Hash Functions in Action

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Lecture 11

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- Today: CRHF construction. Domain Extension.
  Applications of hash functions

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- Collision-Resistant HF: $h \leftarrow \mathcal{H}$; $A(h) \rightarrow (x,y)$. $h(x)=h(y)$ w.n.p

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  - All candidates use mathematical structures that are considered computationally expensive

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  - Hash halves the size of the input

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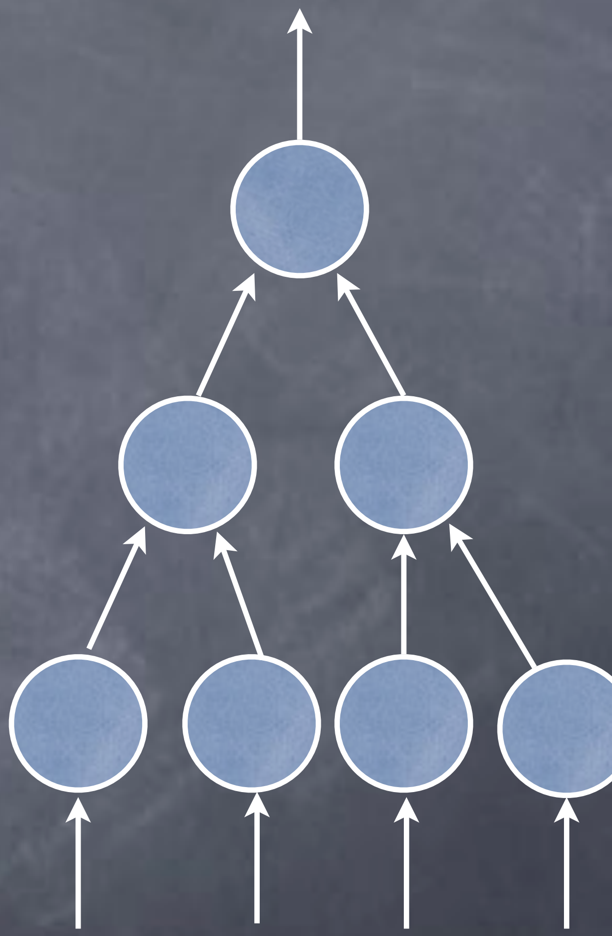

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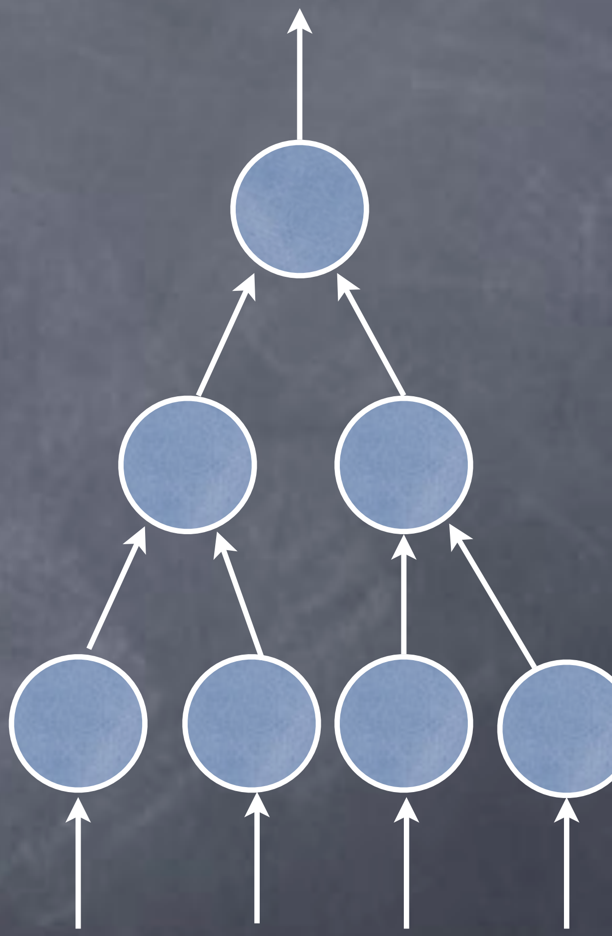

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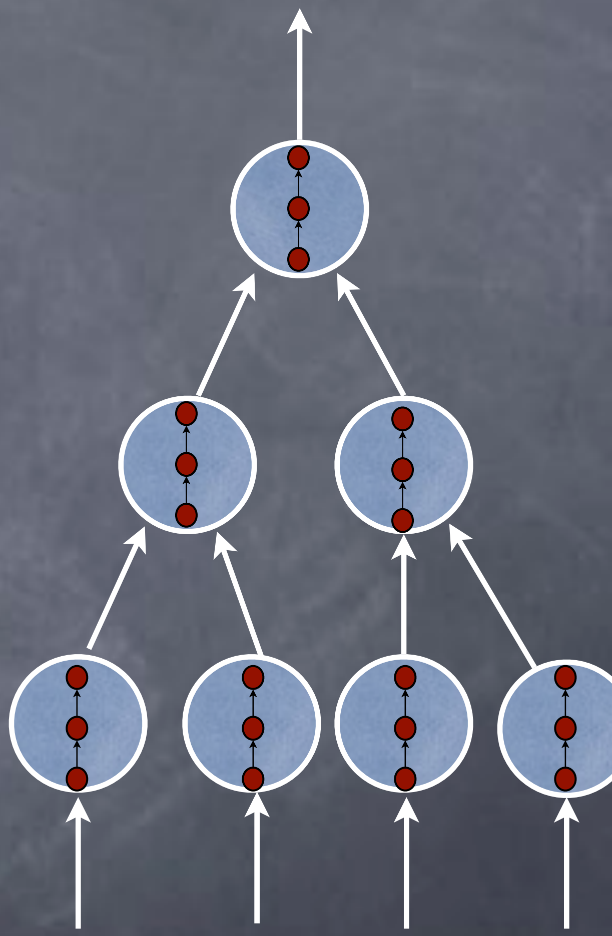

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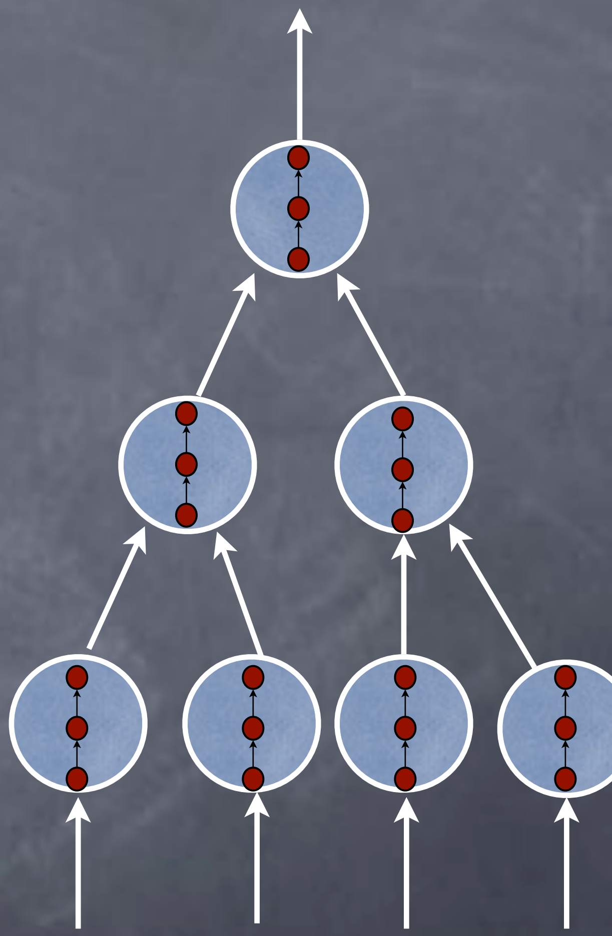

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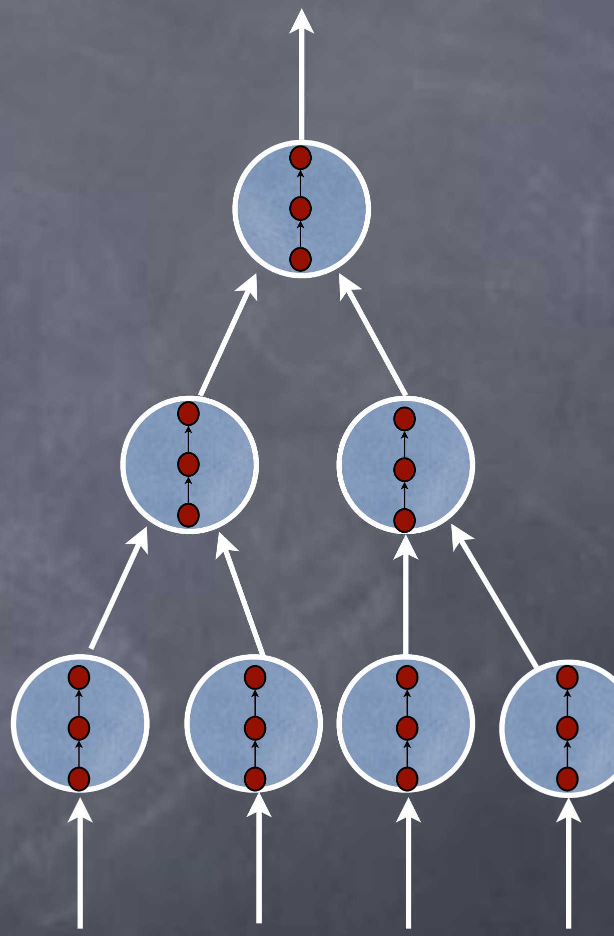

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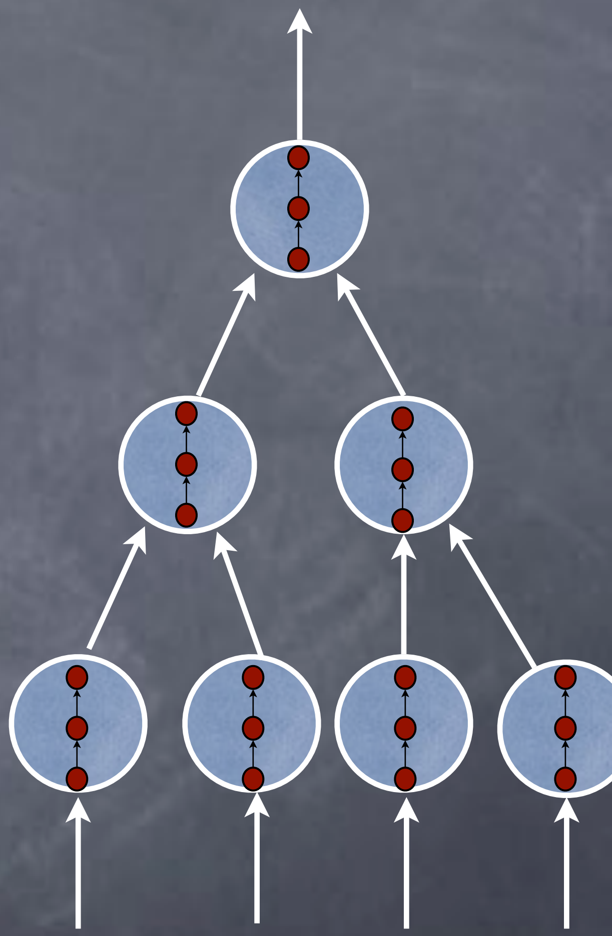

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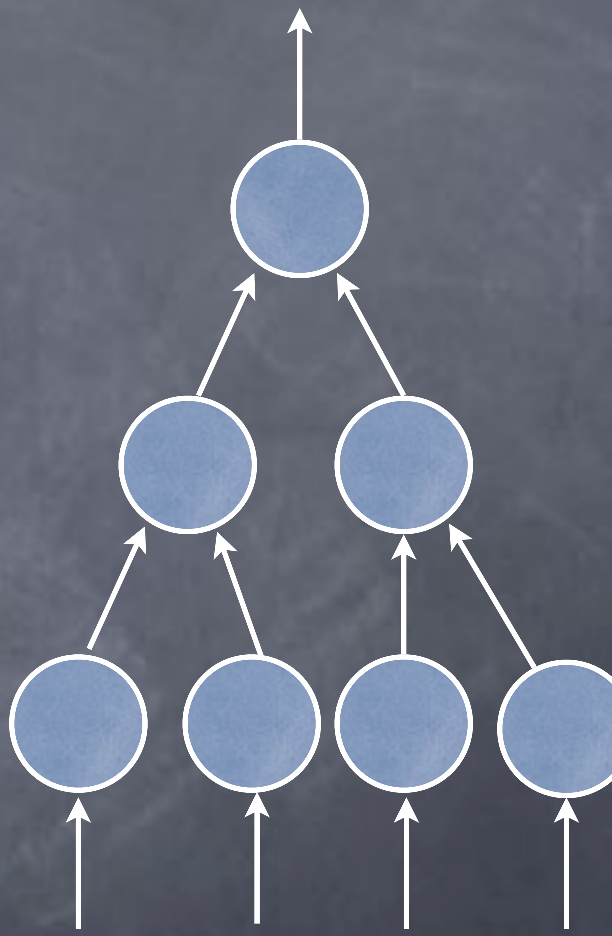

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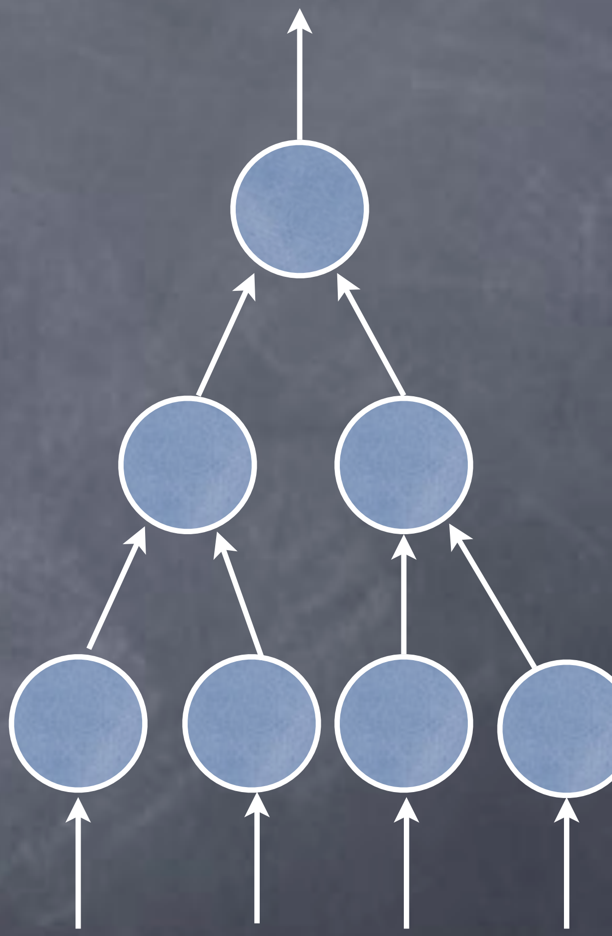

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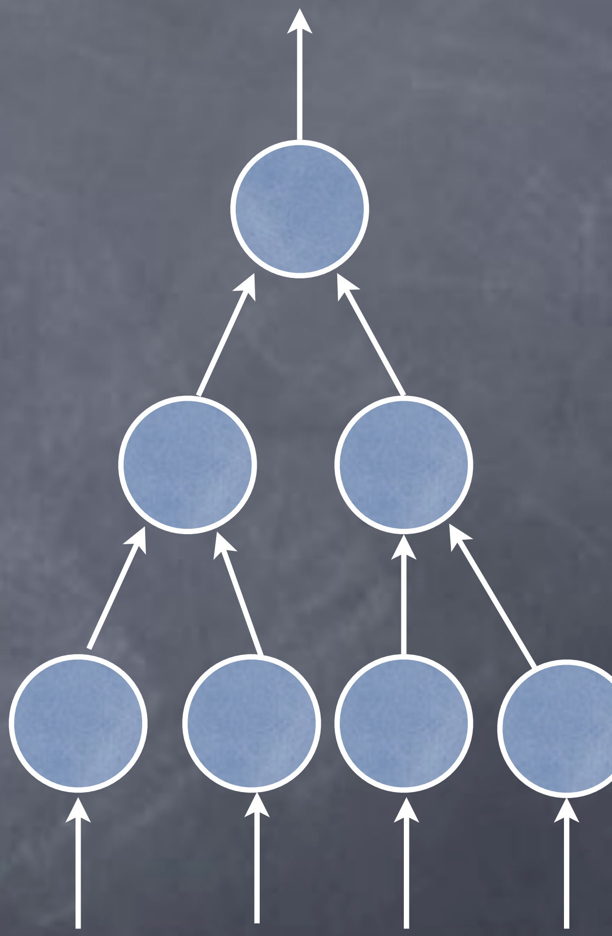

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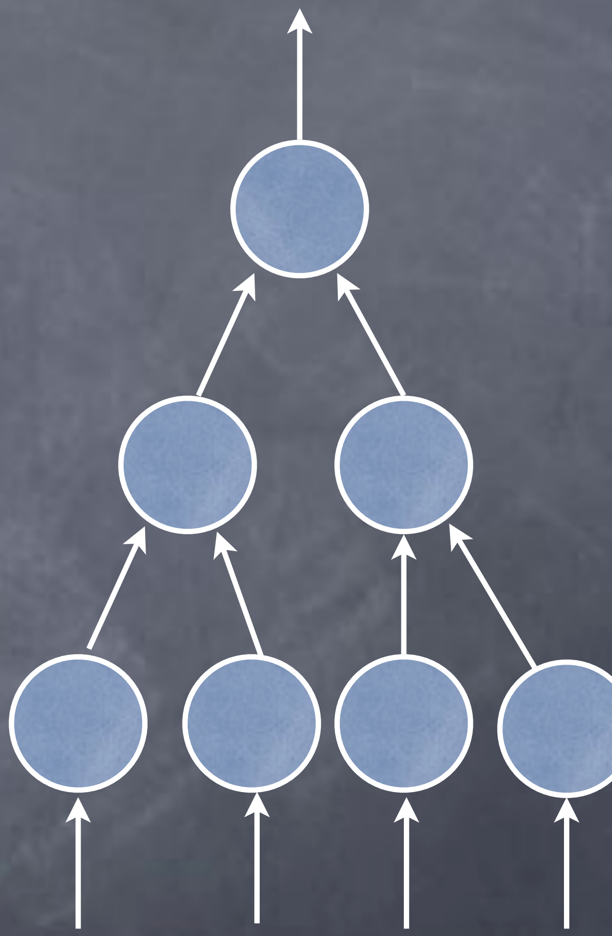

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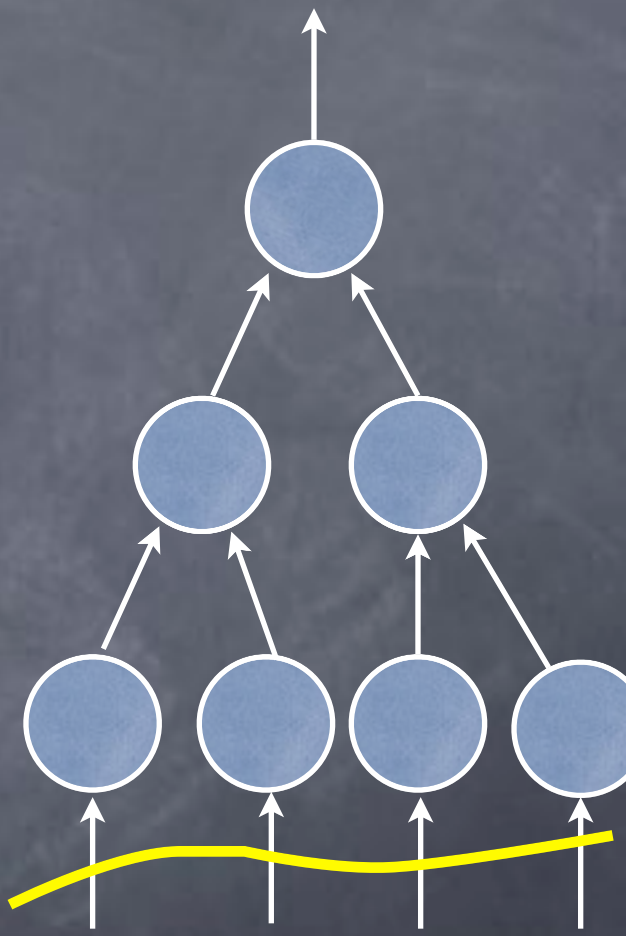

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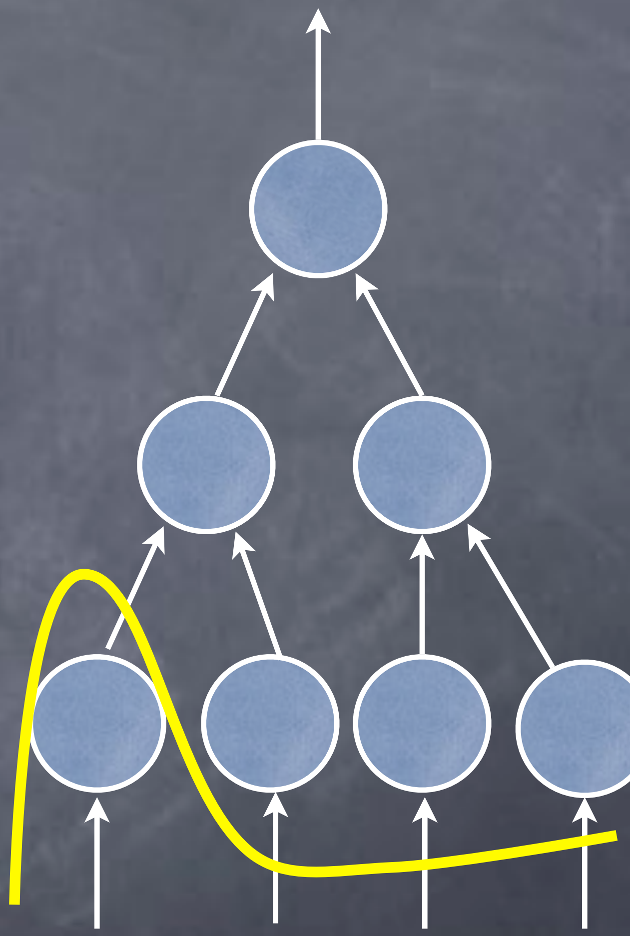

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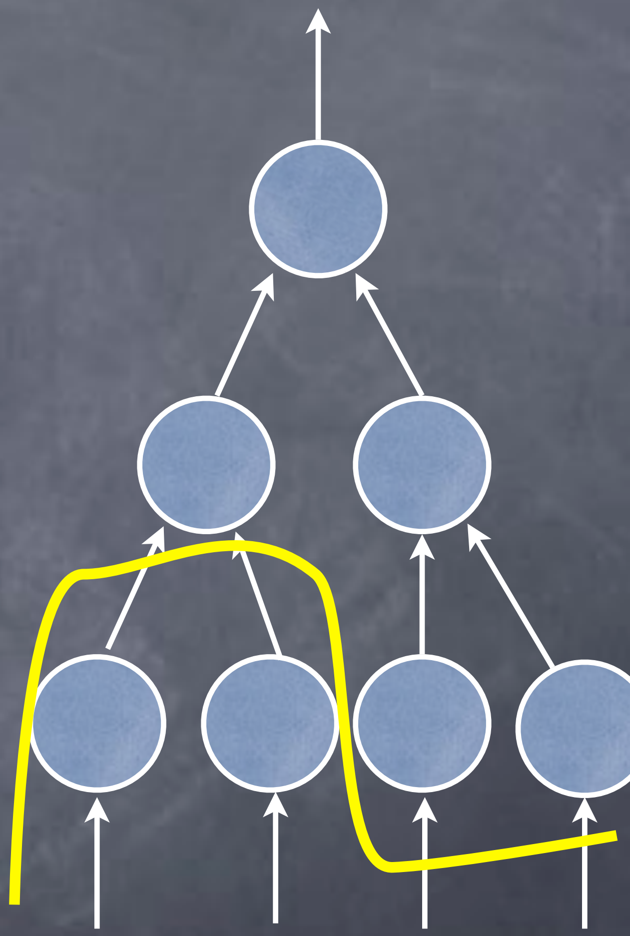

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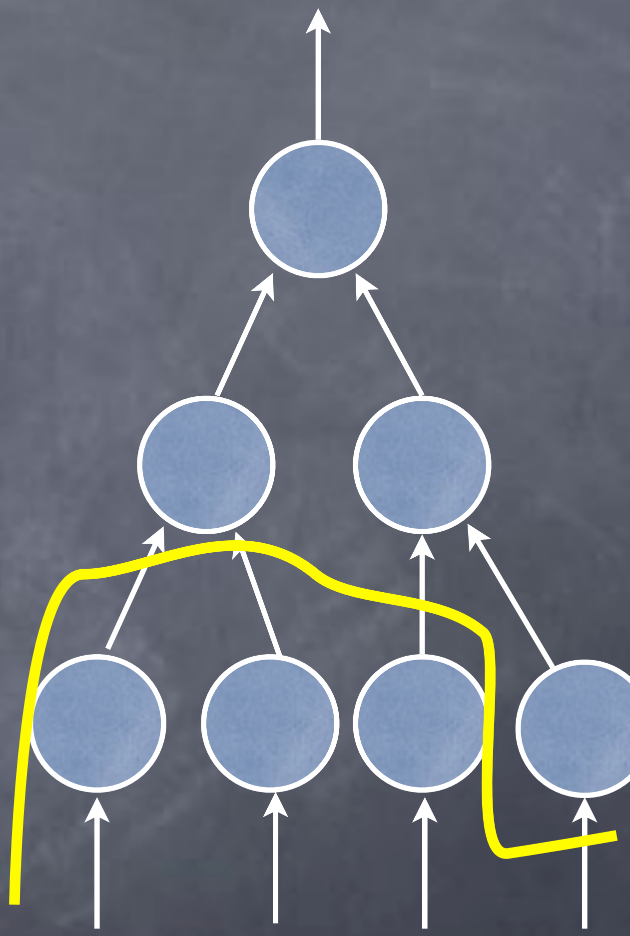

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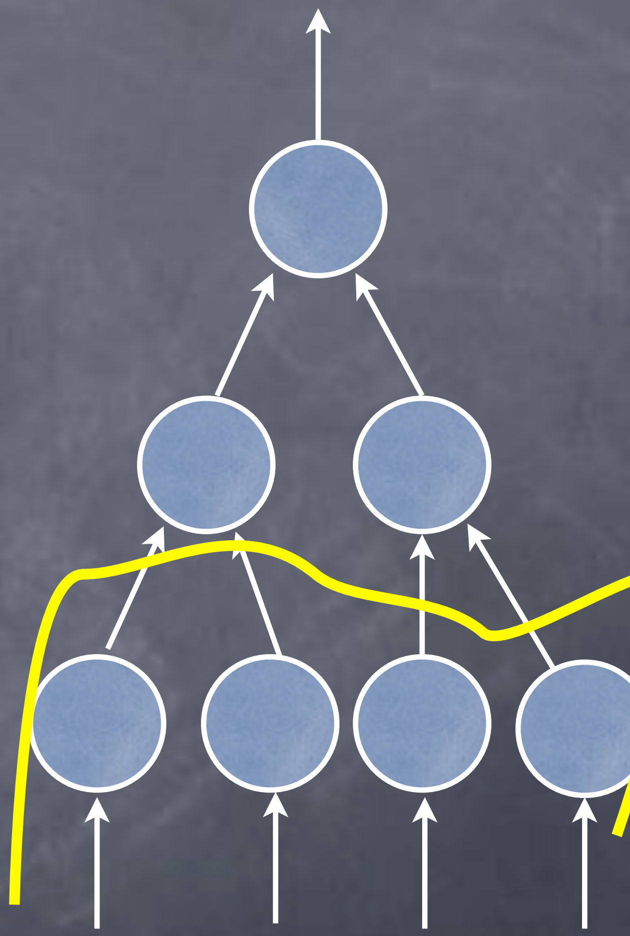

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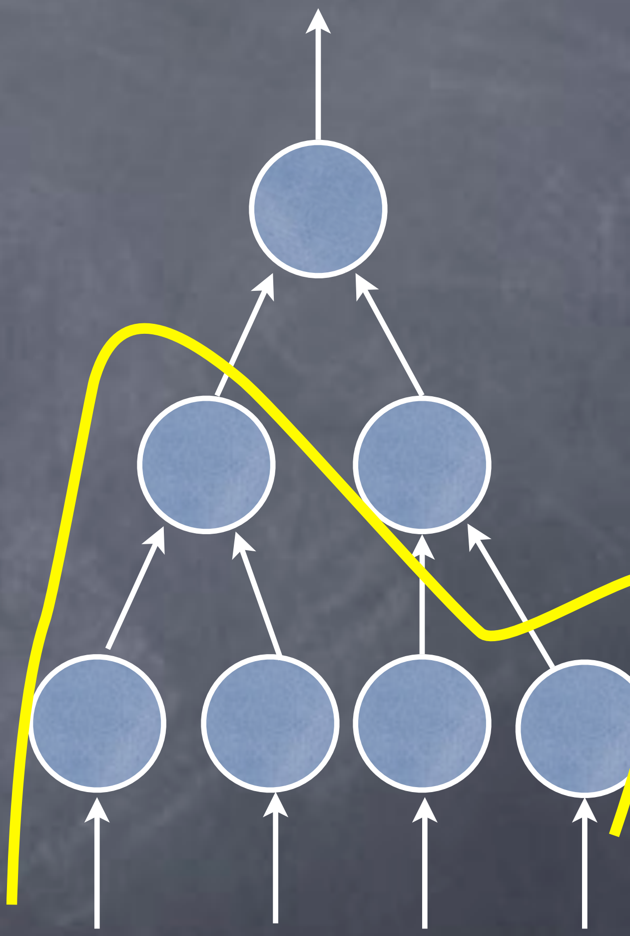

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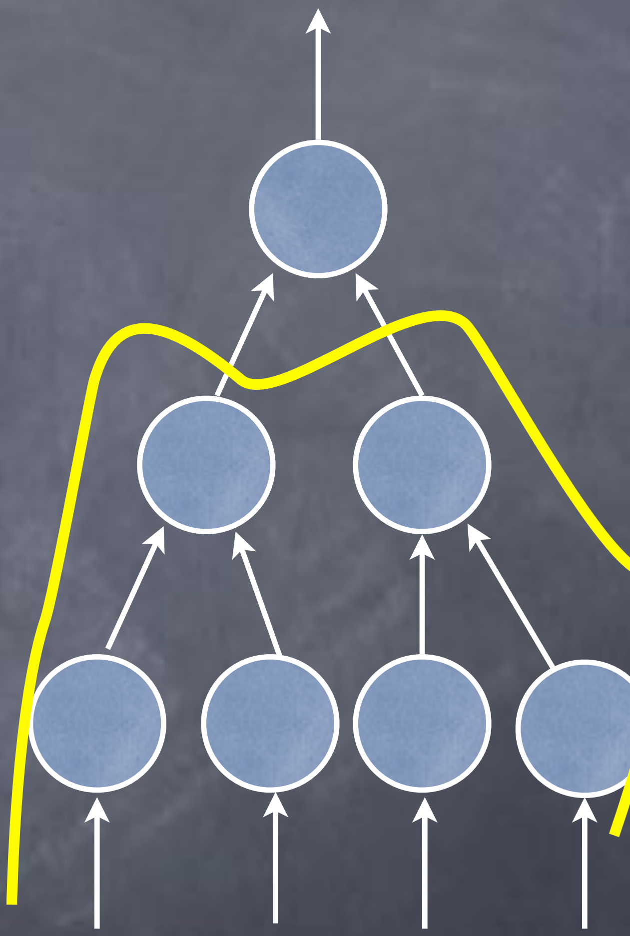

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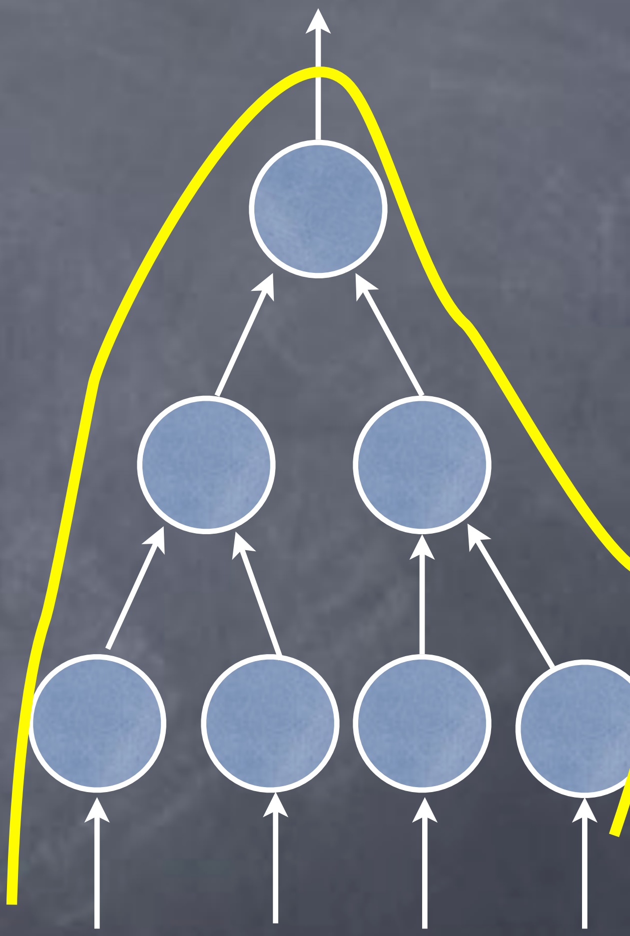

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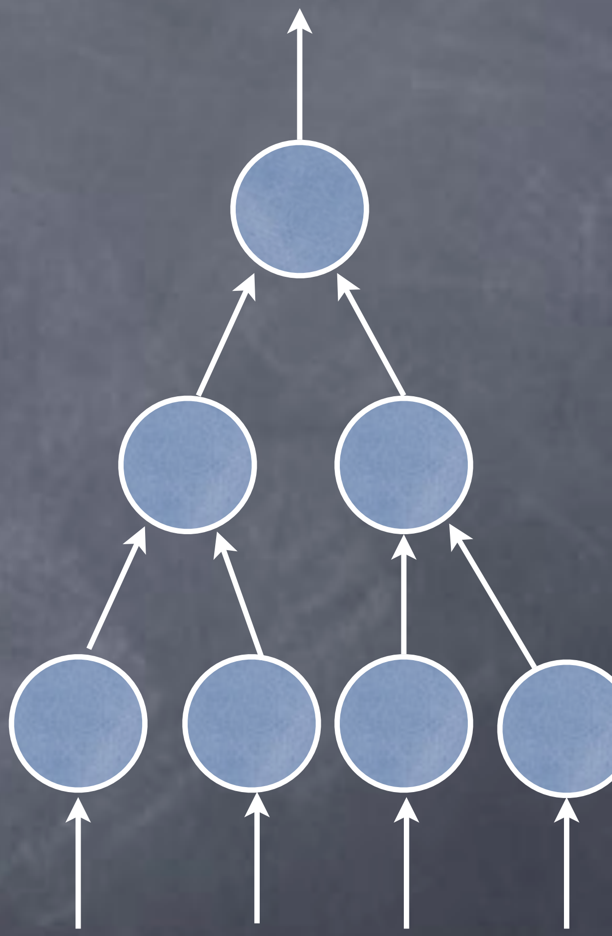

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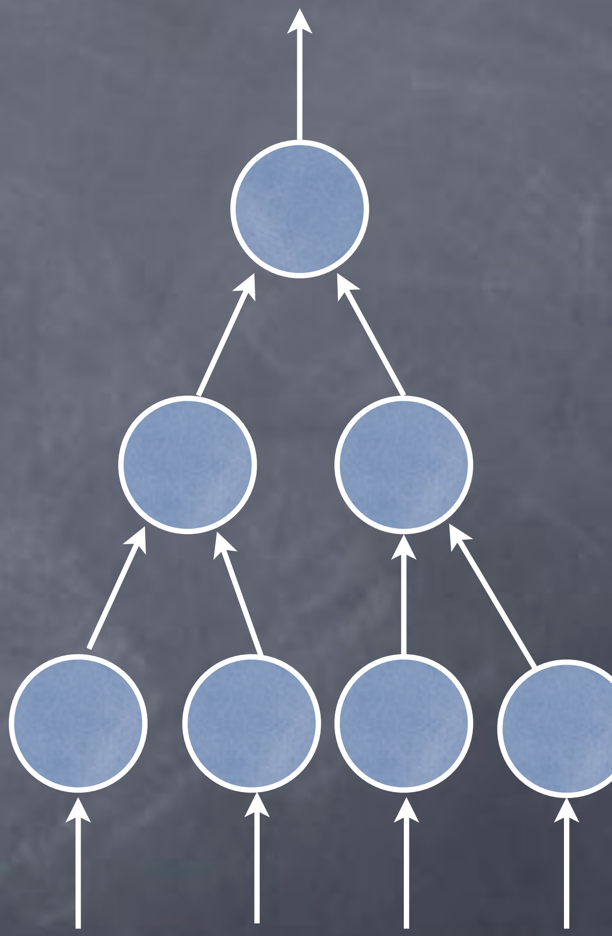

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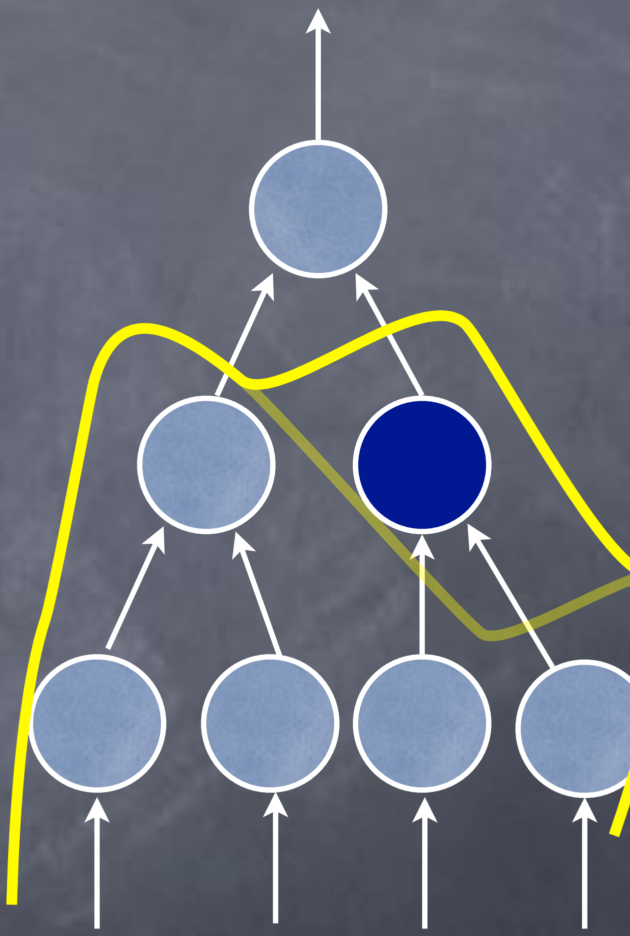

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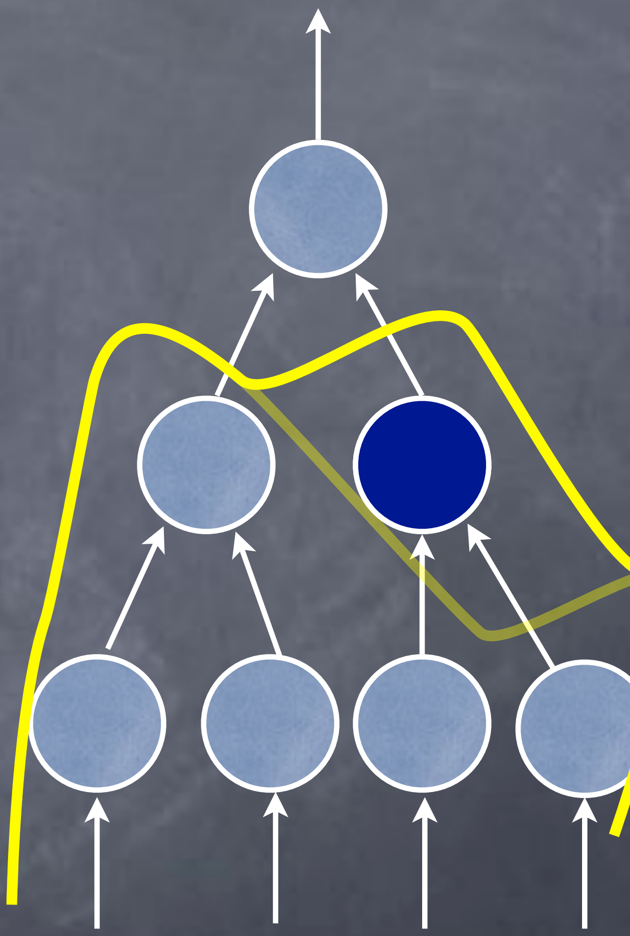

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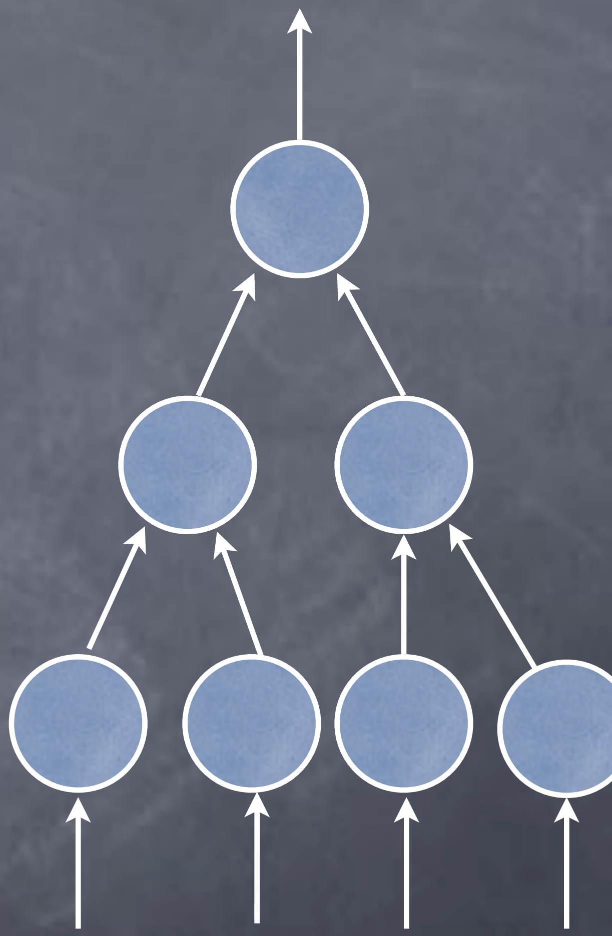

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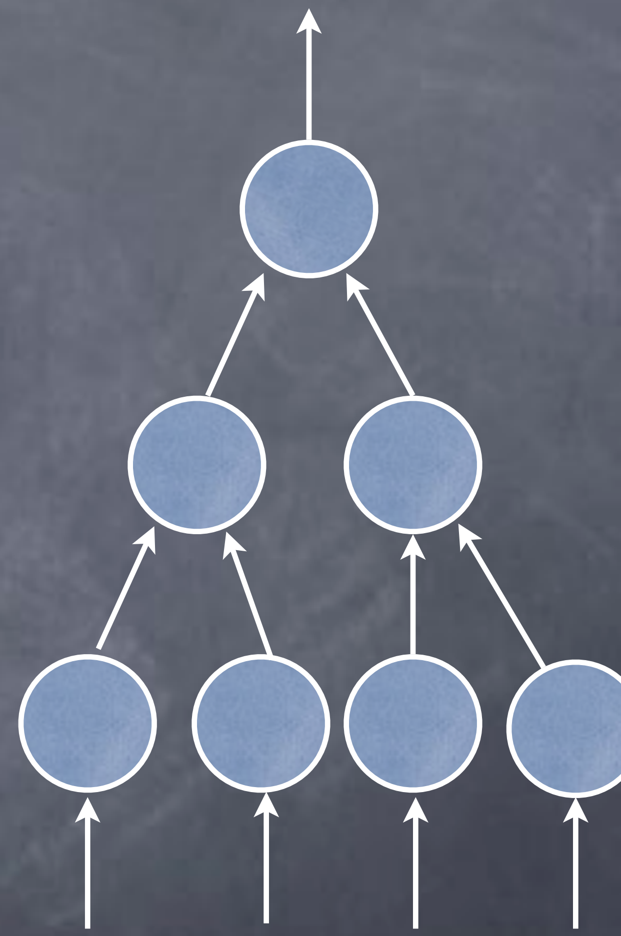

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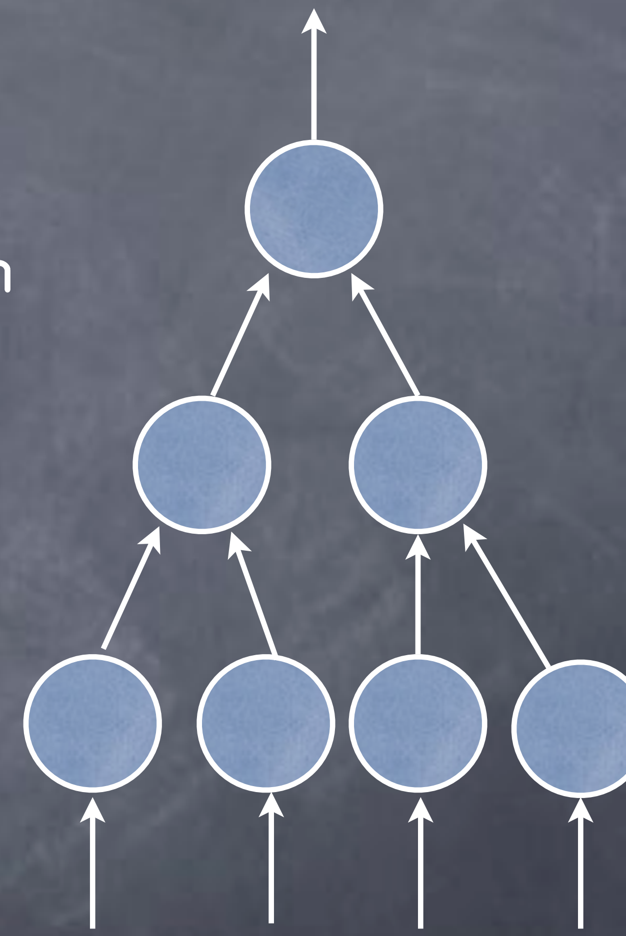

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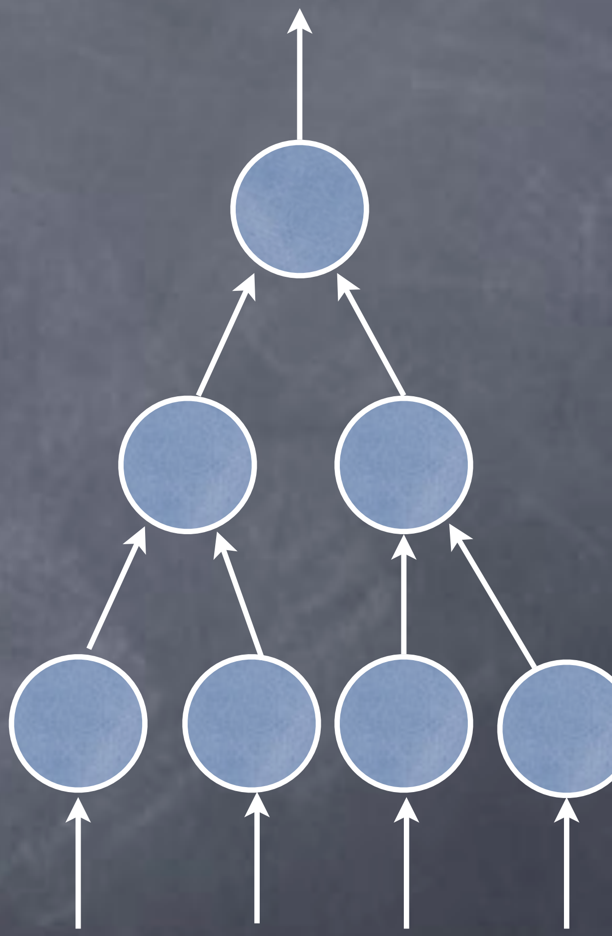

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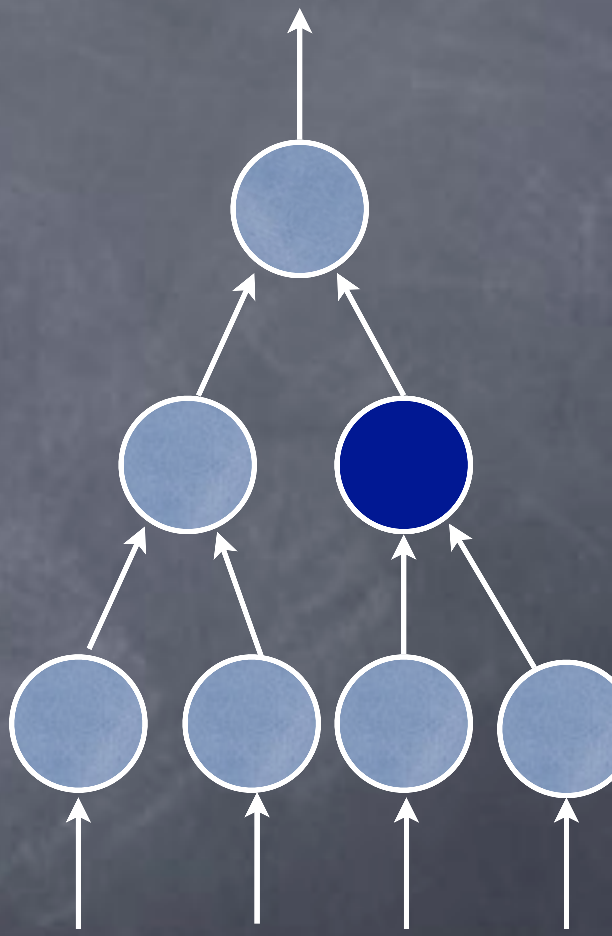

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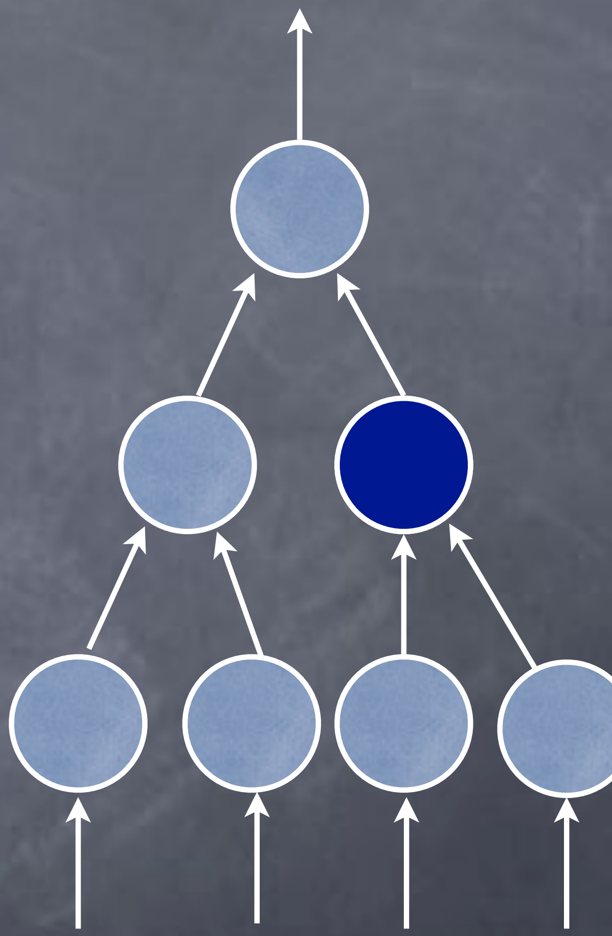

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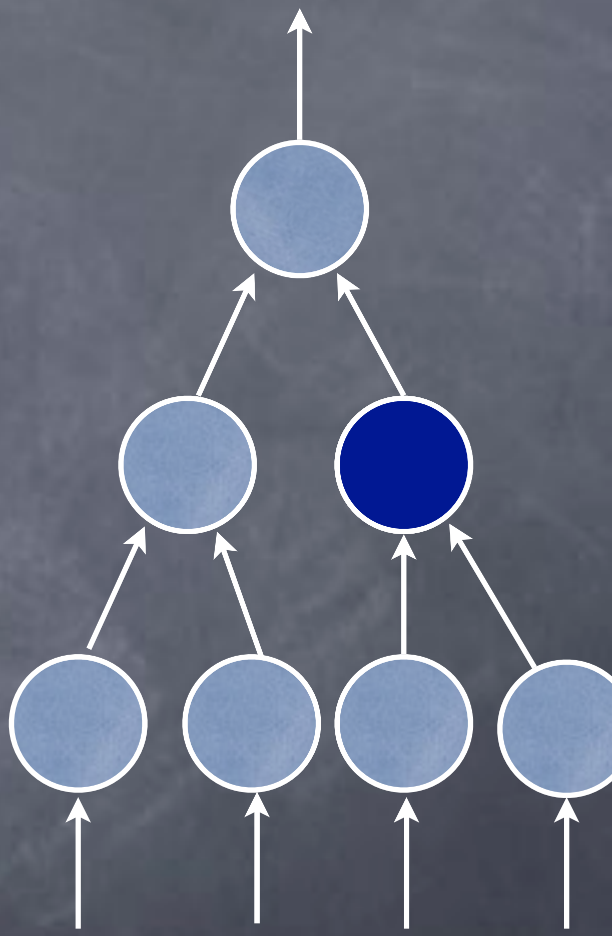

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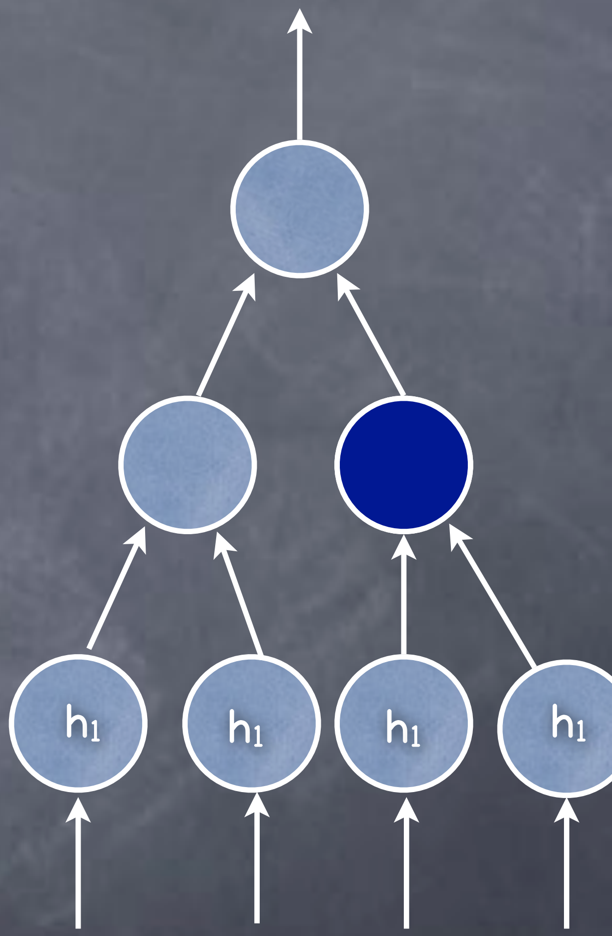

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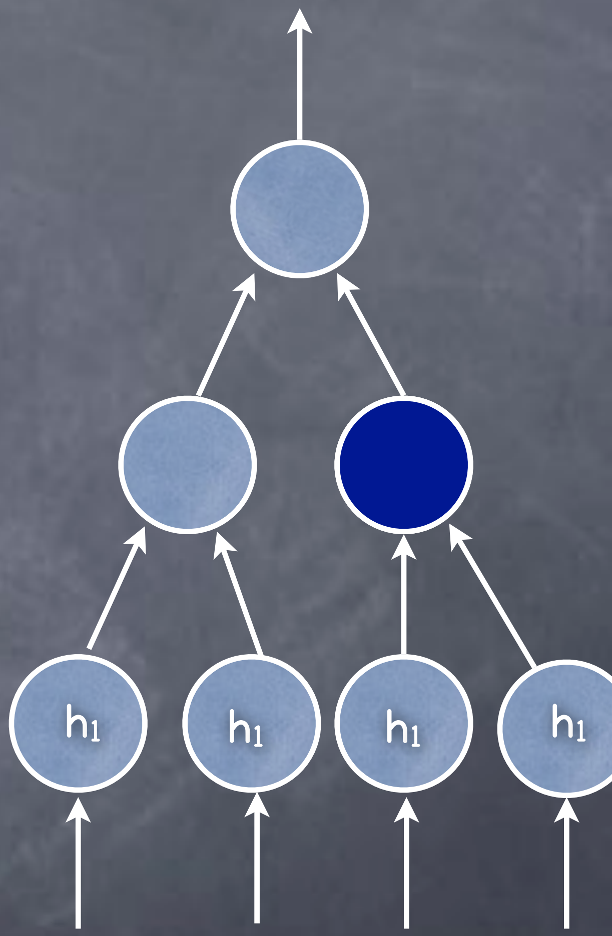

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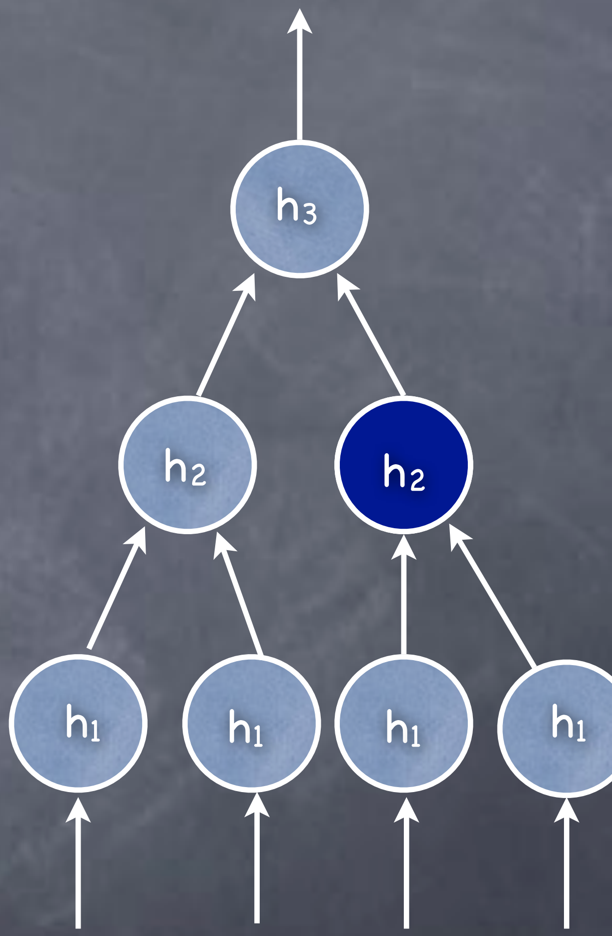

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- UOWHF theoretically important (based on simpler assumptions, good if paranoid), but CRHF can substitute for it
- Current practice: much less paranoid; faith on efficient, ad hoc (and unkeyed) constructions (though increasingly under attack)

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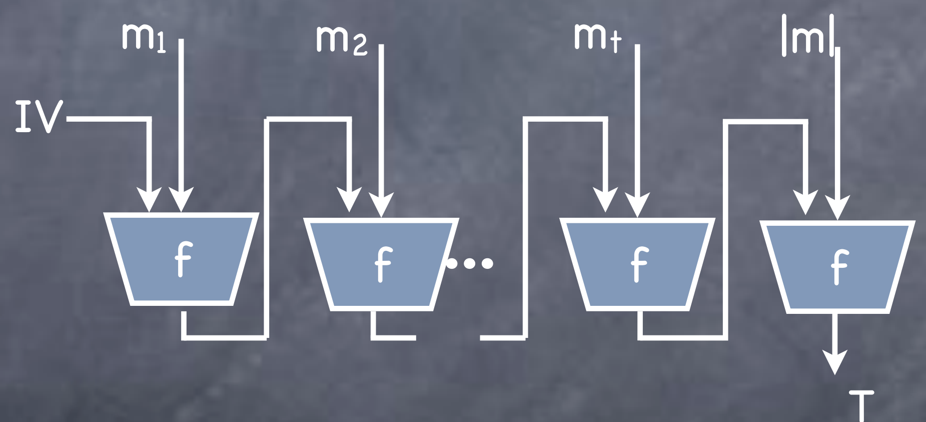

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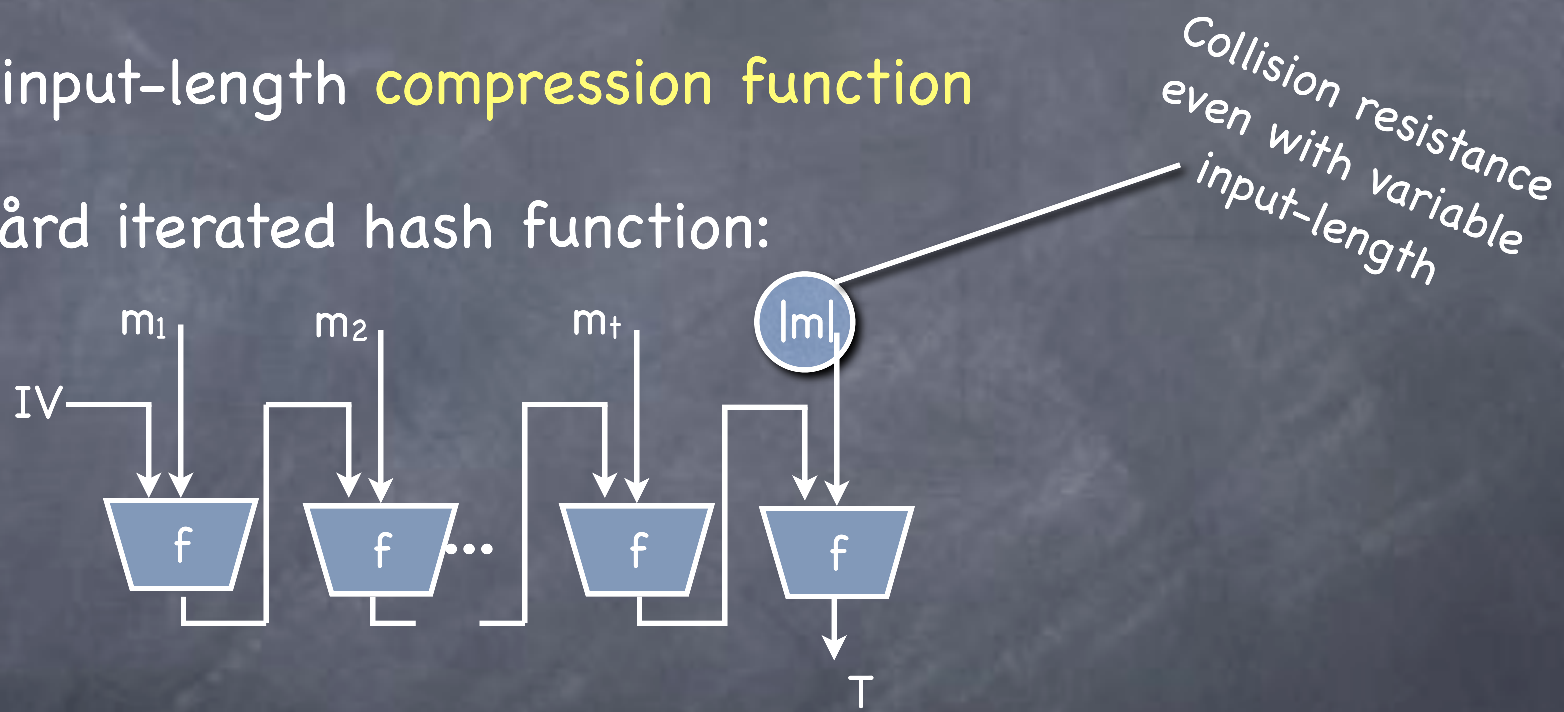

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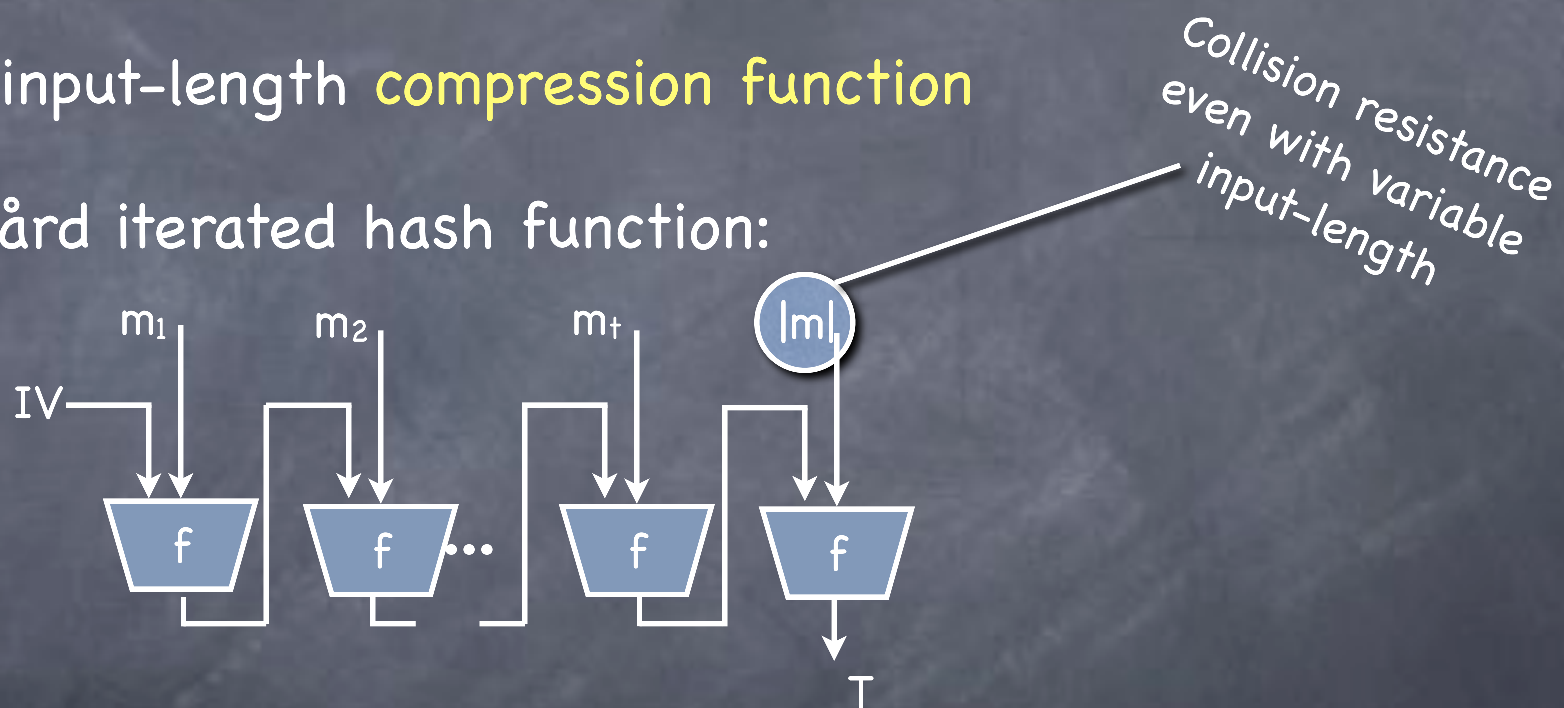

- If f collision resistant (not as "keyed" hash, but "concretely"), then so is the Merkle-Damgård iterated hash-function (for any IV)

MAC

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With 2-Universal Hash Functions

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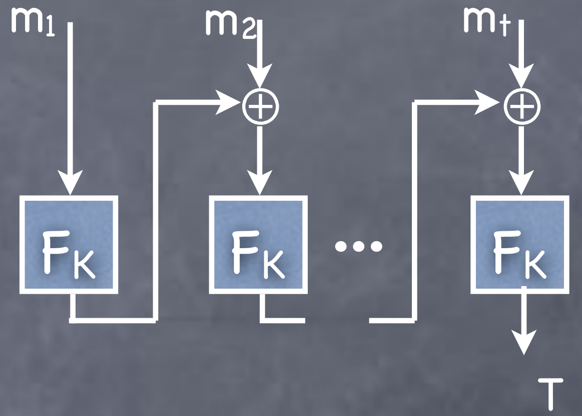

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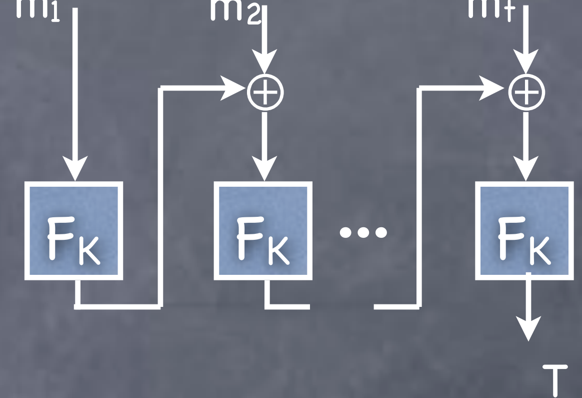

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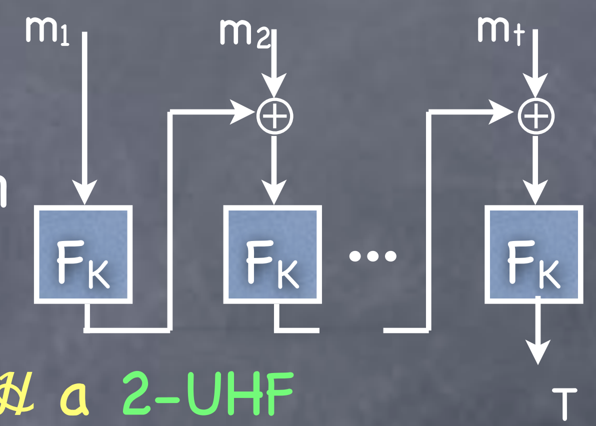

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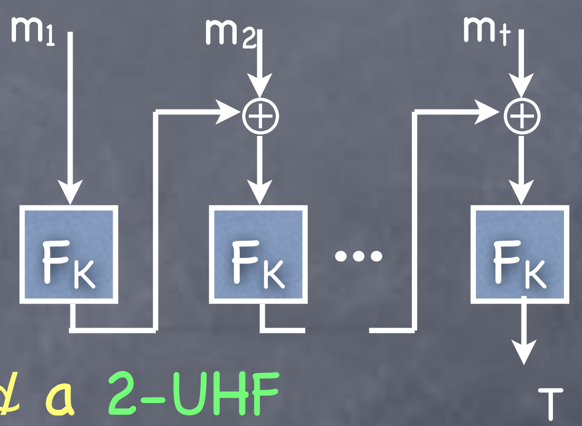

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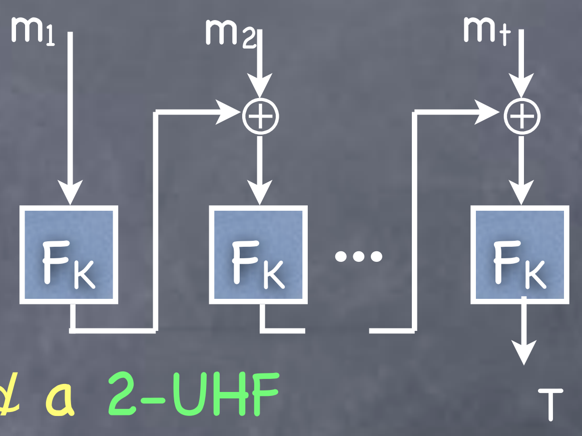

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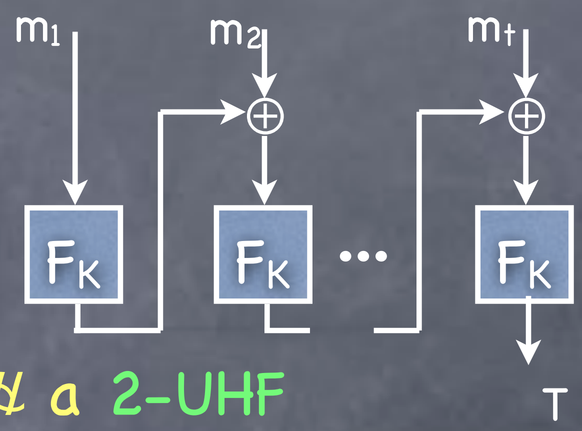

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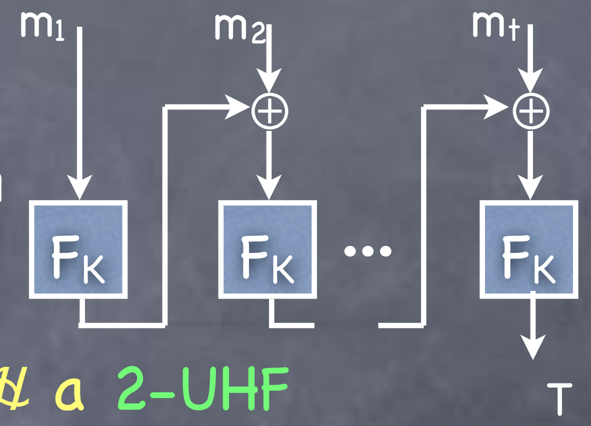

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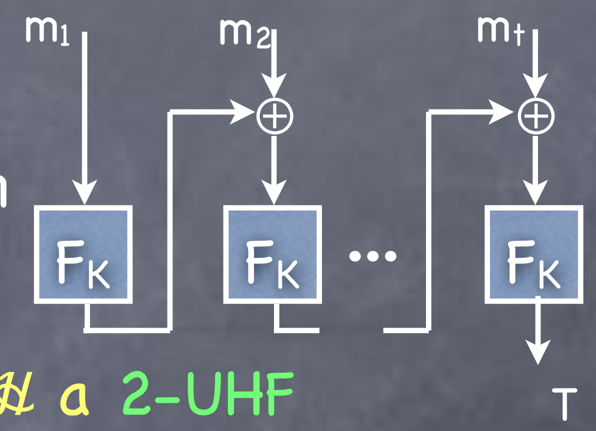

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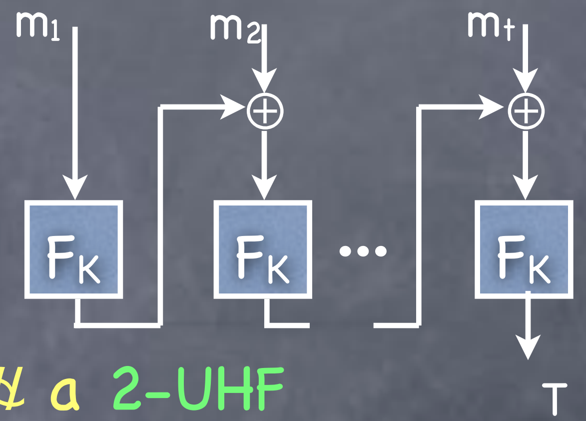

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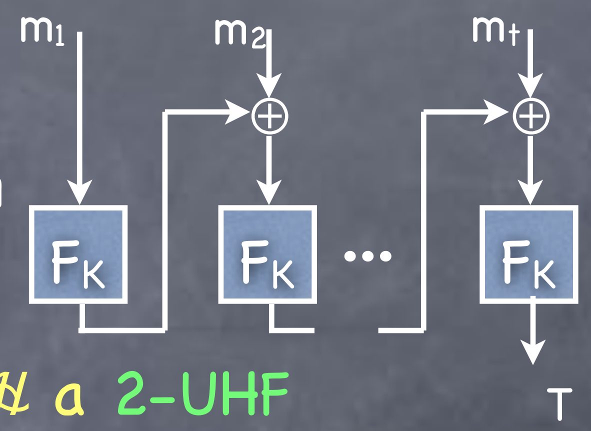

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- Leave variable input-lengths to the hash?

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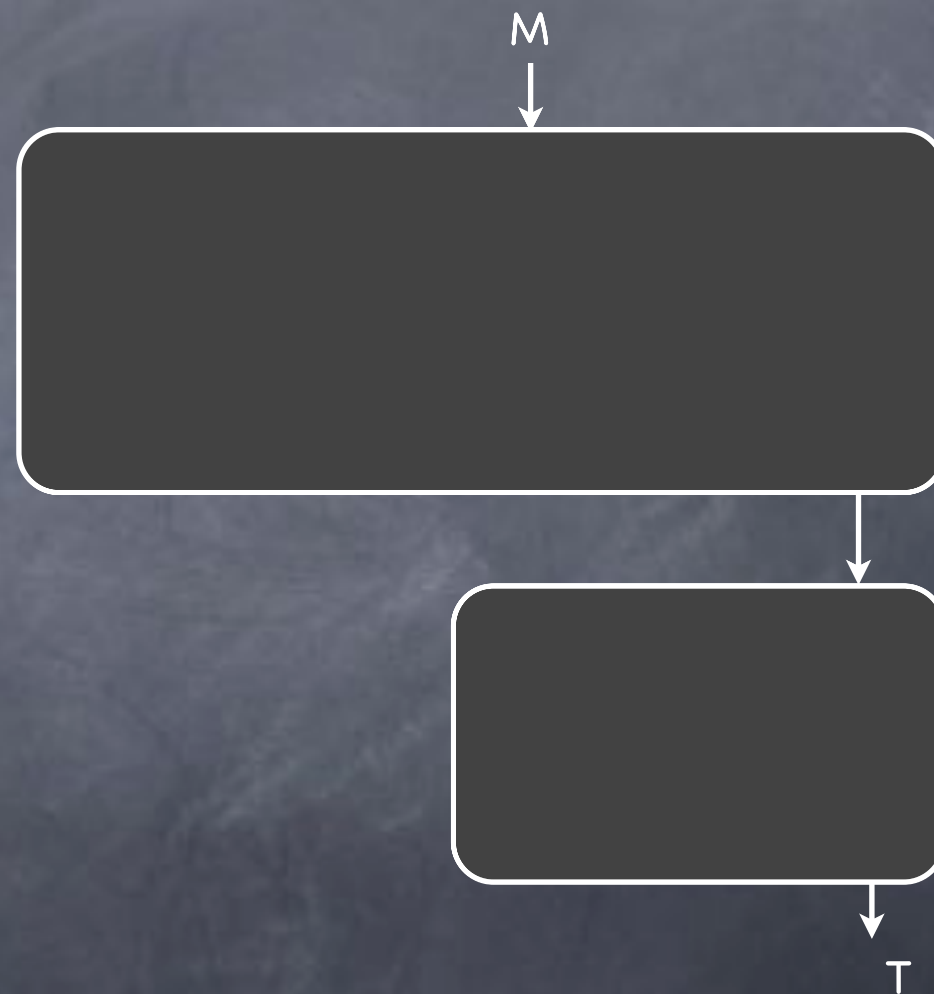
- What are candidate fixed input-length MACs in practice that do not use a block-cipher?
  - Compression functions (with key as IV)

HMAC



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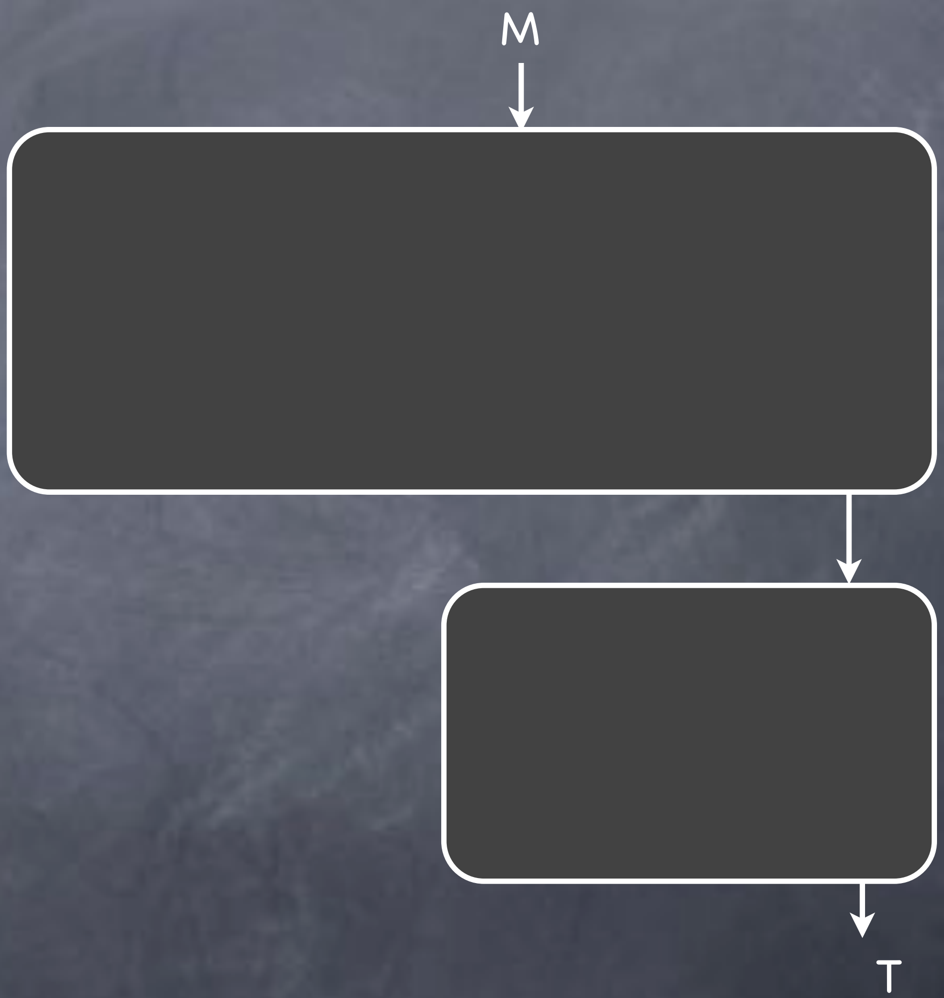
- HMAC: Hash-based MAC



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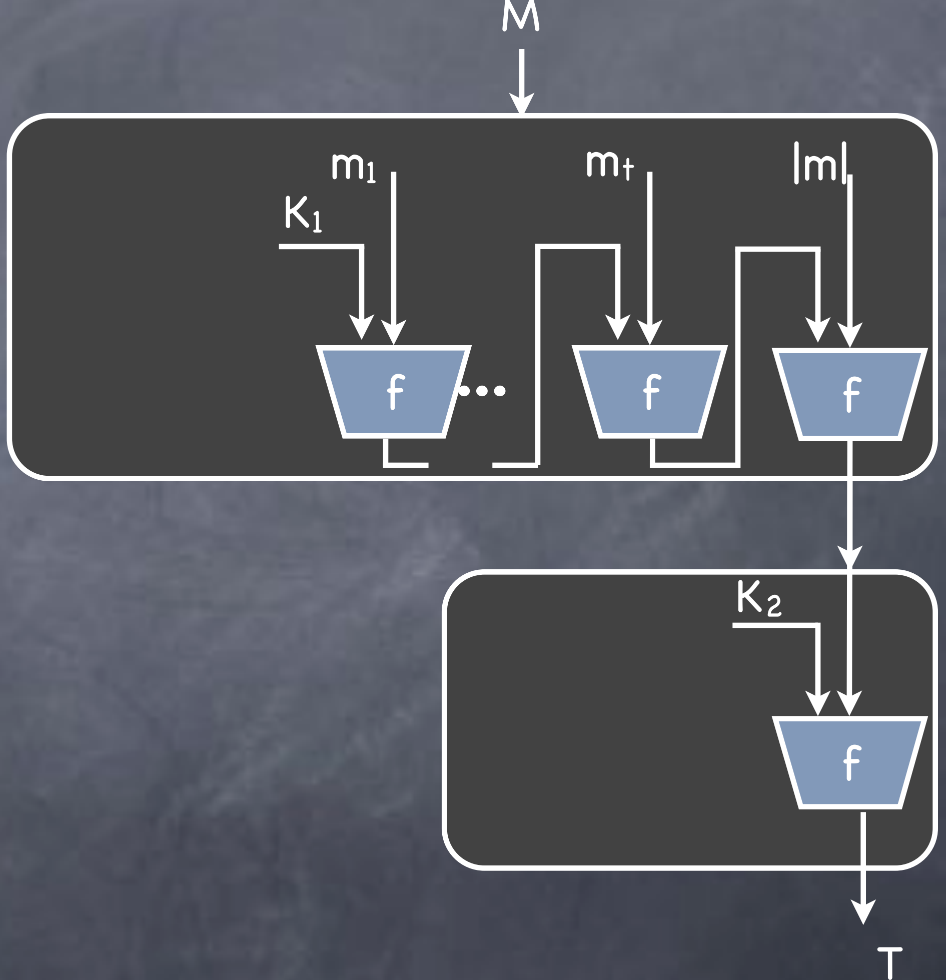

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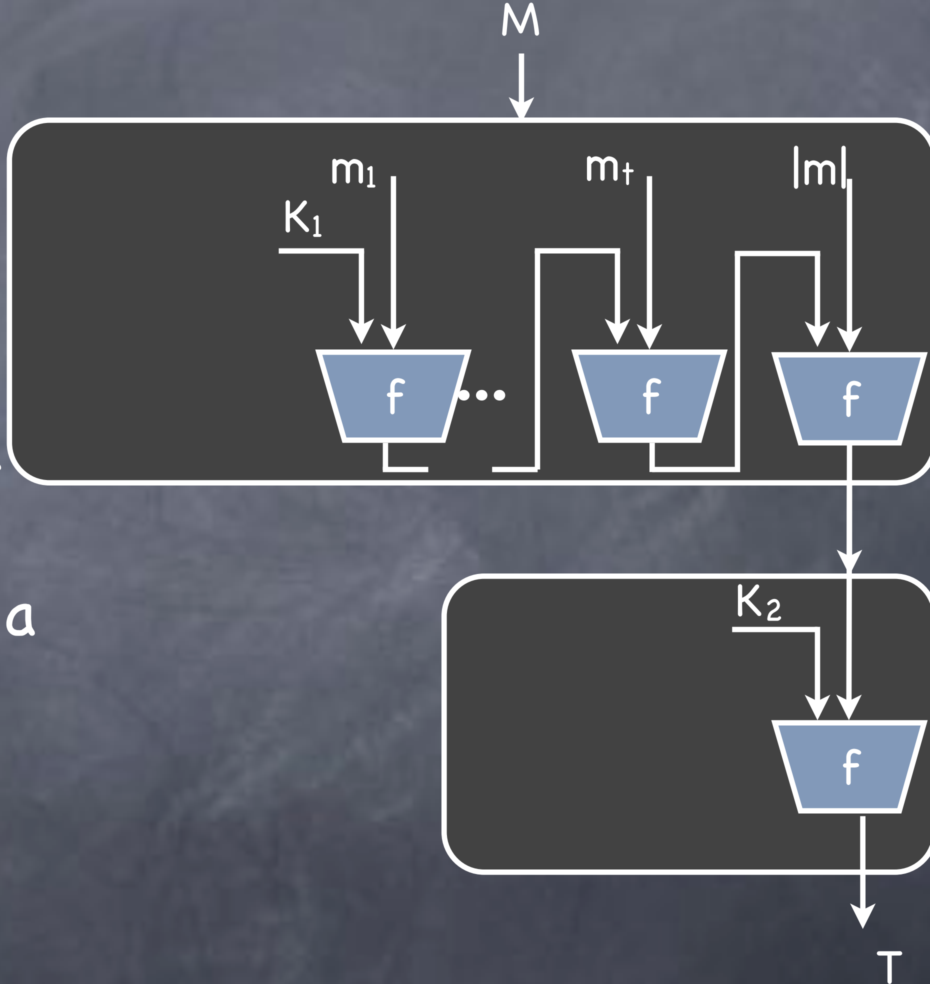

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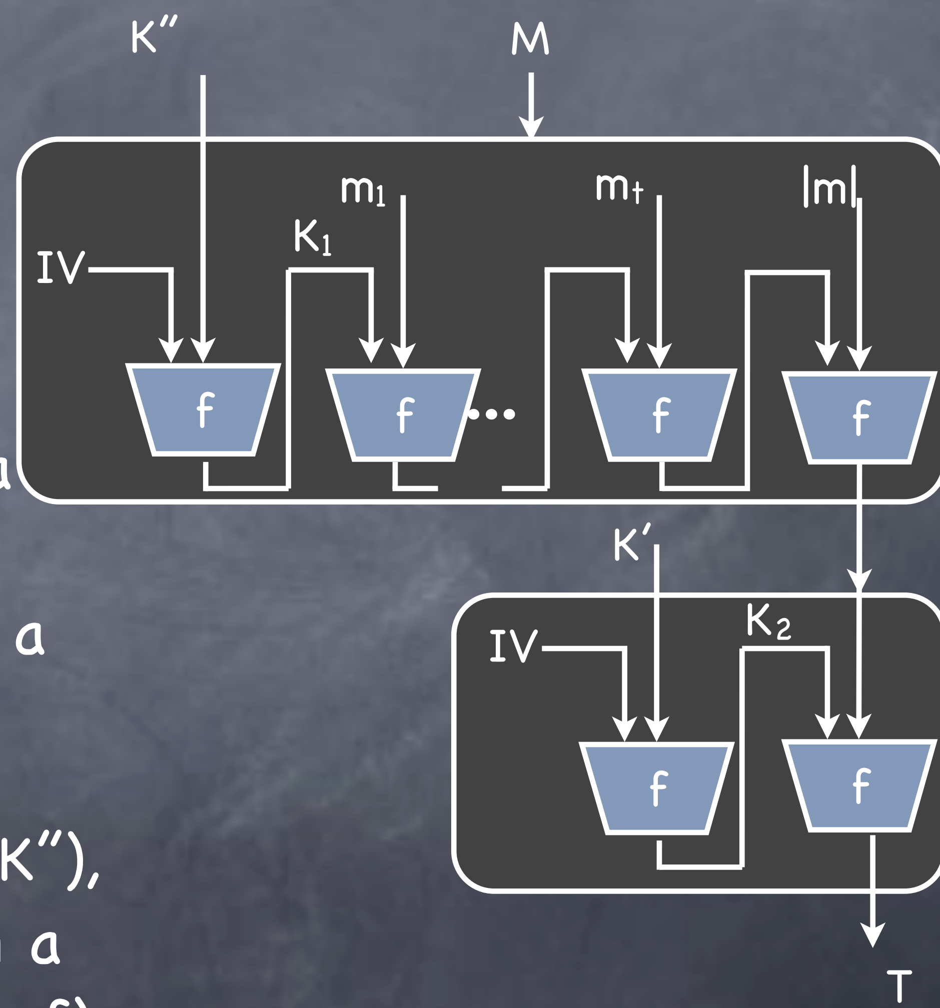
  - In HMAC (K_1,K_2) derived from (K′,K″), in turn heuristically derived from a single key K. If f is a (weak kind of) PRF K_1, K_2 can be considered independent



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    - But if H is a Merkle-Damgård iterated-hash function, then there is a simple length-extension attack for forgery
        - (That attack can be fixed by preventing extension: prefix-free encoding)
    - Other suggestions like SHA1(M||K), SHA1(K||M||K) all turned out to be flawed too

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- Next: Digital Signatures