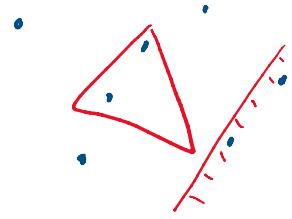


Non-orthogonal Range Searching



triangle range search
(or simplex)

halfplane range search
(or halfspace)

History

⇒ Willard '82

Edelsbrunner-Welzl '86

Hausler-Welzl '88

Welzl '88

Chazelle-Sharir-Welzl '90

⇒ Matoušek '91

Matoušek '92

C'10

	preproc	space	query	in 2D	\log_3^3
		n	$n^{0.793}$		$(n^{3D}, n^{0.936})$
		n	$n^{0.774}$	$\log_6 4$	
		n	$n^{0.695}$	$\log_2 6$	$1 - \frac{1}{d(d+1)}$
		n	$n^{0.667}$	(rand.)	n
		n	$\sqrt{n} \log n$		$(1 - \frac{1}{d}) + \varepsilon$
		n	$n^{\frac{1}{2} + \varepsilon}$		$n^{1-\frac{1}{d}} \log n$
		$n \log n$	$\sqrt{n} \log \frac{O(1)}{n}$		$n^{1-\frac{1}{d}}$
		$n^{1+\varepsilon}$	n		
		$n^{1+\varepsilon}$	\sqrt{n}		
		$n^{1+\varepsilon}$	\sqrt{n}		
		$n \log n$	$\sqrt{n} (\text{rand.})$		

⇒ Clarkson '87

Matoušek '92

$$m \quad \frac{n}{\sqrt{m}} \log^{\frac{O(1)}{n}} n$$

$(n \leq m \leq n^2)$

Method 1: Willard '82

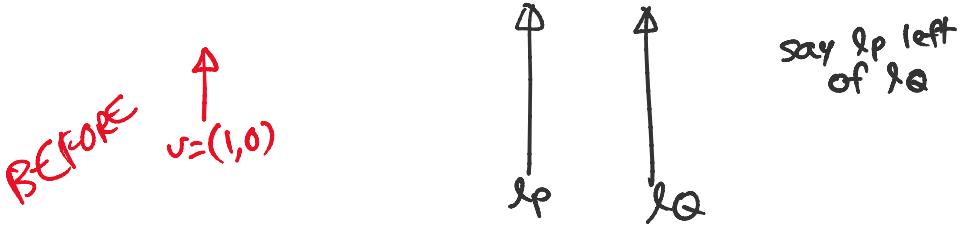
Ham-Sandwich Cut Theorem

Given any 2 point sets P, Q in \mathbb{R}^2 ,
 \exists line that simultaneously bisects P & Q .

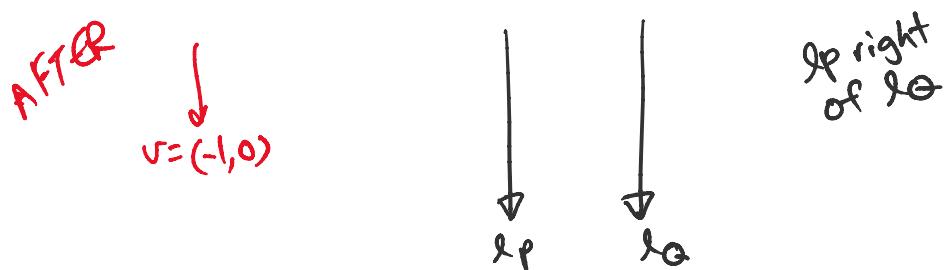
Ham Word

(in \mathbb{R}^d , d sets, \exists hyperplane ...)

Pf: Given direction v ,
let $l_P(v) =$ line bisecting P pointing in dir. v
 $l_Q(v) = \dots$



as v rotates. l_P & l_Q move continuously



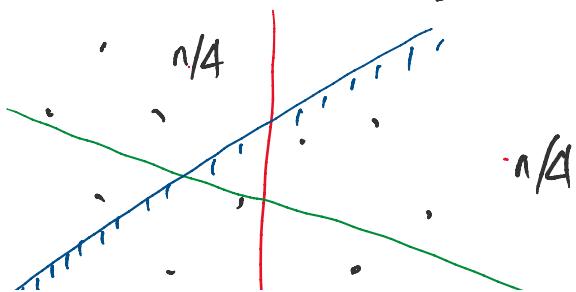
By intermediate value thm,

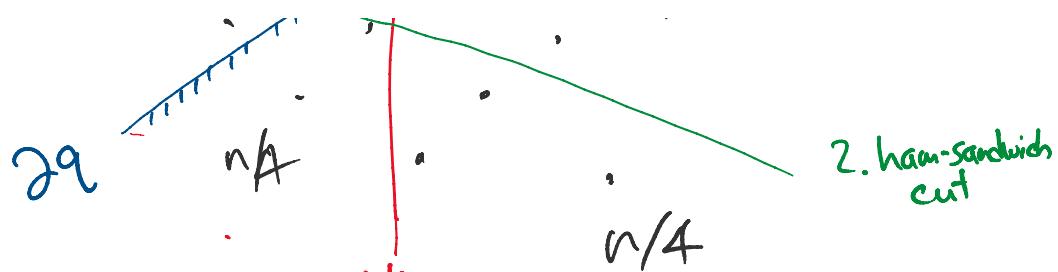
$\exists v$, s.t. $l_P(v) = l_Q(v)$. \square

(Megiddo '87: $O(n)$ time)

(in \mathbb{R}^d , need Borsuk-Ulam Thm)

{ Cor Given any set P of n pts in \mathbb{R}^2 ,
 \exists 2 lines which partition P into 4 subsets
of $\underline{n/4}$ pts





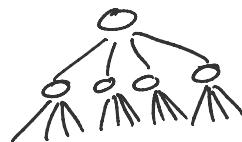
reurse \Rightarrow partition tree

Space $O(n)$

Preproc time

$$P(n) = 4 P\left(\frac{n}{4}\right) + O(n)$$

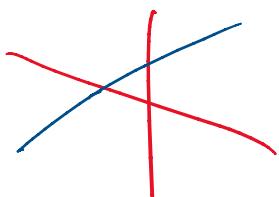
$$\Rightarrow O(n \log n)$$



$$\begin{aligned} &\text{to find} \\ &\text{ham-sand cut} \end{aligned}$$

query: given halfplane q ,
reurse in cells crossed by line $2q$.

\uparrow
 3 out of the 4



$$Q(n) = 3 Q\left(\frac{n}{4}\right) + O(1)$$

$$\Rightarrow O(n^{\log_4 3})$$

$$= O(n^{0.793})$$

given triangle q ,

$$Q(n) = O(\# \text{cells crossed by } 2q).$$

$$= O(3 \cdot \# \text{cells crossing by a line})$$

$$= O(n^{0.793})$$

Rmk - 8-sectioning in 3D
but no 16-sectioning in 4D!

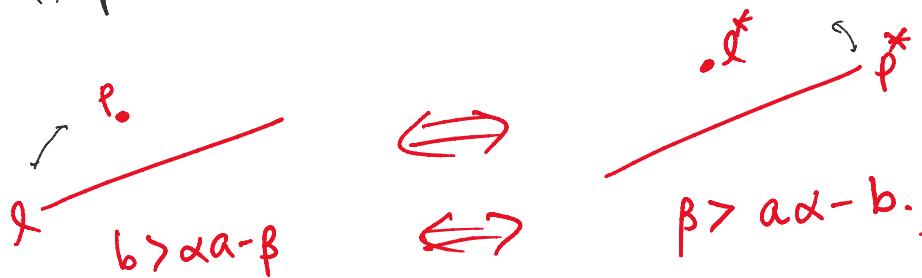
divide into > 4 parts in 2D?

Method 2:

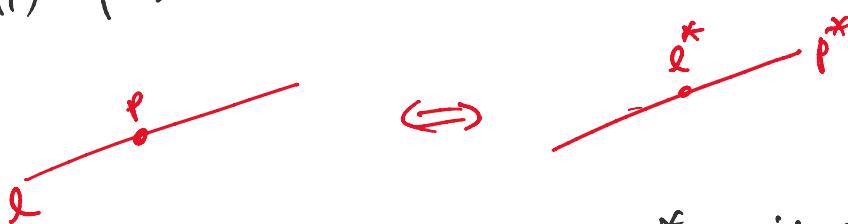
Def (Duality) Given pt $p = (\alpha, \beta)$ in 2D,
define its dual line p^* : $y = \alpha x - \beta$.

Given line ℓ : $y = \alpha x - \beta$,
define its dual pt $\ell^* = (\alpha, \beta)$.

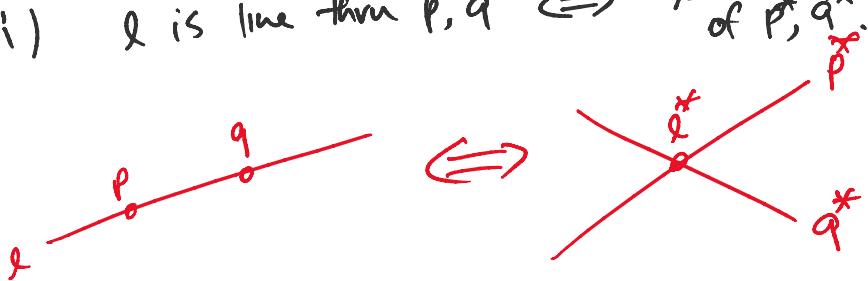
Fact (i) p above $\ell \Leftrightarrow \ell^*$ above p^*



(ii) p is on $\ell \Leftrightarrow \ell^*$ is on p^*



(iii) ℓ is line thru $p, q \Leftrightarrow \ell^*$ is intersection of p^*, q^* .



(iv) given n pts.
Count pts above query line \Leftrightarrow

{ given n lines,
Count # lines below
query pt

