October 21, 2022 10:51 AM

Problems Related to LCS / Edit Distance

1. **LCS:** Given string $A = a_1 \cdots a_n$, $B = b_1 \cdots b_n$

find longest common subsequence

(equiv: # deletes/inserts to go from A to B)

e.g. logarithm, algorithm — 4 changes

$A = 12\cancel{10}3120\cancel{2}0$, $B = 12312\cancel{0}2\cancel{3}\cancel{0}0$ — 6 changes

DP Soln: let $C[i,j]$ = min # changes to go from $a_1 \cdots a_i$ to $b_1 \cdots b_j$

$$\Rightarrow C[i,j] = \min\{\, C[i-1,j]+1,\ C[i,j-1]+1,\ C[i-1,j-1]+d(a_i,b_j)\,\}$$

$$d(a,b) = \begin{cases} 0 & \text{if } a=b \\ \infty & \text{if } a\neq b \end{cases}$$

$O(n^2)$ time

2. **Edit Distance:** min # inserts/deletes/substitutions

Same DP but with $d(a,b) = \begin{cases} 0 & \text{if } a=b \\ 1 & \text{if } a\neq b. \end{cases}$

3. **Dynamic Time Warping Distance (DTW):**

find expansions $\tilde{A}, \tilde{B}$ of A, B to minimize $\sum_i d(\tilde{a}_i, \tilde{b}_i)$

allow chars to repeat

e.g. $A = 12103120220$, $B = 12312123 10$

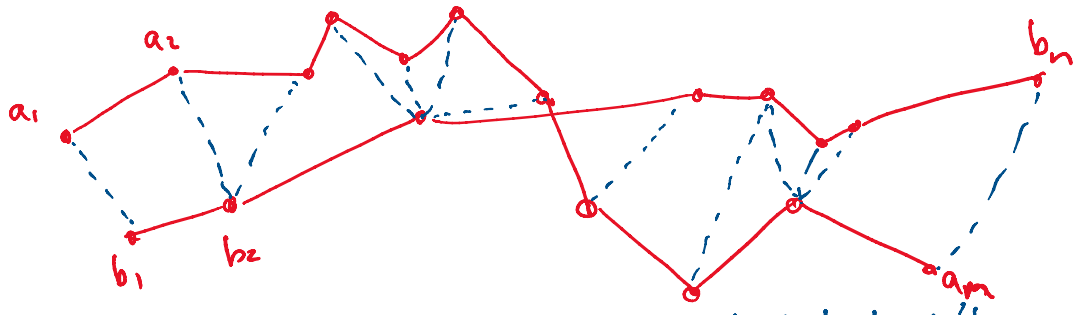
$\tilde{A} = 12103120222 0$

$\tilde{B} = 12223121 23 10$

DP Soln: $C[i,j] = \max(\min\{ C[i-1,j], C[i,j], C[i-1,j-1]\}, \cancel{d(a_i,b_j).}\ \}$ $O(n^2)$ time

4. **Discrete Fréchet distance:**

find expansions $\tilde{A}, \tilde{B}$ to minimize $\max d(\tilde{a}_i, \tilde{b}_i)$.



"walking a dog with shortest leash"

Thm Assuming SETH, no $O(n^{2-\delta})$ alg'm for these problems

History: by Bringmann '14 for Frechet dist. (discrete & continuous)
Backurs, Indyk '15 for edit dist.
Abboud, Backurs, Vassilevska '15 } for LCS
Bringmann, Künnemann '15

Reduce OV → Discrete Frechet (Bringmann '14)

Suppose disc Frechet could be solved in $O(n^{2-\delta})$ time.

Given vectors $a_1, \dots, a_n, b_1, \dots, b_n \in \{0,1\}^d$.

define alphabet $\Sigma = \{0, 1, \&, \$, \#, 0', 1', \&', \$', \#', \%', @'\}$

$d(\cdot,\cdot)$	$0'$	$1'$	$\&'$	$\%'$	$\$'$	$\#'$	$@'$
0	$<r$	$<r$					$<r$
1	$<r$						$<r$
&			$<r$				$<r$
\$	$<r$	$<r$	$<r$	$<r$	$<r$		$<r$
#	$<r$	$<r$	$<r$	$<r$	$<r$	$<r$	$<r$

all other $>r$

Define strings $A = \$f(a_1)\#\$f(a_2)\# \dots \$f(a_n)\#$

$B = @'\$'g(b_1)\%'g(b_2)\%' \dots \%'g(b_n)\#'@'$ ⟸ $O(dn)$

where $f(a) = a[1]\&a[2]\& \dots \&a[d]$

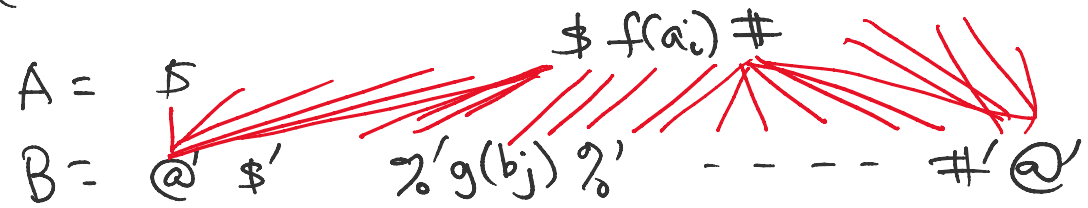
$g(b) = b[1]'\&'b[2]'\&' \dots \&'b[d]'$ $O((dn)^{2-\delta})$

where $f(a) =$ [illegible] $O((dn)$)

$g(b) = b[1]' \$' b[2]' \$' \ldots \$' b[d]'$

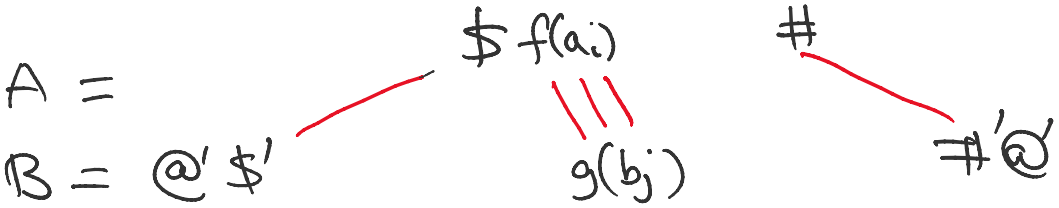
Claim disc Frechet dist $< r \iff \exists_{i,j}$ orth pair.

Pf: ($\Leftarrow$) Suppose $a_i \cdot b_j = 0$.



then Frechet dist $< r$.

($\Rightarrow$) Suppose Frechet dist $< r$.



$\Rightarrow a_i \cdot b_j = 0$ □