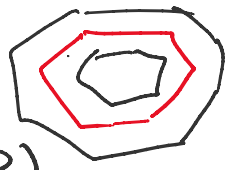
Appl'ns of ε-Kernels:

- Convex-hull-related problems

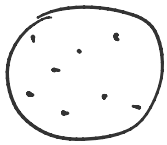


If S is an ε-kernel of P,

$$(1 - O(\varepsilon))\,CH(P) \subseteq CH(S) \subseteq CH(P)$$

$\Rightarrow$ ε-coresets for min-encbs. ball

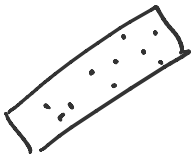
min. enclos. cylinder



⋮

min enclos. box volume

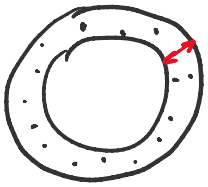
min enclos. simplex volume



⋮

- Some non-convex problems

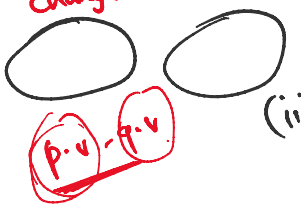
e.g. min enclos. annulus with



- Sketching & streaming

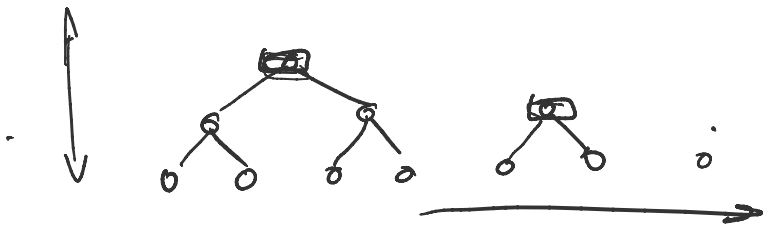
Fact (i) If S_1 is an ε-kernel of P_1,
S_2 is an ε-kernel of P_2,
then $S_1 \cup S_2$ is an ε-kernel of $P_1 \cup P_2$.

(require slight change in Def)



(ii) If S is an ε-kernel of P
S' " ε' " S,
then S' is an $(\varepsilon+\varepsilon')$-kernel of P.

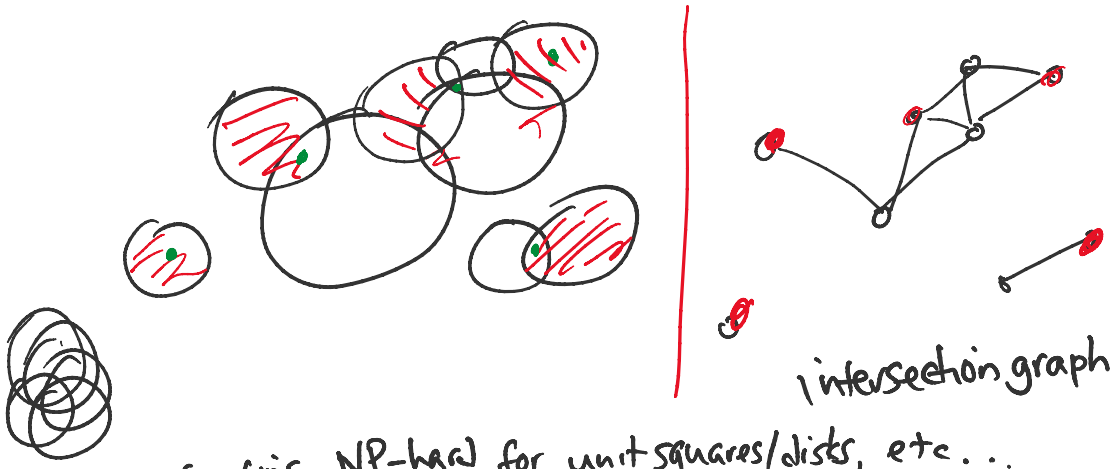
"merge-and-reduce"



$\Rightarrow$ $O(\log^{O(d)} n)$ space
while supporting insertions ....

Geom. Independent Set

Given n objects S,
find max subset $T^* \subseteq S$
s.t. no 2 objects in T^* intersect



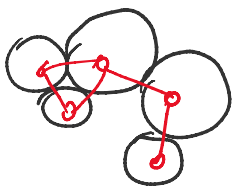
intersection graph

remains NP-hard for unit squares/disks, etc...

general graphs: hard to approx. $n^{1-\varepsilon}$

unweighted vs. weighted

(Koebe's Thm: planar graph is an ~~intersection~~ contact graph of disks.)



Related problem - piercing
Given n objects S, find min # pts X^*
that stab all objects

$\rightarrow$ min vertex #

Obs: piercing # $\geq$ indep #

Technique 0: Greedy

$T = \phi$

repeat {

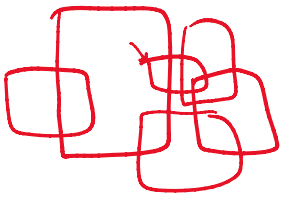
pick smallest (radius) object $t \in S$

insert t to T

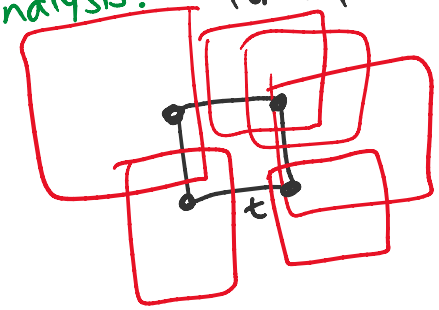
remove t & all objects intersecting t from S

}

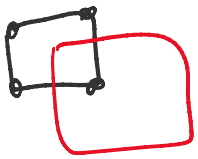
polytime

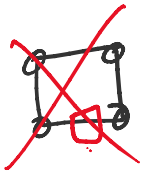


Analysis: For squares



all squares intersecting t bigger than t are stabbed by 4 pts.



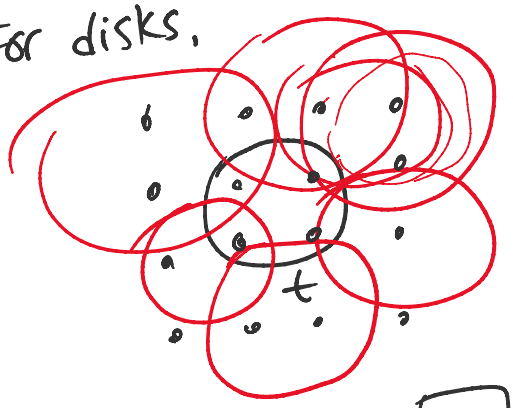


Let X^* be min piercing set.

$\Rightarrow |T^*| \leq |X^*| \leq 4|T|$. $\leq 4|T^*|$

$\Rightarrow$ factor 4

For disks.

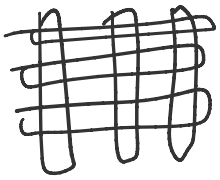


all disks intersecting t bigger than t can be stabbed by ≤ 16 pts

$\Rightarrow$ factor $\leq$ 16

For "fat objects", const factor

Rmk - not work for arbitrary rectangles

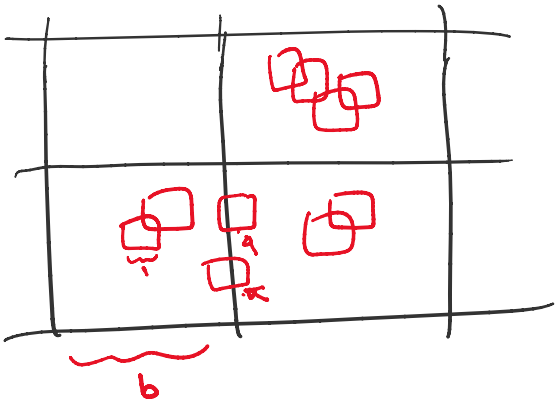


- not work for weighted squares

Technique 1: Shifted Grid (Hochbaum-Maass '85)

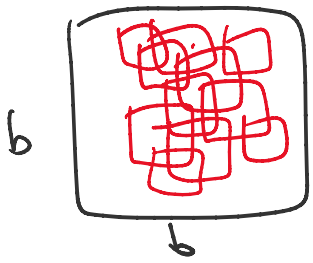
for unit squares/disks

1. Shift all objects by rand. vector
2. form unif. grid of side length b
3. remove all objs that crosr grid cell boundaries
4. solve problem inside each grid cell separately



by brute force

$\leq \sum_i n_i^{O(b^2)} = O(n^{O(b^2)})$ time



What is approx factor?