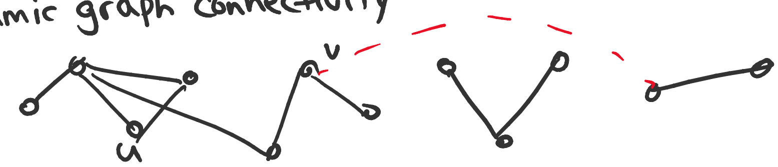
November 4, 2022 10:51 AM

Appl'n to Dynamic Data Structures

e.g. dynamic graph connectivity

undir



query: given u, v, decide $\exists$ path $u \to v$

edge inserts/deletes

$O(\text{polylog}\, n)$ update/query time known

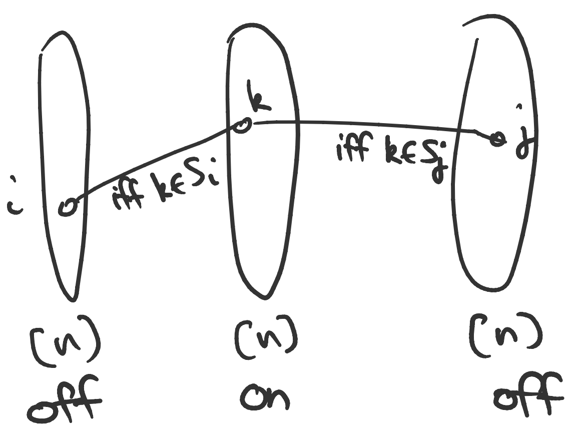
vertex (re)inserts/deletes, i.e. turn vertex on/off

(called dyn subgraph connectivity)

C.-Patrascu-Roditty '10 $\tilde{O}(m^{2/3})$

why no $O(\text{polylog}\, n)$ possible?

(Int)3SUM $\rightarrow$ Triangle Listing $\downarrow$ Set Intersection Queries $\downarrow$ Set Disjointness Queries $\rightarrow$ Dyn Graph Conn with Vertex Updates



turn on i & j

query (i, j)

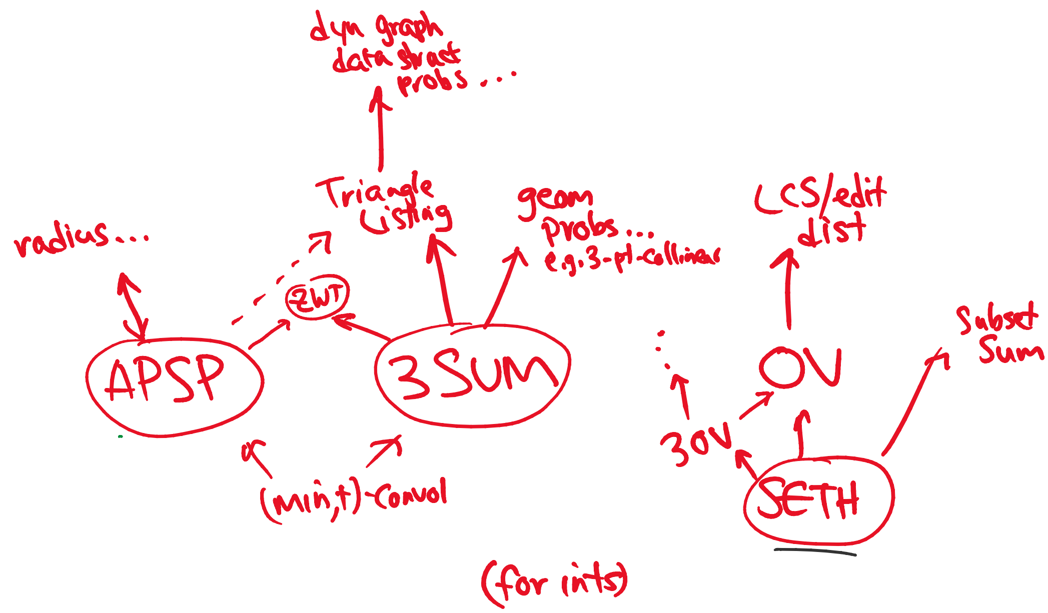
turn off i, j

update/query

$\Rightarrow$ lower bd of $\Omega(m^{\frac{1}{3}-\delta})$ per op.

⋮

dyn graph data struct probs ...



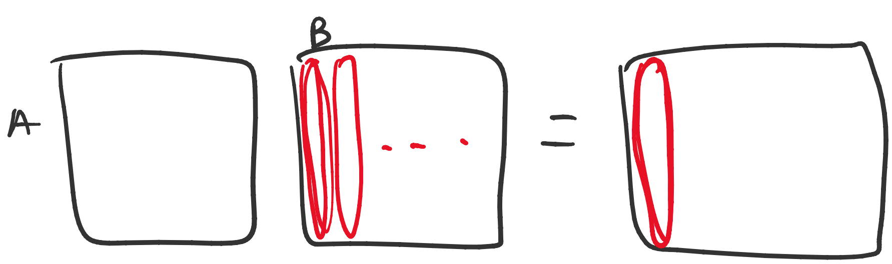
Cond. Lower Bds via Other Conjectures

"combinatorial" ← no precise def'n!

"Comb." BMM Conj — no $O(n^{3-\delta})$ alg'm for Boolean matrix mult.

Online-Matrix-vector-prod.

OMv Conj — no $O(n^{3-\delta})$ alg'm for Bool. matrix mult of A & B when columns of B arrive online



useful for data struct. problems with online queries/updates

kSUM Conj — no $O(n^{\lceil k/2 \rceil - \delta})$ alg'm for k-SUM

"Comb." k-Clique Conj — no comb. $O(n^{k-\delta})$ alg'm

"Comb." **k-Clique Conj** no Comb. $O(n^{k-\delta})$ alg'm
for deciding $\exists$ k-clique in graph

(known: $\sim n^{k\omega/3}$ time non comb.)

e.g. Jin, Xu '22: dyn graph conn. with vertex updates
$\Omega(m^{2/3-\delta})$ tight lower bd
under Comb. 4-Clique Conj.

Weighted k-Clique Conj no $O(n^{k-\delta})$ alg'm
for finding k-clique with min-wt
in edge-weighted graphs.

k-Hyperclique Conj no $O(n^{k-\delta})$ alg'm
for deciding $\exists$ k-hyperclique
in d-unif. hypergraph
for any fixed $d \geq 3$.

⋮

or ... invent new conj!

e.g. **Conj** dir unwted APSP does not
have $O(n^{2.5-\delta})$ alg'm

(recall: Zwick's alg $O(n^{2.529})$
or $\tilde{O}(n^{2.5})$ if $\omega = 2$)

Let $T_{\text{dir-unwt-APSP}}(n)$ be complexity of dir unwt APSP.

Let $M^*(n, d, n \mid \ell)$ be complexity of
(min,+)-MM of $n \times d$ & $d \times n$ matrix
with entries from $[\ell]$.

Thm (essentially by Zwick's alg)

$$T_{\text{dir-unwt-APSP}}(n) \le \tilde{O}\left(\max_{\ell} M^*\left(n, \frac{n}{\ell}, n \mid \ell\right)\right)$$

Thm (C., Vassilevska W., Xu '21)

$$\max_{\ell} M^*\left(n, \frac{n}{\ell}, n \mid \ell\right) \le O\left(T_{\text{dir-unwt-APSP}}(n)\right)$$

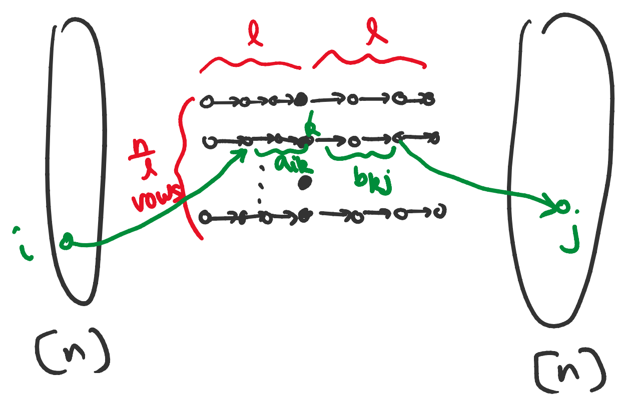
equivalence!

Pf: Very simple!

Given $n \times \frac{n}{\ell}$ matrix A all entries in $[\ell]$

$\frac{n}{\ell} \times n$ matrix B

Construct dir. graph



#verts $\frac{n}{\ell} \cdot O(\ell) = O(n)$

$\square$

Cor The following problems are equiv:

- APSP for dir unwted graphs
- APSP for unwted DAGs
- APSP for dir graphs in wts in $\{-1,0,1\}$
  $\{-c, \dots, c\}$
- APLP for unwted DAGs

$\vdots$