October 23, 2020 10:39 AM

Problems Related to LCS & Edit Distance

1. LCS:

Given strings $A = a_1 \cdots a_n$
$B = b_1 \cdots b_n$,

find longest common subseq.

equiv: min # inserts/deletes to transform A to B

e.g. l~~o~~g~~a~~rithm   1210312020
algorithm   1231212310

DP Sol'n: let $C[i,j]$ = min # changes for $a_1 \cdots a_i$ to $b_1 \cdots b_j$.

$$\Rightarrow C[i,j] = \min\{C[i-1,j]+1,\ C[i,j-1]+1,\ C[i-1,j-1]+d(a_i,b_j)\}$$

where $d(a,b) = \begin{cases} 0 & \text{if } a=b \\ \infty & \text{if } a \neq b \end{cases}$

$\Rightarrow$ $\boxed{O(n^2)}$ time

2. Edit Distance:

min # inserts/deletes/substitutions

e.g. l~~o~~garithm
~~a~~lgorithm
edit dist 3

12~~10~~312020
1231212~~31~~0
edit dist 4

Same DP, but $d(a,b) = \begin{cases} 0 & \text{if } a=b \\ 1 & \text{if } a \neq b. \end{cases}$

3. Dynamic Time Warping Distance (DTW)

find expansions $\tilde{A}$ and $\tilde{B}$ of A and B
to minimize $\sum_i d(\tilde{a}_i, \tilde{b}_i)$

to minimize $\sum_i d(\tilde{a}_i, \tilde{b}_i)$

allow to duplicate chars

$\tilde{A}$ 1 2 1 0 3 1 2 0 2 2 0

$\tilde{B}$ 1 2 2 2 3 1 2 1 2 3 0

DP Sol'n:

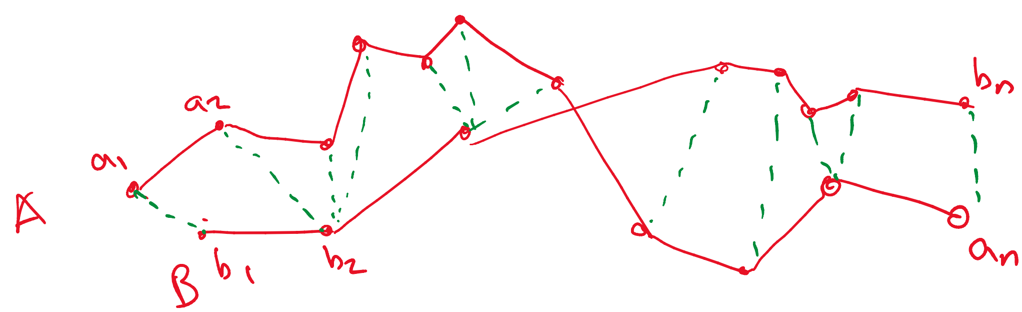
$$C[i,j] = \min\{C[i-1,j],\ C[i,j-1],\ C[i-1,j-1]\} + d(a_i, b_j)$$

4. Discrete Fréchet Distance:

find expansions $\tilde{A}$ and $\tilde{B}$

to minimize $\max_i d(\tilde{a}_i, \tilde{b}_i)$

geom. interpretation:
"walking a dog" with shortest leash



DP: $C[i,j] = \max\{\min(C[i-1,j], C[i,j-1], C[i-1,j-1]),\ d(a_i, b_j)\}$

$\Rightarrow O(n^2)$ time

Thm Assuming SETH,
no $O(n^{2-\delta})$ alg'm for all these problems

History: by Bringmann '14 for Frechet dist. (discrete & continuous)
Backurs & Indyk '15 for edit dist.
Abboud, Backurs, Vassilevska '15
Bringmann & Künnemann '15 } for LCS

Reduction: OV → Discrete Frechet (Bringmann '14)

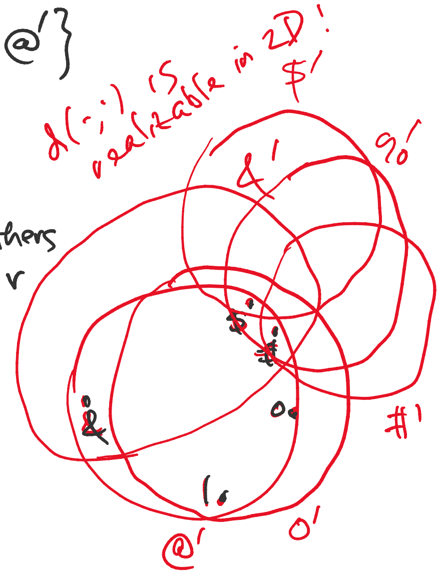
Suppose discrete Frechet could be solved in $O(n^{2-\delta})$ time.

To solve OV for vectors $a_1, \dots, a_n, b_1, \dots, b_n \in \{0,1\}^d$,

define $\Sigma = \{0, 1, \&, \$, \#, 0', 1', \&', \$', \#', \%', @'\}$

$d(\cdot,\cdot)$	$0'$	$1'$	$\&'$	$\%'$	$\$'$	$\#'$	$@'$
0	$<r$	$<r$					$<r$
1	$<r$						$<r$
$\&$			$<r$				$<r$
$\$$	$<r$	$<r$	$<r$	$<r$	$<r$		$<r$
$\#$	$<r$	$<r$	$<r$	$<r$		$<r$	$<r$

all others $> r$



Define strings

$A = \$ f(a_1) \# \$ f(a_2) \# \cdots \$ f(a_n) \#$

$B = @' \$' g(b_1) \%' g(b_2) \%' \cdots \%' g(b_n) \#' @'$

where $f(a) = a[1] \& a[2] \& \cdots \& a[d]$

$g(b) = b[1]' \&' b[2]' \&' \cdots \&' b[d]'$

Claim: discrete Frechet dist $< r \iff \exists\, a_i, b_j$ s.t. $a_i \cdot b_j = 0$.

Pf: ($\Leftarrow$) Suppose $a_i \cdot b_j = 0$.

$A = \cdots \$ f(a_i) \# \cdots$

$B = @' \$' \cdots \% g(b_j) \% \cdots \#' @'$

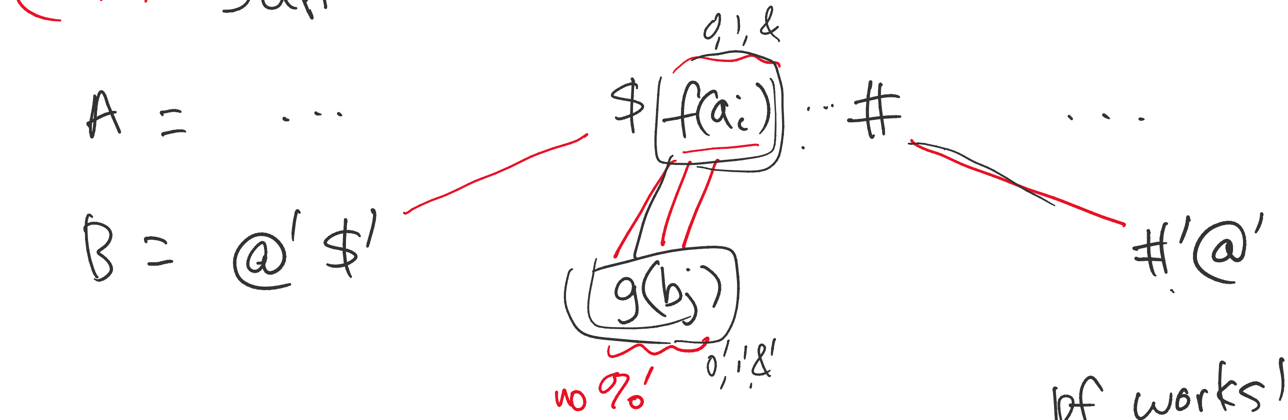
$f(a_i) = a_i[1] \& a_i[2] \& \cdots \& a_i[d]$

$g(b_j) = b_j[1]' \&' b_j[2]' \&' \cdots \&' b_j[d]'$

$$g(b_j) = b_j[1]' \,\&'\, b_j[2]' \,\&'\, \cdots \,\&'\, b_j[d]$$

So Frechet dist $< r$.

$(\Rightarrow)$ Suppose frechet dist $< r$.



pf works!

$\exists a_i, b_j, \quad a_i \cdot b_j = 0.$ $\square$

Rmks:

- Similar pf holds for c-approx. for some const c.
- DTW / edit dist / LCS similar but replace separators with complicated gadgets
- still hard for $|\Sigma| =$ const
- hardness in other params
- recent work on approx alg'ms for edit dist.

  ⋮

- Abboud et al. '16: lower bd holds for much ~~weaker~~ weaker version of SETH!

  SAT w/ o(n) depth)

nower we ...

of SETH!

(for circuit-SAT w. $o(n)$ depth)

or stronger LBs $n^2/\log^{100} n$ ...