November 2, 2022 10:48 AM

More Cond. Lower Bds from Int 3SUM

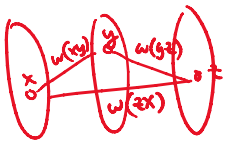
(Int) Zero-Wt Triangle in $O(n^{3-\delta})$ ←

↑ (Int) 3SUM in $O(n^{2-\delta})$

APSP in $O(n^{3-\delta})$

Rmk: can reduce Zero-Wt Triangle back to 3SUM but with much weaker bds

[Given weighted tripartite graph $G=(V,E)$,



$\forall xy \in E$ create number $w(xy) + xM - yM^2$

$\forall yz \in E$, " " $w(yz) + yM^2 - zM^3$

$\forall zx \in E$, " " $w(zx) + zM^3 - xM$ ← $n^{3/2-\delta}$

If 3SUM could be solved in $T(n)$ time,

Zero-Wt Triangle could be solved & hence APSP in $O(T(n^2))$ time ← $n^{3-\delta'}$]

Triangle Listing Problem Given unweighted sparse graph $G=(V,E)$ with m edges, report all K triangles

Recall: Alon, Yuster, Zwick '97 (detection) $O(m^{\frac{2\omega}{\omega+1}}) \le O(m^{1.41})$ (if $\omega=2$, $O(m^{4/3})$)

Bjorklund et al. '14 (report all) same bound for $K \le m$.

Thm (Patrascu '10) Assuming Int 3SUM Conj, no $O(m^{4/3-\delta})$ algm for triangle listing for $K=m$.

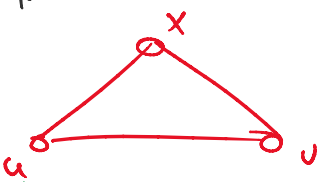
(lots of appl'ns to data structure lower bds ...)

Problem: Set Intersection Queries

Given N sets $S_1, \dots, S_N$, build data structure to answer queries: report all elems in $S_i \cap S_j$.

Given N sets $S_1, \ldots, S_N$,
build data structure to answer queries:
given i, j, report all elems in $S_i \cap S_j$.

(in offline case, all queries are given in advance)



Reduce Convol3SUM → Offline Set Intersect. Queries

To solve Convol3SUM for $a_0, \ldots, a_{n-1}$, ← decide $\exists i,j$: $a_i + a_j = a_{i+j}$

use hash fn $h(x) = x \bmod p$ for rand $p \in (R/2, R)$

define bucket $B_\ell = \{ j : h(a_j) = \ell \}$ $(\ell \in [R])$

(call bucket B_ℓ good if $|B_\ell| = \tilde{O}(\frac{n}{R})$)

(Prob [ans in 3 good bucket] $= \tilde{\Omega}(1)$).

1. for $i \in [n]$ do
2. for $\ell \in [R]$ do ← $j \in B_\ell$
3. find all j s.t. $h(a_j) = \ell$ and $h(a_{i+j}) = \ell + h(a_i)$ → $i+j \in B_{\ell + h(a_i)}$ ← $j \in B_{\ell + h(a_i)} - i$
   or $= \ell + h(a_i) - p$.
4. for each such j, if $a_i + a_j = a_{i+j}$, exit & return yes
5. return no

want $a_{i+j} = a_i + a_j$ ⇒ $h(a_{i+j}) = h(a_i) + h(a_j)$

Def: $S + x = \{ s + x : s \in S \}$
"shifted set"

line 3 reduces intersecting B_ℓ and $B_{\ell + h(a_i)} - i$

but # possible sets $= O(Rn)$ too big

idea - write $i = xd + y$, $x \in [n/d]$, $y \in [d]$

reduces to intersecting
$B_\ell + y$ and $B_{\ell + h(a_i)} - xd - y$
possible sets # possible sets

possible sets $= O(Rd)$ — # possible sets $= O(R\frac{n}{d})$

Choose $d = \sqrt{n}$

$\Rightarrow$ $N = \#\text{ sets} = O(R\sqrt{n})$

total set size $= \tilde{O}(R\sqrt{n} \cdot \frac{n}{R}) = O(n^{3/2})$

$\#\text{ queries} = O(nR)$ ←

expected total output size of queries

$=$ expected # false positives $(+1)$

$=$ # (i,j) with $a_i + a_j \neq a_{i+j}$ but $h(a_i + a_j) = h(a_{i+j})$

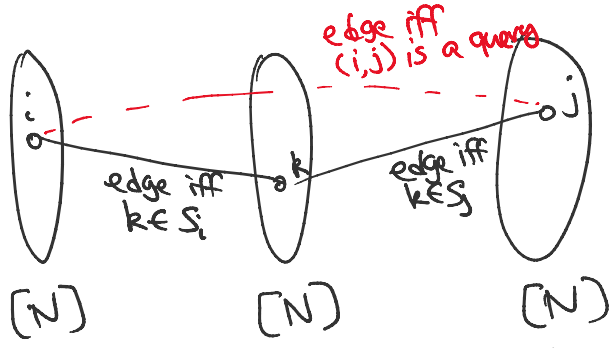
$\leq \tilde{O}(n^2 \cdot \frac{1}{R})$ ←

Choose $R = \sqrt{n}$ $\Rightarrow$ total set size $= \tilde{O}(n^{3/2})$
queries $= \tilde{O}(n^{3/2})$
total output size $= \tilde{O}(n^{3/2})$

Reduce Offline Set Intersect. Queries → Triangle Listing

Given sets $S_1, \ldots, S_N$, & queries,

create tripartite graph



$m = \#\text{ edges} =$ total set size + total # queries

$= \tilde{O}(n^{3/2})$

$V = \#\text{ triangles} =$ total output size $= \tilde{O}(n^{3/2})$

$= O(\cdot\cdot)$

K = # triangles = total output size $= \tilde{O}(n^{3/2})$

$\Rightarrow$ time $\tilde{O}\left(T_{\text{tri-list}}\left(\tilde{O}(n^{3/2})\right)\right)$

$\leq \tilde{O}\left(\left(n^{3/2}\right)^{4/3-\delta}\right)$

$= \tilde{O}\left(n^{2-\delta'}\right)$. □

Rmk: Vassilevska W. & Xu '20:
same lower bd for triangle listing under 4th APSP conj

CUX '22: .. .. real APSP conj.