November 11, 2020 11:00 AM

Shaving Logs

Last Time:

BMM

LCS / edit dist.

integer 3SUM $\quad O\left(\frac{n^2}{\log^2 n} \log^2 \log n\right)$

3SUM for reals?
APSP for reals?

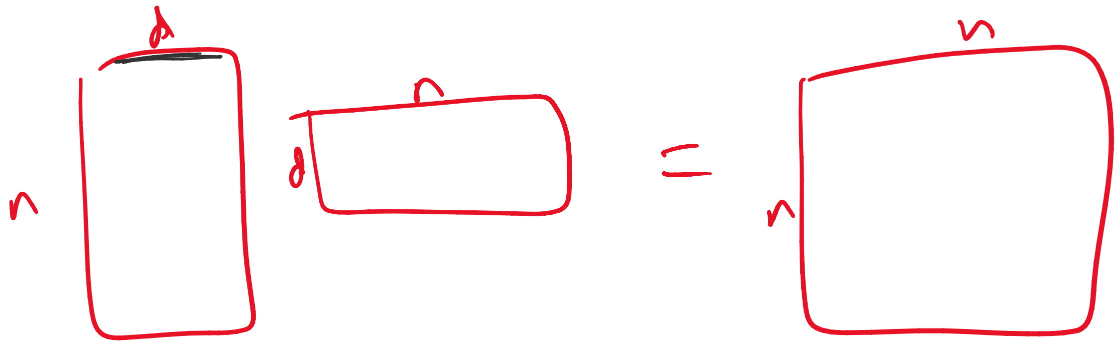
Decision tree complexity = # comparisons on real #s

APSP for Reals

Fredman '75: APSP can be solved in $O(n^{2.5})$ comps !!

subcubic

Lemma can compute $(\min,+)$-MM of $n \times d$ and $d \times n$ matrices with $\tilde{O}(d^2 n)$ comps. (instead of $O(dn^2)$)



Pf: idea - $\quad a_{ik} + b_{kj} \overset{?}{\leq} a_{ik'} + b_{k'j}$

$\iff \quad a_{ik} - a_{ik'} \overset{?}{\leq} b_{k'j} - b_{kj}$

$$\iff \quad a_{ik} - a_{ik'} \geq b_{k'j} - b_{kj}$$

("Fredman's trick")

Just sort $\{ a_{ik} - a_{ik'} : i \in [n],\ k,k' \in [d] \}$

$\cup \{ b_{k'j} - b_{kj} : j \in [n],\ k,k' \in [d] \}$

$\Rightarrow O((d^2 n)\log(d^2 n))$ time. $\leftarrow$

Afterwards, no more comps. $\square$

Cor Can compute $(\min,+)$-MM of 2 $n \times n$ matrices in $\tilde{O}(n^{5/2})$ comps.

Pf: reduce to $\frac{n}{d}$ products of $n \times d$ and $d \times n$

$$\Rightarrow \tilde{O}\left(\frac{n}{d} \cdot d^2 n + \underbrace{\frac{n}{d} \cdot n^2}_{\text{take min of } \frac{n}{d}\ n\times n \text{ matrices}} \right)$$

$$= \tilde{O}\left(dn^2 + \frac{n^3}{d} \right)$$

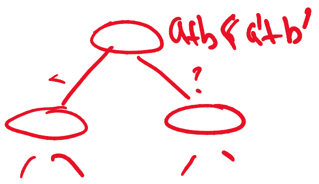
Set $d = \sqrt{n}$. $\square$

But runtime?

reduce to $O\left(\left(\frac{n}{s}\right)^3\right)$ $(\min,+)$-MM of $s \times s$ matrices

build decision tree

size/time $2^{\tilde{O}(s^{5/2})} \ll n$



time α

$$\Rightarrow \text{ total time } O\left(\left(\frac{n}{s}\right)^3 \cdot s^{5/2}\right) = O\left(\frac{n^3}{\sqrt{s}}\right)$$

$$\text{Set } s \simeq \delta\left(\frac{\log n}{\log\log n}\right)^{2/5} \qquad = \boxed{O\left(\frac{n^3 (\log\log n)^{1/5}}{\log^{1/5} n}\right)}$$

for (min,+)-MM & APSP.

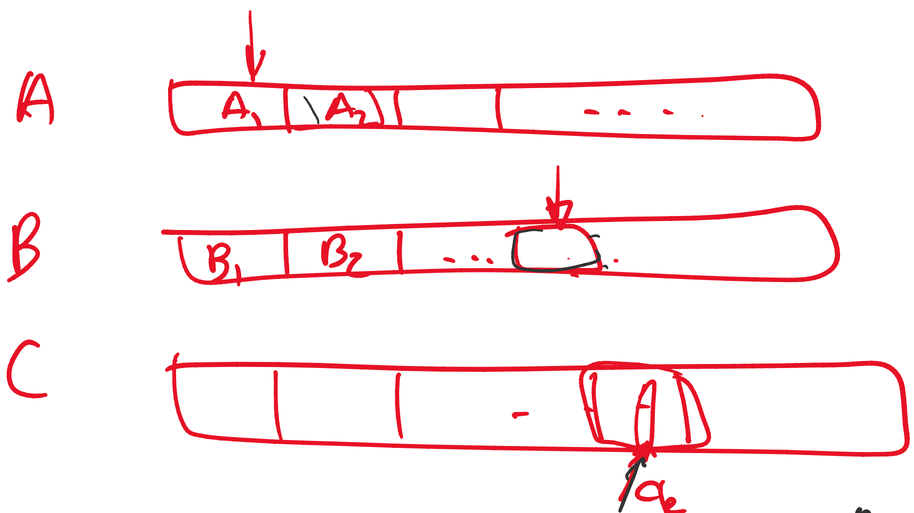
3SUM for Reals

Grönlund-Pettie '14: 3SUM can be solved in $\tilde{O}(n^{1.5})$ comps !!

subquadratic

~~$a+b<c$~~
$a+b < a'+b'$

Warm-up with Convol 3SUM and (min,+)-Convol:
[BCDEHILPT '06]



divide A into blocks $A_1, \ldots, A_{n/d}$ of size d
B " " $B_1, \ldots, B_{n/d}$
C " " $C_1, \ldots, C_{n/d}$

idea - Fredman trick

idea - Fredman trick

$$a_{i-k} + b_k \overset{?}{\lesssim} a_{i-k'} + b_{k'}$$

$$\iff a_{i-k} - a_{i-k'} \overset{?}{\lesssim} b_{k'} - b_k .$$

Preproc:

Sort $\{ a_i - a_j : i,j \text{ are in same block}\}$

$\cup \; \{ b_i - b_j : i,j \text{ in same block}\}$

in $\tilde{O}\left(\frac{n}{d} \cdot d^2\right) = \tilde{O}(dn)$ time.

For each c_i:

need to search $O\left(\frac{n}{d}\right)$ pairs of blocks A_s, B_t.

& for each such A_s, B_t,

Cost is 0. (no new comps)

for $(\min,+)$-Convol

cost is $O(\log n)$ for Convol 3SUM

by binary search.

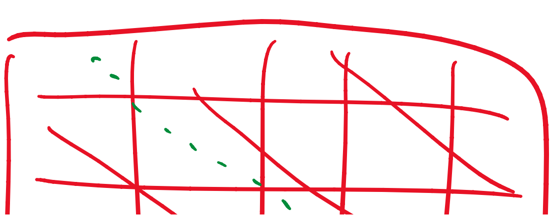
$\Rightarrow$ Cost per c_i is $\tilde{O}\left(\frac{n}{d}\right)$.

$\Rightarrow$ total cost is $\tilde{O}\left(n \cdot \frac{n}{d}\right) = \tilde{O}\left(\frac{n^2}{d}\right)$

Total: $\tilde{O}\left(dn + \frac{n^2}{d}\right)$

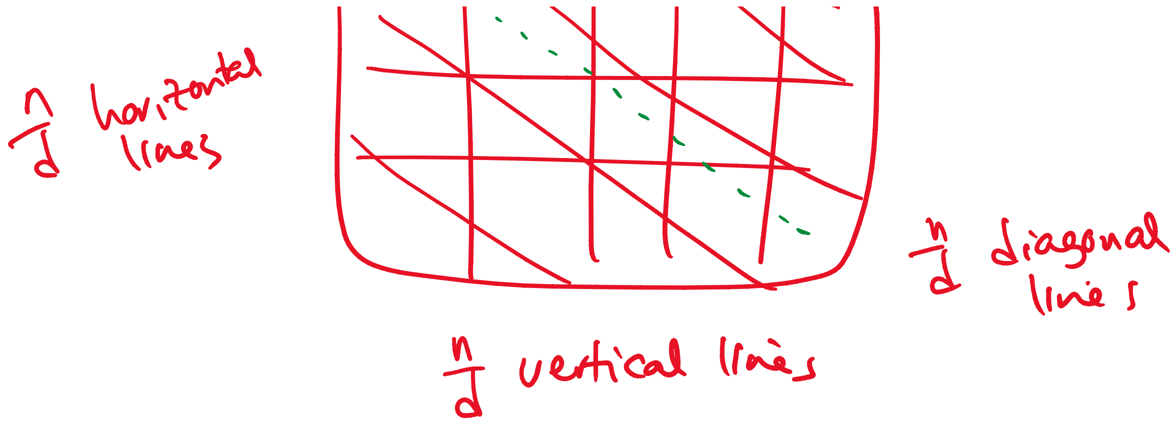
Set $d = \sqrt{n}$ $\Rightarrow$ $\boxed{\tilde{O}(n^{3/2})}$.

3SUM: Similar



$\Rightarrow O\left(\left(\frac{n}{d}\right)^2\right)$ subprobs of size d.

of size d.



But runtime?

reduce to $O((\frac{n}{s})^2)$ instances of sequences of size s

build decision tree size/time $2^{\tilde{O}(s^{3/2})} \ll n$

$\Rightarrow$ total time $O\left(\left(\frac{n}{s}\right)^2 \cdot s^{3/2}\right) = O\left(\frac{n^2}{\sqrt{s}}\right)$

Set $s = \delta \frac{\log^{2/3} n}{\log^{2/3} \log n}$ $= \boxed{O\left(\frac{n^2 (\log\log n)^{1/3}}{\log^{1/3} n}\right)}$

Rmk: Kane, Lovett, Moran '18:

3SUM in $O(n \log^2 n)$ comps!!

of the form $a+b+c \overset{?}{\leq} a'+b'+c'$

4SUM . . .

kSUM

& APSP in $\tilde{O}(n^2)$ comps.

Slightly Faster APSP

Fredman '76 $O((n^3/\log^{1/3} n) \log\log n)$

Fredman '76	$O((n^3/\log^{1/3} n)\ \log\log n)$
'90	$O(n^3/\log^{1/2} n)$
'04	$O(n^3/\log^{5/7} n)$
'04	$O((n^3/\log n)\log^2\log n)$
Zwick '04	$O((n^3/\log n)\log\log n)$
C '05	$O(n^3/\underline{\log n})$
Han '06	$O((n^3/\log^{5/4} n)\ \log^{5/4}\log n)$
C '07	$O((n^3/\log^2 n)\ \log^3\log n)$
Williams '14	$O(n^3/2^{\Omega(\sqrt{\log n})})$.

use geometry in $\underline{\log n}$ dims