CS 598TMC, Fall 2021
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Homework 3 (due Nov 10 Wednesday 5pm)

Instructions: see previous homework.

1. [*50 pts*] Given a set P of n points in $\mathbb{R}^d$ for a constant dimension d, a *k-hop c-spanner* is a subgraph H of the complete graph with vertex set P such that for every $p, q \in P$, there exists a path from p to q in H with at most $k+1$ edges and total length at most $c\|p-q\|$ (in the Euclidean metric). Describe an algorithm to construct a 1-hop c-spanner with $O(n \log n)$ edges for some constant c.

   [Hint: one approach is to use shifted, balanced quadtrees; another approach is to use shifted Z-ordering.]

   [Bonus: make c arbitrarily close to 1. Also, how many edges for a 2-hop spanner?]

2. [*50 pts*] In the *discrete 1-median* problem, we are given a set P of n points in 2D, and want to find a point $q^* \in P$ that minimizes the sum of Euclidean distances $\sum_{p \in P} \|p - q^*\|$. (Note that q^* must be in P.) There is a naive $O(n^2)$-time exact algorithm. Describe a more efficient approximation algorithm to solve the problem.

   [Note: $O(n \log n)$ run time and constant approximation factor would be fine, though $O(n)$ time and $1 + \varepsilon$ approximation would be even better (and worth bonus points).]

   [Hint: one approach is to change the metric...]