

Assignment 1 (due Feb 15 Wednesday 1pm (in class))

Note: Acknowledge any discussions you have with other students (and any references you have consulted). Solutions must be written individually *in your own words*.

1. [15 marks] We want to store a set S of n points in 2D in a data structure so that given a query axis-aligned rectangle q and an integer k , we can quickly find the k -th *leftmost point* in q , i.e., the point in q with the k -th smallest x -coordinate. (You may assume that k is smaller than the number of points inside q .)

Describe how to solve this problem with $O(n)$ space and $O(\log n)$ query time, by adapting Chazelle's compressed range tree method with bit packing.

Hint: first it suffices to consider the case when q is a horizontal slab (why?)...

2. [10 marks]

- (a) [5 marks] Consider the following problem: store a set S of n (axis-aligned) rectangles in 2D so that for a given query point q , we can quickly decide whether q is contained in at least one rectangle of S . (This is in some sense the “dual” of 2D orthogonal range searching, with input rectangles and query points instead of input points and query rectangles.)

Show that this 2D problem can be reduced to 4D orthogonal range emptiness.

- (b) [5 marks] Next consider the following problem: store a set S of n vertical line segments in 2D so that for a given query horizontal line segment q , we can quickly decide whether q intersects at least one segment of S .

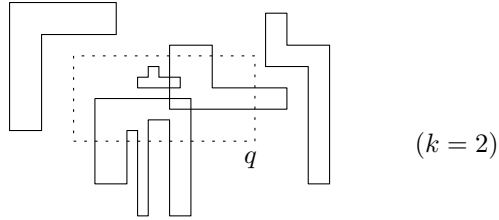
Show that this 2D problem can be reduced to 3D orthogonal range emptiness.

3. [20 marks]

- (a) [15 marks] Given a collection of m polygons in 2D with a total of n vertices ($m \leq n$) where all edges are horizontal or vertical, design a data structure with $O(n \log^c n)$ space so that for a given query rectangle q , we can report (the labels of) all polygons whose boundaries intersect the boundary of q in $O((1+k) \log^c n)$ time for some constant c , where k is the number of reported polygons. (Note that k may be much smaller than the number of edges intersecting the boundary of q .)

Hint: recursively divide into two subcollections of $m/2$ polygons, and use Question 2(b)...

- (b) [5 marks] Describe how to modify your solution in (a) to report all polygons whose interior intersect the interior of q (including polygons that are completely contained in q or contain q).



4. [15 marks] Solve the counting version of the problem in Question 3: Given a collection of m polygons in 2D with a total of n vertices where all edges are horizontal or vertical, design a data structure with $O(n \log^c n)$ space (or better), so that for a given query rectangle q , we can *count* the number of polygons that intersect the boundary of q in $O(n^{3/4} \log^c n)$ time for some constant c .

Hint: form an $r \times r$ grid for some choice of r ; precompute the answer for each grid-aligned rectangle; in addition, use Question 2(b)... Bonus marks may be given for a significantly better solution (though off-hand I don't know of one).