Technique 1: Shifted Grid (Hochbaum-Maass '85)

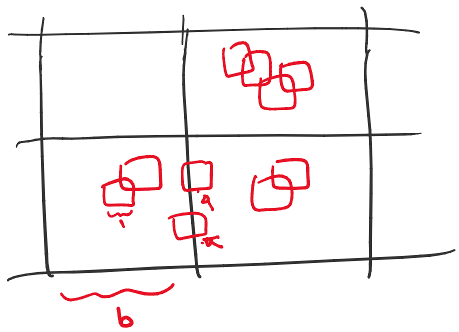
for unit squares/disks

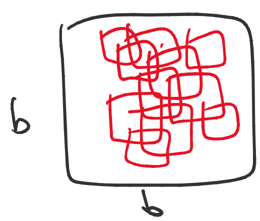
1. shift all objects by rand. vector
2. form unif. grid of side length b
3. remove all objs that cross grid cell boundaries
4. solve problem inside each grid cell separately

by brute force

$$\le \sum_i n_i^{O(b^2)} = O(n^{O(b^2)})$$

time





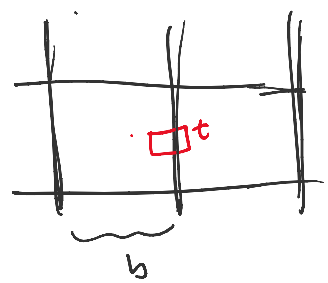
What is approx factor?

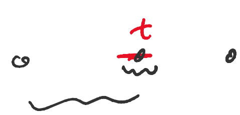
Analysis: Fix $t \in T^*$.

$\Pr[\,t$ crosses grid boundary$]$

$$\le \frac{1}{b} + \frac{1}{b}$$

$$= \frac{2}{b}.$$





$\Rightarrow E[\#\text{objs of } T^* \text{ crossing grid bdry}]$

$$\le \frac{2}{b}|T^*|.$$

$\Rightarrow E(|T|) \ge E(\#\text{objs of } T^* \text{ not crossing grid bdry})$

$$\ge \left(1 - \frac{2}{b}\right)|T^*|$$

Set $b := \frac{2}{\varepsilon}$ $\Rightarrow$ $(1 \pm \varepsilon)$-approx. in time $n^{O(1/\varepsilon^2)}$

polynomial-time approx scheme (PTAS)

Rmk - can be derandomized

weighted ✓

fat objs of similar size $\frac{\text{max rad}}{\text{min rad}} \le c$.

piercing

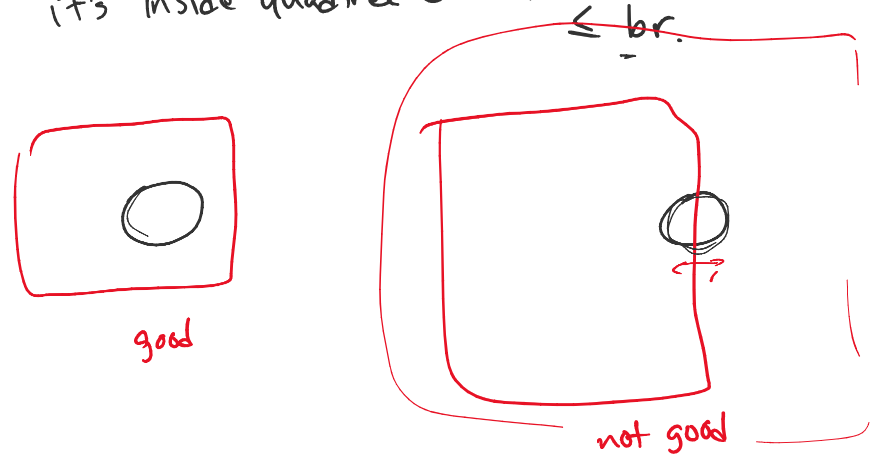
What about arbitrary squares/disks?

Technique 2: Shifted Quadtrees (Erlebach-Jansen-Seidel '01 / C. '01).

indep. set for arb. squares/disks, or "fat" objs.

Fix b.

Def An obj of radius r is good if it's inside quadtree cell of side length $\le br$.



Lemma If all objs are good & fat, indep set problem can be solved exactly in polytime $n^{O(b)}$.

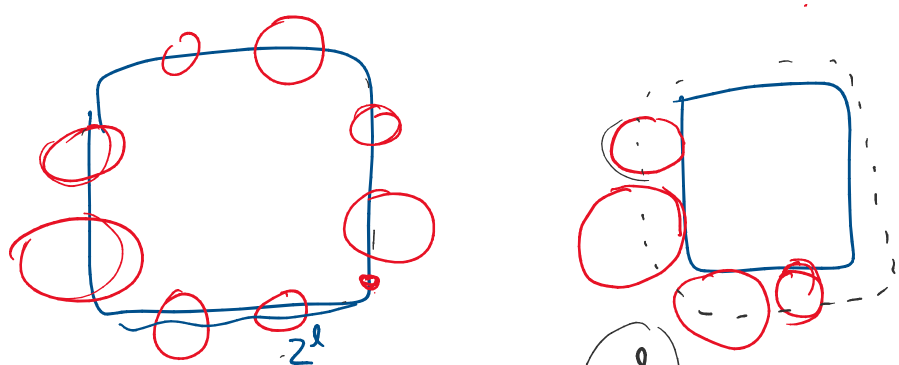
... dynamic programming.

Pf: idea - dynamic programming.

Subproblem Given a compressed quadtree cell B of side length 2^{ℓ}, & a set T_0 of disjoint objs intersecting ∂B,

"interface"

find size of largest subset T of objs inside B s.t. $T \cup T_0$ is disjoint.

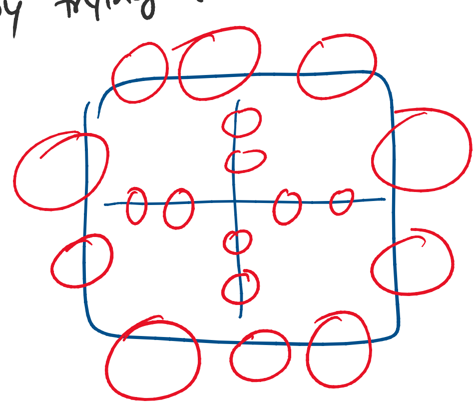


Note: each obj in T_0 has radius $> \frac{2^{\ell}}{b}$. by goodness

$\Rightarrow \quad |T_0| \le O(b)$

$\Rightarrow \quad$ total # subprobs at $B \le n^{O(b)}$

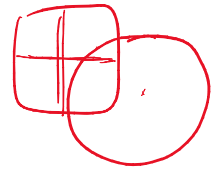
Given sol'n to all subproblems at 4 children of B, can compute sol'n to all subprobs at B by trying all interface combinations



$$\left(n^{O(b)}\right)^4 n = n^{O(b)}.$$

□

Approx Alg'm:

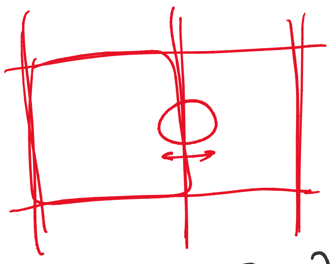


1. shift all objs by rand vector
2. Solve prob. on the good objs.

Analysis: Let $t \in T^*$ of radius r

Let ℓ be s.t. $2^\ell \le br < 2^{\ell+1}$.

$$\Pr[t \text{ not good}] = \Pr[t \text{ crosses grid cell boundary with side length } 2^\ell]$$



$$\le \frac{2r}{2^\ell} + \frac{2r}{2^\ell}$$

$$= O\left(\frac{1}{b}\right).$$

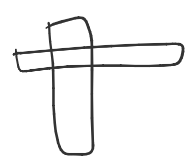
$$\Rightarrow \quad E(|T|) \ge E(\underline{\#\text{good objs in } T^*})$$

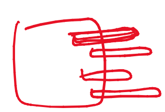
$$= \left(1 - O\left(\tfrac{1}{b}\right)\right)|T^*|.$$

Set $b = \Theta\left(\frac{1}{\varepsilon}\right) \Rightarrow$ factor $1 \pm \varepsilon$ in polytime $\underline{n^{O(1/\varepsilon)}}$.

PTAS.

Rmk · can be derand.
- weighted ✓   fat objs ✓
- not working for piercing ...
- space is bad

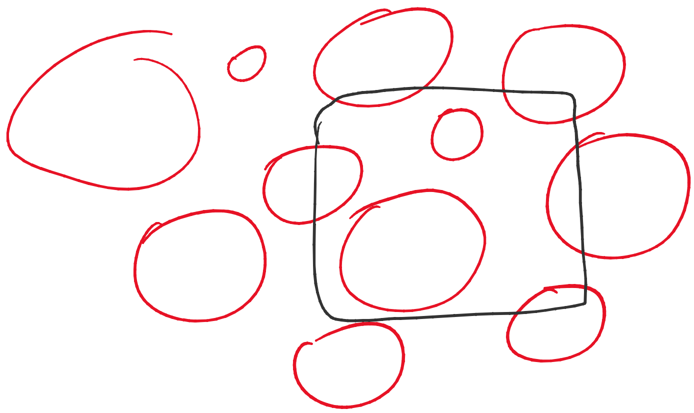




Technique 3: Separators

before, planar graph separators

before, planar graph separators



Geometric Separator Thm (Smith-Wormald '98)

Given n disjoint disks/squares/"fat" objs in $\mathbb{R}^2$,

$\exists$ square B s.t.

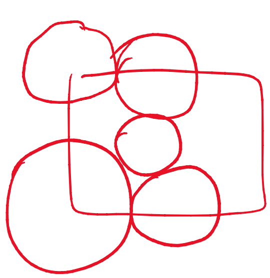
objs inside $B \leq \frac{4}{5}n$

objs outside $B \leq \frac{4}{5}n$.

objs intersecting $\partial B \leq O(\sqrt{n})$

$n^{1-\frac{1}{d}}$ in $\mathbb{R}^d$

(extends planar graph case)

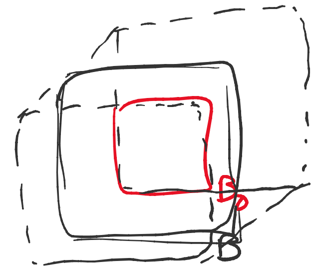


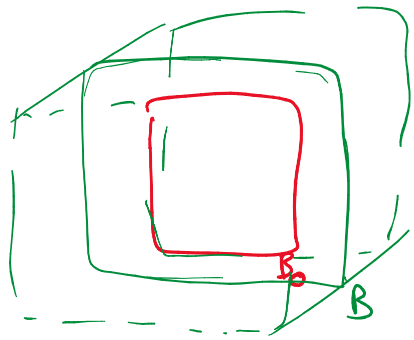
Pf: Let B_0 be smallest square containing $\geq \frac{n}{5}$ center pts.

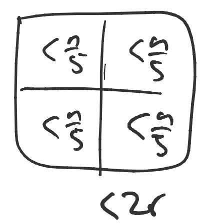
side length r

Pf: Let B_0 be <u>smallest</u> square containing $\geq \frac{n}{5}$ center pts.

Let B be a <u>randomly shifted</u> square of side length $(2-\delta)r$ containing B_0







$$\frac{n}{5} \leq \text{\# center pts inside } B \leq \frac{4n}{5}$$

$\vdots$

$\square$