

ZK Proofs (cntd.)

Universal Composition



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# Universal Composition

Lecture 16



RECALL

# An Example





RECALL

# An Example

- Graph Isomorphism





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- **Graph Isomorphism**
- $(G_0, G_1)$  in L iff there exists an isomorphism  $\sigma$  such that  $\sigma(G_0) = G_1$





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 $\pi^*(G_b) = G^*$





RECALL

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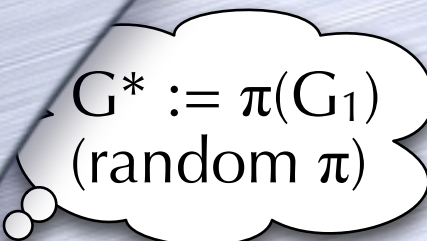
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RECALL

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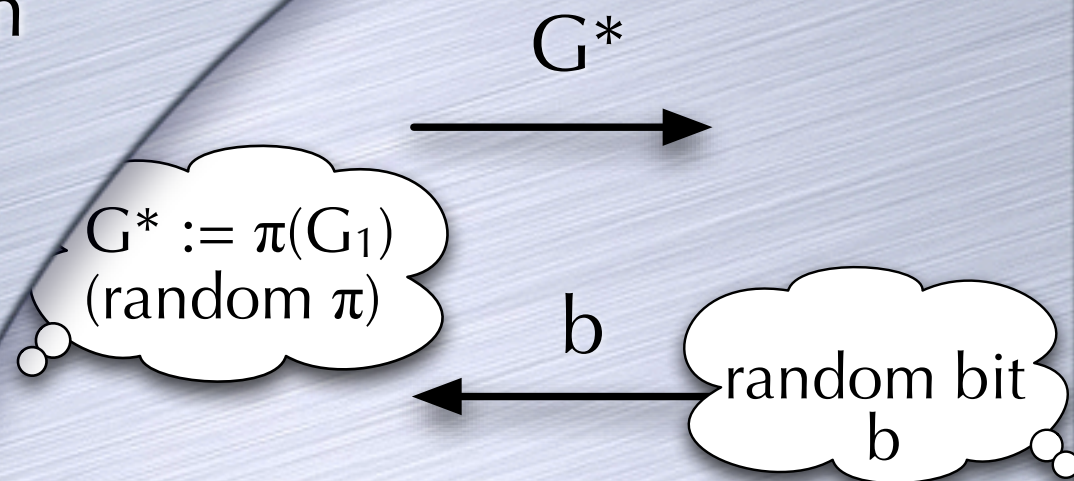
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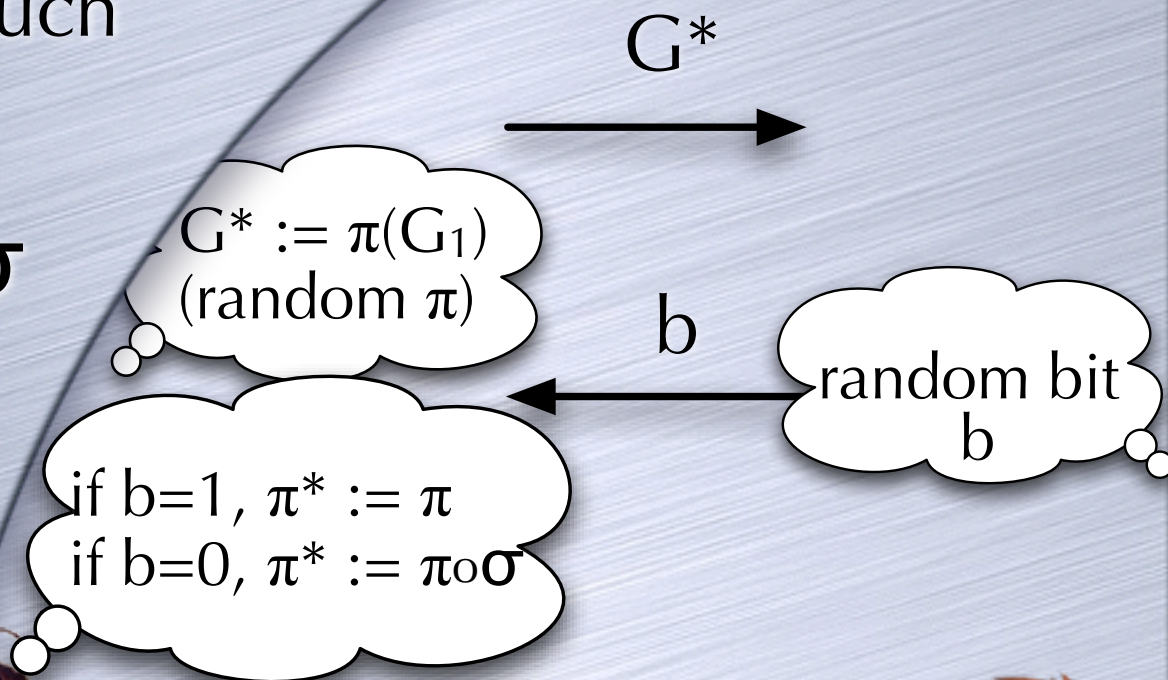
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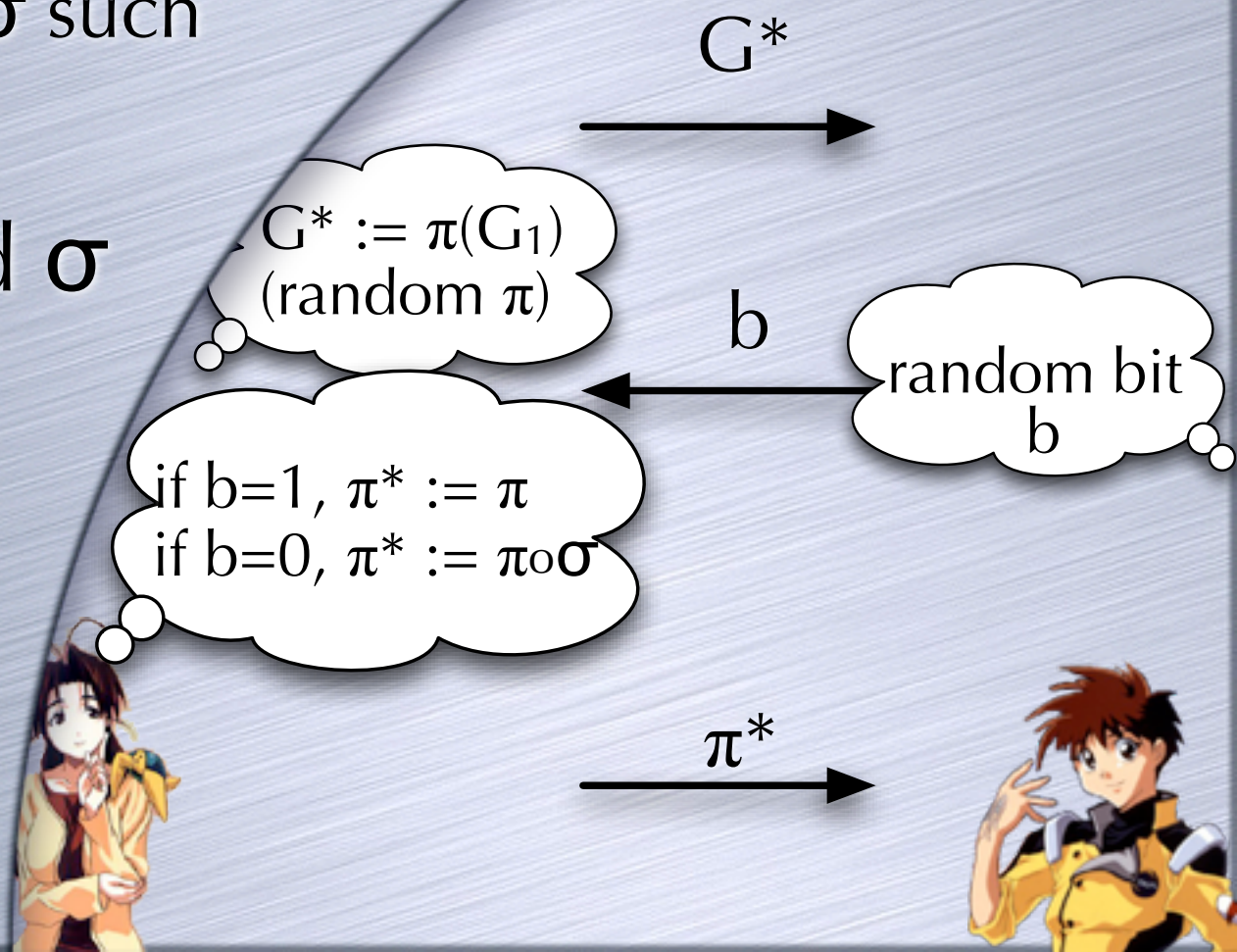
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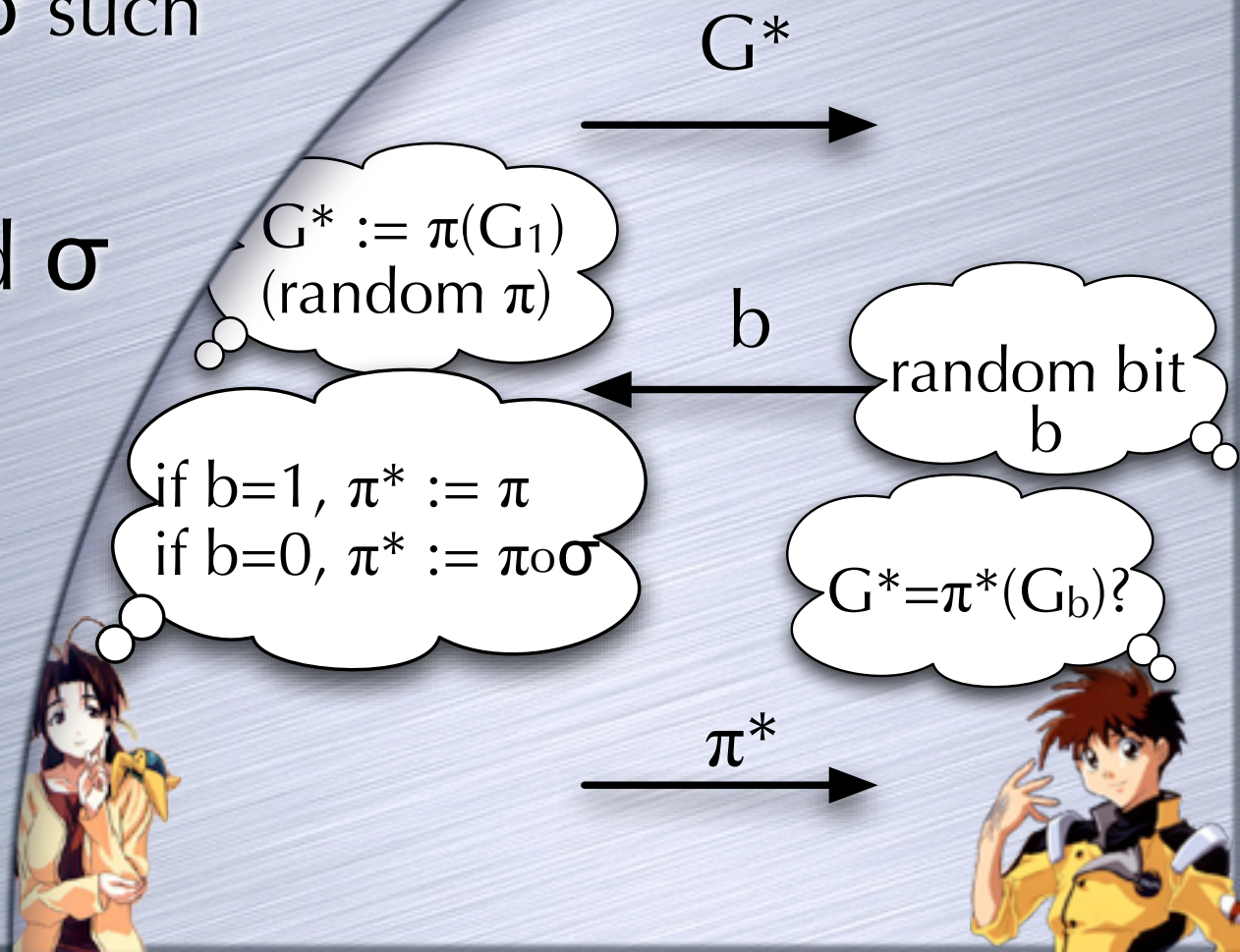
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**Bob:** But I picked  $b$  at random and she had no trouble answering me...



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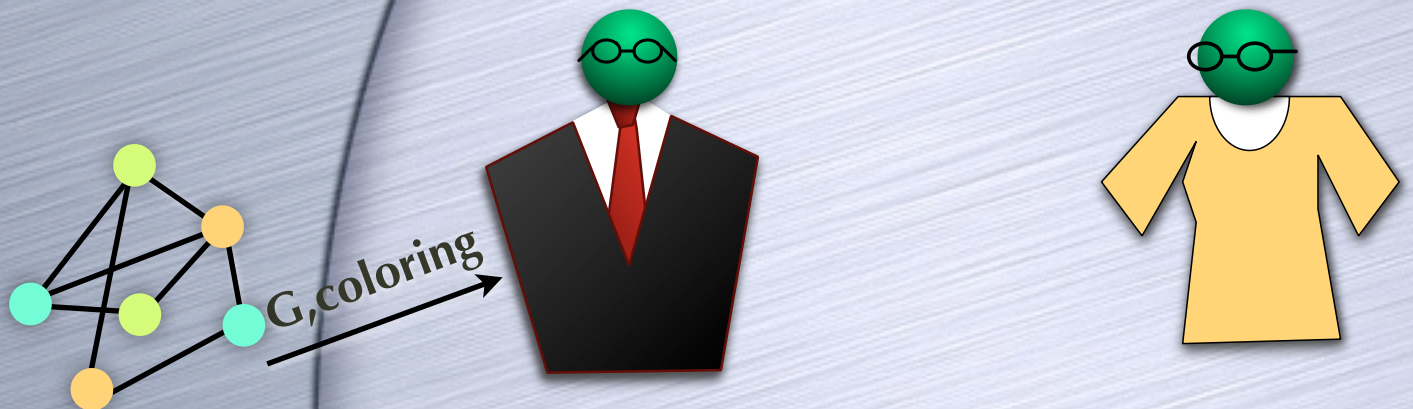
# A ZK Proof for Graph Colorability





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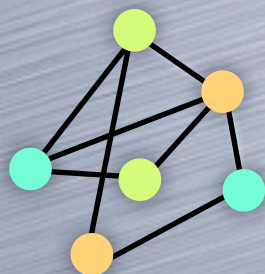




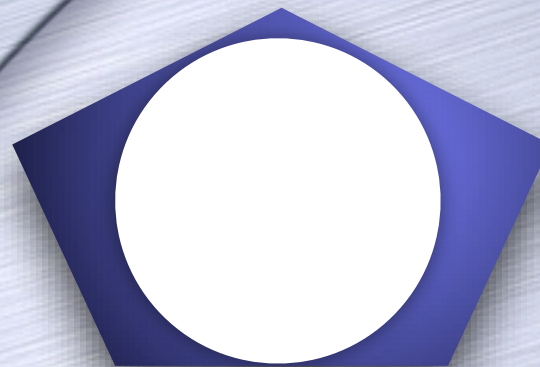
RECALL

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- Uses a commitment protocol as a subroutine



$G, \text{coloring}$

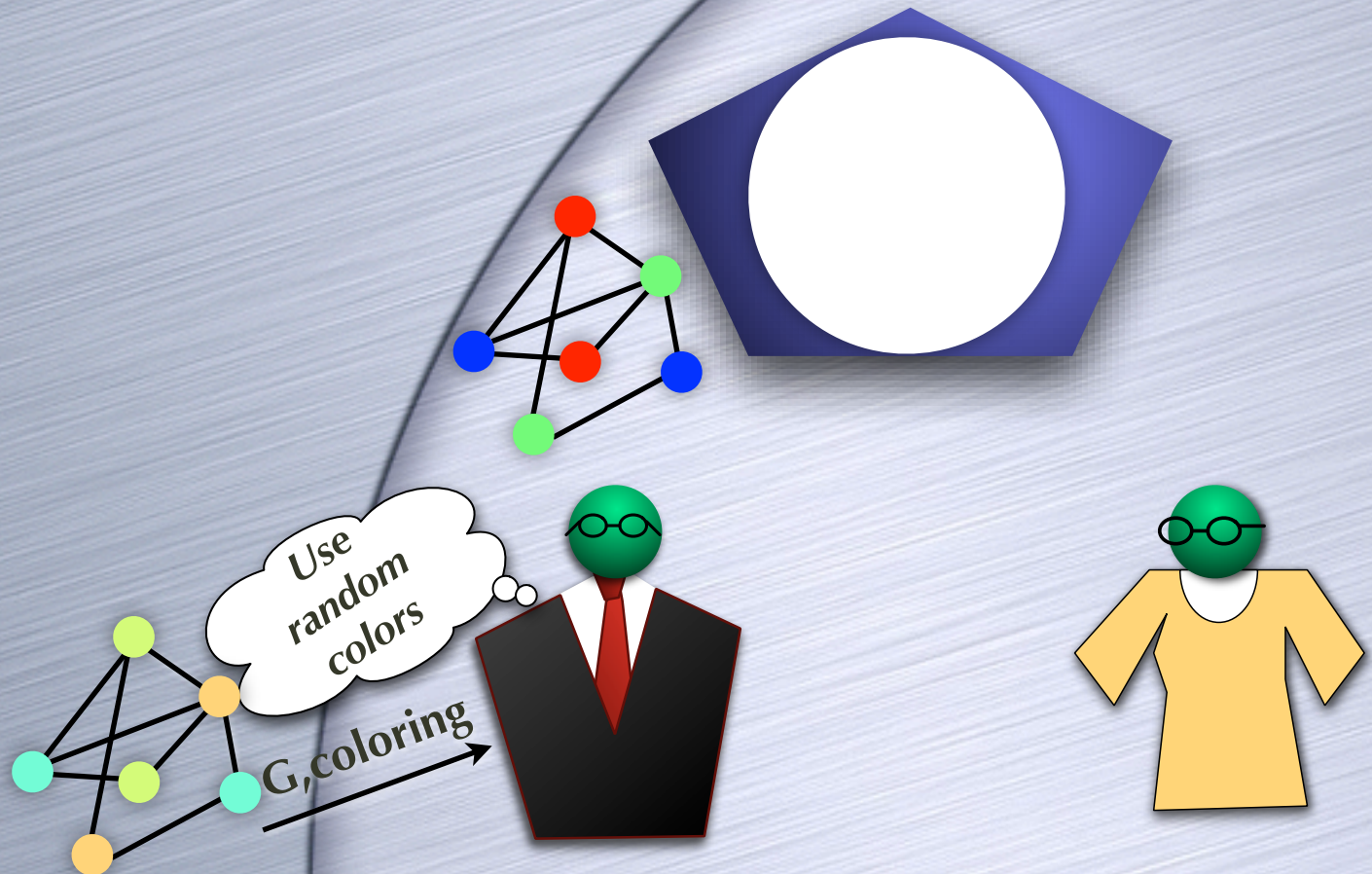




RECALL

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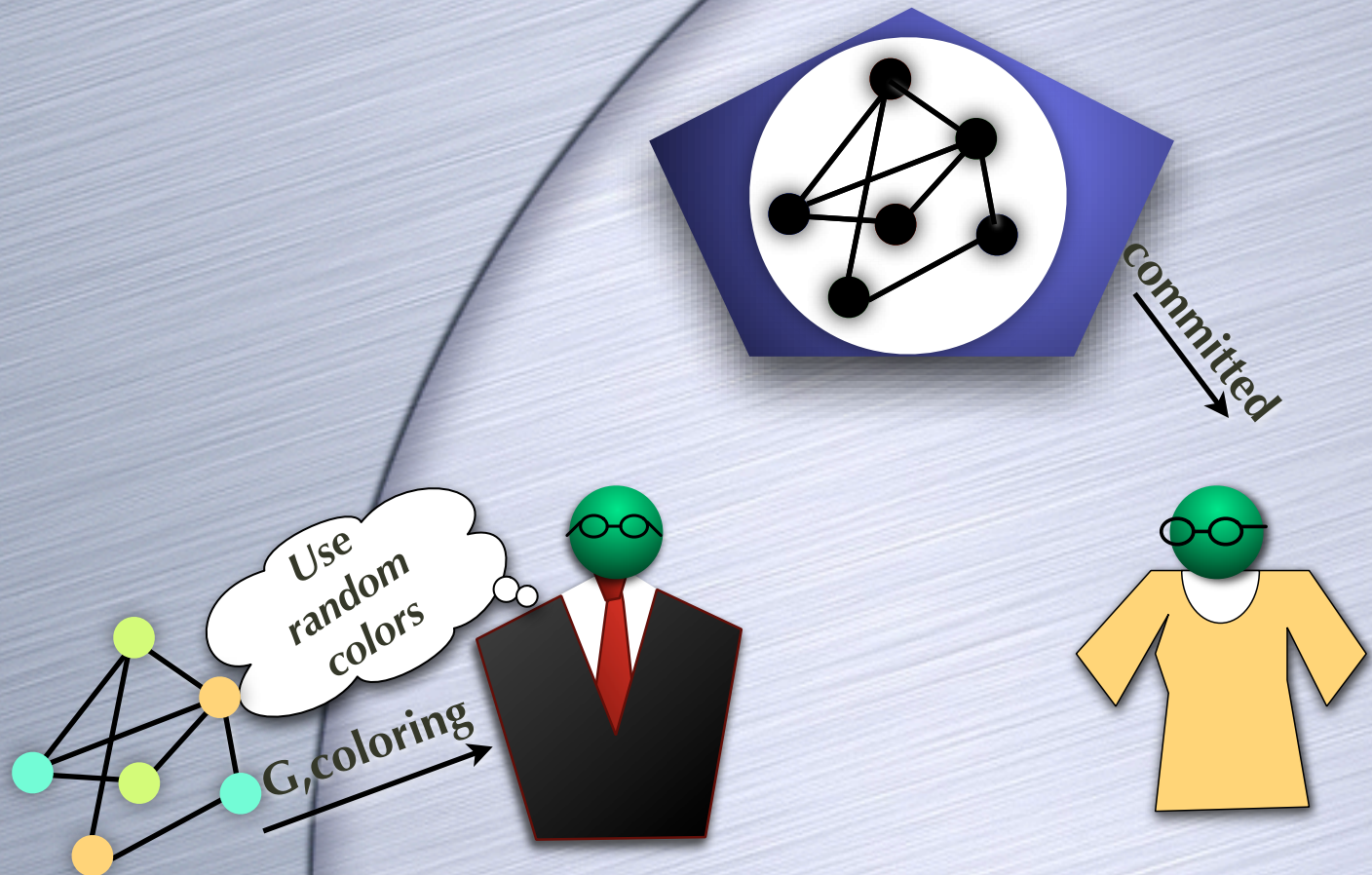




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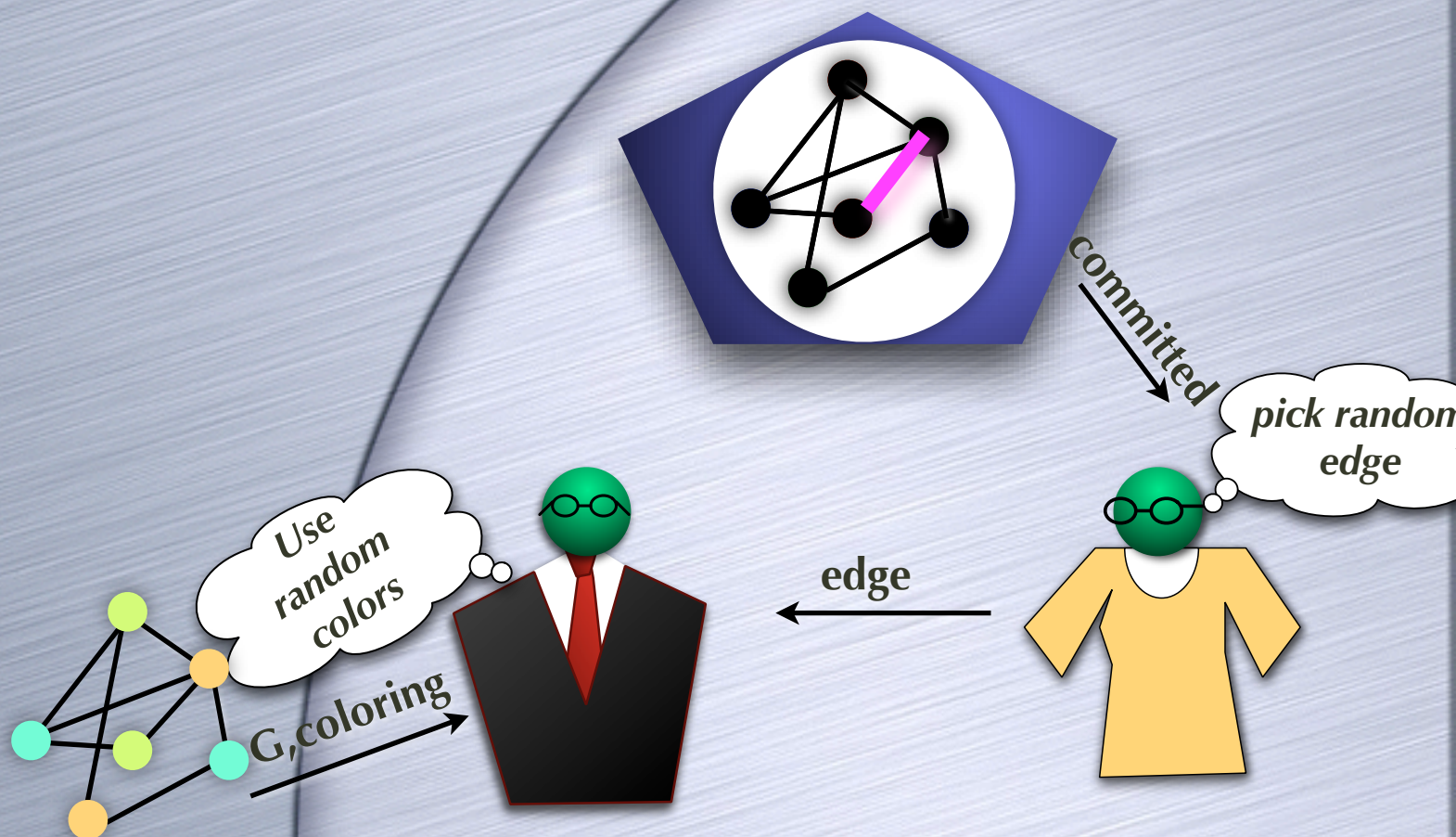




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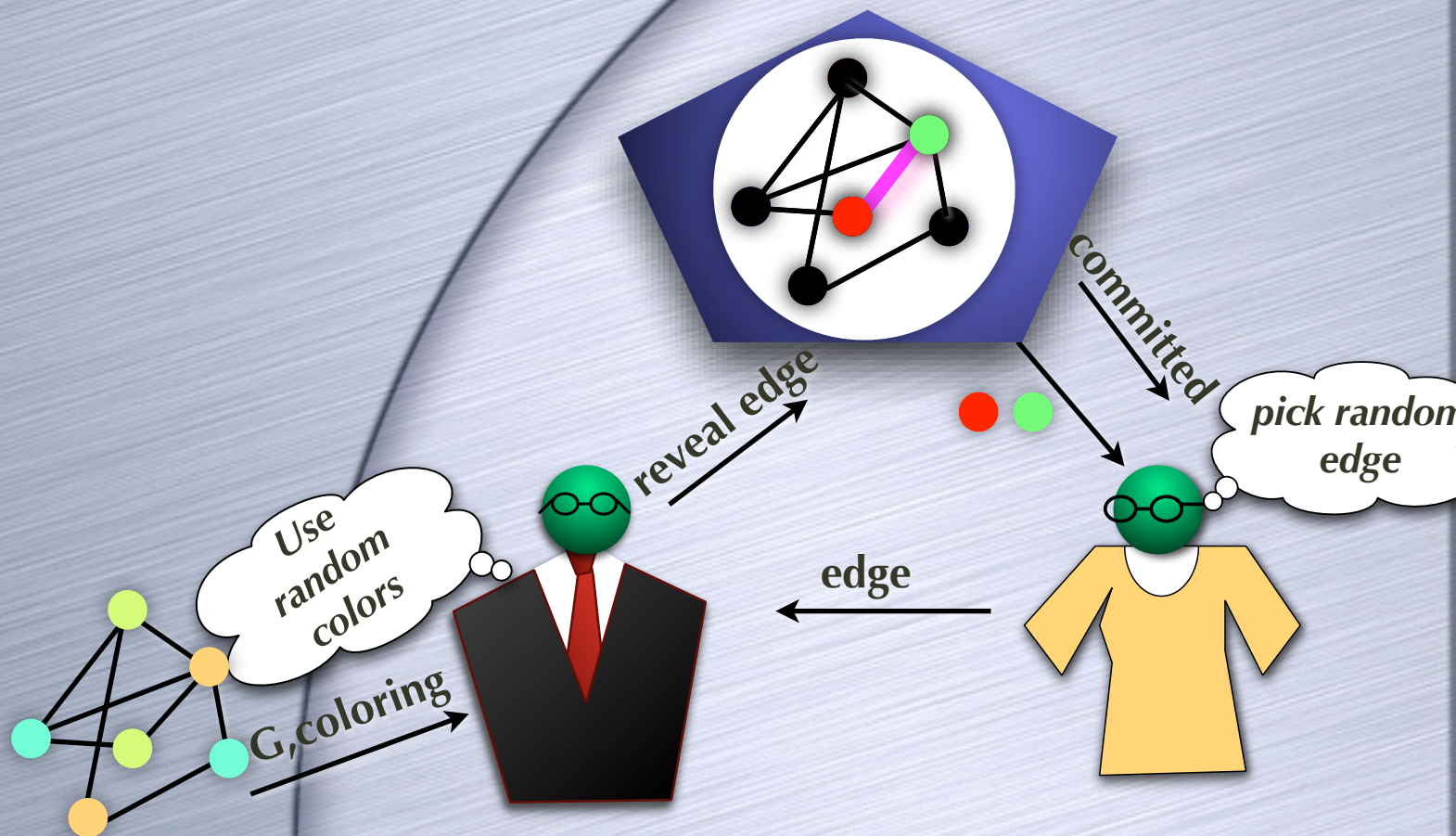




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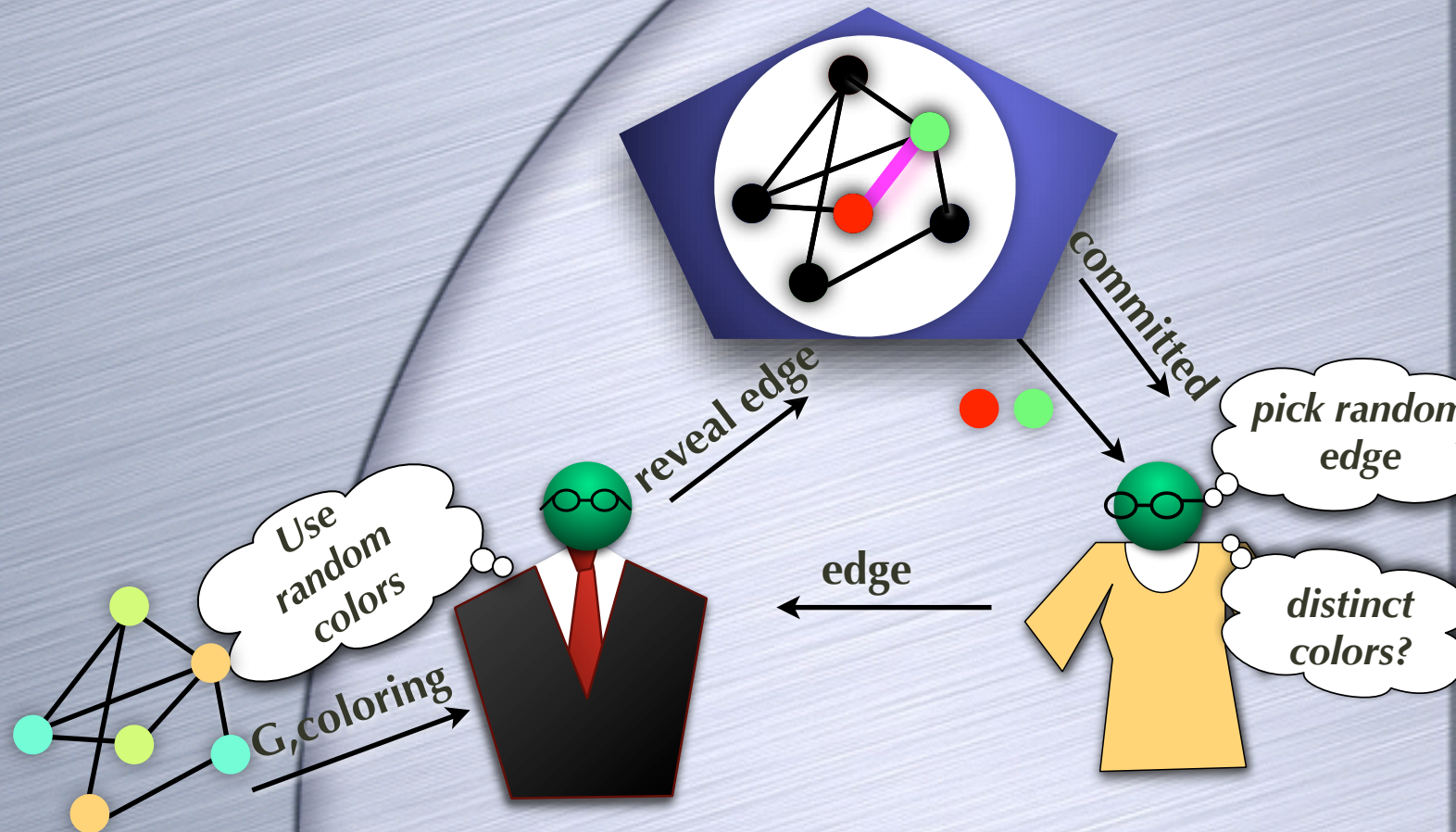




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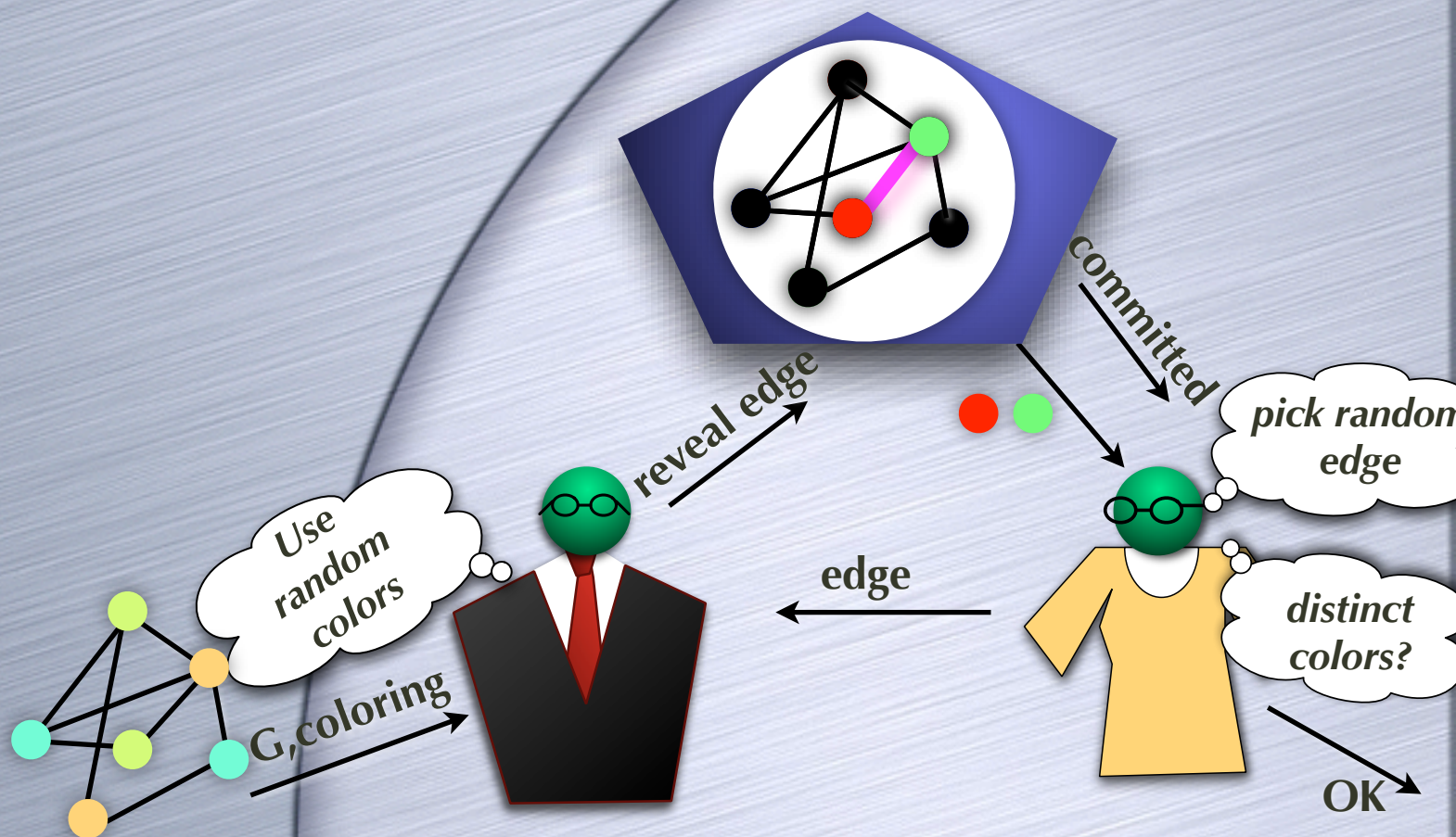




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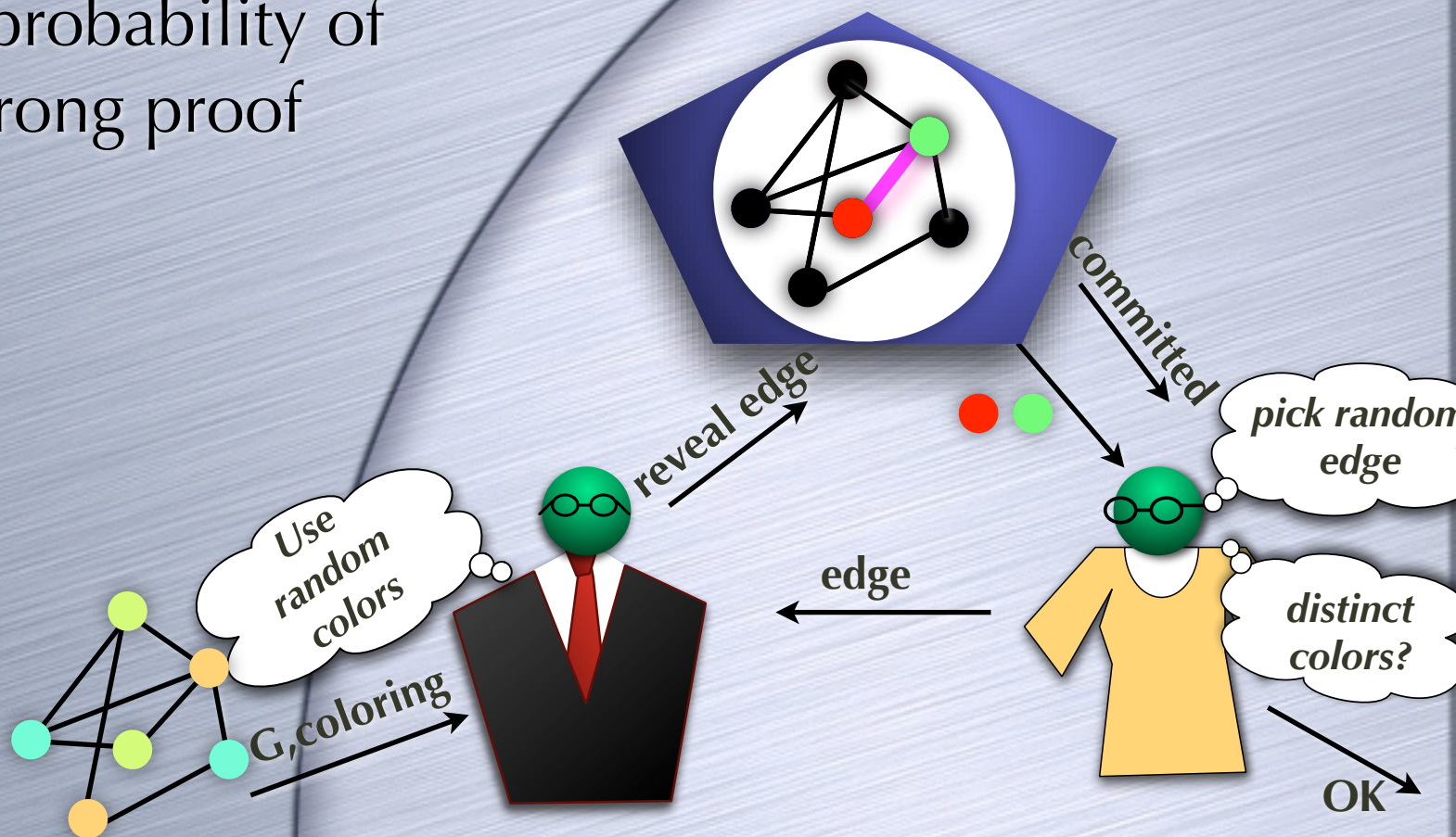




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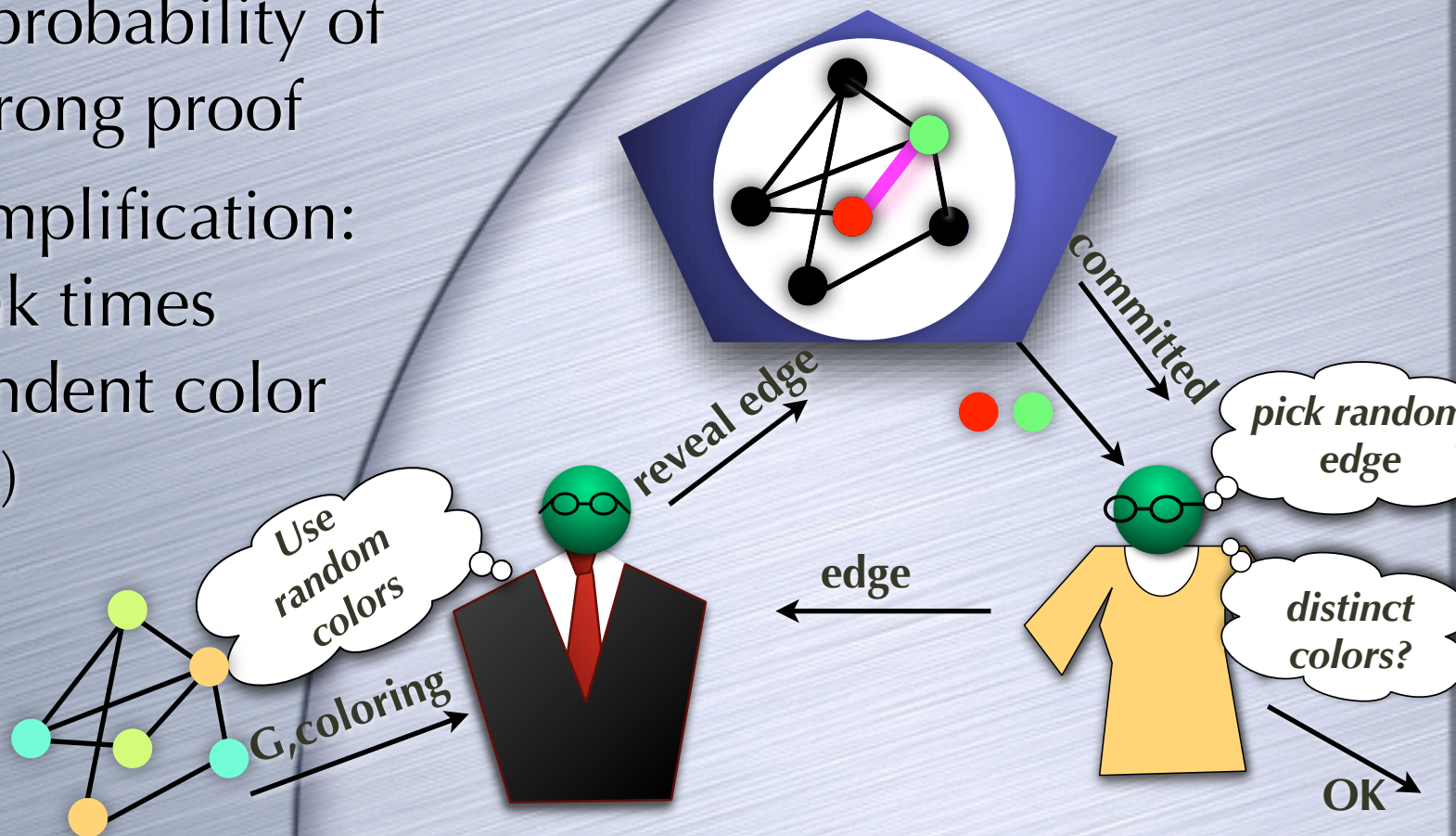




RECALL

# A ZK Proof for Graph Colorability

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- Soundness amplification: Repeat say  $mk$  times (with independent color permutations)





# A Commitment Protocol





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- Using a OWP  $f$  and a hardcore predicate for it  $B$





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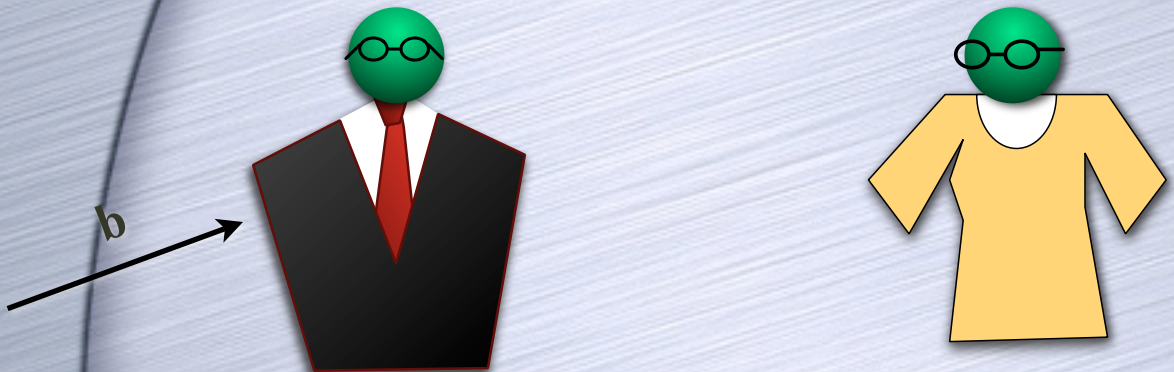
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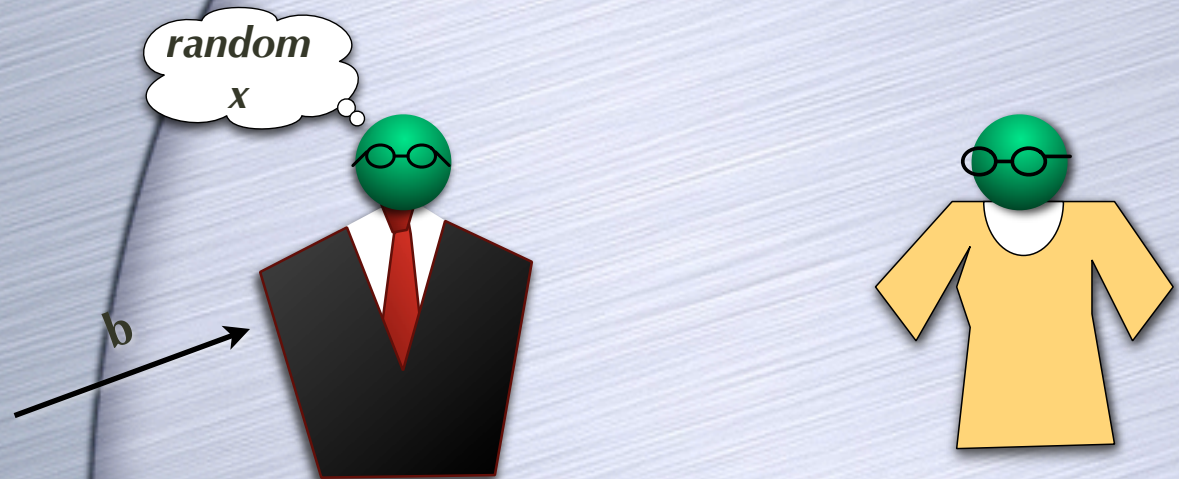
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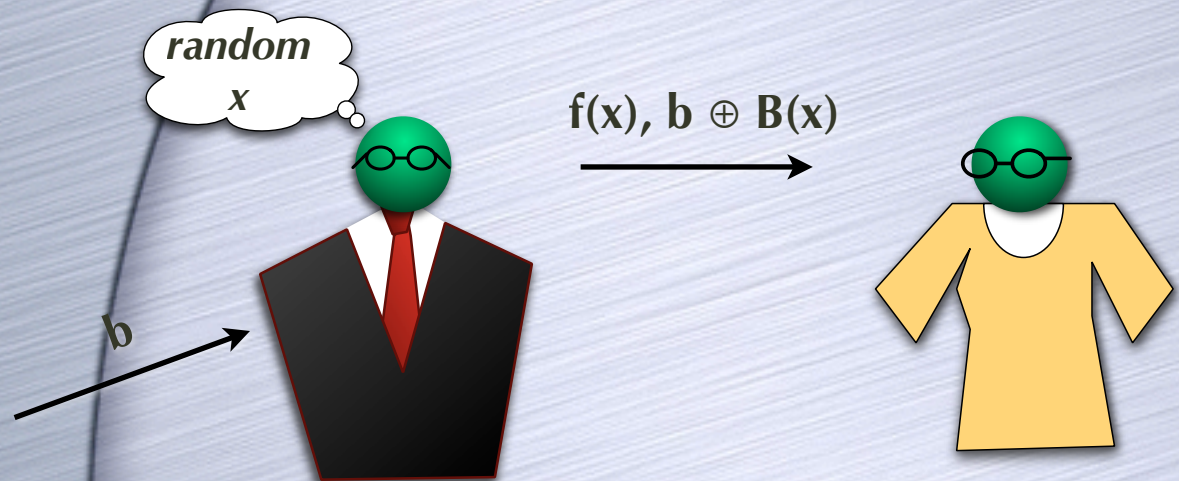
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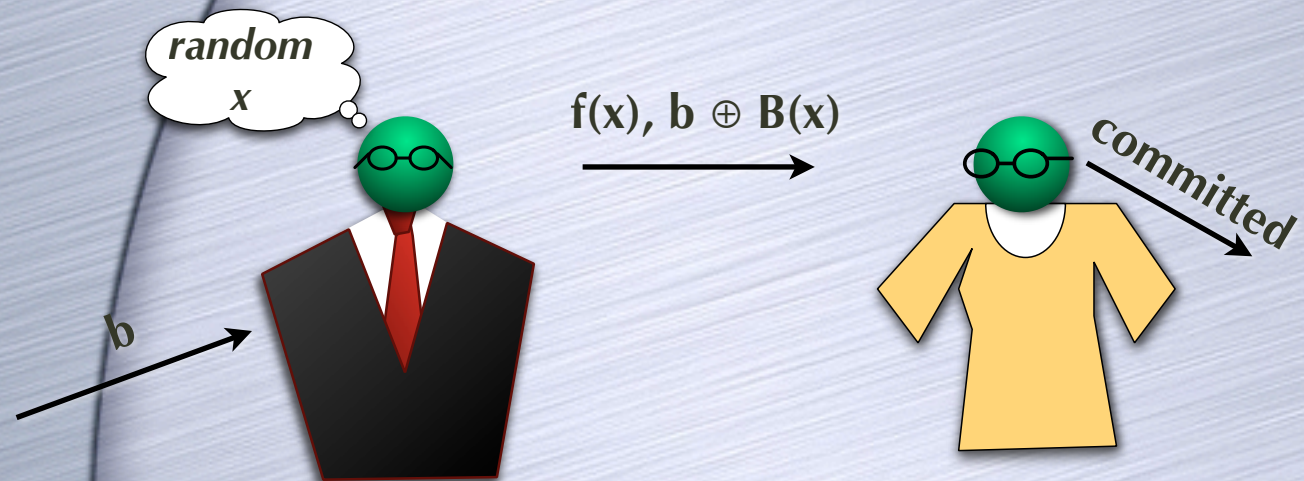
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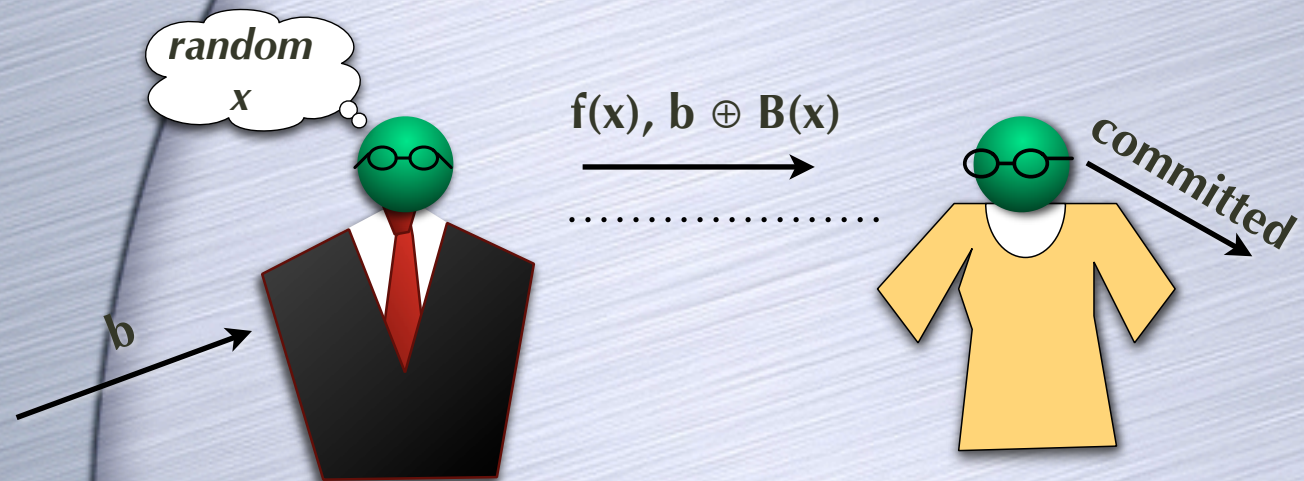
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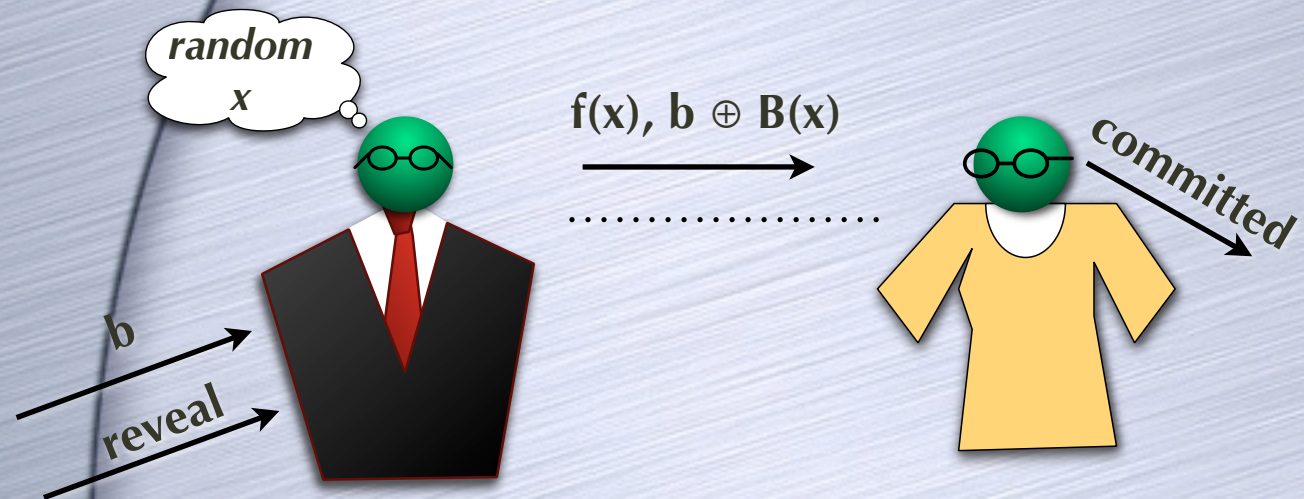
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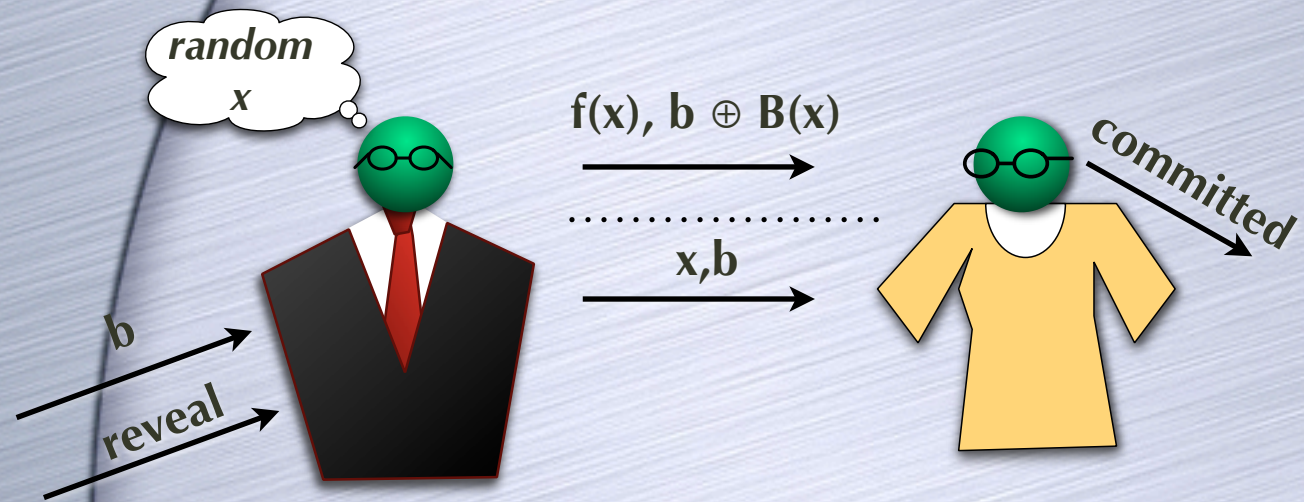
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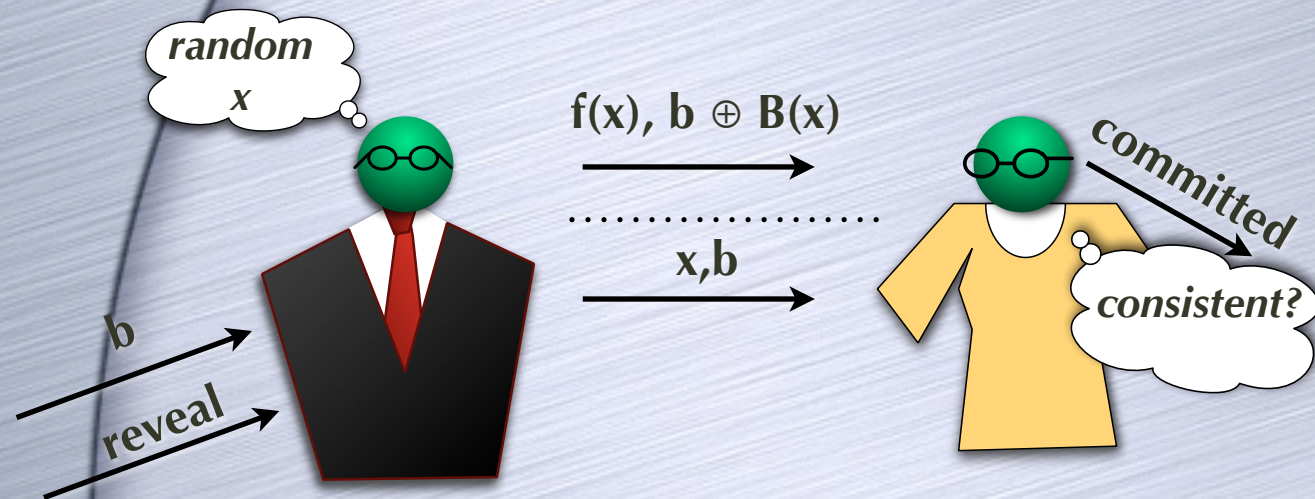
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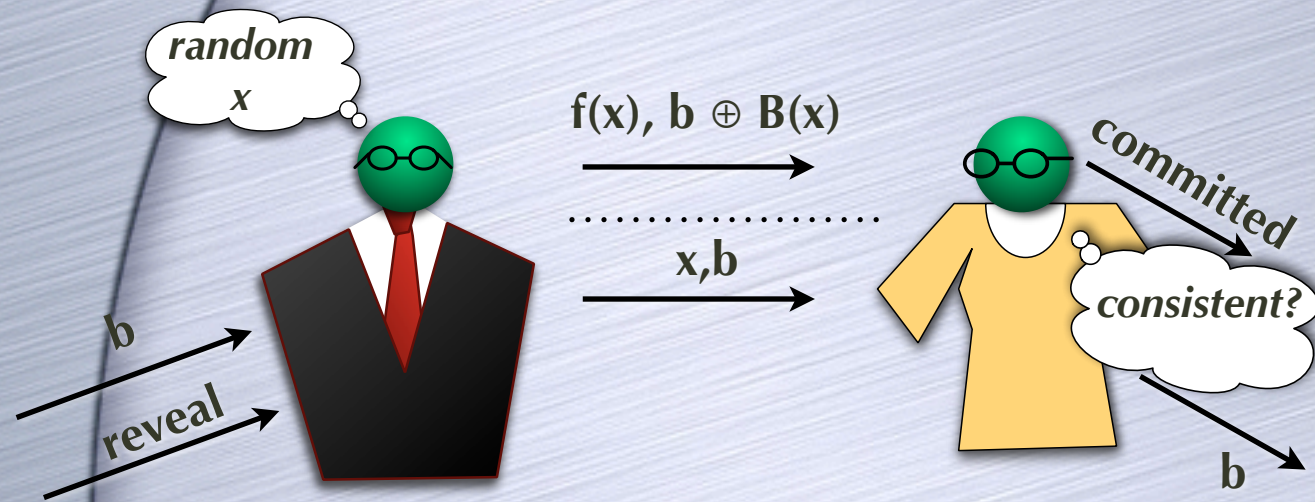
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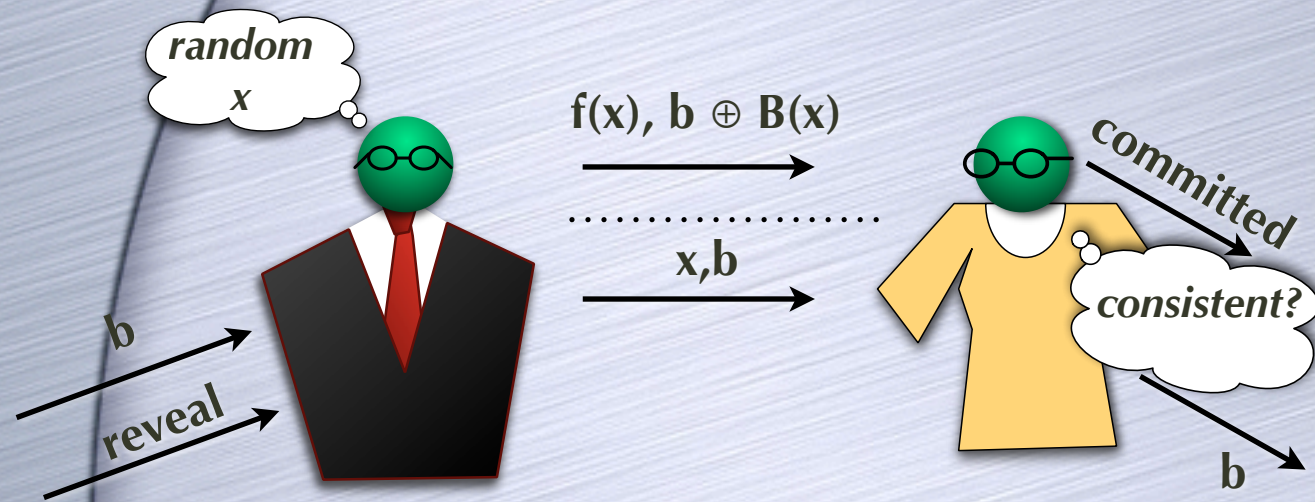
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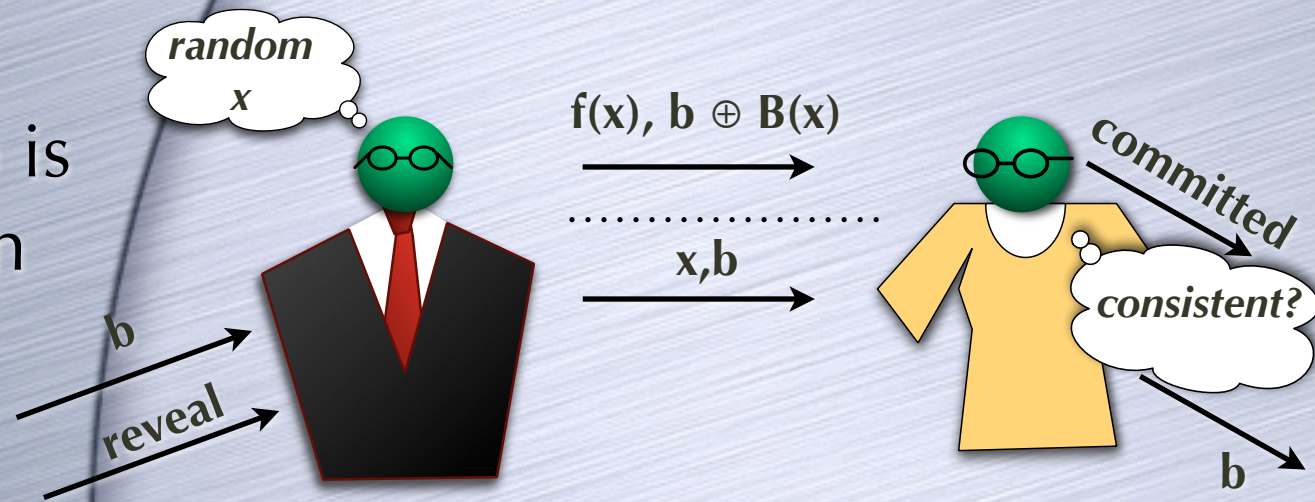
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- Using a OWP  $f$  and a hardcore predicate for it  $B$
- Satisfies only classical (IND) security, in terms of hiding and binding
- Perfectly binding because  $f$  is a permutation
- Hiding because  $B(x)$  is pseudorandom given  $f(x)$





# ZK Proofs: What for?





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$x_1$

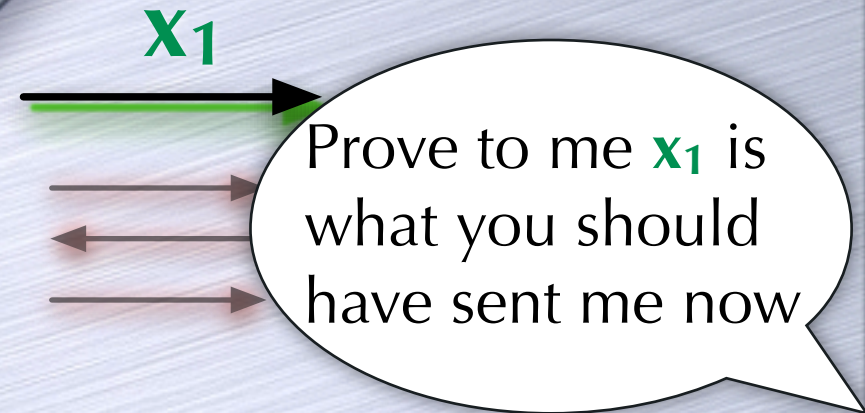
Prove to me  $x_1$  is what you should have sent me now





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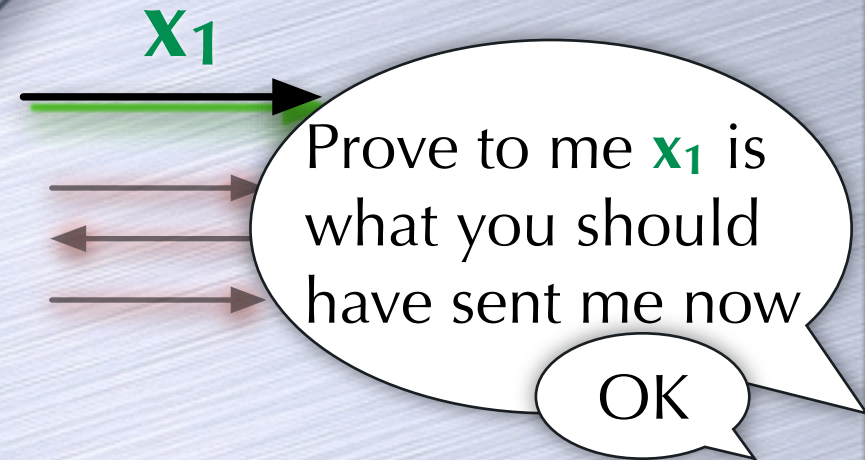
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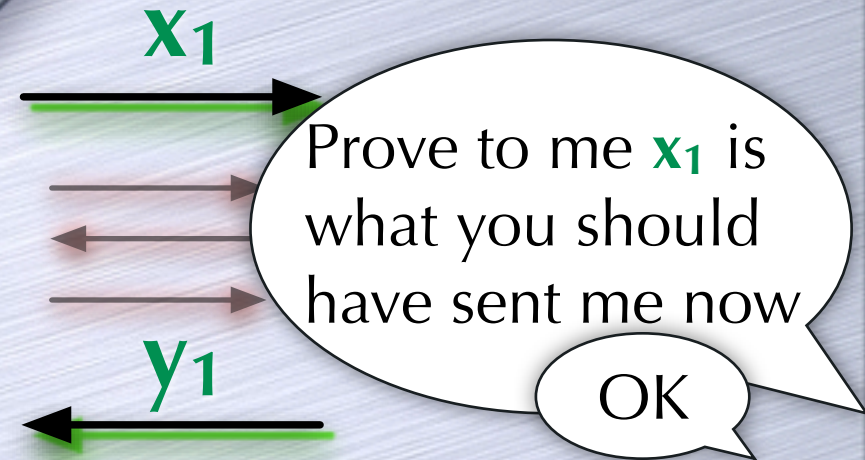
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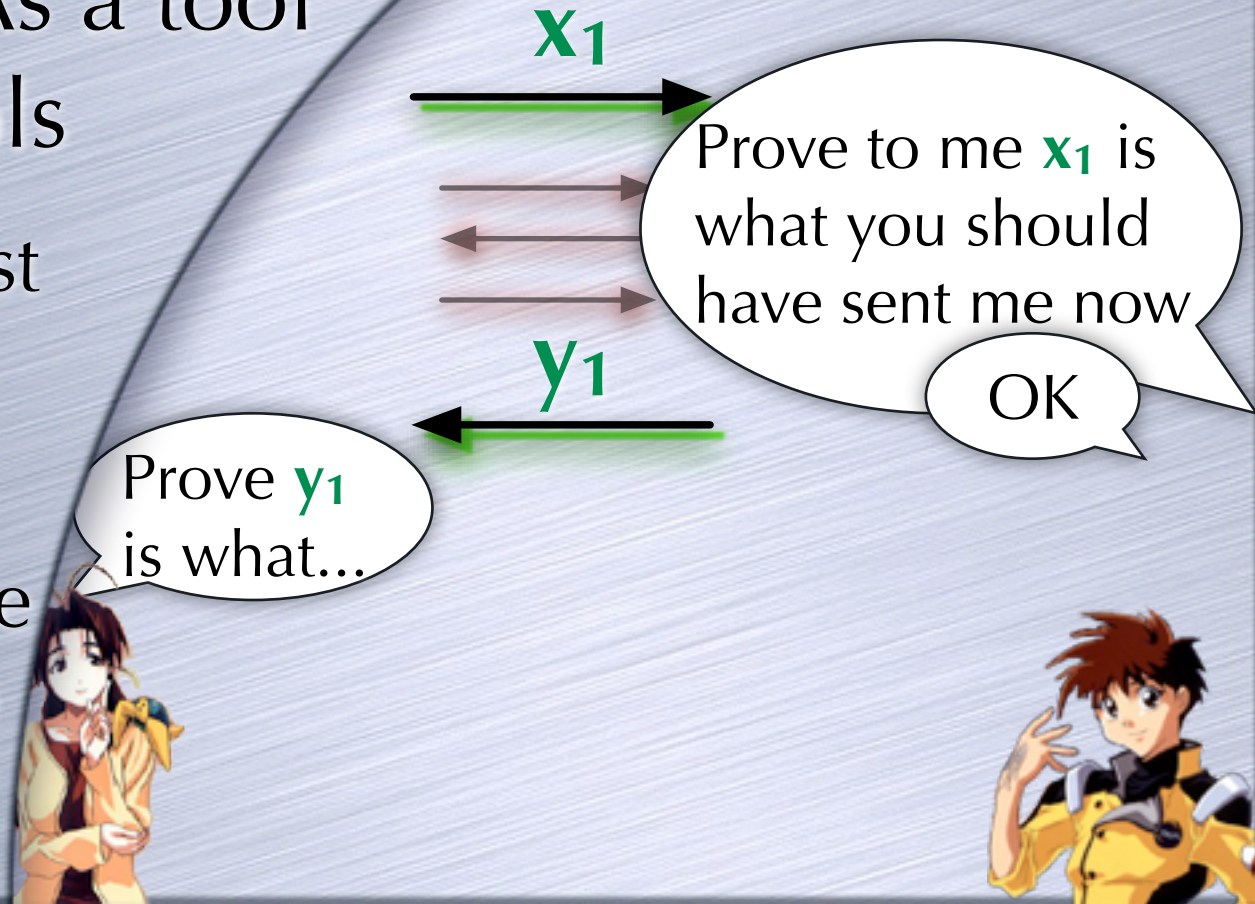
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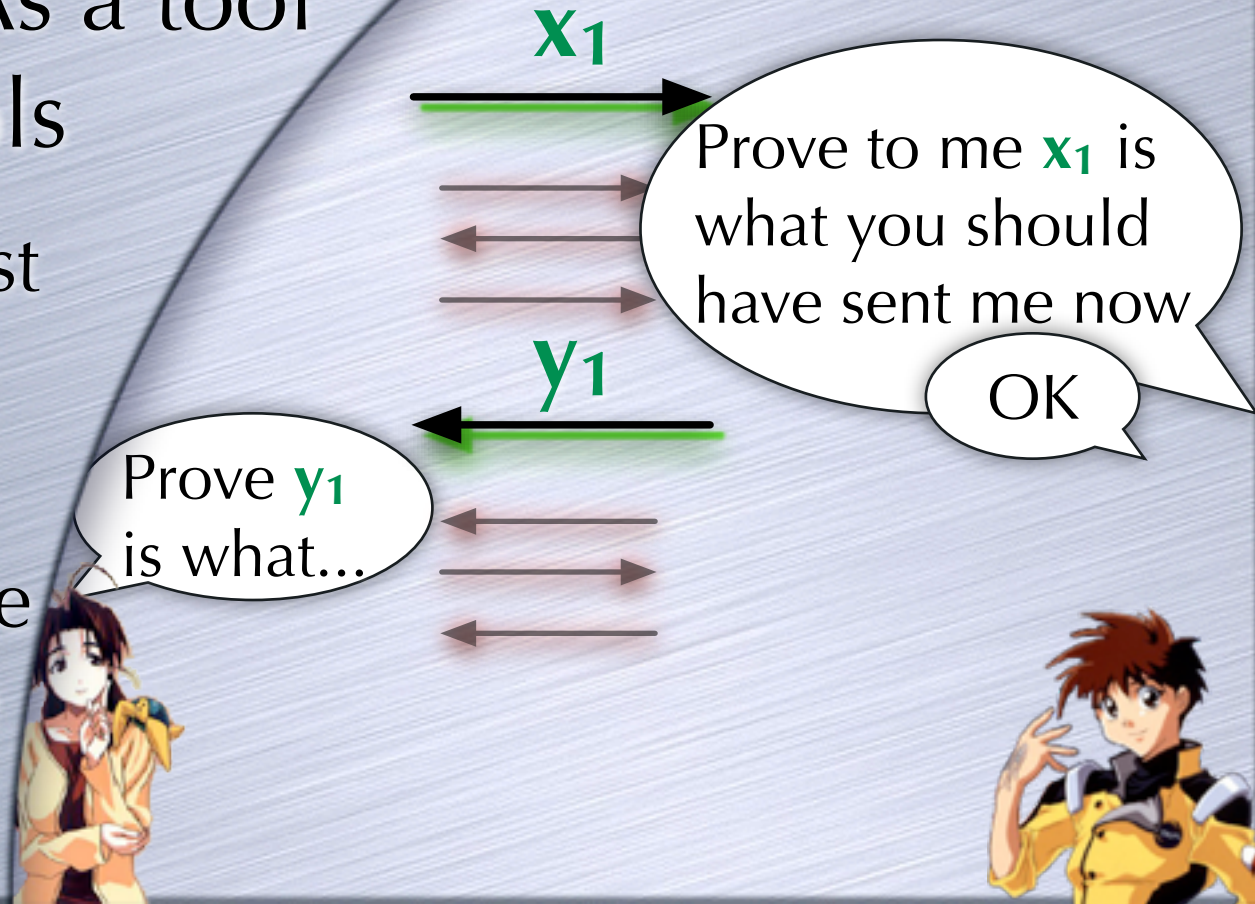
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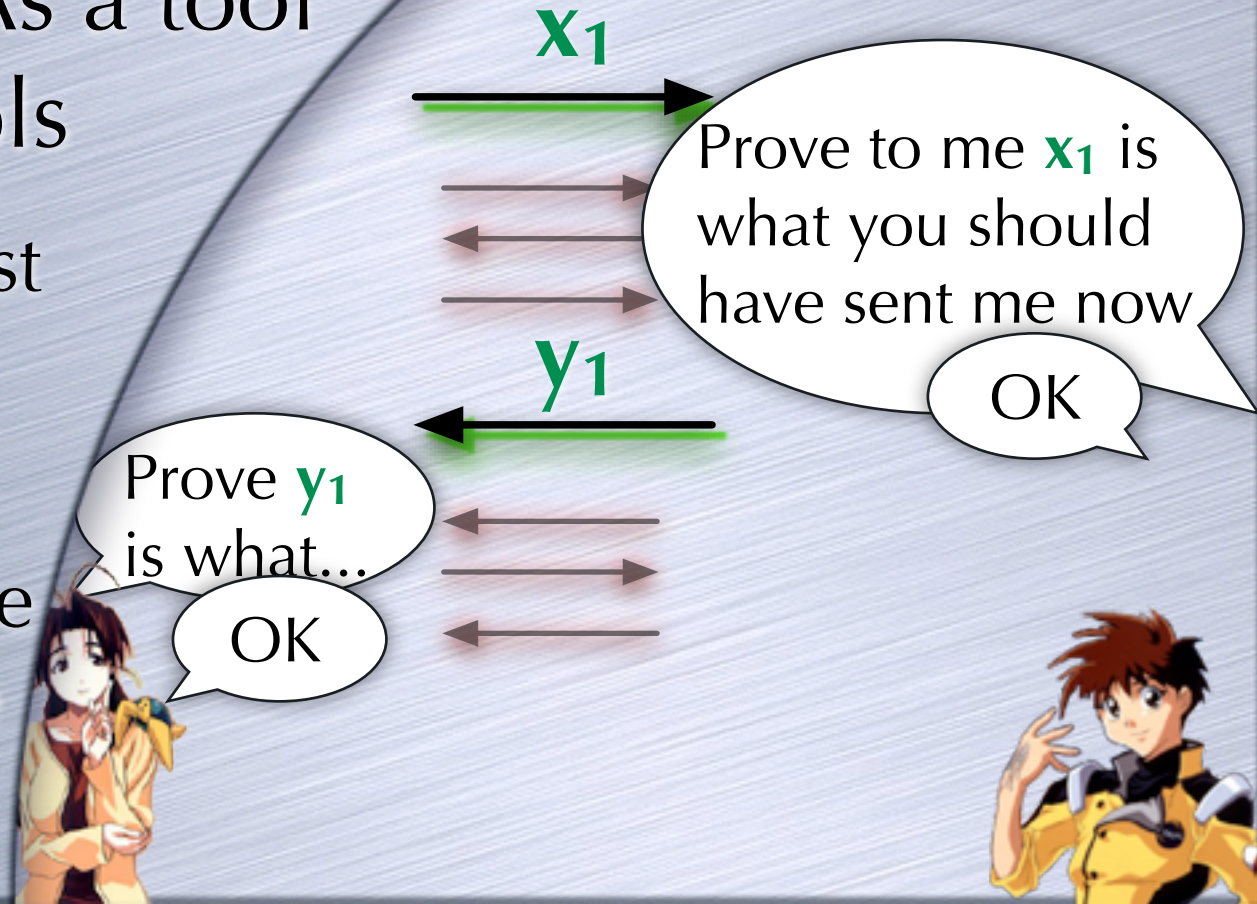
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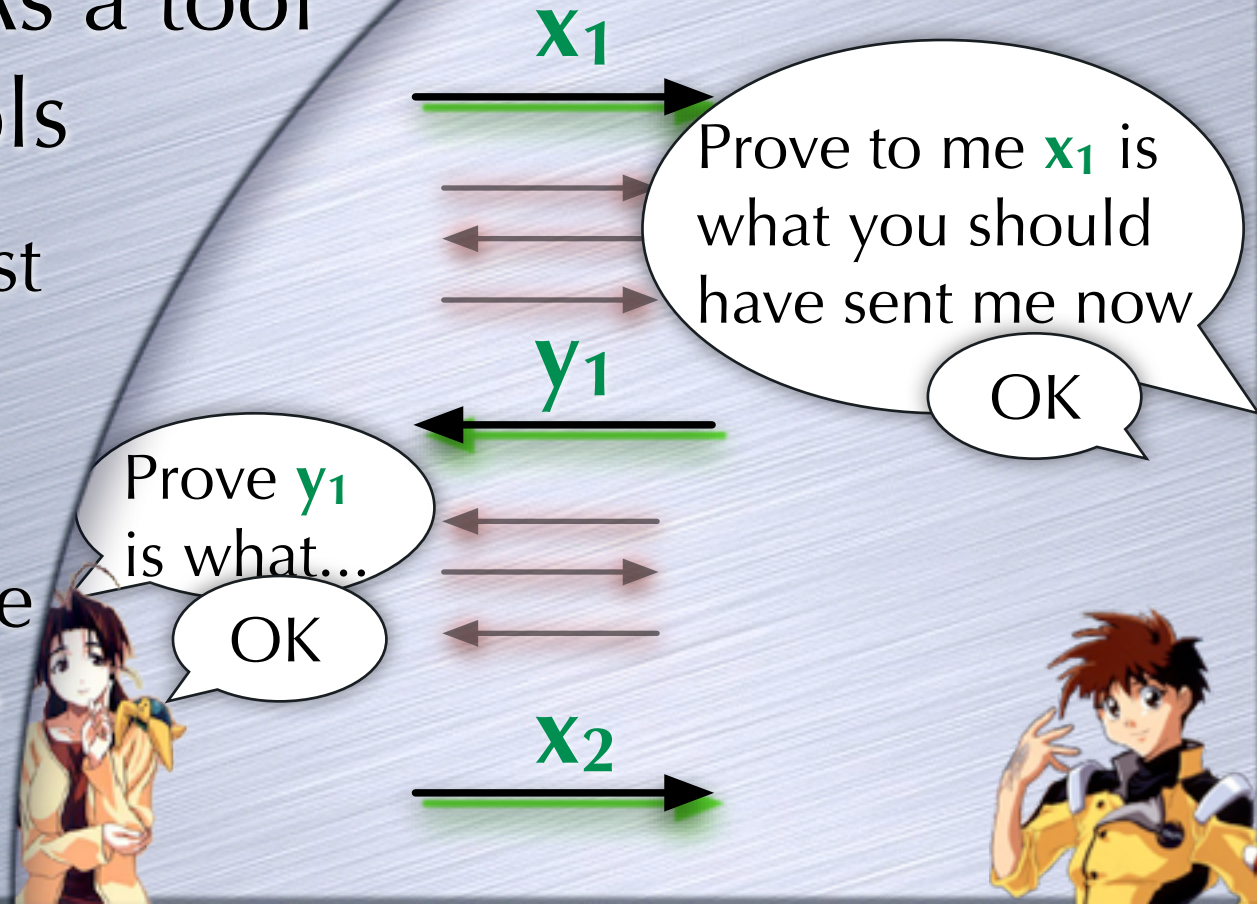
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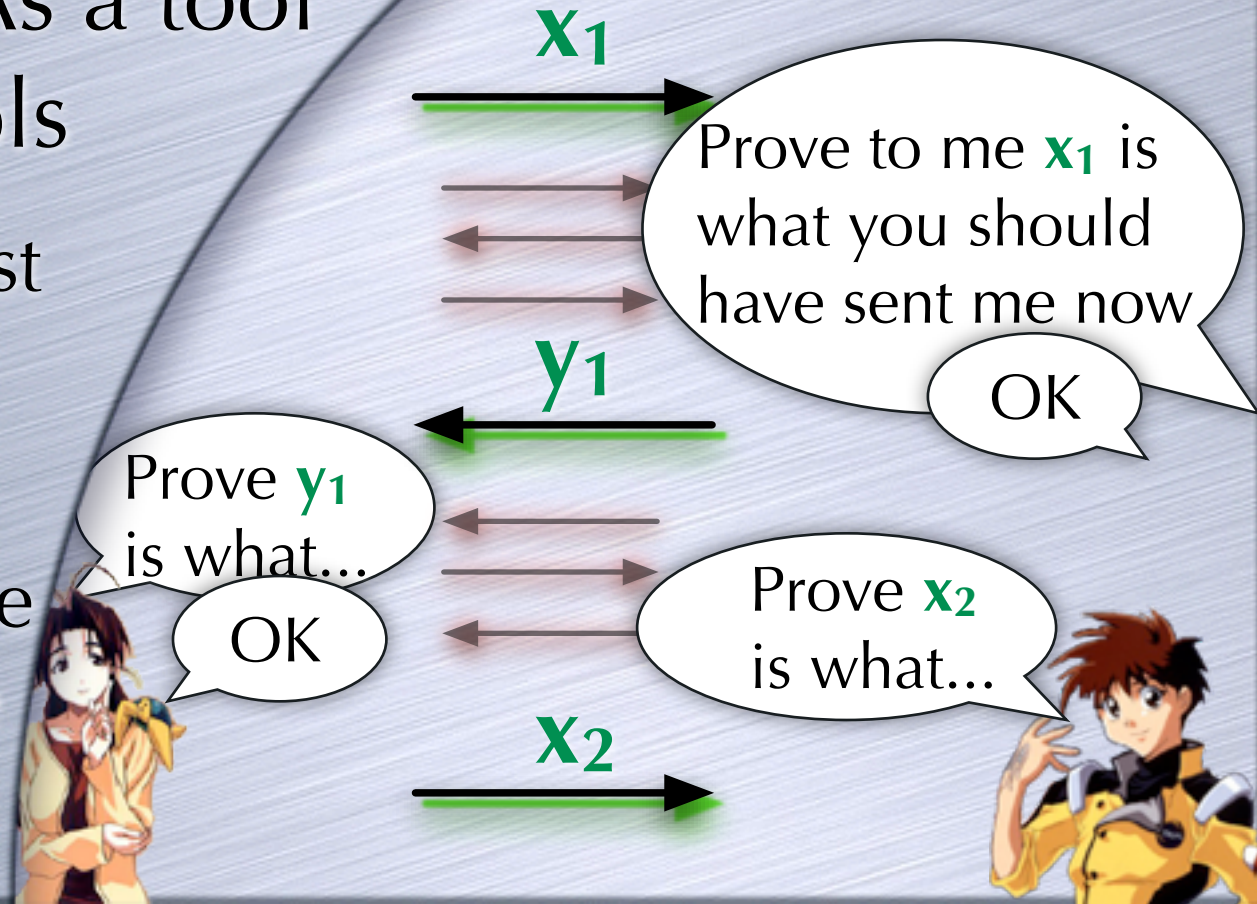
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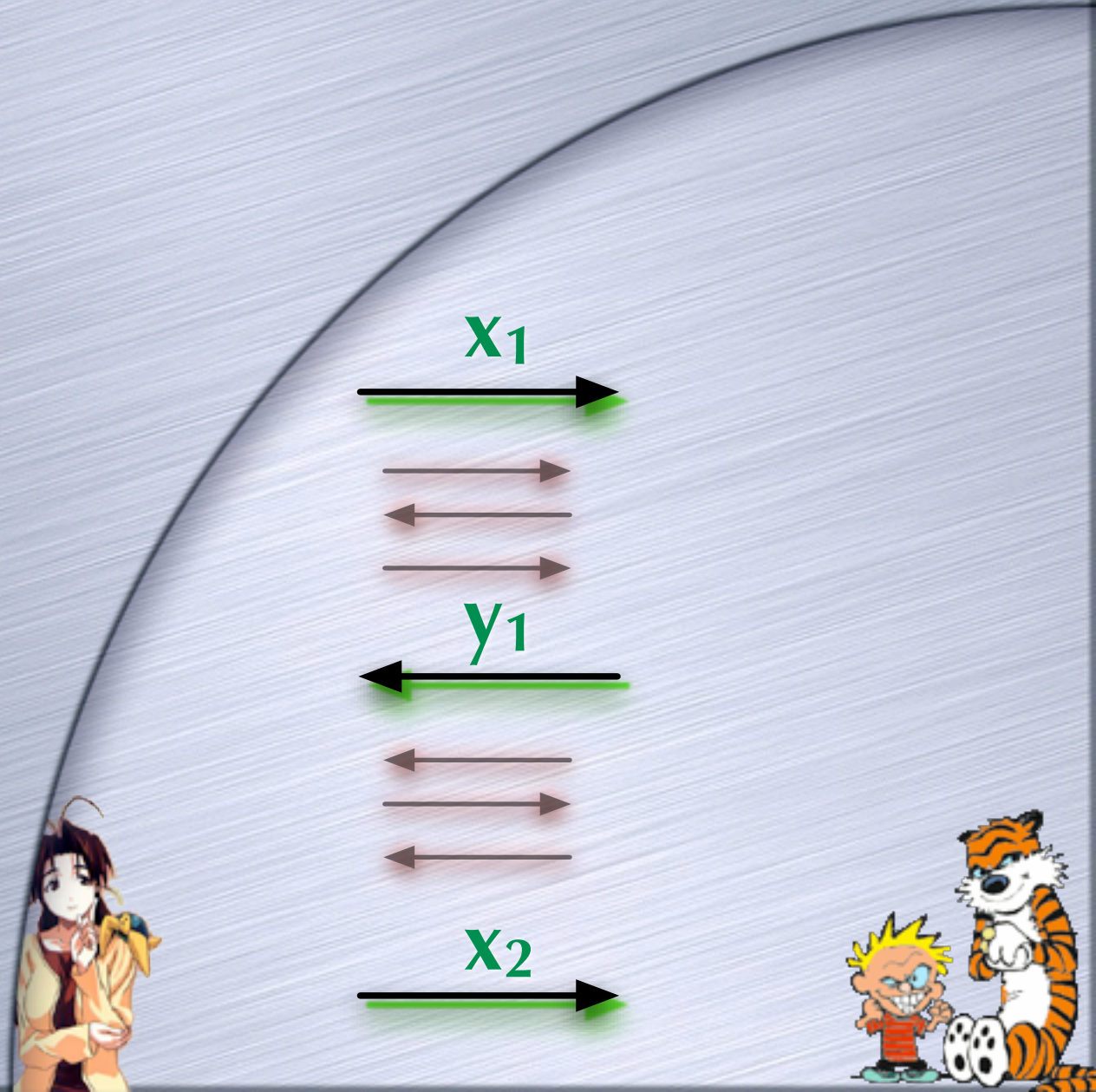
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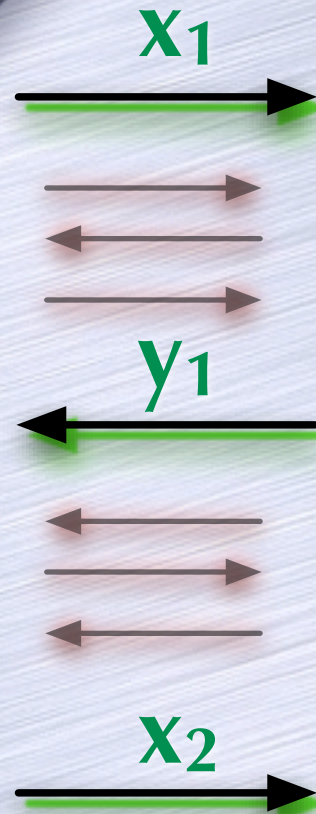
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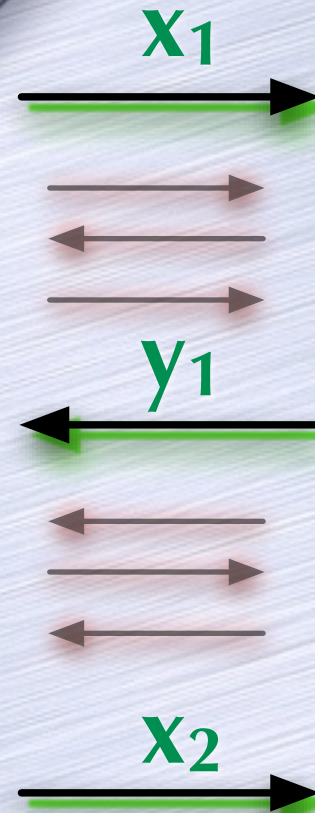
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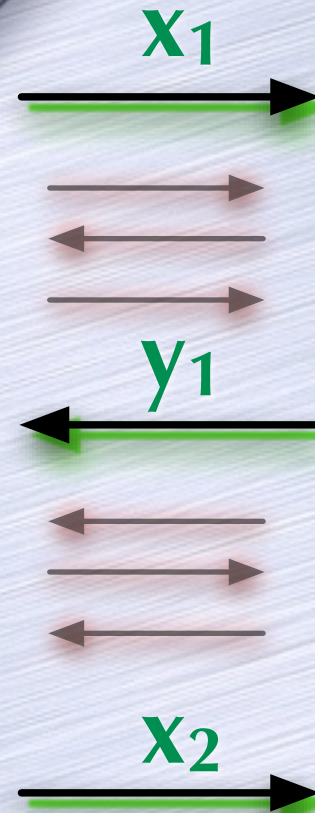
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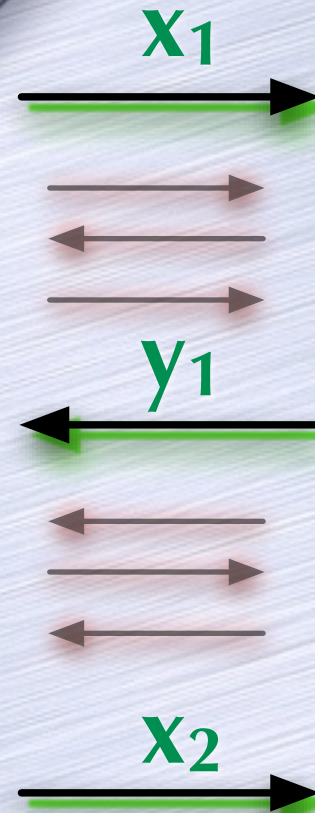
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- Does the proof stay ZK in the big picture?
- Composition
  - Several issues: auxiliary information from previous runs, concurrency issues, malleability/man-in-the-middle
- In general, to allow composition more complicated protocols





# Non-Interactive ZK



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- Can the prover just give a written proof (no interaction) which any one can verify and can simulate too?



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- Defined in standalone setting, but WI property is preserved under "parallel composition"



# Composition



# Composition Issues



GM1 vs. Hacker

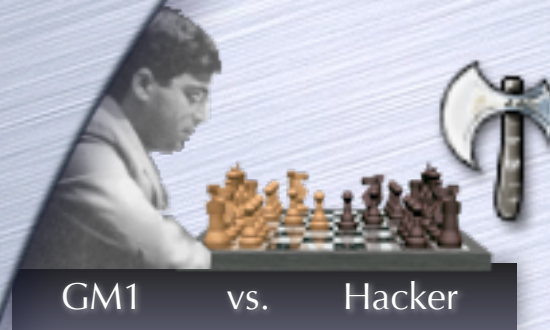


Hacker vs. GM2



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Play the GM's against each other  
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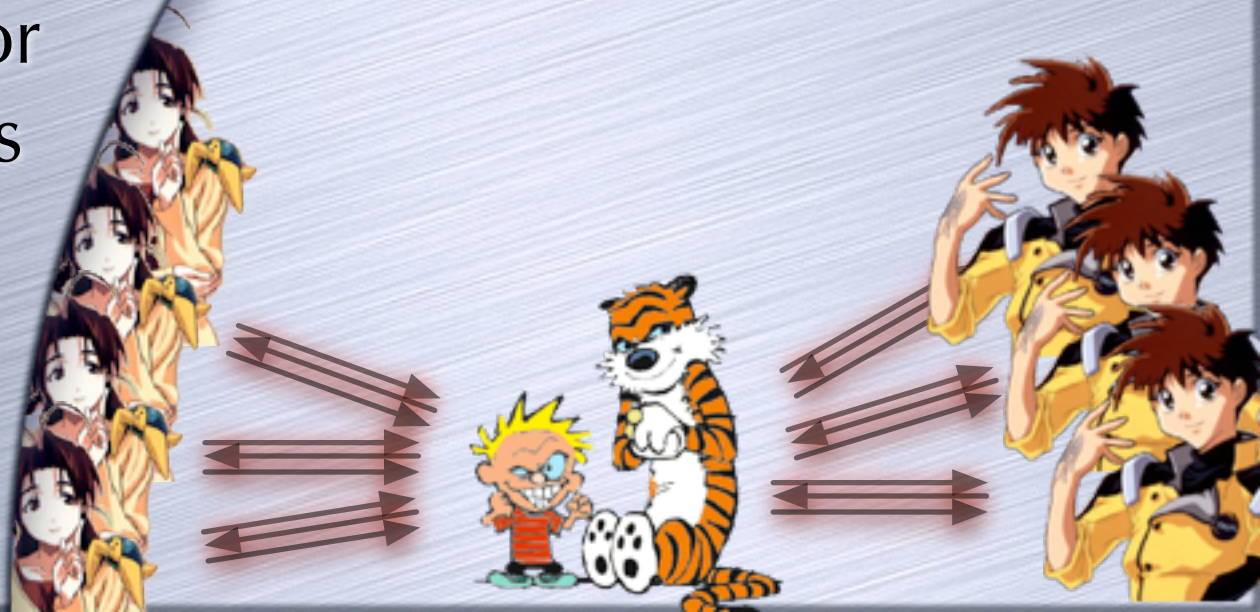


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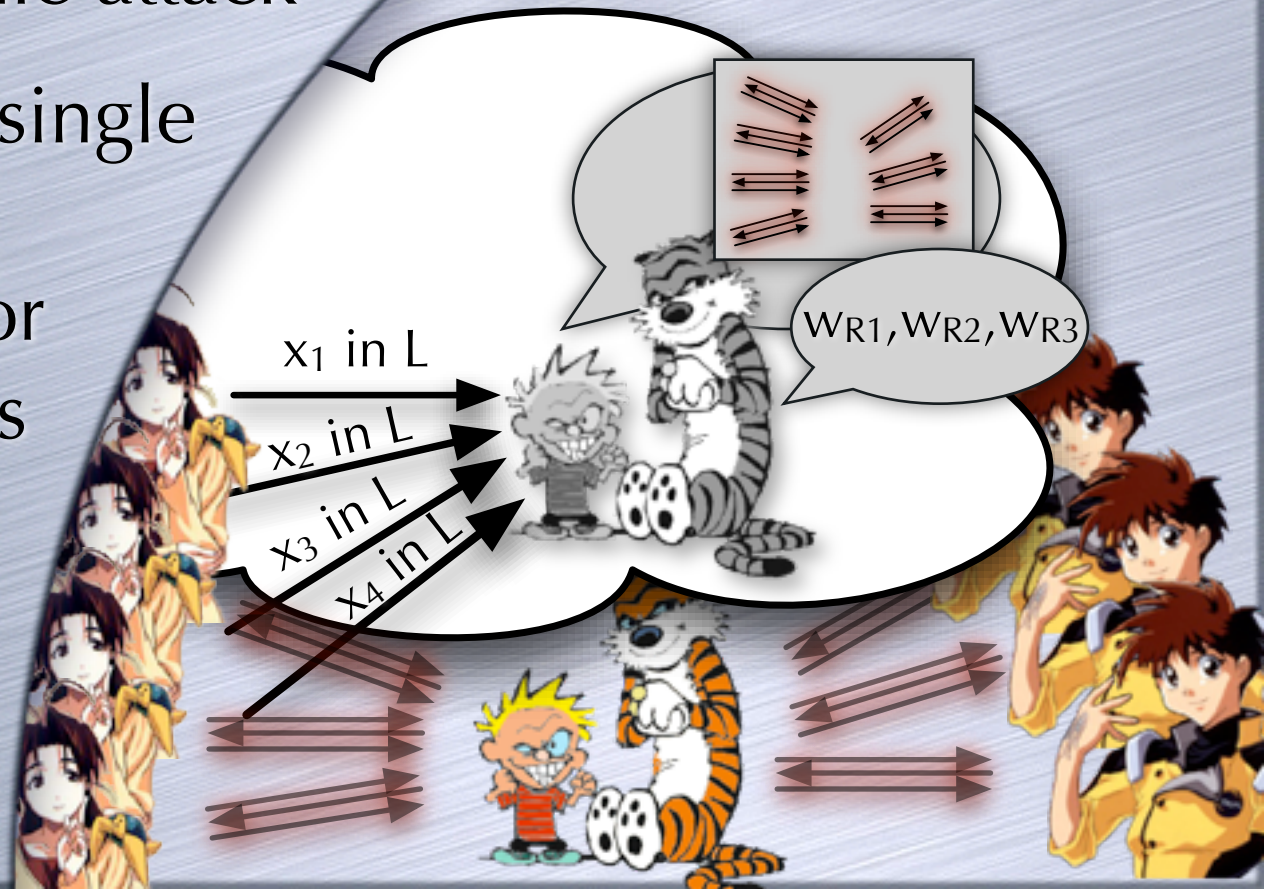
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# Universal Composition



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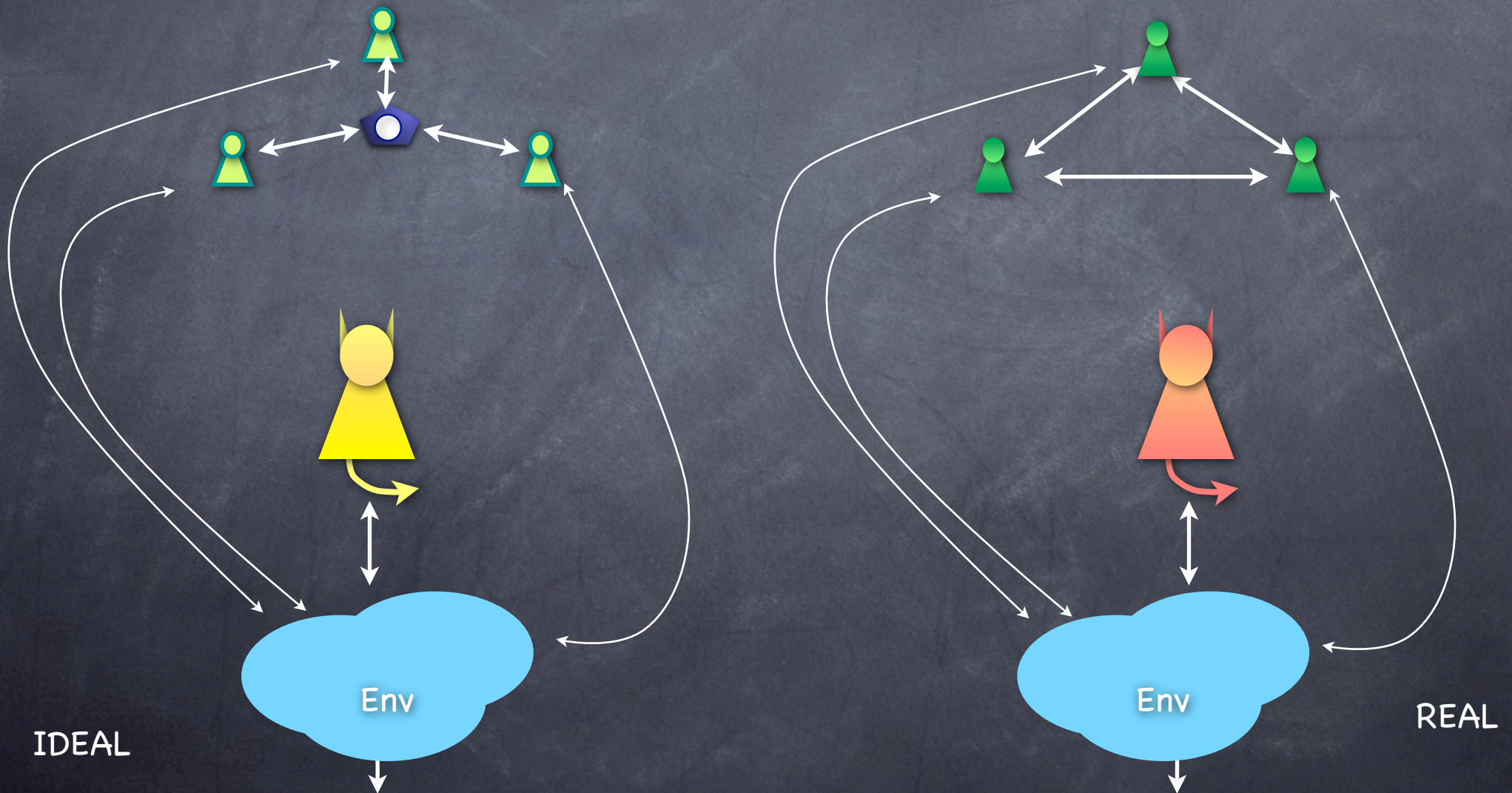
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RECALL

# Security

REAL (with protocol) is as secure as IDEAL (with functionality) if:

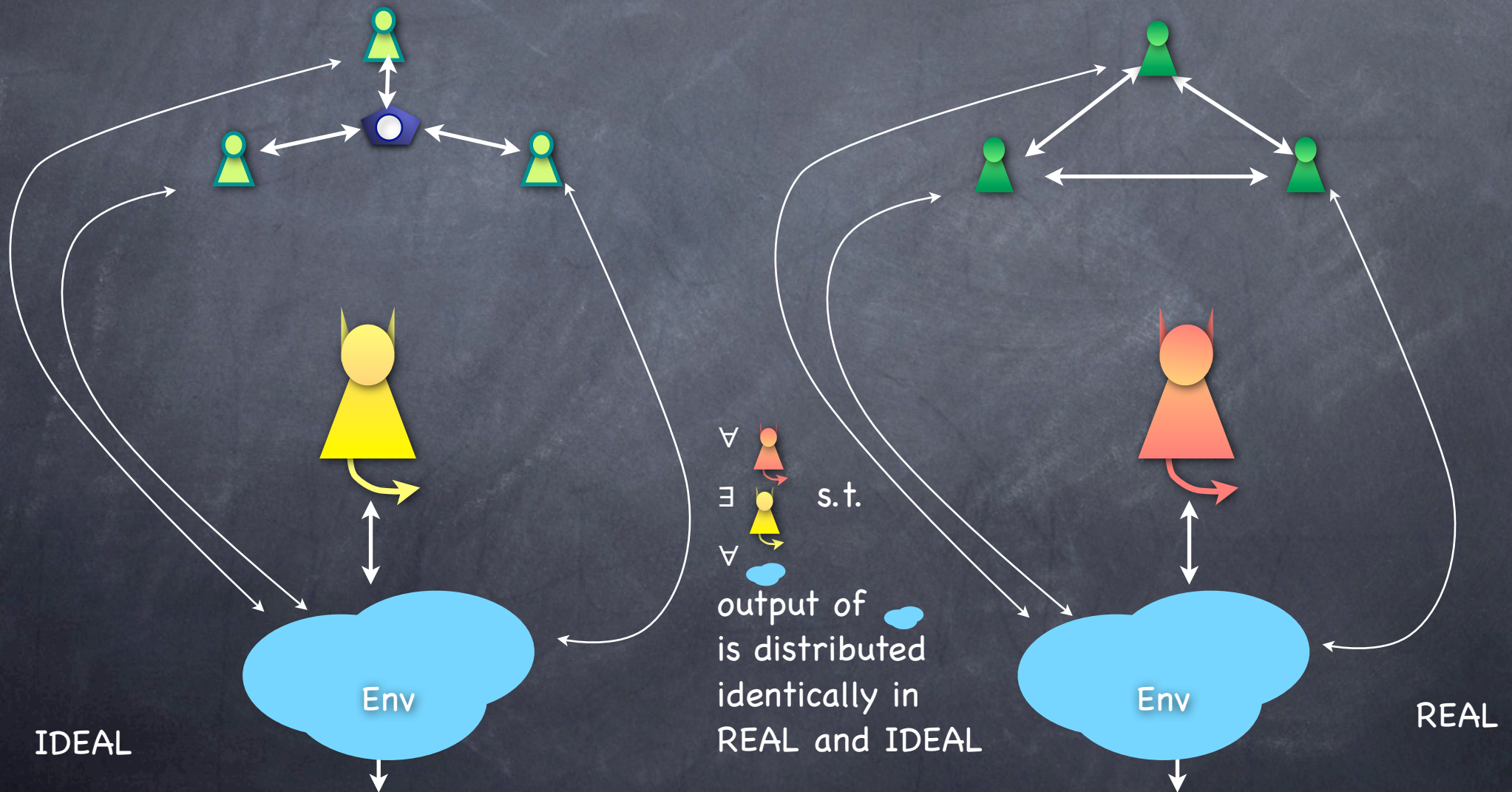




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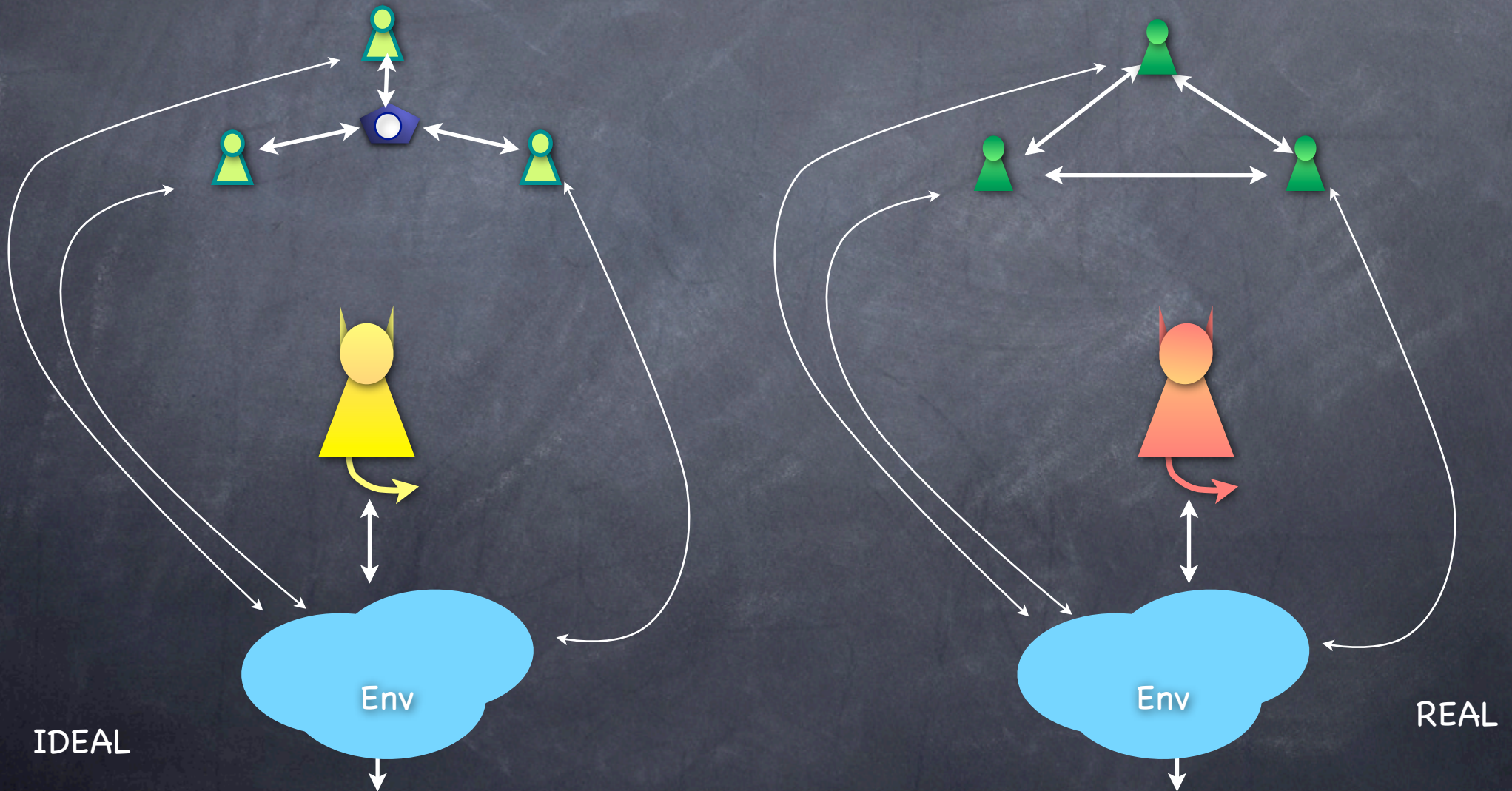
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# Security of Composed Systems

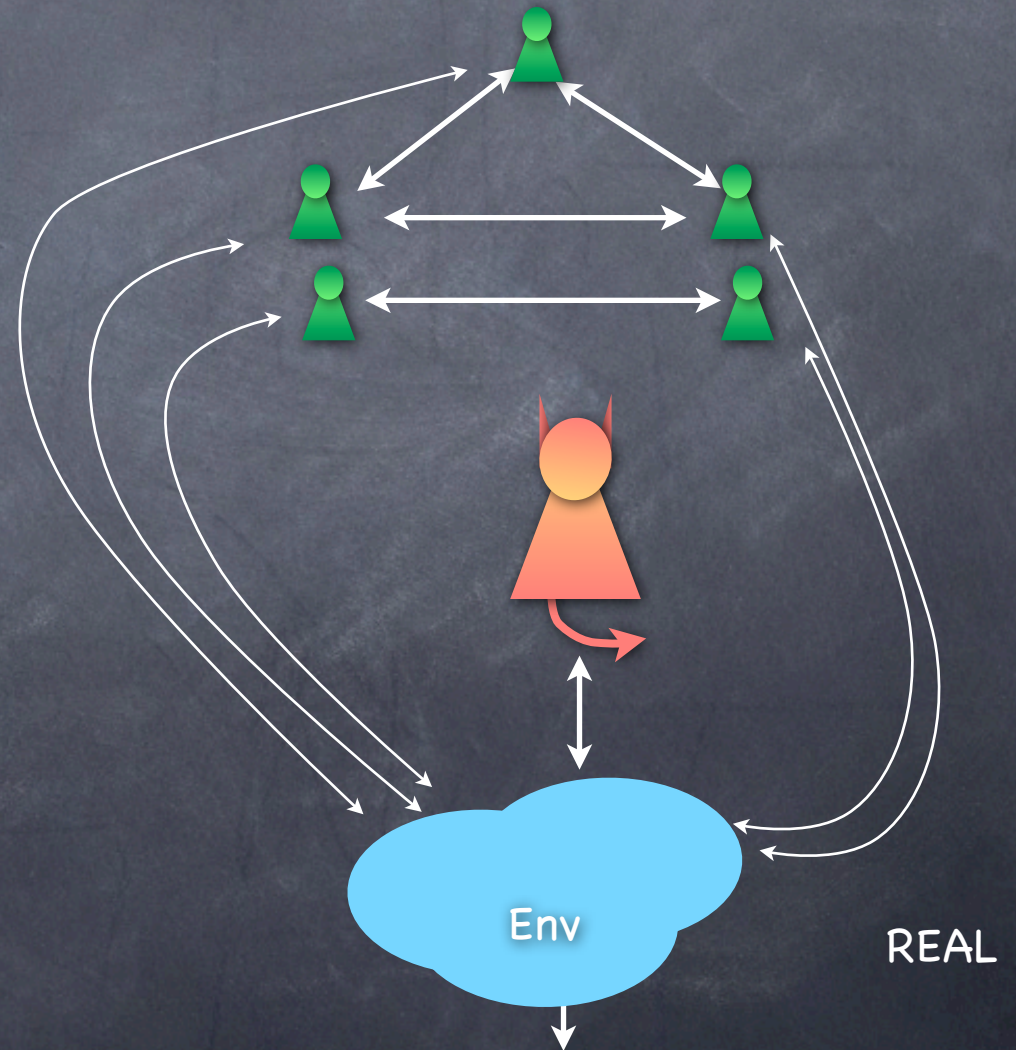
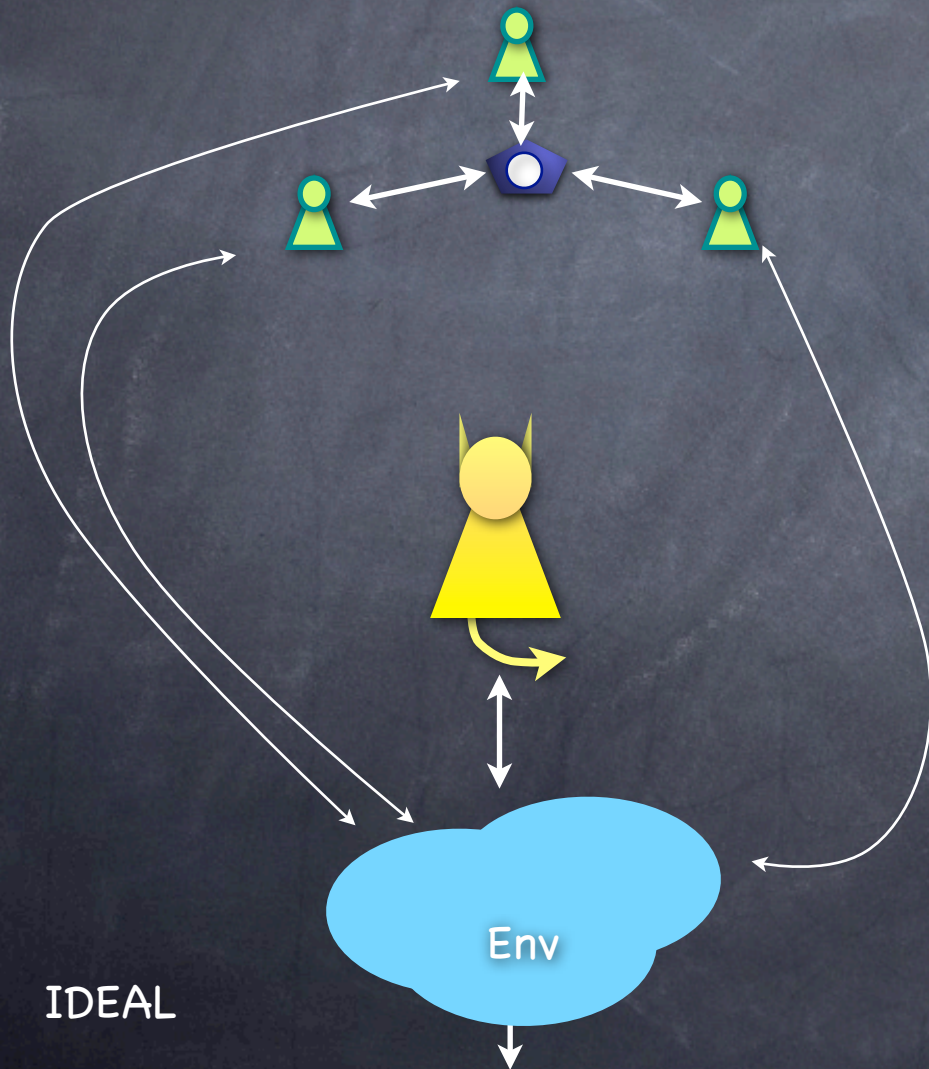
- Extend to allow a “composed system” with multiple functionalities





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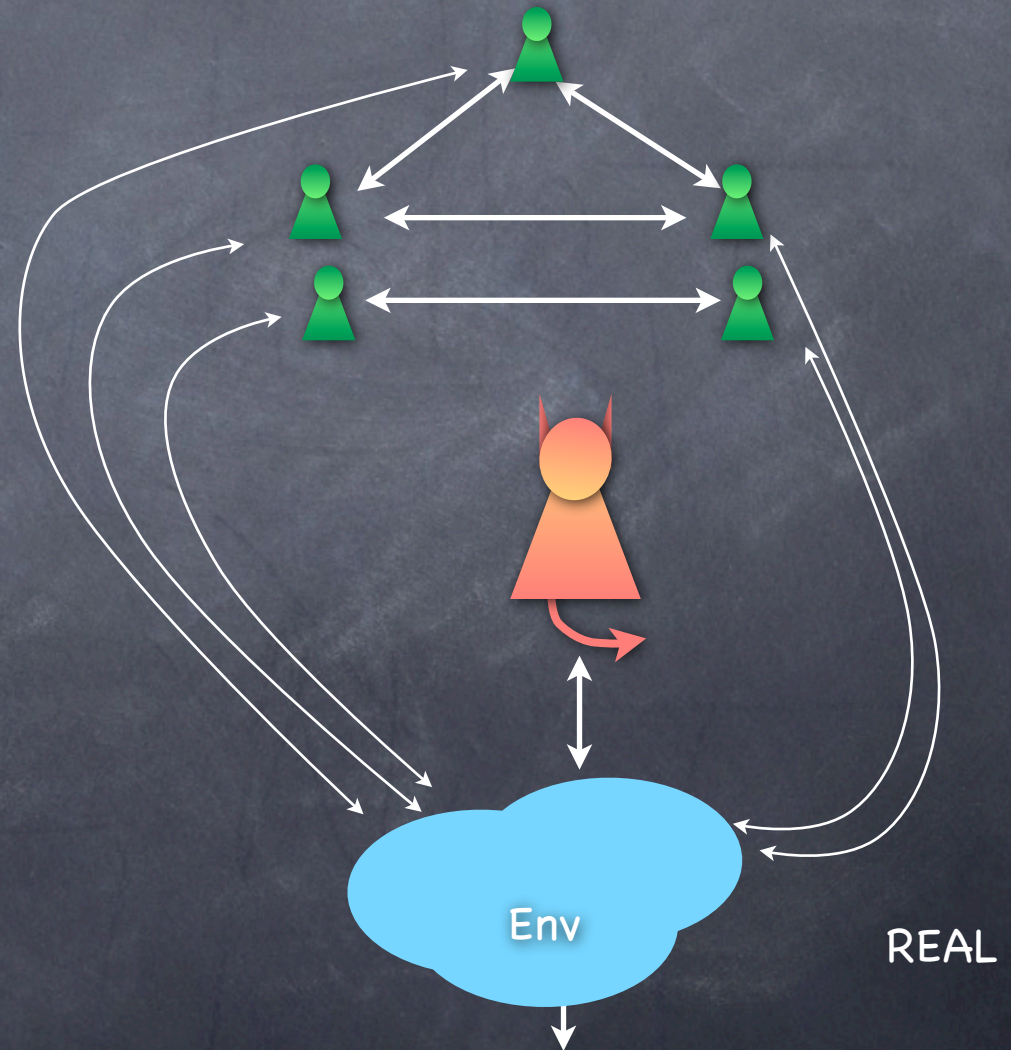
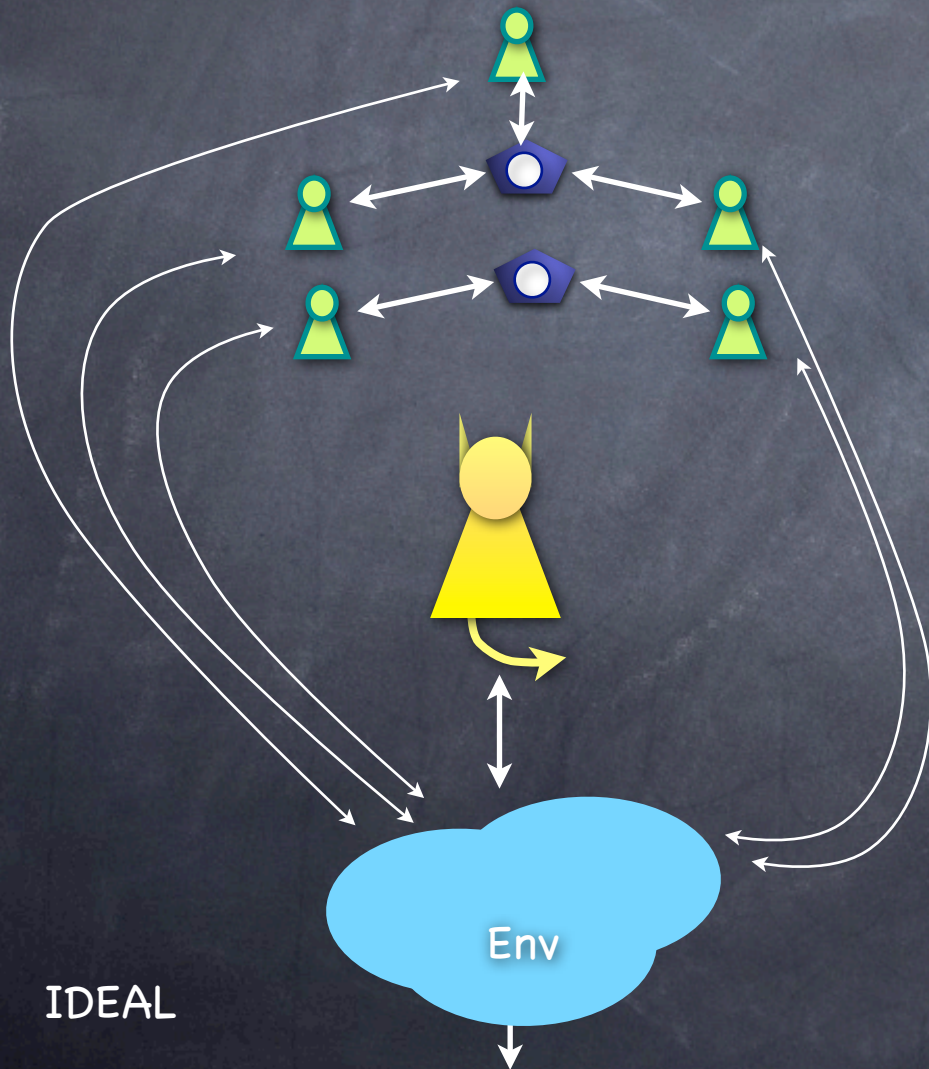
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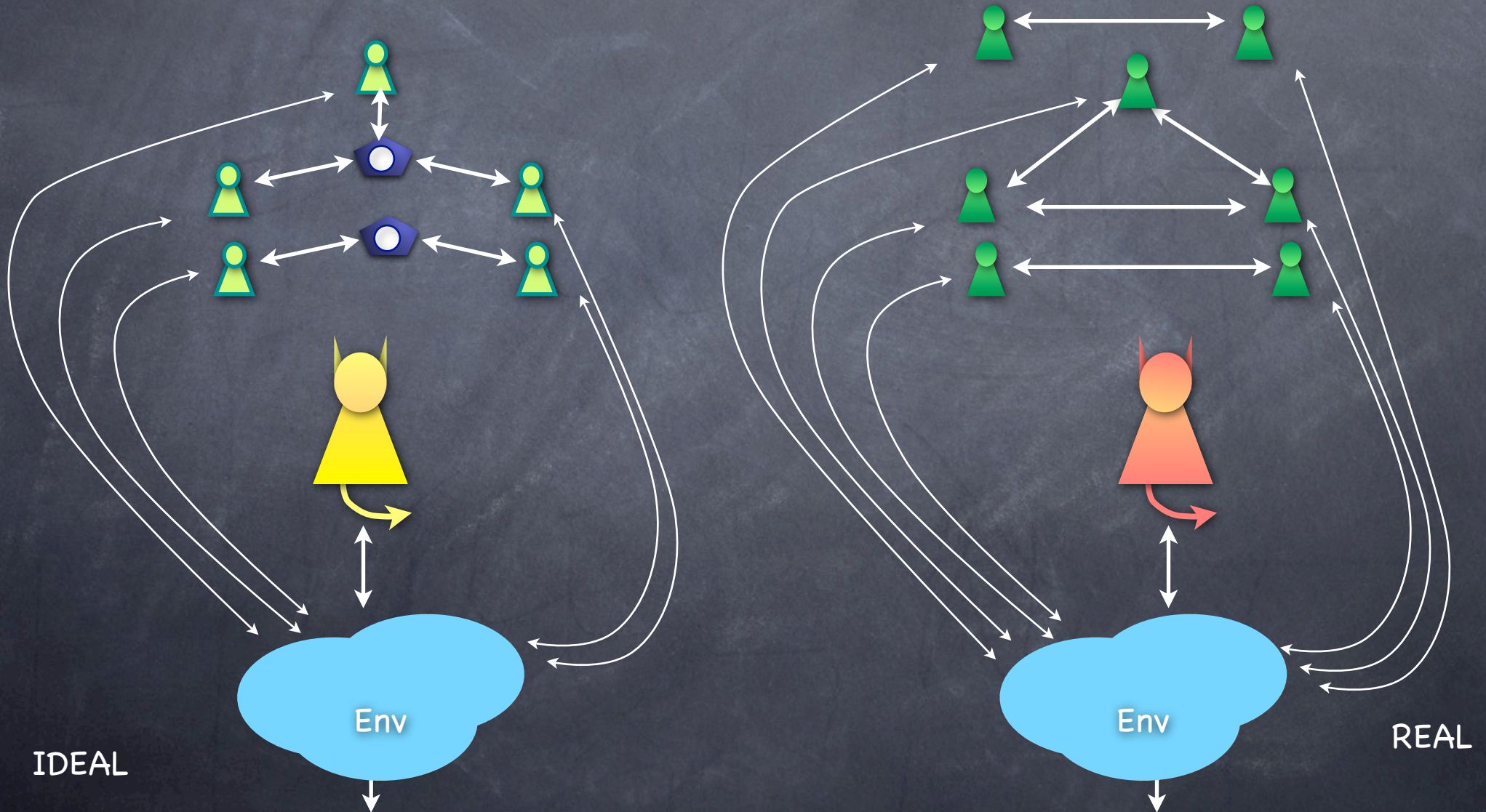
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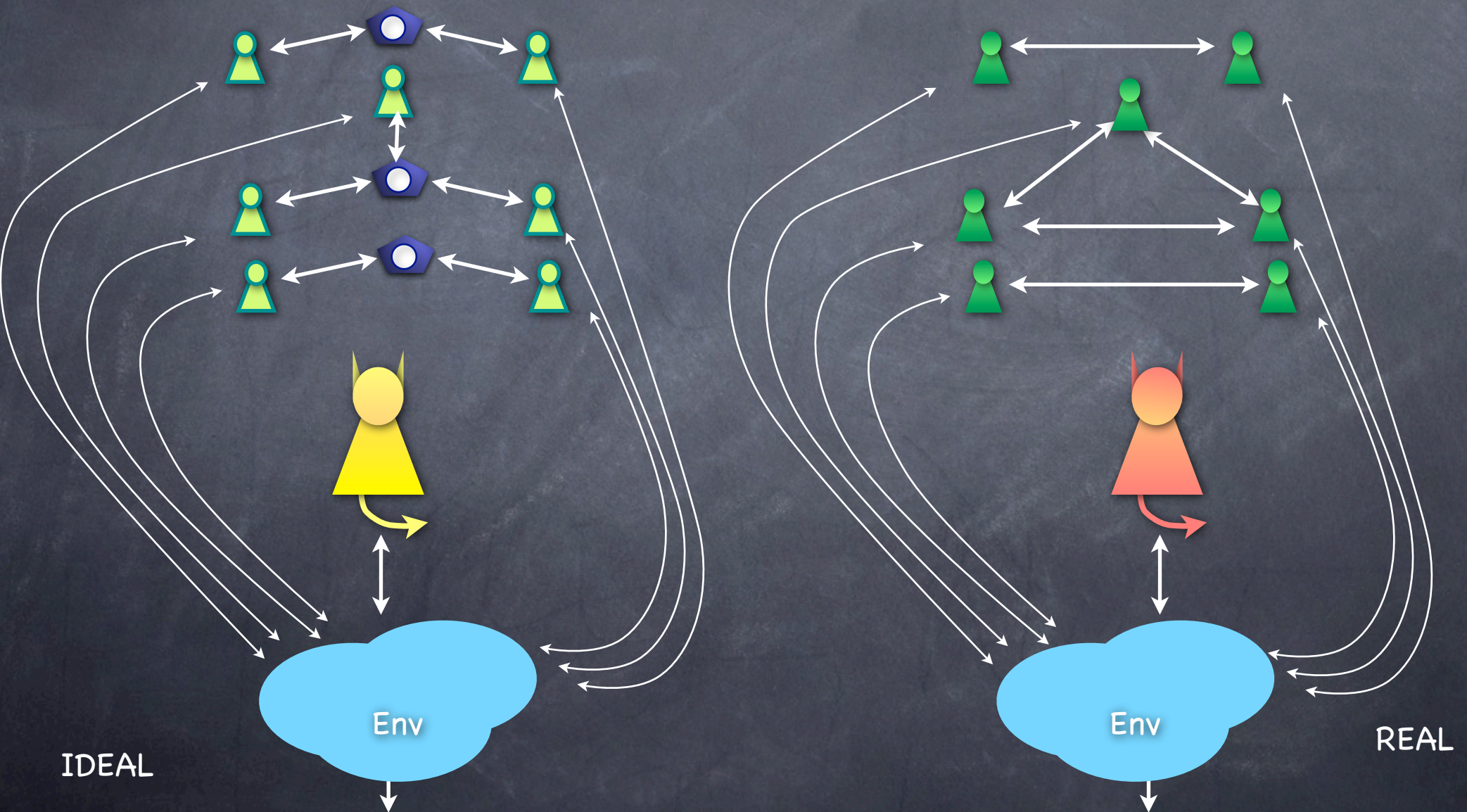
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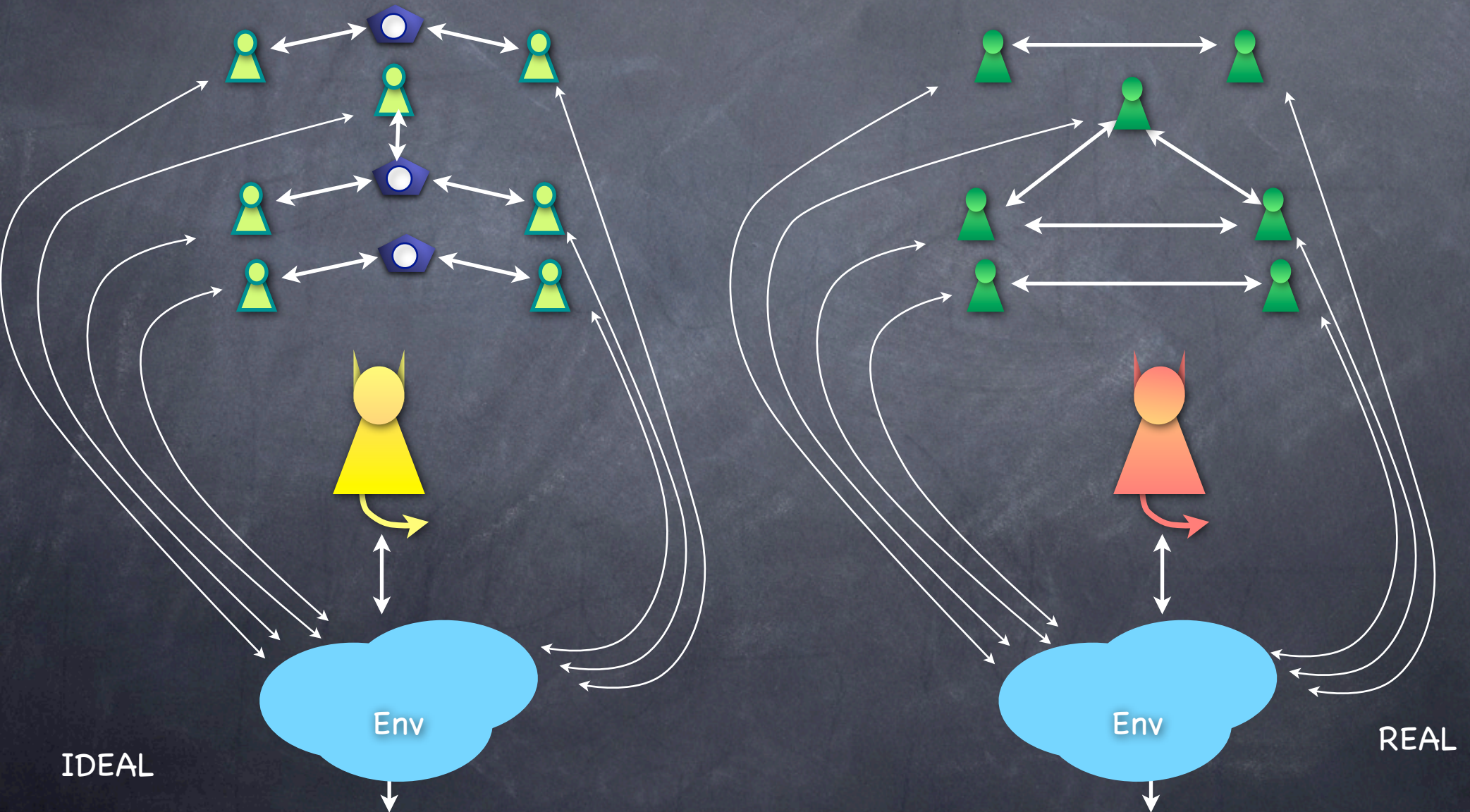
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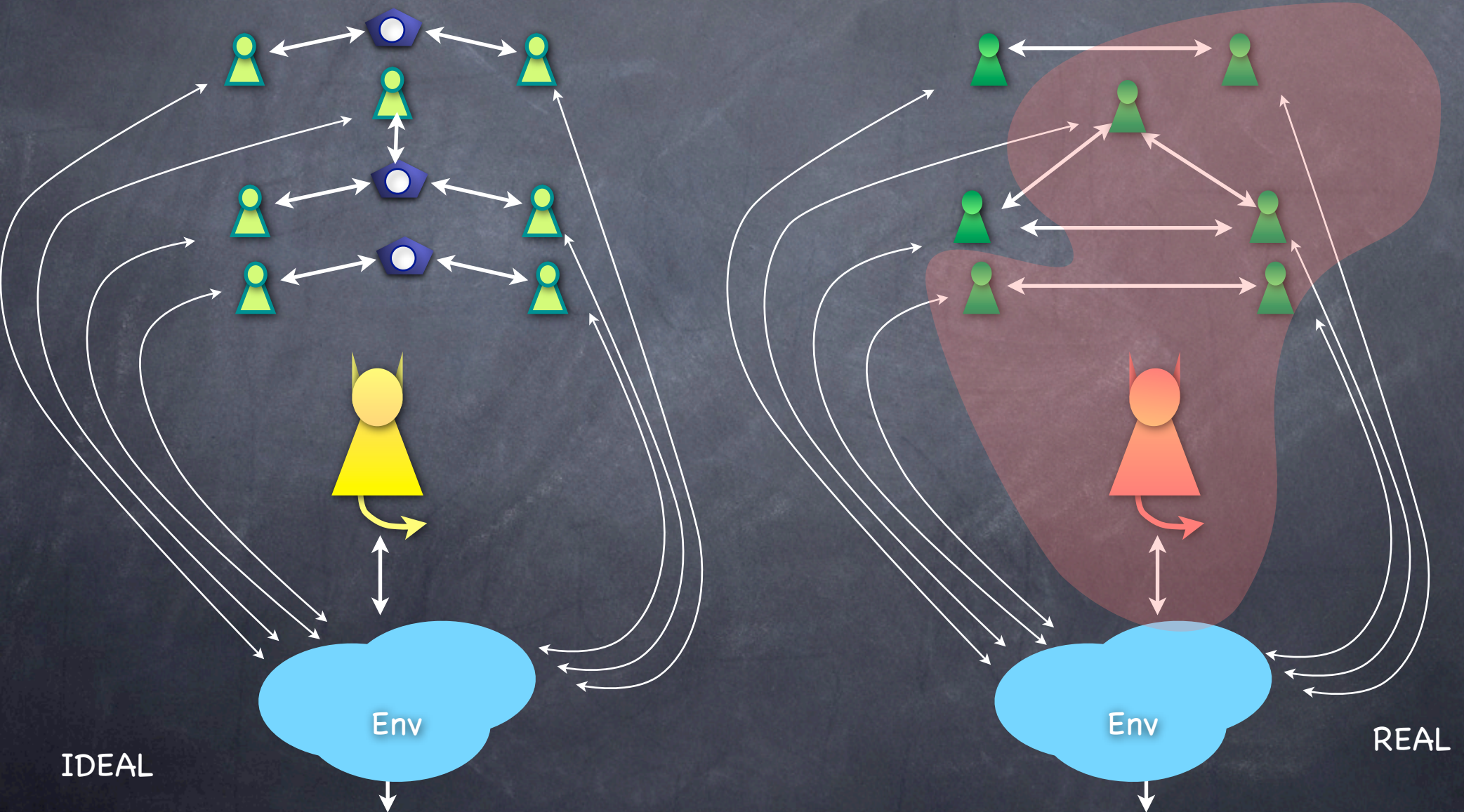
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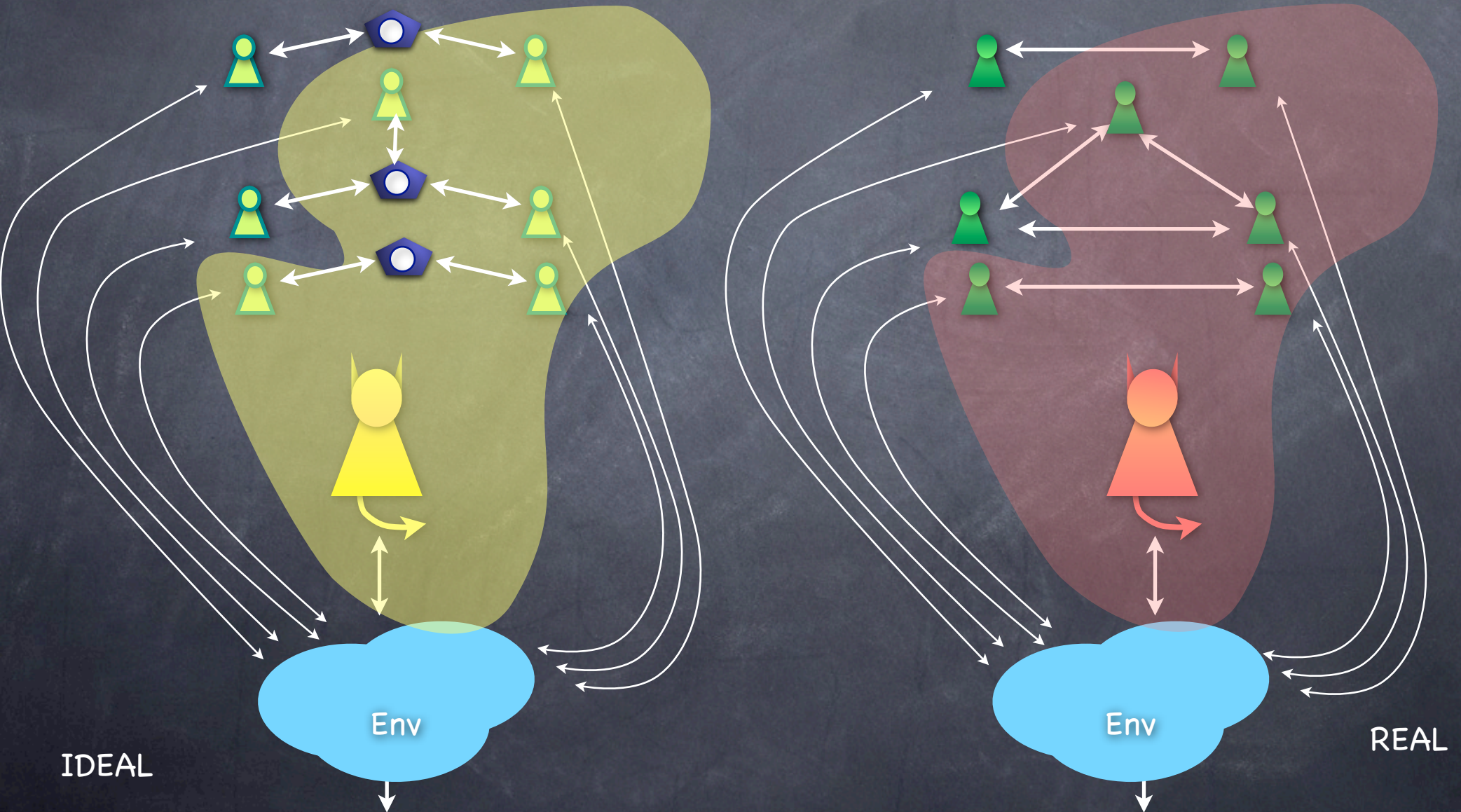
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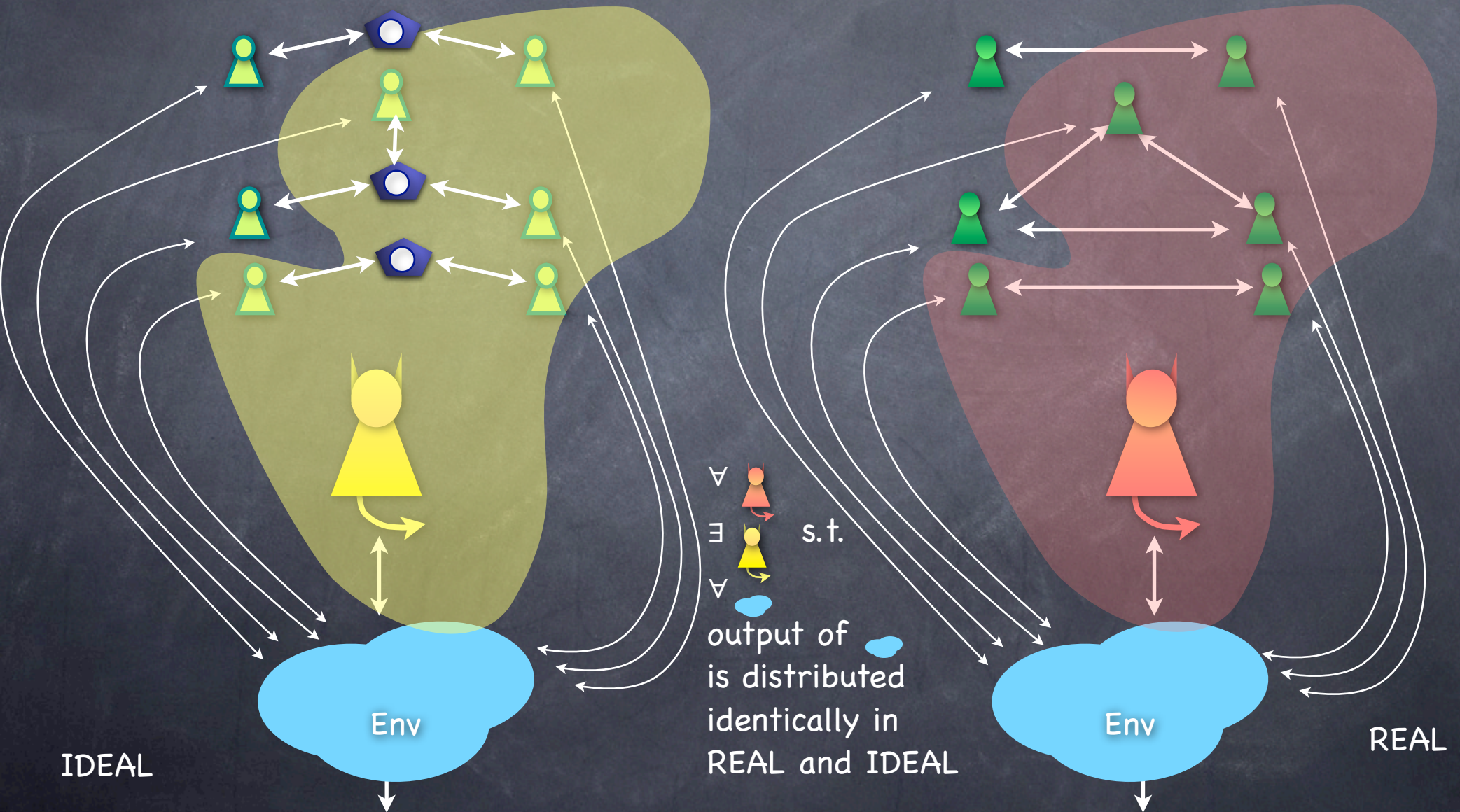
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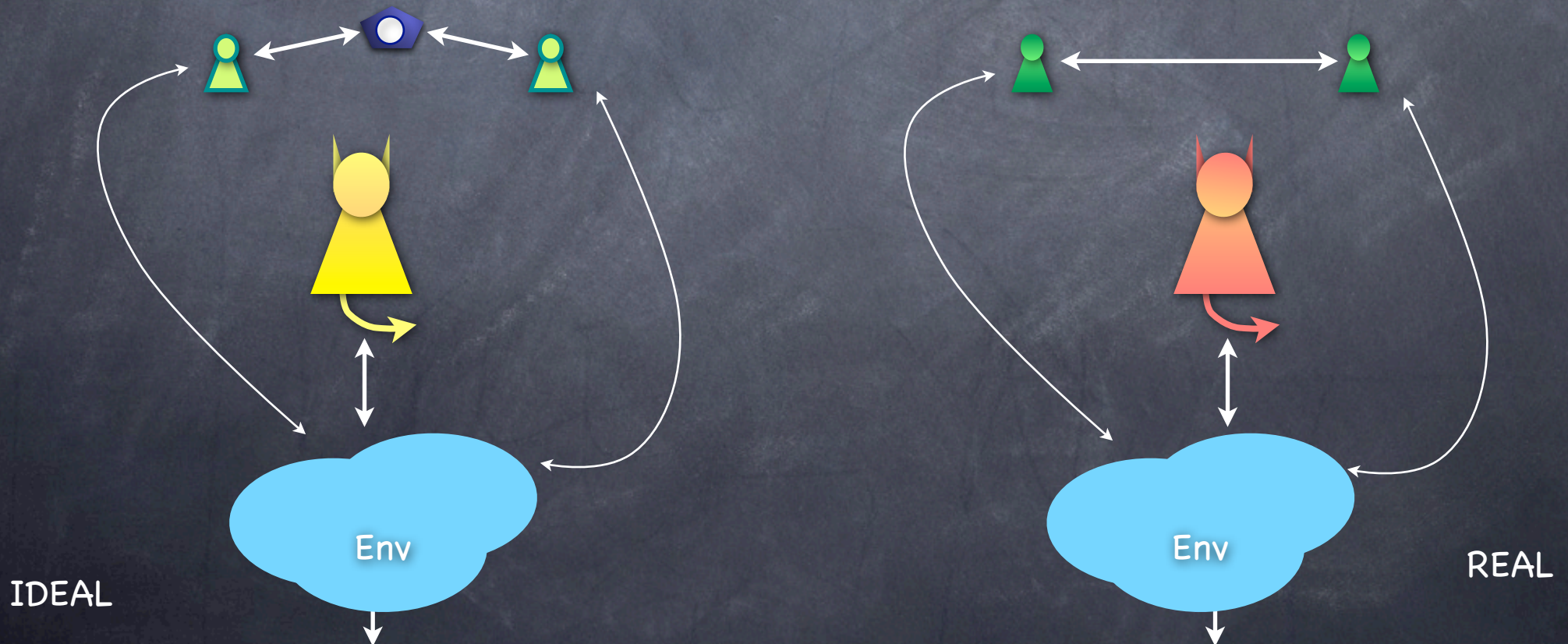
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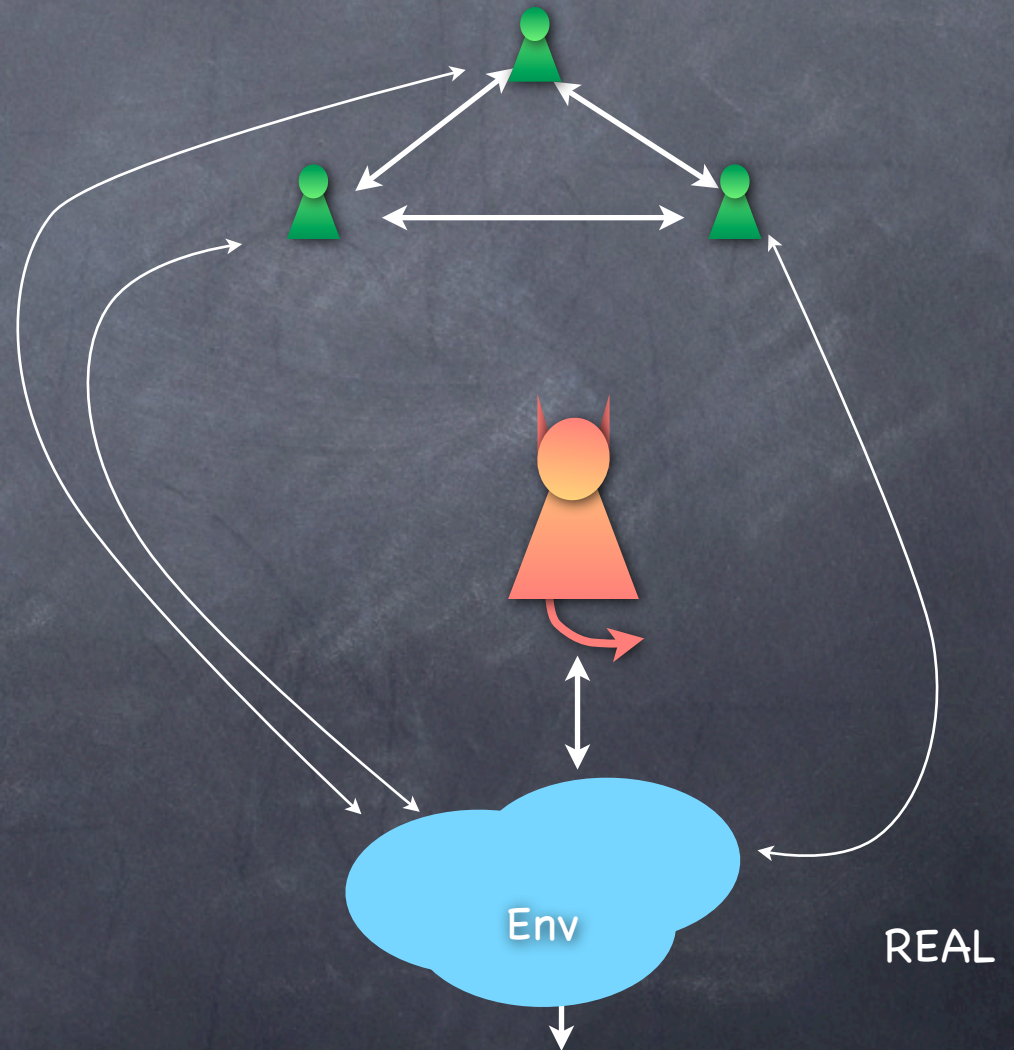
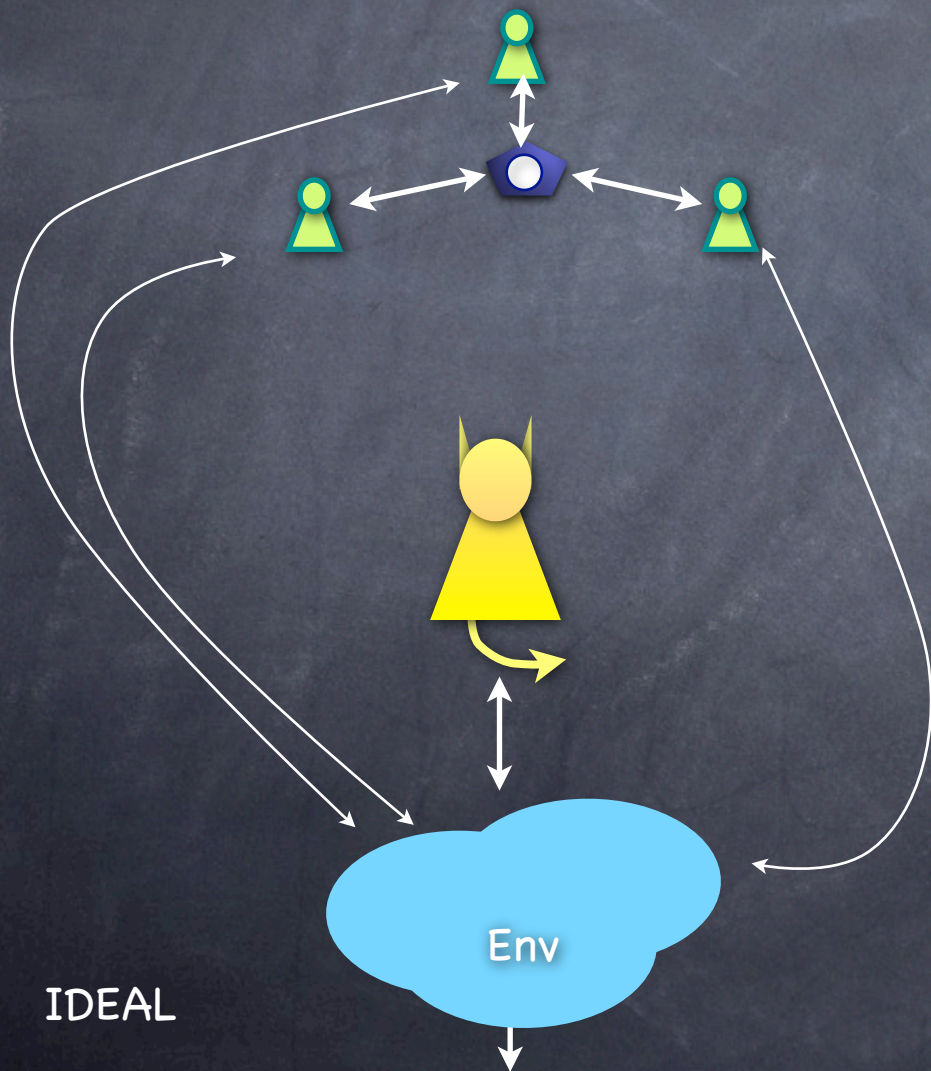
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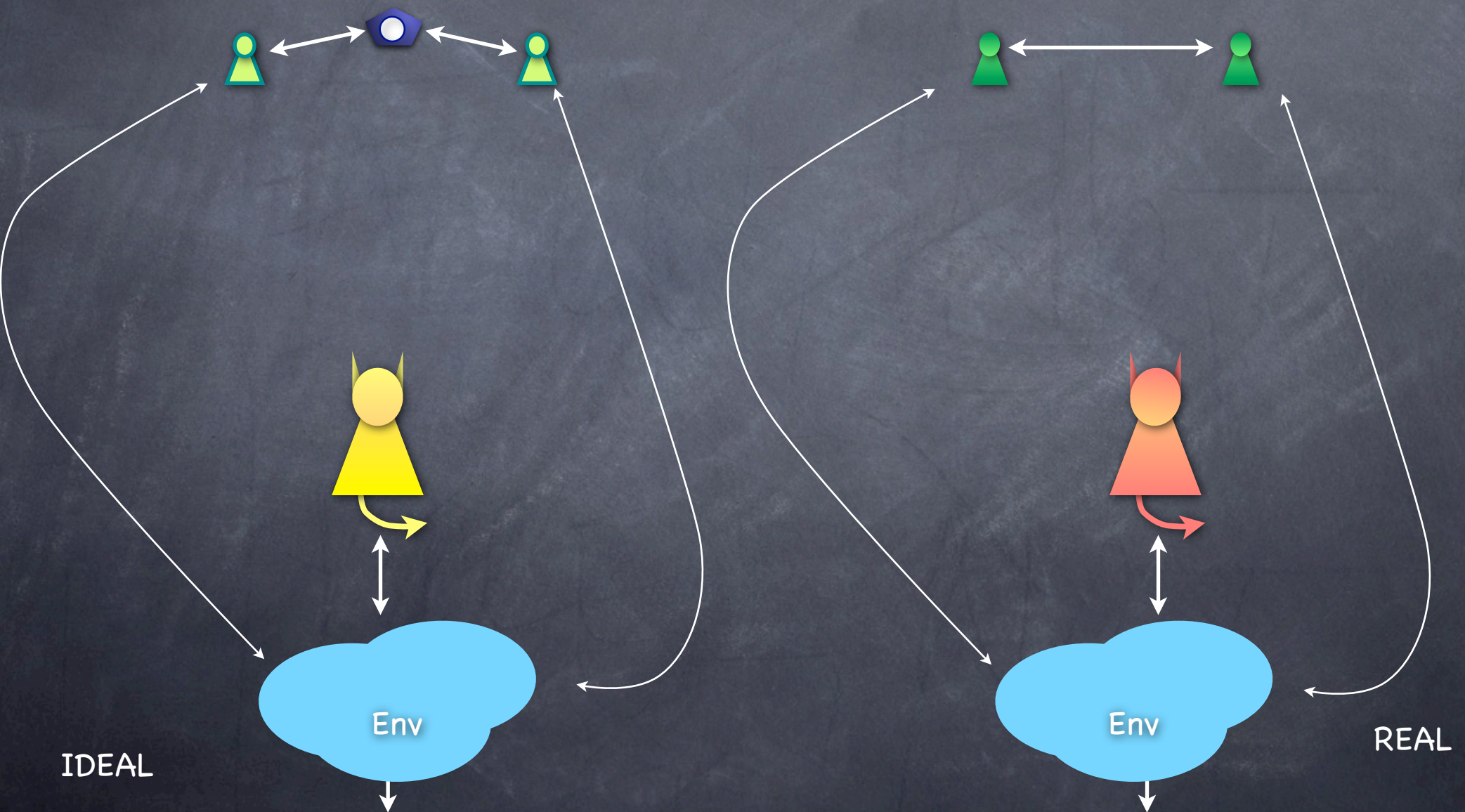
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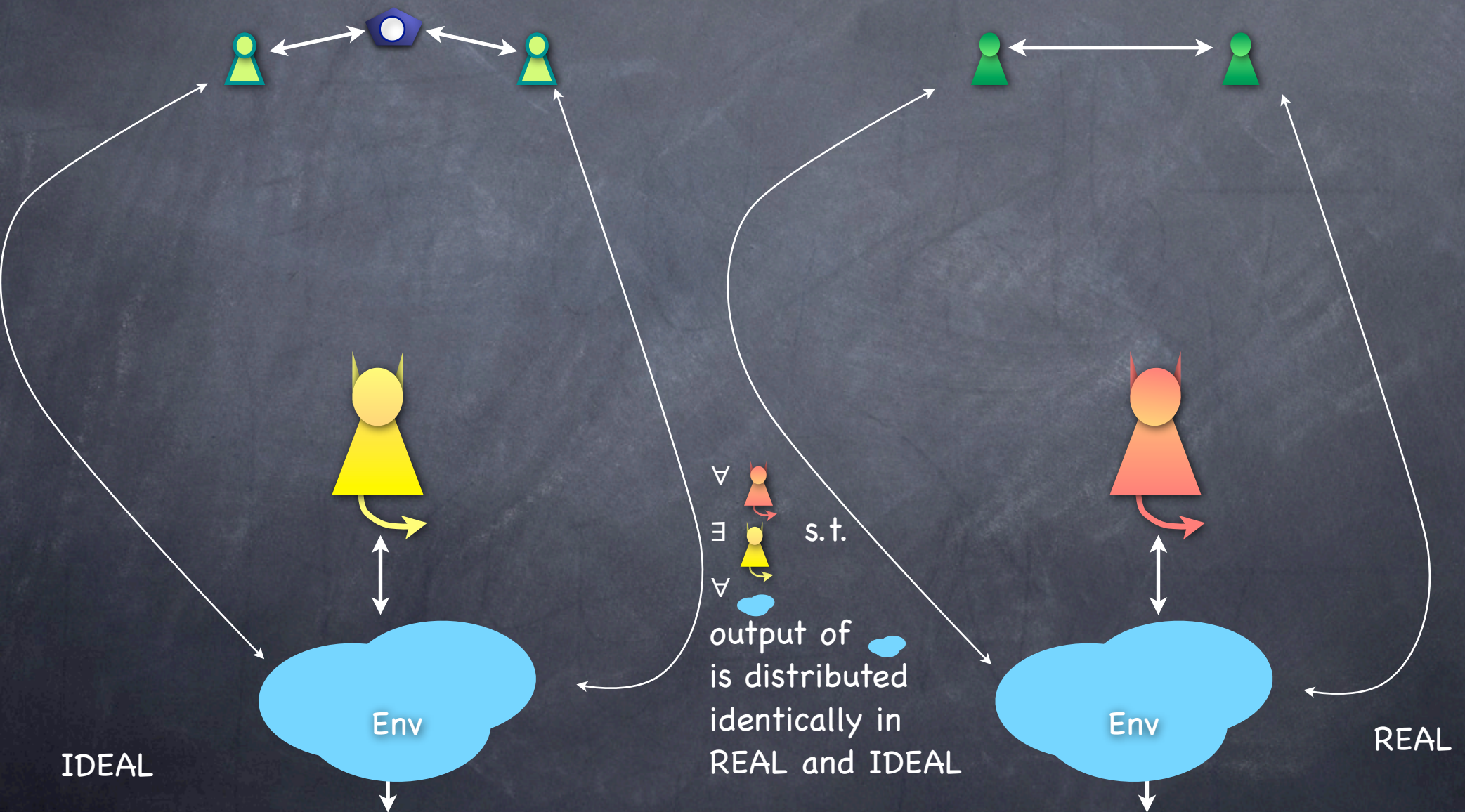
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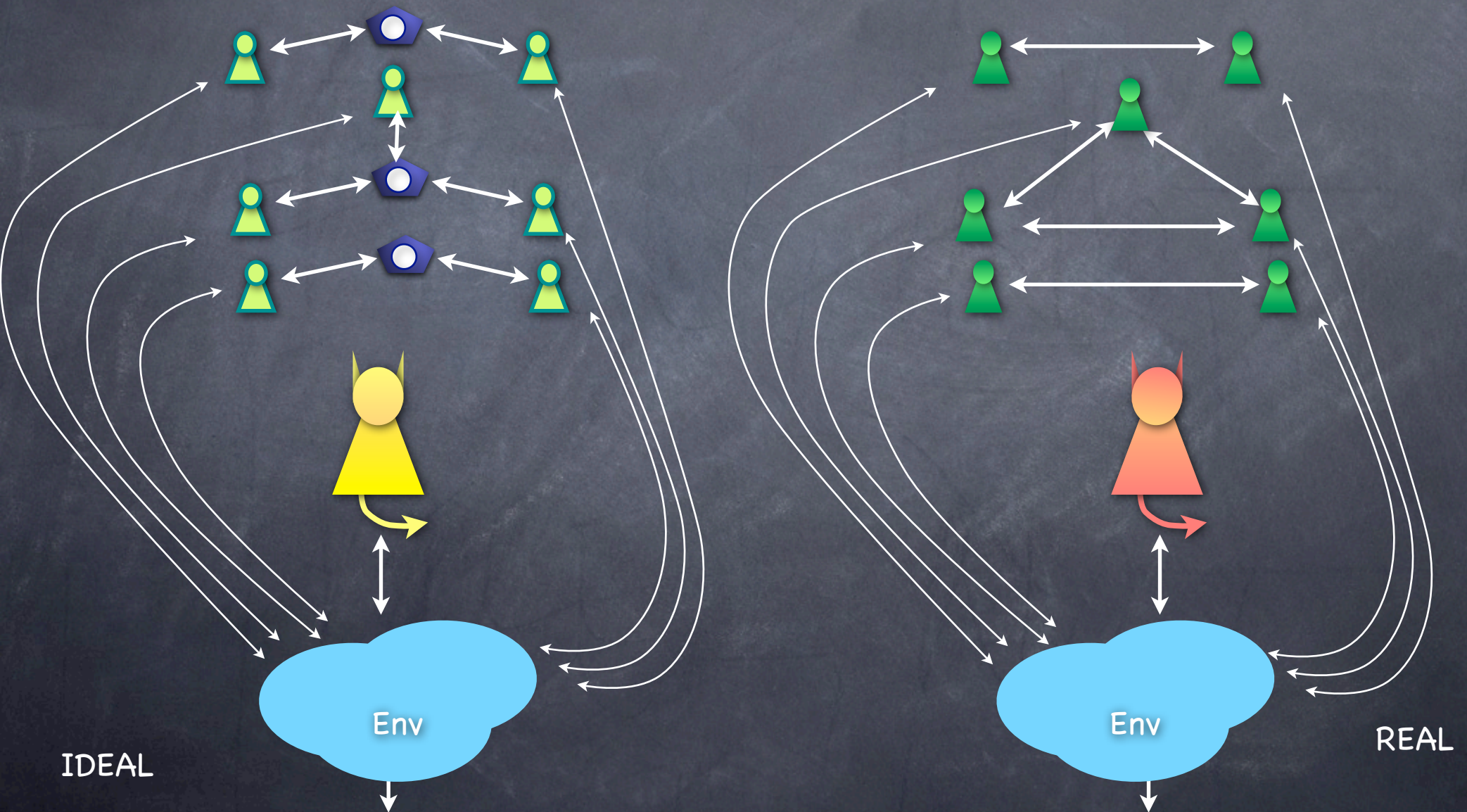
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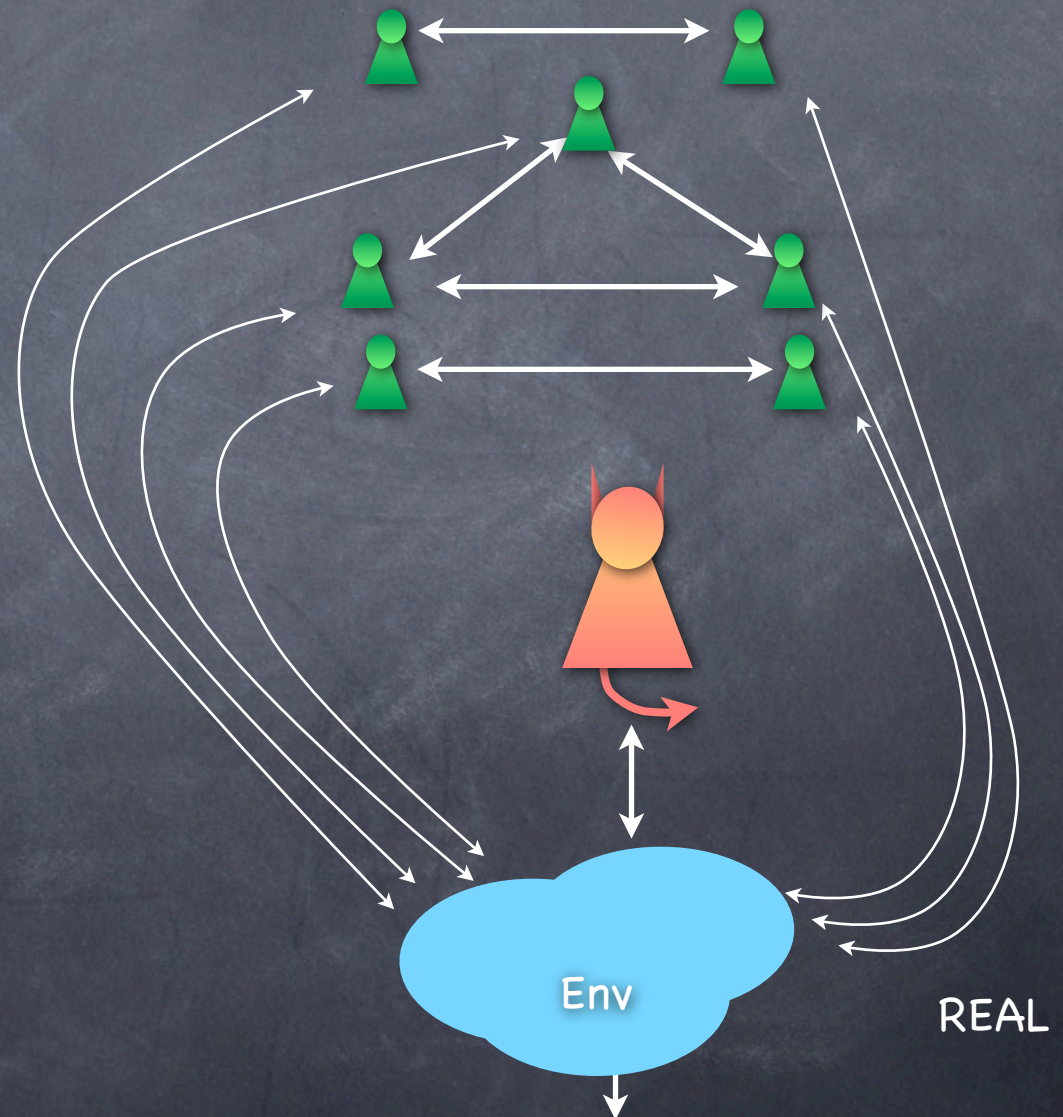
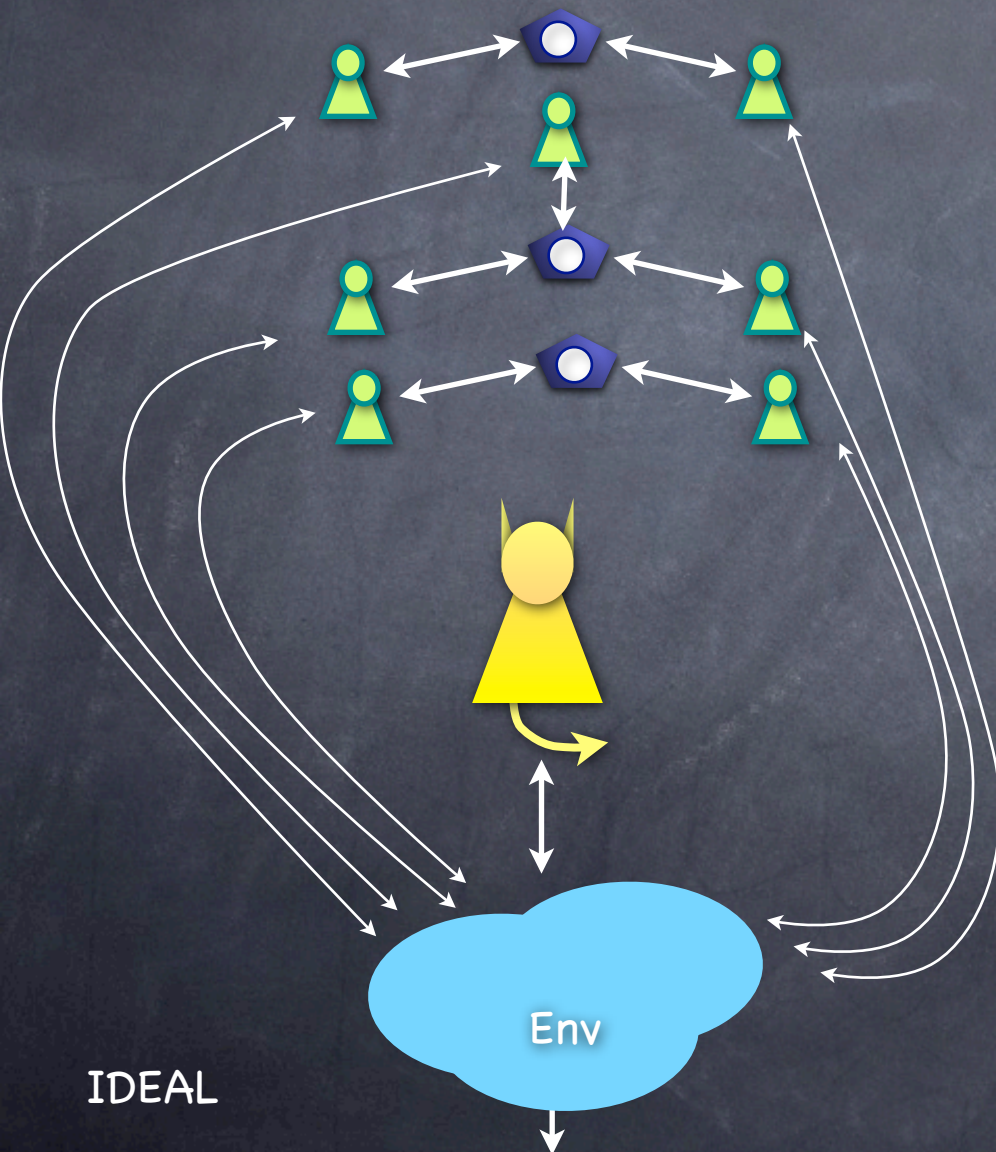
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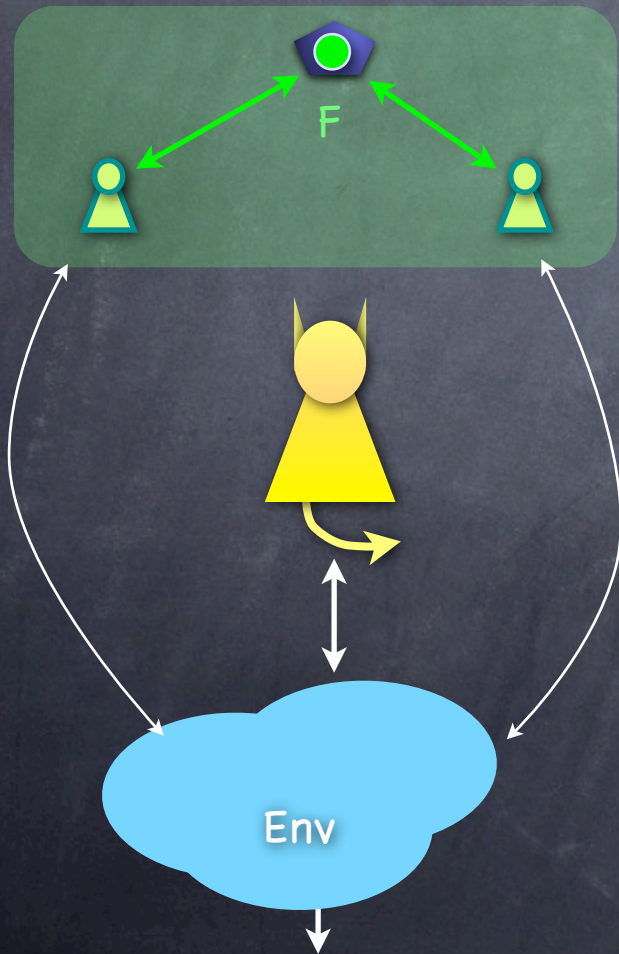
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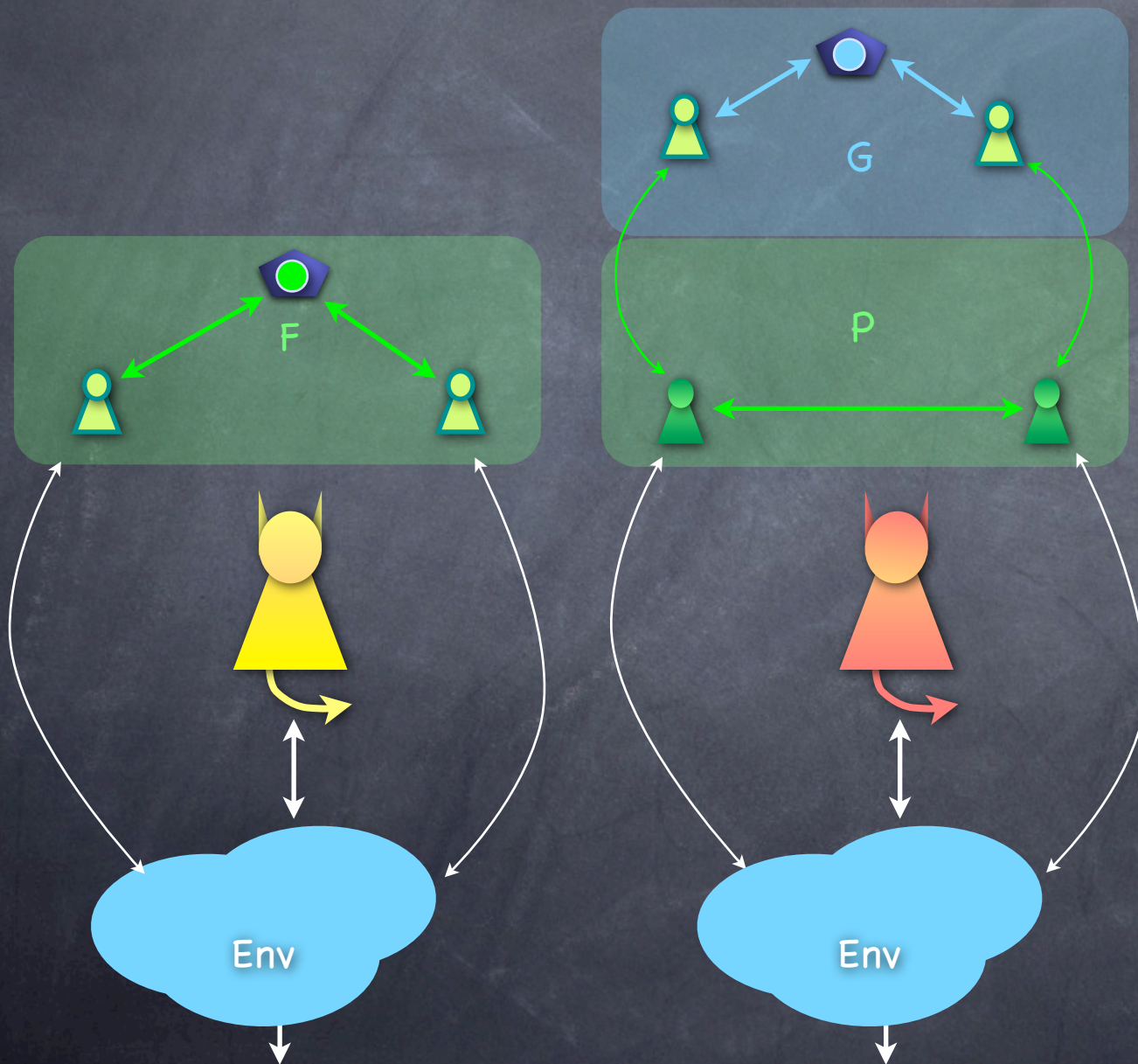


# Universal Composition – 2





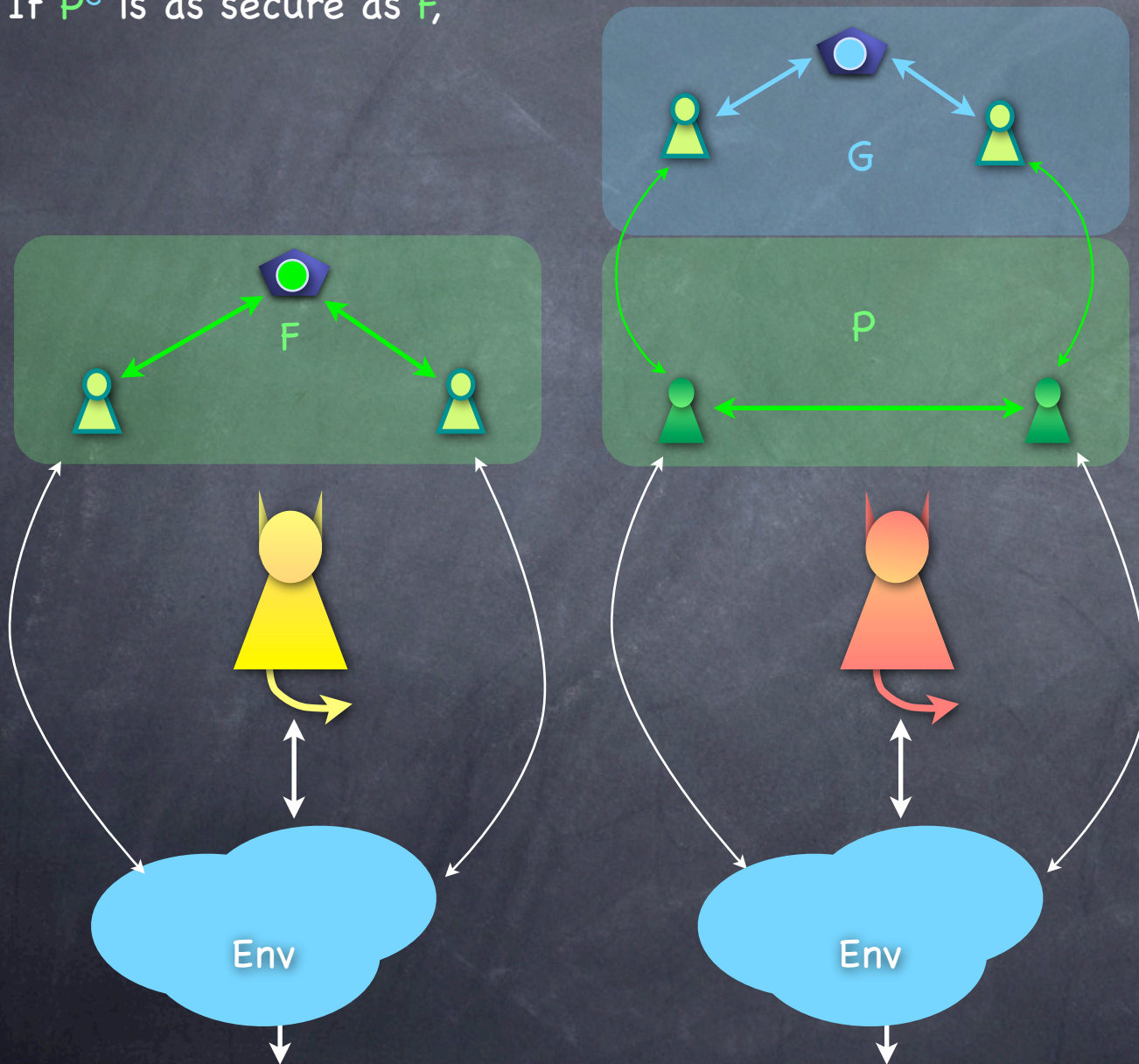
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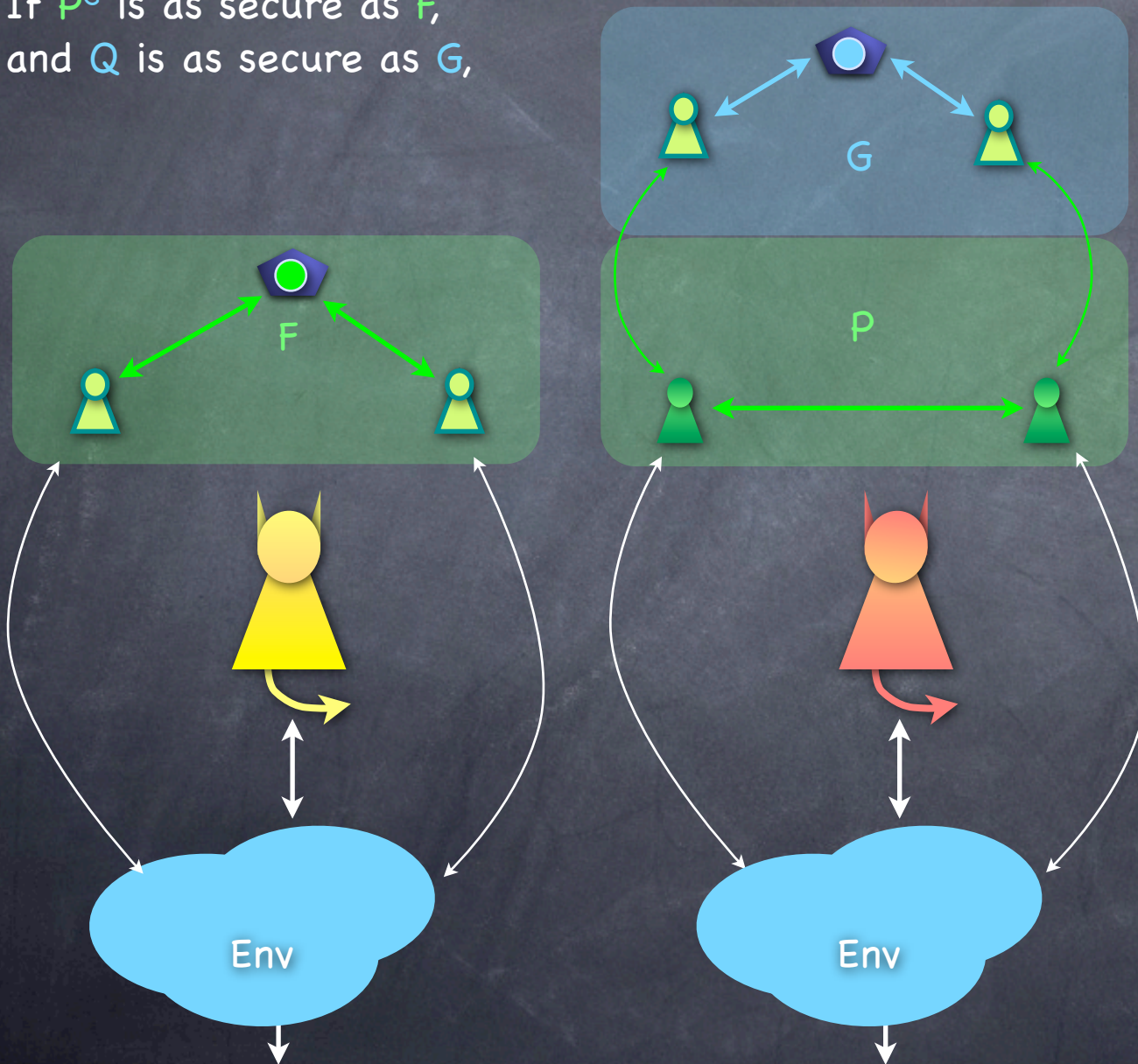
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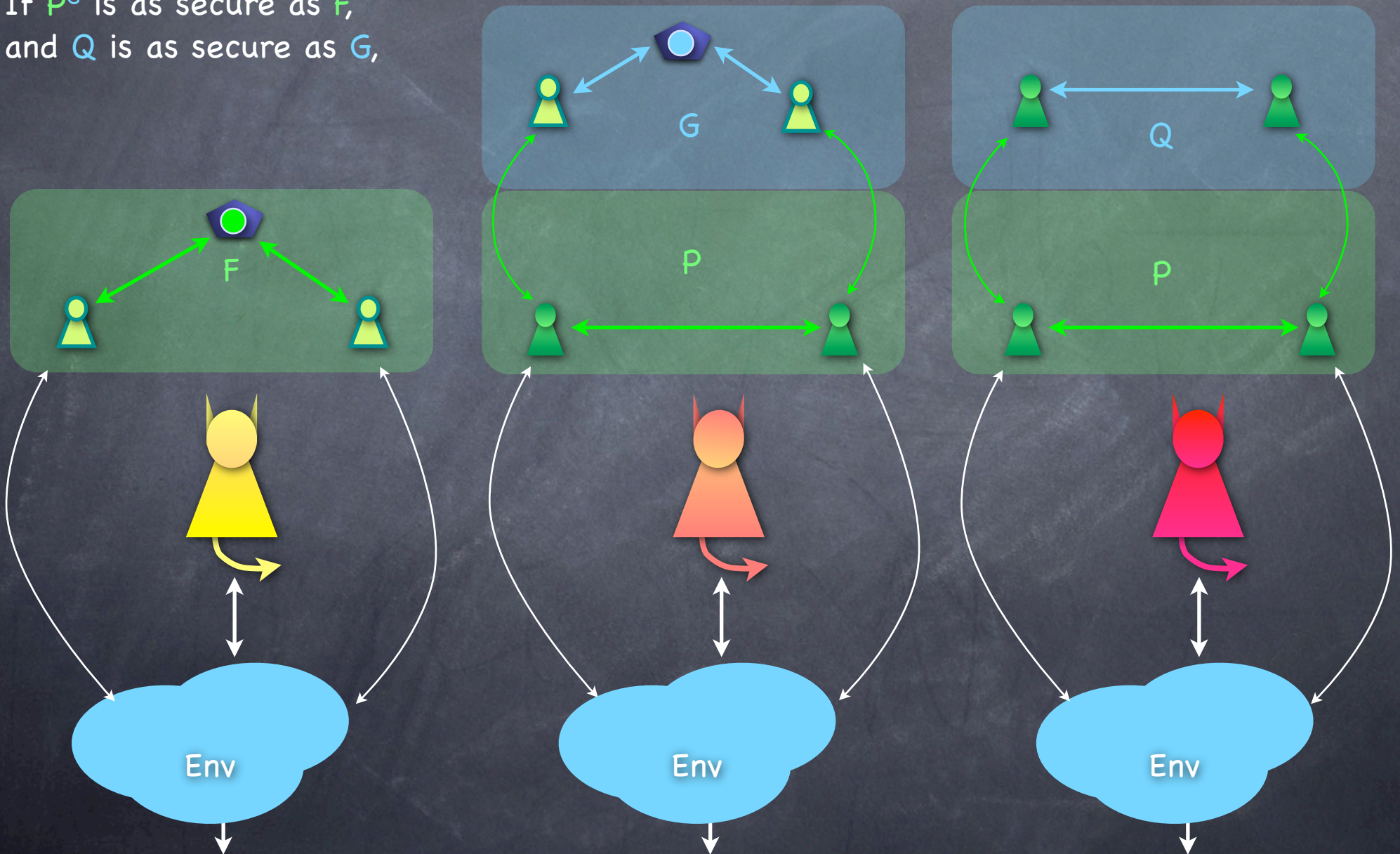
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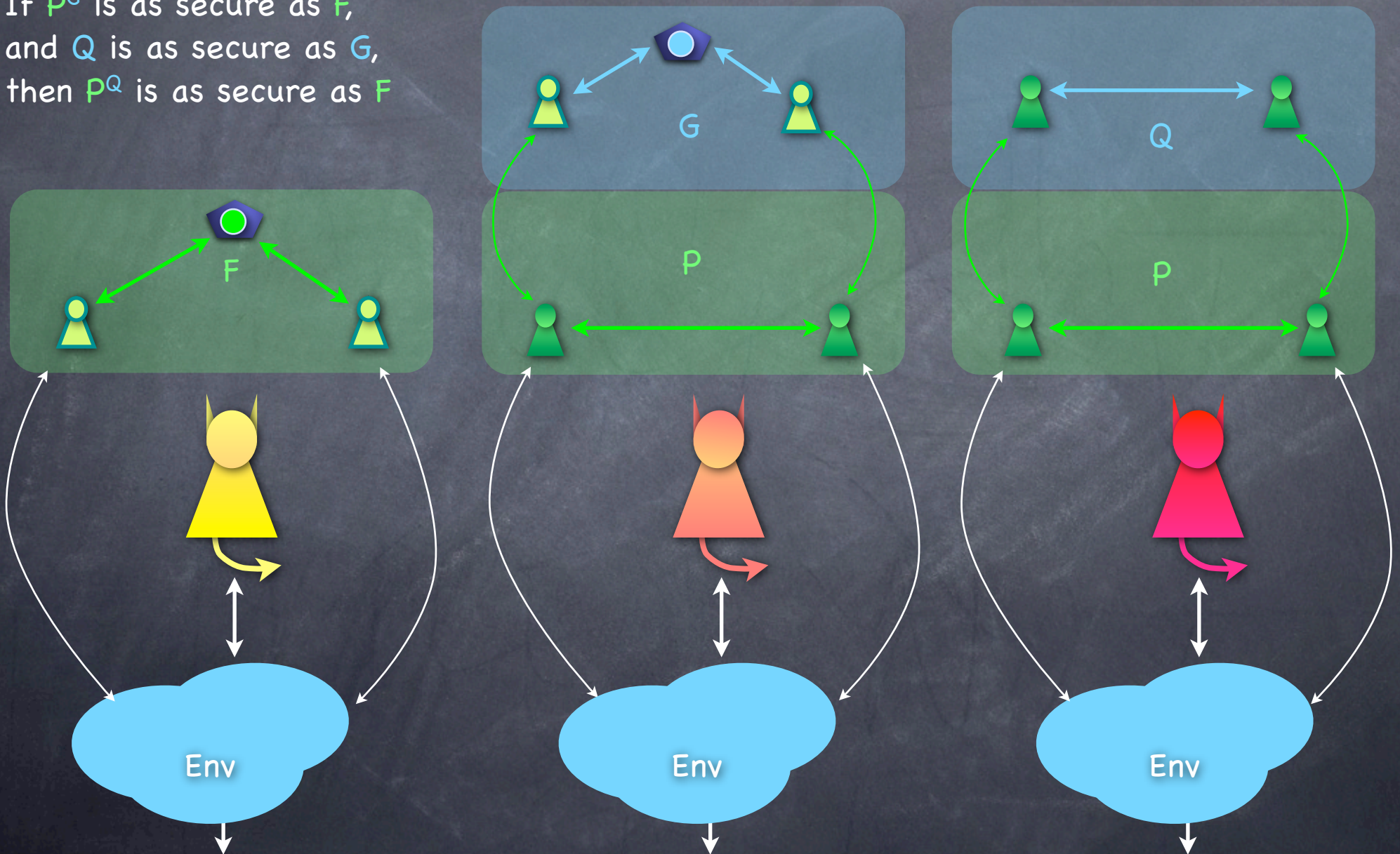
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  - Gives a modular implementation of the IDEAL world



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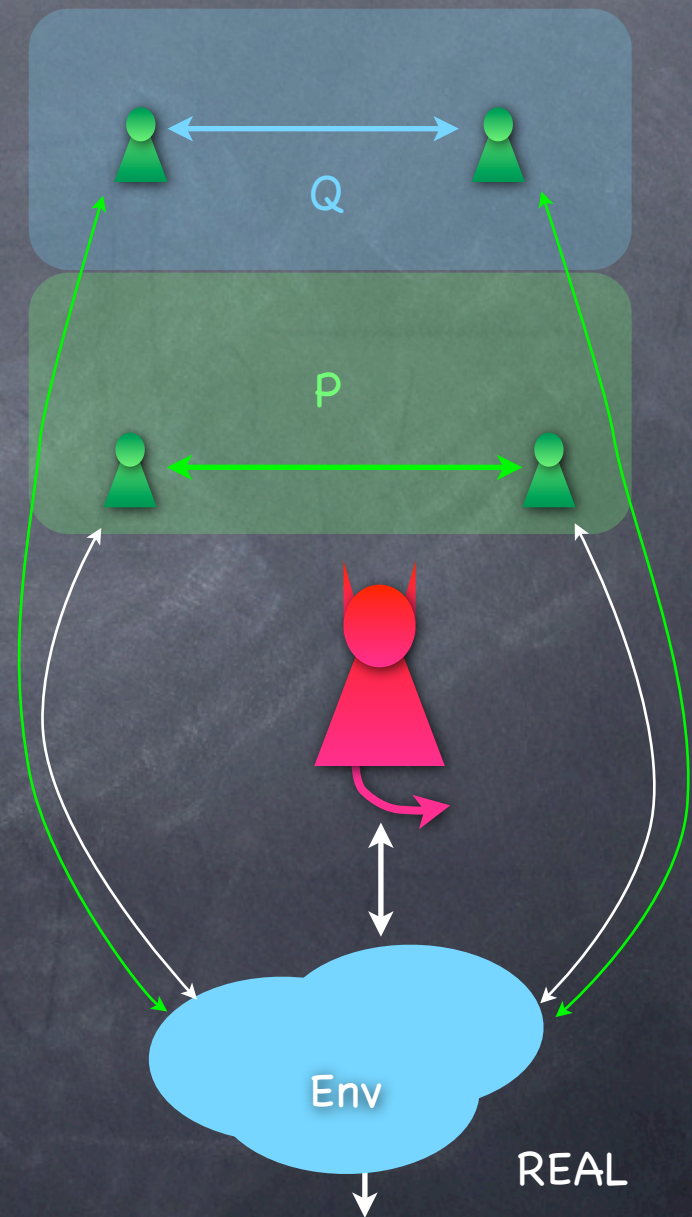


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    - e.g. Define security of composed system as security of each individual component; Or, define everything secure.

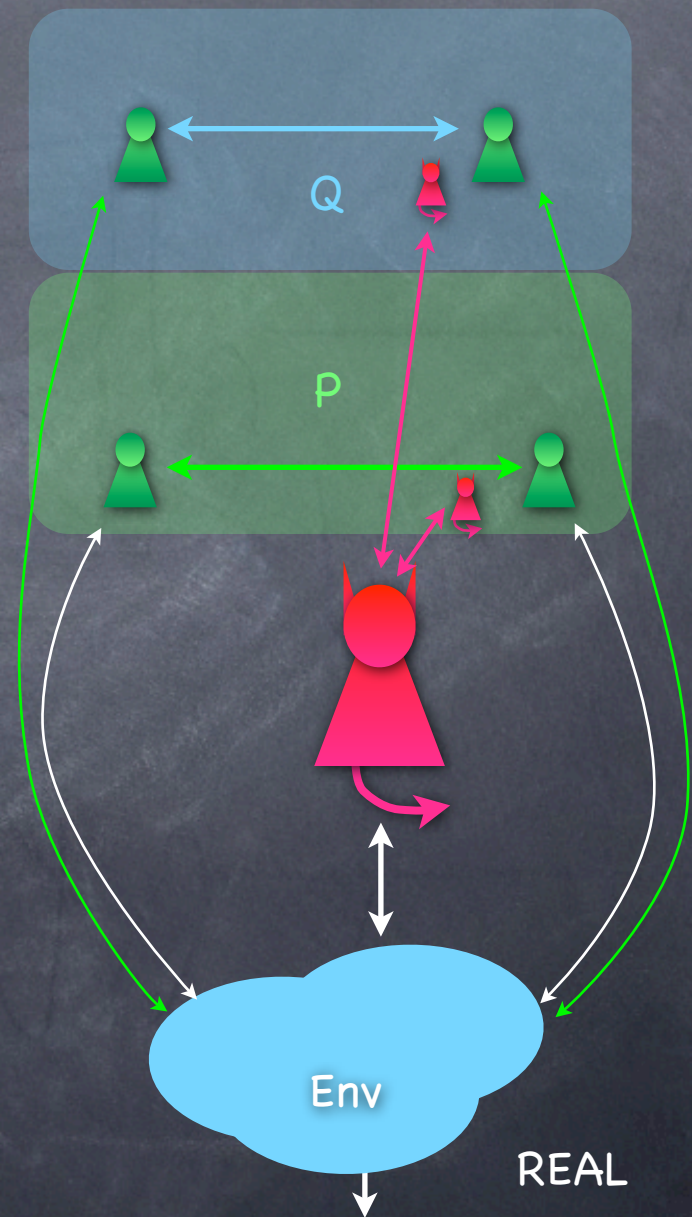


# Proving the UC theorem





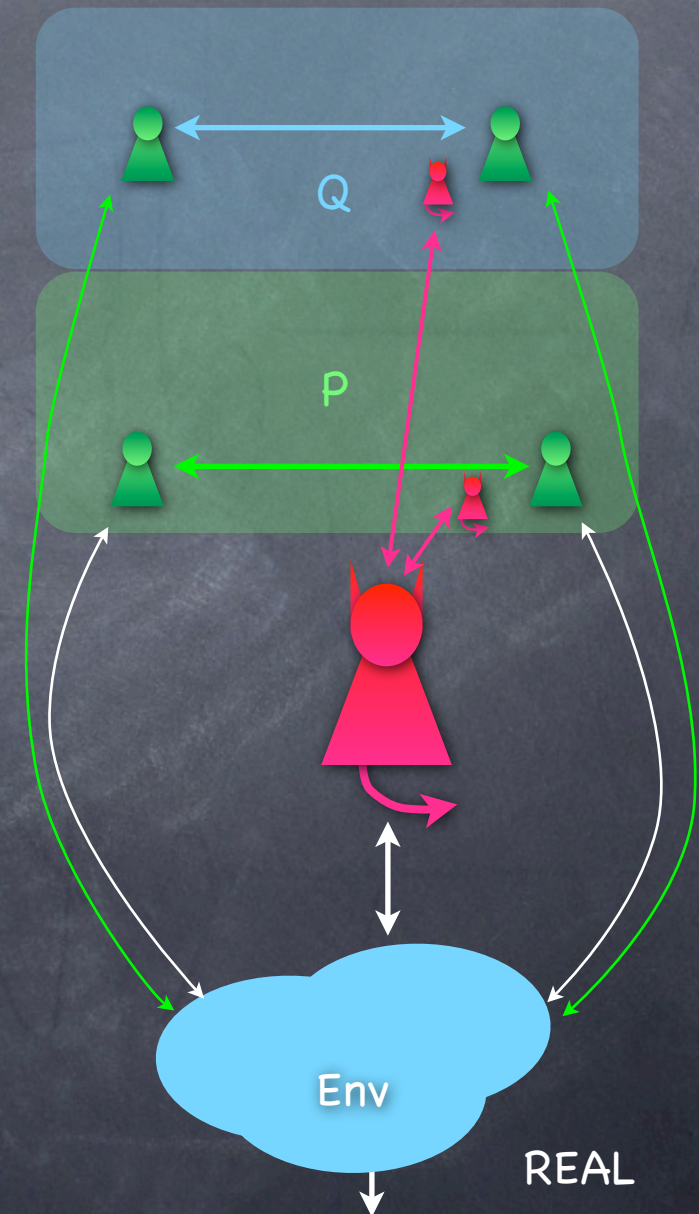
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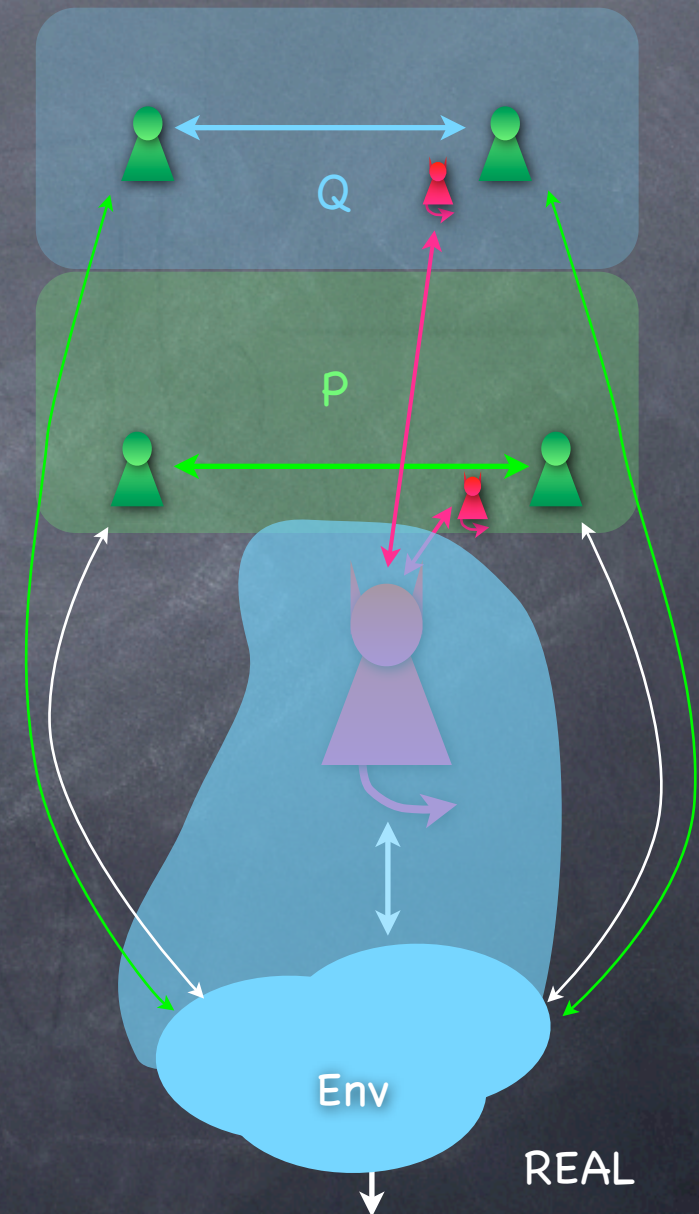
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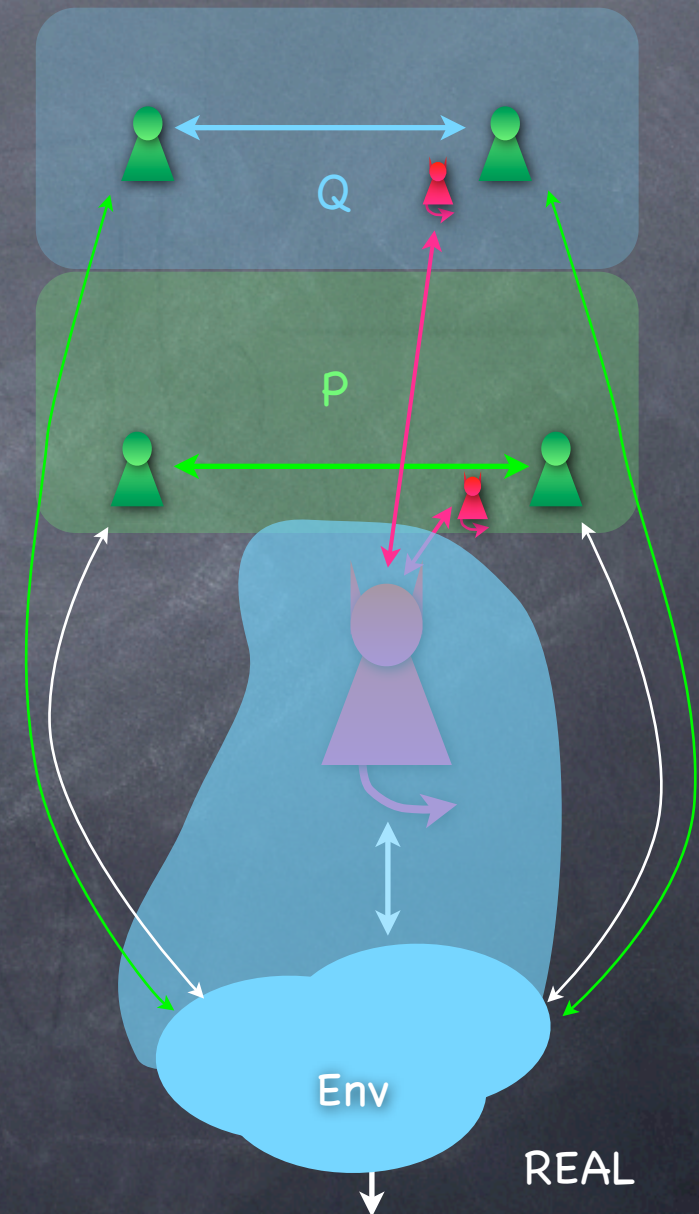
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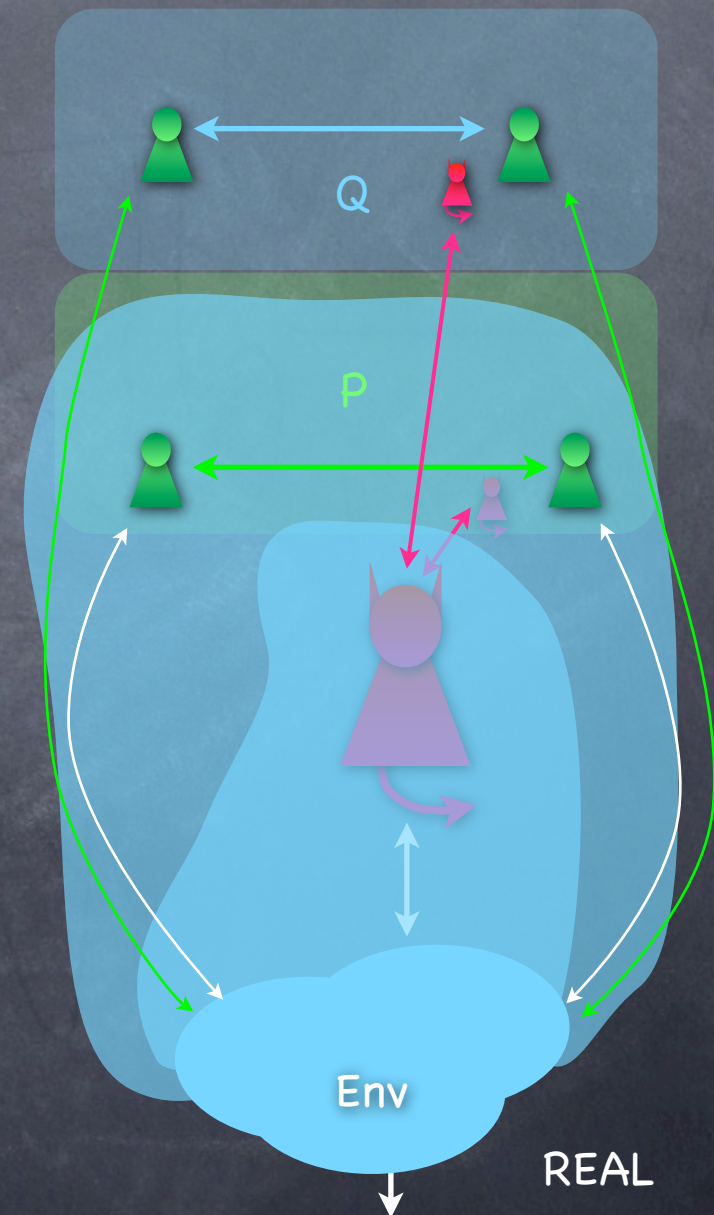
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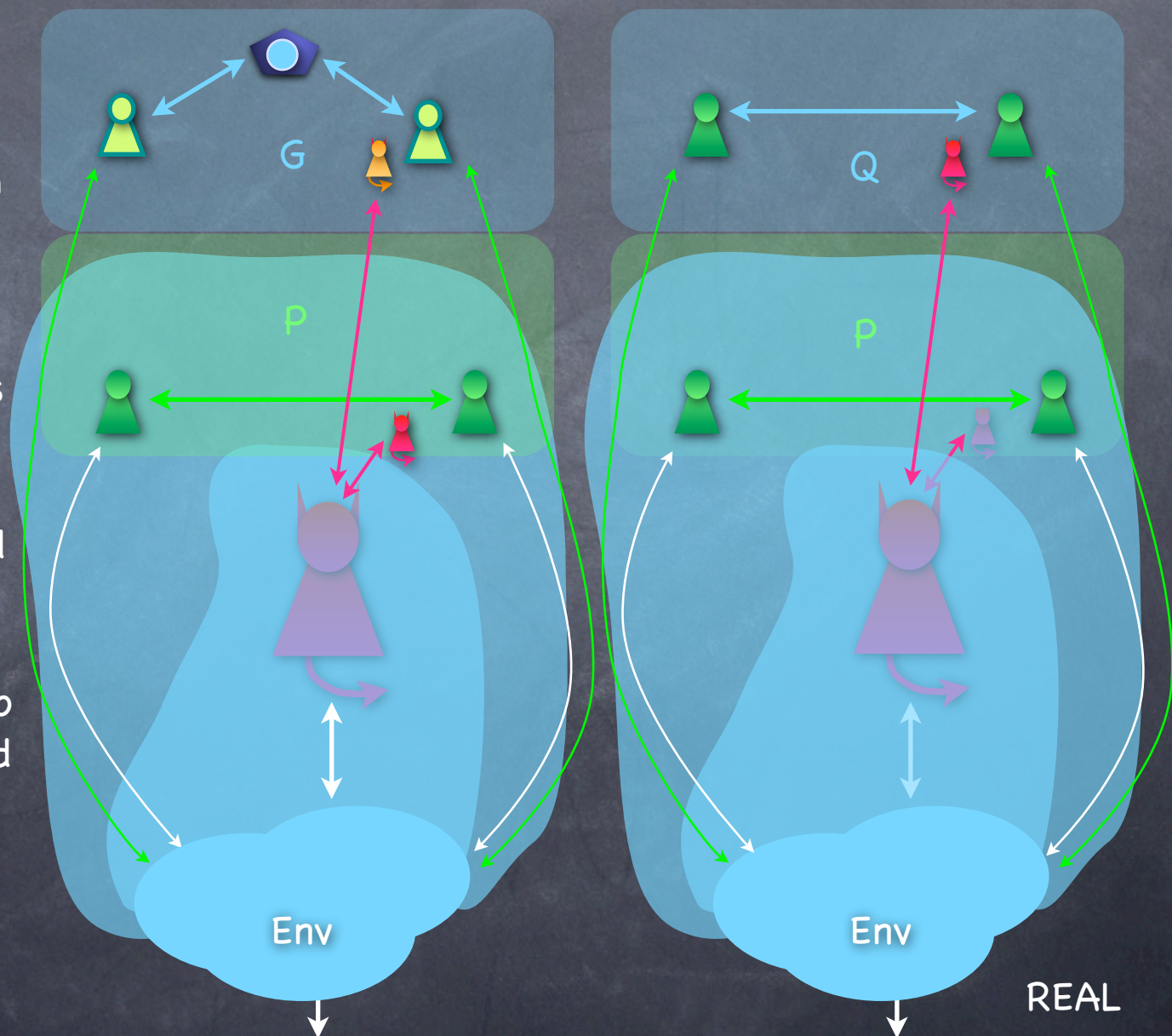
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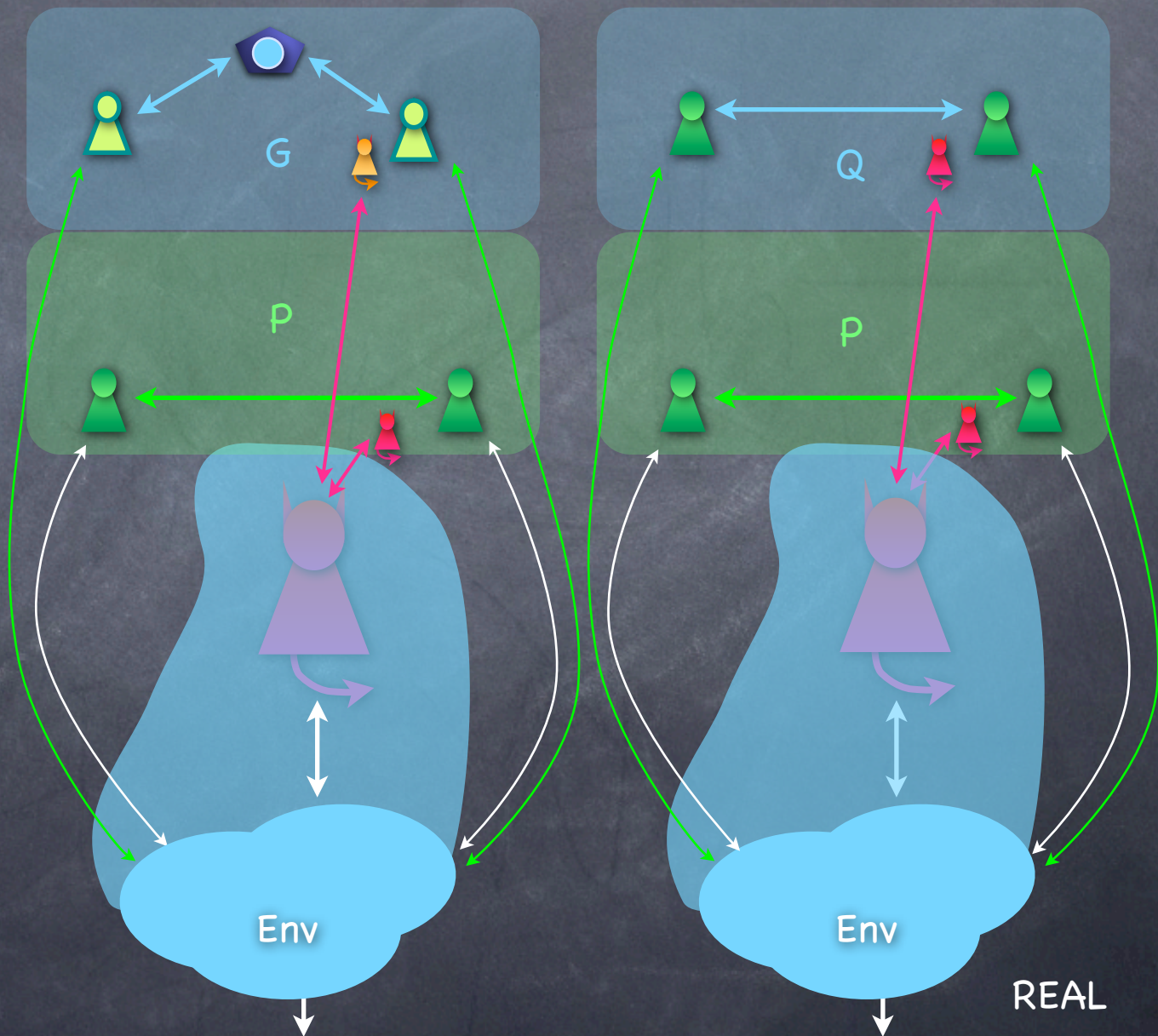
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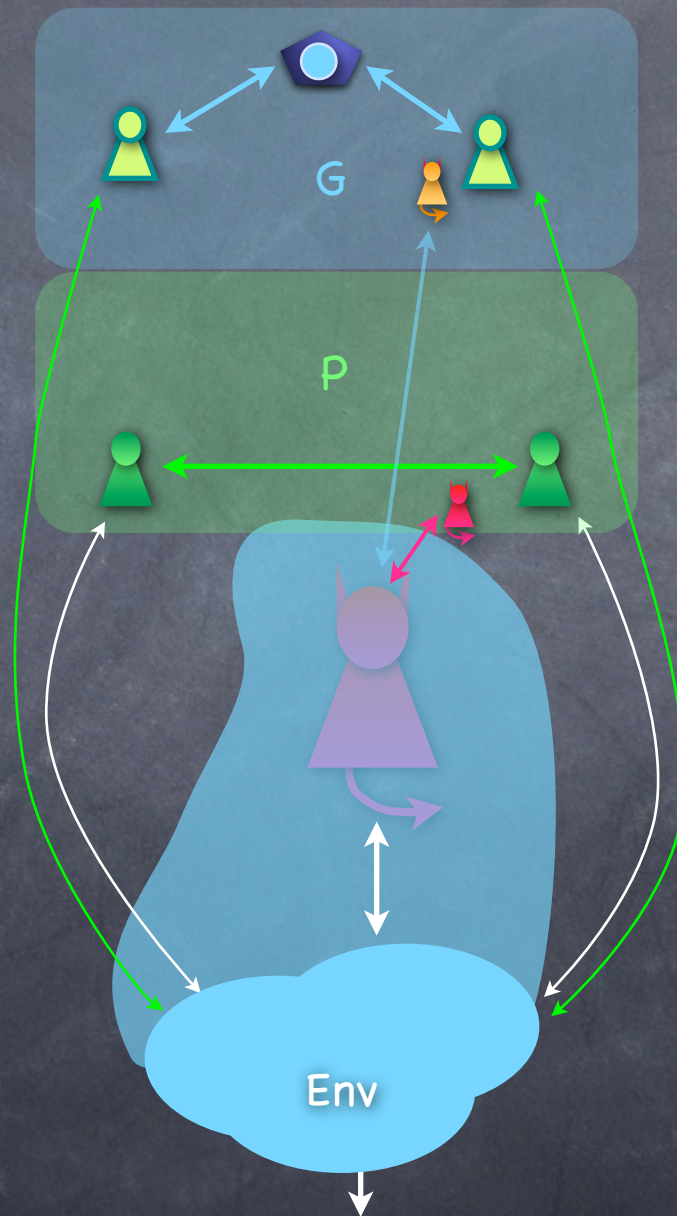
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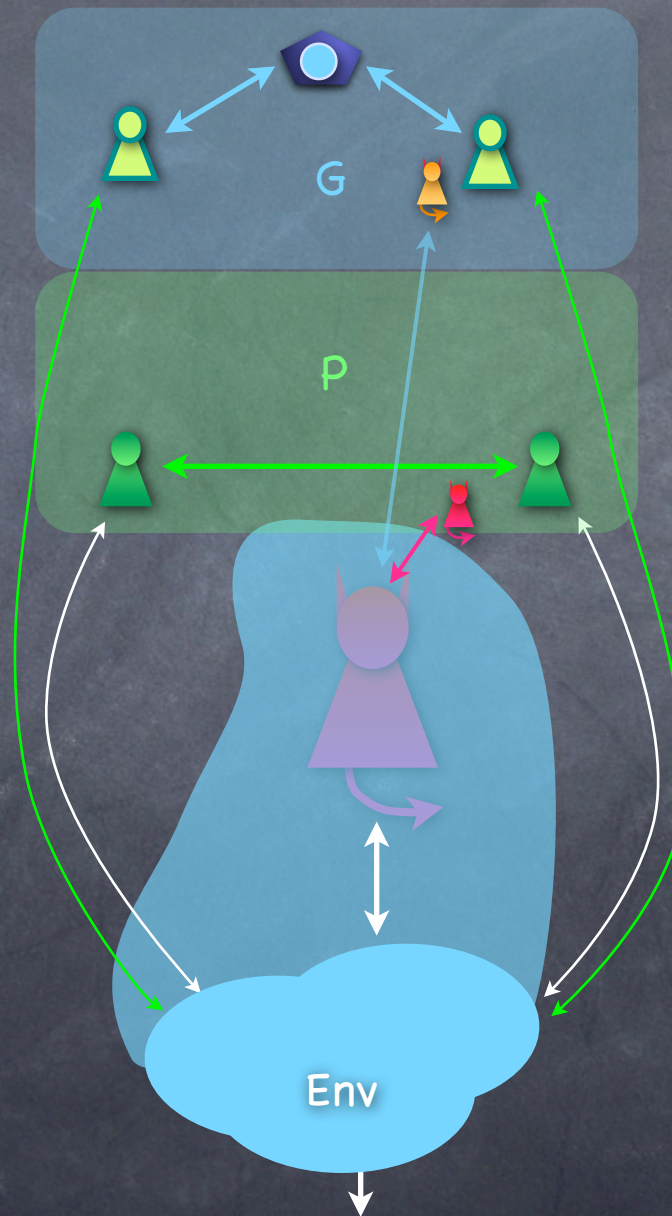


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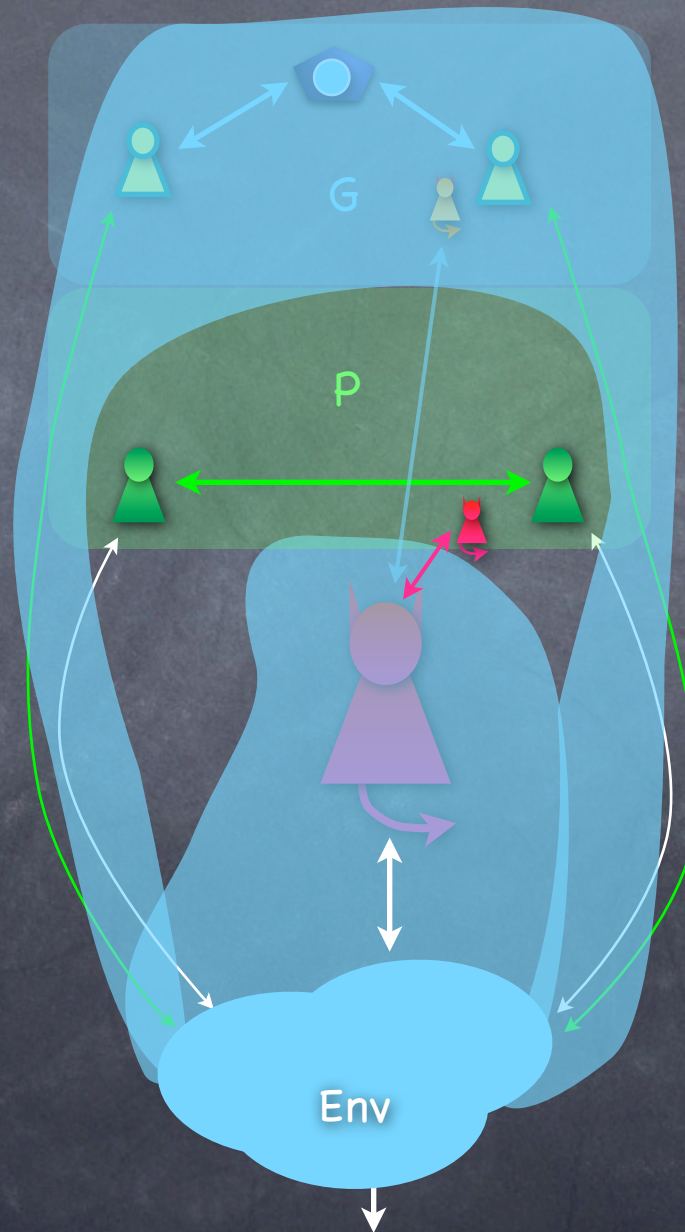
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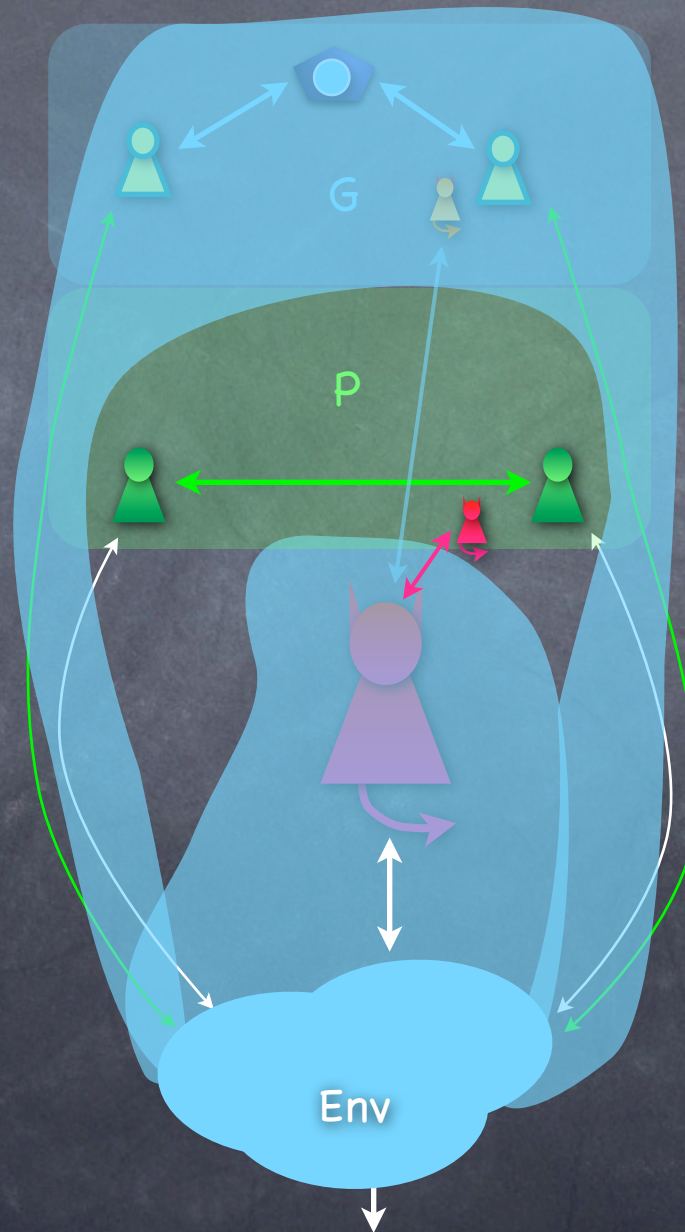
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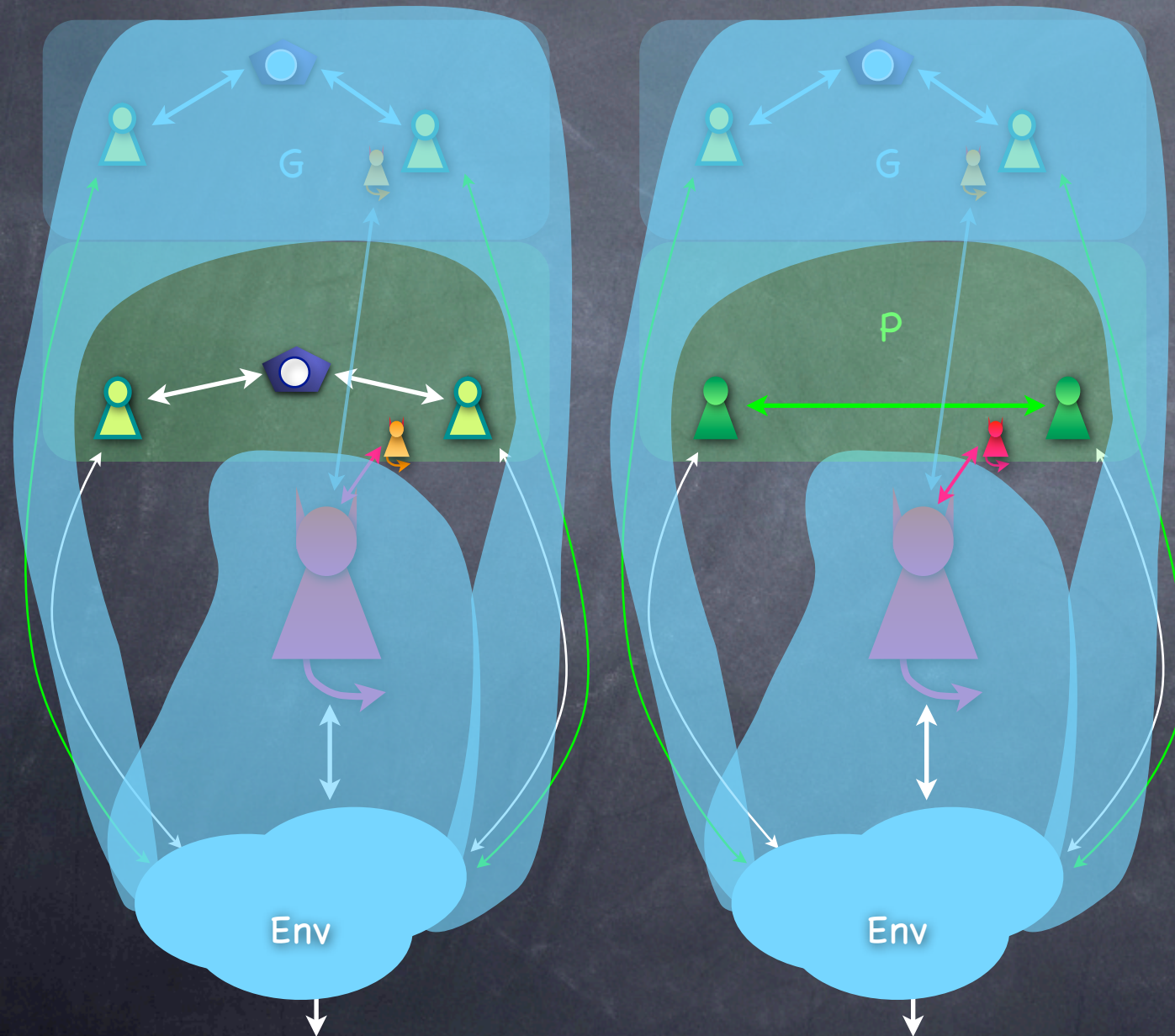
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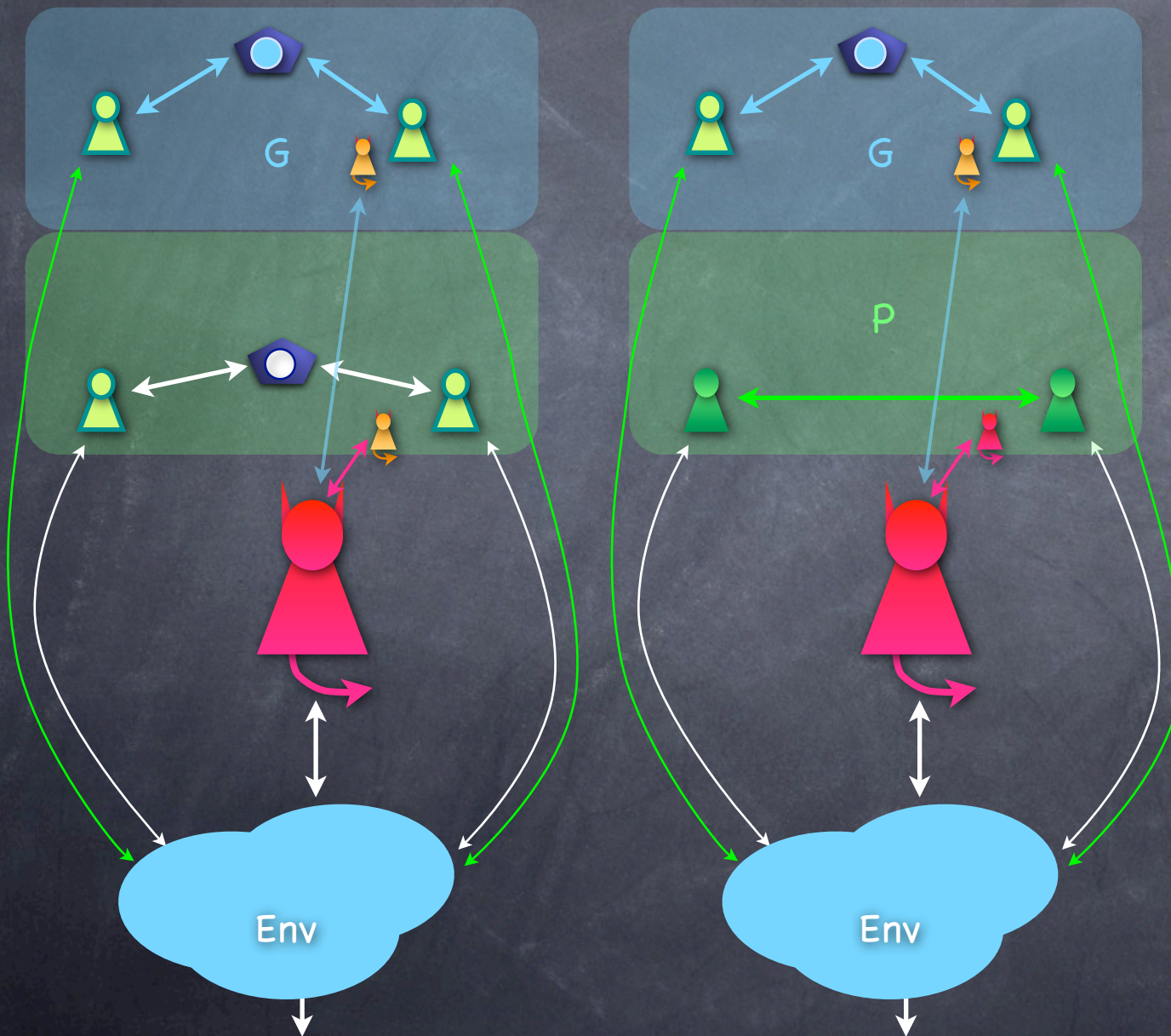
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- Use " $P$  is as secure as  $F$ " to get a new world with  $F$  and a new adversary



# Proving the UC theorem



- Now consider new environment s.t. only P (and adversary) is outside it
- Note: G and simulator for Q/G are inside the new environment
- Use "P is as secure as F" to get a new world with F and a new adversary