CS 598TMC, Fall 2021
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Homework 4 (due Dec 8 Wednesday 5pm)

Instructions: as before. I will drop the lowest grade of your four homeworks. (This effectively makes HW4 optional, if you have done well on your first three HWs.)

1. [*33 pts*] Given a set P of n points in $\mathbb{R}^2$ and an integer k, we want to find the smallest set S^* of unit disks in $\mathbb{R}^2$ such that the number of points of P that are covered by S^* is at least k. Describe a PTAS (a polynomial-time $(1+\varepsilon)$-approximation algorithm) for the problem, by using the shifted grid technique and dynamic programming.

2. [*33 pts*] We are given a sequence of n points $p_1, \ldots, p_n$ in $\mathbb{R}^d$, and want to build a data structure so that for any two indices $i < j$, we can output a $(1+\varepsilon)$-factor approximation to the width of the subset $P_{i,j} = \{p_i, p_{i+1}, \ldots, p_j\}$. Describe a solution with near-linear space and polylogarithmic query time.

   Note: you may use results in class about ε-kernels (in particular, the properties related to "merge-and-reduce").

3. [*34 pts*] Let S be a set of n (axis-parallel) rectangles in 2D of maximum depth d (i.e., the maximum number of rectangles containing any point is d). We say that two rectangles s and t intersect *regularly* if their boundaries intersects two times or one is contained in the other (thus, s contains a vertex of t or t contains a vertex of s). We say that s and t intersect *irregularly* if their boundaries s and t intersect four times.

   (a) [*7 pts*] Prove that for any set of n rectangles with maximum depth d, then the number of regular intersecting pairs is at most $O(nd)$.

   (b) [*7 pts*] Prove that we can color the rectangles with $O(d)$ colors so that no two regularly intersecting rectangles have the same color.

   [Hint: use (a) to find a rectangle s that regularly intersects at most $O(d)$ other rectangles. Color $S - \{s\}$ first, then try to find a color for $s \ldots$]

   (c) [*10 pts*] Prove that we can color the rectangles with $O(d)$ colors so that no two irregularly intersecting rectangles have the same color.

   [Hint: define the *level* of a rectangle s to be the number of rectangles t that irregularly intersects s such that t has larger height than s (with respect to y-coordinates). For two rectangles s and t with the same level, how can s and t intersect?]

   (d) [*5 pts*] Now prove that we can color the rectangles with $O(d^2)$ colors so that no two intersecting rectangles have the same color.

   (e) [*5 pts*] Give an algorithm for maximum independent set with approximation factor $O(d^2)$ (which is thus constant in the special case that d is bounded by a constant).