if exist
set ans for (z,x) to yes
remove (z,x) from E
else break
}

Total time: # oracle calls $= O(r^3 + K)$

"yes" edges

$$\Rightarrow O\left((r^3 + K)\, T\left(\frac{n}{r}\right) \right)$$

$$\leq O\left((r^3 + n^2)\, T\left(\frac{n}{r}\right) \right)$$

Set $r = n^{2/3}$ $\Longrightarrow$

$$O\left(n^2 \cdot T(n^{1/3}) \right)$$

$$\leq O\left(n^2 \cdot (n^{1/3})^{3-\delta} \right)$$

$$= O\left(n^{3 - \frac{\delta}{3}} \right). \quad \square$$

Last Time:

APSP → (min,+)-MM

↓

(min,+)-MM-Decis ≡ NWT (in AE vers)

↓ (*) highlight

NWT in Report-One

↓ today

NWT in orig versi.

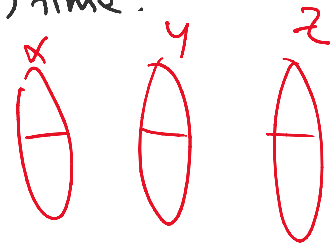
↓

min-wt triangle

Step 3. Reduce NWT in Report-one vers. $\rightarrow$ NWT in Orig vers.

Suppose NWT could be solved in $T(n) = O(n^{3-\delta})$ time.

To solve Report-One NWT:



Divide X into X_1, X_2 of size $\frac{n}{2}$
Y into Y_1, Y_2
Z into Z_1, Z_2

for each $i, j, k \in \{1, 2\}$
call oracle to decide $\exists$ neg-wt triangle in $X_i \times Y_j \times Z_k$

if yes, recurse in X_i, Y_j, Z_k & exit

Runtime

$$T'(n) \le T'\left(\frac{n}{2}\right) + \underbrace{8\,T\left(\frac{n}{2}\right)}_{O(n^{3-\delta})}$$

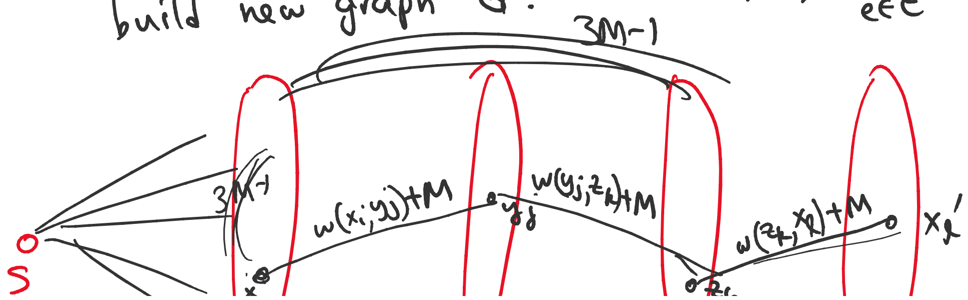
$$\Rightarrow T'(n) \le O(n^{3-\delta}). \qquad \square$$

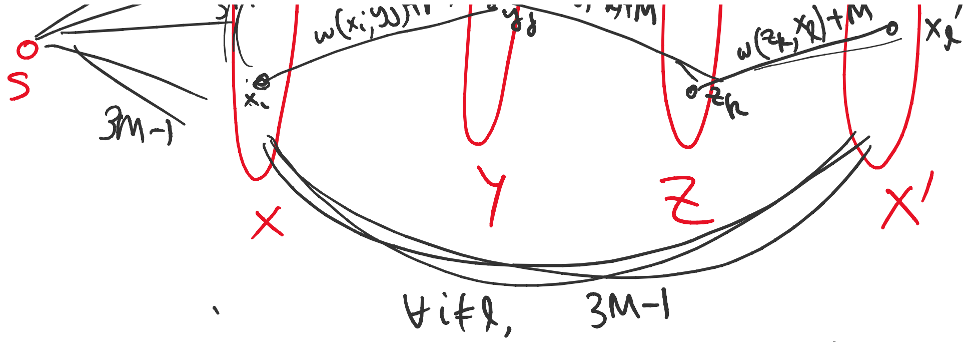
Reduce NWT $\rightarrow$ Radius:

Suppose graph radius could be computed in $T(n) = O(n^{3-\delta})$ time.

To solve NWT for a given weighted tripartite graph $G = (X \cup Y \cup Z, E)$:

build new graph $\hat{G}$: $\qquad M > \max_{e \in E} |w(e)|$





compute radius of $\vec{G}$ by oracle
in $T(O(n))$ time
$\leq O(n^{3-\delta})$

Claim G has neg-wt triangle
$\iff$ $\hat{G}$ has radius $< 3M$.

Pf: ($\Rightarrow$) Suppose $x_i y_j z_k$ is neg-wt tri in G.
dist from x_i to any vertex in $\hat{G}$
$< 3M$.

($\Rightarrow$) Suppose $\hat{G}$ has radius $< 3M$.
Center must be in X, say it is x_i.
dist from x_i to x_i' $< 3M$.

$\Rightarrow \exists y_j, z_k,\ (w(x_i,y_j)+\cancel{M}) + (w(y_j,z_k)+\cancel{M})$
$+ (w(z_k,x_i)+\cancel{M}) < \cancel{3M}\ 0$

$\Rightarrow \exists$ neg-wt tri in G. $\square$

Open: APSP $\to$ diam?

Problem Zero-Weight Triangle (ZWT)
Given tripartite graph,
$x \in X,\ y \in Y,\ z \in Z$ s.t.

Given tripartite graph,
decide $\exists\, x \in X,\ y \in Y,\ z \in Z$ s.t.
$$w(x,y) + w(y,z) + w(z,x) = 0.$$

Thm for int wts,
if ZWT has $O(n^{3-\delta})$ alg'm,
then APSP has $O(n^{3-\delta'})$ alg'm.

Pf: Reduce NWT $\to$ ZWT:

Lemma (turn ineq. to equality) given 3 ints a, b, c

$$a+b+c > 0$$
$$\iff \exists i,\ \left\lfloor \frac{a}{2^i} \right\rfloor + \left\lfloor \frac{b}{2^i} \right\rfloor + \left\lfloor \frac{c}{2^i} \right\rfloor = 1 \text{ or } 2 \text{ or } 3 \text{ or} \ldots \text{or } 7.$$

Pf: ($\Leftarrow$) Suppose $\left\lfloor \frac{a}{2^i} \right\rfloor + \left\lfloor \frac{b}{2^i} \right\rfloor + \left\lfloor \frac{c}{2^i} \right\rfloor \in \{1, \ldots 7\}$
$$\Rightarrow \frac{a}{2^i} + \frac{b}{2^i} + \frac{c}{2^i} > 0$$
$$\Rightarrow a+b+c > 0.$$

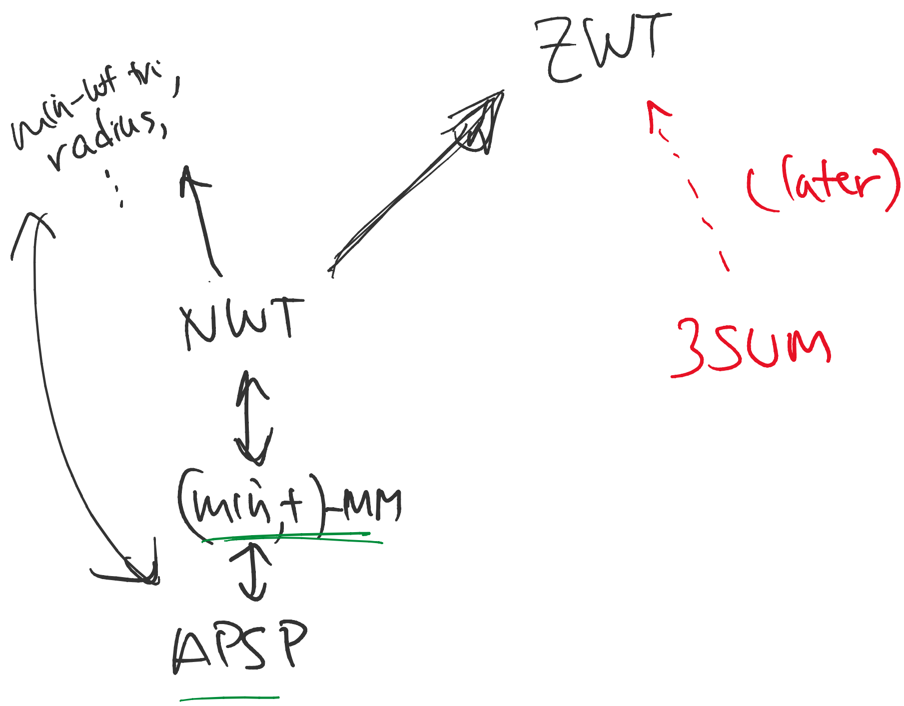
($\Rightarrow$) Suppose $a+b+c > 0$.
$$2^{i+2} \leq a+b+c < 2^{i+3} \quad (i \geq 0)$$

$$0 \leq \frac{a+b+c}{2^i} - 3 < \left\lfloor \frac{a}{2^i} \right\rfloor + \left\lfloor \frac{b}{2^i} \right\rfloor + \left\lfloor \frac{c}{2^i} \right\rfloor$$
$$\leq \frac{a+b+c}{2^i} < 8 \quad \square$$

Suppose ZWT could be solved in $T(n)$ time.
$O(T(n) \log W)$ time

Suppose ZWT could be solved in $T(n)$ time.
Then NWT " " ... $O(T(n) \log U)$ time.

Summary:



Next:

Cond. LBs from (min,+)-Convolution

Problem Given $a_0, \dots, a_{n-1}, b_0, \dots, b_{n-1}$,
Compute $c_k = \min_i (a_i + b_{k-i})$

Conjecture No $O(n^{2-\delta})$ alg'm for (min,+)-Convol.