# Universal Composition

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Lecture 17

And the GMW-Paradigm for MPC Protocols

# Turing Award!

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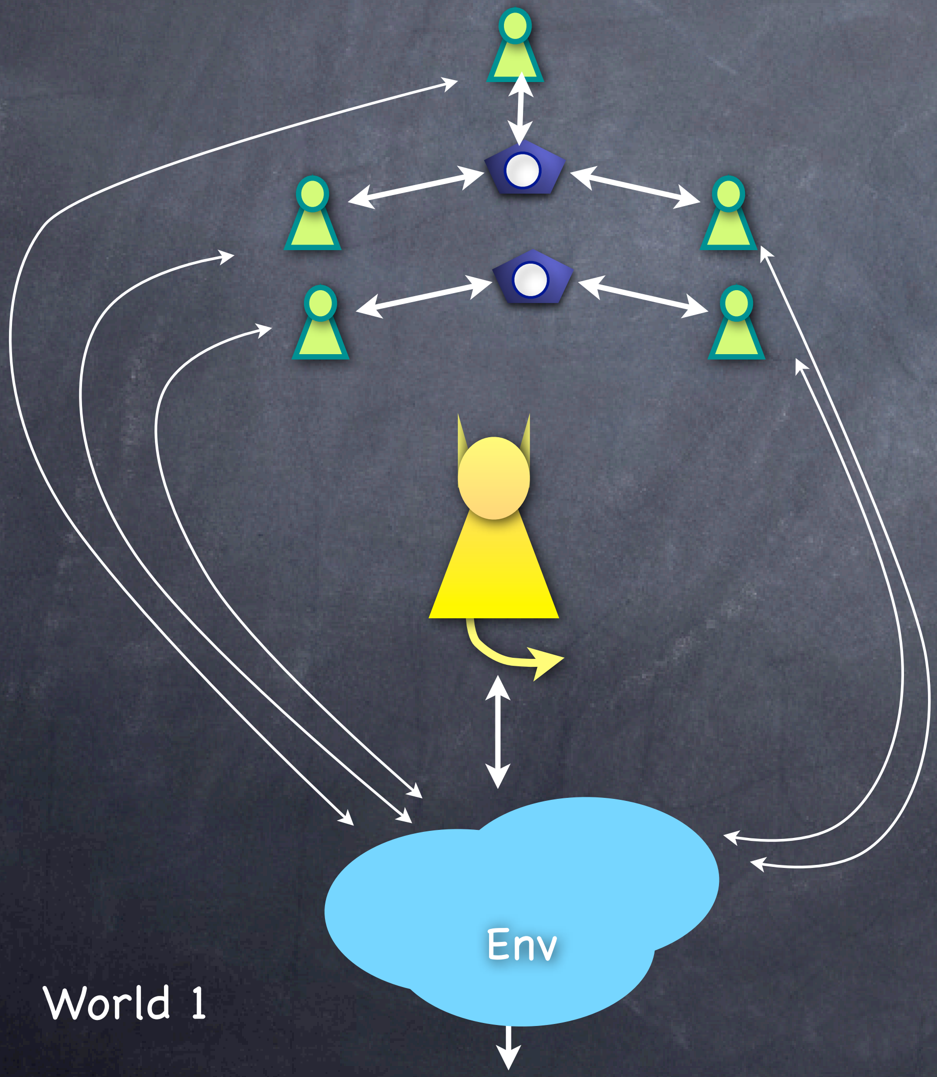

World 1

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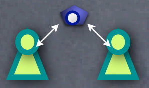
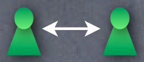
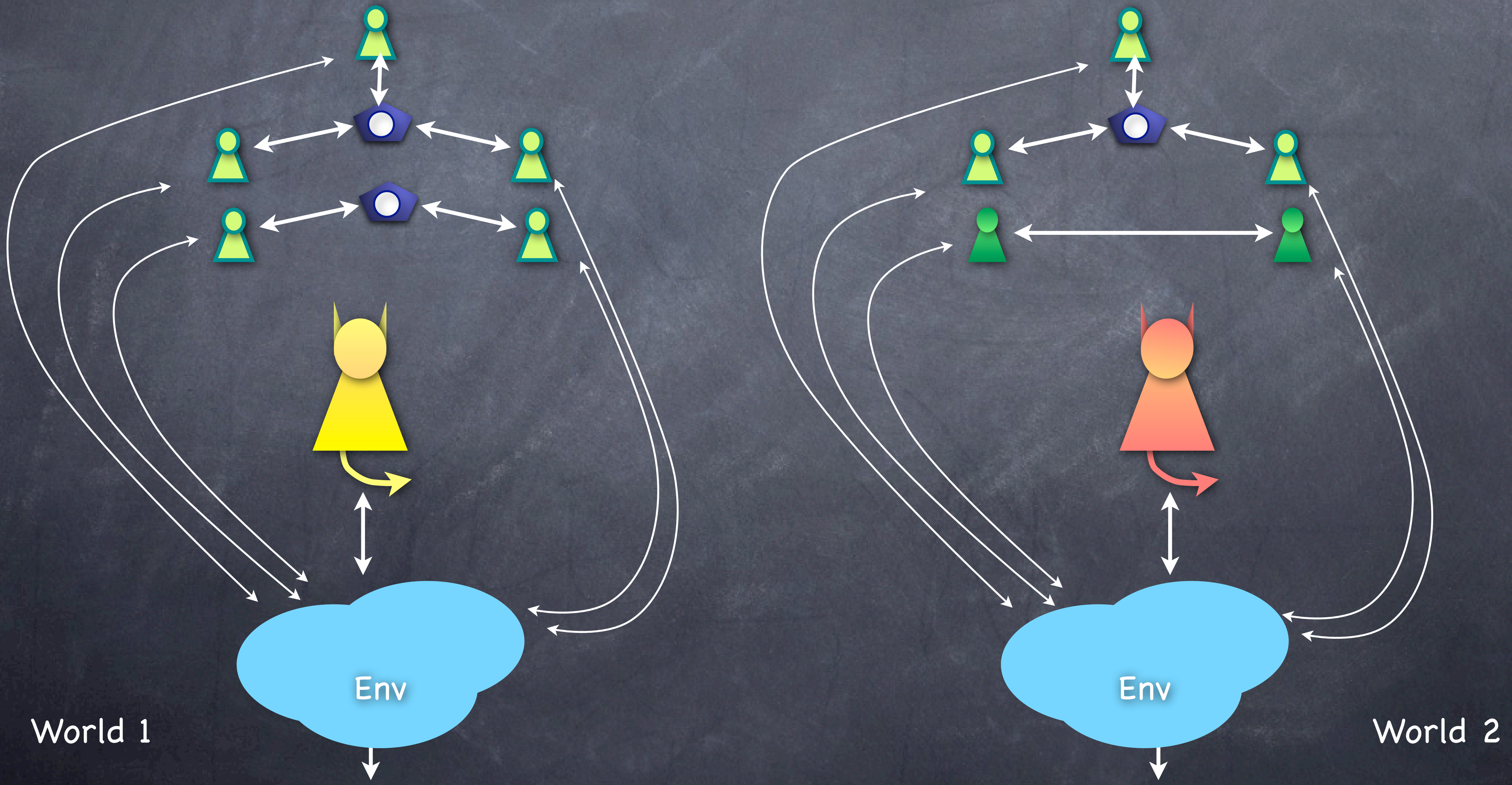
Replace protocol  with  which is as SIM-secure, etc.



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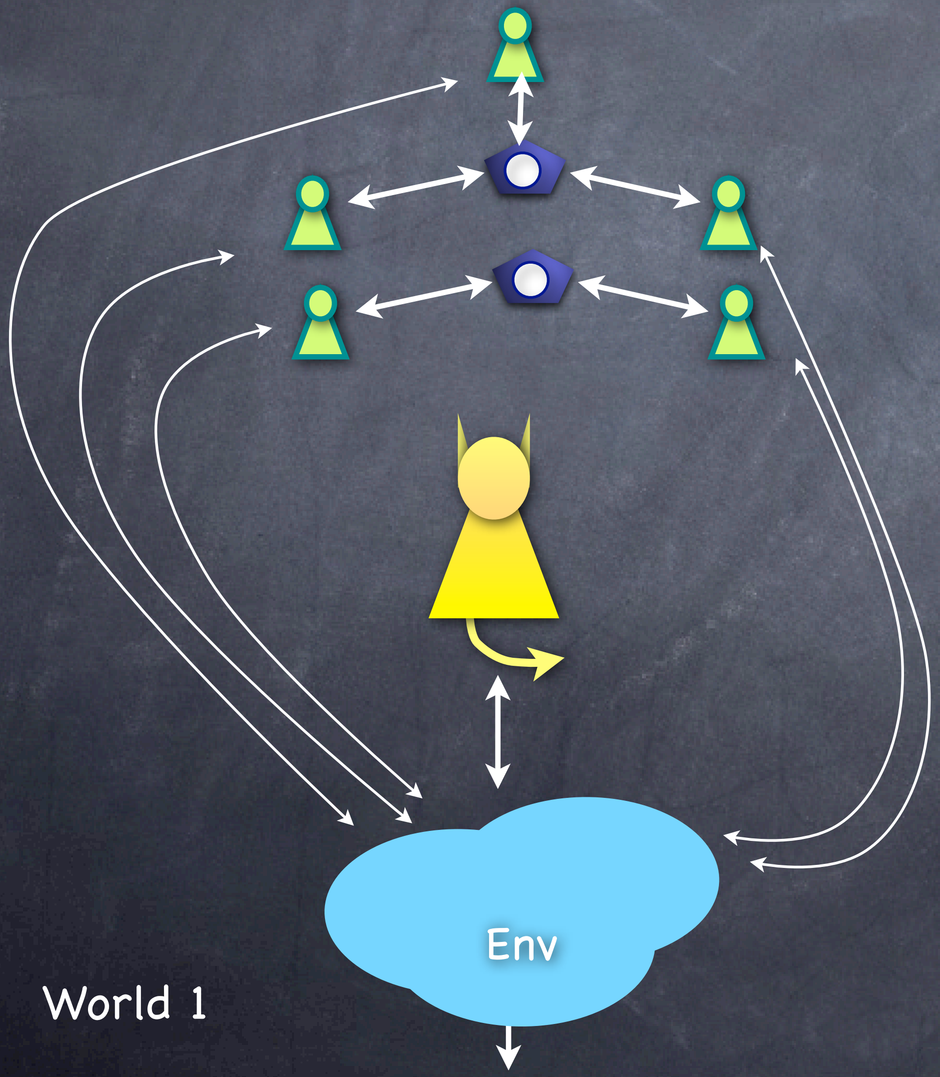

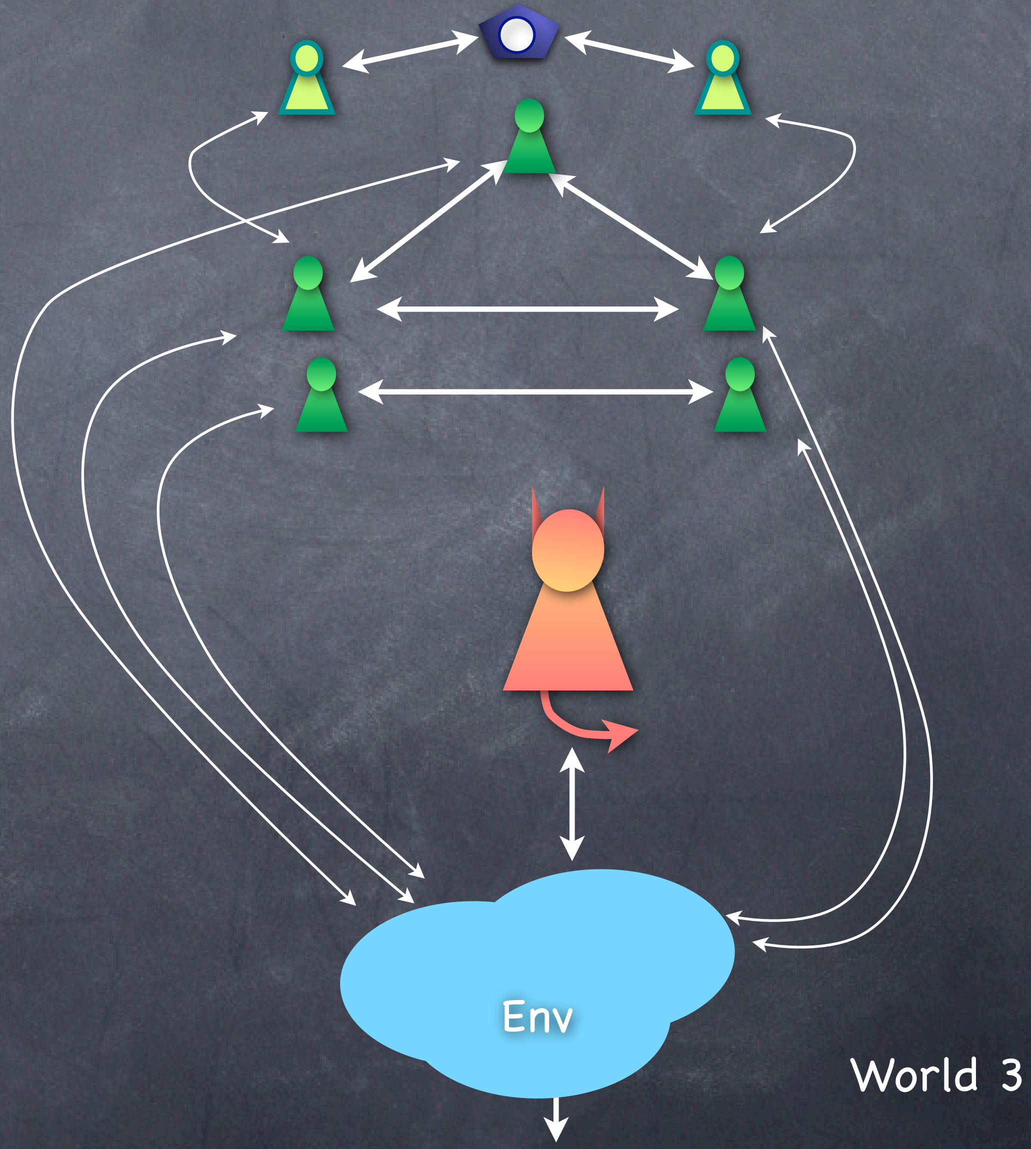

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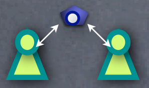
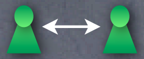
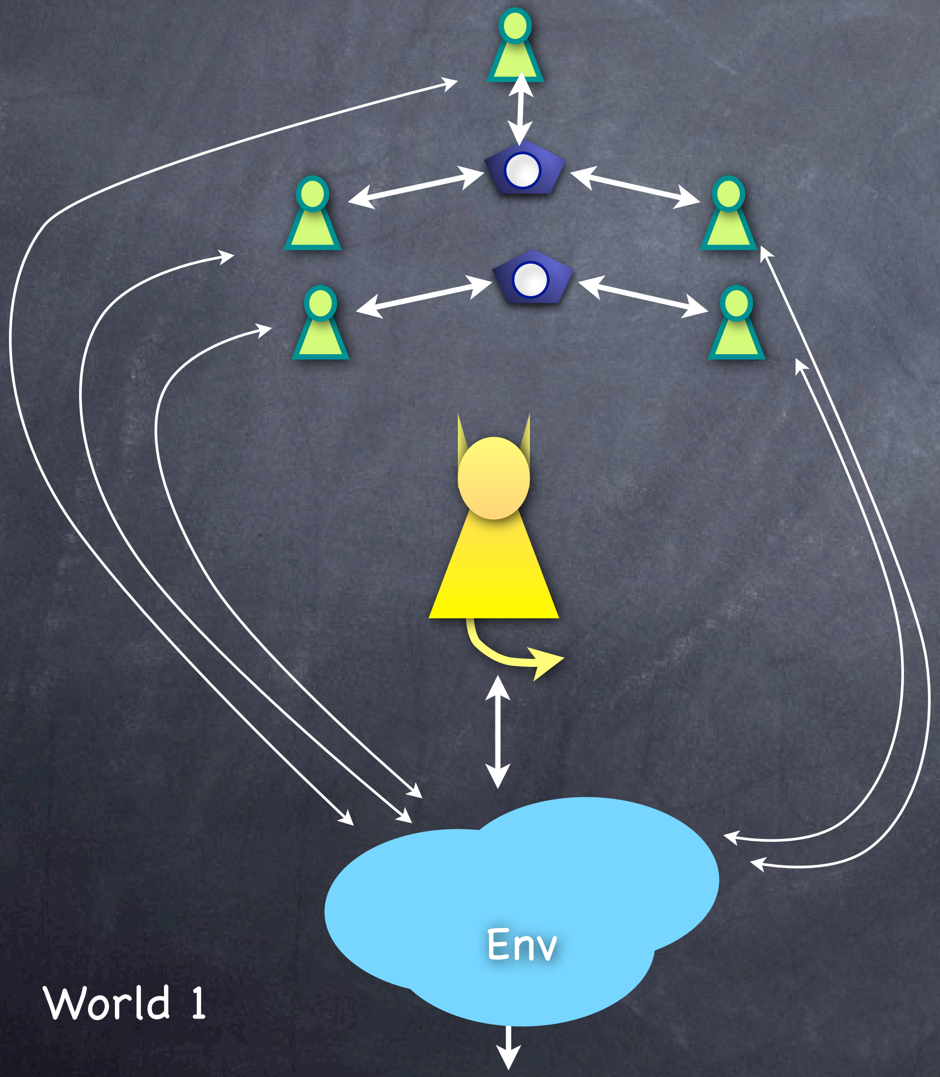
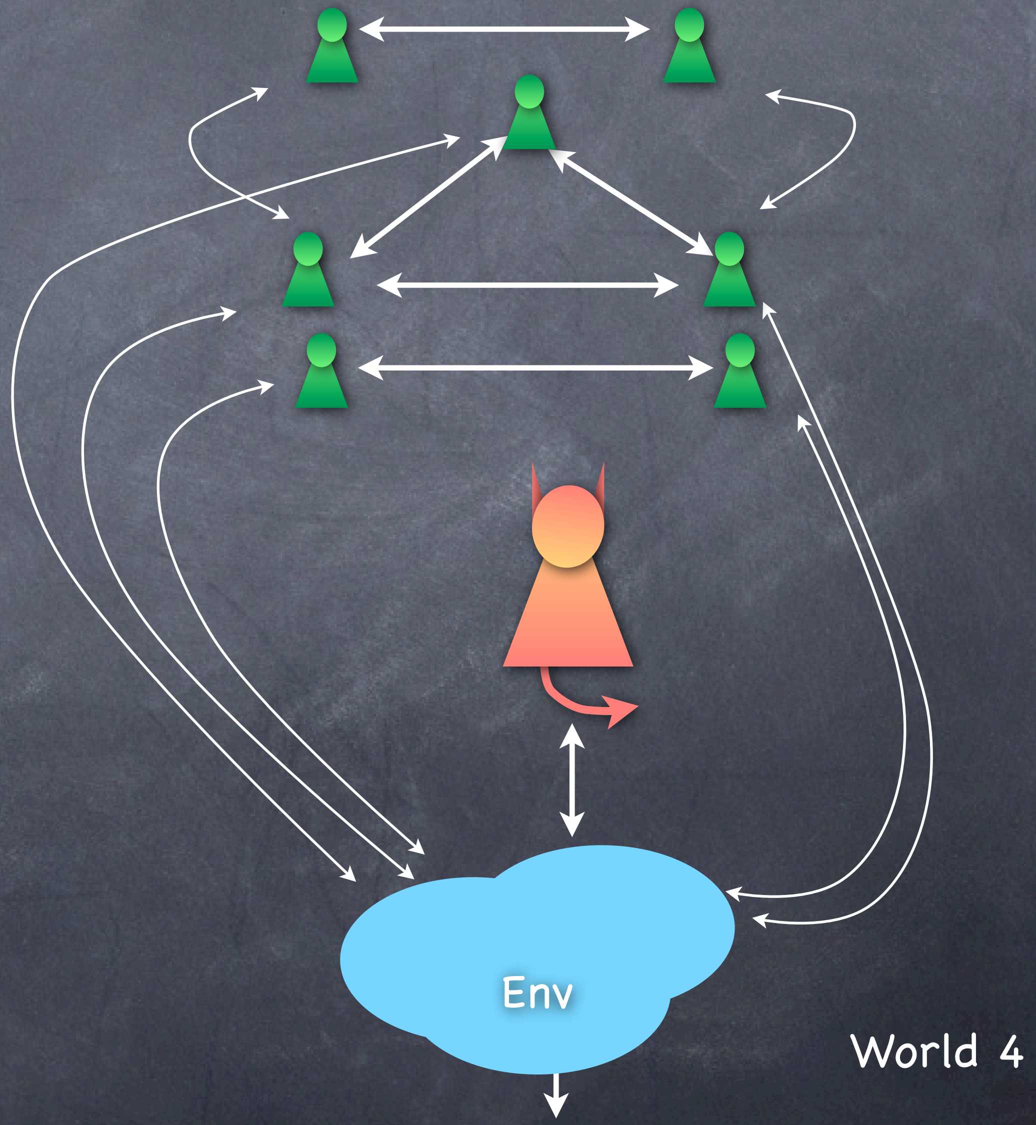
Replace protocol  with  which is as SIM-secure, etc.





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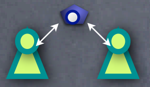
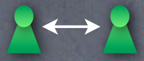
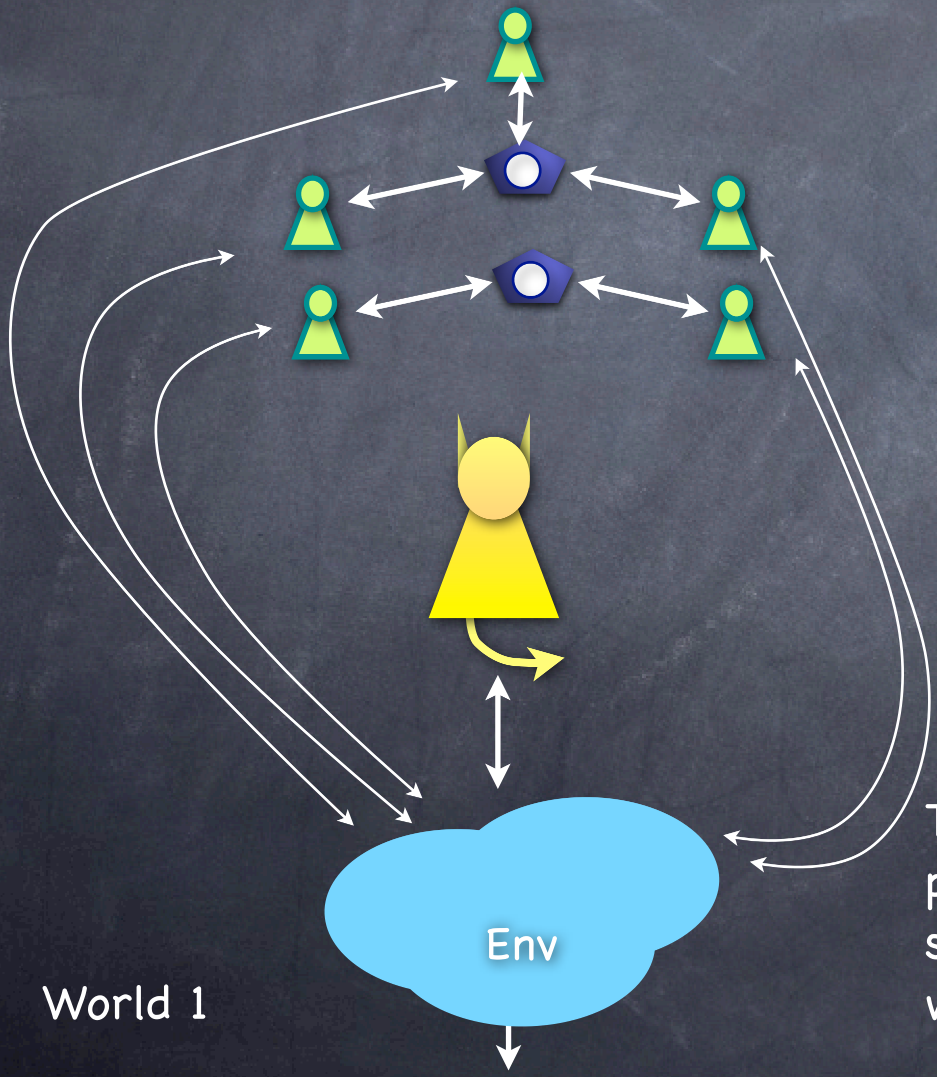
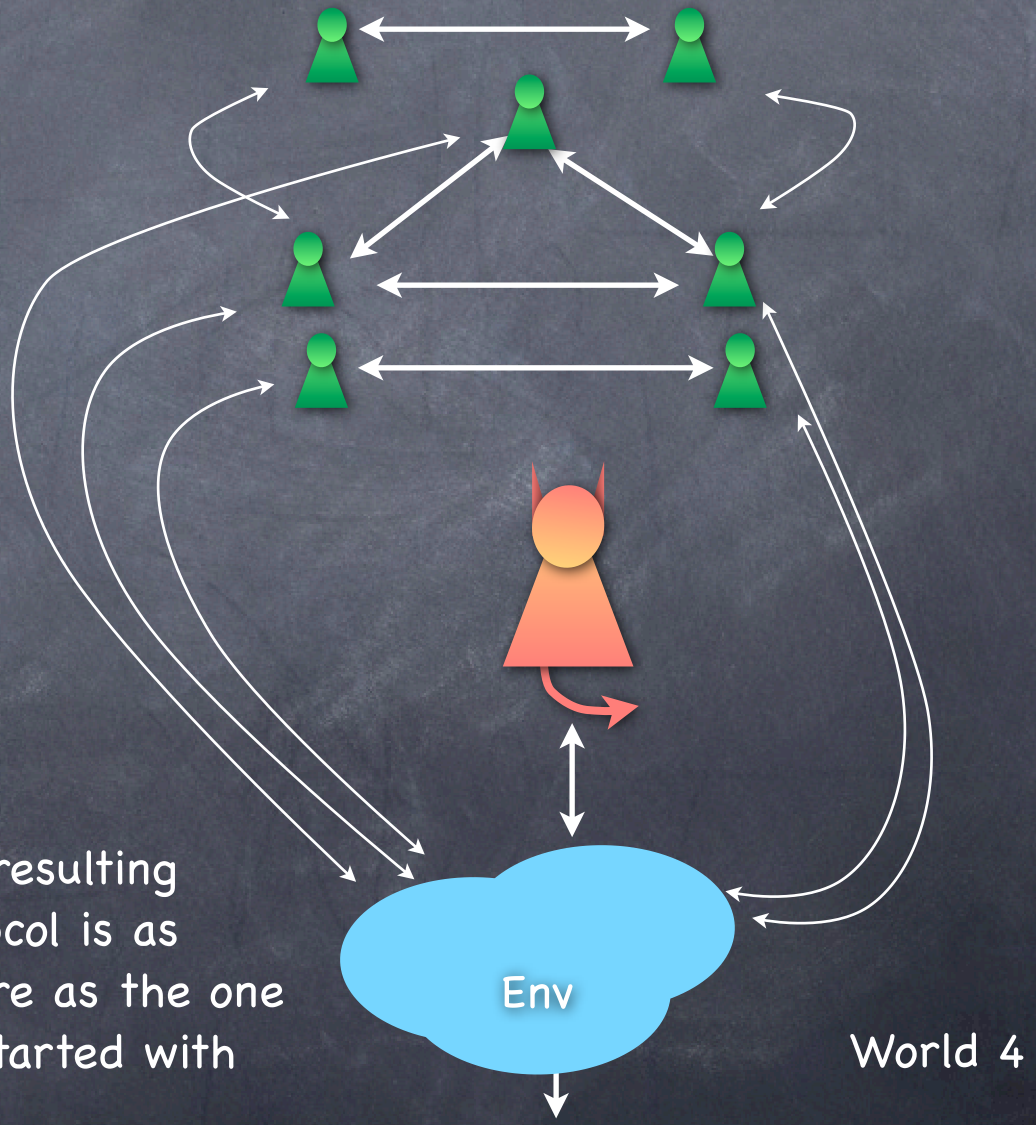
Replace protocol  with  which is as SIM-secure, etc.



The resulting protocol is as secure as the one we started with



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  - UC Theorem: Then world B is as secure as world A
- Gives a modular implementation of the IDEAL world

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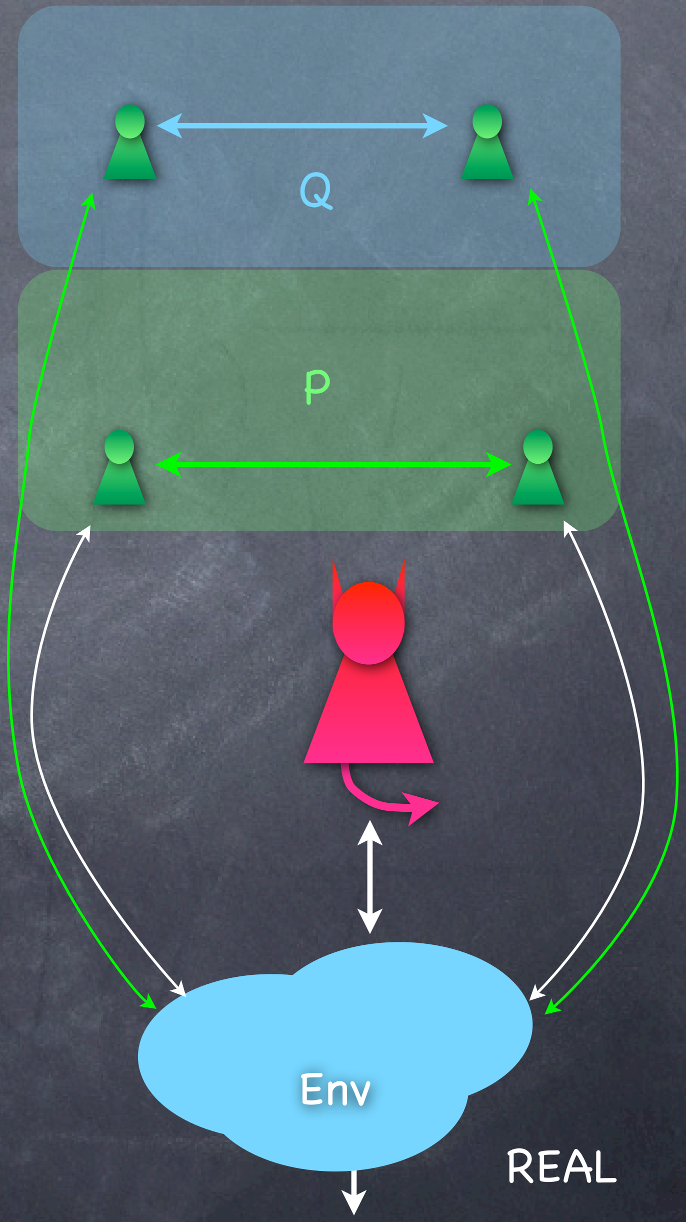

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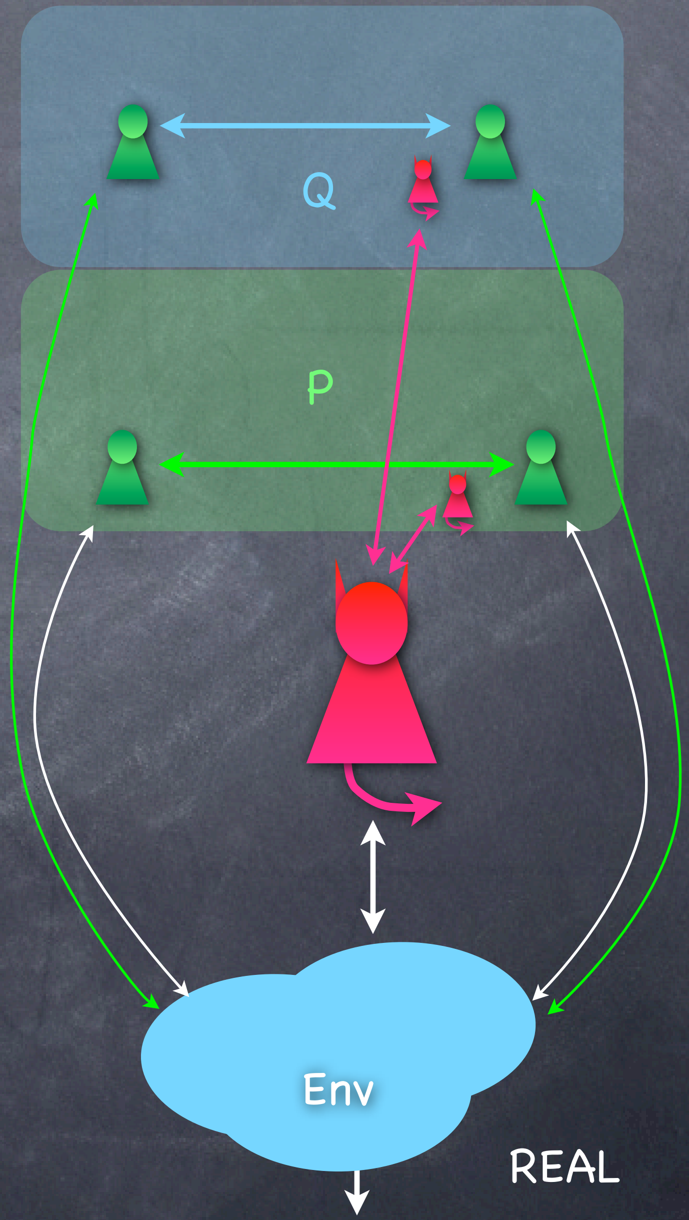

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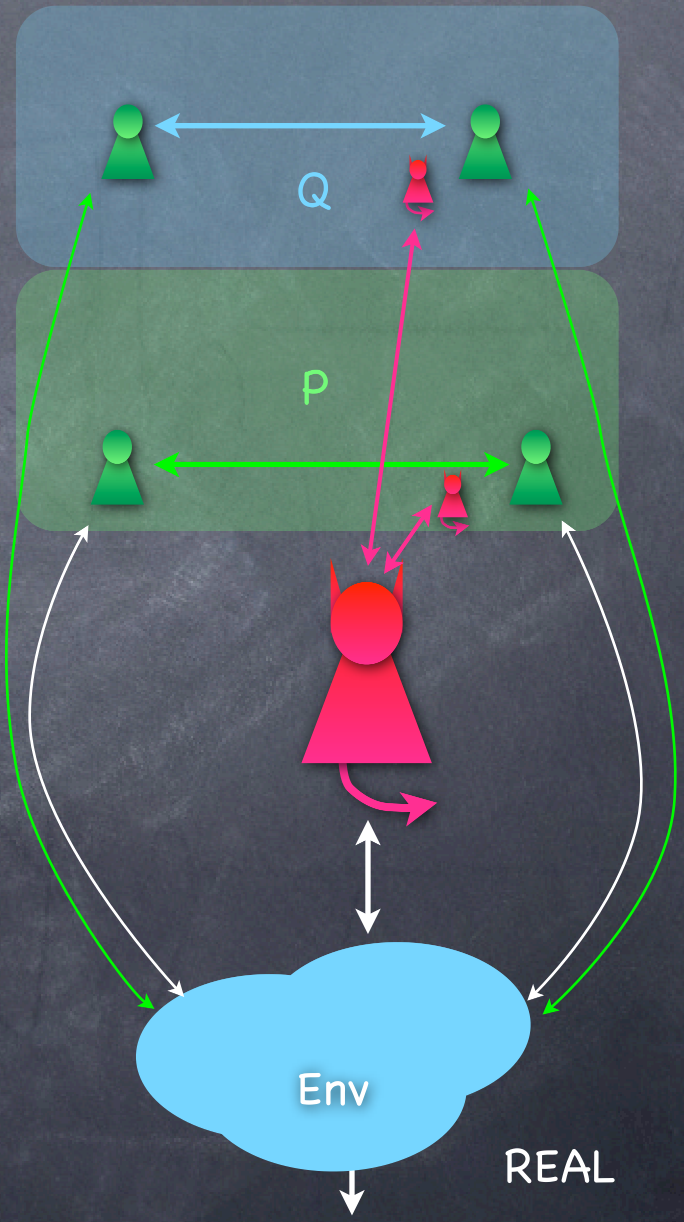
- Consider environment which runs the adversary internally, and depends on "dummy adversaries" to interface with the protocols



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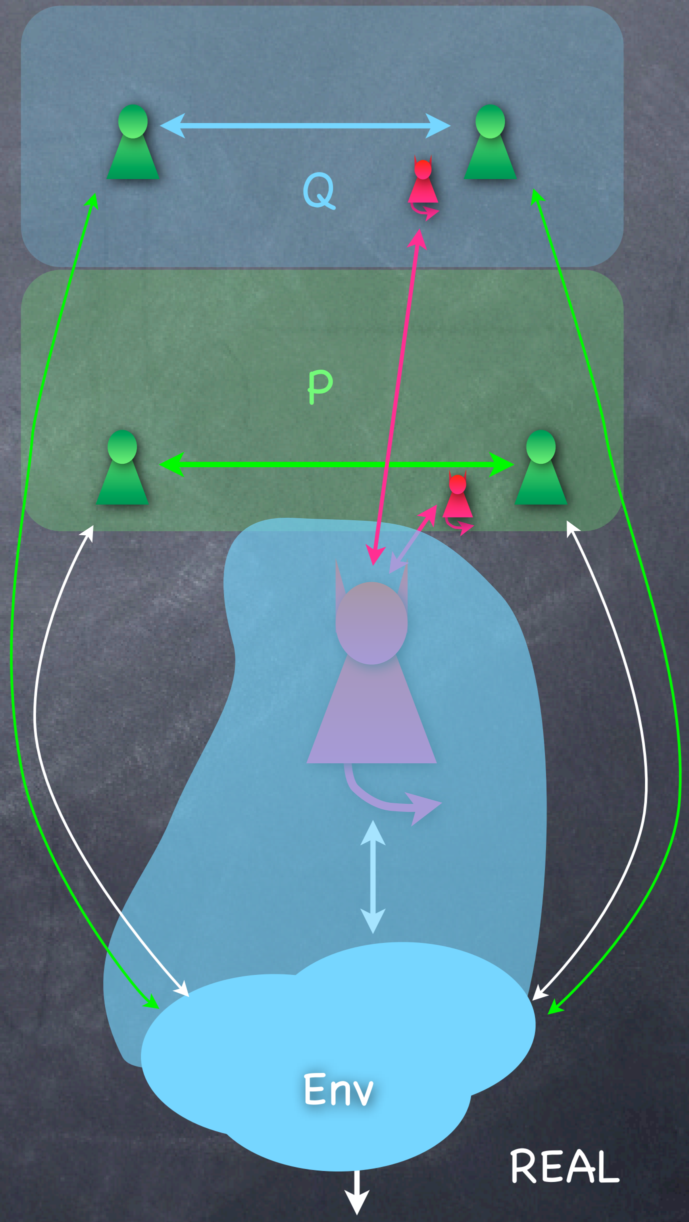

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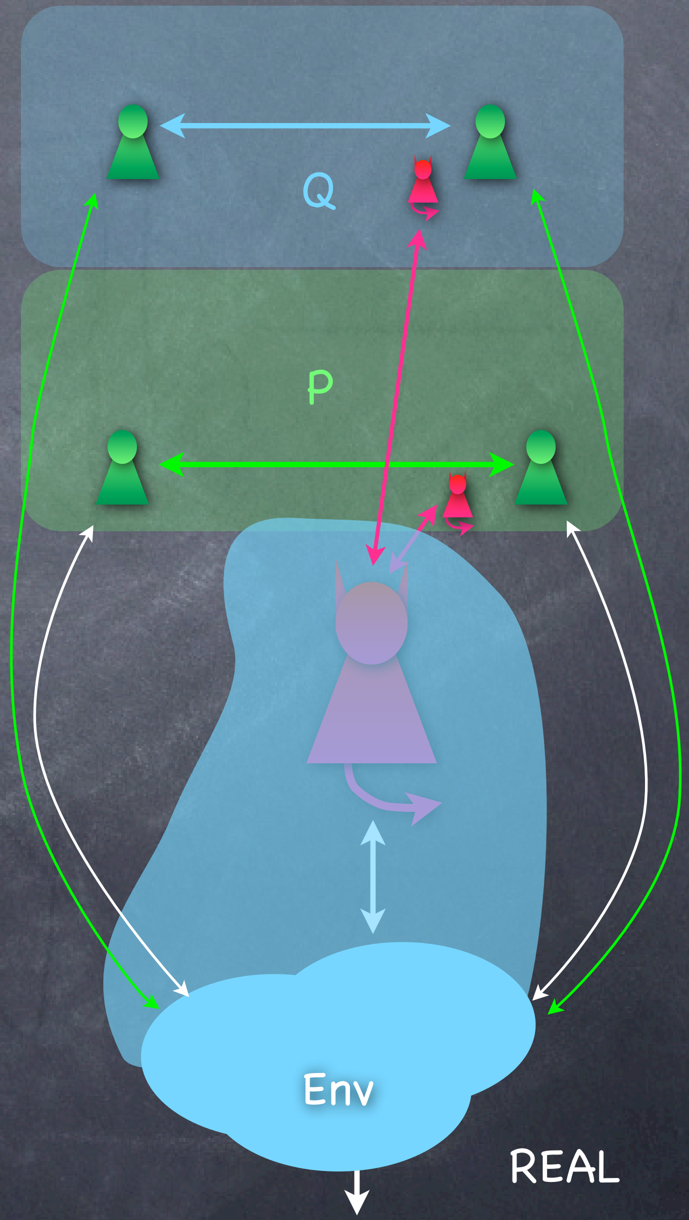

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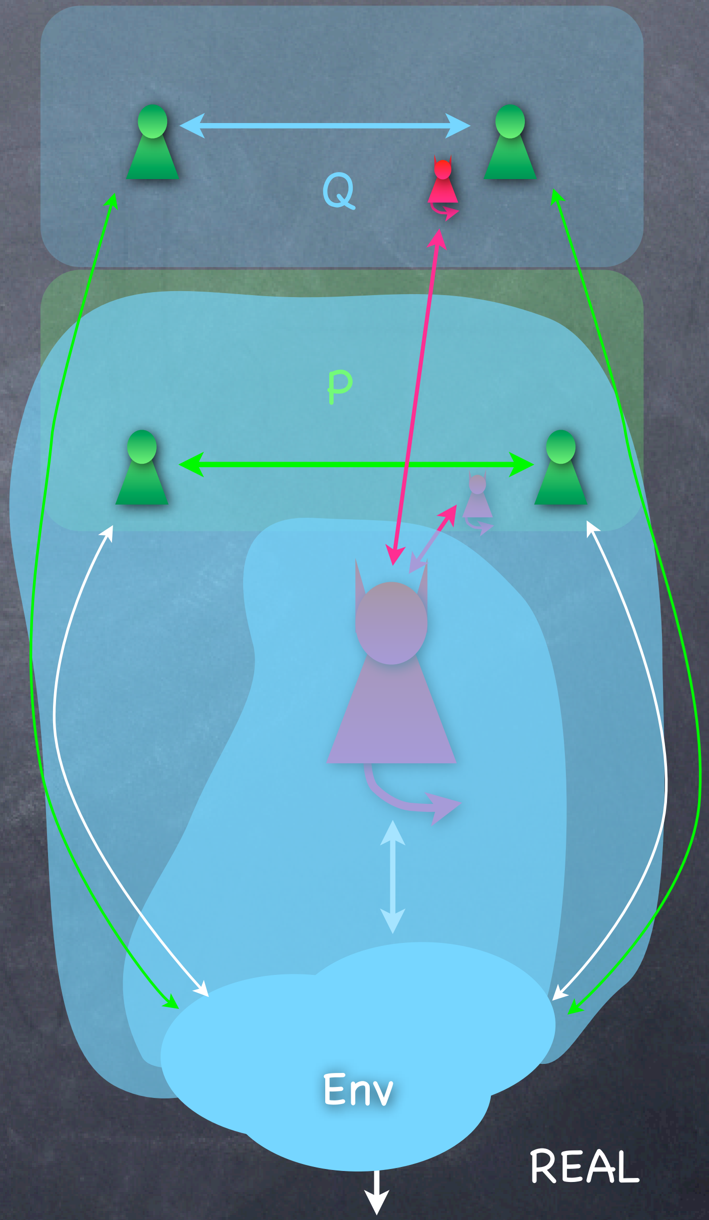

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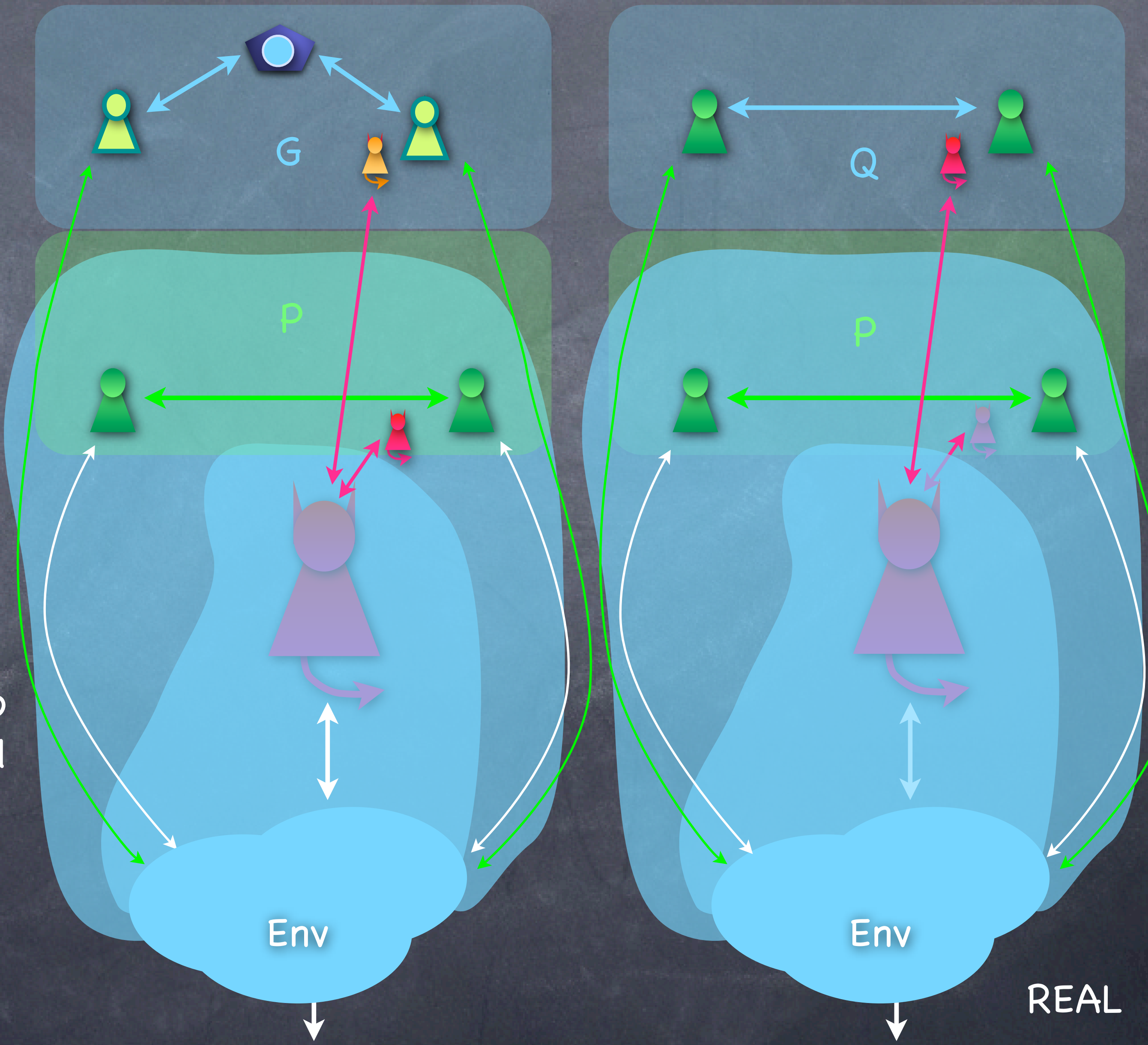

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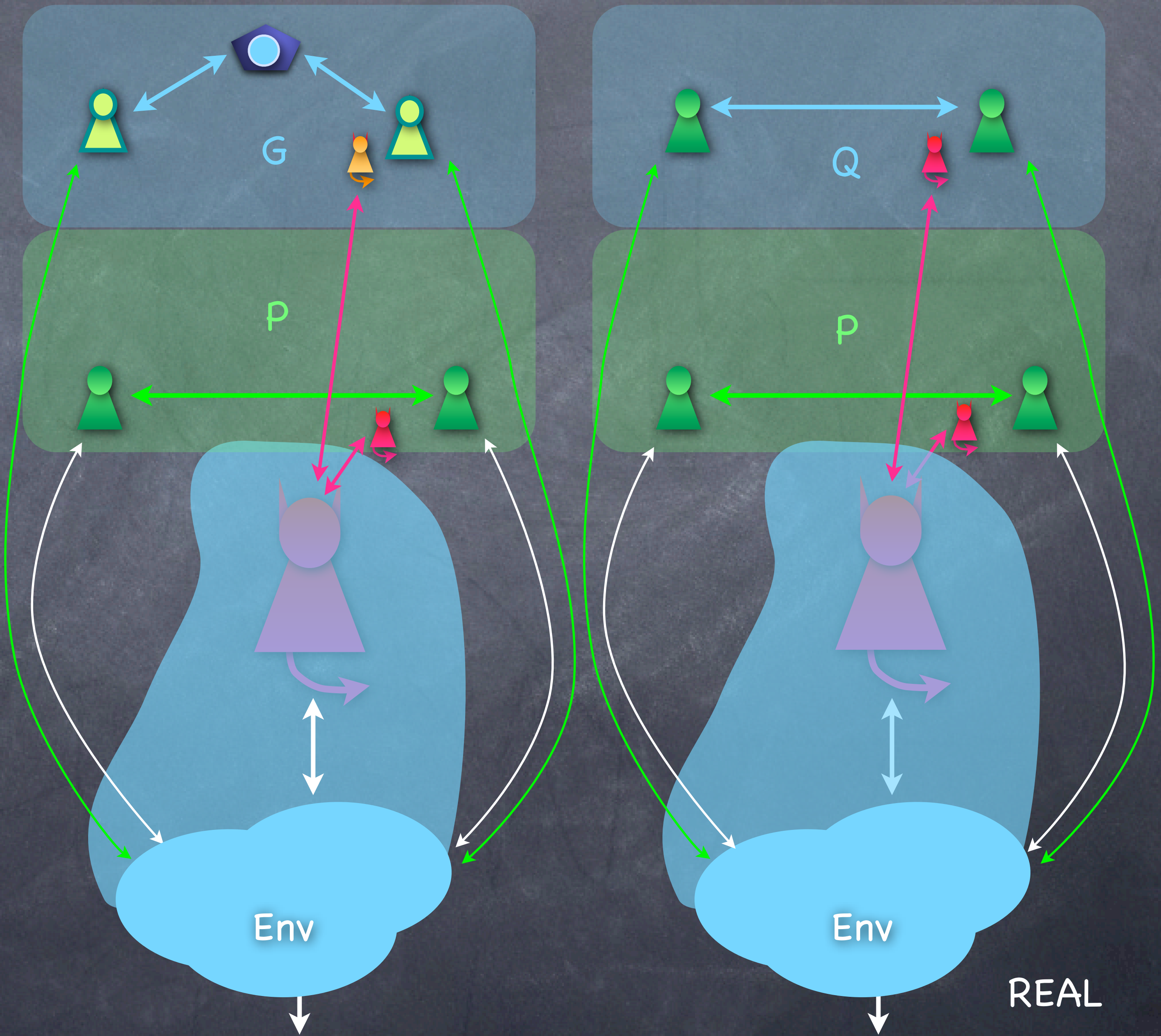

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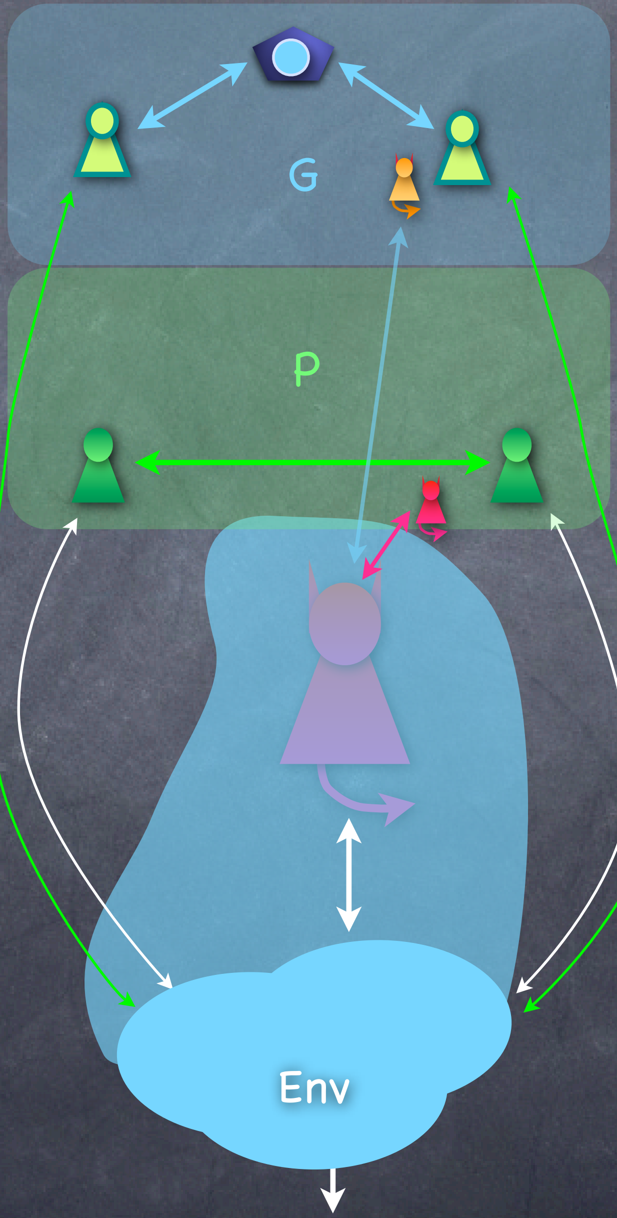

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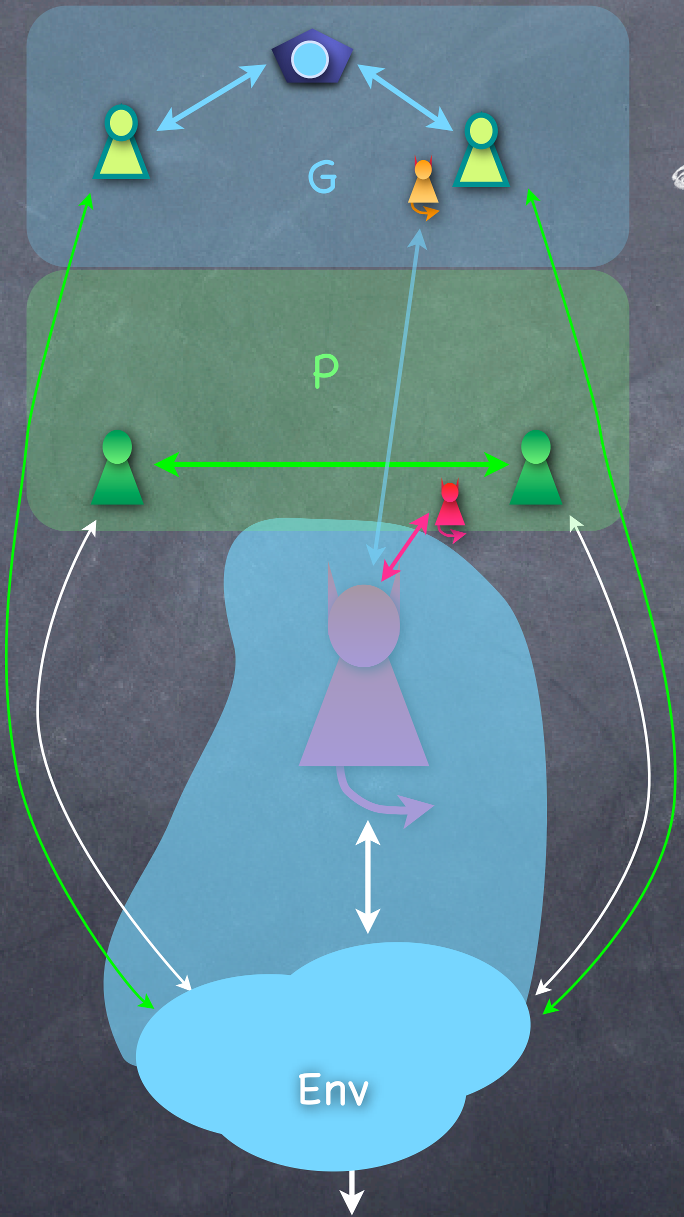

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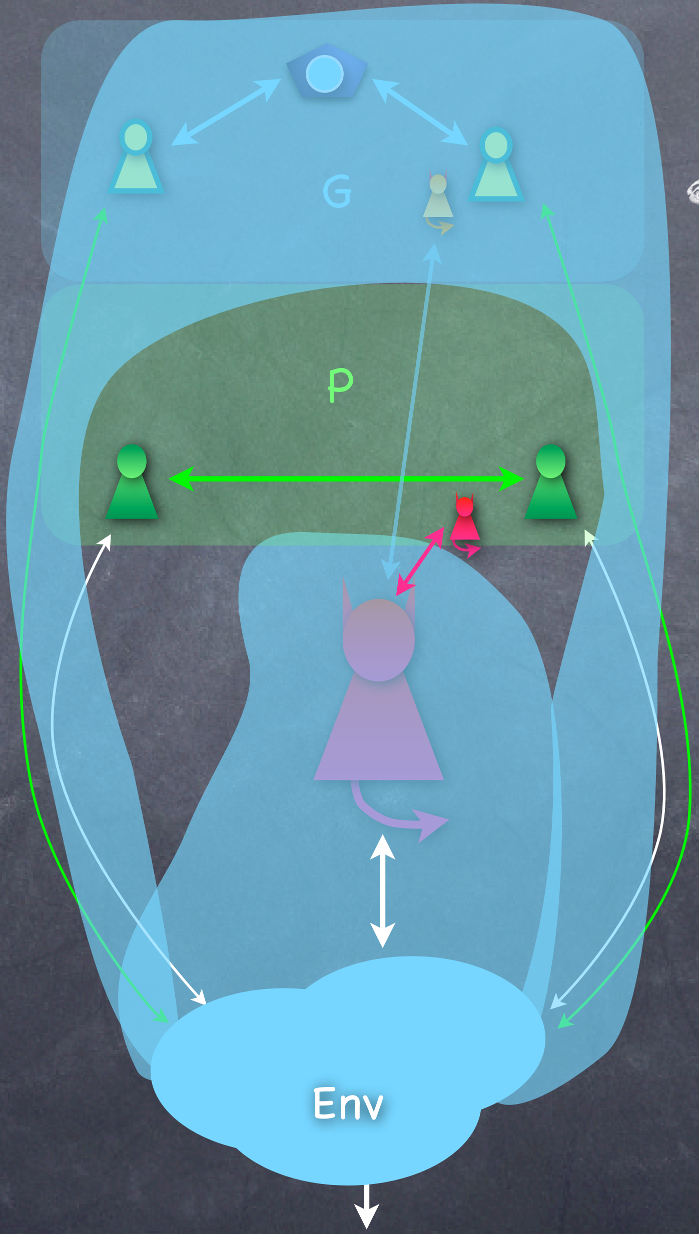

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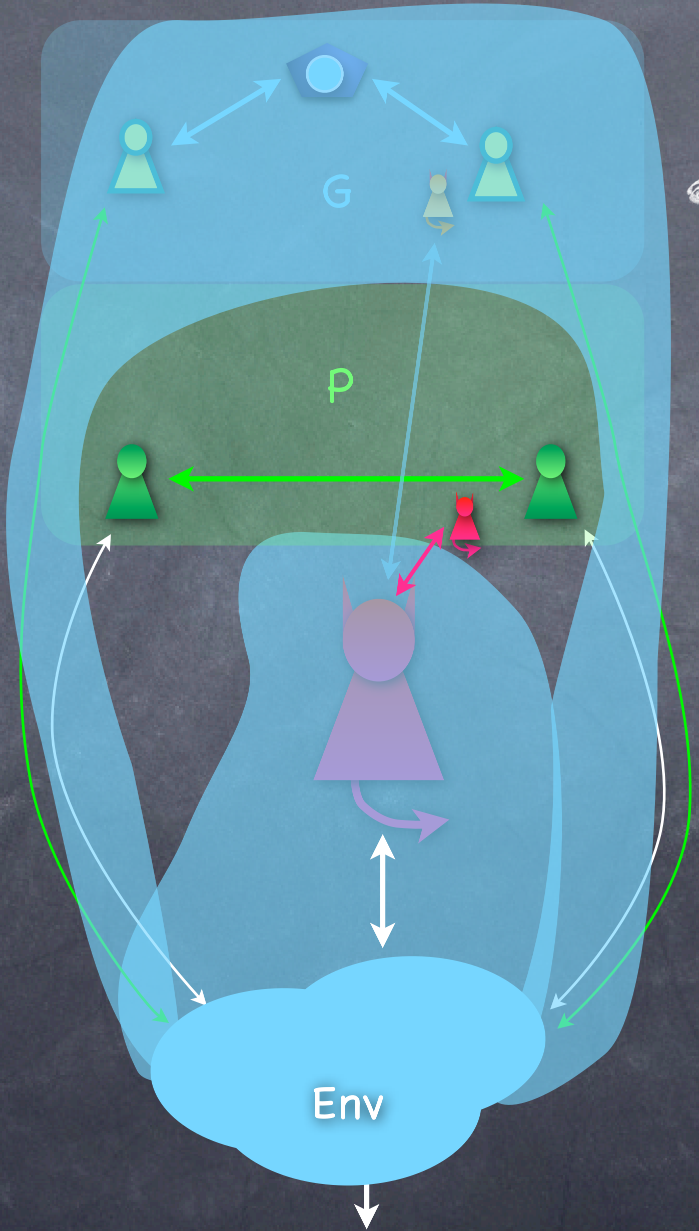

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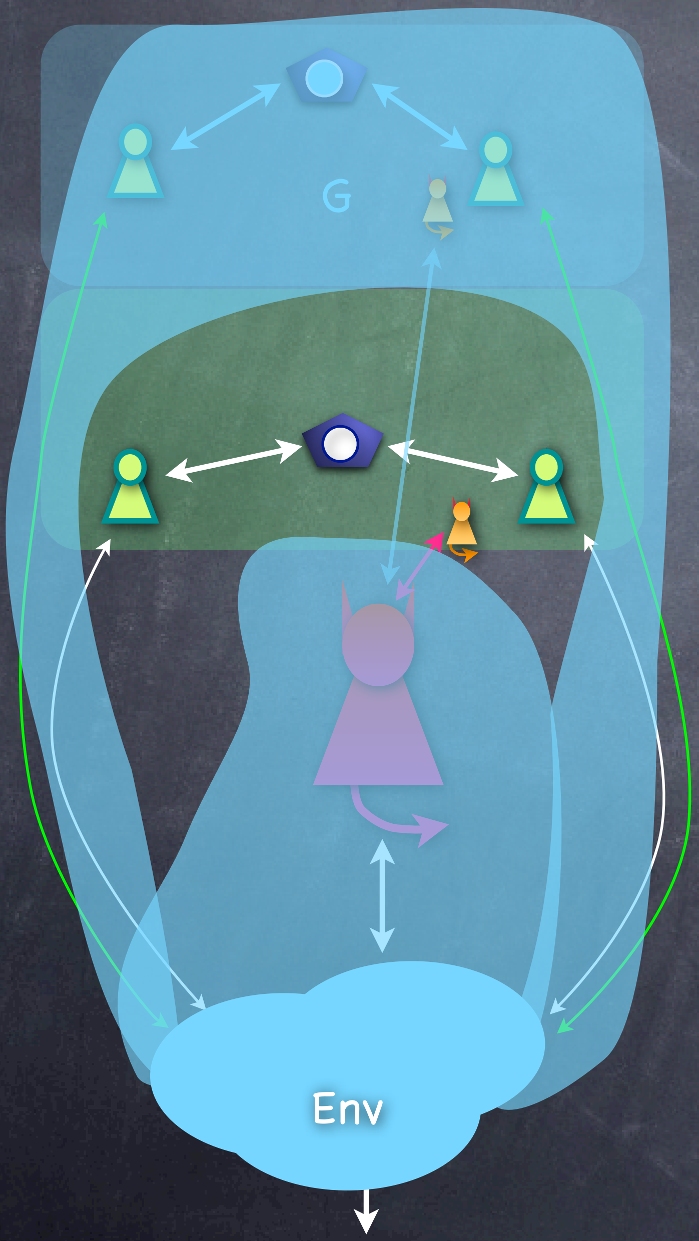

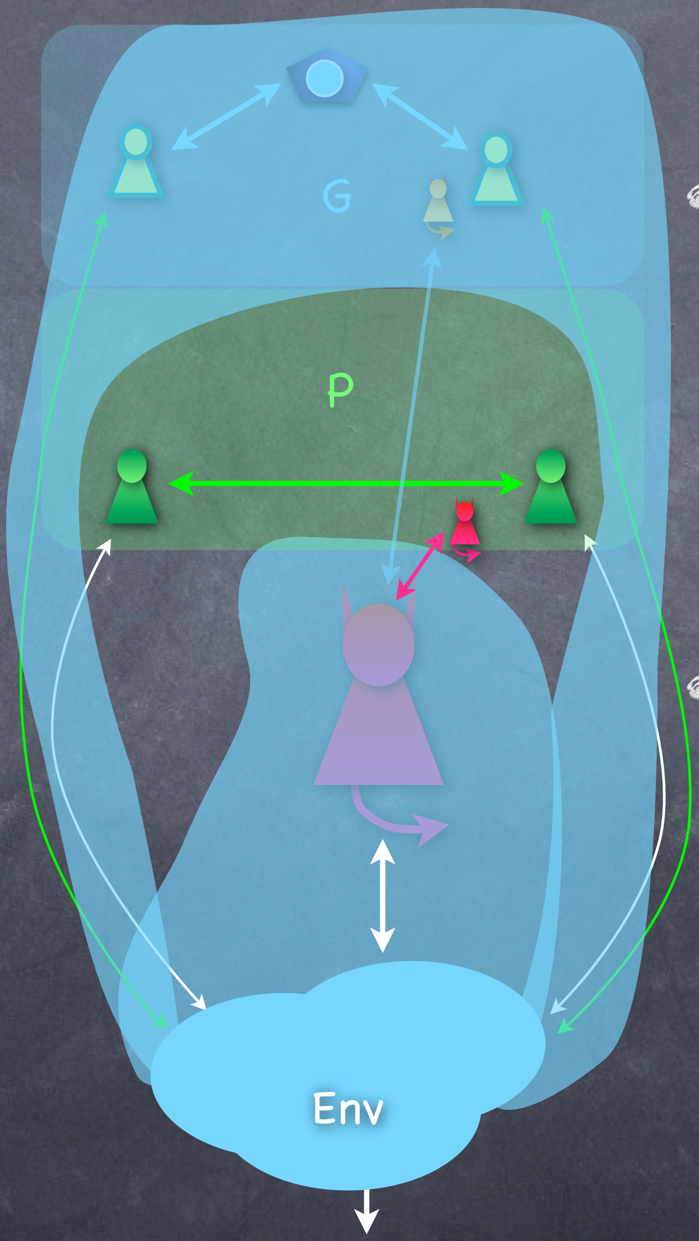

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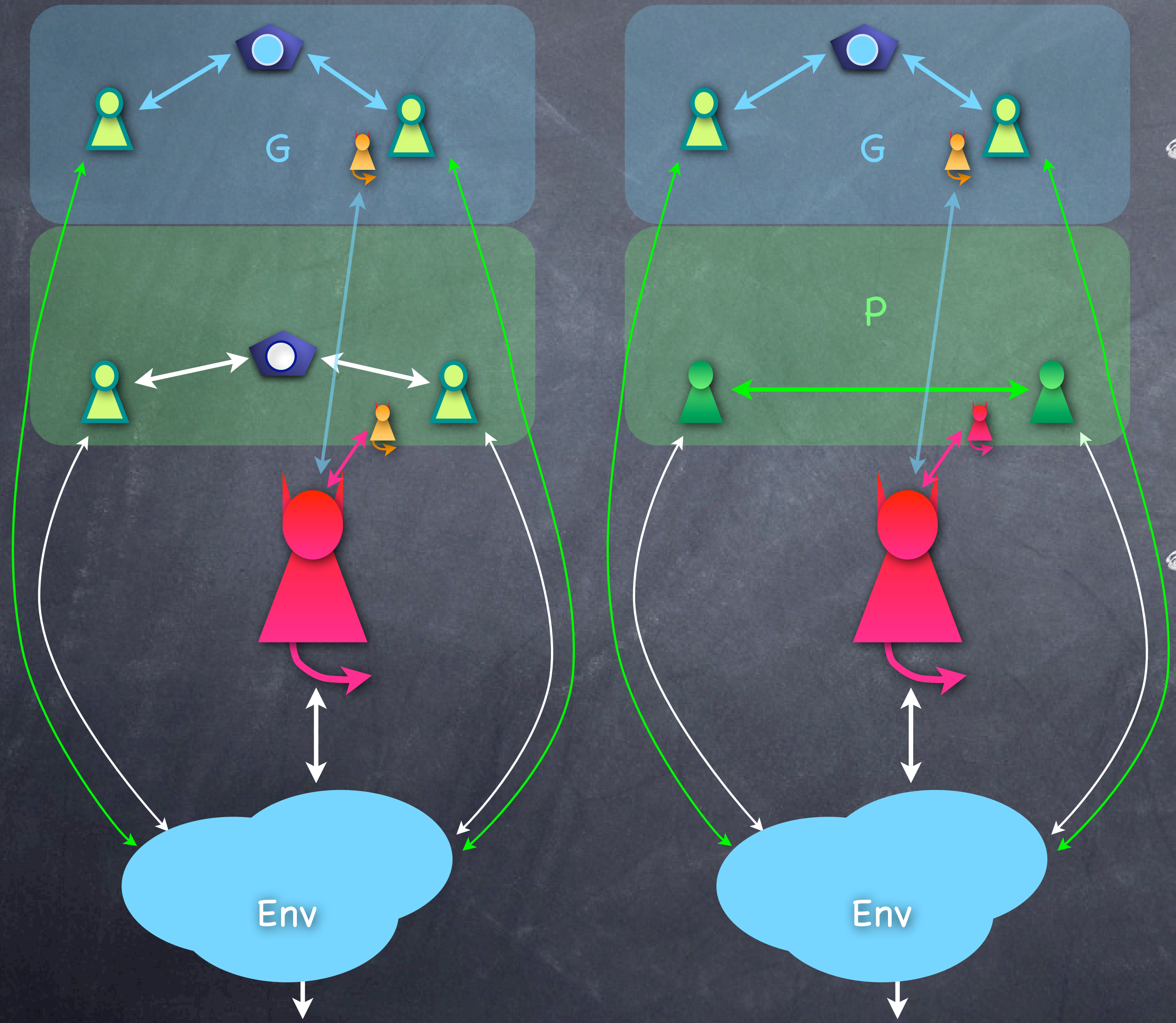

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- For AND: need $z^{(1)} + z^{(2)} +\ldots+ z^{(m)} = [x^{(1)}+\ldots+x^{(m)}]\ [y^{(1)}+\ldots+y^{(m)}]$ (and $z^{(i)}$ random otherwise). Will use OT.

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- Reconstruct the output: all parties send their shares of the output wire for party i to that party. Party i adds up all the shares of that output wire.

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  - General Multi-party: "Shared Evaluation"
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    - This is a statement about the messages so far (publicly known) and randomness and input (committed using CaP)

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