August 26, 2020 12:09 AM

CS 598 TMC Algorithms from the Fine-Grained Perspective

http://courses.engr.illinois.edu/cs598tmc

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off hr on zoom: Thu 3p-4p
or by appointment

online, synchronous, lectures recorded (mediaspace)
scribbles on website

Course Work

4 HWs	40%	(may work in groups of ≤ 3)
Presentation	20%	
Project	40%	

Prerequisite: undergrad alg'ms (CS374)
(CS 473 not required but helpful ...)

Theme - understand the fine-grained complexity of basic algorithmic problems

but go beyond polynomial vs. NP-hard

Ex1 all-pairs shortest paths (APSP)
for dense weighted graph with n vertices

Floyd-Warshall (DP) $O(n^3)$ time
Dijkstra n times $O(n^3)$ time

better?

n^3 ... (n^3) ... $O(n^3)$

better?

$$\sim O\left(\frac{n^3}{\log^{1/3} n}\right) \quad \cdots \quad O\left(\frac{n^3}{\log^2 n}\right) \quad O\left(\frac{n^3}{c^{\sqrt{\log n}}}\right)$$

Fredman '75 ... C'07 ... Williams '14

$$c^{\sqrt{\log n}} > \log^{1{,}000000} n$$

$$c^{\sqrt{\log n}} < n^{0.0001}$$

Conj: no truly subcubic alg'm e.g. $O(n^{2.9999})$?

(many related graph problems: diameter, radius, shortest cycles, ...)

Ex2 longest common subseq (LCS) of 2 strings $a_1 \dots a_n$, $b_1 \dots b_n$

DP $\Rightarrow$ $O(n^2)$ time

$$L[i,j] = \max \begin{cases} L(i-1, j) \\ L[i, j-1] \\ L[i-1, j-1] + 1 \text{ if } a_i = b_j \end{cases}$$

better? current record $\sim O\left(\frac{n^2}{\log^2 n}\right)$

(many similar problems: edit dist, Frechet dist, ...)

Ex3 3SUM

Given n numbers S & target t.

Ex's 3SUM

Given n numbers S & target t,
$\exists\, a,b,c \in S$ s.t. $a+b+c=t$?

trivial: $O(n^3)$
std HW prob: $O(n^2)$
(one way: sort all $a+b$ ...)
(another way: n instances of 2SUM)
↑ $O(n)$ time after sorting

better?

Grønlund-Pettie '14 $\sim O\left(\frac{n^2}{\log^{1/2} n}\right)$

C '18 $\sim O\left(\frac{n^2}{\log^2 n}\right)$

Conj: no truly subquad alg'm
(many related problems from geometry ...)

kSUM: trivial $O(n^k)$

4SUM: $(a+b)+(c+d)=t$

"meet-in-middle" $O(n^{k/2}\log n)$ if k even
$O(n^{\lceil k/2 \rceil})$ if k odd

better?

Subset-Sum:

$\exists$ subset of S summing to t?

trivial: $O(2^n)$ time
"meet-in-middle" $\sim O(2^{n/2})$ time
($2^{n/4}$ space)

"meet-in-middle" ~ $O(2^{n/2})$ time ($\sim 2^{n/4}$ space)

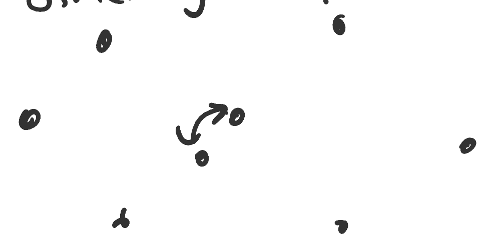
better?

for positive integers:

DP $O(nt)$ time (can be improved to $\tilde{O}(t)$)

better?

Ex4 closest pair in moderate dims $d \sim \log n$ & other geom problems



trivial: $\sim O(dn^2)$

(related prob: "orthogonal vectors" (OV))

Proving lower bds in general models is very difficult

Idea - prove conditional lower bd via reductions (under conjectures that certain basic probs are hard)

Surprisingly recent development

Similar to NP-completeness pf but fine-grained

e.g. Abboud-Grandoni-Vassilevski W.'14:

if we could compute graph radius in $O(n^{2.99})$ time,

... $O(n^{2.999})$ time

if we could compute … in $O(n^{2.99})$ time,

we could solve APSP in $O(n^{2.999})$ time

Bringmann '14:

if we could compute Frechet dist (or edit dist, LCS) in $O(n^{1.99})$ time,

we could solve CNF-SAT in $O(1.999999^n)$ time

↳ which refutes Strong Exponential Time Hypothesis (SETH)

⋮

Course Outline

I. Basic Alg'mic Tools:
   Convolution / FFT, matrix mult,

II. Conditional LBs
   (reductions ... APSP / 3SUM / SETH ...)

III. Advanced Alg'm Techniques
   - log shaving
   - polynomial method
   - additive combinatorics

   ⋮