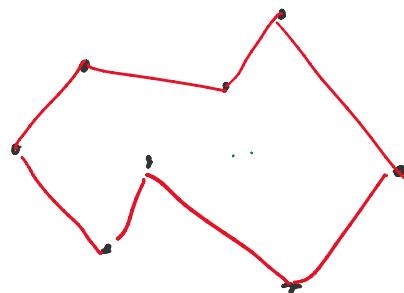


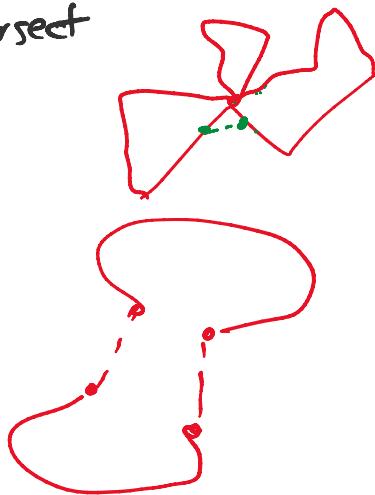
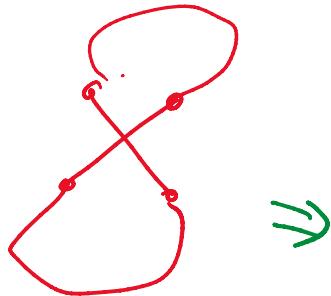
Euclidean Traveling Salesman Problem (TSP)

Given n pts $P \subset \mathbb{R}^2$,

find shortest tour C^∞
thru all pts

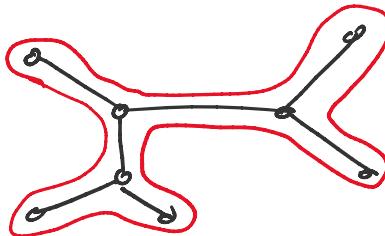


Note: opt tour is a polygon
with vertices at P
& does not self-intersect



General metric case:

2-approx. by MST



$(3/2)$ -approx. by Christofides '76

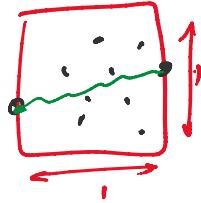
Karlin et al. '21: $\frac{3}{2} - 10^{-36}$ approx.

Geom. Case:

Arora '96 : $(1+\varepsilon)$ -approx, i.e. PTAS!!
Mitchell '96 :

idea- shifted
quadtree + DP

Say min bounding square has side length 1.



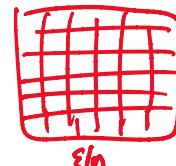
Round pts to grid of side length $\frac{\epsilon}{n}$

\Rightarrow total additive error

$$\leq n \cdot O\left(\frac{\epsilon}{n}\right)$$

$$= O(\epsilon)$$

$$\leq O\left(\epsilon \left|\frac{C^*}{\text{off}}\right|\right)$$



$\frac{n}{\epsilon} \times \frac{n}{\epsilon}$ grid.

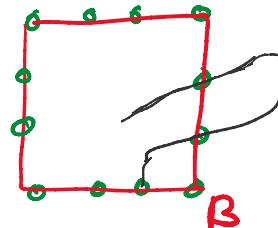
Build quadtree (no need to compress)

\Rightarrow depth $O\left(\log \frac{n}{\epsilon}\right)$

$$\#\text{nodes } O(n \log \frac{n}{\epsilon})$$

fix k.

Def for each quadtree cell B ,
place k evenly spaced pts on ∂B
called portals

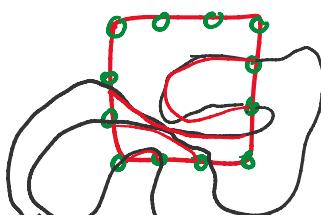


Def A tour T is portal-respecting
if \forall quadtree cell B ,
 T crosses ∂B only thru portals ✓
 $\triangle T$ visits each portal at most ≤ 2 times. ✓

Lemma can find shortest portal-respecting tour exactly
in $O(2^{O(k)} n \log n)$ time

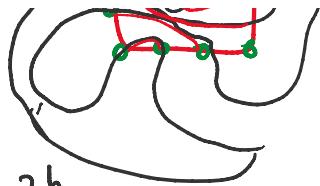
Pf: By DP.

Subproblem Given quadtree cell B



Subproblem

Given quadtrees cell B

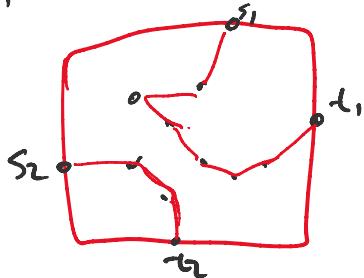


& list of portal pairs

$(s_1, t_1), \dots, (s_b, t_b)$, $b \leq 2^k$.

call this
an interface

find shortest set of b portal-respecting paths
from s_1 to t_1 , ..., s_b to t_b
thru all pts in $P \cap B$.



interfaces $\leq O(k^{O(k)})$.

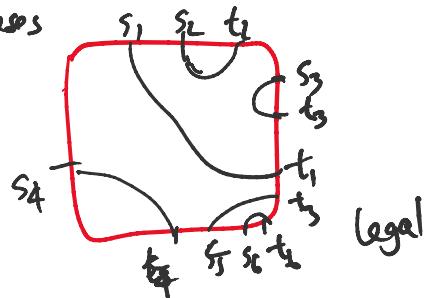
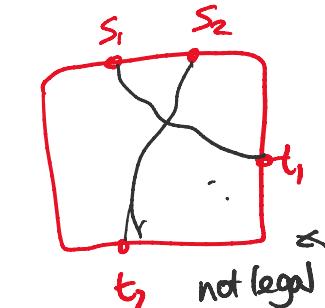
legal interfaces $O(k)$

\approx strings of balanced parentheses

\approx Catalan's number

$\leq 2^{O(k)}$

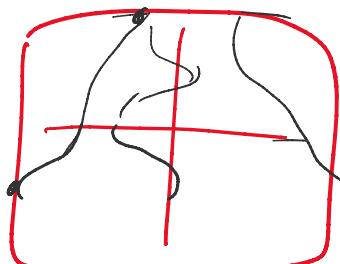
..

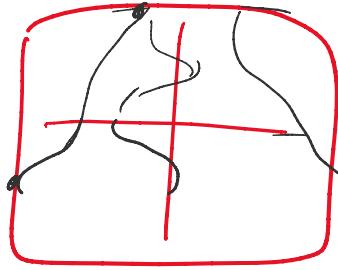


total # subprobs $\leq O(2^{O(k)} n \log n)$.

total time $O((2^{O(k)})^t n \log n)$.

□



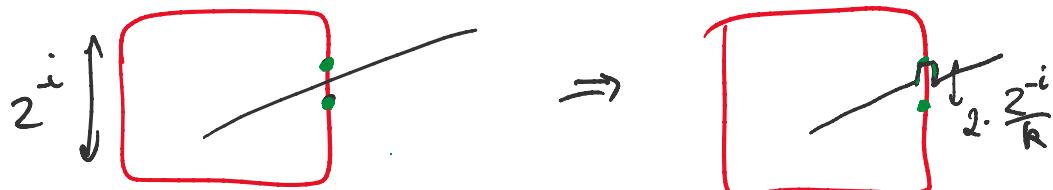


Arora's Alg'm:

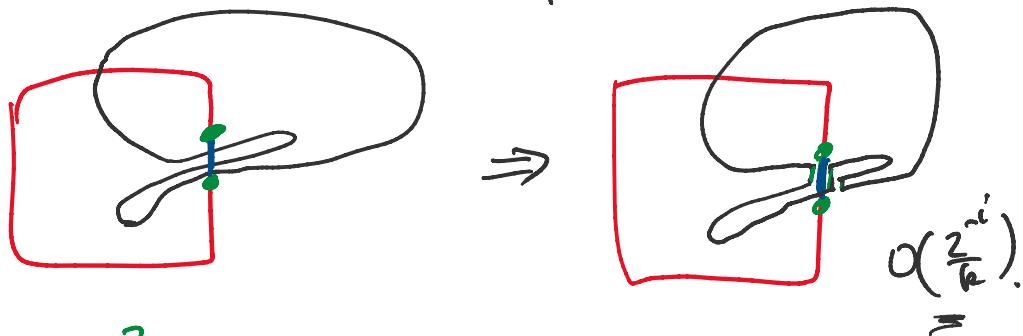
1. randomly shift P
2. return shortest portal-resp. tour

Analysis:

Round C^* into portal-resp. tour by adding detours
(length $O(\frac{2^{-i}}{k})$)



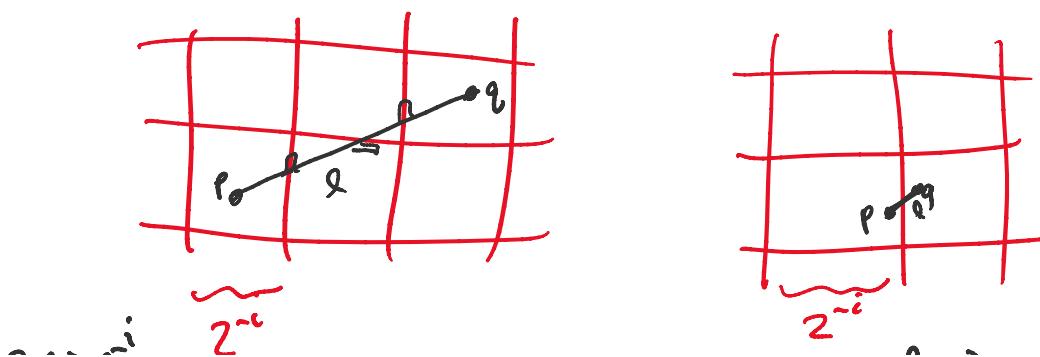
if portal is visited > 2 times, patch first



Total error?

Fix edge pq of C^* , of length l .

Consider grid of side length 2^{-i} .



If $\lambda > \tilde{2}^i$, $\frac{\lambda}{\tilde{2}^i}$

$\# \text{times } pq \text{ crosses grid bdry} = O\left(\frac{\lambda}{\tilde{2}^i}\right)$

If $\lambda < \tilde{2}^i$,

$E(\# \text{times } pq \text{ crosses grid bdry}) = O\left(\frac{\lambda}{\tilde{2}^i}\right)$

$$\Rightarrow E(\# \text{times } C^* \text{ crosses grid bdry}) = O\left(\frac{|C^*|}{\tilde{2}^i}\right).$$

$$\begin{aligned} \Rightarrow E(\text{total err}) &= O\left(\sum_{i=0}^{O(\log n)} \frac{|C^*|}{\tilde{2}^i} \cdot \frac{\tilde{2}^i}{k}\right) \\ &= O\left(\underline{\log n} \cdot \frac{|C^*|}{k}\right). \\ &= O(\varepsilon \cdot |C^*|) \end{aligned}$$

by setting $k = \frac{1}{\varepsilon} \log n$.

Runtime: $2^{O\left(\frac{1}{\varepsilon} \log n\right)} n \log n = n^{O\left(\frac{1}{\varepsilon}\right)}$

Avora '97: $O\left(n \left(\frac{\log n}{\varepsilon}\right)^{O(1/\varepsilon)}\right)$

$$2^{O(1/\varepsilon)^c} \downarrow n \log n$$