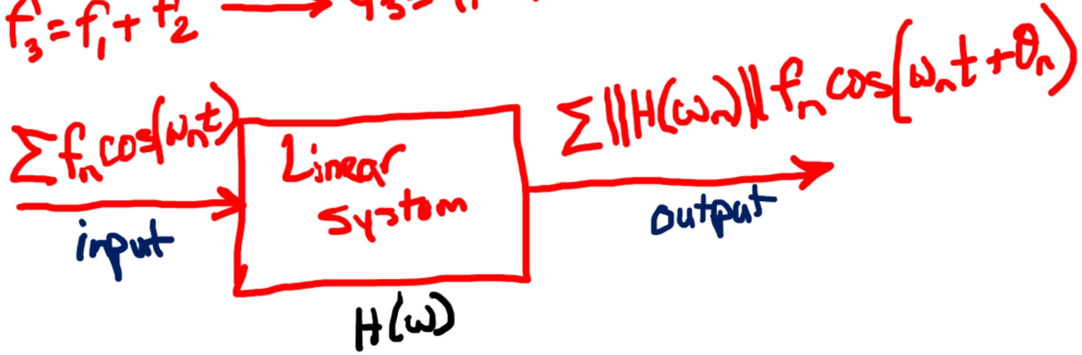
Frequency Response to multiple inputs

Linear System

$$f_1 \longrightarrow y_1$$

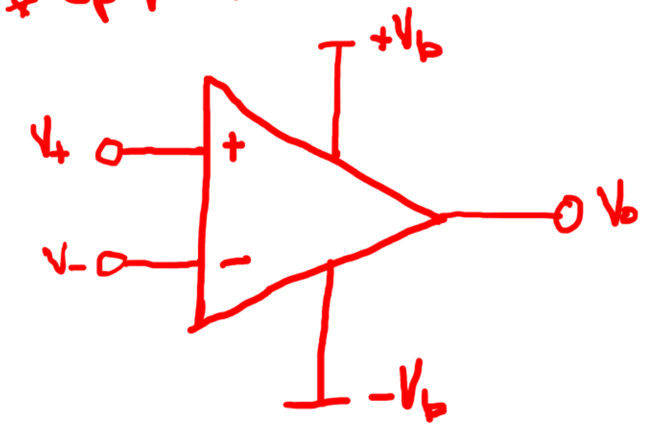
$$f_2 \longrightarrow y_2$$

$$f_3 = f_1 + f_2 \longrightarrow y_3 = y_1 + y_2$$

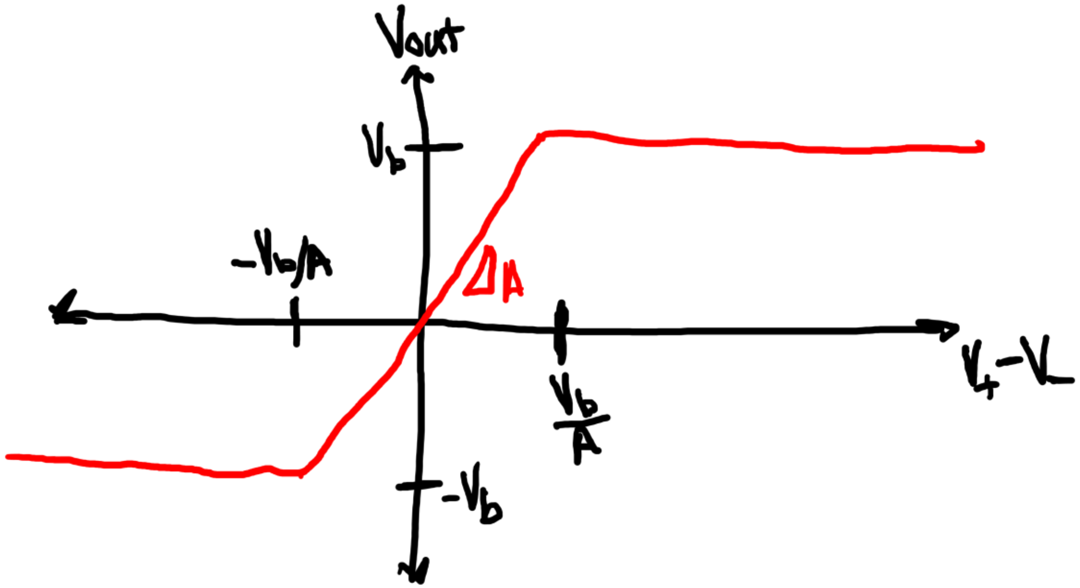


Operational Amplifiers

* Op-Amps

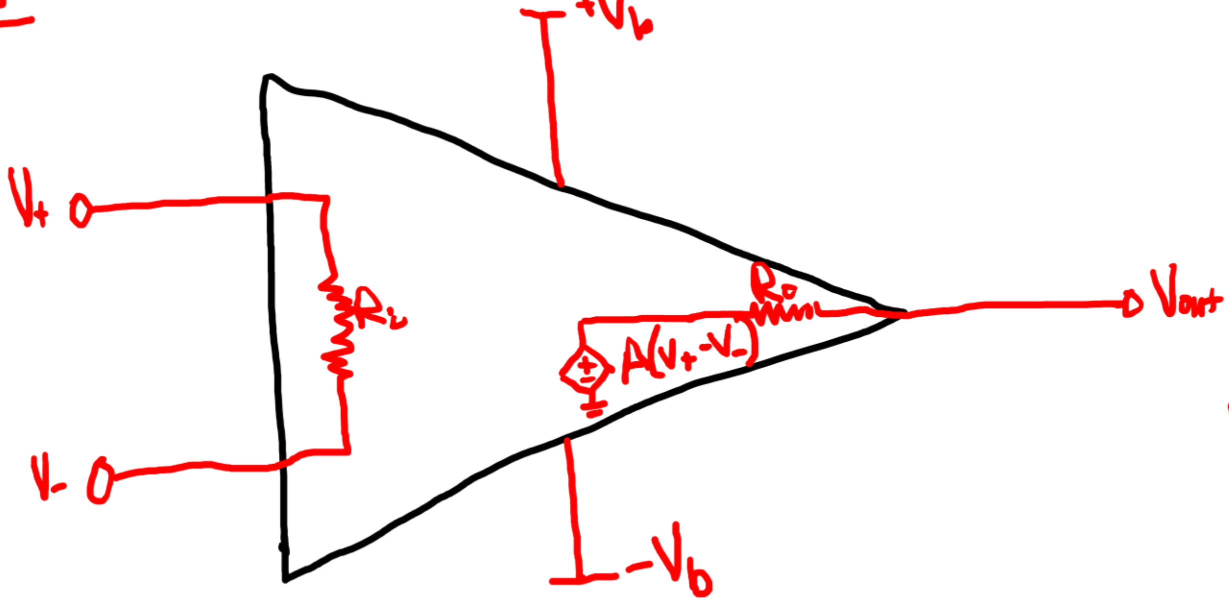


A = Op-Amp Gain

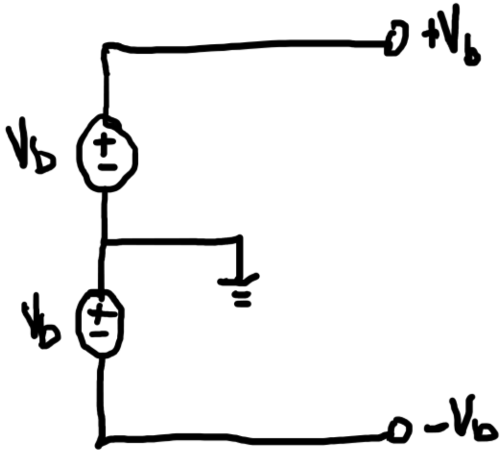


Need $|V_+ - V_-| < \frac{V_b}{A}$ to be in linear region of op-Amp.

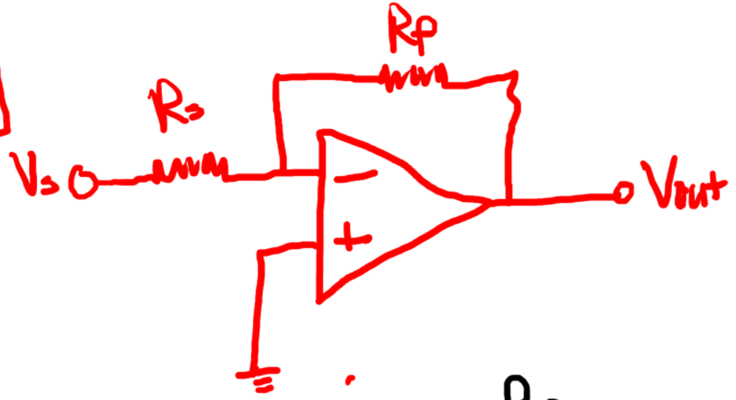
Op-Amp Model



$$V_{out} = A(V_+ - V_-) - i_o R_o$$



Ex

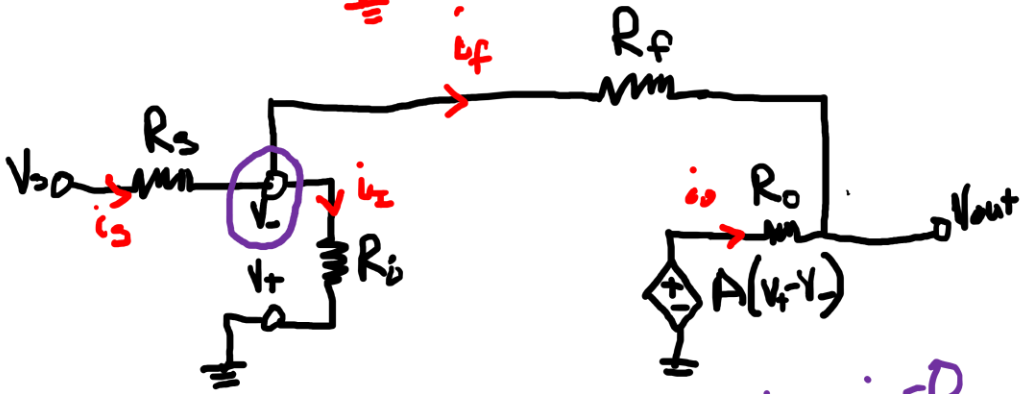


Gain: A

Assume: $R_i \to \infty$ (very large), $i_I \to 0$

$R_o = 0$ (very small)

$A >> 1$ (1×10^6)



$V_+ = 0$

$i_I = \dfrac{V_- - V_+}{R_i}$

$i_I = \dfrac{V_-}{R_i}$

KCL: $i_s - i_I - i_f = 0$

$i_s = i_I + i_f$

$$\frac{V_s - V_-}{R_s} = \frac{V_-}{R_i} + \frac{V_- - V_{out}}{R_f}$$

$$\frac{V_s + \frac{1}{A}V_{out}}{R_s} = \frac{-\frac{1}{A}V_{out} - V_{out}}{R_f}$$

$$\Downarrow$$

$$V_{out} = \frac{-V_s}{\frac{R_s}{R_f} + \frac{1}{A}\left(1 + \frac{R_s}{R_f}\right)}$$

$V_{out} = A(V_+ - V_-) - i_o R_o$

$V_{out} = -AV_- - i_o R_o$

$-AV_- = V_{out} + i_o R_o$

$V_- = -\frac{1}{A}V_{out} - \frac{i_o R_o}{A}$

$V_- = -\frac{1}{A}V_{out}$

$$\approx \boxed{V_{out} = \frac{-R_f}{R_s} V_s}$$

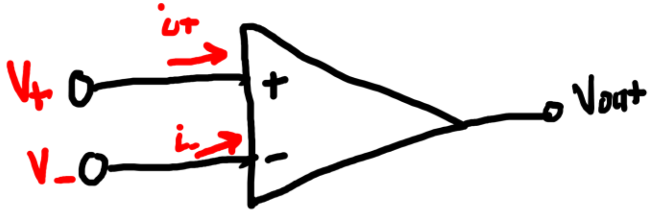
* Inverting Op-Amp

Ideal Linear Op-Amp Rules

* $R_i \to \infty$

Rule: $i_+ = i_- = 0$

Rule: $V_+ = V_-$



$A \gg 1$

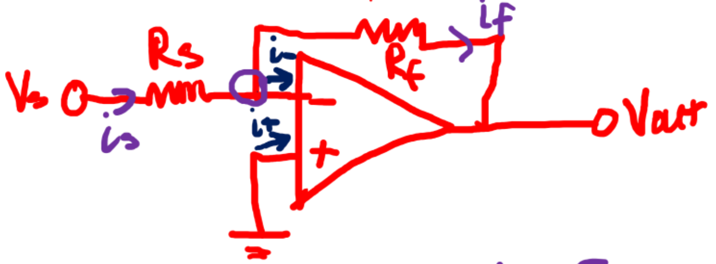
$$V_{out} = A(V_+ - V_-)$$

$$= \frac{V_+ - V_-}{\frac{1}{A}}$$

$A \to \infty, \ \frac{1}{A} \to 0$

* Remain Bounded

$V_+ - V_- \to 0$



Rules

$i_+ = i_- = 0$

$V_+ = V_-$

KCL: $i_s - i_- - i_f = 0$

$i_s = i_f$

$$\frac{V_s - V_-}{R_s} = \frac{V_- - V_{out}}{R_f}$$

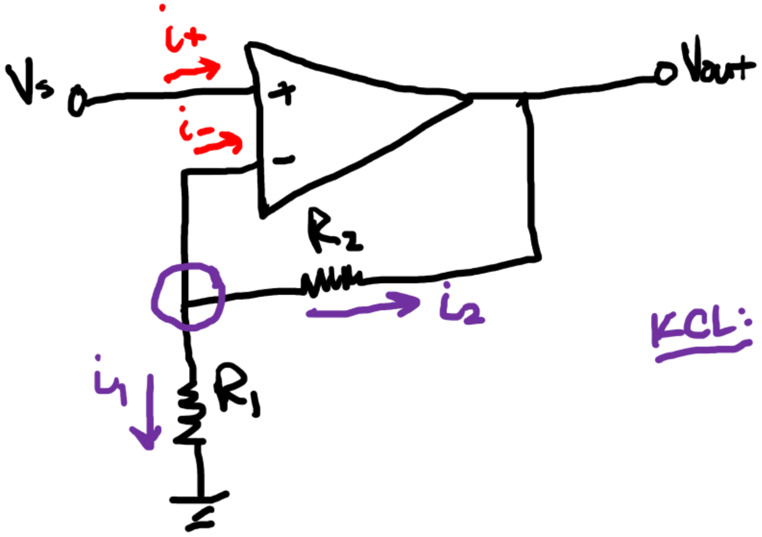
$V_+ = 0$

$V_- = 0$

$$\frac{V_s - 0}{R_s} = \frac{0 - V_{out}}{R_f}$$

$$\boxed{V_{out} = -\frac{R_f}{R_s} V_s}$$

* Inverting Op-Amp



Rules

$i_+ = i_- = 0$

$V_+ = V_-$

$V_+ = V_s \qquad V_- = V_s$

KCL: $-i_- - i_1 - i_2 = 0$

$$0 = i_1 + i_2$$

$$0 = \frac{V_- - 0}{R_1} + \frac{V_- - V_{out}}{R_2}$$

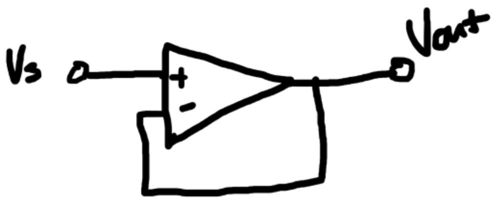
$$0 = \frac{V_s}{R_1} + \frac{V_s}{R_2} - \frac{V_{out}}{R_2}$$

$$\frac{V_{out}}{R_2} = V_s\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$$\boxed{V_{out} = V_s\left(1 + \frac{R_2}{R_1}\right)}$$

* Non-Inverting Op-Amp

Buffer



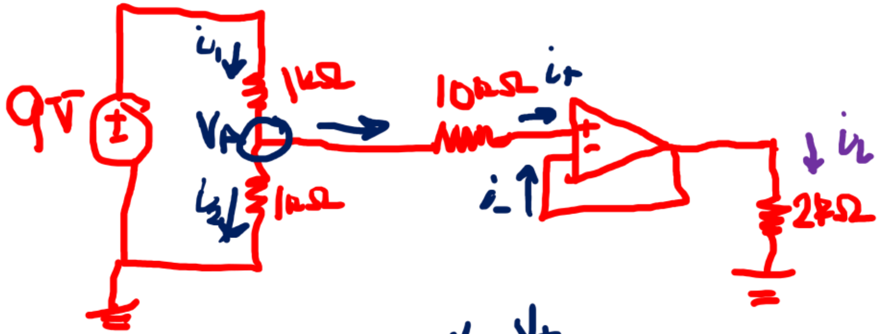
$V_+ = V_-$

$V_+ = V_s$

$V_- = V_s$

$V_{out} = V_- \Rightarrow \boxed{V_{out} = V_s}$

* Helpful for isolating part of your circuit.



KCL: $i_1 - i_+ - i_2 = 0$

$i_1 = i_2$

VDR: $V_A = \left(\frac{1k}{2k}\right)9 \Rightarrow \boxed{V_A = 4.5\ V}$

$V_+ = V_-$

$i_+ = i_- = 0$

$i_+ = 0 = \frac{V_A - V_+}{10k}$

$V_+ = V_A$

$V_+ = 4.5\ V$

$V_- = 4.5\ V$

$V_{out} = 4.5\ V$

$i_L = \frac{V_{out} - 0}{2k}$

$= \frac{4.5 - 0}{2k}$

$\boxed{i_L = 2.25\ mA}$