CS 598TC, Spring 2017
Timothy Chan

Assignment 4 (due April 21 Friday 2pm (in class))

Note: Acknowledge any discussions you have with other students (and any references you have consulted). Solutions must be written entirely *in your own words*.

1. [*10 marks*] Given a set of red points and a set of blue points in a constant dimension d, consider the problem of finding a pair of points (r, b), where r is red and b is blue, with the smallest Euclidean distance. Describe a (simple!) dynamic data structure to solve this problem with constant approximation factor, while supporting insertions and deletions of red and blue points in $O(\log n)$ (or $O(\log \log U)$) time.

   [Hint: use Z-ordering...]

   [Bonus/research problem: could we get the same $O(\log n)$ update time but with $1+\varepsilon$ approximation factor? I don't think this is known.]

2. [*10 marks*] Given a set P of n points in a high (nonconstant) dimension d, and two points $s, t \in P$, we want to find a path from s to t minimizing the maximum edge length, i.e., a sequence $p_0, p_1, \ldots, p_\ell \in P$ with $p_0 = s$ and $p_\ell = t$, minimizing $\max\{d(p_0, p_1), d(p_1, p_2), \ldots, d(p_{\ell-1}, p_\ell)\}$, where $d(\cdot, \cdot)$ denotes the Euclidean distance.

   Give a c-approximation algorithm to solve the problem in about $d^{O(1)} n^{1+1/c} \log U$ time.

   [Hint: first focus on the corresponding approximate decision problem. One approach is to simulate breadth/depth-first search, using (possibly dynamic versions of) data structures from class...]