Mix-Nets

Lecture 16

Some tools for electronic-voting (and other things)

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- Corruption model: Active adversary can corrupt a limited number of servers

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- Active adversary can corrupt a limited number of servers
- Ideal: Same as for SIM-CPA, but with servers also getting the message (if the receiver decides to get it); if number of corrupted servers above threshold, adversary can block (but not substitute) output to others

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  - Proof using an Honest-Verifier ZK proof
    - Using a special purpose proof (Chaum-Pederson), rather than ZK for general NP statements

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  - Can amplify soundness using parallel repetition: still 3 rounds

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      - Removes need for interaction!

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- Special soundness: given answers for $v \neq v'$ either $v_1 \neq v_1'$ or $v_2 \neq v_2'$. By special soundness, extract witness for stmnt_1 or stmnt_2

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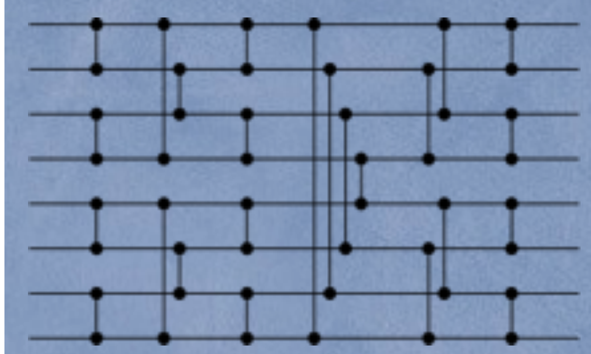

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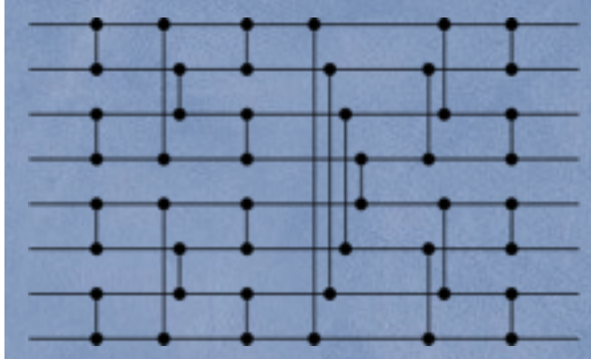

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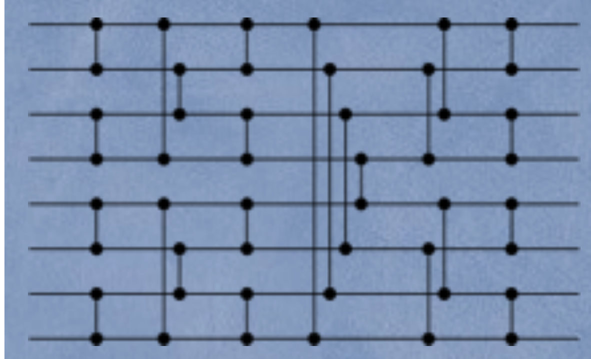

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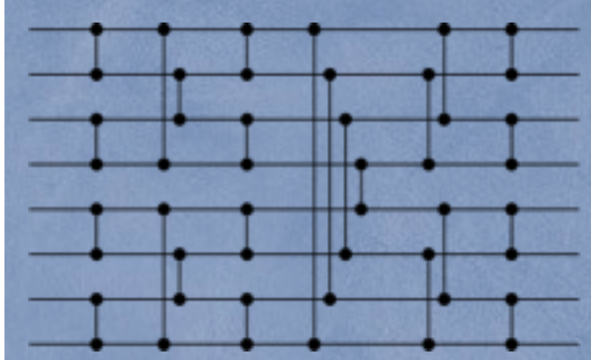

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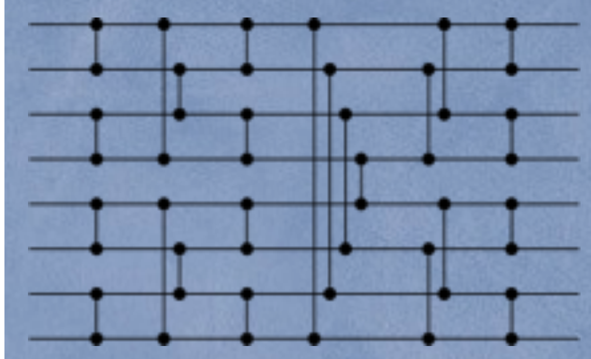

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  - 3 rounds: Parallel composition of HVZK proofs

Alternate Verifiable-Shuffles

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- Improved efficiency: $H_{g1,..,gn,h}(x_1,\ldots,x_n,r) = g_1^{x1} \ldots g_n^{xn}\, h^r$

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    - Use homomorphic properties of the commitments to carry out equality proofs w.r.t committed permutation (omitted)

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