OFDM Introduction:

Orthogonal Frequency Division Multiplexing (OFDM) is a digital multi-carrier modulation scheme that extends the concept of single subcarrier modulation by using multiple subcarriers within the same single channel. Rather than transmit a high-rate stream of data with a single subcarrier, OFDM makes use of a large number of closely spaced orthogonal subcarriers that are transmitted in parallel. Each subcarrier is modulated with a conventional digital modulation scheme (such as QPSK, 16QAM, etc.) at low symbol rate. However, the combination of many subcarriers enables data rates similar to conventional single-carrier modulation schemes within equivalent bandwidths.

OFDM is based on the well-known technique of Frequency Division Multiplexing (FDM). In FDM different streams of information are mapped onto separate parallel frequency channels. Each FDM channel is separated from the others by a frequency guard band to reduce interference between adjacent channels.

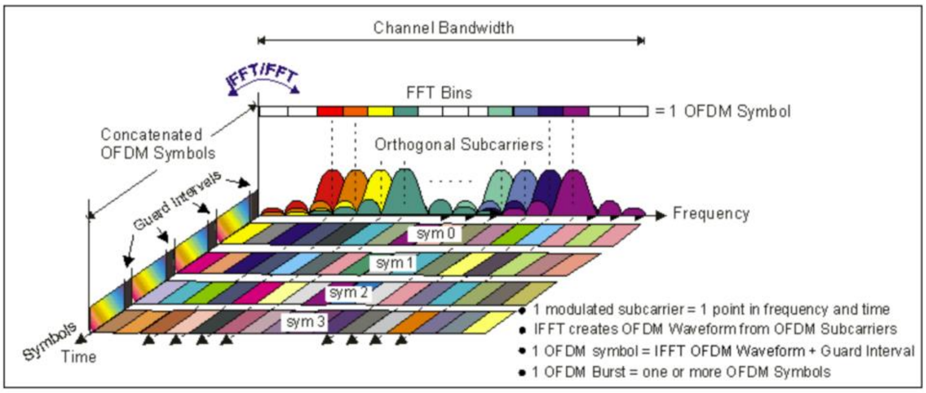
The OFDM scheme differs from traditional FDM in the following interrelated ways:

1. Multiple carriers (called subcarriers) carry the information stream,

2. The subcarriers are orthogonal to each other, and

3. A guard interval is added to each symbol to minimize the channel delay spread and inter-symbol interference.

The following figure illustrates the main concepts of an OFDM signal and the inter-relationship between the frequency and time domains. In the frequency domain, multiple adjacent tones or subcarriers are each independently modulated with complex data. An Inverse FFT transform is performed on the frequency-domain subcarriers to produce the OFDM symbol in the time-domain. Then in the time domain, guard intervals are inserted between each of the symbols to prevent inter-symbol interference at the receiver caused by multi-path delay spread in the radio channel. Multiple symbols can be concatenated to create the final OFDM burst signal. At the receiver an FFT is performed on the OFDM symbols to recover the original data bits.



Frequency-Time Representative of an OFDM signal

The Importance of Orthogonally Spaced Subcarriers in OFDM:

The OFDM signal can be described as a set of closely spaced FDM subcarriers. In the frequency domain, each transmitted subcarrier results in a sinc function spectrum with side lobes that produce overlapping spectra between subcarriers, see "OFDM Signal Frequency Spectra" figure below. This results in subcarrier interference except at orthogonally spaced frequencies. At orthogonal frequencies, the individual peaks of subcarriers all line up with the nulls of the other subcarriers. This overlap of spectral energy does not interfere with the system's ability to recover the original signal. The receiver multiplies (i.e., correlates) the incoming signal by the known set of sinusoids to recover the original set of bits sent.

OFDM signal generation:

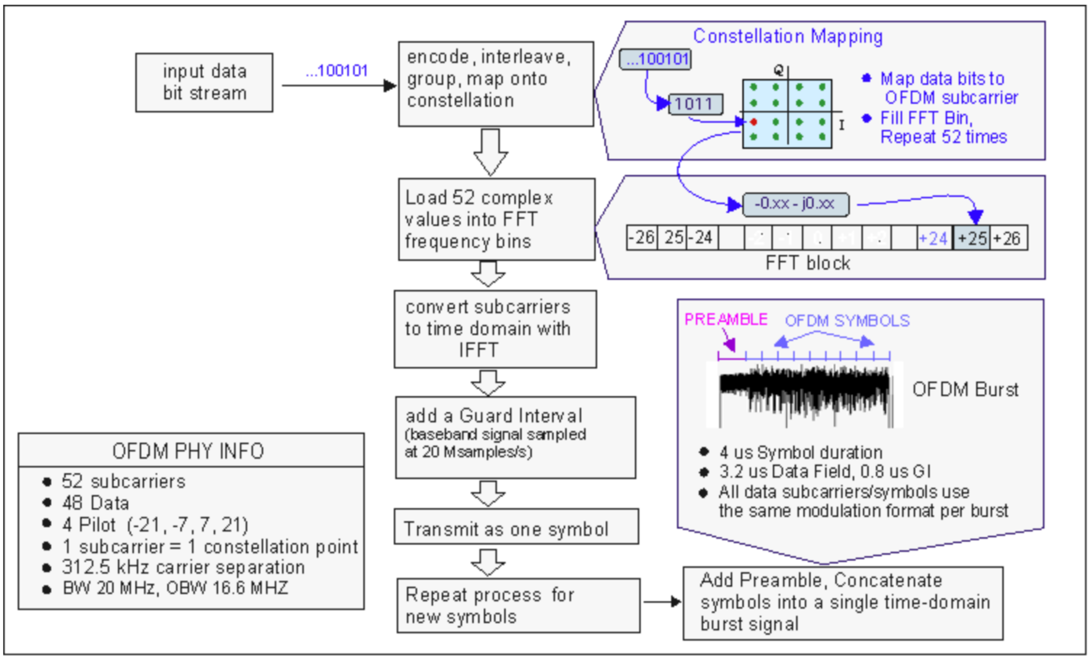
An 802.11a OFDM carrier signal (burst type) is the sum of one or more OFDM symbols each comprised of 52 orthogonal subcarriers, with baseband data on each subcarrier being independently modulated using quadrature amplitude modulation (available formats: BPSK, QPSK, 16-QAM, or 64-QAM). This composite baseband signal is used to modulate a main RF carrier.

To begin the OFDM signal creation process, the input data bit stream is encoded with convolutional coding and Interleaving. Each data stream is divided into groups of "n" bits (1 bit -BPSK, 2 bits -QPSK, 4 bits -16QAM, or 6 bits -64QAM) and converted into complex numbers (I+jQ) representing the mapped constellation point. Note that the bit-rate will be different depending on the modulation format, a 64-QAM constellation (6 bits at a time) can have a bit rate of 54 Mbps while a QPSK constellation (2 bits at time) may only be 12 Mbps.

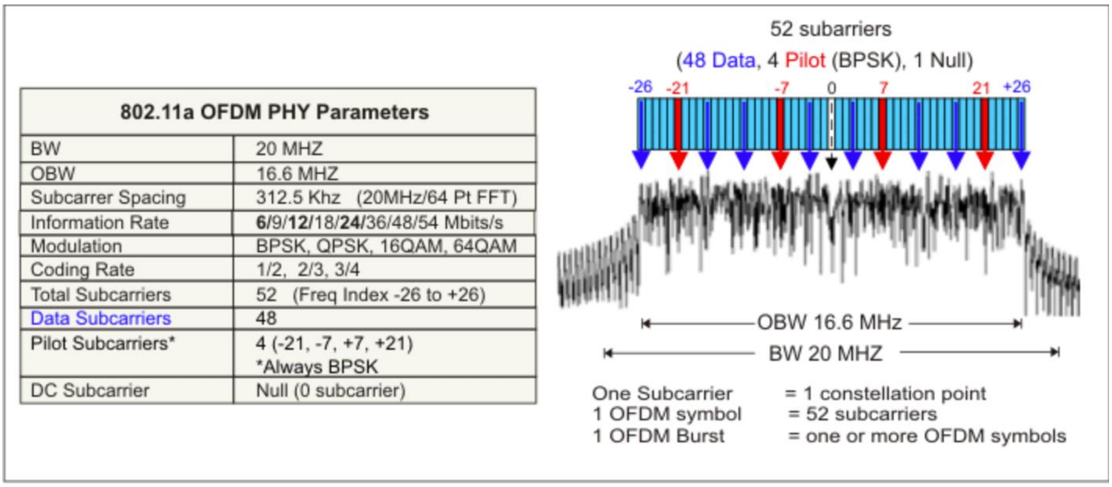
Then 52 bins of the IFFT block are loaded. 48 bins contain the constellation points which are mapped into frequency offset indexes ranging from -26 to +26, skipping the 4 Pilot and zero bins. There are 4 Pilot subcarriers inserted into frequency offset index locations -21, -7, +7, and +21. The zero bin is the Null or DC subcarrier and is not used; it contains a 0 value (0+j0).

When the IFFT block is completely loaded, the Inverse FFT is computed, giving a set of complex time-domain samples representing the combined OFDM subcarrier waveform. The samples are clocked out at 20 Msps to create a 3.2 us (20Msps/64) duration OFDM waveform. To complete the OFDM symbol, a 0.8 us duration Guard Interval (GI) is then added to the beginning of the OFDM waveform. This produces a "single" OFDM symbol with a time duration of 4 us in length, (3.2 us + 0.8 us). The process is repeated to create additional OFDM symbols for the remaining input data bits.

To complete the OFDM frame structure, the single OFDM symbols are concatenated together and then appended to a 16 us **Preamble (used for synchronization)** and a 4 us **SIGNAL symbol (provides Rate and Length information).** This completes the OFDM frame and is ready to be transmitted as an OFDM Burst.



802.11a OFDM Signal Generation Process



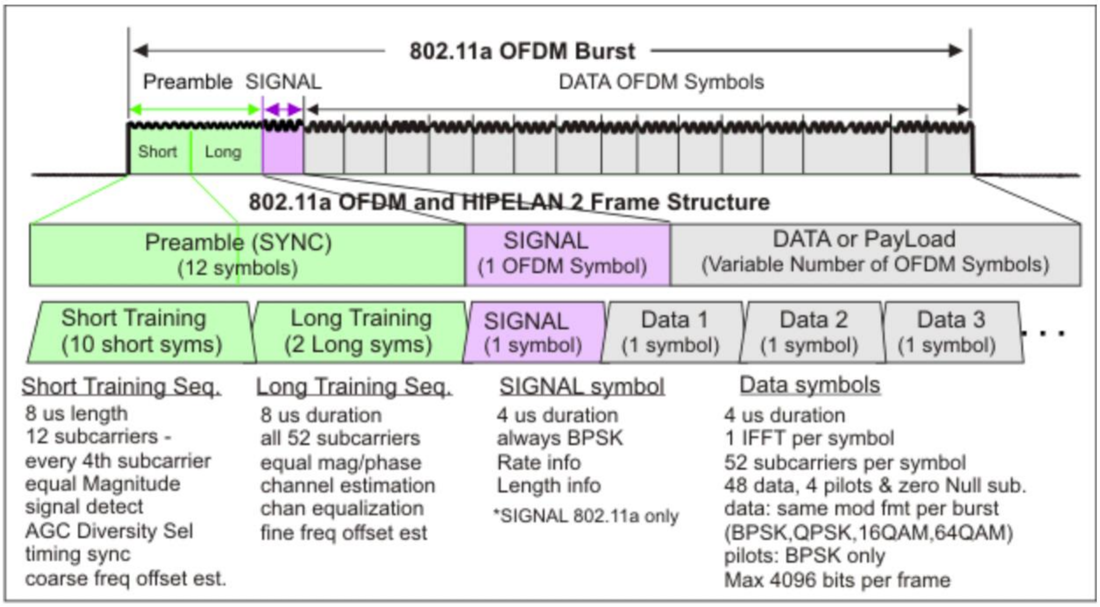
802.11a OFDM PHY Parameters	
BW	20 MHZ
OBW	16.6 MHZ
Subcarrer Spacing	312.5 Khz (20MHz/64 Pt FFT)
Information Rate	**6**/9/**12**/18/**24**/36/48/54 Mbits/s
Modulation	BPSK, QPSK, 16QAM, 64QAM
Coding Rate	1/2, 2/3, 3/4
Total Subcarriers	52 (Freq Index -26 to +26)
Data Subcarriers	48
Pilot Subcarriers*	4 (-21, -7, +7, +21) *Always BPSK
DC Subcarrier	Null (0 subcarrier)

One Subcarrier = 1 constellation point
1 OFDM symbol = 52 subcarriers
1 OFDM Burst = one or more OFDM symbols

802.11a OFDM Physical Parameters

802.11a OFDM Frame Structure

The basic frame structure of an 802.11a burst contains a preamble field followed by a SIGNAL field and multiple data fields. At the start of the burst, a preamble is transmitted at a well-known magnitude and phase. The preamble is used for synchronization and channel equalization. The SIGNAL field (not used in HIPERLAN 2 signals) is transmitted using BPSK, and contains the length, modulation type, and data rate information. Then multiple OFDM symbols containing the input data bits are appended to complete the burst.



802.11a and HIPERLAN/2 Frame Structure

Practical Transmission Model

A practical transmission system can be modeled as shown below:

The sampling frequency of the Digital-to-Analog Converter (DAC) at the transmitter fs and the sampling frequency of the Analog-to-Digital Converter (ADC) at the receiver f's is different. The Sampling Frequency Offset (SFO) ζ is defined as follows:

$$\begin{aligned} \zeta &= \frac{T_s' - T_s}{T_s} \\ T_s &= \frac{1}{f_s} \\ T_s' &= \frac{1}{f_s'} \end{aligned}$$

At the transmitter, there is an oscillator which modulates the baseband signal to passband. Similarly, at the receiver, there is an oscillator which demodulates the passband signal to baseband.

(a) δf is the Carrier Frequency Offset (CFO) between the transmitter and the receiver oscillator.

(b) $\Theta(t)$ represents the phase noise in the oscillators. The frequency generated by practical oscillators varies over time. This variation can be modeled as noise in the phase of the received signal.

$n(t)$ is the Additive White Gaussian Noise seen by the receiver. The thermal noise faced by the RF front end as well as the quantization noise of the data converters contributes to $n(t)$.

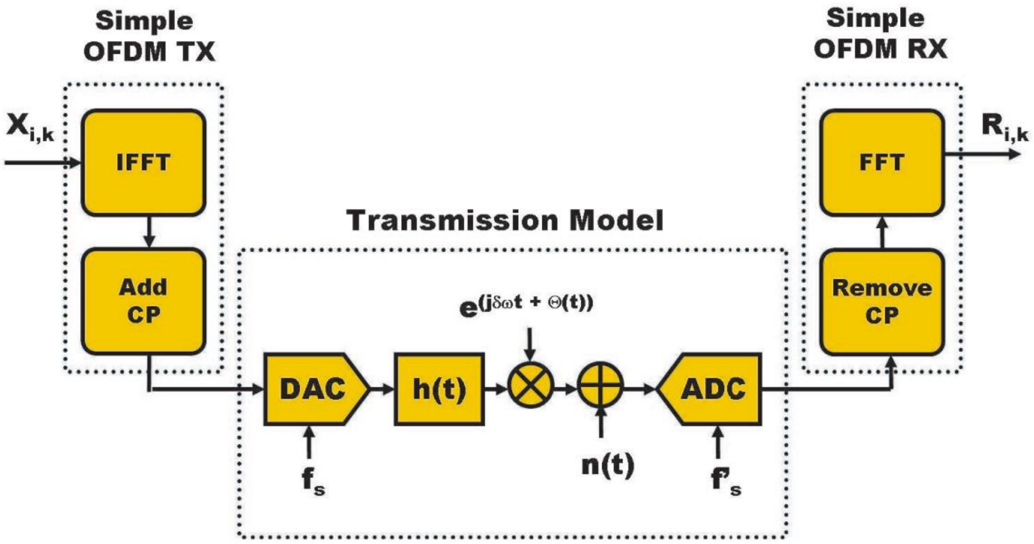


Fig.1 Complete baseband transmission model

Receiver Design:

the received signal $r[n]$ is passed through the following sequence of blocks:

- Packet Detection: This module is responsible for detecting the start of the packet.
- CFO Estimator/Corrector: Once the packet is detected, the CFO is estimated and corrected to minimized the effects of ICI in the later stages.
- CP Removal: The Cyclic Prefix (CP) that is inserted to guard against Inter symbol Interference (ISI) is removed prior to the FFT.
- FFT: Perform a transform on each OFDM symbol in time to obtain the sub carrier symbols.
- Channel Estimator/Corrector: Estimate and correct the channel-induced attenuation and phase rotation on the sub-carrier symbols.
- Phase Tracking: Estimate and correct for time variant phase rotation on the sub-carrier symbols caused by CFO, SFO and phase noise.
- SFO Estimator/Corrector: Estimate the SFO by processing side-products of phase tracking. This module will correct for the OFDM Symbol Window Drift by indicating to the

CP Removal Module to advance or delay by a sample when appropriate. This module also indicates to the channel corrector module to compensate the phase changes in the Frequency domain from the advance or delay of a sample.

Packet Detection:

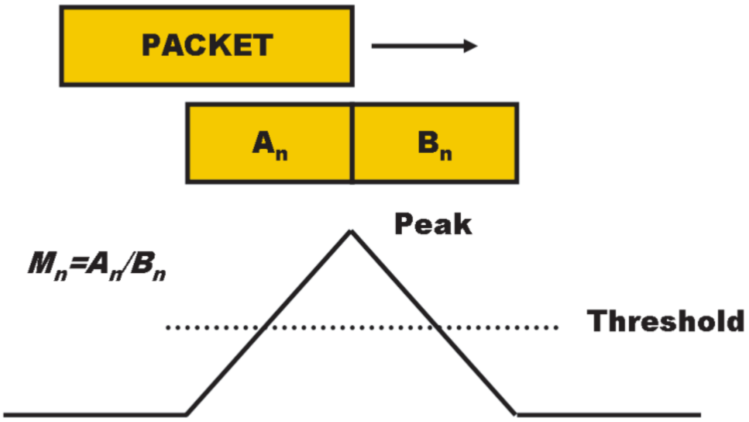
The variables An and Bn are obtained from the received signal r[n] by:

$$\begin{aligned} A_n &= \sum_{i=0}^{N_{FFT}-1} |r[n-i]|^2 \\ B_n &= \sum_{i=1}^{N_{FFT}} |r[n+i]|^2 \end{aligned}$$

The peak E[Mn] occurs when index n is exactly the start of the packet (i.e. An would consist of signal and noise and Bn would consist purely of noise):

$$E[M_n]_{peak} = SNR + 1$$

Thus, locating the peak of Mn would provide the expected start point of the packet.



Double Sliding Window Packet Detection Algorithm

CFO Estimation and Correction:

If there are two identical symbols each of NFFT samples, the Maximum Likelihood (ML) CFO estimate δf can be represented as a function of the cross-correlation z between these two symbols:

$$\begin{aligned} -2\pi\hat{\delta}fT_s &= \frac{\angle z}{N_{FFT}} \\ z &= \sum_{n=0}^{N_{FFT}-1} r^*[n]r[n-N_{FFT}] \\ &= \sum_{n=0}^{N_{FFT}-1} |r[n]|^2 e^{-j2\pi\delta f T_s N_{FFT}} \end{aligned}$$

Channel Estimation and Correction:

The channel estimation stage occurs after the CFO correction as the effect of ICI is minimized. However, since the CFO estimation is not perfect, there will be a residual CFO on the signal. Received signal in Fig.1 Complete baseband transmission model, can be re-expressed as:

$$R_{i,k} = H_{eff;k} \cdot X_{i,k}^{\varphi_{i,k}(t)} + n_{eff;i,k}$$

Where

- $H_{eff;k}$ represents a time-invariant multiplicative factor.
- $n_{eff;i,k}$ represents all the possible noise sources inclusive of ICI.
- $X_{i,k}^{\varphi}$ represents the sent symbol $X_{i,k}$ with timing varying rotation $\varphi_{i,k}$ caused by residual CFO, SFO and the CPE.

Channel Estimation seeks to estimate Hef f;k. If a pilot OFDM symbol at i = −1 is sent before the data OFDM symbols (corresponding to i = 0, 1, 2…), R−1,k can be expressed as the following:

$$R_{-1,k} = H_{eff;k} \cdot X_{-1,k} + n_{eff;-1,k}$$

(Equ.1)

Heff;k can be estimated from the Least Squares (LS) method:

$$\hat{H}_{eff;k} = \frac{R_{-1,k}}{X_{-1,k}}$$

The estimator can be improved through averaging by sending the same pilot OFDM symbol twice at i = −1 and −2:

$$\hat{H}_{eff;k} = \frac{1}{2} \cdot \frac{R_{-1,k} + R_{-2,k}}{X_{-1,k}}$$

This assumes that the residual CFO and the SFO is relatively small such that that the time varying rotation can be ignored when comparing R−1,k and R−2,k.

Phase Tracking

Looking at Equation 1:

$$R_{-1,k} = H_{eff;k} \cdot X_{-1,k} + n_{eff;-1,k}$$

there remains the issue of the time-varying phase rotation ϕi,k that needs to be compensated after removing the effects of Heff;k. Let δϕi,k be phase accumulation that occurred going from OFDM symbol i − 1 to OFDM symbol i i.e:

$$\delta\varphi_{i,k} = \varphi_{i,k} - \varphi_{i-1,k}$$

From Section 3.3.1 and 3.3.2, δϕi,k is a linear function of k where the intercept is a function of CPE and CFO and the slope is a function of the SFO.

Reference:

1. Concepts of Orthogonal Frequency Division Multiplexing (OFDM) and 802.11 WLAN (keysight.com)
2. Tan, Jit Ken. *An adaptive orthogonal frequency division multiplexing baseband modem for wideband wireless channels*. Diss. Master's thesis, MIT, 2006.