Symmetric-Key Encryption: constructions

Lecture 4

OWF, PRG, Stream Cipher

RECALL

One-Way Function, Hardcore Predicate

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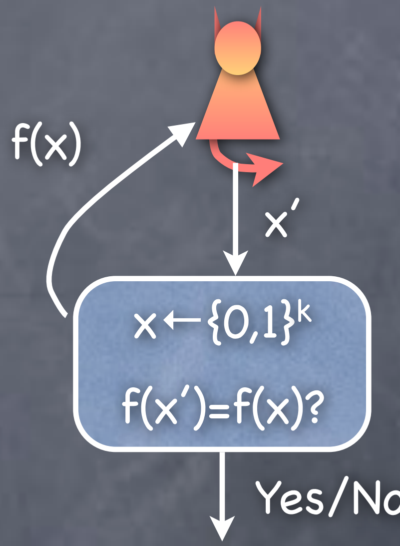
- f_k: $\{0,1\}^k \rightarrow \{0,1\}^{n(k)}$ is a one-way function (OWF) if
  - f is polynomial time computable
  - For all (non-uniform) PPT adversary, probability of success in the "OWF experiment" is negligible
  - But x may not be completely hidden by f(x)



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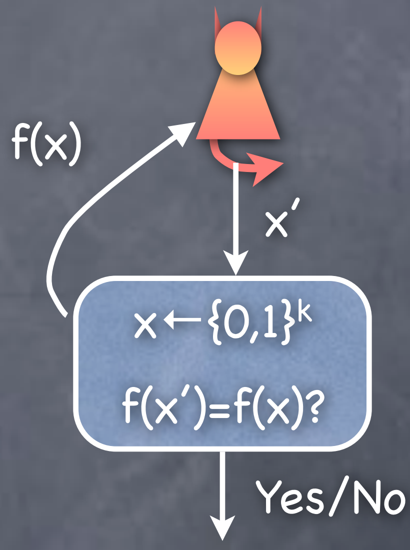
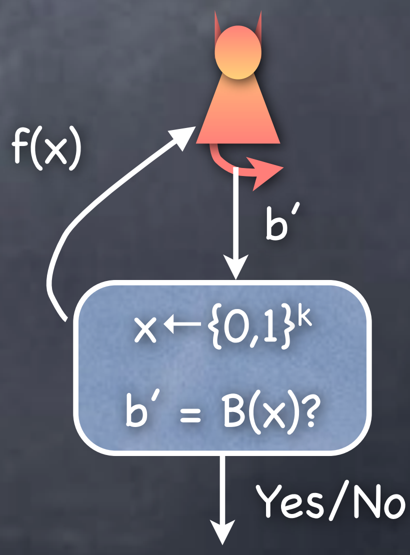


- B is a hardcore predicate of a OWF f if
  - B is polynomial time computable
  - For all (non-uniform) PPT adversary, advantage in the Hardcore-predicate experiment is negligible
  - B(x) remains "completely" hidden, given f(x)



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  - Fact: input distribution (x,y) random k-bit integers will also work (if k-bit primes distribution works)
    - Important that we require $|x|=|y|=k$, not $|x \cdot y|=k$ (otherwise, 2 is a factor of x.y with 3/4 probability)

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  - Inverting f_{subsum} known to be NP-complete, but assuming that it is a OWF is "stronger" than assuming $P \neq NP$

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    - Reduction: Given an algorithm for finding LSB(x) from $f_{Rabin}(x;n)$ for random x, show how to invert f_{Rabin}

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    - Predictor for B(x,r) is a "noisy channel" through which x, encoded as $(\langle x,0 \rangle, \langle x,1 \rangle \ldots \langle x,2^{|x|}-1 \rangle)$ (Walsh-Hadamard code), is transmitted. Can recover x by error-correction (local list decoding)

RECALL

Pseudorandomness Generator (PRG)

- Expand a short random seed to a "random-looking" string
  - So that we can build "stream ciphers" (to encrypt a stream of data, using just one short shared key)
- First, PRG with fixed stretch: $G_k: \{0,1\}^k \rightarrow \{0,1\}^{n(k)}$, $n(k) > k$
- Random-looking:
  - Next-Bit Unpredictability: PPT adversary can't predict i^{th} bit of a sample from its first (i-1) bits (for every $i \in \{0,1,\ldots,n-1\}$)
  - A "more correct" definition:
    - PPT adversary can't distinguish between a sample from $\{G_k(x)\}_{x \leftarrow \{0,1\}^k}$ and one from $\{0,1\}^{n(k)}$
  - Turns out they are equivalent!

$| \Pr_{y \leftarrow PRG}[A(y)=0] - \Pr_{y \leftarrow rand}[A(y)=0] |$ is negligible for all PPT A

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- If X, X′ are short (say a single bit), $X \approx X'$ iff X, X′ are statistically indistinguishable (Exercise)

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  - Note: $\{G_k(x)\}_{x \leftarrow \{0,1\}^k}$ cannot be statistically indistinguishable from $U_{n(k)}$ unless $n(k) \leq k$ (Exercise)

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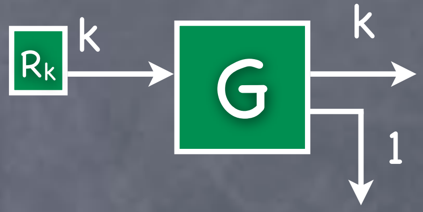

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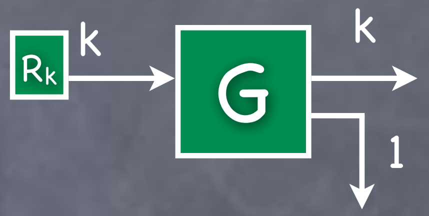

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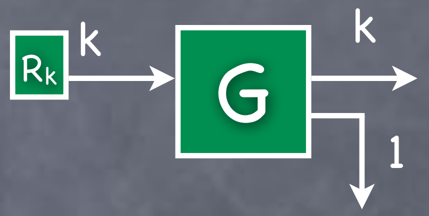

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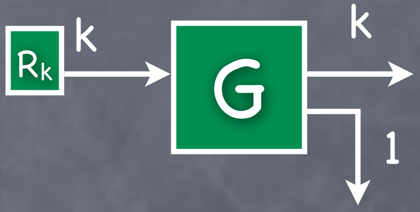

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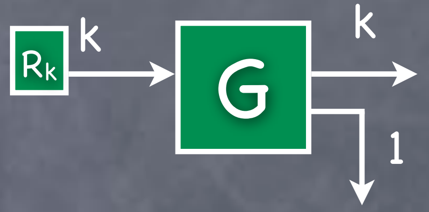

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    - ... or pseudorandom

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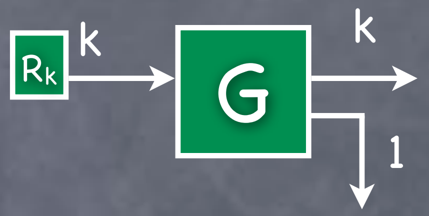

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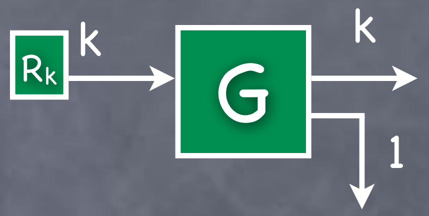

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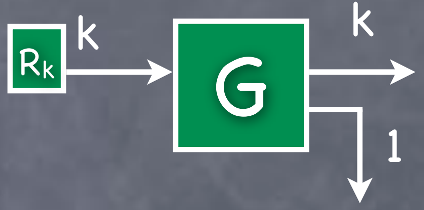

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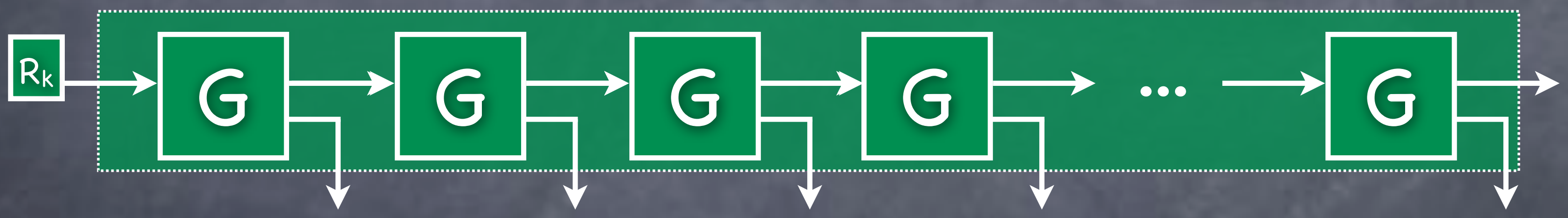

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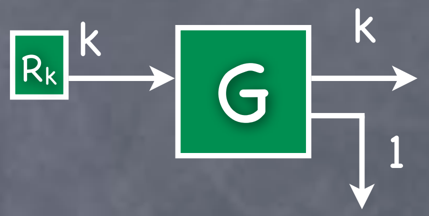

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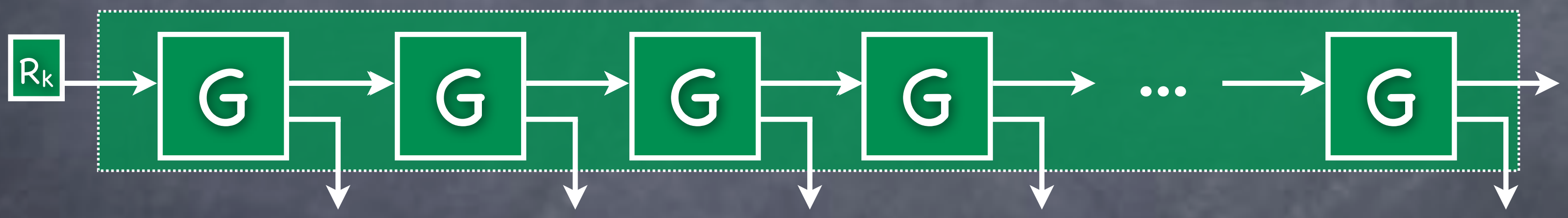

  - If the intermediate seeds are never output, can keep stretching on demand (for any "polynomial length")

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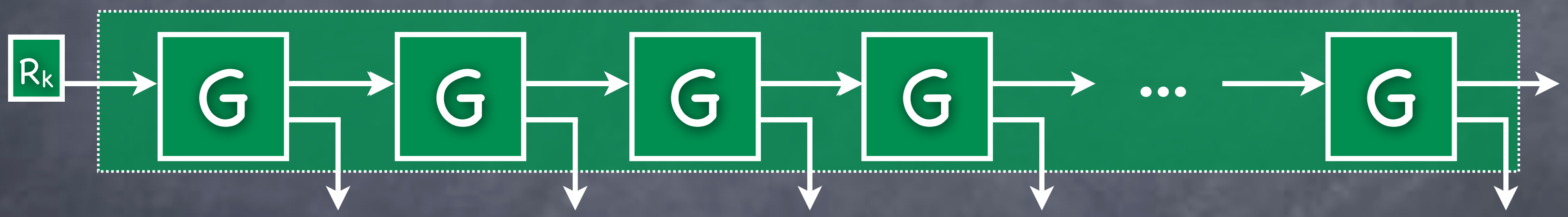

  - If the intermediate seeds are never output, can keep stretching on demand (for any "polynomial length")
  - A stream cipher

One-time CPA-secure SKE with a Stream-Cipher

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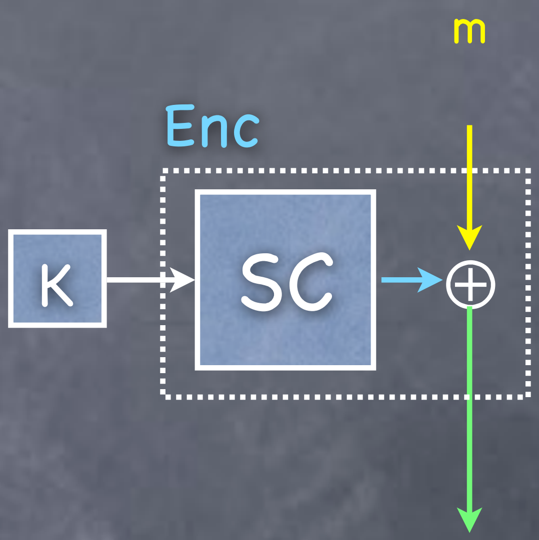

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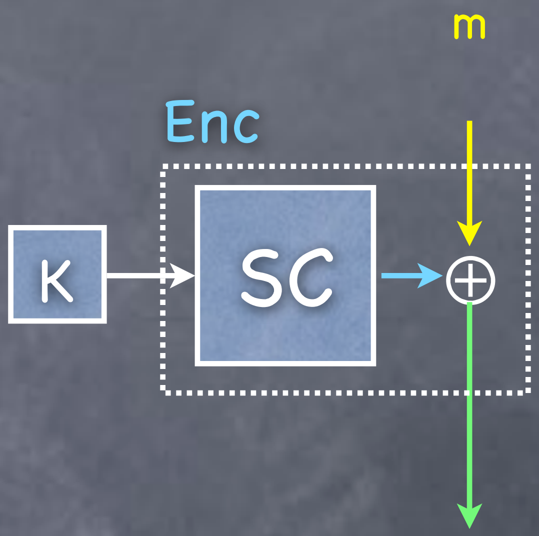

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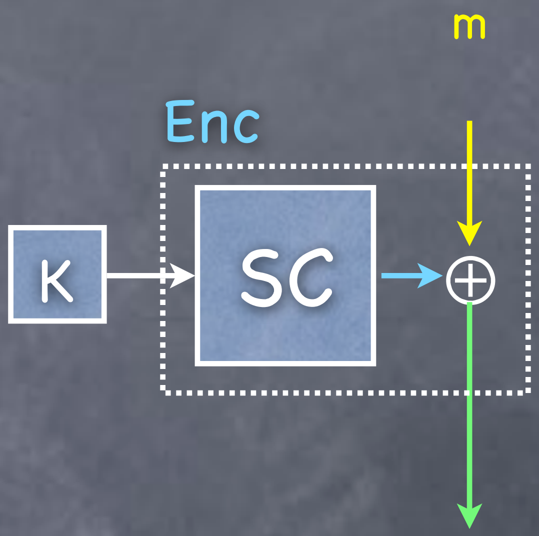

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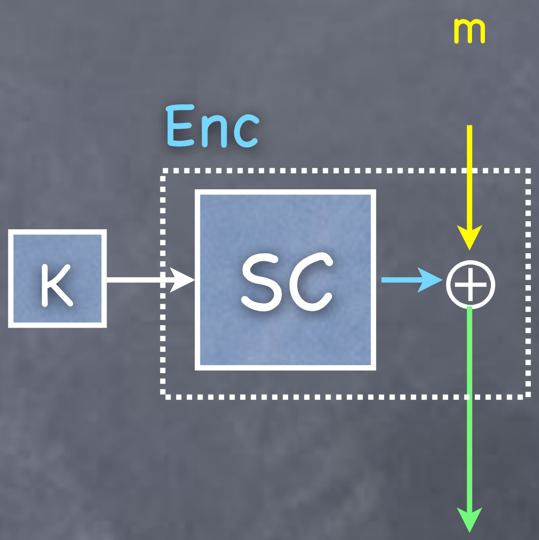

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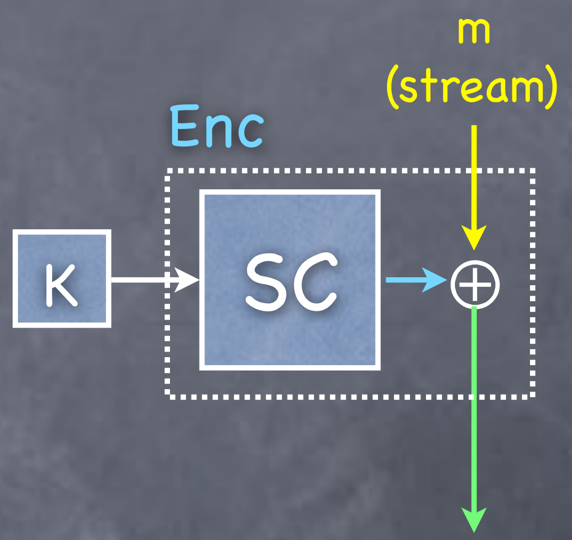

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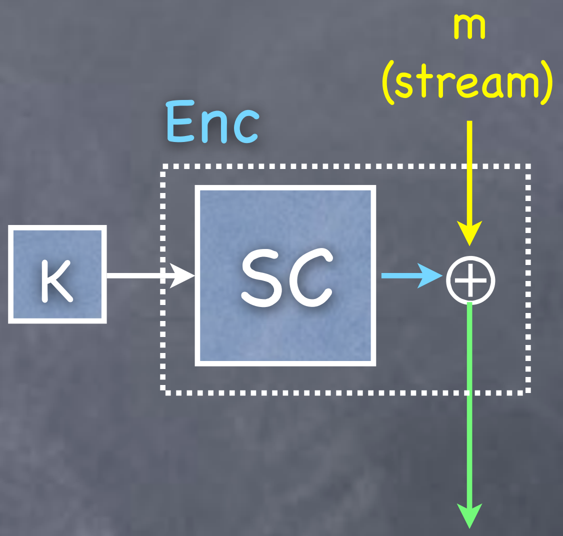

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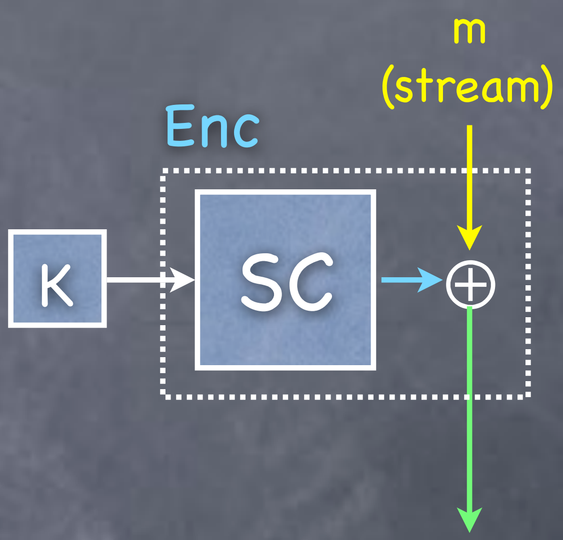

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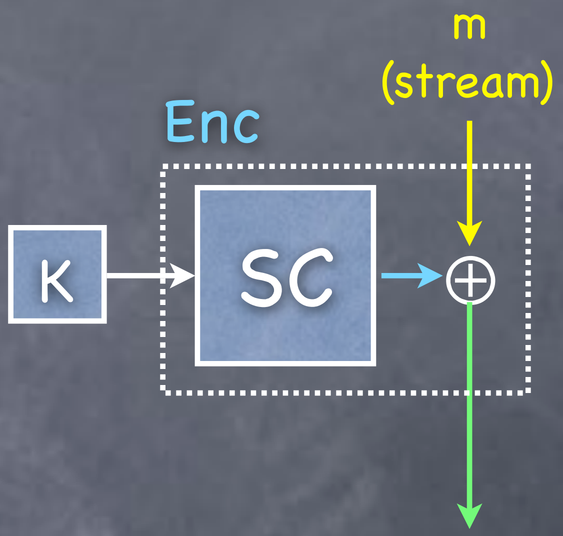

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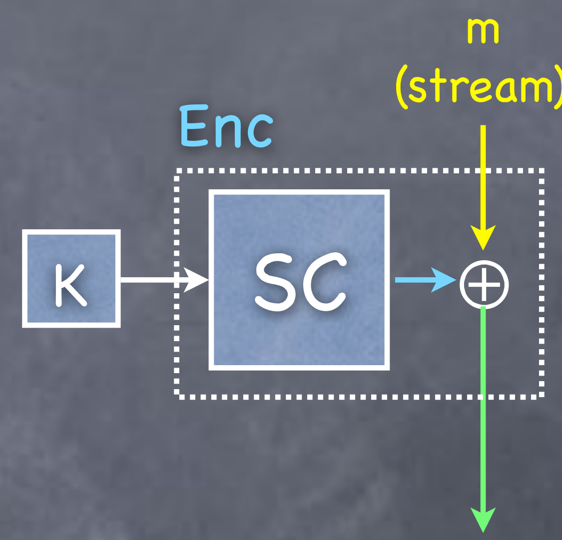

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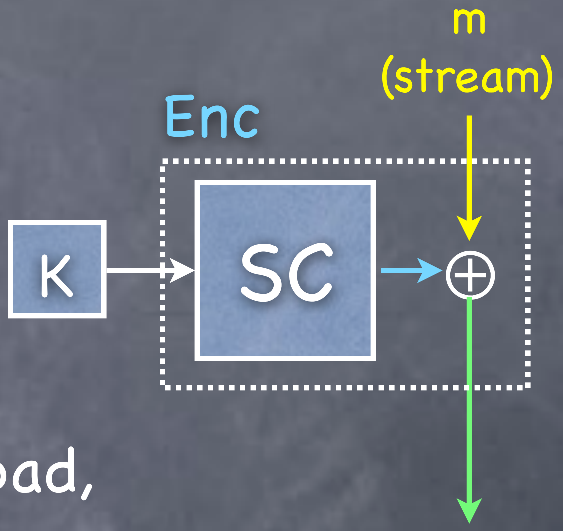

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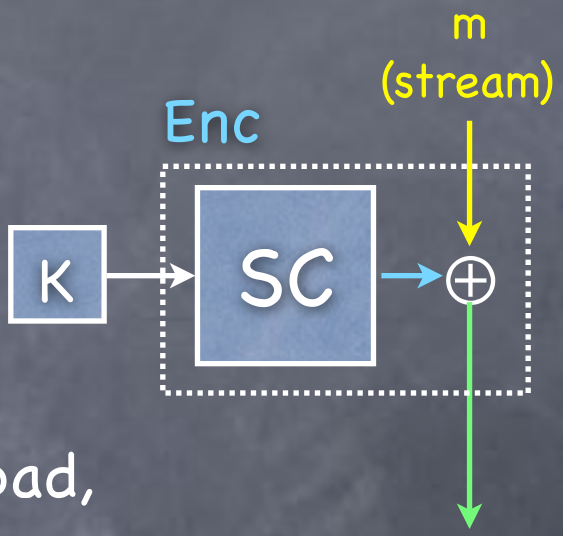

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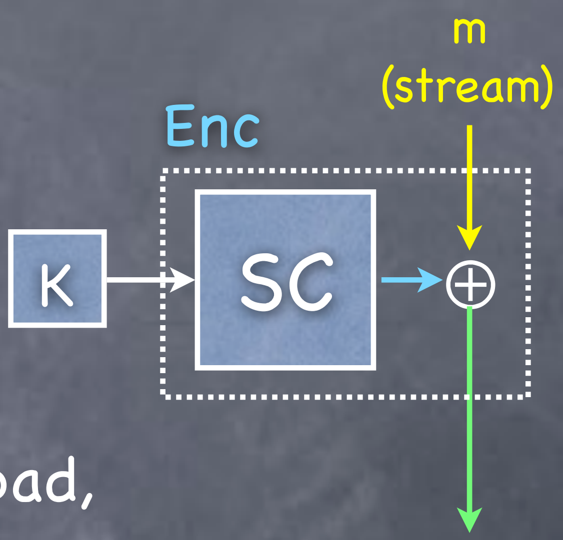

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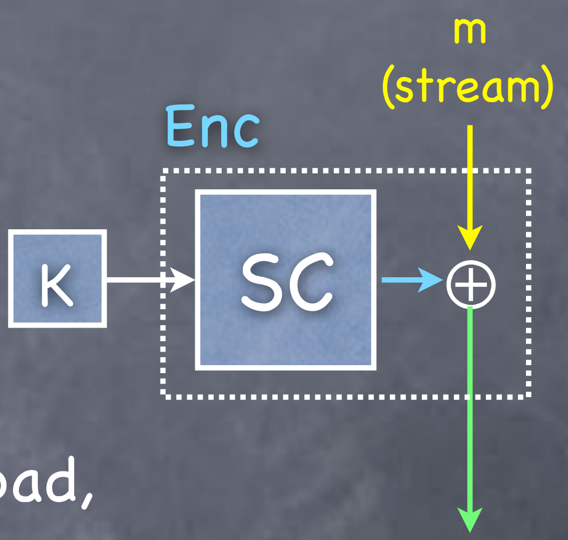

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  - <u>Claim</u>: REAL ≈ HYBRID.
    - Consider the experiments as a system that accepts a pad from outside (R' = SC(K) for a random K, or truly random R) and outputs the environment's output. This system is PPT, and so can't distinguish pseudorandom from random.

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  - Next: Constructing a proper (multi-message) SKE scheme