if exist
  Set ans for (z, x) to yes
  remove (z, x) from E
else break
}

Total time:  # oracle calls = \( O(r^3 + K) \)

\[ \Rightarrow O\left((r^3 + K) \cdot T\left(\frac{n}{r}\right)\right) \]

\[ \leq O\left((r^3 + n^2) \cdot T\left(\frac{n}{r}\right)\right) \]

Set \( r = n^{4/3} \)  \[ \Rightarrow O\left(n^2 \cdot T\left(n^{1/3}\right)\right) \]

\[ \leq O\left(n^2 \cdot (n^{1/3})^{3-s}\right) \]

\[ = O\left(n^{3\cdot \frac{5}{3}}\right). \]  \( \Box \)

---

Last Time:

APSP \( \rightarrow (\min^+)\)-MM

(\(\min^+\))-MM-Decis \( \equiv \) NWT (in AE ver)

\( (*) \) highlight

NWT in Repeat-One

today

NWT in orig

\( \min \)-wt triangle
Step 3. Reduce NWT in original vers. \rightarrow NWT in orig vers.

Suppose NWT could be solved in 
\[ T(n) = O(n^{3-\delta}) \] time.

To solve Report one NWT:

Divide \( X \) into \( X_1, X_2 \) of size \( \frac{n}{2} \)

\( Y = Y_1, Y_2 \)

\( Z = Z_1, Z_2 \)

for each \( i, j, k \in \{1, 2\} \)
call oracle to decide 3 neighbour triangle in \( X_i \times Y_j \times Z_k \)
if yes, recurse in \( X_i, Y_j, Z_k \) & exit

Runtime
\[ T'(n) \leq T'(\frac{n}{2}) + 8 \frac{T(\frac{n}{2})}{O(n^{3-\delta})} \]
\[ \Rightarrow T'(n) \leq O(n^{3-\delta}) \]

---

Reduce NWT \rightarrow Radius:

Suppose graph radius could be computed
\[ T(n) = O(n^{3-\delta}) \] time.

To solve NWT for a given weighted tripartite graph \( G = (X \cup Y \cup Z, E) \):

build new graph \( G' \):

\[ M \geq \max_{e \in E} |w(e)| \]
Compute radius of $G$ by oracle in $T(\omega(n))$ time
\[ \leq O(n^{3.5}) \]

**Claim**

$G$ has neg-wt triangle $\iff \hat{G}$ has radius $< 3M$.

**Pf:** ($\Rightarrow$) Suppose $x_i y_j z_k$ is neg-wt tri in $G$.

\[ \text{dist from } x_i \text{ to any vertex } x' \in \hat{G} < 3M. \]

($\Leftarrow$) Suppose $G$ has radius $< 3M$. Center must be in $X_1$, say it is $x_i$.

\[ \text{dist from } x_i \text{ to } x'_i < 3M \]

\[ = \exists y_j, z_k, (w(x_i, y_j) + M) + (w(y_j, z_k) + M) + (w(z_k, x_i) + M) < 3M \]

\[ = \exists \text{ neg-wt tri in } G. \qquad \Box \]

**Open:** $\text{APSP} \rightarrow \text{diam}$. 

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**Problem** Zero-Weight Triangle ($ZWT$)

Given tripartite graph,

\[ -v \quad \leq y \leq z \quad \text{st.} \]
Given tripartite graph,

\[ \exists x \in X, y \in Y, z \in Z \text{ s.t.} \]
\[ w(x,y) + w(y,z) + w(z,x) = 0. \]

**Thm**: for int wts,

if \( ZWT \) has \( O(n^{3.8}) \) alg'm, 

then \( APSP \) has \( O(n^{3.8}) \) alg'm.

**Pf**:

Reduce \( NWT \rightarrow ZWT \):

**Lemma** (turn ineq. to equality) given 3 wts \( a, b, c \)

\[ a + b + c > 0 \]

\[ \iff \exists i, \left\lfloor \frac{a}{2^i} \right\rfloor + \left\lfloor \frac{b}{2^i} \right\rfloor + \left\lfloor \frac{c}{2^i} \right\rfloor = 1 \text{ or } 2 \text{ or } 3 \text{ or } \ldots \text{ or } 7. \]

**Pf**: (\( \leq \)) Suppose \( \left\lfloor \frac{a}{2^i} \right\rfloor + \left\lfloor \frac{b}{2^i} \right\rfloor + \left\lfloor \frac{c}{2^i} \right\rfloor \in \{1, \ldots, 7\} \)

\[ \Rightarrow \frac{a}{2^i} + \frac{b}{2^i} + \frac{c}{2^i} > 0 \]

\[ \Rightarrow a + b + c > 0. \]

(\( \geq \)) Suppose \( a + b + c > 0 \).

\[ 2^{i+2} \leq a + b + c < 2^{i+3} \quad (i \geq 0) \]

\[ 0 < \frac{a+b+c}{2^i} - 3 < \left( \left\lfloor \frac{a}{2^i} \right\rfloor + \left\lfloor \frac{b}{2^i} \right\rfloor + \left\lfloor \frac{c}{2^i} \right\rfloor \right) \]

\[ \leq \frac{a+b+c}{2^i} < 8 \quad \Rightarrow \]

Suppose \( ZWT \) could be solved in \( T(n) \) time.

\( \in \Theta(n \log n) \) time.
Suppose ZWT could be solved in $T(n)$ time.
Then NWT $\in \Omega(T(n) \log U)$ time.

**Summary:**

ZWT \(\leftarrow\) \(\text{(later)}\)

3SUM

\(\text{min+} \rightarrow\)

radius,

NWT

\((\text{min},+)\text{-MM}\)

\(\uparrow\)

APSP

Next:

Cond. LBs from \((\text{min},+)\text{-Convolution}\)

**Problem**

Given \(a_0, \ldots, a_{n-1}, b_0, \ldots, b_{n-1}\),

compute \(q_k = \min_i (a_i + b_{k-i})\)

**Conjecture**

No $O(n^{2.5})$ algorithm for \((\text{min},+)\text{-Convol.}\)