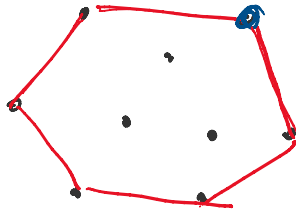


Dynamic Convex Hull in 2D

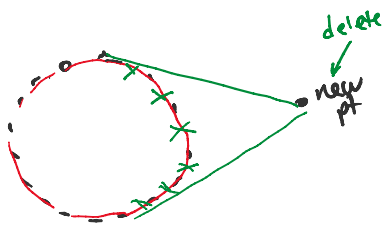


static: $O(n \log n)$ time &
e.g. Graham's scan

dynamic?

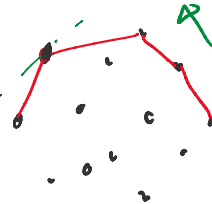
insert pt
delete pt
query (e.g. find extreme pt
along dir)

$O(\log n)$ time?

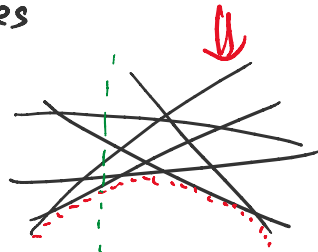


difficulty - each insert/delete
can cause $\Omega(n)$ changes
to CH.

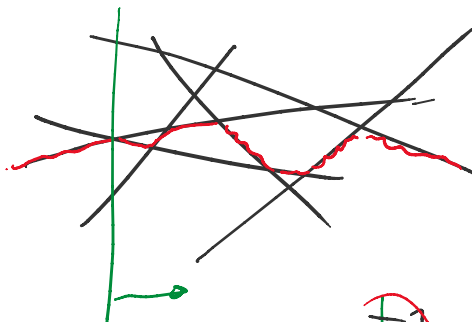
Suffice to consider upper hull (UH)



by duality: equiv. to lower envelope
of lines



appl'n - compute k-level of n lines



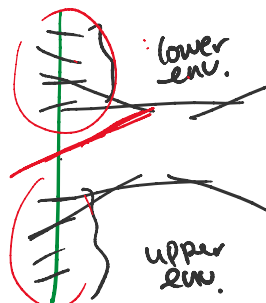
if k-level has
 h vertices,



$O(h)$ insert/delete
queries

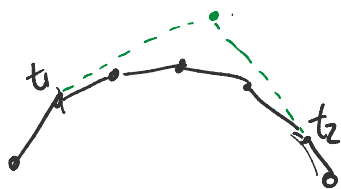


$O(h \log n)$
tree algo?



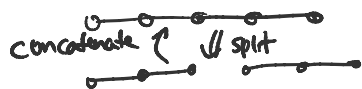
Sweep
line

Warm-Up: Insert-Only



insert reduces to
find 2 tangents
($O(\log n)$ time
by binary search)

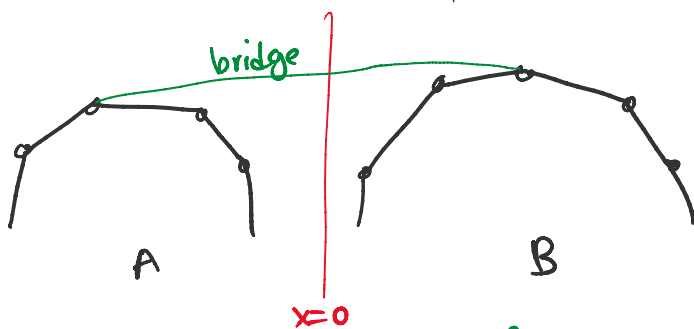
{split/concat lists
($O(\log n)$)



$\Rightarrow O(\log n)$ time

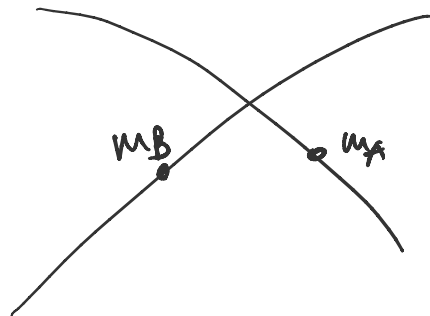
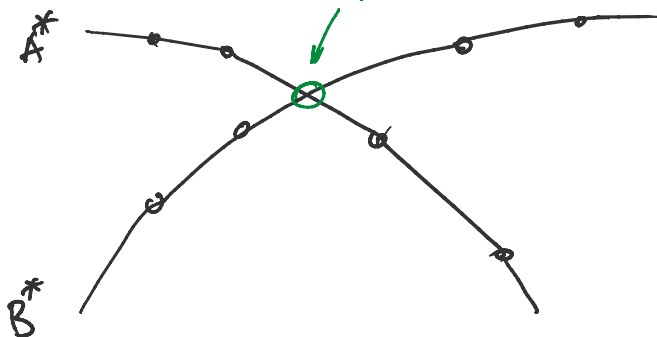
Overmars - van Leeuwen '80: Fully dynamic

Lemma Given 2 vertically separated UHs A, B,
can find the bridge between A, B
in $O(\log n)$ time



Pf:

dual



by careful binary search

Case 1.



Case 2.



Case 3.



Case 4.

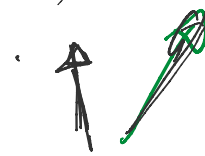
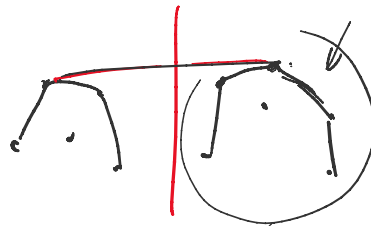


$\Rightarrow \leq 2 \log n$ iterations.

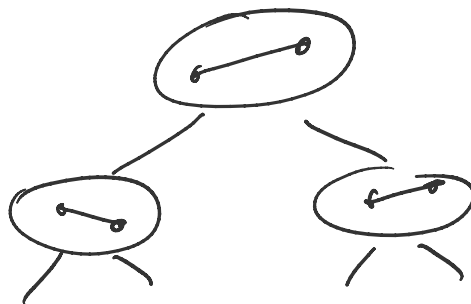
D

idea- hull tree

divide by median x
recurse on left & right
store bridge



$\Rightarrow O(n)$ space



extreme-pt query:

$O(\log n)$ time

insert/delete(p):

if p is on left side
insert/delete on left subtree
... "

insert/delete(p): if p is on left side
 insert/delete on left subtree
 right "
 recompute bridge from scratch

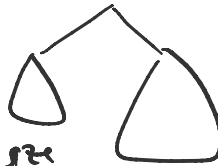
$O(\log n)$
 time
 by Lemma

$\Rightarrow O(\log^2 n)$ time

issue - balancing?

option 1 - weight balance

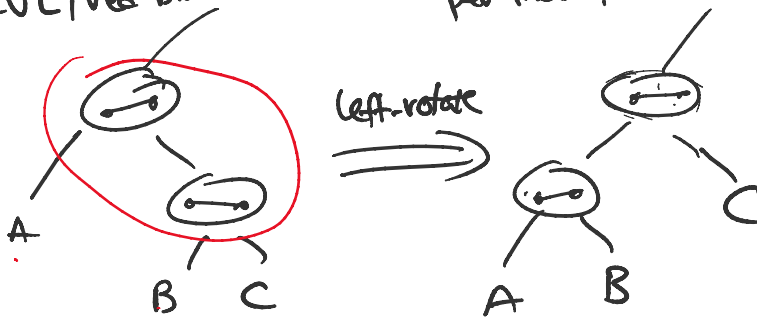
if left subtree size $\gg$ c. right subtree size
 rebuild subtree



$\Rightarrow$ amortization

option 2 - rotation

AUL/red-black tree - $O(1)$ rotations per insert/delete



Improvements:

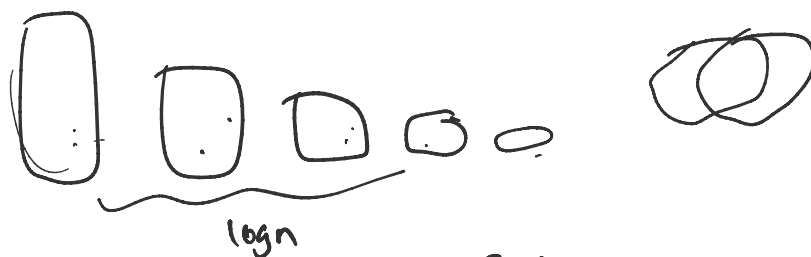
Chazelle '83: $O(\log n)$ amort. update time but delete-only
 (null tree w. linear search & amort.)

fully dynamic

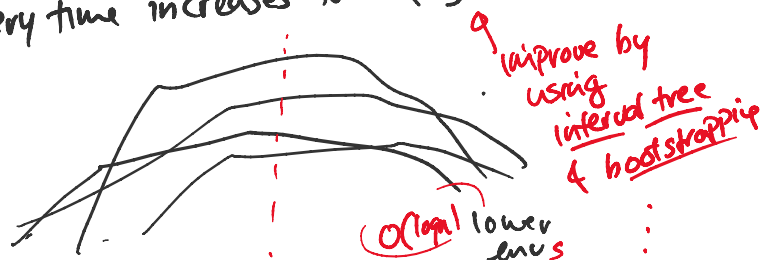
C. '99: $O(\log^{HE} n)$ amort. update, $O(\log n)$ extract query
 Brodal; Jacob '00: $O(\log n \log \log n)$ " , $O(\log n)$ "
 Brodal-Jacob '02: $O(\log n)$ " , $O(\log n)$ "
 ... - in progress!

Brodal, Jacob '00: $O(\log n)$ " ; $O(\log n)$ "
 Brodal-Jacob '02: $O(\log n)$ "
 very complicated, > 100 pages!

idea. Start w. delete-only
 apply the logarithmic method to support insert



query time increases to $O(\log^2 n)$



Open Qs:

dynamic convex hull area in 2D
 ($O(\log^2 n)$ update)

dynamic convex hull in 3D
 ($O(\log^4 n)$ update,
 $O(\log^2 n)$ extreme-pt query) [C.'19]

dynamic convex hull volume in 3D
 ($\tilde{O}(n^{11/12})$)