Searching on/Testing Encrypted Data

Lecture 23

Searchable Encryption

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- e.g. Application: delegating e-mail filtering
  - Sender attaches a list of (searchably) encrypted keywords to the (normally encrypted) mail. Receiver hands the mail-server test keys for keywords of its choice. Mail-server filters mails by checking for keywords and can forward them appropriately.

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  - May require perfect or statistical correctness. Or, should hold w.h.p against computationally bounded environments choosing w, w' (after seeing PK, and for w', possibly after seeing C, T_w also).
- Secrecy: CPA or CCA security against adversary with oracle access to TestKeyGen(SK, .), as long as adversary doesn't query w_0,w_1

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- Compact keys, but ciphertext is still long

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  - Or add such "decryption recognition" directly to Anonymous IBE

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  - Can use IBE to shorten keys. Ciphertext still too long.

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  - Extends to range checking

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      - Check if binary $[X^a_{ij}]$ defined as $X^a_{ij} = 1$ iff $j \leq a_i$, matches with $[T^r_{ij}]$ defined as $T^r_{ij} = 1$ if $j \leq r_i$, and * otherwise

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- More efficient set membership?

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    - Or h a CRHF with range being indices of a "cover free set system"

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    - For encrypting to identity id use attribute $a_{id} = (1,id)$. SK_{id} is the test key for predicate with $v_{id} = (-id,1)$. Anonymity: attribute remains hidden if no matching SK given

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  - Can support * in both the pattern and the hidden vector

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    - Map V to v as $v_0=1$ and for i=1 to n, $v_i = 1$ iff $i \in V$. Map A to a vector a where $a_0 = -t$, for i=1 to n, $a_i = 1$ iff $i \in A$.

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