

# Symmetric-Key Encryption: constructions

Lecture 5  
PRF, Block Cipher

RECALL

# PRG from One-Way Permutations

- One-bit stretch PRG,  $G_k: \{0,1\}^k \rightarrow \{0,1\}^{k+1}$



- Increasing the stretch

- Can use part of the PRG output as a new seed



- If the intermediate seeds are never output, can keep stretching on demand (for any “polynomial length”)

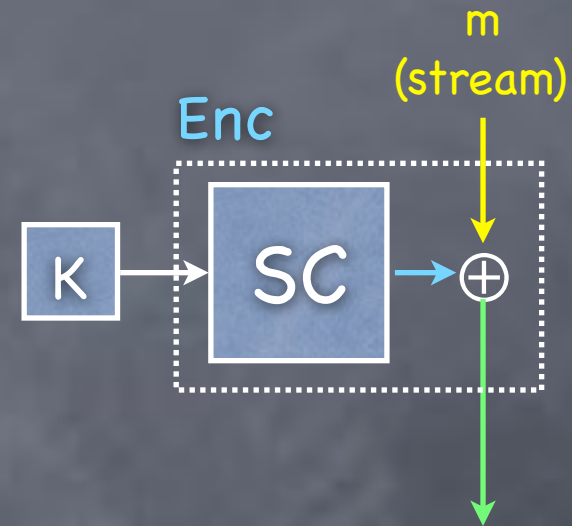
- A stream cipher



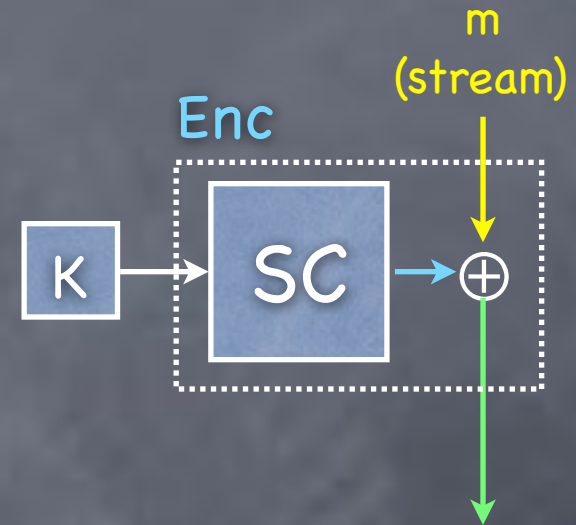
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# One-time CPA-secure SKE with a Stream-Cipher

- One-time Encryption with a stream-cipher:
  - Generate a one-time pad from a short seed
  - Can share just the seed as the key
  - Mask message with the pseudorandom pad
- Decryption is symmetric: plaintext & ciphertext interchanged
- SC can spit out bits on demand, so the message can arrive bit by bit, and the length of the message doesn't have to be a priori fixed
- Security: indistinguishability from using a truly random pad

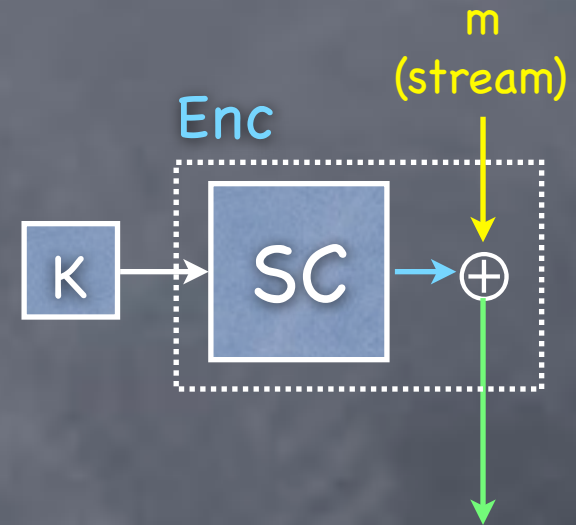


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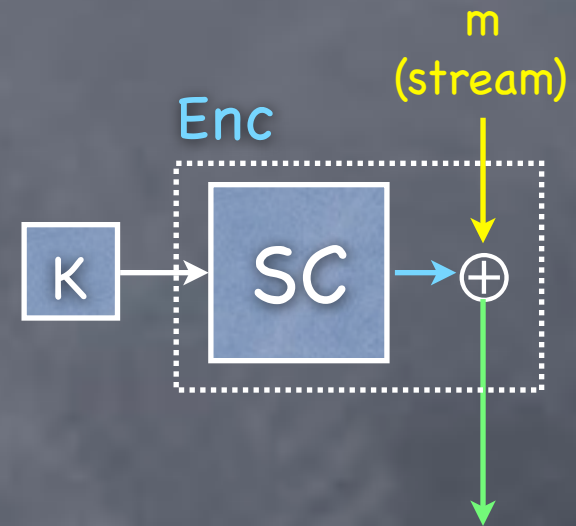
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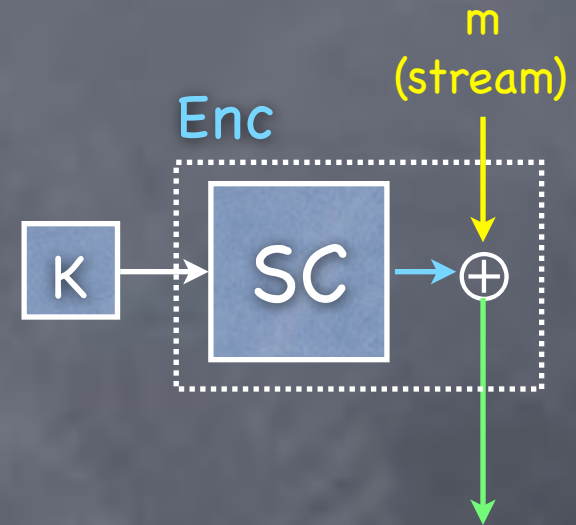
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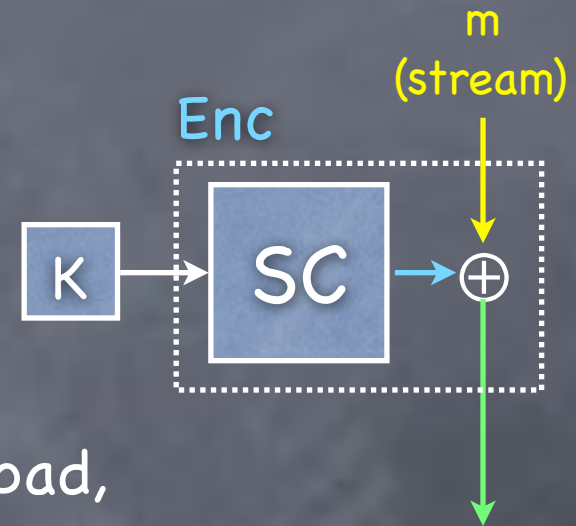
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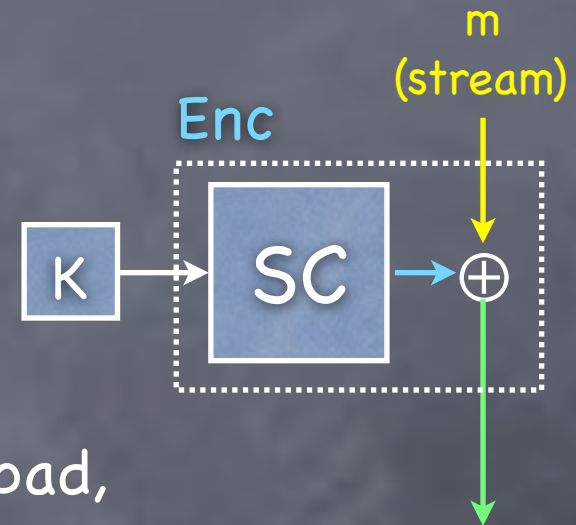
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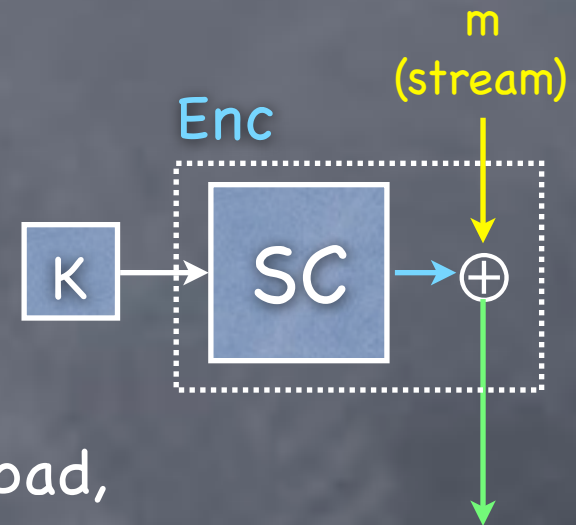
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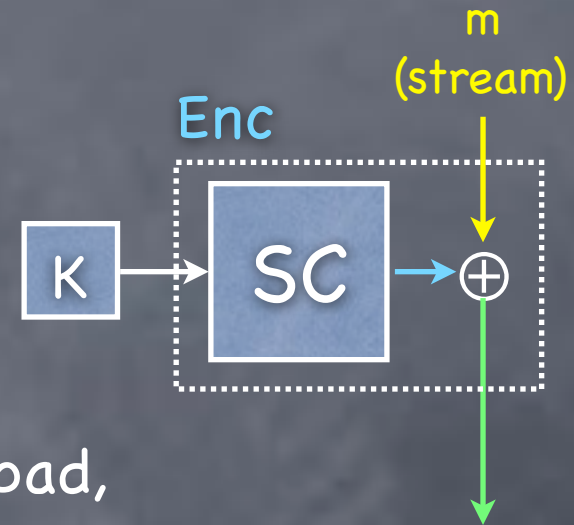
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  - Claim:  $\text{REAL} \approx \text{HYBRID}$ .
    - Consider the experiments as a system that accepts a pad from outside ( $R' = \text{SC}(K)$  for a random  $K$ , or truly random  $R$ ) and outputs the environment's output. This system is PPT, and so can't distinguish pseudorandom from random.



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  - Need to define pseudorandomness for a function (not a string)

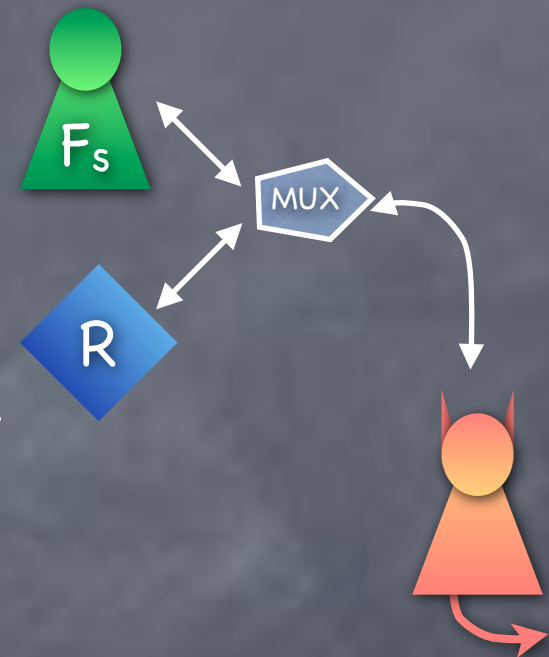
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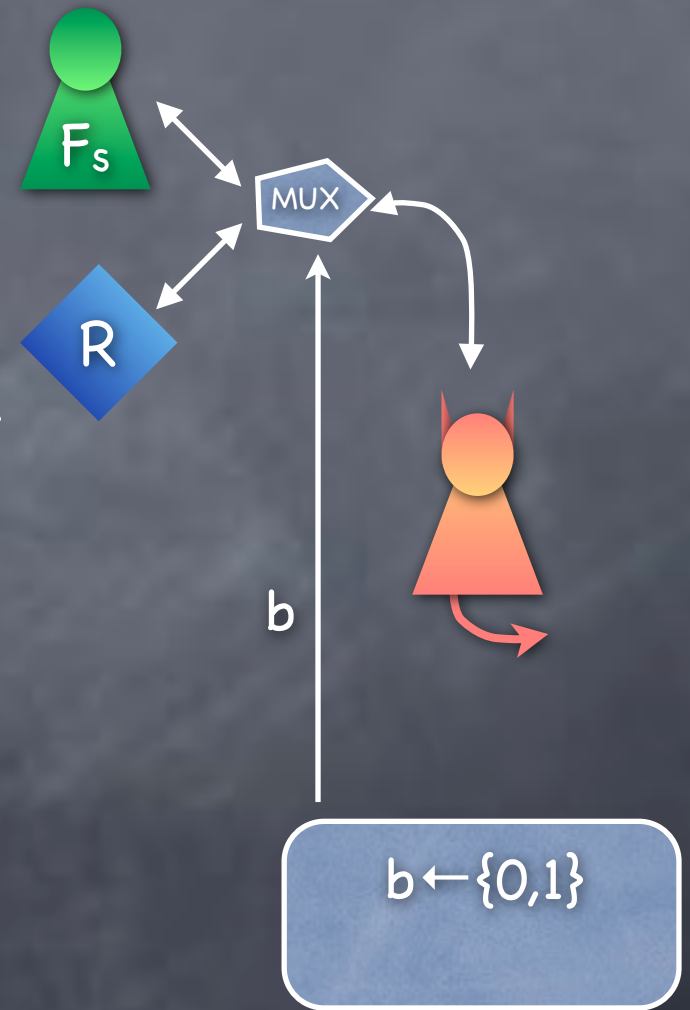
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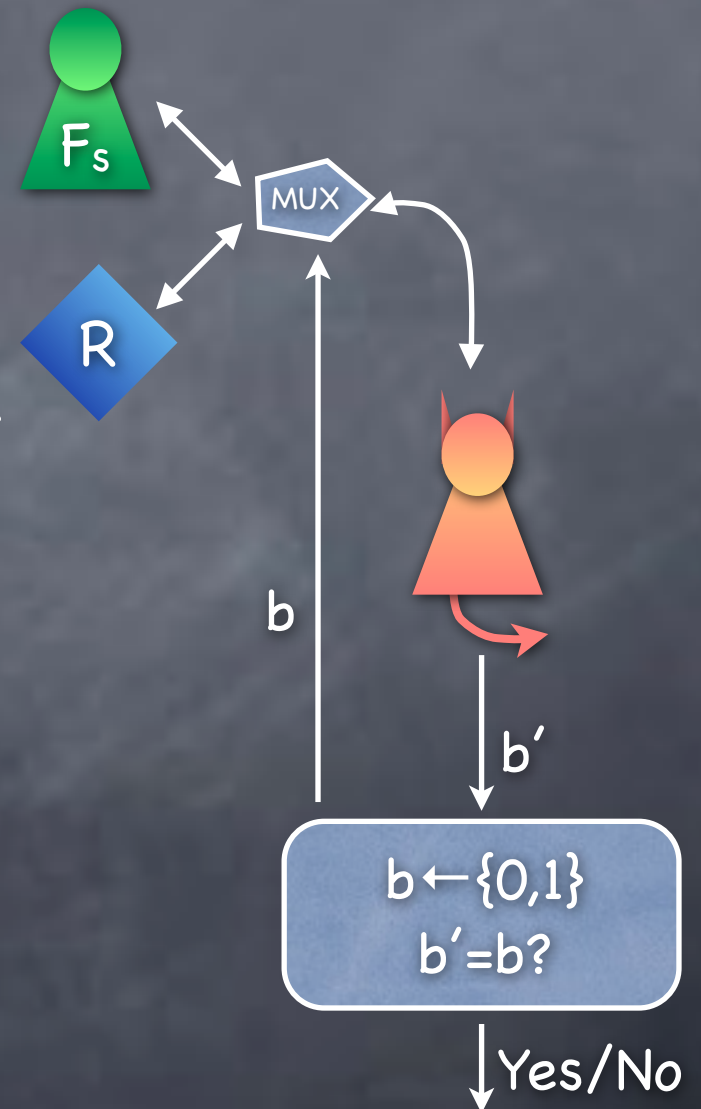
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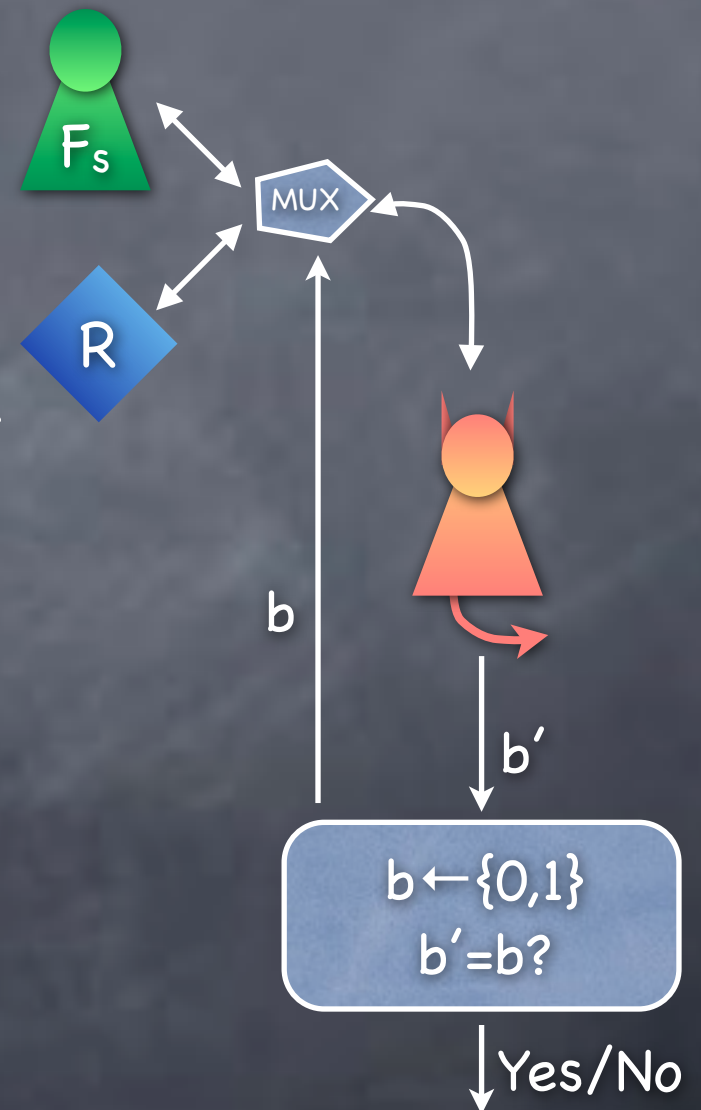
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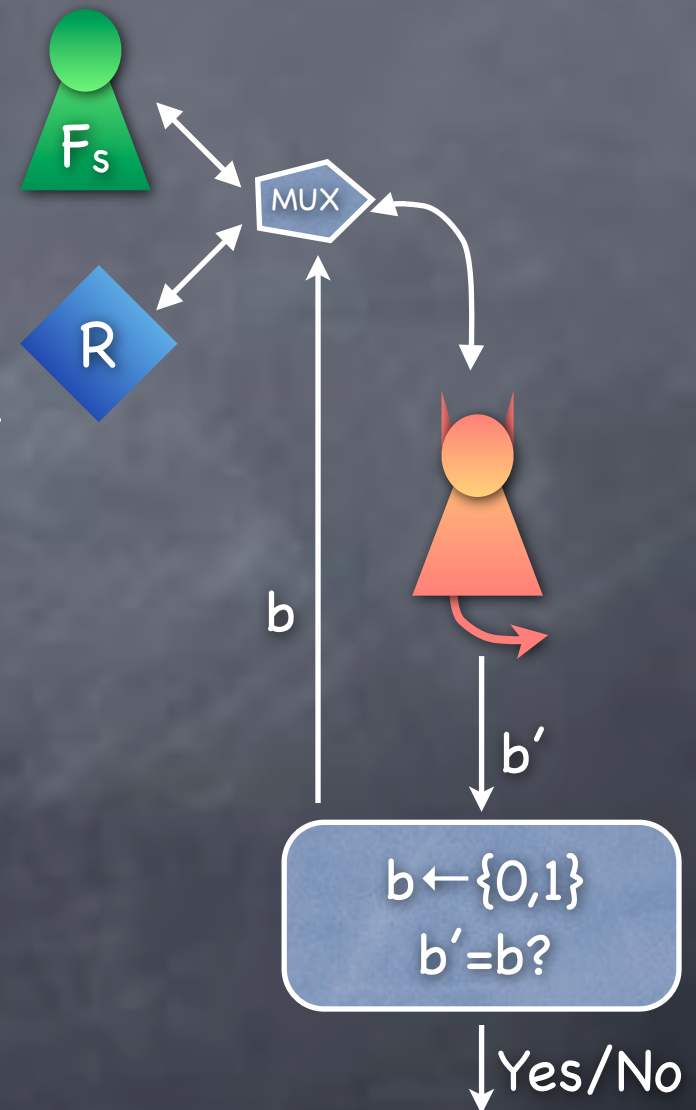
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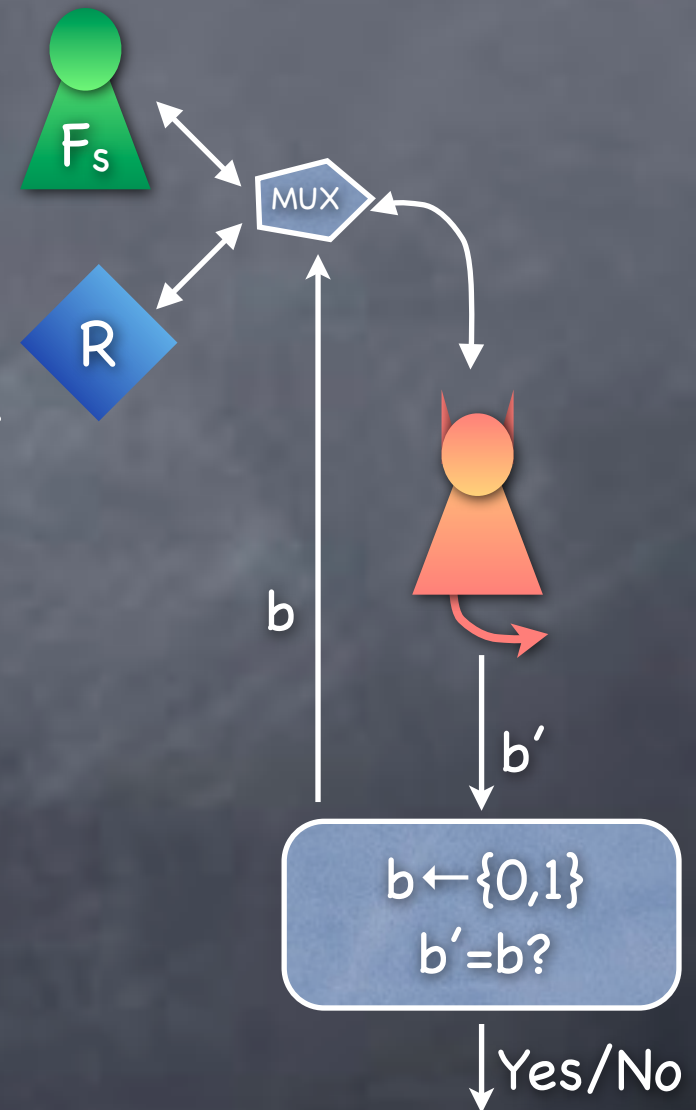
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- PRF stretches  $k$  bits to  $n2^m$  bits



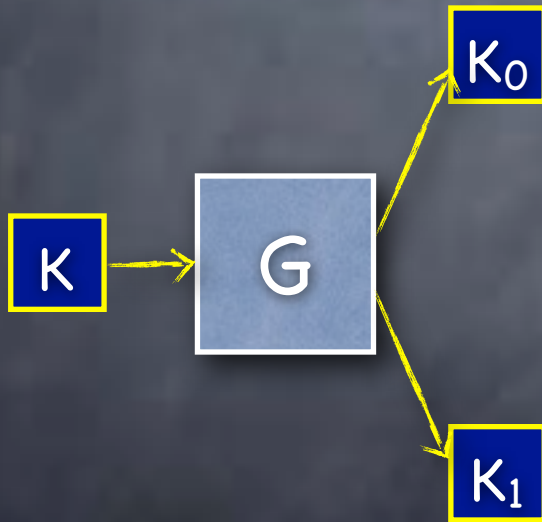
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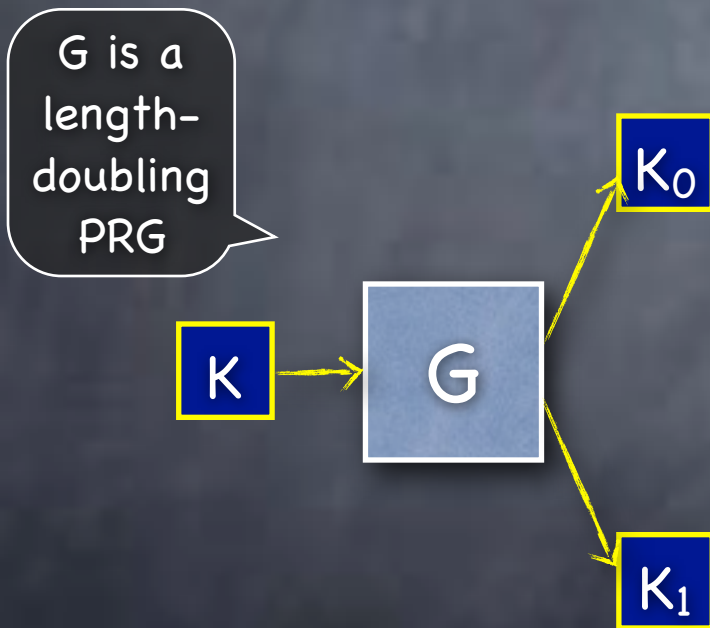
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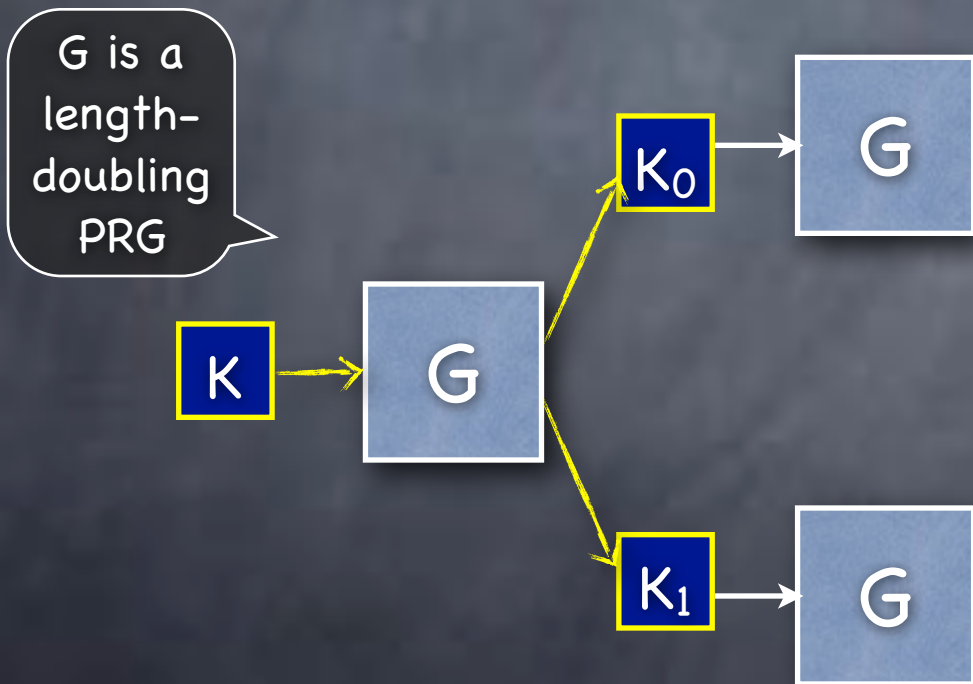
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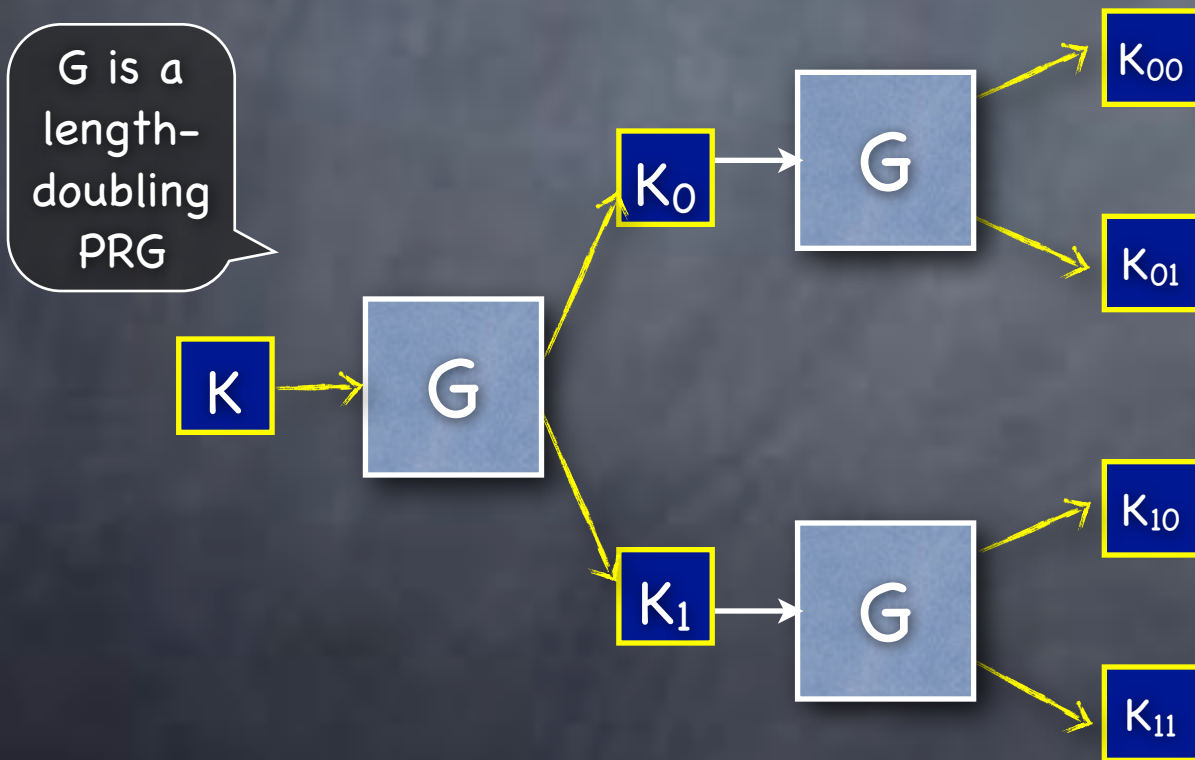
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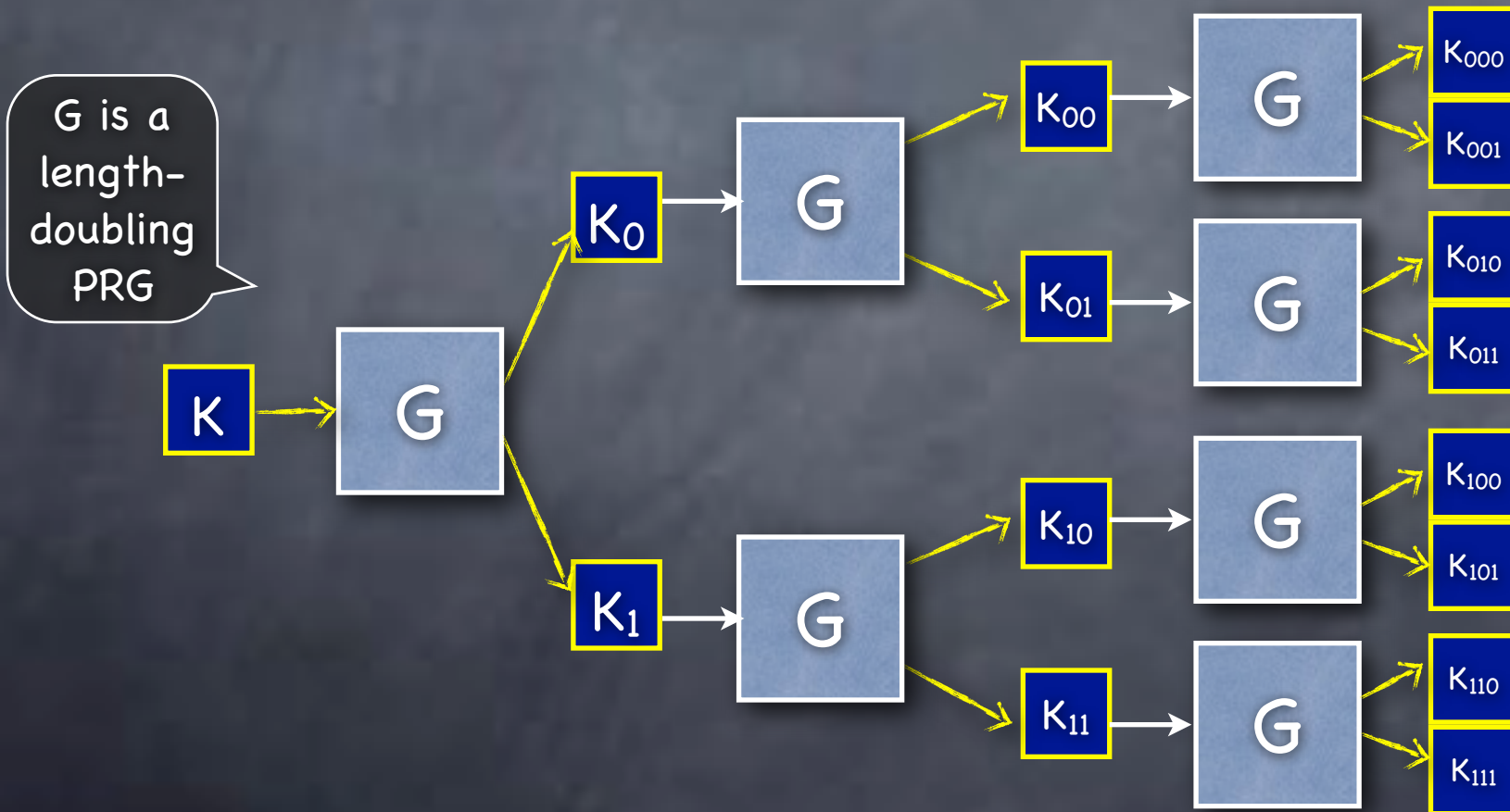
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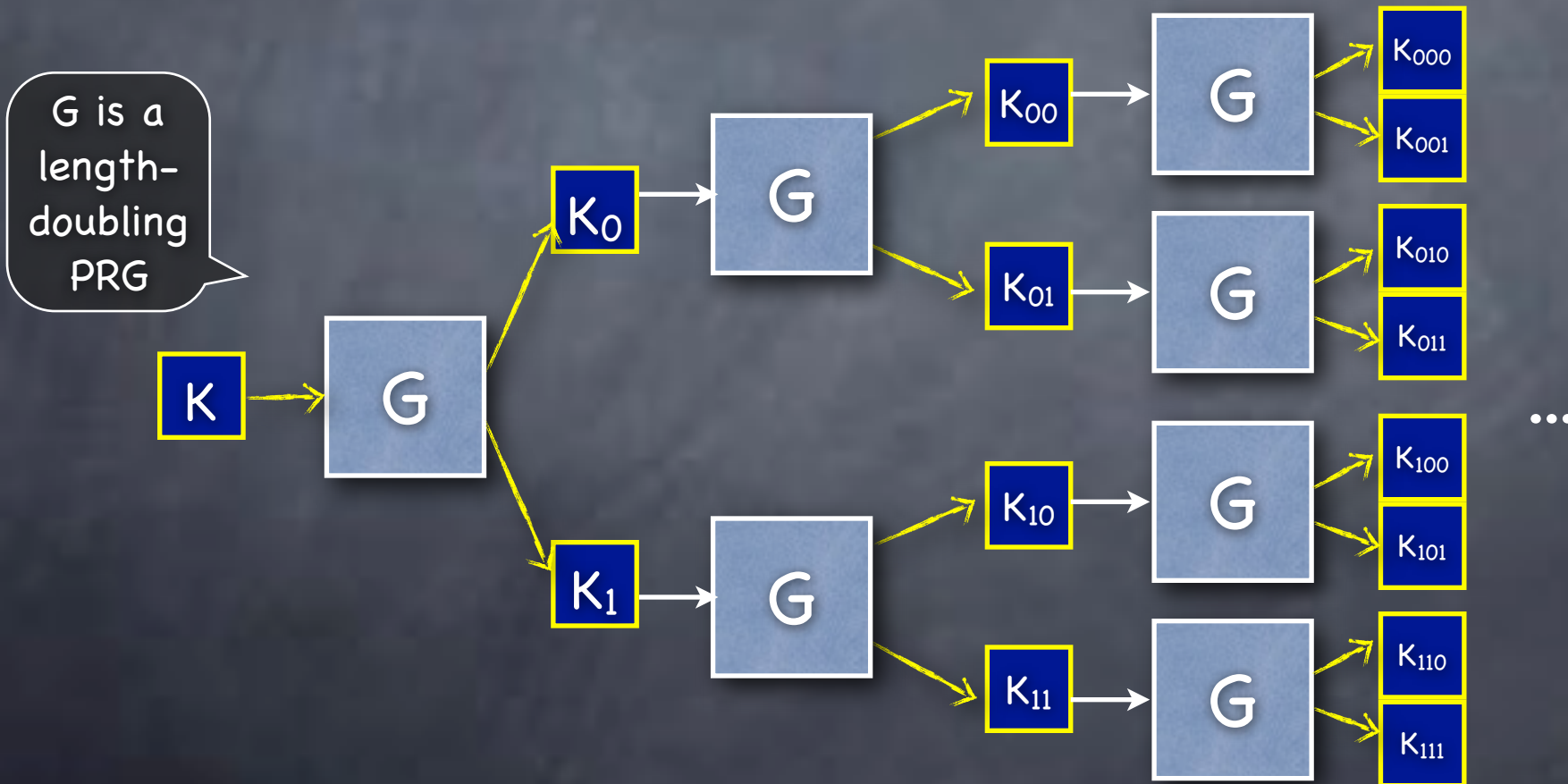
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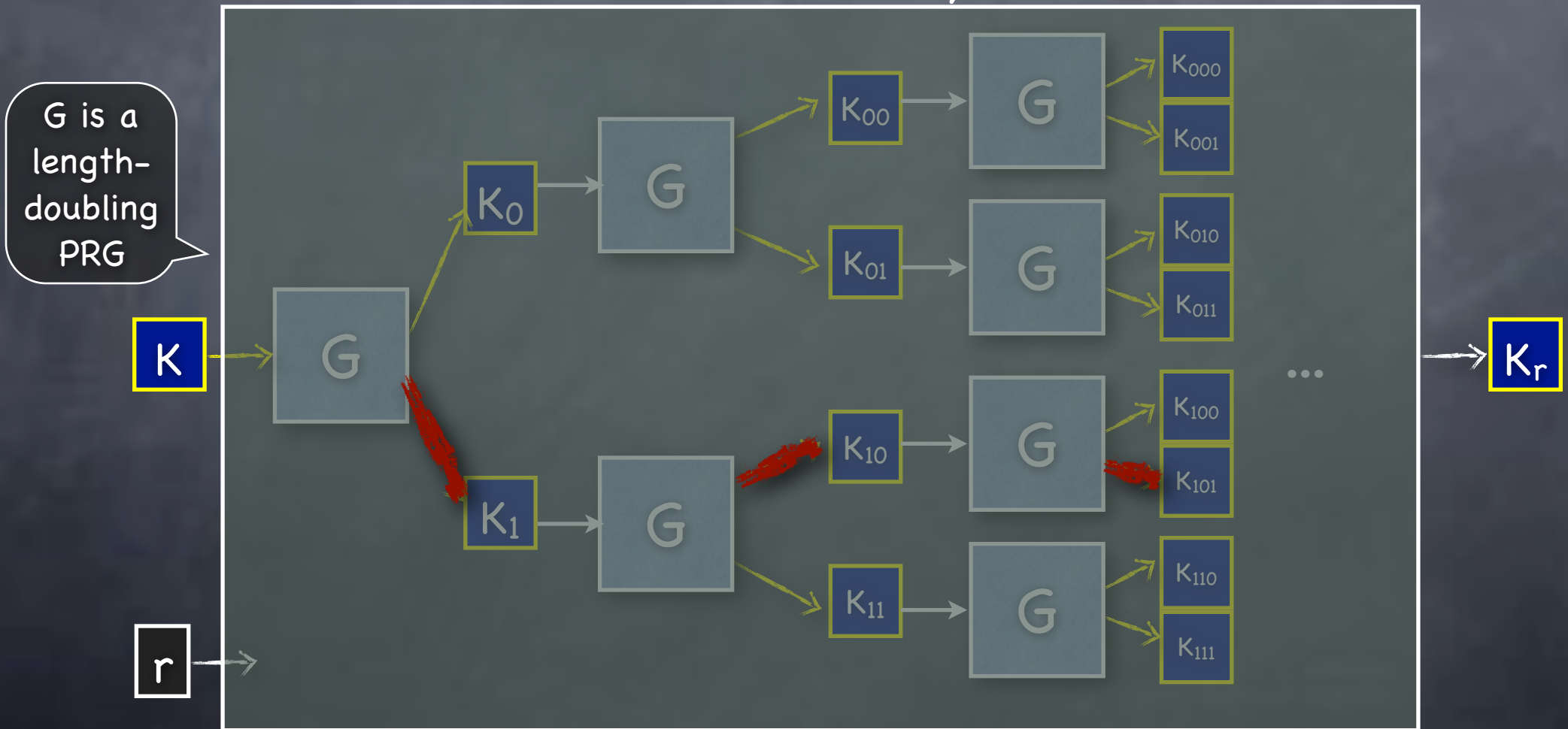
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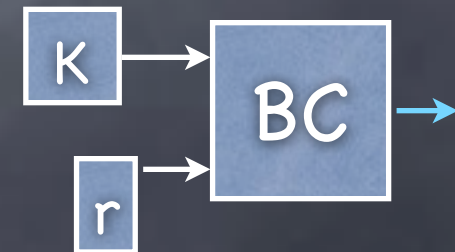
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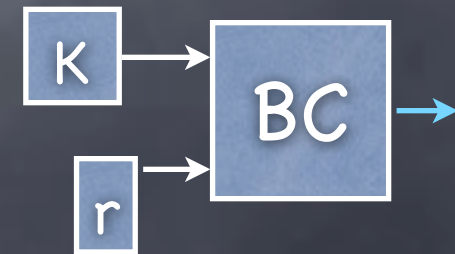
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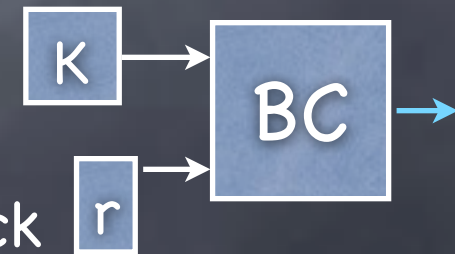
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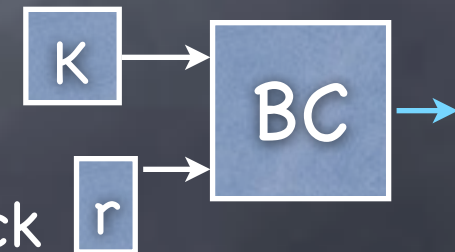
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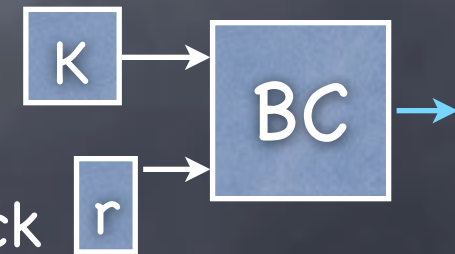
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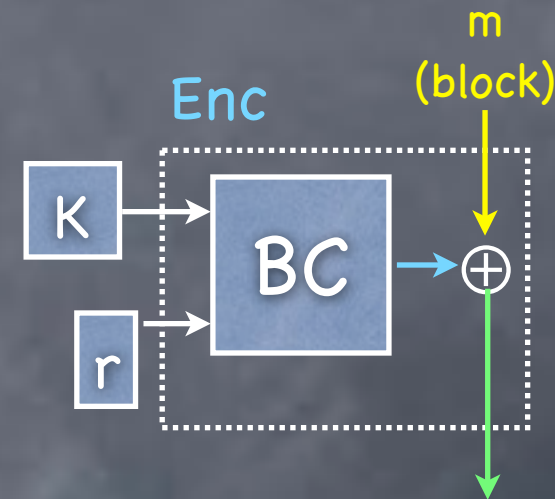
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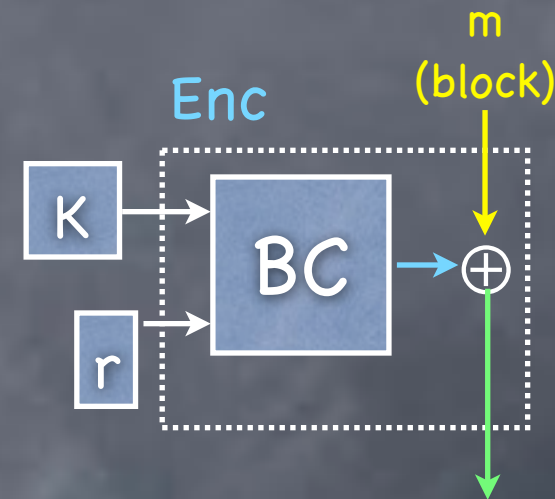
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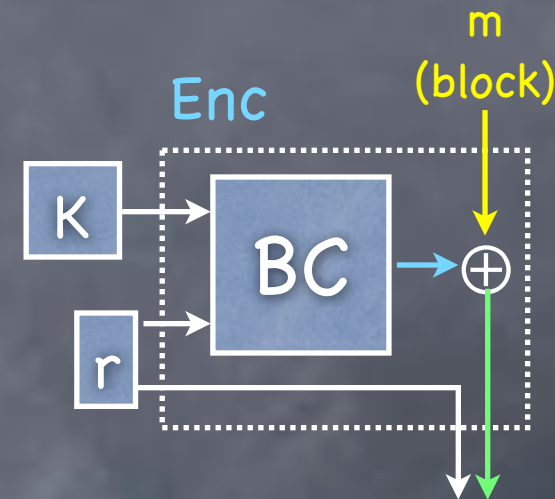
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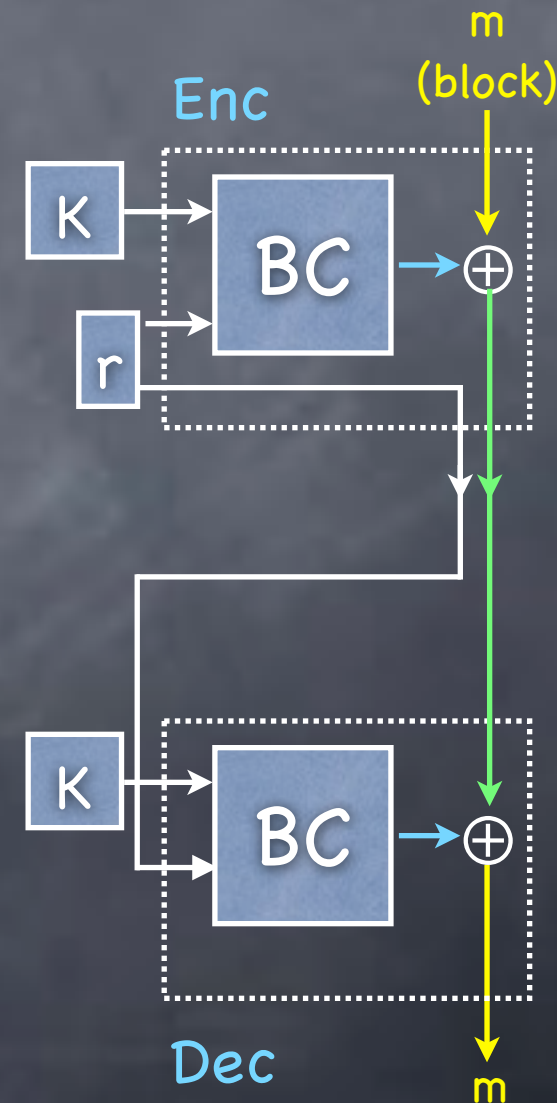
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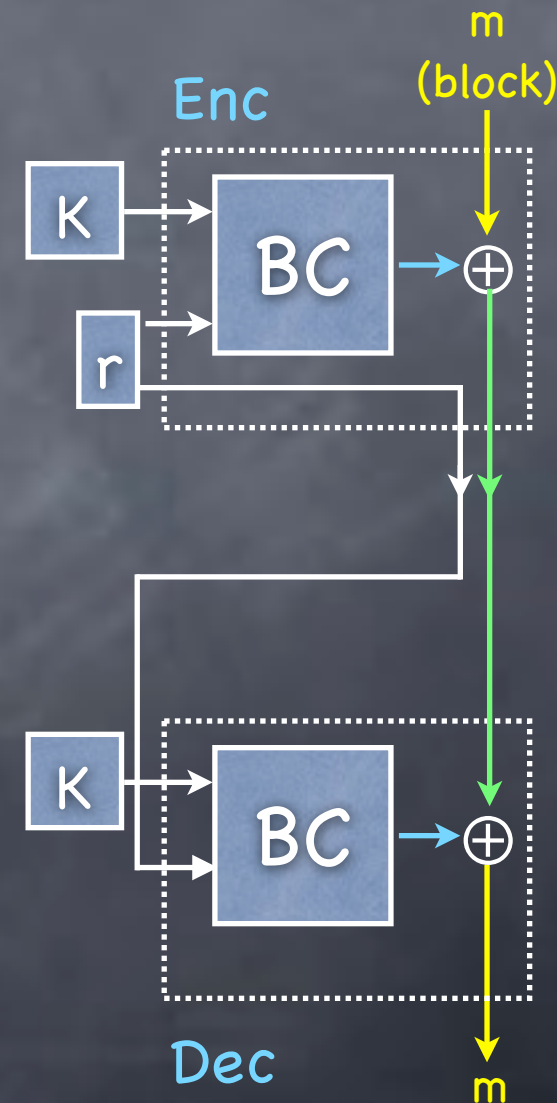
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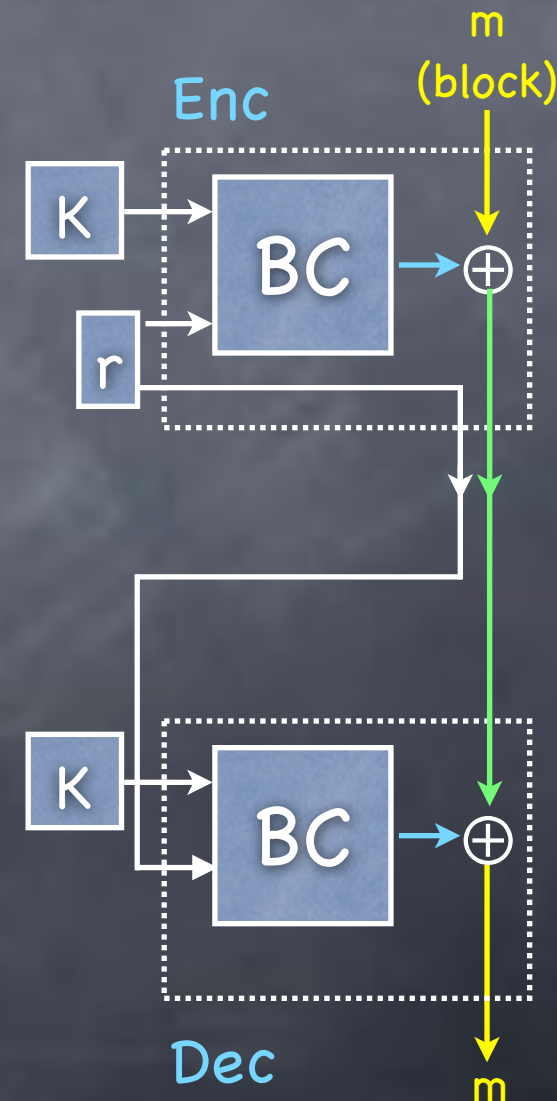
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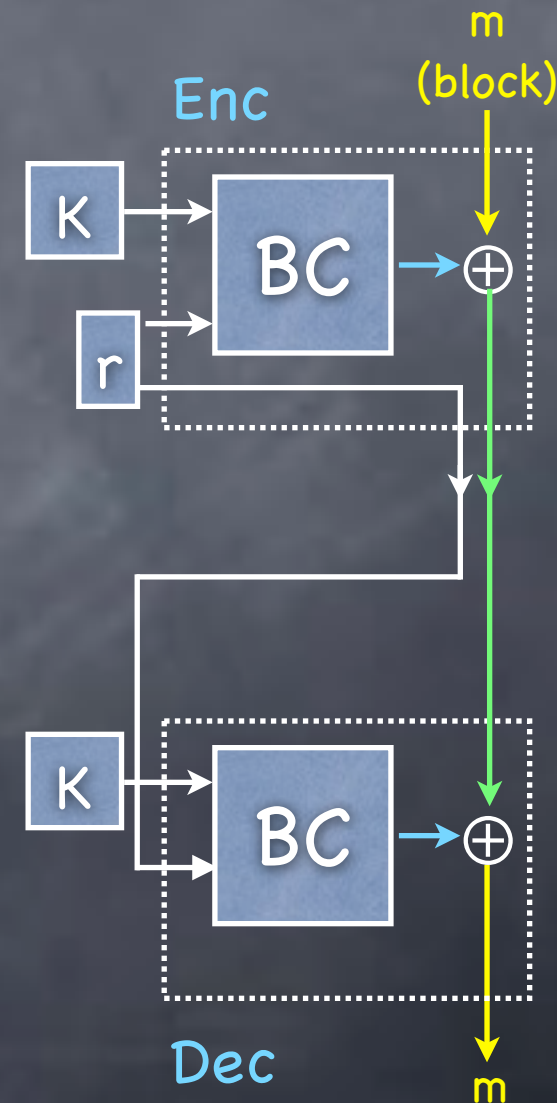
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  - Pick at random!



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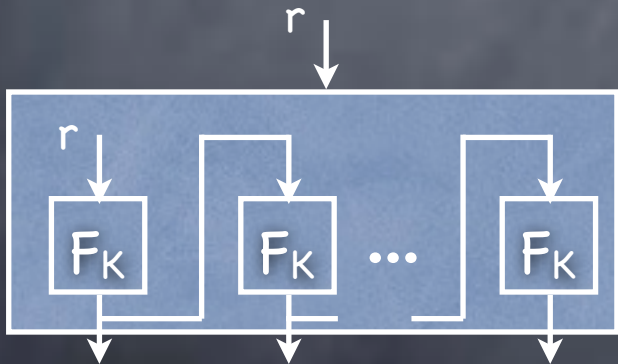
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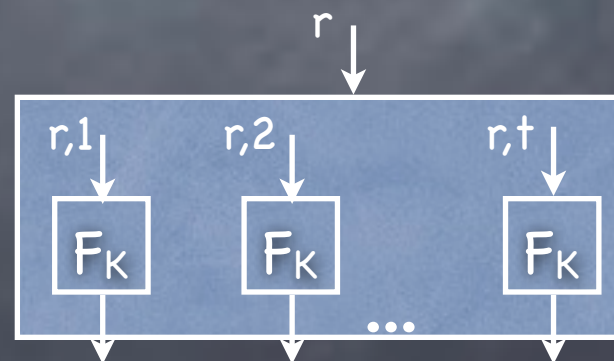
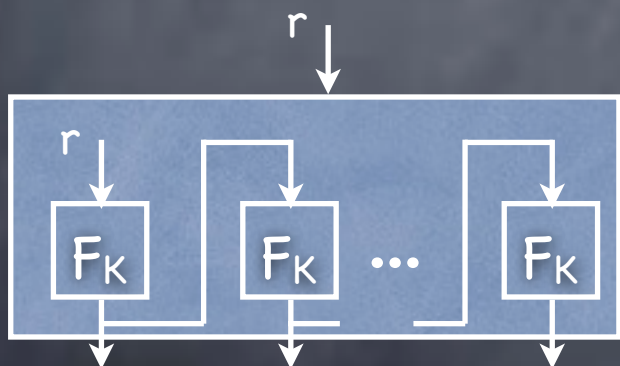
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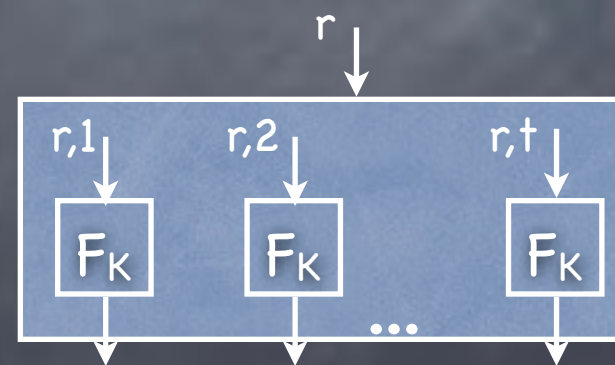
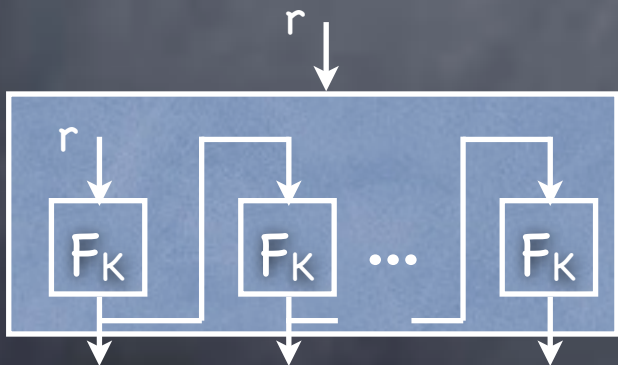
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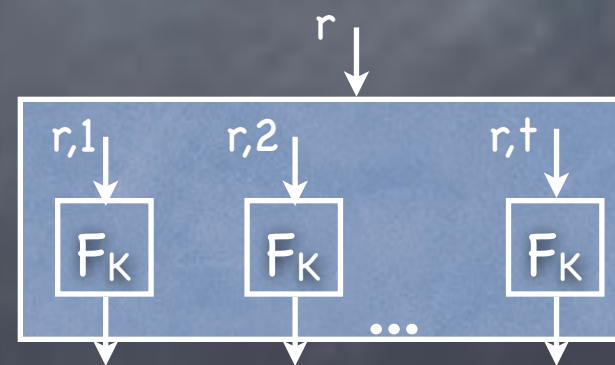
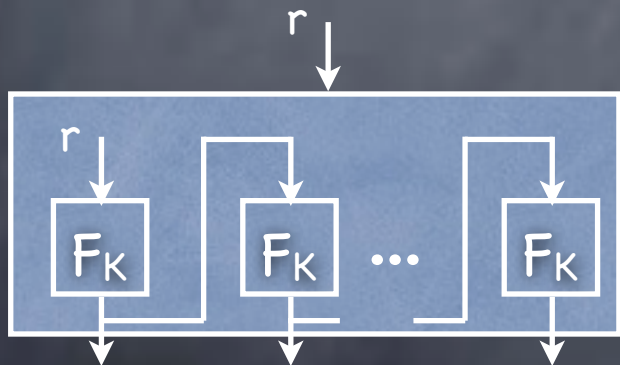
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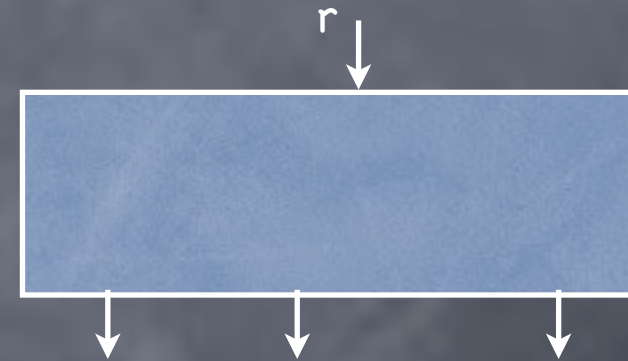
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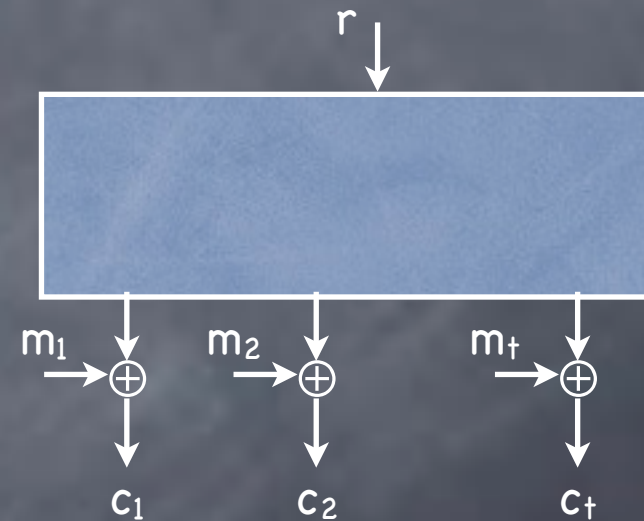
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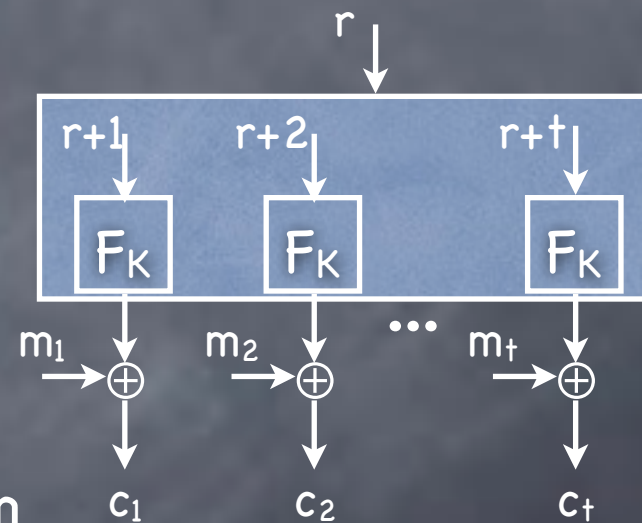
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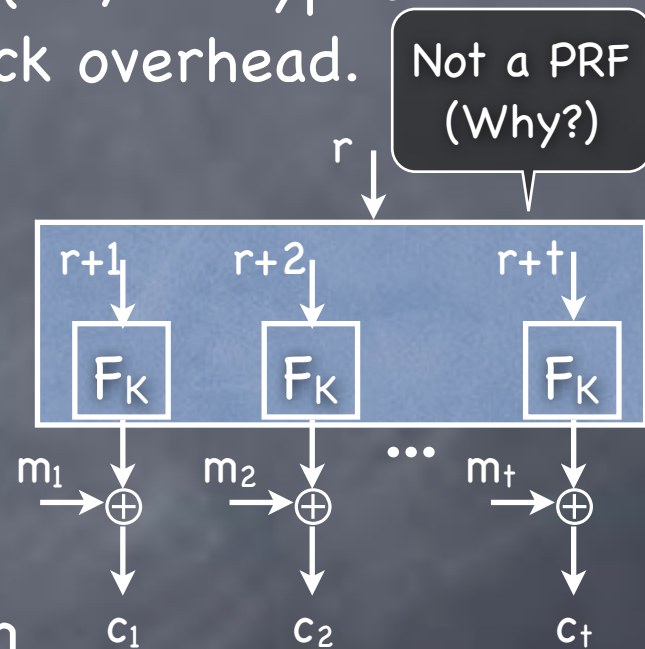
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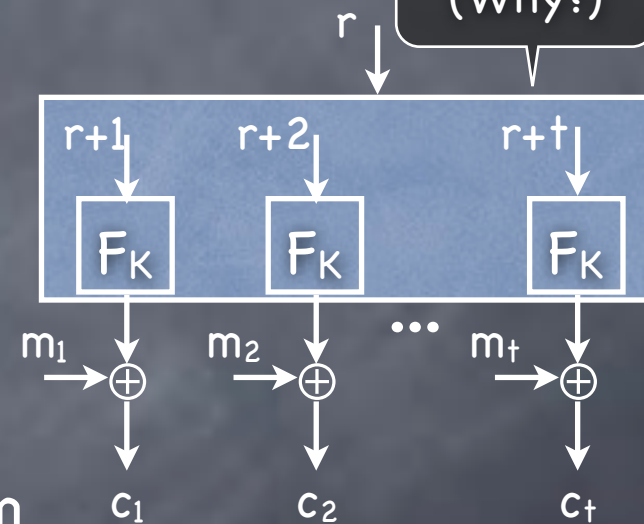


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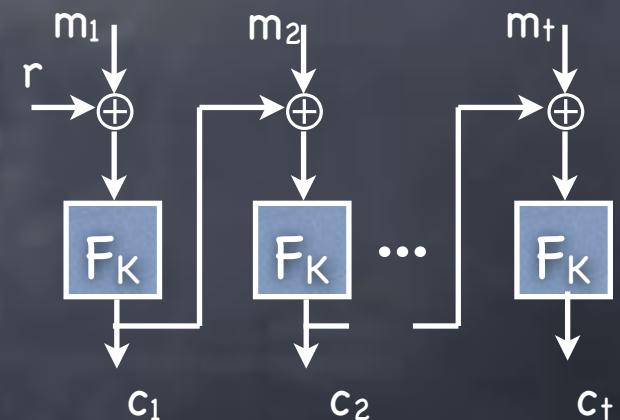
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- Cipher Block Chaining (CBC) mode:** Sequential encryption. Decryption uses  $F_K^{-1}$ . Ciphertext an integral number of blocks.



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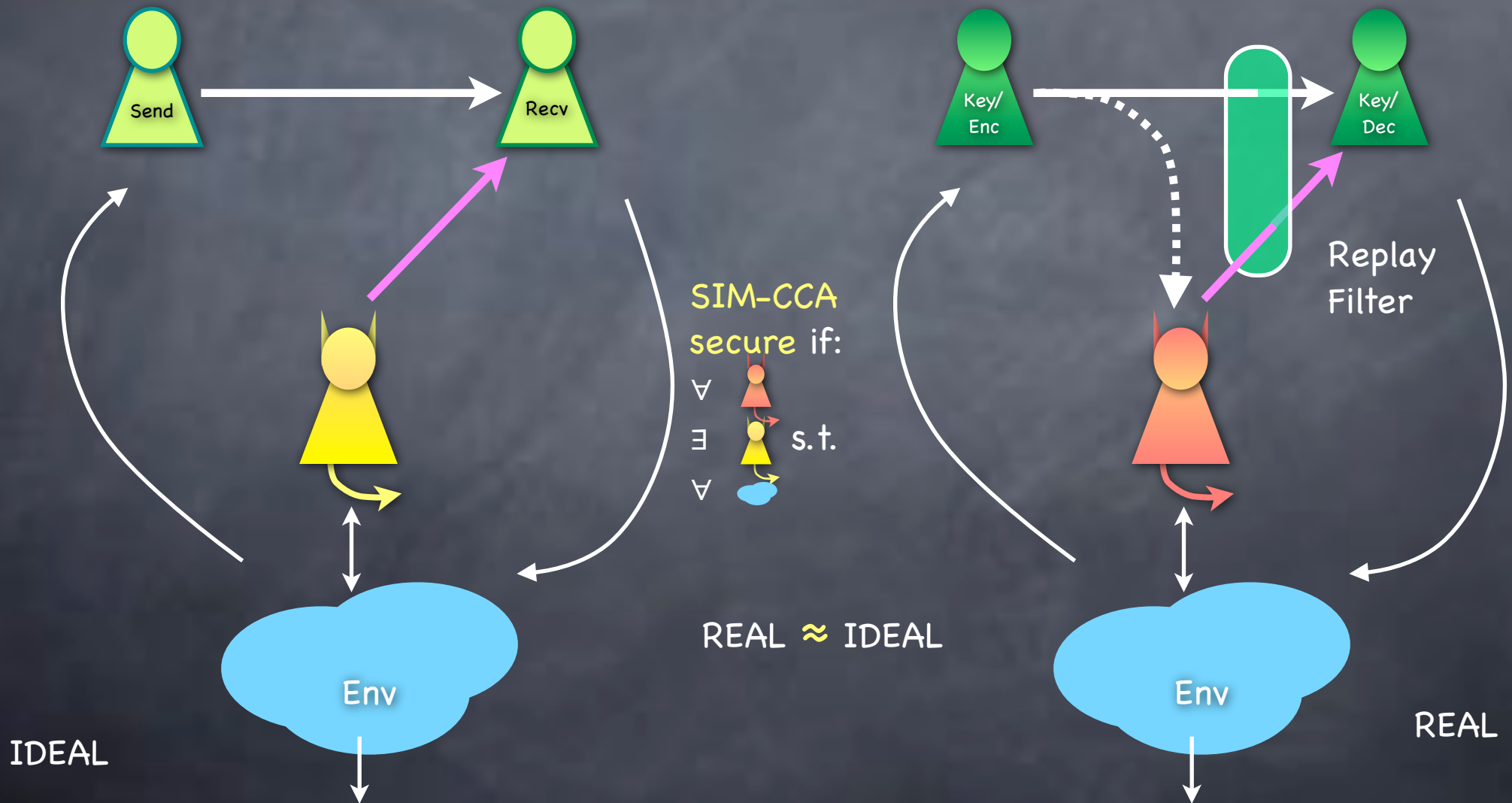
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  - What can Bob do?

RECALL

# Symmetric-Key Encryption

## SIM-CCA Security



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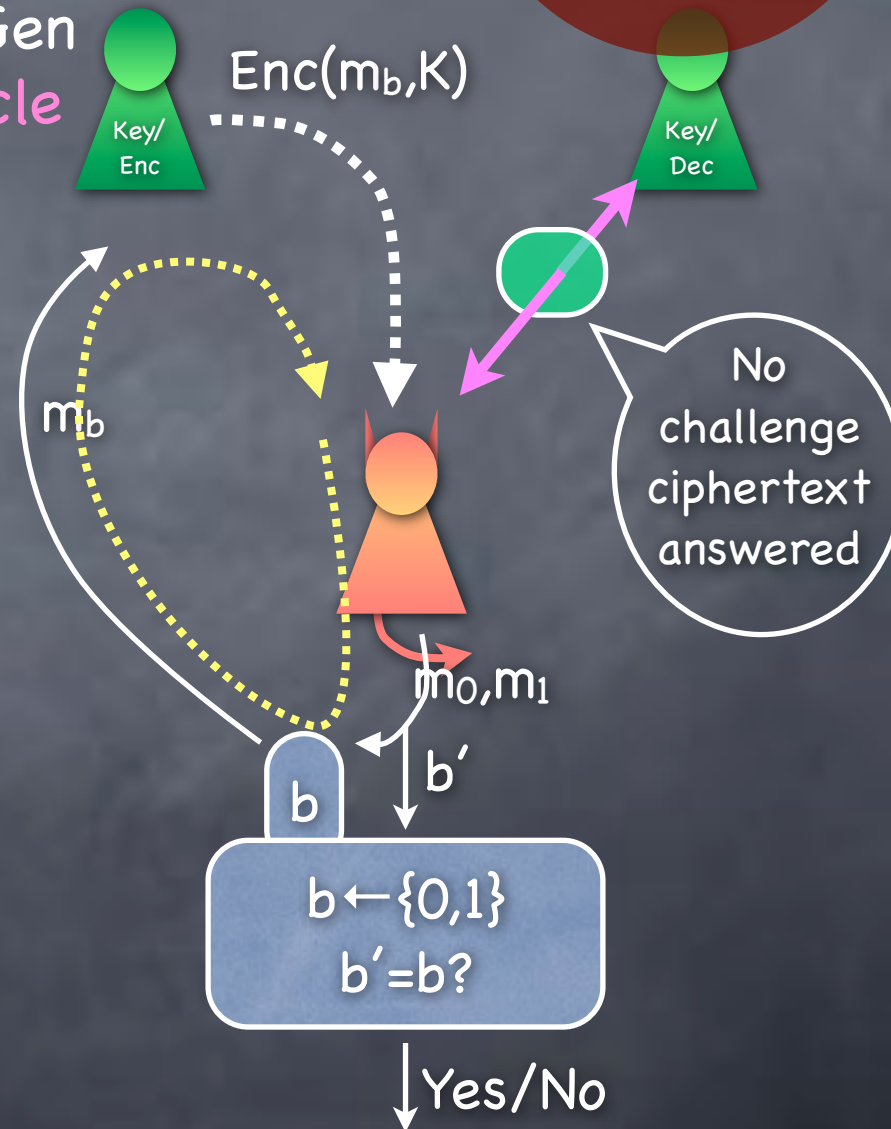
IND-CCA +  
~correctness  
equivalent to  
SIM-CCA

- Experiment picks  $b \leftarrow \{0,1\}$  and  $K \leftarrow \text{KeyGen}$   
Adv gets (guarded) access to  $\text{Dec}_K$  oracle

- For as long as Adversary wants

- Adv sends two messages  $m_0, m_1$  to the experiment
- Expt returns  $\text{Enc}(m_b, K)$  to the adversary

- Adversary returns a guess  $b'$
- Experiment outputs 1 iff  $b' = b$
- IND-CCA secure if for all feasible adversaries  $\Pr[b' = b] \approx 1/2$



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  - **MAC**: Message Authentication Code

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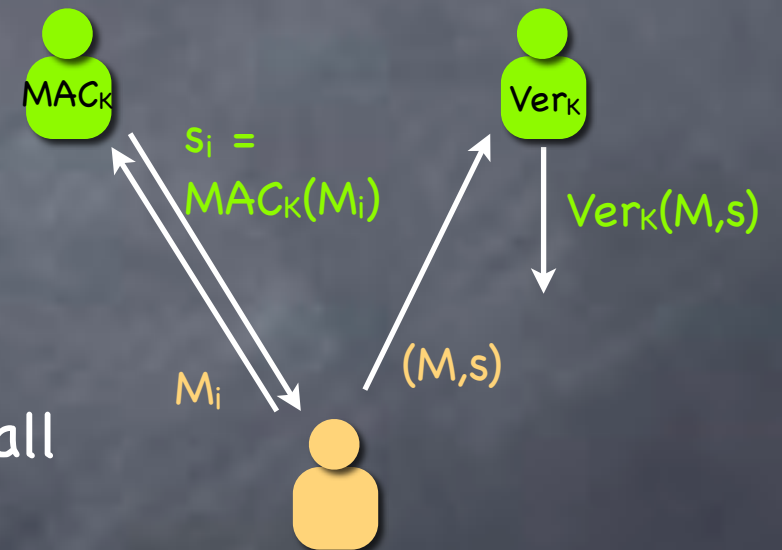
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- Security: probability that an adversary can produce  $(M, s)$  s.t.  $\text{Verify}_K(M, s) = 1$  is negligible unless Alice produced an output  $s = \text{MAC}_K(M)$



Advantage

$$= \Pr[ \text{Ver}_K(M, s) = 1 \text{ and } (M, s) \notin \{(M_i, s_i)\} ]$$

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