

Further Collapses in TFNP

1. TFNP classes

$$CLS = PPAD \cap PLS$$

2. New collapses

3. Proof sketches

TFNP: search problems with:

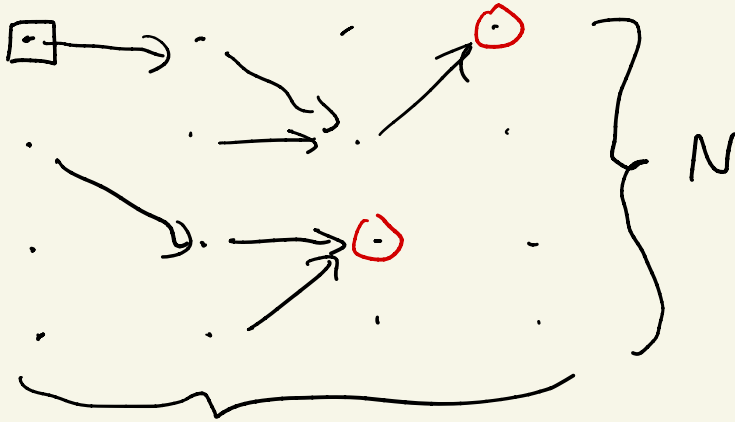
1. \exists solution (totality)
2. check solutions efficiently.

PLS

Intuition: Every DAG has a sink.

sink-of-DAG (SoD):

$$N \times N \rightarrow \boxed{C} \rightarrow N \times N$$

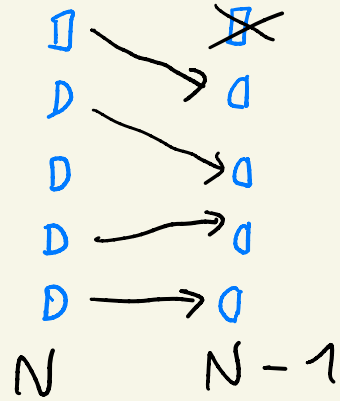
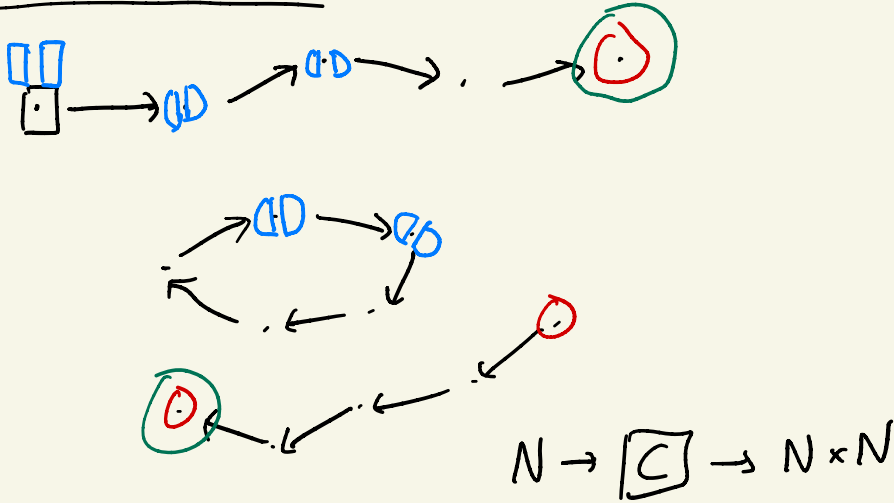


$$\underline{N = 2^u}$$

PPAD

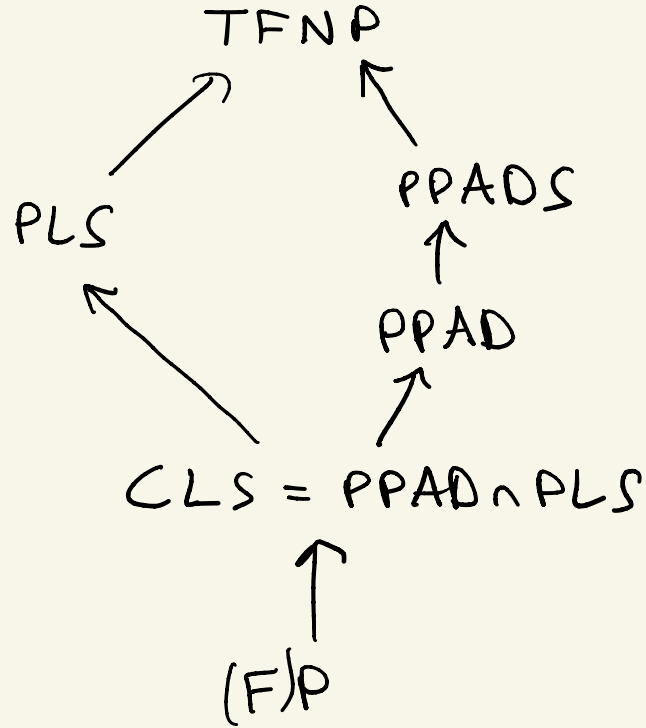
Intuition: Every directed graph with an unbalanced node must have another one.

End-of-Line (EoL):



Bijjective - PHP

PPADS: Sink-of-Line (Sol)



PPAD \cap PLS

Complete problem: EOL \wedge SOD

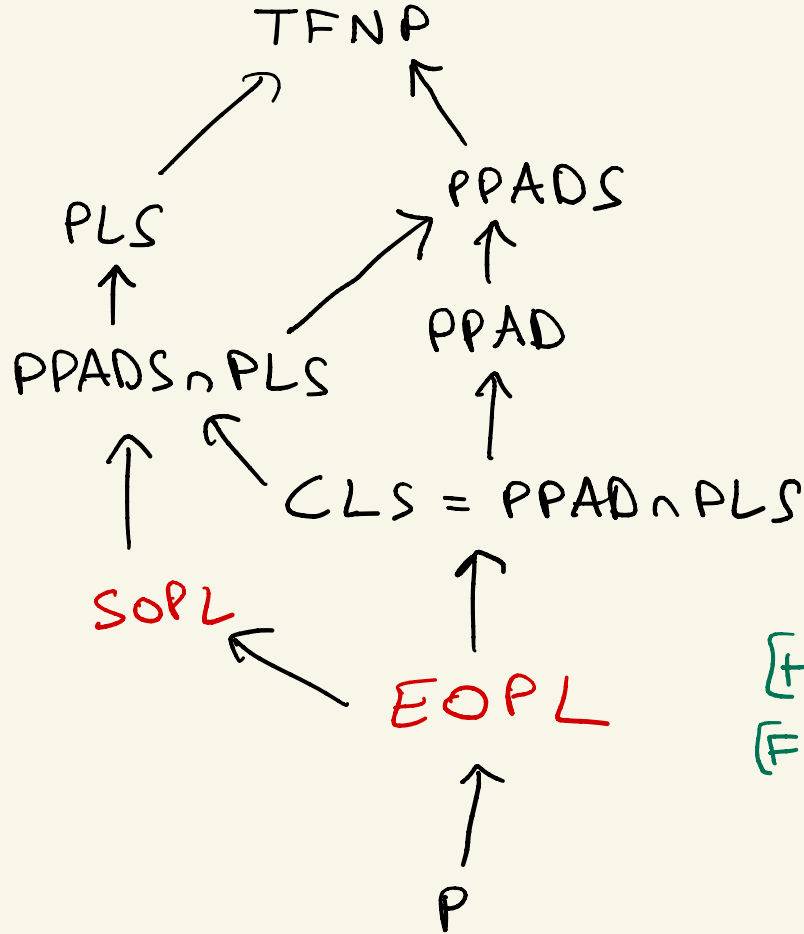
Given: instance I_{EOL} of EOL

and instance I_{SOD} of SOD

Output: solution of I_{EOL} or of I_{SOD} .

Exercise: Show this works for any TFNP
classes A and B.

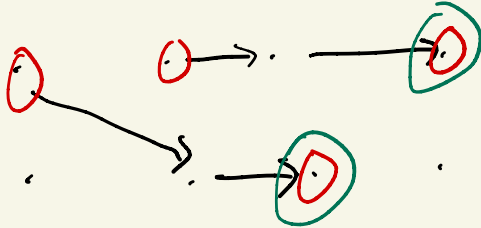
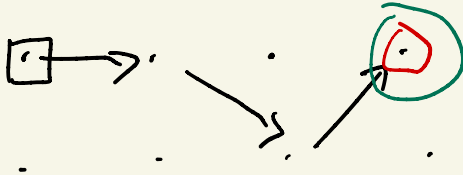
P_A/P_B complete for A/B \Rightarrow $P_A \wedge P_B$ complete A \cap B



[GKRS18]

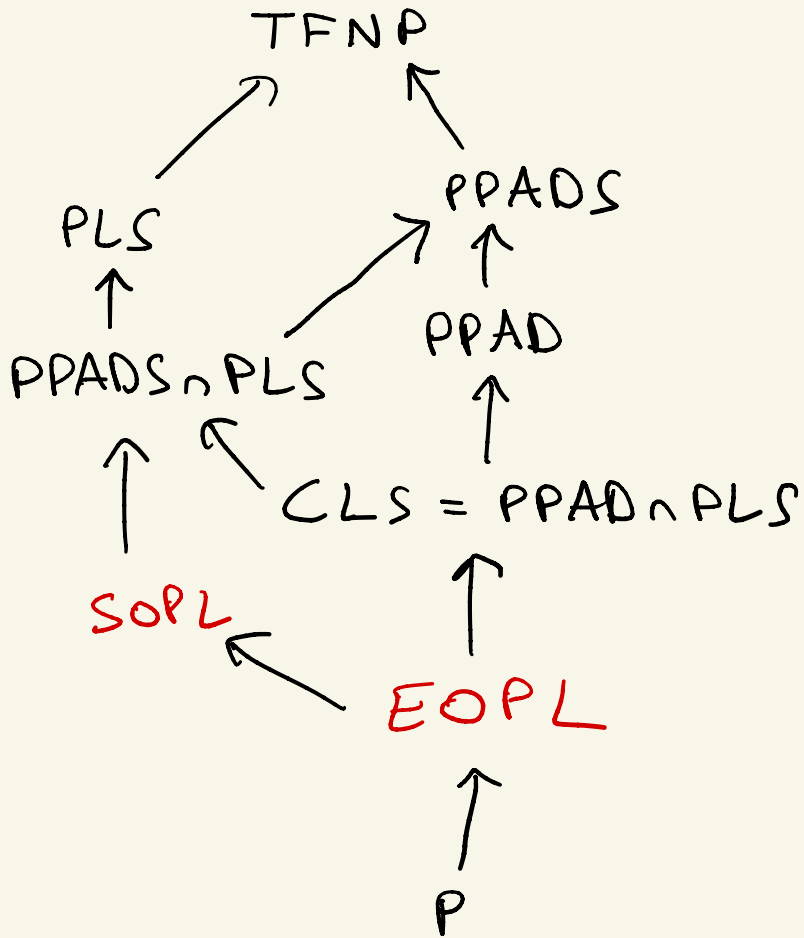
[HY20]
[FGMS20]

End-of-Potential-Line (EOPL)



Sink-of-Potential-Line (SOPL)

[GH]MPRT 22]

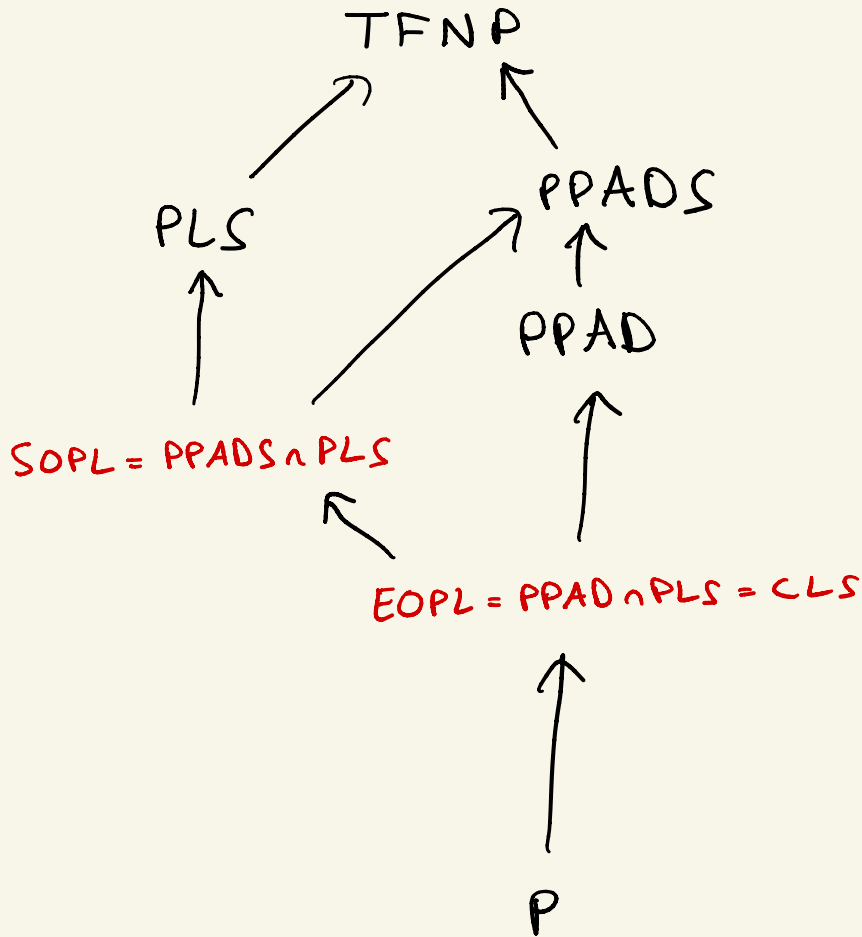


Theorem 1:

$$EOPL = PPAD \cap PLS$$

Theorem 2:

$$SOPL = PPADS \cap PLS$$



Theorem 1:

$$EOPL = PPAD \cap PLS$$

Theorem 2:

$$SOPL = PPADS \cap PLS$$

$$\underline{EOPL = PPAD \wedge PLS}$$

Proof: assume $SOPL = PPADS \wedge PLS$

Reduction from $PPAD \wedge \overset{SOPL}{\cancel{PLS}}$ to EOPL.

$$PPAD \wedge PLS = (PPAD \wedge PLS) \wedge PPADS$$

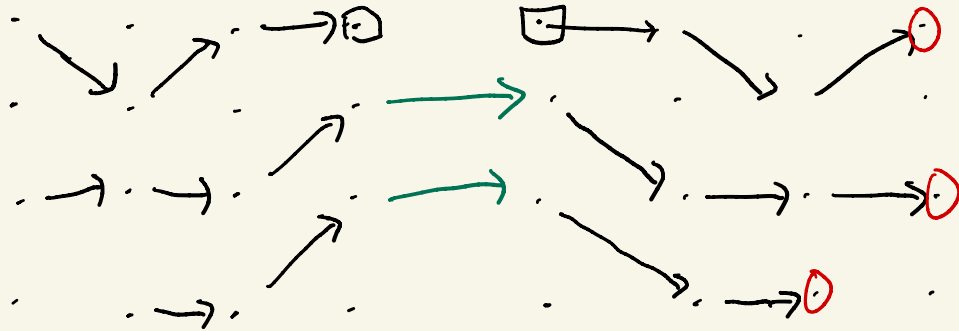
↑
 $PPAD \subseteq PPADS$

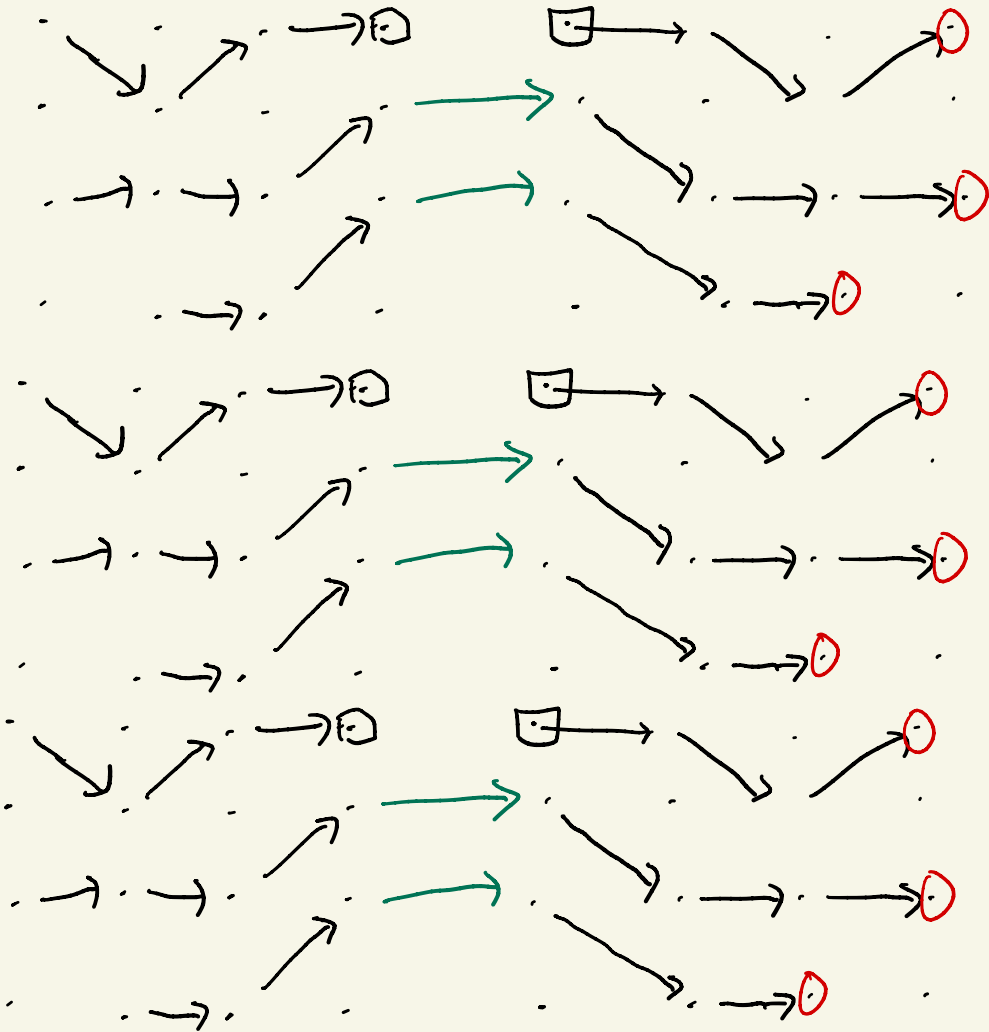
$$= PPAD \wedge (PLS \wedge PPADS)$$

$$= PPAD \wedge SOPL$$

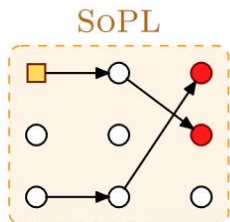
Reduce $PPAD_n$ SOPL to EOPL.

Attempt: $SOPL \leq EOPL$.

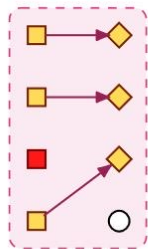




N copies



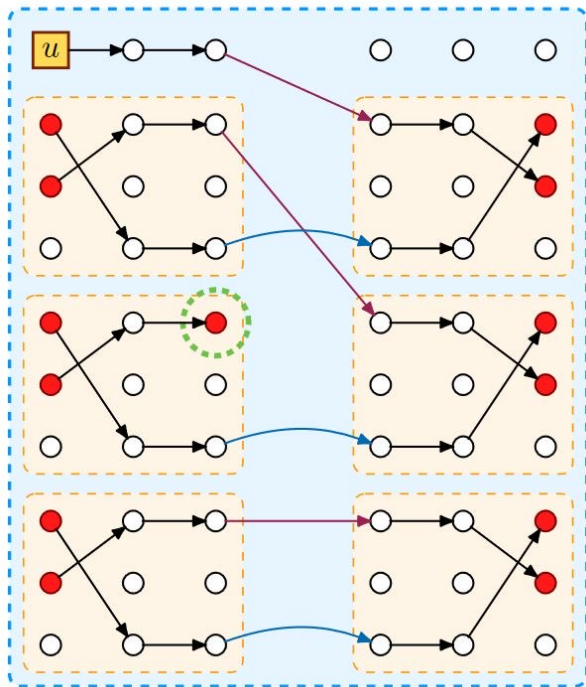
\wedge



BIJ-PHP

\geq

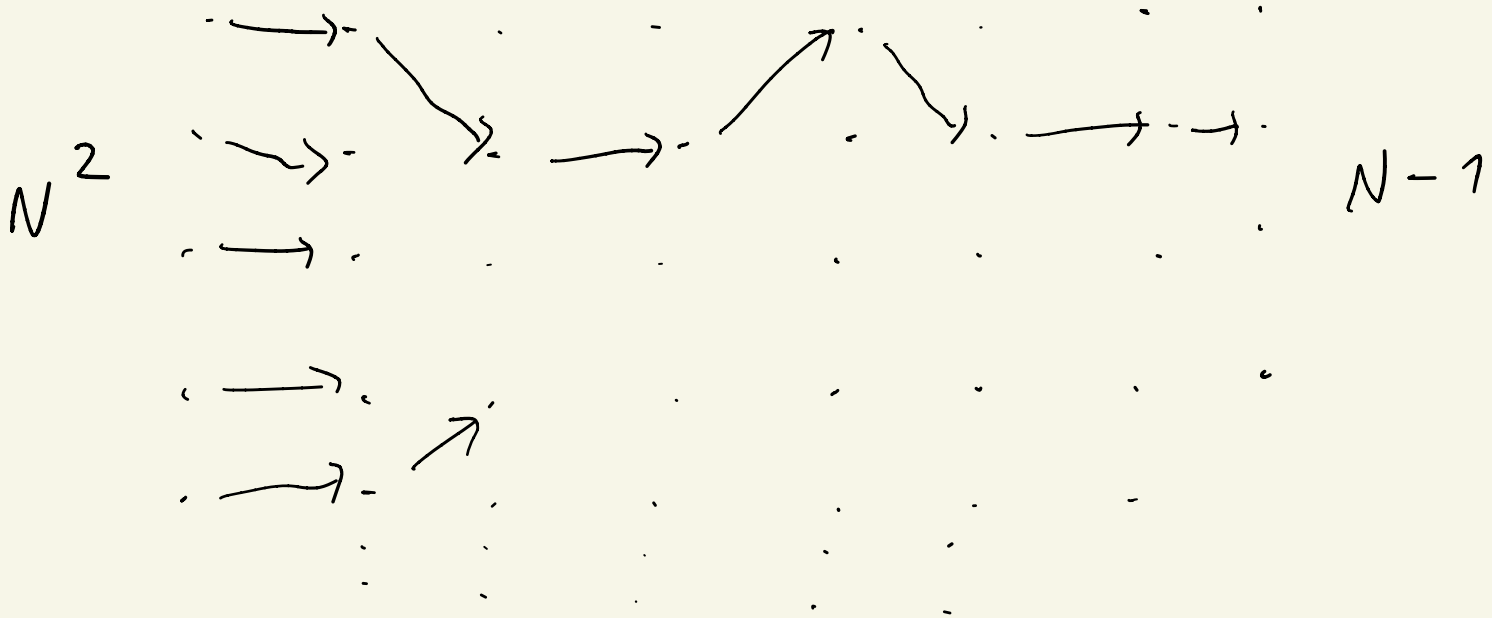
EoPL

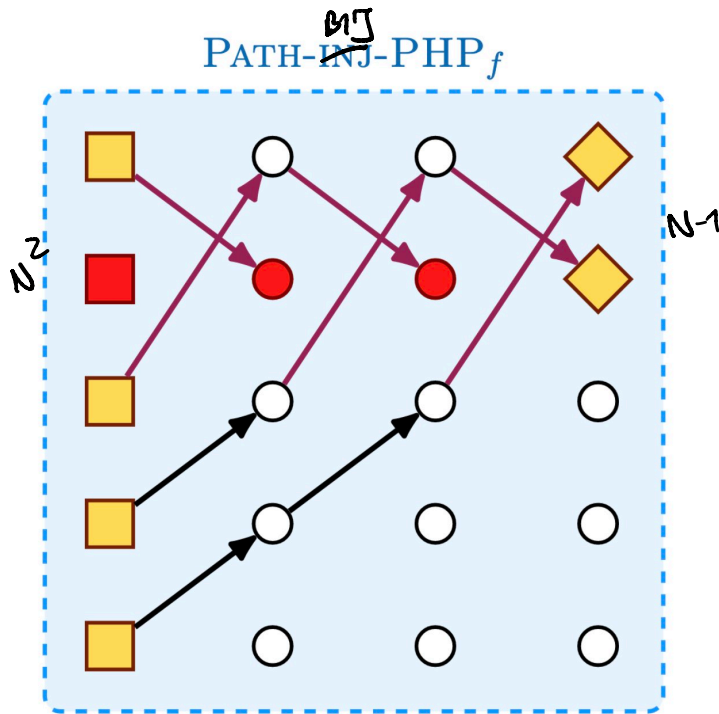
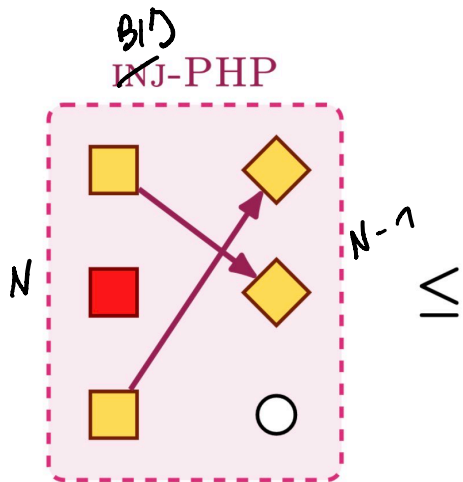


Theorem 2:

$$\text{SOPL} = \text{PPADS} \cap \text{PLS}$$

Detour: Path - PHP problems.



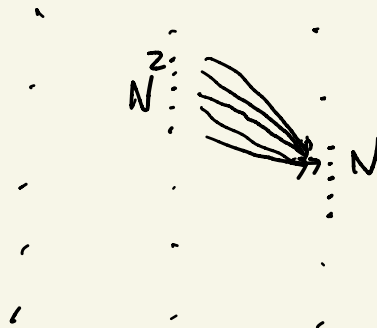
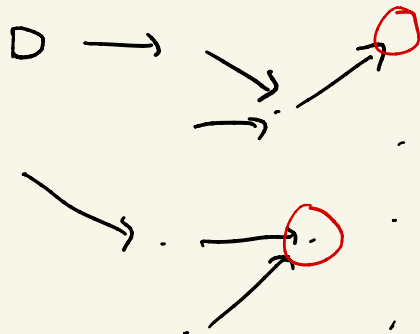


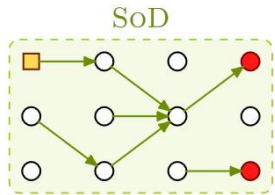
Theorem 2:

$$\text{SOPL} = \text{PPADS} \cap \text{PLS}$$

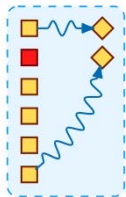
Attempt: $\text{SoD} \leq \text{SoPL}$

$$N = S$$





λ

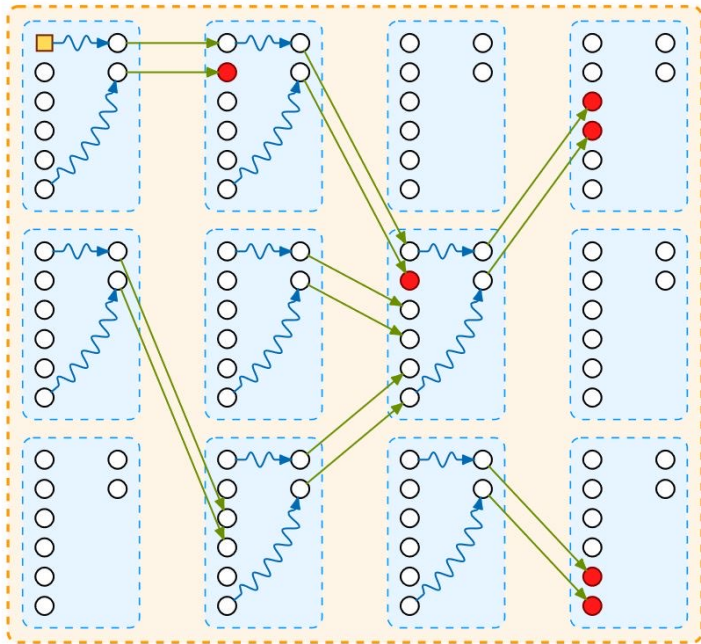


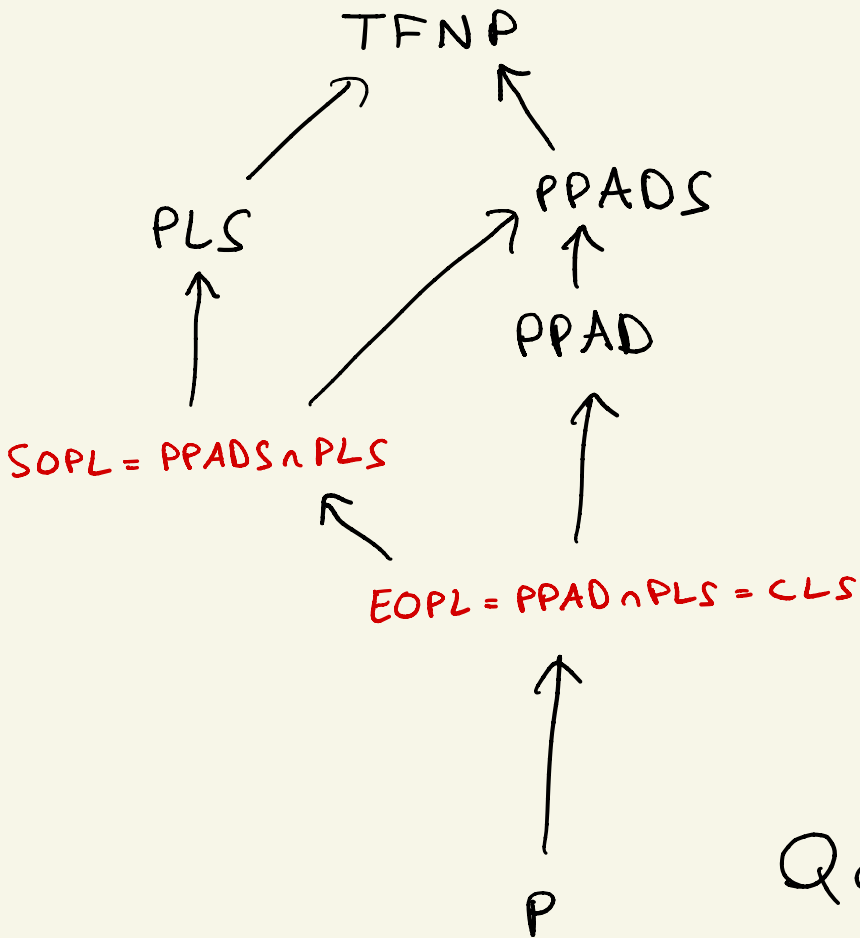
PATH-INJ-PHP

N^2
 \rightarrow
 N

\vee

SoPL





Theorem 1:

$$EOPL = PPAD \cap PLS$$

Theorem 2:

$$SOPL = PPADS \cap PLS$$

Question:

More collapses?