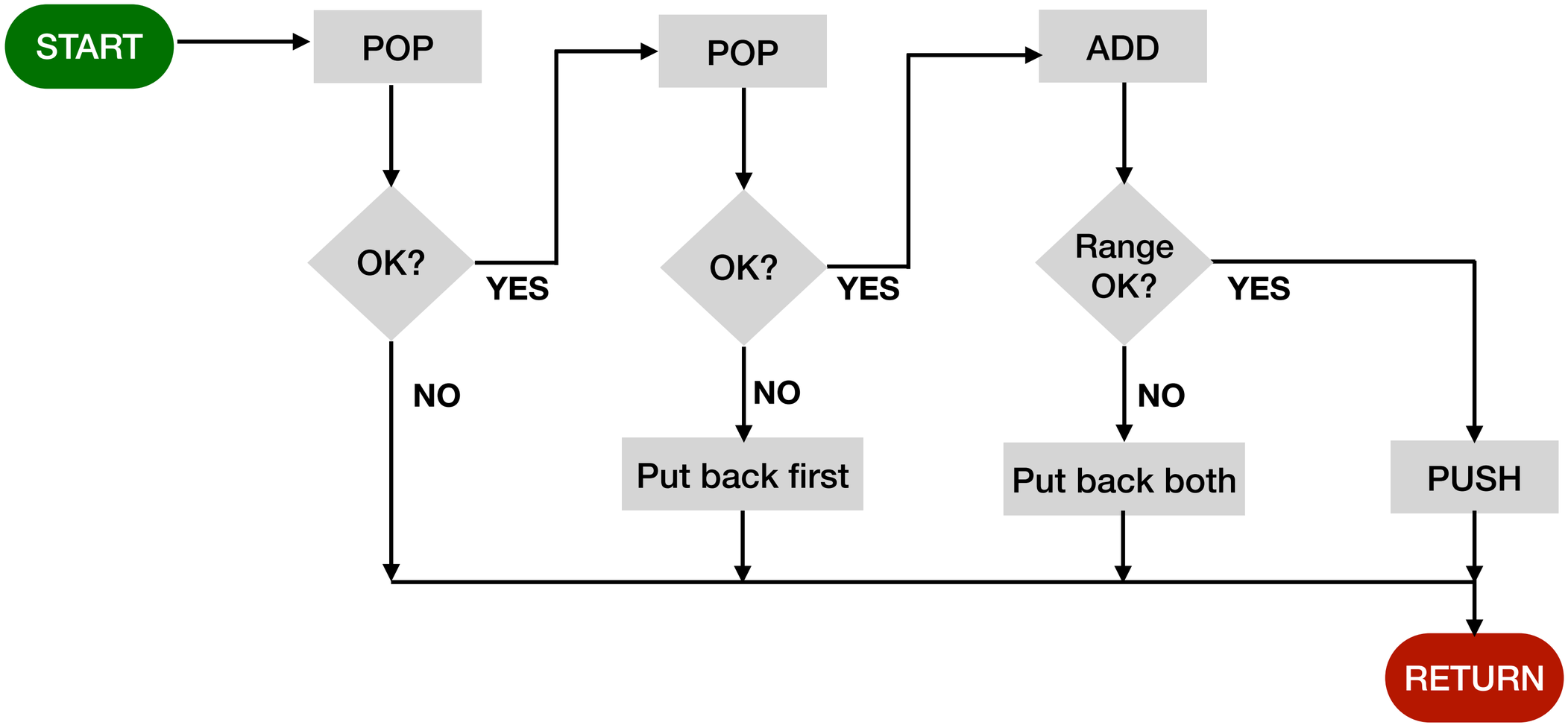
Flowchart - ADD subroutine



Required reading: Section 10.2 and 10. 3 of Patt & Patel

Implement ADD subroutine

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```
;PUSH
;Input: R0 (value to store on stack)
;Output: R5 (0-success, 1-fail)



;POP
;Output: R0 (value to load from stack)
;Output: R5 (0-success, 1-fail)



;CHECK_RANGE
;Input: R0 (value to be checked)
;Output: R5 (0-success, 1-fail)
```

Implement ADD subroutine

```
;PUSH
;Input: R0 (value to store on stack)
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;POP
;Output: R0 (value to load from stack)
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;CHECK_RANGE
;Input: R0 (value to be checked)
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```

- Save `R7` before calling other subroutines.

Implement ADD subroutine

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;PUSH
;Input: R0 (value to store on stack)
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;CHECK_RANGE
;Input: R0 (value to be checked)
;Output: R5 (0-success, 1-fail)
```

- Save `R7` before calling other subroutines.
- Save registers that will be altered in this subroutine

Implement ADD subroutine

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;POP
;Output: R0 (value to load from stack)
;Output: R5 (0-success, 1-fail)



;CHECK_RANGE
;Input: R0 (value to be checked)
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- Save `R7` before calling other subroutines.
- Save registers that will be altered in this subroutine
- `R6` is stack pointer (points to the next available spot on the stack)

Implement ADD subroutine

```
;PUSH
;Input: R0 (value to store on stack)
;Output: R5 (0-success, 1-fail)



;POP
;Output: R0 (value to load from stack)
;Output: R5 (0-success, 1-fail)



;CHECK_RANGE
;Input: R0 (value to be checked)
;Output: R5 (0-success, 1-fail)
```

- Save `R7` before calling other subroutines.
- Save registers that will be altered in this subroutine
- `R6` is stack pointer (points to the next available spot on the stack)
- Assume `PUSH`, `POP` and `CHECK_RANGE` subroutines are provided to you

```
; ADD subroutine – pop two numbers from stack,
; perform '+' operation and then push result back to
the stack

ADD_OP
; save registers




; initialize R5


; first pop


; check return value of first pop, go to EXIT if it
failed (R5 = 1)


;save value in R1 before second pop


; second pop


; check result of second pop, go to RESTORE_1 if it
failed


  ; add two numbers: R0 <- R0 + R1
```

```
; check range of sum, go to RESTORE_2 if it failed




; everything is good, push sum (already in R0) to
stack
;


RESTORE_1     ; put back first number

    ; Load STACK_TOP
    ; Put back item
    ; Update STACK_TOP
    ; Go to exit


RESTORE_2     ; put back both numbers

    ; Load STACK_TOP
    ; Put back item(s)
    ; Update STACK_TOP
;
EXIT
; update stack top pointer
; restore registers


  RET
```

Check Gitlab

https://gitlab.engr.illinois.edu/itabrah2/ece220-fa24.git

Postfix evaluation

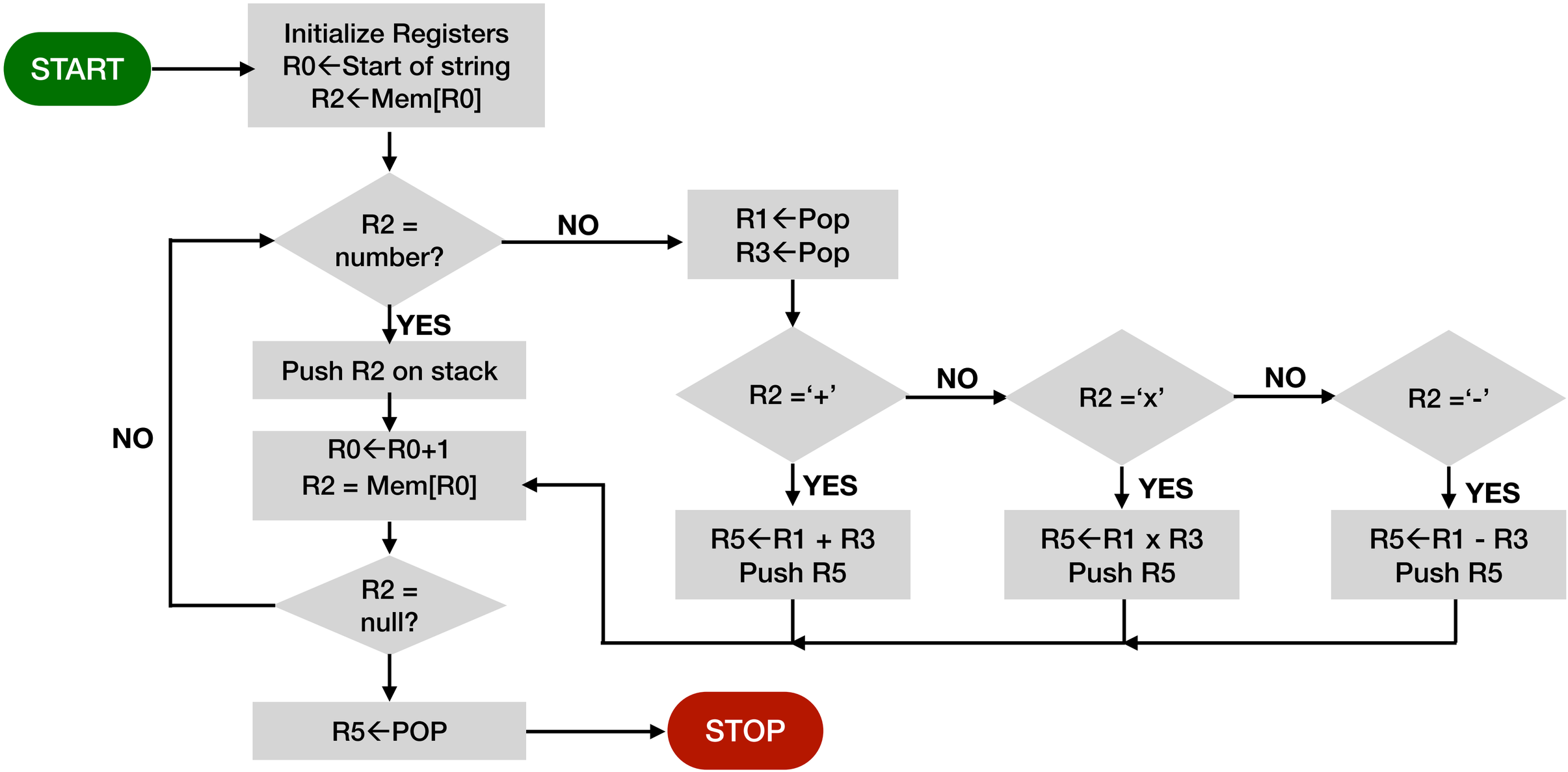
Postfix evaluation

Problem: Given a postfix expression with numerals and ‘+’, ‘-’, ‘*’ in the form of a string, evaluate it and store the answer in `R5`. Each numeral is a single character.

Algorithm:

- Read the string (postfix expression) left to right
- Push the numbers in the expression on the stack
- For an operator, pop the top two elements, compute the answer and push it on the stack

Example decomposition



Next time

- Introduction to C
- Compiling a C program on EWS
- Running the GNU debugger etc.
- Bring your laptop!