CS 598 TMC
Algorithms from the Fine-Grained Perspective

courses.grainger.illinois.edu/cs598tmc

Course Work: 4 HWs 45%
presentation 15%
project 40%

(may work in groups of ≤ 3)

Prereq: undergrad alg'ms (CS374)

Theme - understand fine-grained complexity of basic algorithmic problems

beyond polynomial vs. NP-hard

e.g. n^3 vs. n^2 etc.

Ex1 **All-Pairs Shortest Paths (APSP)** for dense weighted graphs

Floyd-Warshall (by DP) $O(n^3)$ time

Dijkstra n times $O(n^3)$

better? Fredman '75 $\sim O\left(\frac{n^3}{\log^{1/3} n}\right)$

$\vdots$

C '07 $\sim O\left(\frac{n^3}{\log^2 n}\right)$

Williams '14 $O\left(\frac{n^3}{c^{\sqrt{\log n}}}\right)$

Conjecture no truly subcubic alg'm
e.g. $n^{2.9999}$

Ex2 Longest Common Subsequence (LCS)
of 2 strings $a_1 \cdots a_n$
$b_1 \cdots b_n$

DP $\Rightarrow$ $O(n^2)$ time

$$L[i,j] = \max \begin{cases} L[i-1,j] \\ L[i,j-1] \\ L[i-1,j-1]+1 & \text{if } a_i = b_j \end{cases}$$

better? current record $\sim O\left(\frac{n^2}{\log^2 n}\right)$

related problems: edit dist, Frechet dist, ...

Ex3 3SUM
Given set S of n numbers & target t,
? $\exists\, a,b,c \in S$ s.t. $a+b+c = \cancel{t}\ 0$?

trivial: $O(n^3)$
standard HW prob: $O(n^2)$

better? Grönlund-Pettie '14: $\sim O\left(\frac{n^2}{\log n}\right)$
C'18: $\sim O\left(\frac{n^2}{\log^2 n}\right)$

Conjecture no truly subquad alg'm

6SUM
$\underbrace{a+b+c}+\underbrace{d+e+f}=0$

k-SUM: trivial $O(n^k)$
"meet-in-middle": $O(n^{k/2}\log n)$ for k even
$O(n^{(k+1)/2})$ for k odd

meet-in-middle:

$O(n^{(k+1)/2})$ for k odd

better?

Subset-Sum: given S, & t,

$\exists$ subset summing to t

trivial $\sim O(2^n)$

meet-in-middle: $\sim O(2^{n/2})$

better?

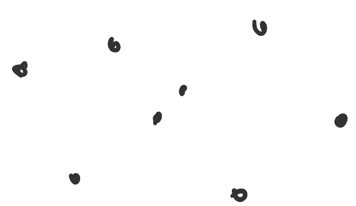
DP: $O(nt)$ time

assuming positive integers

current record: $O(\underline{(n+t)} \log^c n)$

better?

Ex closest pair in d dims



trivial: $O(dn^2)$ time

known: $O(d^{O(d)} n \log n)$

bad when $d \sim \log n$

(orthogonal vector)

Proving lower bds is very difficult in general comp. model

idea: prove conditional lower bds

under conjectures that certain basic probs are hard

via fine-grained reductions

under conj ... are hard
via fine-grained reductions

Similar to NP-hardness (conj $P \neq NP$)

Exs of Cond. Lower Bd. Results:

1. Gajentaan-Overmars '93:

   if 3-point-collinearity could be solved in $O(n^{1.99})$ time,
   then 3SUM " " " $O(n^{1.99})$ time.

2. Abboud et al. '14:

   if graph radius could be computed in $O(n^{2.99})$ time, $n^{3-\varepsilon}$
   APSP - - - - - .. $O(n^{2.9999})$ time $n^{3-\varepsilon'}$

3. Patrasca '10:

   if triangle listing could be solved in $O(m^{4/3-\epsilon})$ time,
   then 3Sum for integers - - - - - $O(n^{2-\varepsilon'})$ time.

4. Bringmann '14:

   if Frechet dist (edit dist, LCS) could be solved in $O(n^{\cancel{1.99}\ 2-\varepsilon})$ time,
   then SAT " " " $O(\cancel{1.99999}^n\ (2-\varepsilon')^n)$ time

   which contradicts Strong Exponential Time Hypothesis (SETH)

etc.

Strong Exponential
(SETH)

etc.

Course Outline

I. Basic Alg'mic Tools (Upper Bds)

Convolution / FFT, matrix mult.

II. Conditional Lower Bds using conjs on APSP / 3SUM / SETH / ...

lots of reductions ...

III. Adv. Alg'mic Techniques (Back to UBs)