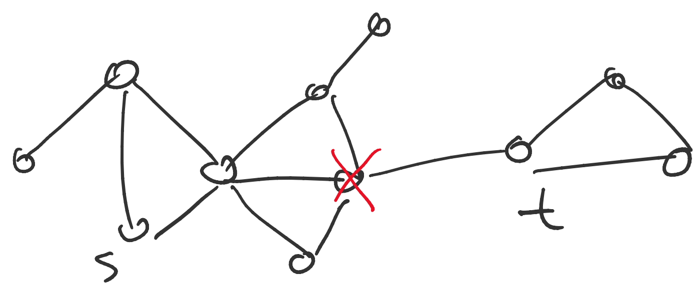
November 4, 2020 11:00 AM

Last Time:

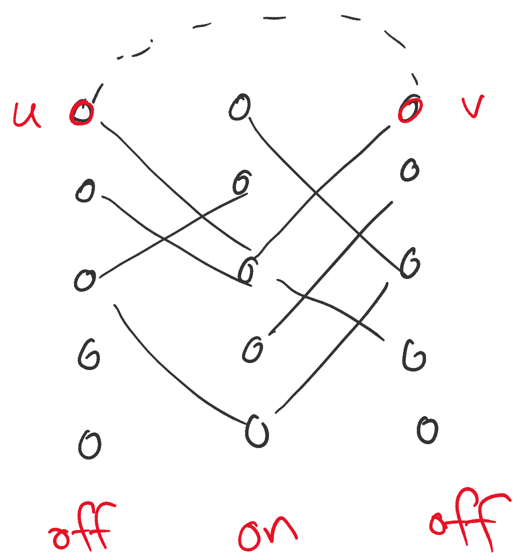
triangle listing $\Omega(\underline{m^{4/3-\delta}})$. δ

appl'ns to dynamic graph DSs

e.g. dynamic graph connectivity with vertex deletes/ re-inserts



C.-Patrasca-Roditty '11: $\underline{\tilde{O}(m^{2/3})}$.



$\Omega(m^{1/3-\delta})$.

Jumbled Text Indexing

build data structure for a text string $t = t_1 \cdots t_n \in \Sigma^*$,
$\Sigma = [\sigma]$ σ = const

s.t. given any query pattern $p = p_1 \cdots p_\ell$,
can determine if $\exists i$ s.t.
$p_1 \cdots p_\ell$ matches $t_{i+1} \cdots t_{i+\ell}$.
up to permutation!

e.g. t = "algorithmisfun" $t = 011010110$

e.g. t = algorithmistun $t = 011010\ldots$

p = "him" p = 2 0's, 3 1's

offline version: queries are given in advance

offline version reduces to a special case of 3SUM:

for each $\alpha \in \Sigma$,

let $a_i(\alpha)$ = frequency of α in $t_1 \cdots t_i$

$c_k(\alpha)$ = frequency of α in k^{th} query

for each k, want $\exists\, i, j$ with

$$a_j - a_i = c_k.$$

like 3SUM

(u, v)

$uM + v$

3SUM for σ-dimensional vectors reduces to standard 3SUM . . .

C.-Lewenstein '16:

$\sigma = 2$: $O(n^{1.86})$ time for $O(n)$ queries

$\sigma \geq 3$: $\tilde{O}\left(n^{2 - \frac{2}{\sigma + 13}}\right)$ time

Thm (Amir, C., Lewenstein, Lewenstein '14)

Assuming 3SUM conj,

no alg'm for $O(n)$ queries for jumbled indexing in $O\left(n^{2 - \frac{4}{\sigma} - \delta}\right)$ time.

Pf:

$\exists\, i, j$ with $a_j - a_i = c_{j-i}$

Reduction: Convol3SUM $\rightarrow$ Convol3SUM in $[U]$

$U = \tilde{O}(n^2)$

Reduction: Convol3SUM → Convol3SUM in [U] with $U = \tilde{O}(n^2)$

by hashing

($h(x) = x \bmod p$ for rand. prime $p \in [R]$

err prob $\le \tilde{O}\left(n^2 \cdot \frac{1}{R}\right)$)

Reduction: Convol3SUM in [U] → d-dim Convol3SUM in $[U^{1/d}]^d$

map number to d-dim vector by just writing each number in base $B = U^{1/d}$ with d "digits"

$$a_i = a_i[d-1] \ldots a_i[0]$$

$$c_k = c_k[d-1] \ldots c_k[0]$$

e.g. $B=10$

$$\begin{array}{r} 738 \\ -\ 214 \\ \hline 524 \end{array} \qquad \begin{array}{r} \overset{-1\ 0}{738} \\ -\ 256 \\ \hline 482 \\ \downarrow\ \downarrow\ \ \\ 5,-2,2 \end{array}$$

technicality - carry

guess carry bits $\Delta_0, \Delta_1, \ldots, \Delta_{d-1} \in \{0,1\}$

let $c_k'[\ell] = \begin{cases} c_k[\ell] + \Delta_{\ell-1} & \text{if } \Delta_\ell = 0 \\ c_k[\ell] - B + \Delta_{\ell-1} & \text{if } \Delta_\ell = 1 \end{cases}$

$\Rightarrow$ 2^d instances

Reduction: d-dim Convol3SUM in $[U^{1/d}]^d$

Reduction: d-dim Convol3SUM in $[U^2]$ $\rightarrow$ Jumbled Indexing

Suppose Jumbled Indexing could be solved in
$T(n) = \underline{O(n^{2-4/\sigma-\delta})}$ time.

Given d-dim vectors $a_1, \ldots, a_n, c_1, \ldots, c_n$,

let $\Sigma = [d] \cup \{\$, \%\}$. $\sigma = d+2$

define text string

$t = \%\$\% f_1 \%\$\% f_2 \%\$\% \ldots\ldots \%\$\% f_n \%\$\%$

where $\boxed{f_i = 1^{a_i[1]-a_{i-1}[1]+B}\ 2^{a_i[2]-a_{i-1}[2]+B} \ldots\ d^{a_i[d]-a_{i-1}[d]+B}}$

$B = U^{1/d}$

for each c_k,

find pattern with $c_k[\alpha]+Bk$ occurrences of α for $\alpha \in [d]$

$k+1$ \$'s

$2k$ %'s.

Claim for each k, pattern exists $\iff \exists i, j$, with $a_j - a_i = c_k$ and $j - i = k$.

Pf: ($\Leftarrow$) take substring

$\$\underbrace{\% f_{i+1} \%\$\% \ldots \%\$\% f_j \%}\$$

$\#$ occurrences of α $= a_{i+1}[\alpha] - a_i[\alpha] + a_{i+2}[\alpha] - a_{i+1}[\alpha] + \ldots + a_j[\alpha] - a_{j-1}[\alpha]$

Telescoping sum! $= a_j[\alpha] - a_i[\alpha] = c_k[\alpha]$

$\#$ \$'s $= j - i - 1 + 2 = k+1$

$'s $= j-i-1+2 = k+1$
%'s $= 2(j-i-1)+2 = 2k$.

($\Rightarrow$) pattern must be of the form

% \$ %f_{i+1}% . . . . %f_j% \$

Since # \$'s $= \underline{k+1}$, # %'s $= 2k$.

for some i,j with $j-i = k$.

So, $c_h[\alpha] = a_j[\alpha] - a_i[\alpha] \quad \forall \alpha \in [d]$. $\square$

$$\text{text length} = O(n\, U^{1/d}) = \tilde{O}(n^{1+2/d})$$

$$= \tilde{O}(n^{\frac{d+2}{d}})$$

$(d = \sigma - 2)$

$$= \tilde{O}(n^{\frac{\sigma}{\sigma-2}}).$$

$$\Rightarrow \text{time } \tilde{O}\left(\left(n^{\frac{\sigma}{\sigma-2}}\right)^{2 - \frac{4}{\sigma} - \delta} \right)$$

$$= \tilde{O}\left(\left(n^{\frac{\sigma}{\sigma-2}}\right)^{\frac{2\sigma-4}{\sigma} - \delta} \right)$$

$$= \tilde{O}(n^{2-\delta'}). \quad \square$$

Final Rmks on Conditional LBs

- other conjectures:

  kSUM for larger k (no $n^{\lceil k/2 \rceil - \delta}$ alg'm)

  (no $n^{k-\delta}$ alg'm)

min-wt k-clique (no n^{k-o} alg'm)

⋮

- OMv: online matrix-vector mult.
- ⋮