$$T(2, \underline{cn})$$
$$= O\left((cn)^{O(1)} \cdot \underline{(2^{n/2})^{2-\delta}}\right)$$
$$= O^*\left(2^{(1-\delta/2)n}\right). \quad \square$$

Rmk: generalizes to the k-OV problem:

Given $A_1, \ldots, A_k \subseteq \{0,1\}^d$,

decide if $\exists\, a_1 \in A_1, \ldots, a_k \in A_k$ s.t.

$$\sum_{i=1}^{d} a_1[i] \cdots a_k[i] = 0.$$

<u>k-OV Conj:</u> no $O(d^{O(1)} n^{k-\delta})$ alg'm for k-OV.

<u>OV → Diameter for Sparse Unweighted Graphs</u>

Diameter: $\underline{\max_{s,t \in V} d(s,t)}$

Sparse — naive: $O(mn)$ time $\le O(m^2)$

<u>Thm</u> (Roditty - Vassilevska W. '11)

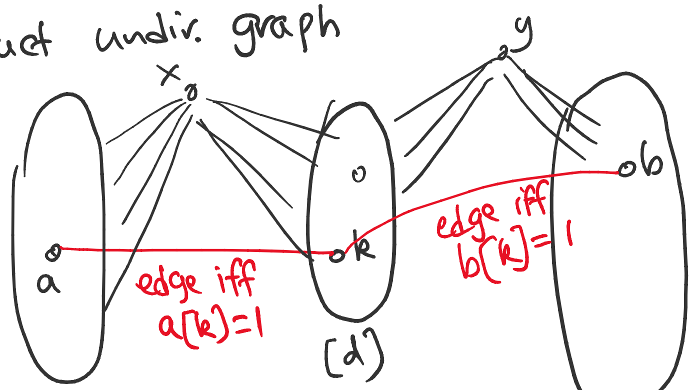
Assuming SETH, no $O(m^{2-\delta})$ alg'm to compute diameter of unweighted undir graphs

Pf: Suppose there is diam alg'm in $T(m) = O(m^{2-\delta})$ time.

Given sets A, B of n vectors in $\{0,1\}^d$,

construct undir. graph ... g

Construct undir. graph



S = A   B = T

Correctness: $d(a,b) = 2$ iff $\exists k,\ a[k] = b[k] = 1$

iff $a \cdot b > 0$

i.e. a, b not orthogonal

else $d(a,b) \geq 3$

diam $= 2$ if no orthogonal pairs of vectors

≥ 3 else

Runtime: # vertices $O(n+d)$

edges $O(dn)$

OV in time $O((dn)^{2-\delta})$. $\square$

Consequence Assuming SETH,

no $O(m^{2-\delta})$ alg'm for $(\frac{3}{2}-\varepsilon)$-factor approx of diam

& for $(2-\varepsilon)$-factor approx of "S-T diameter"

$\max_{s \in S, t \in T} d(s,t)$

Rmk: Chechik et al. '14 gave $\tilde{O}(m^{1.5})$ alg'm
of diam

Rmk: Chechik et al. '14 gave $\tilde{O}(m^{1.5})$ alg'm for $(3/2)$-approx of diam.

Backurs et al. '18 for 2-approx of S-T diam

Thm (Backurs et al. '18)

Assuming SETH, no $O(m^{1.5-\delta})$ alg'm ← $m^{1+\frac{1}{k}-\delta}$

for $(8/5-\varepsilon)$-approx diam ← $\frac{5k-2}{3k-1}-\varepsilon$

or $(7/3-\varepsilon)$-approx S-T diam. ← $3-\frac{2}{k+1}-\varepsilon$

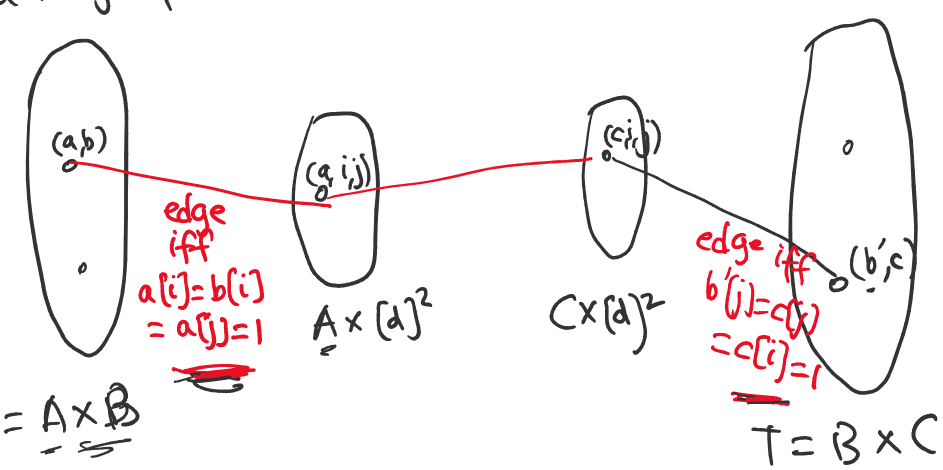
Pf for S-T diam:

Reduce 3-OV to S-T diam:

Suppose S-T diam could be solved in $O(m^{1.5-\delta})$ time.

Given sets A, B, C of n vectors in $\{0,1\}^d$.

construct graph



Correctness: if no orthogonal triple,

$\forall (a,b) \in S,\ (b',c) \in T,$

$$d((a,b),(b',c)) = 3$$

because $\exists i,\ a[i]=b[i]=c[i]=1$

$\exists j,\ a[j]=b'[j]=c[j]=1.$

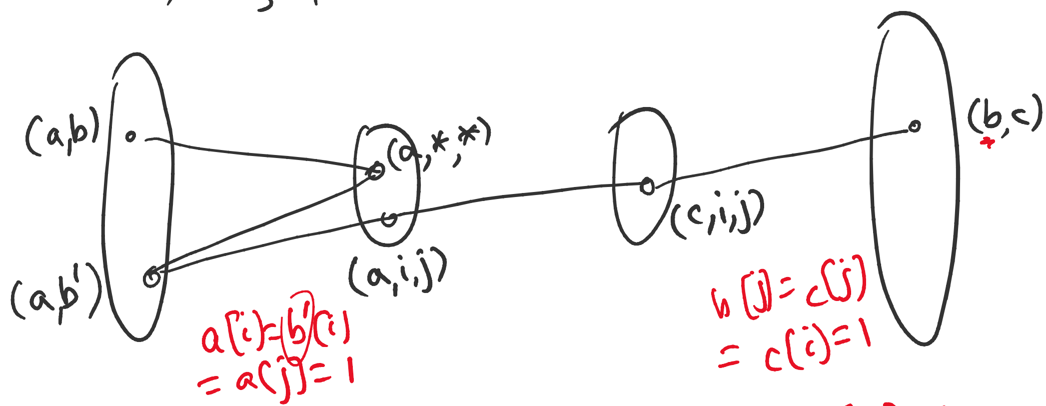
$\Rightarrow$ S-T diam $= 3$

$\Rightarrow$ S-T diam = 3

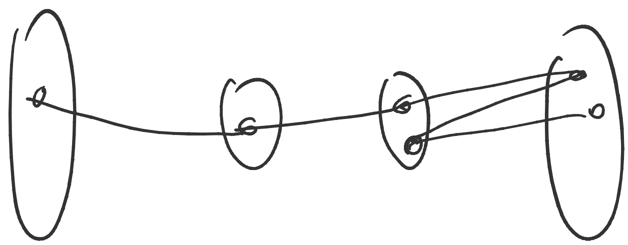
Conversely, if there is orth triple (a,b,c),

$$d\big((a,b),(b,c)\big) \geq 7$$

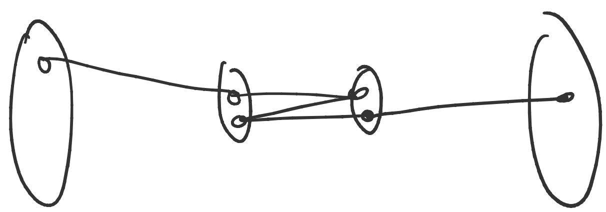
$\Rightarrow$ S-T diam ≥ 7.



$\Rightarrow$ $a[j] = b[j] = c[j] = 1$.
contra!



no paths of length 5



Runtime: # vertices $O(n^2 + d^2 n)$
edges $O(d^2 n^2)$

$\Rightarrow$ 3-OV in time $O\big((d^2n^2)^{1.5-\delta}\big)$

$= O\big(d^{O(1)} n^{3-2\delta}\big)$. $\square$

Problems Related to LCS & Edit Distance

1. LCS: Given strings $A = a_1 \dots a_n$

1. LCS: Given strings $A = a_1 \cdots a_n$
$B = b_1 \cdots b_n$,

find longest common subseq.

equiv: min # inserts/deletes to transform A to B

e.g. ~~l~~ ~~o~~ g ~~a~~ rithm

a l g o rithm

1 2 1 0 3 1 2 0 2 0

1 2 3 1 2 1 2 3 1 0

DP Sol'n: let $C[i,j]$ = min # changes for $a_1 \cdots a_i$ to $b_1 \cdots b_j$.

$$\Rightarrow C[i,j] = \min \begin{cases} C[i-1,j]+1, \\ C[i,j-1]+1, \\ C[i-1,j-1] + d(a_i,b_j) \end{cases}$$

where $d(a,b) = \begin{cases} 0 & \text{if } a=b \\ \infty & \text{if } a \neq b \end{cases}$

$\Rightarrow \boxed{O(n^2)}$ time

2. Edit Distance: min # inserts/deletes/substitutions

e.g. l~~o~~garithm

~~a~~lgorithm

edit dist 3

Same DP, but $d(a,b) = \begin{cases} 0 & \text{if } a=b \\ 1 & \text{if } a \neq b. \end{cases}$

HW3 posted, project guidelines