Defining Encryption (ctd.)

Lecture 3

CPA/CCA security

Computational Indistinguishability

Pseudo-randomness, One-Way Functions

Security of Encryption

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- Coming up: Formalizing notions of "computational" security (as opposed to perfect/statistical security)
  - Then, security definitions used for encryption of multiple messages

Symmetric-Key Encryption

The Syntax

- Shared-key (Private-key) Encryption
  - Key Generation: Randomized
    - $K \leftarrow \mathcal{K}$, uniformly randomly drawn from the key-space (or according to a key-distribution)
  - Encryption: Randomized
    - Enc: $\mathcal{M} \times \mathcal{K} \times \mathcal{R} \rightarrow \mathcal{C}$. During encryption a fresh random string will be chosen uniformly at random from $\mathcal{R}$
  - Decryption: Deterministic
    - Dec: $\mathcal{C} \times \mathcal{K} \rightarrow \mathcal{M}$

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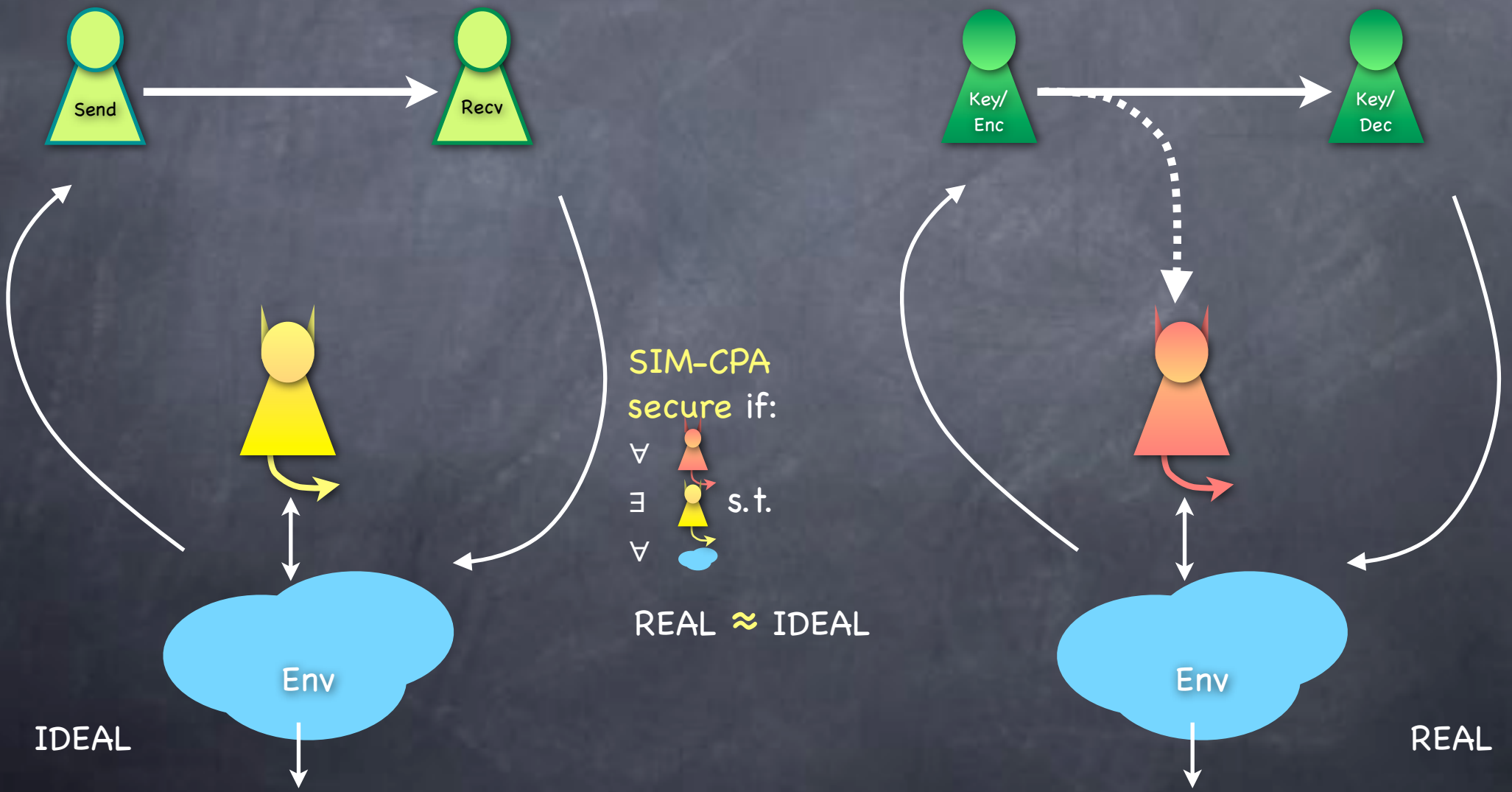
SIM-CPA Security



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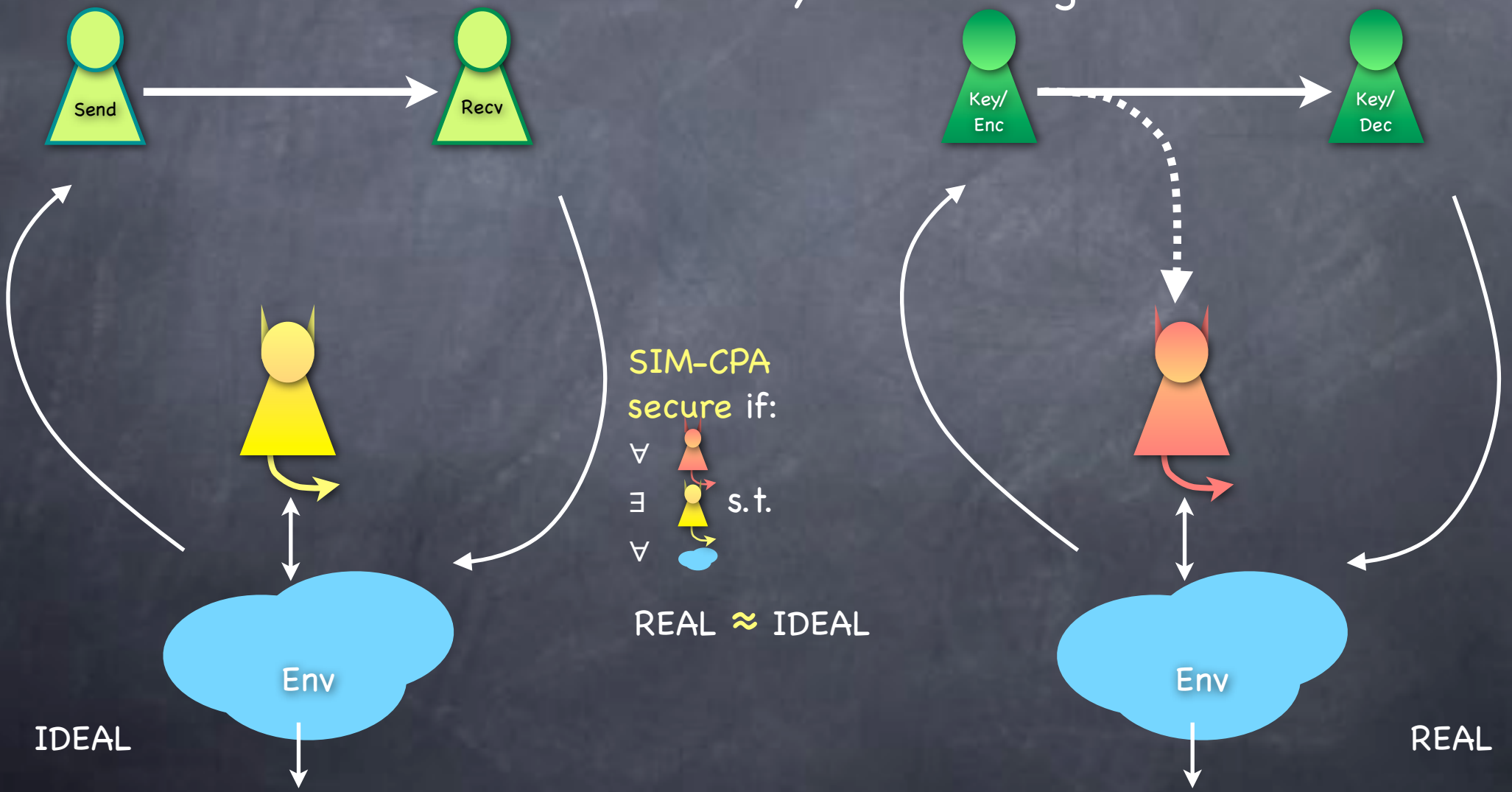

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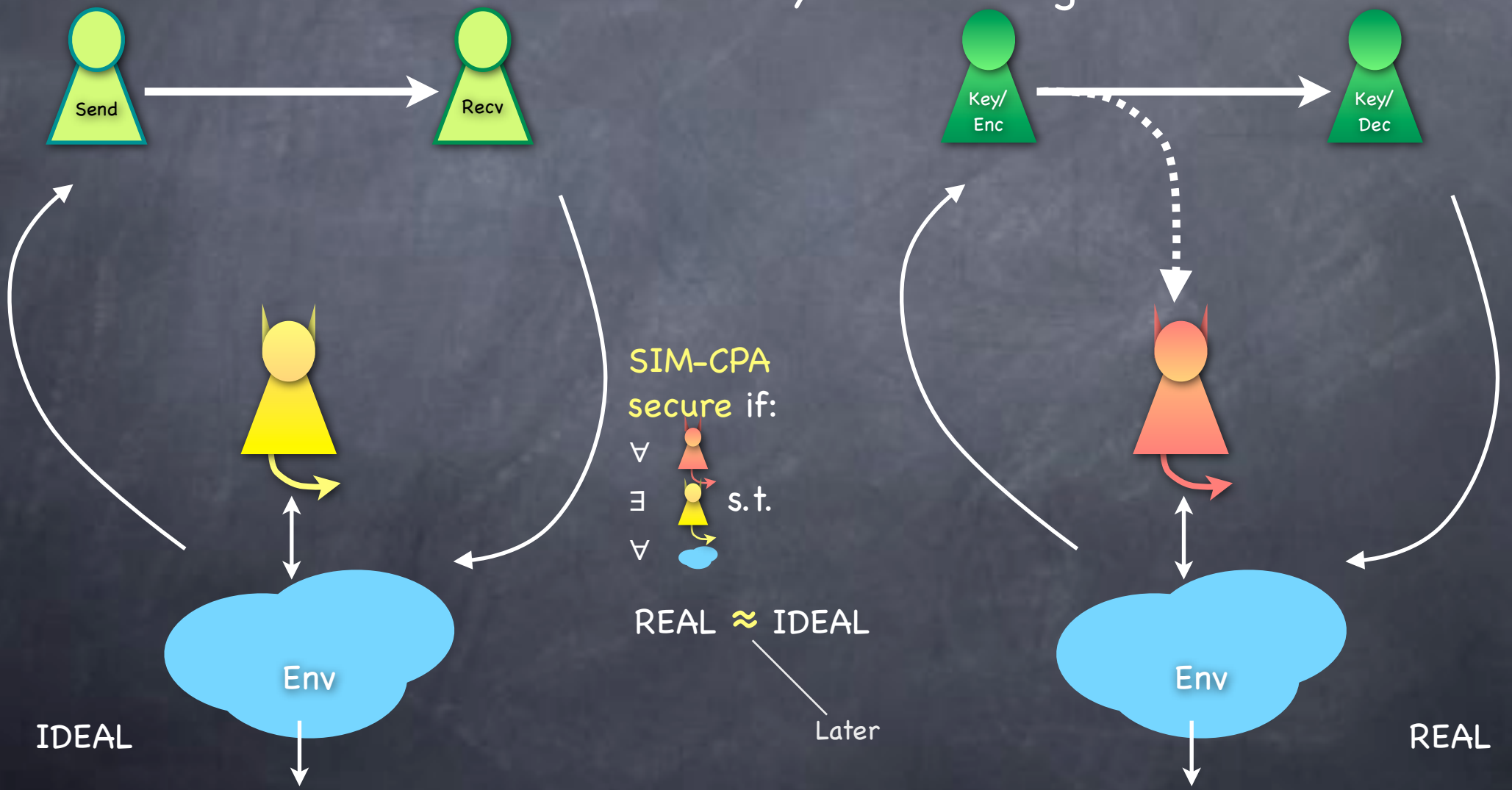

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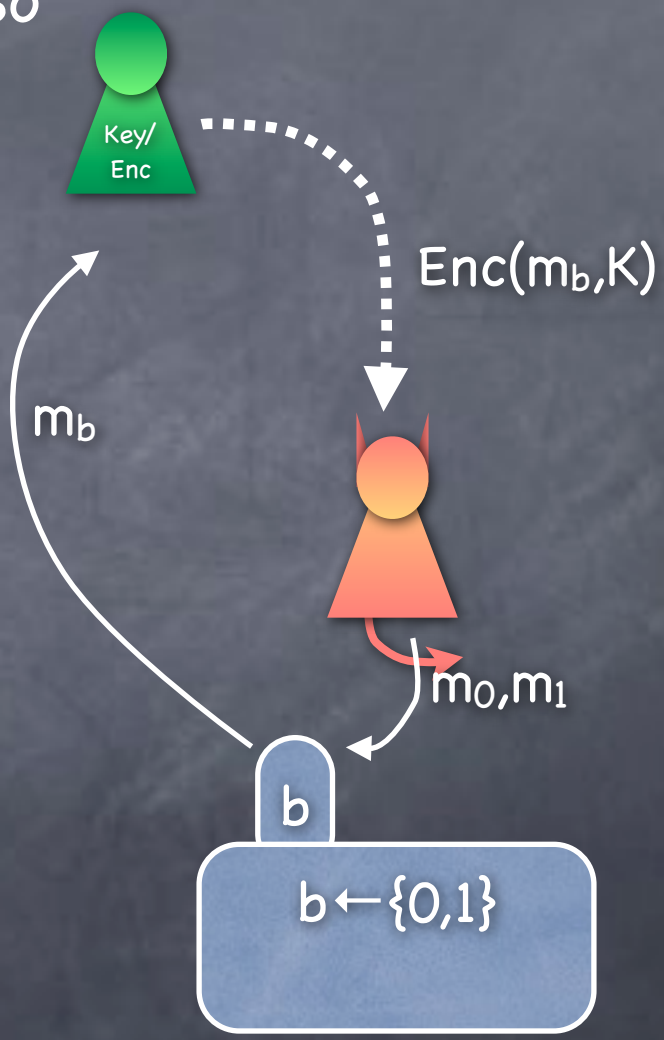

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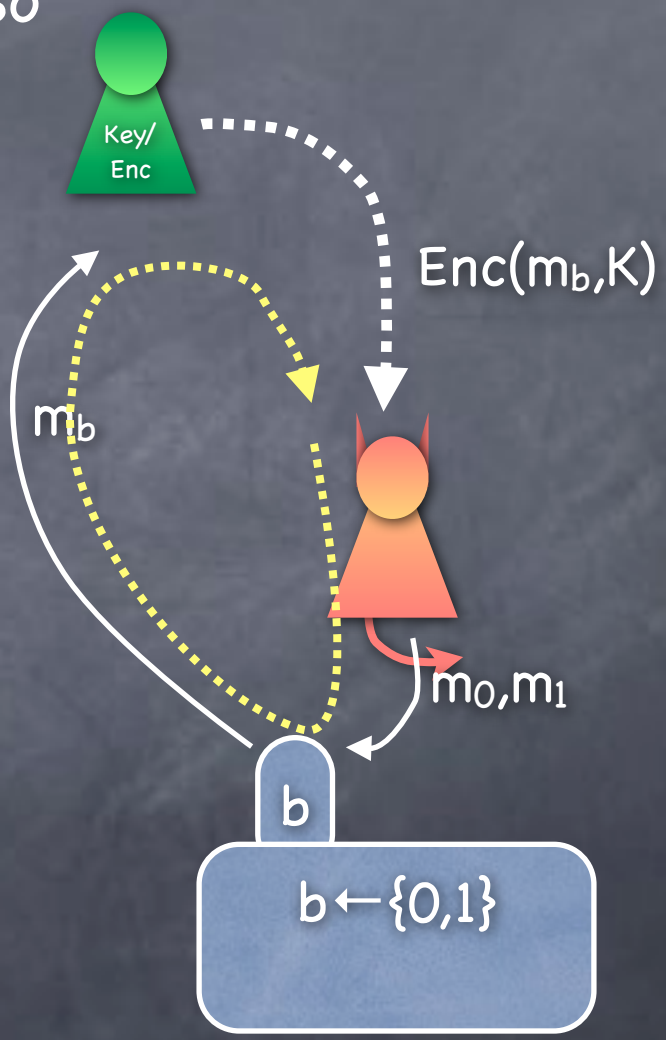

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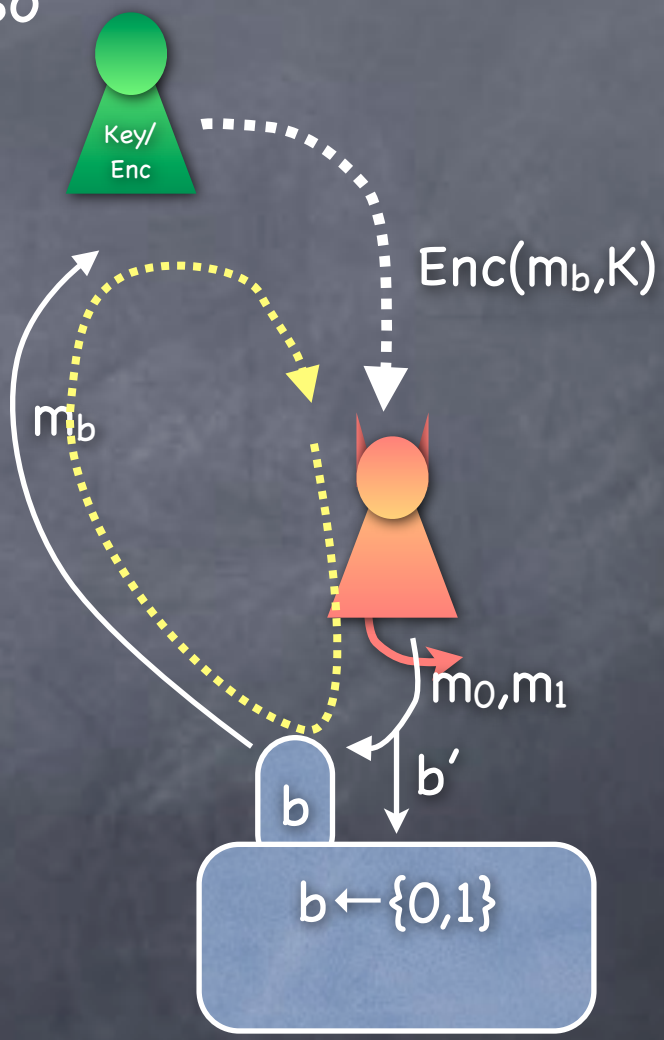

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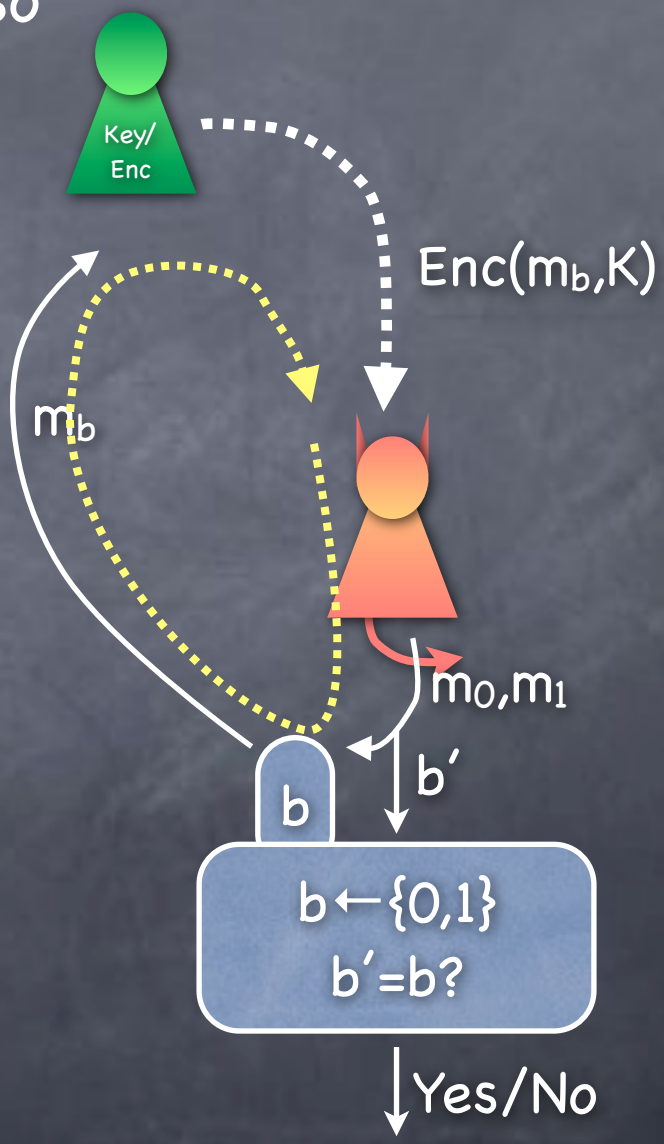

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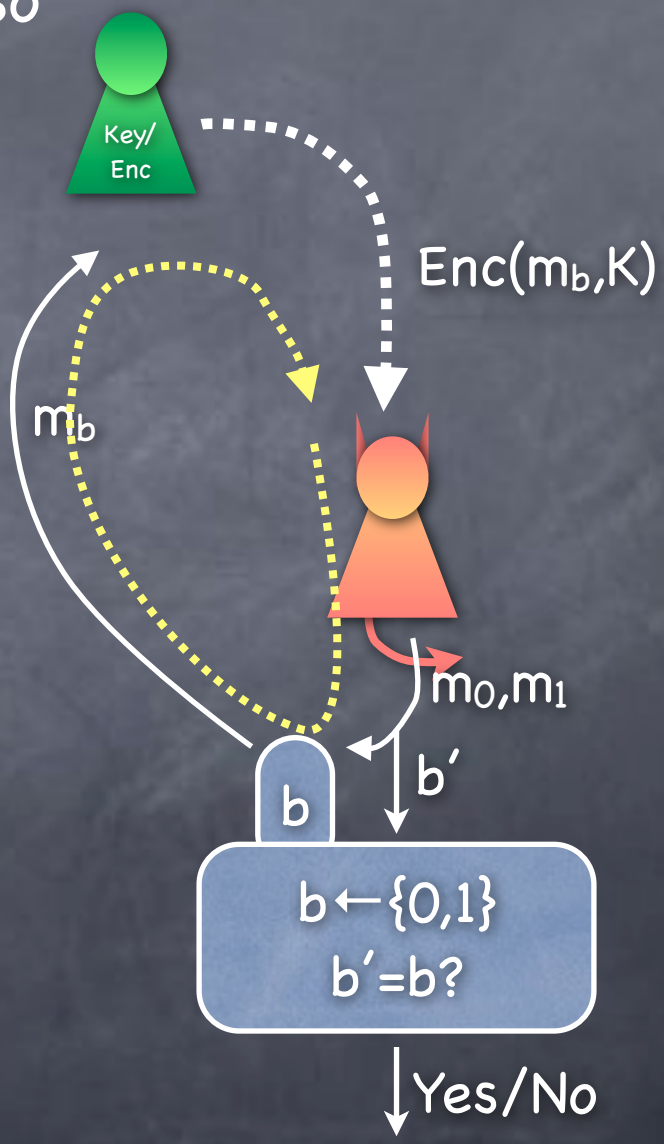

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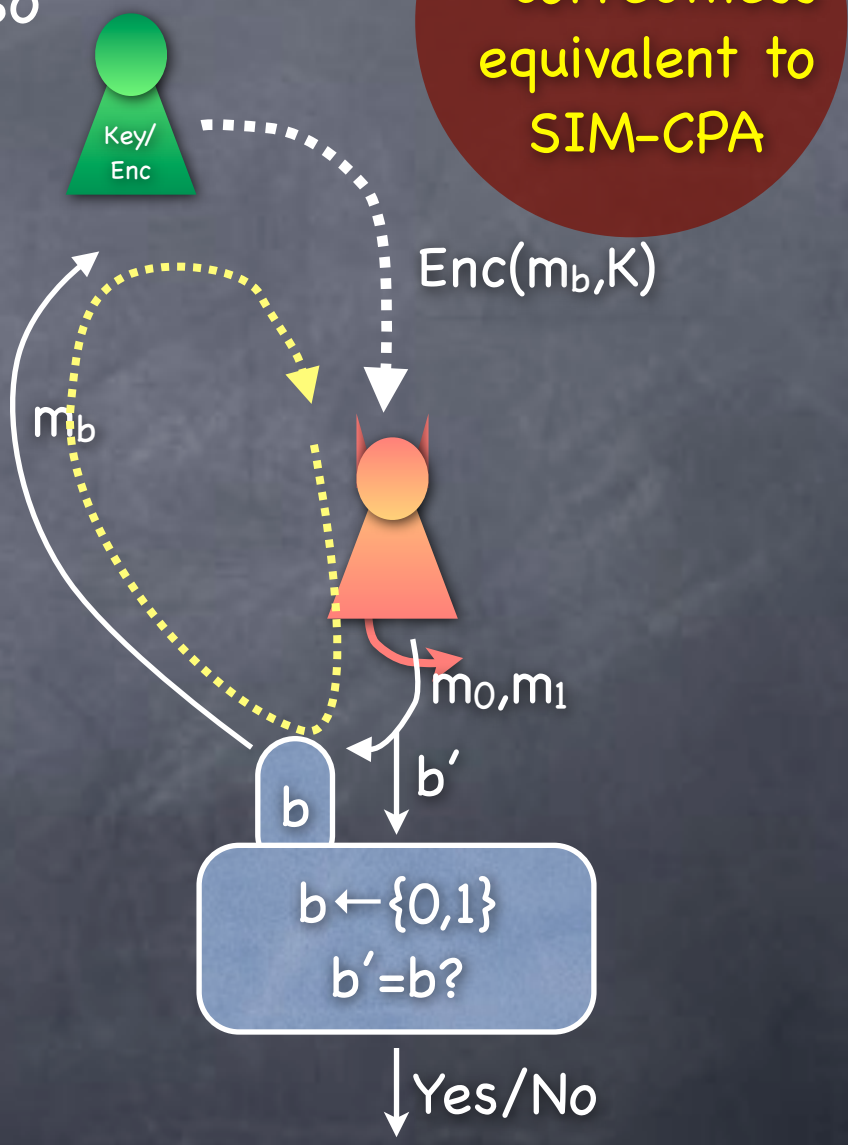

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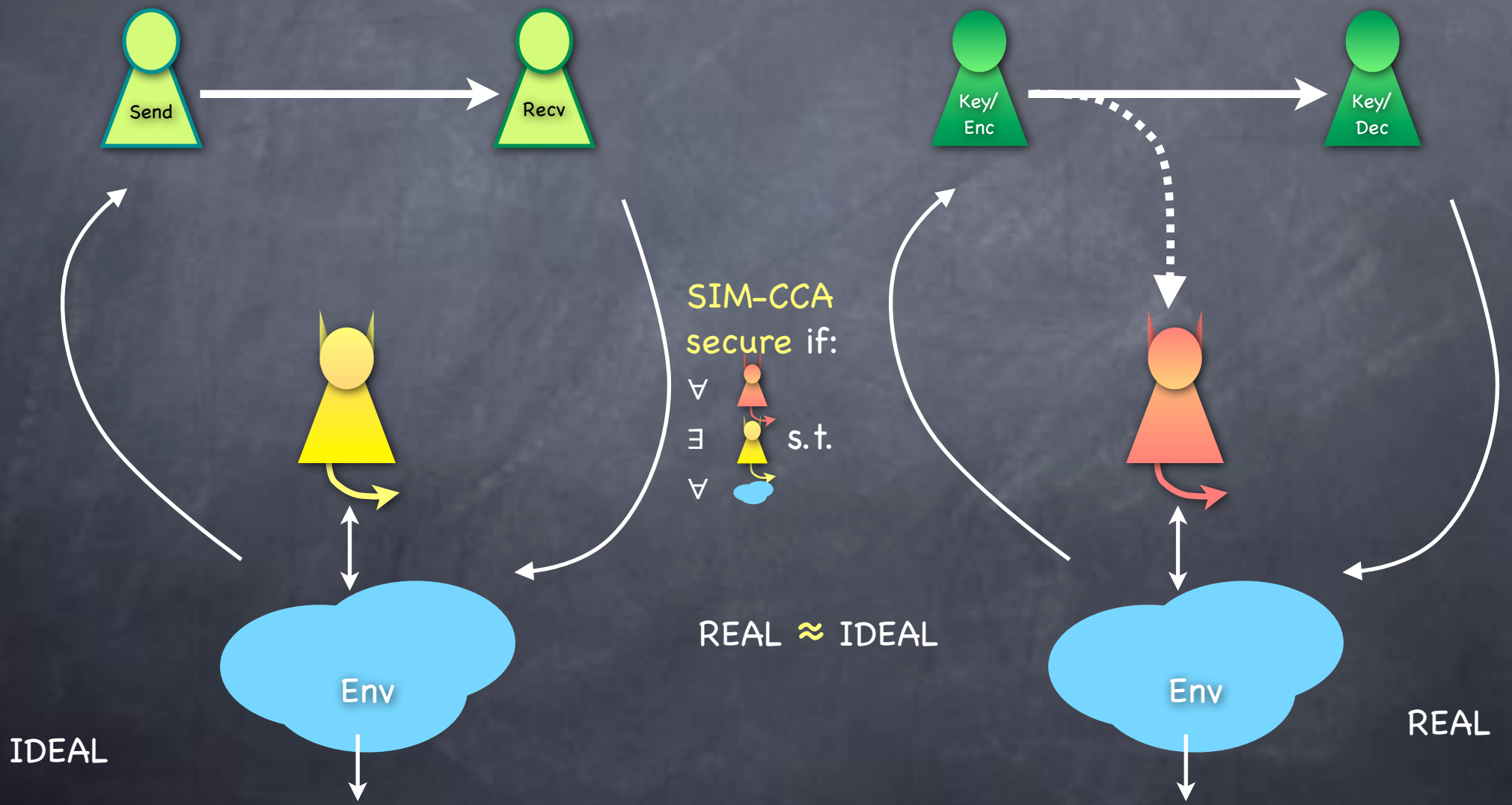

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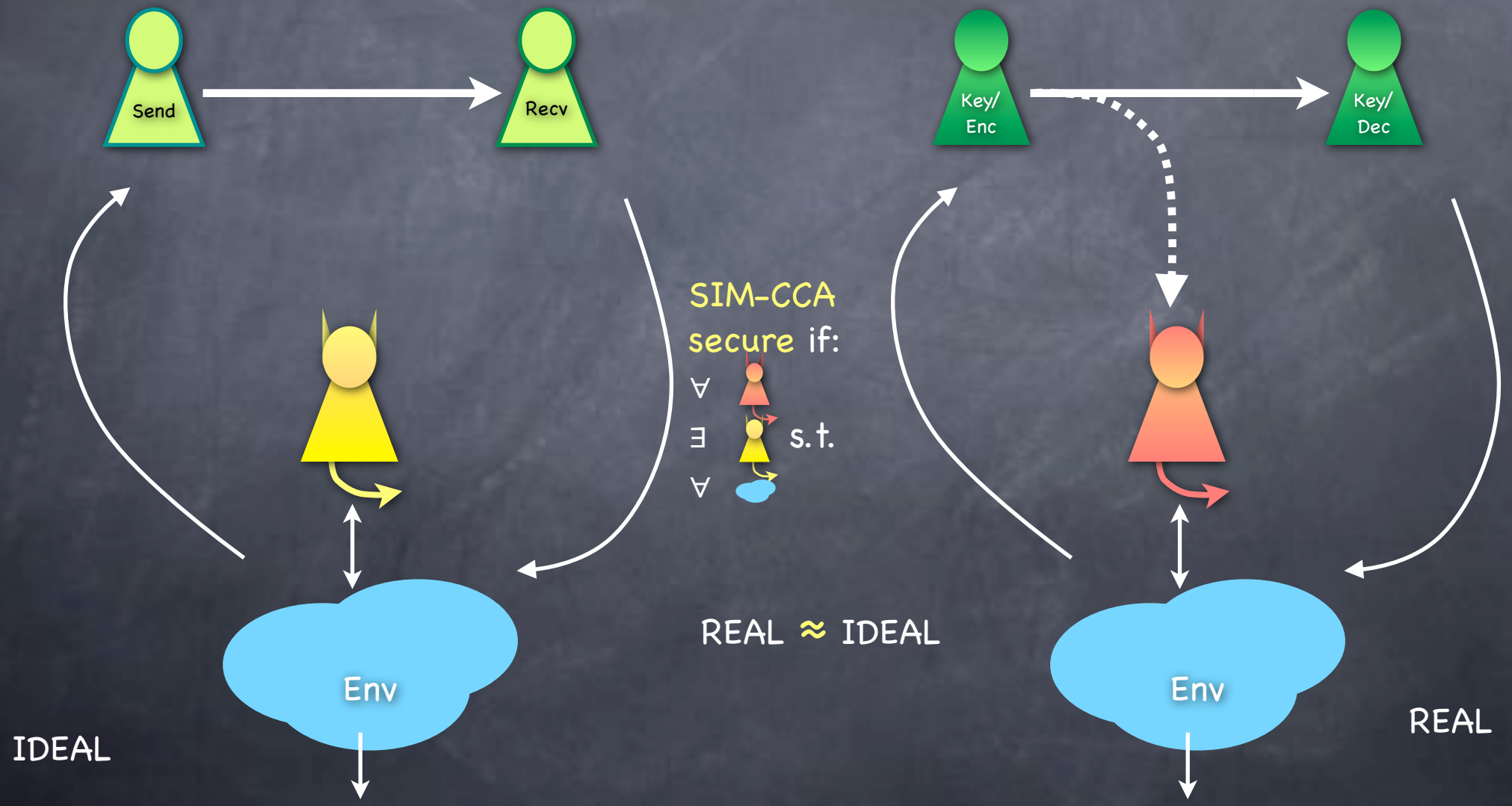

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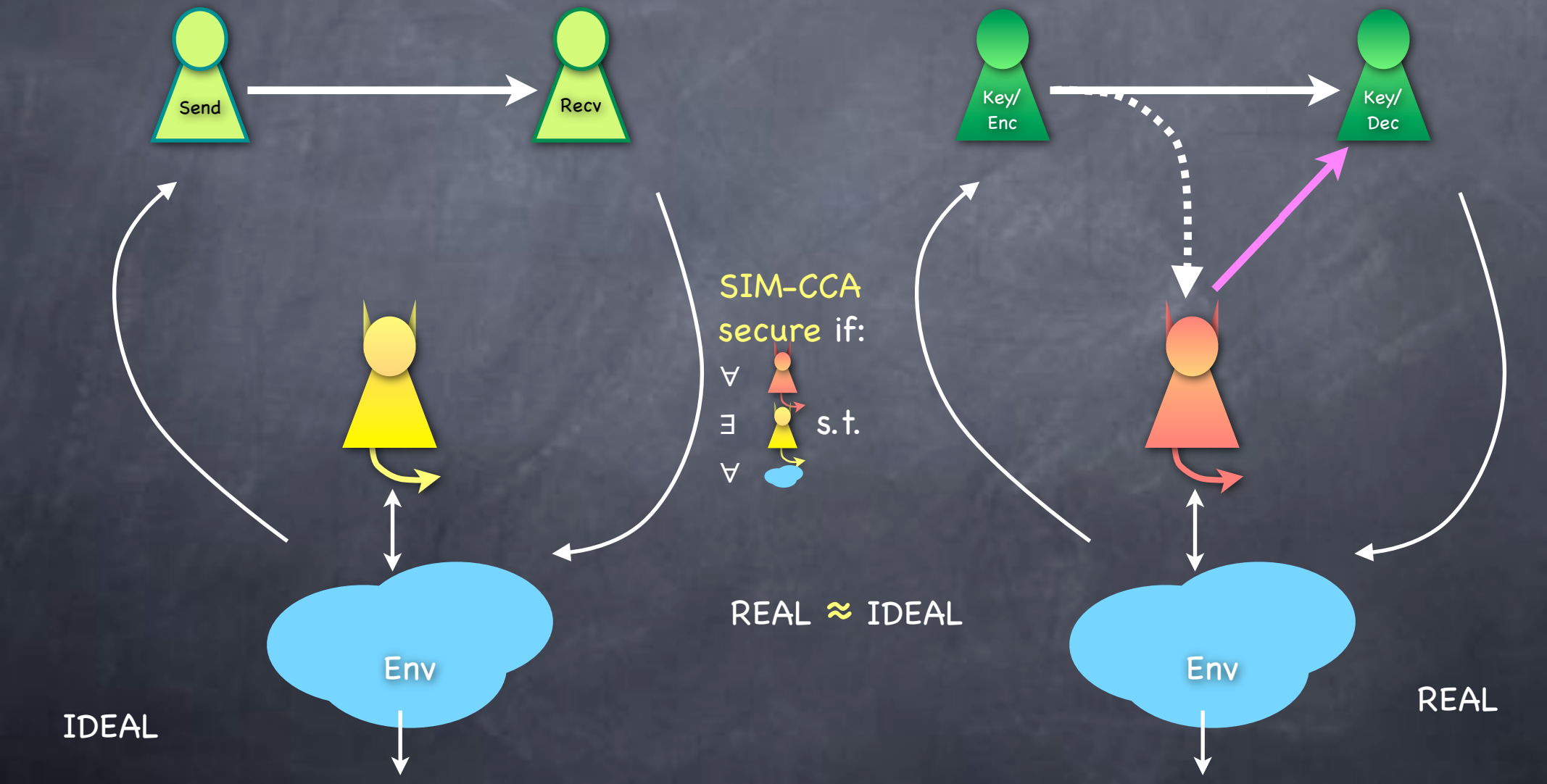

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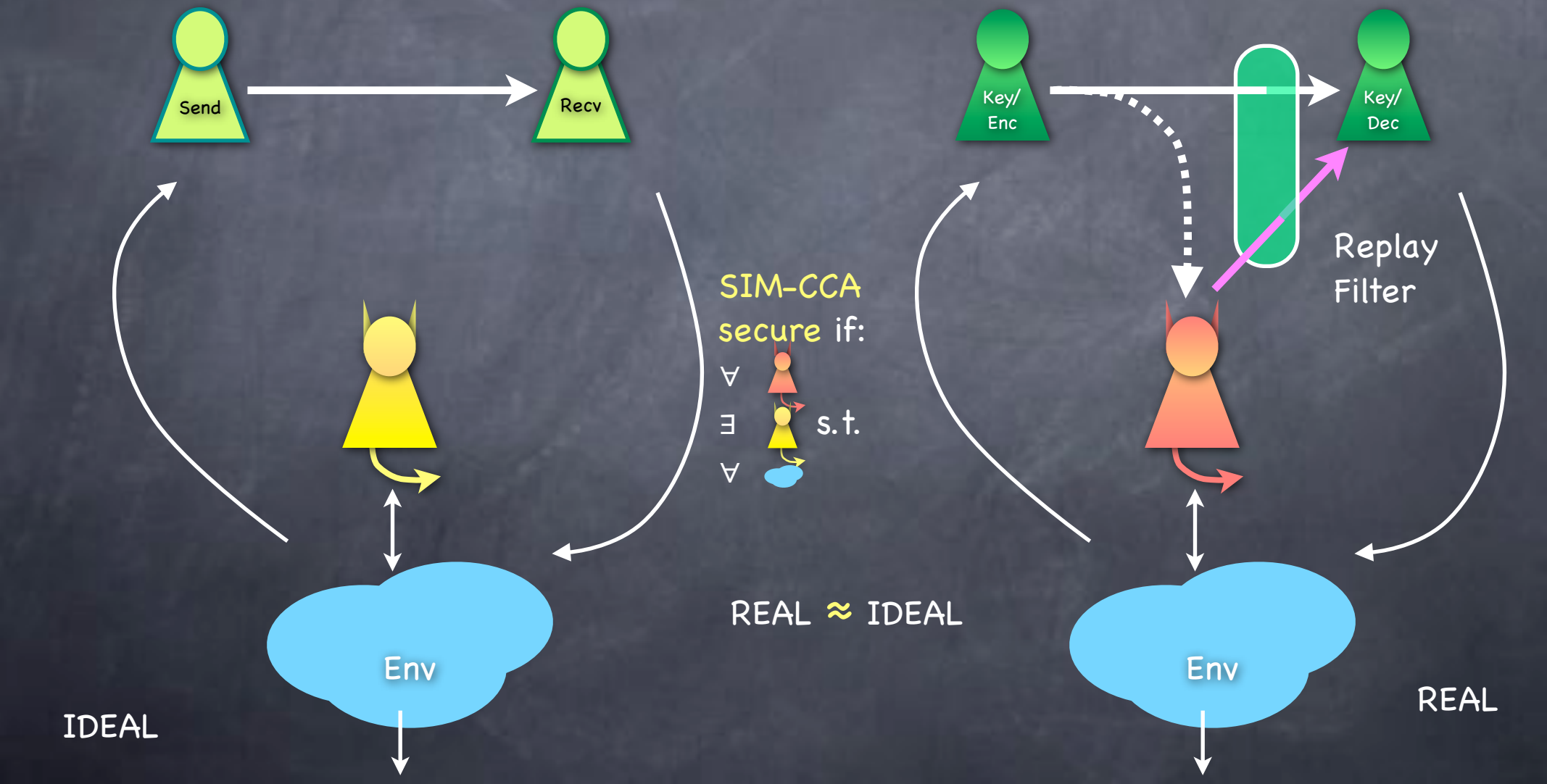

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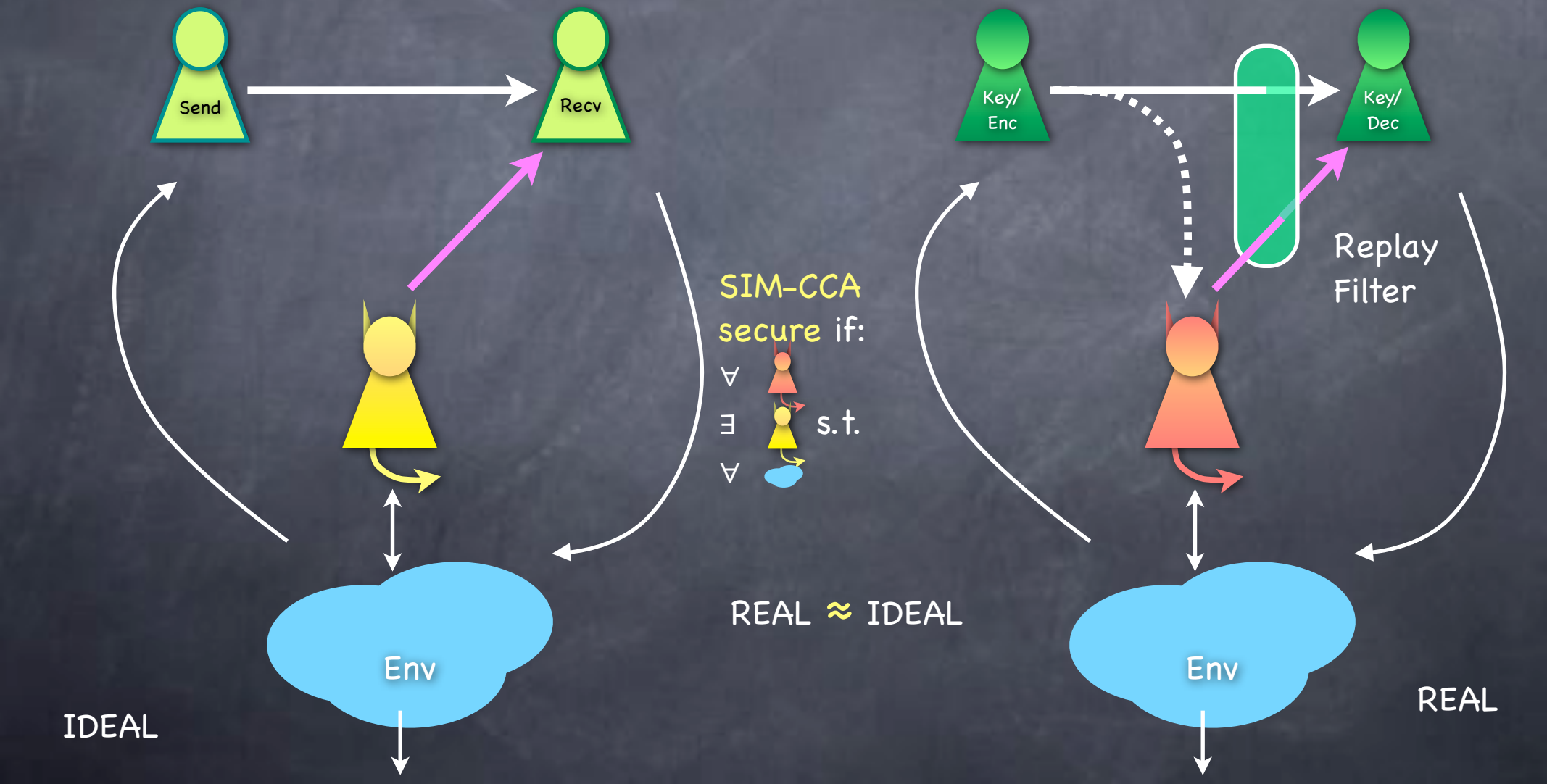

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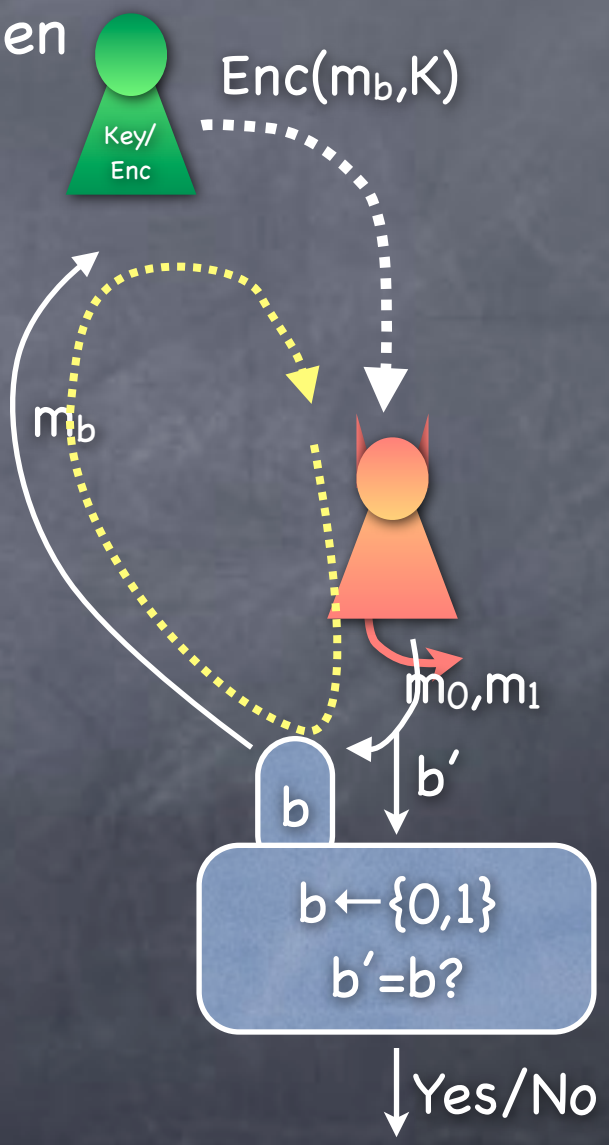

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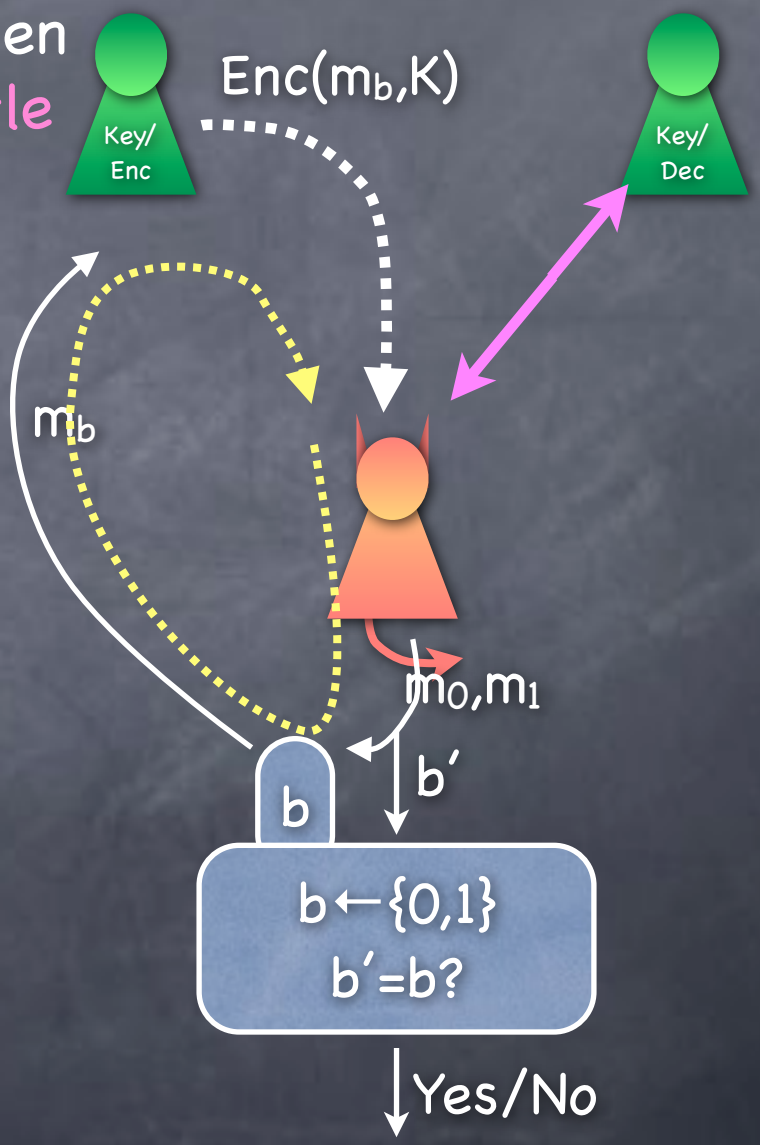

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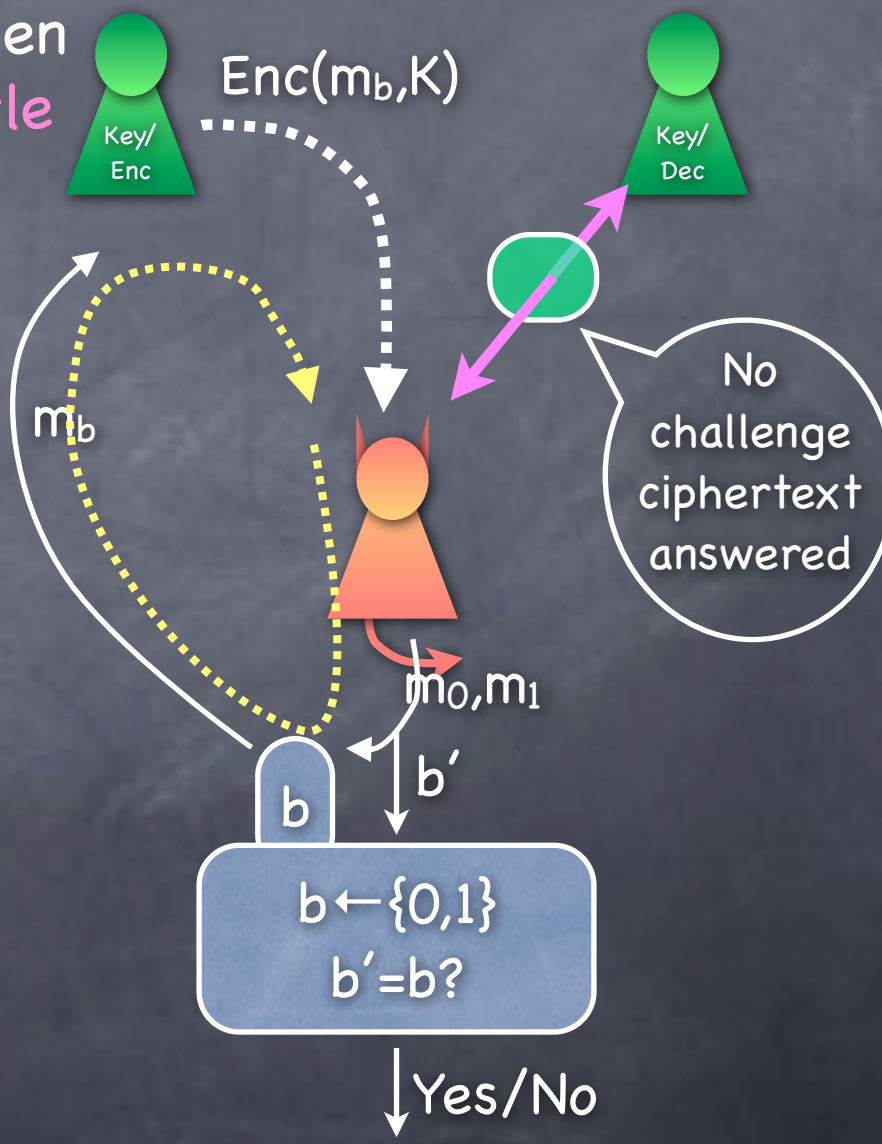

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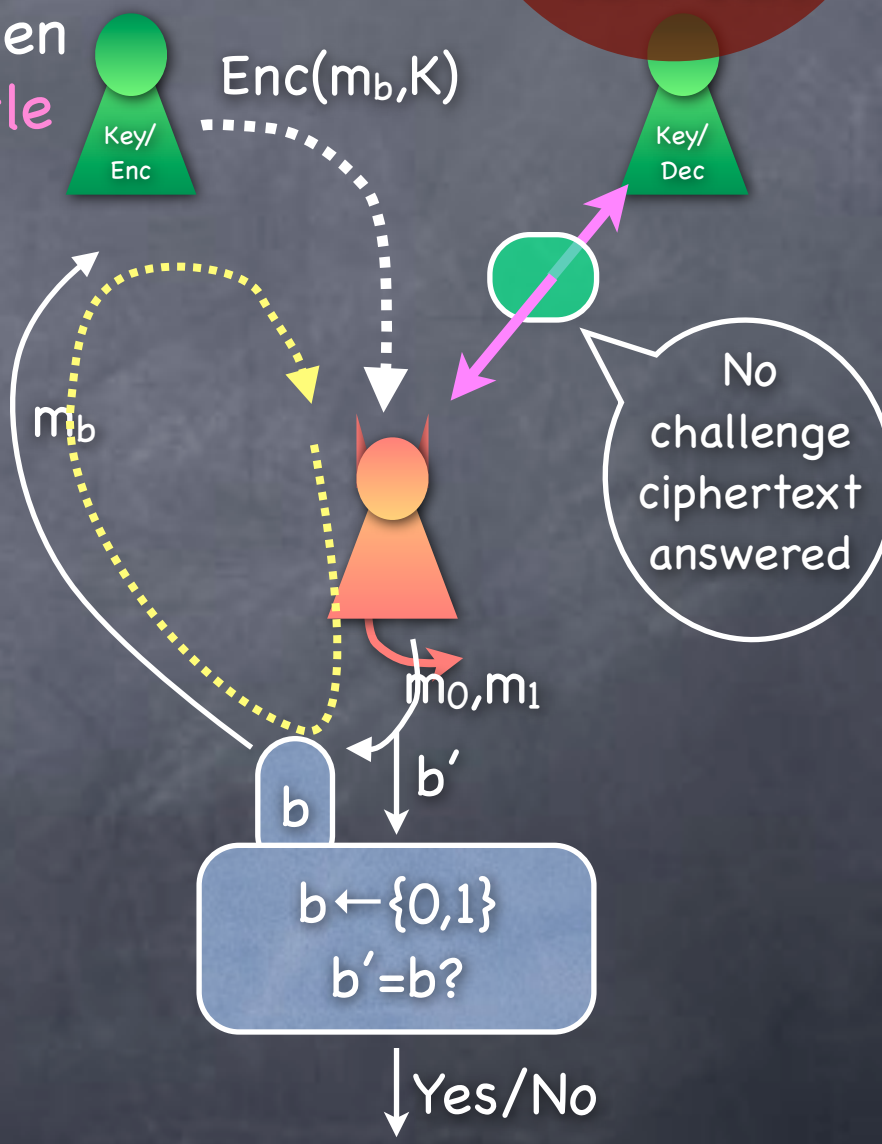

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  - But what is ≈ ?

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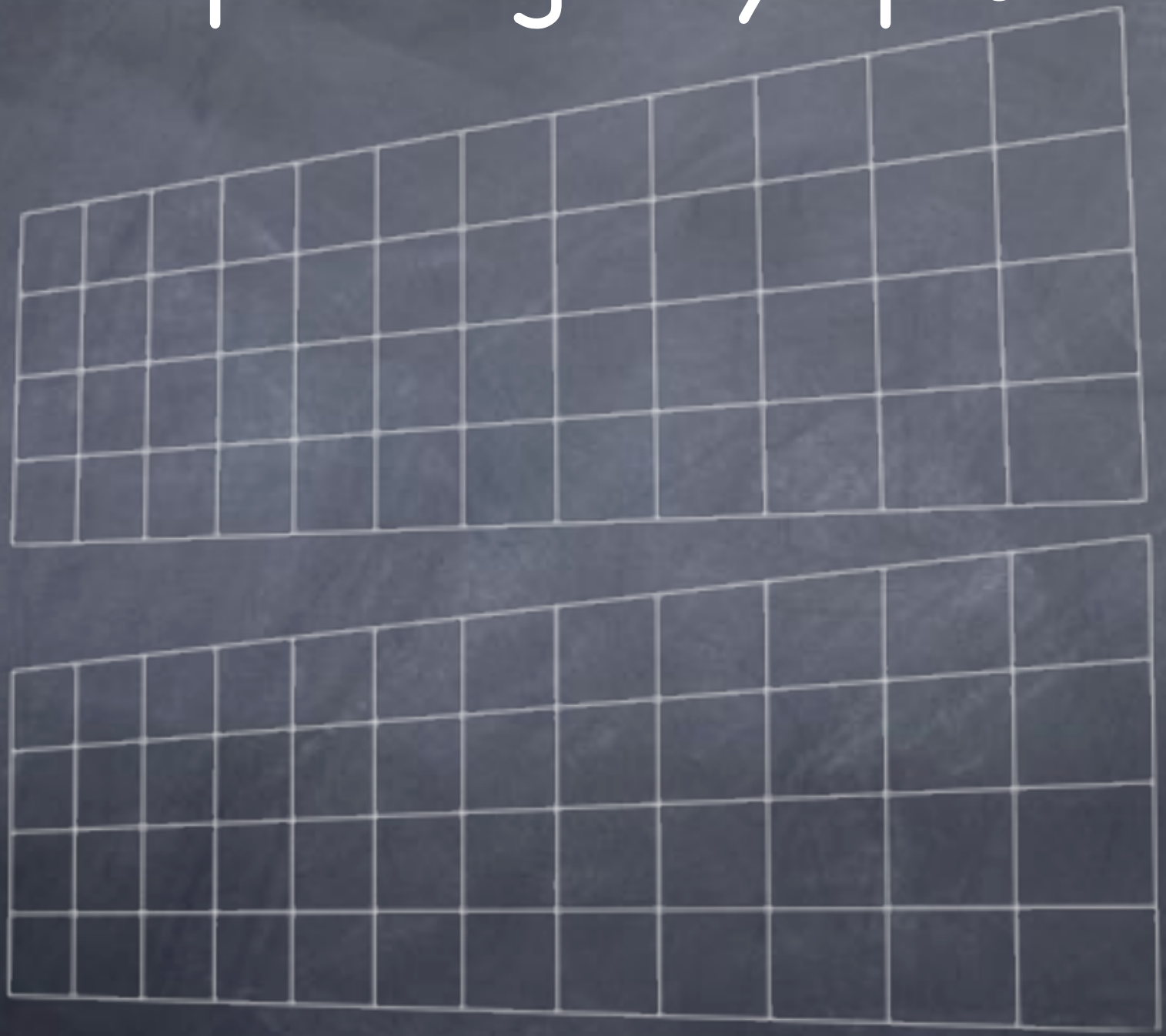

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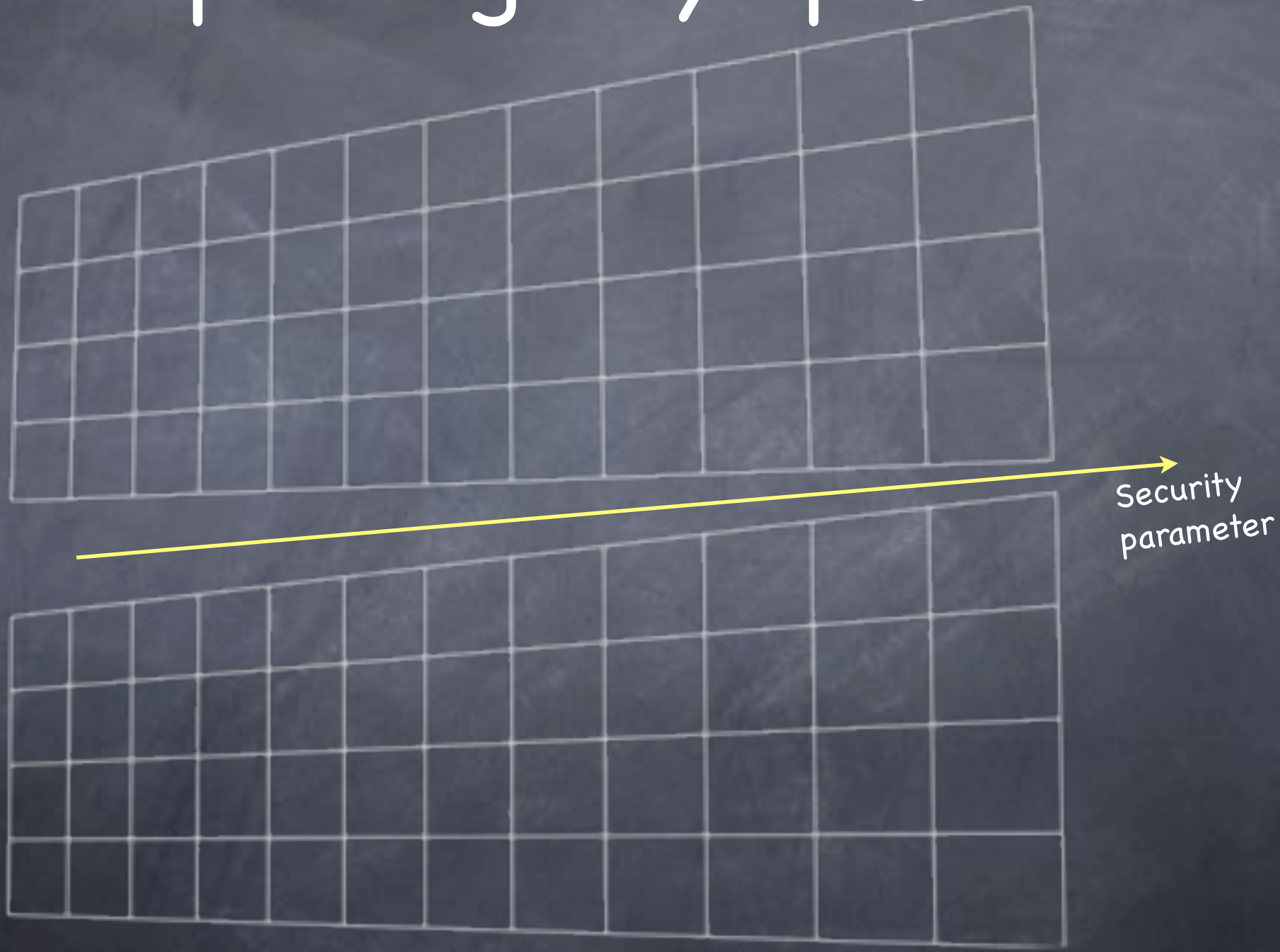

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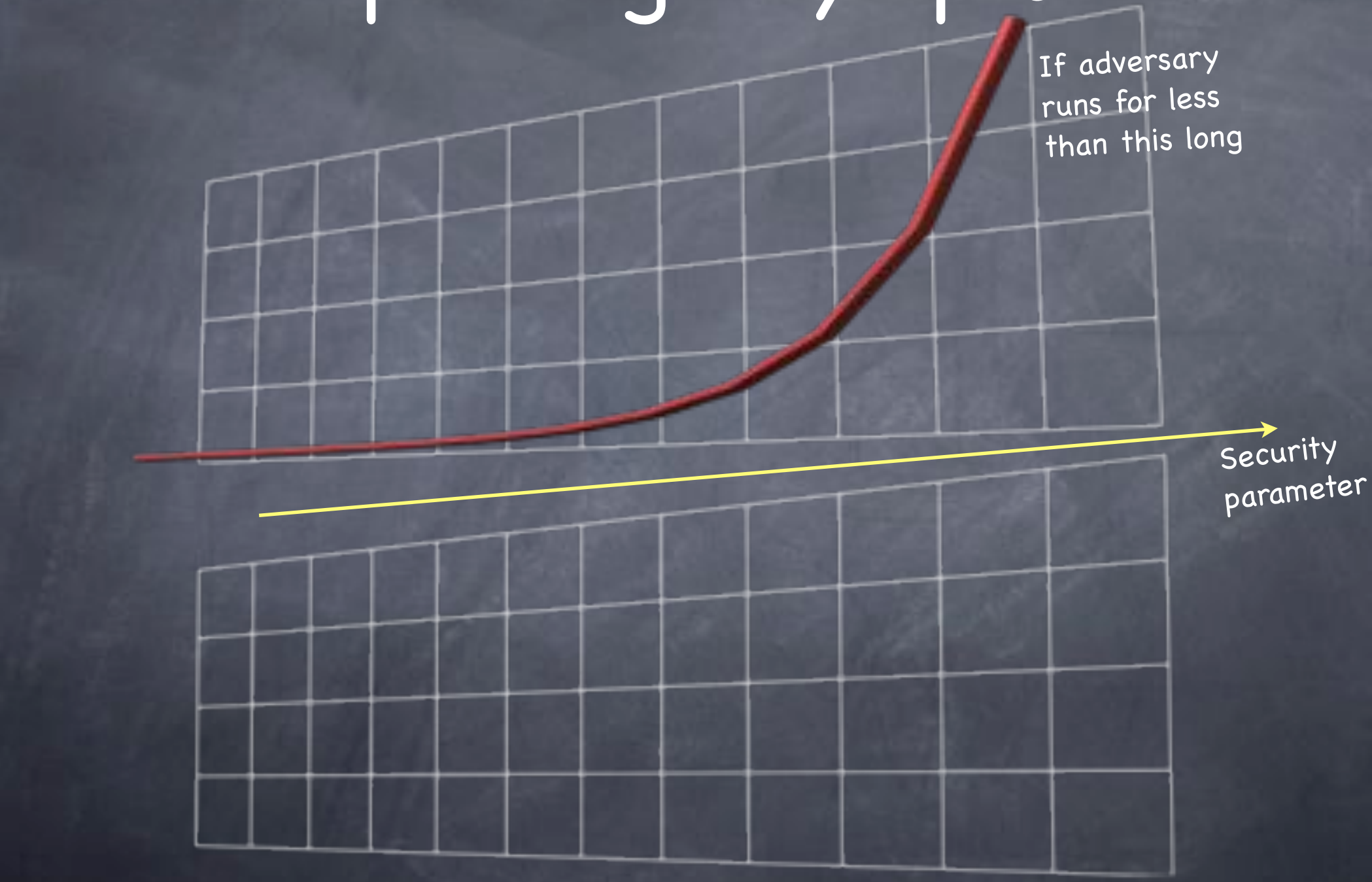

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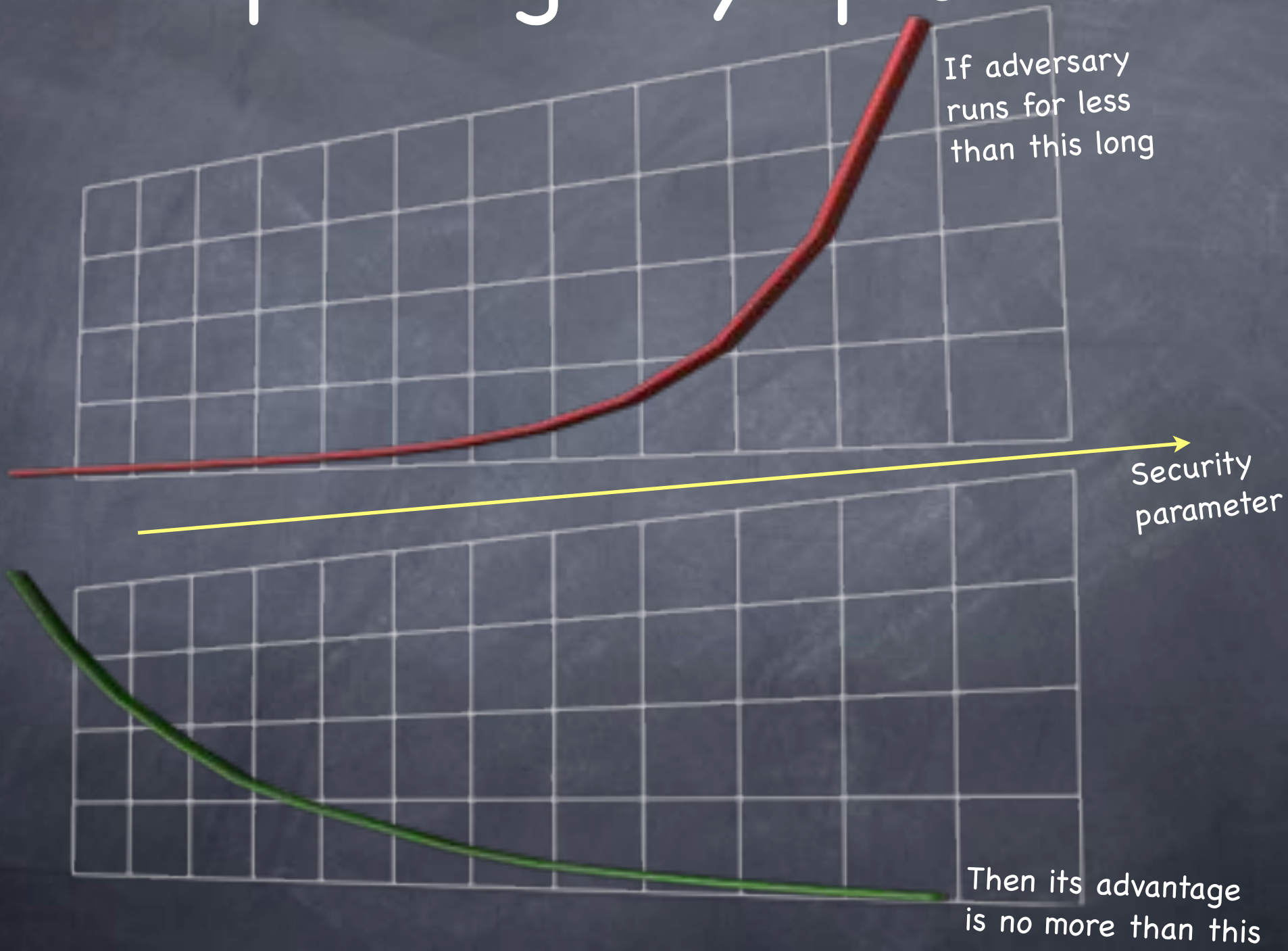

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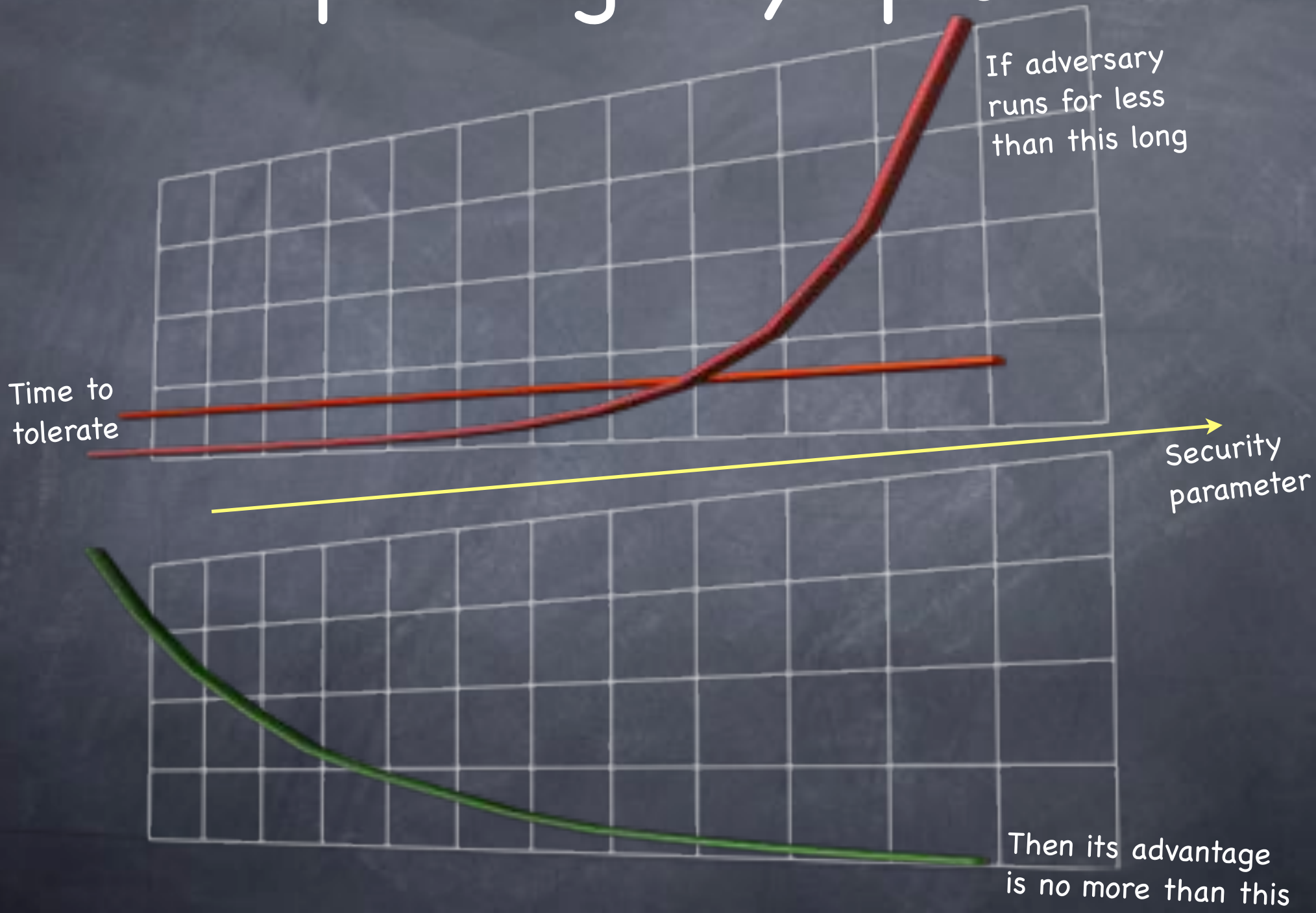

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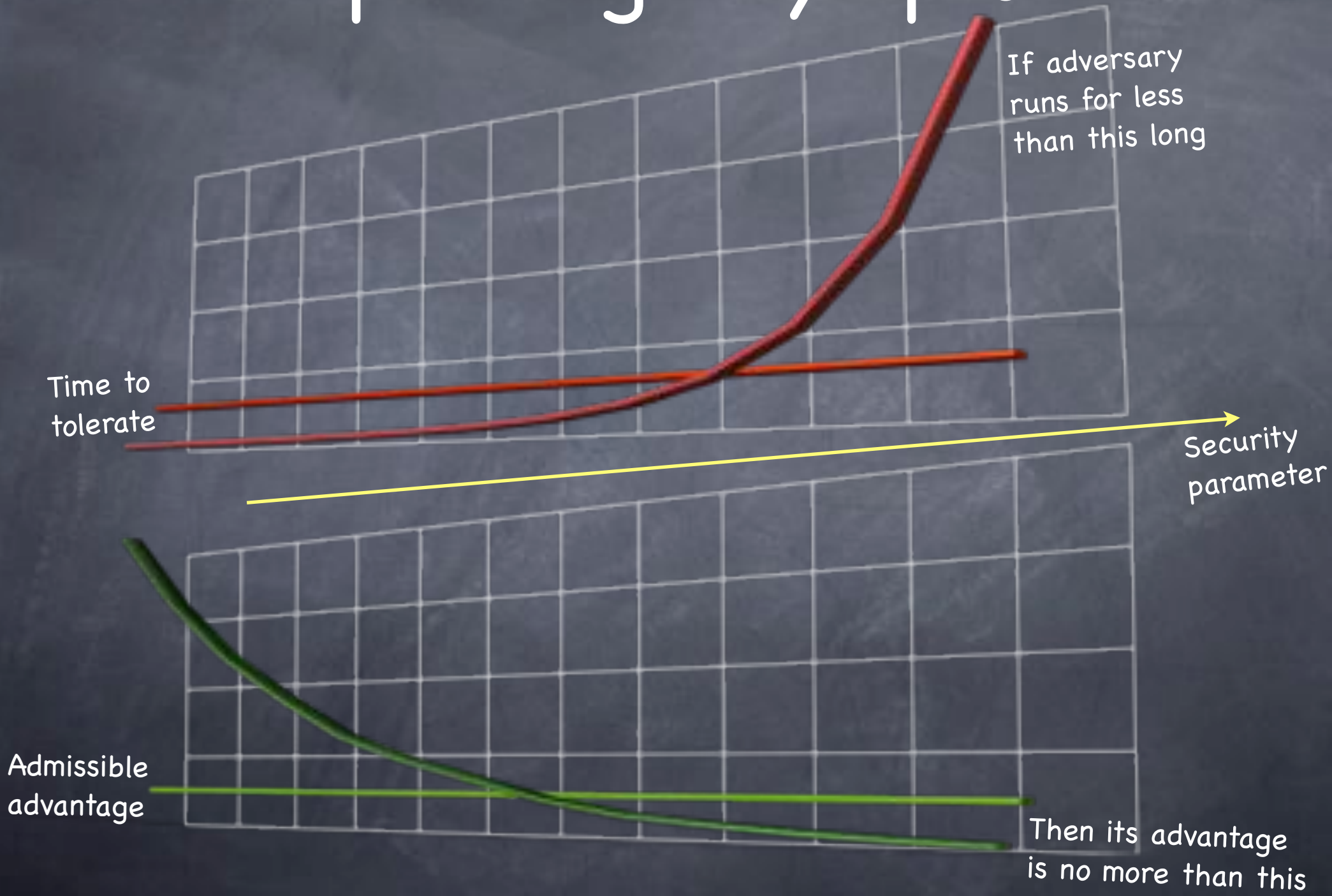

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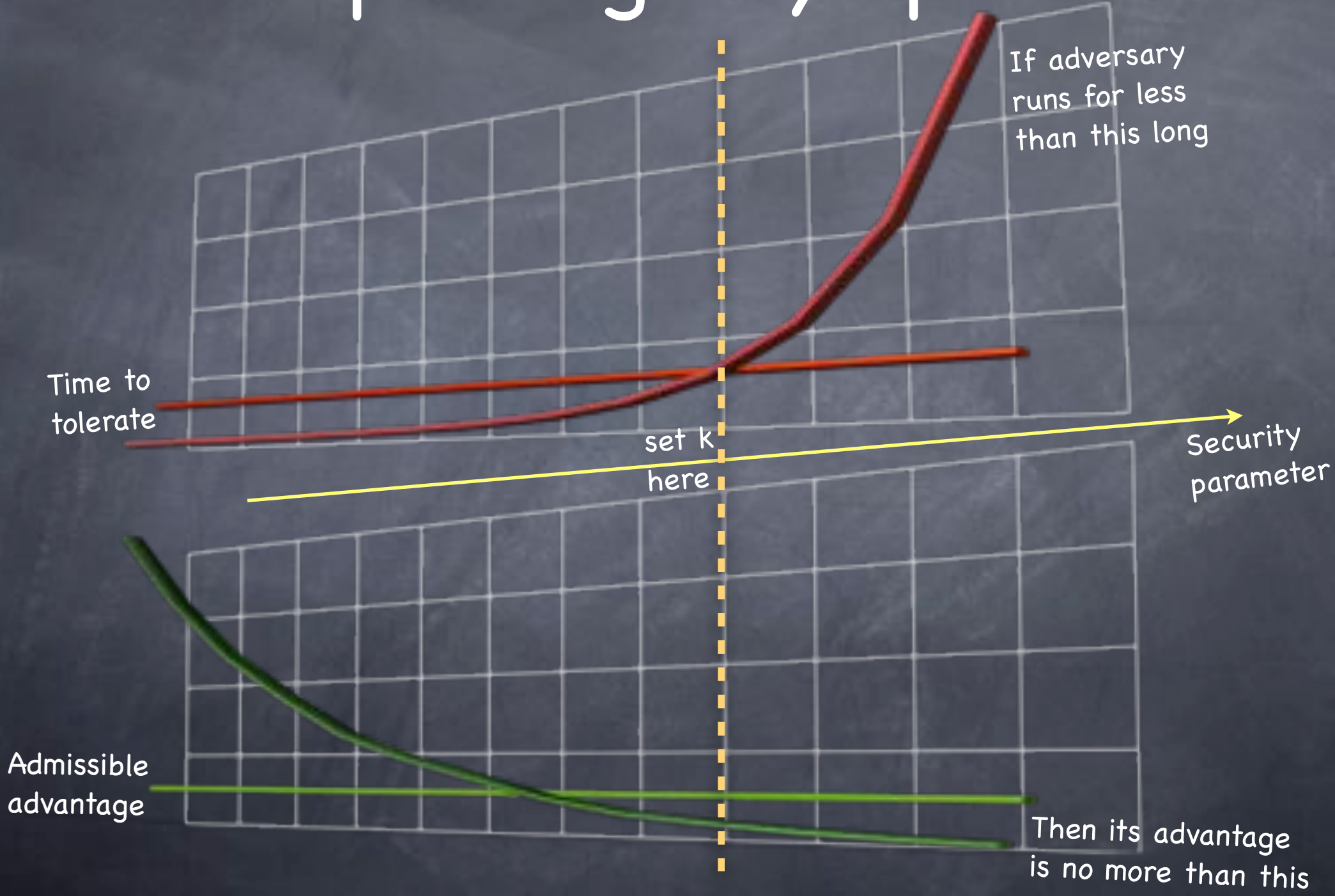

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    - Formally: T negligible if $\forall c>0\ \exists k_0\ \forall k>k_0\ \ T(k) < 1/k^c$

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Negligibly Small

- A negligible quantity: As we turn the knob the quantity should "decrease extremely fast"
  - Negligible: decreases as 1/superpoly(k)
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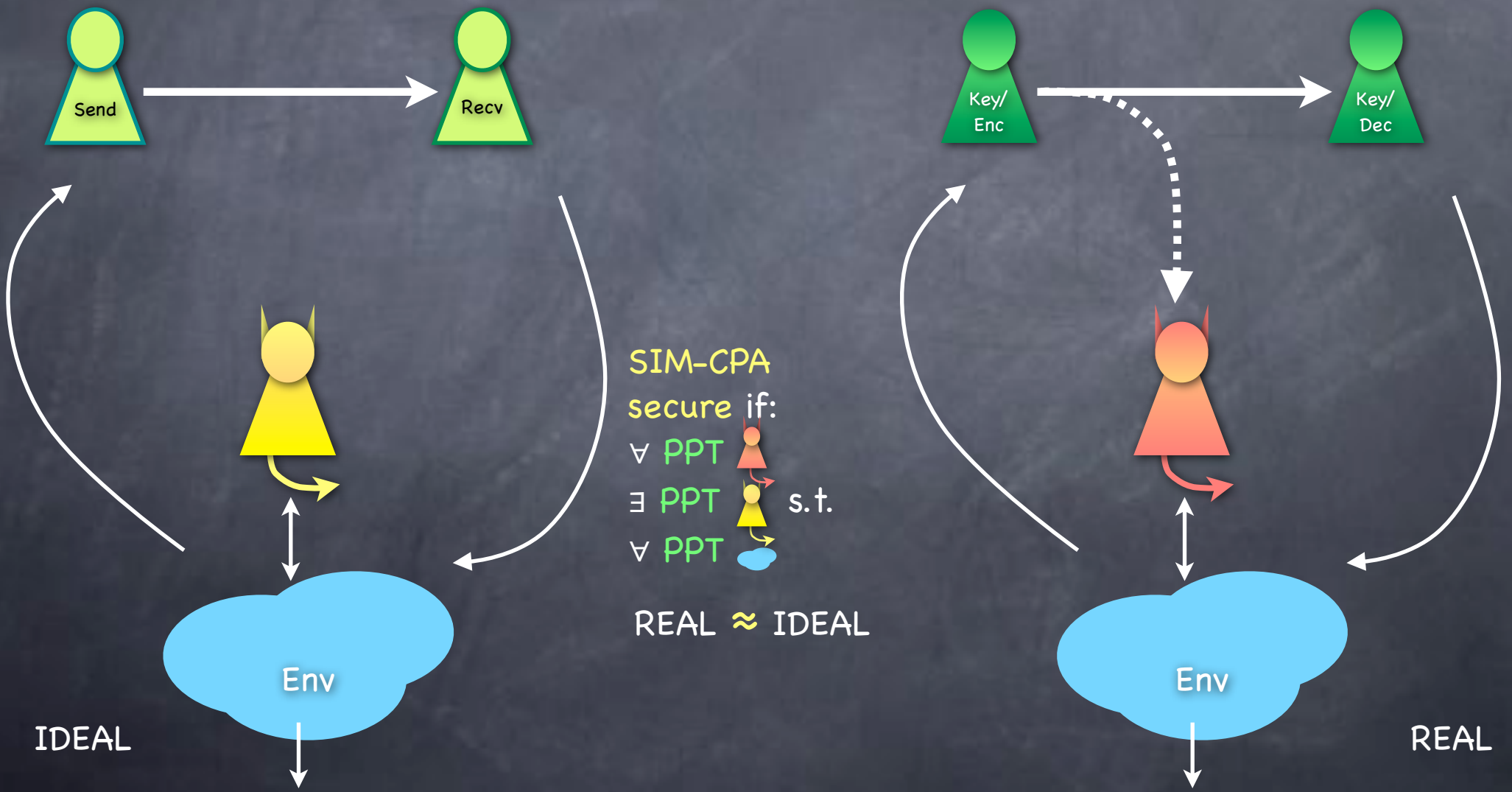
    - Needed, because Eve can often increase advantage polynomially by spending that much more time/by seeing that many more messages

Symmetric-Key Encryption

SIM-CPA Security



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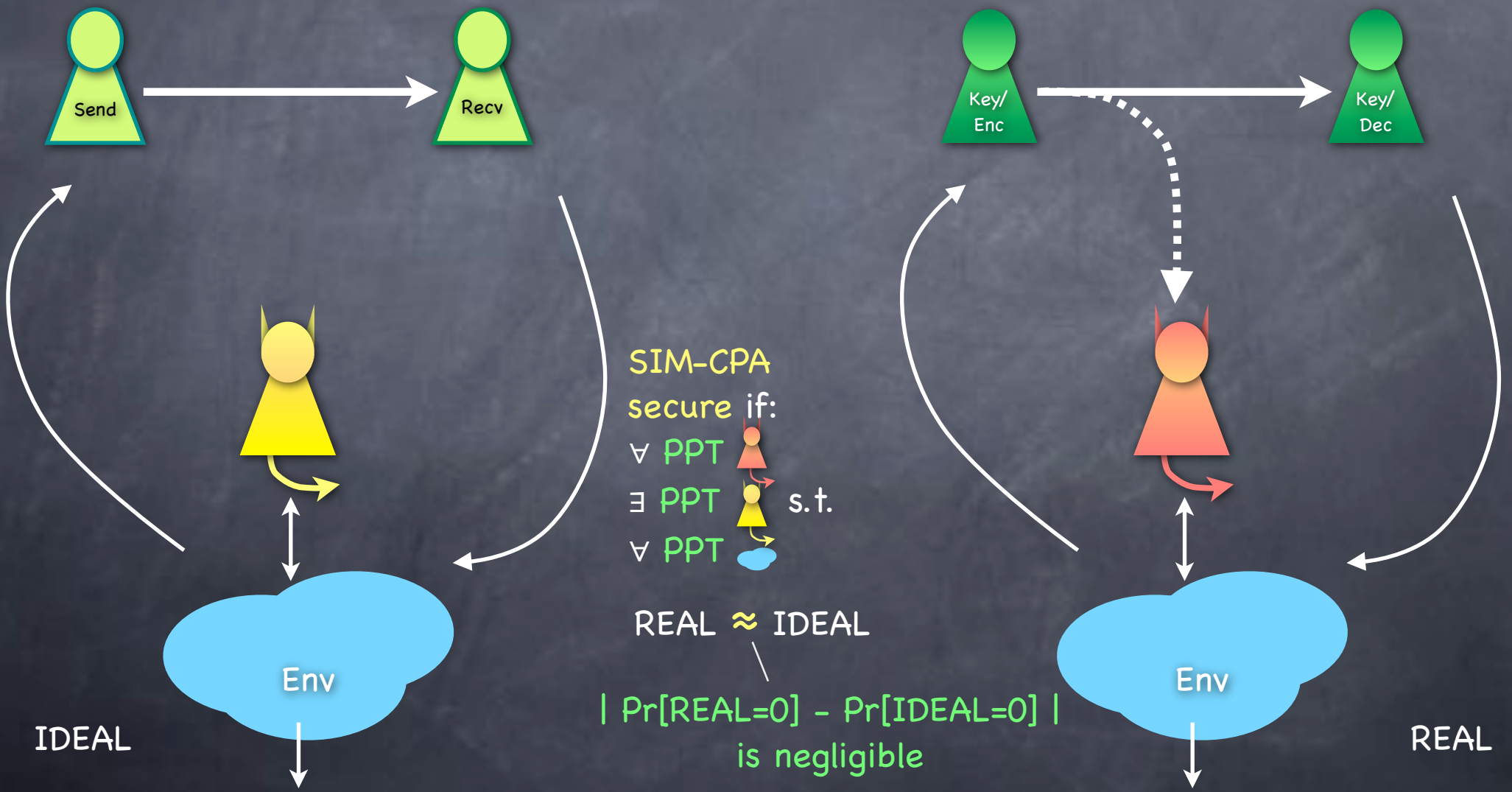

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  - Coming up: One-Way Functions, Hardcore predicates, PRG, ...

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  - Turns out they are equivalent!

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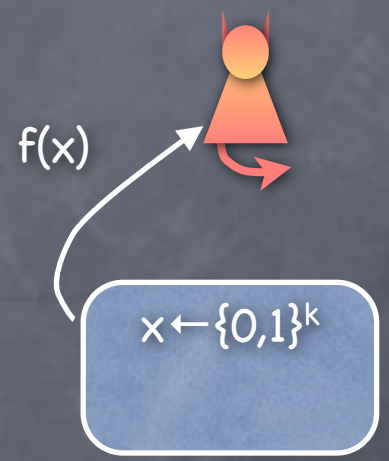

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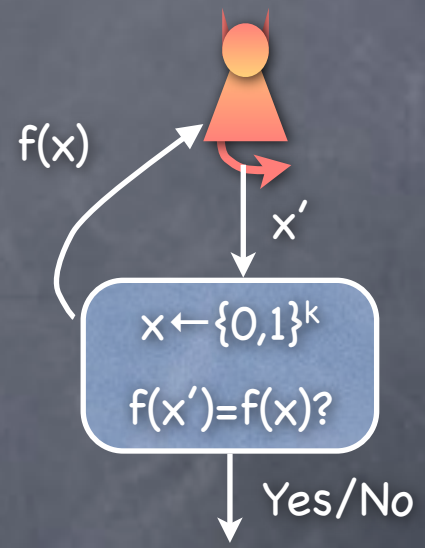

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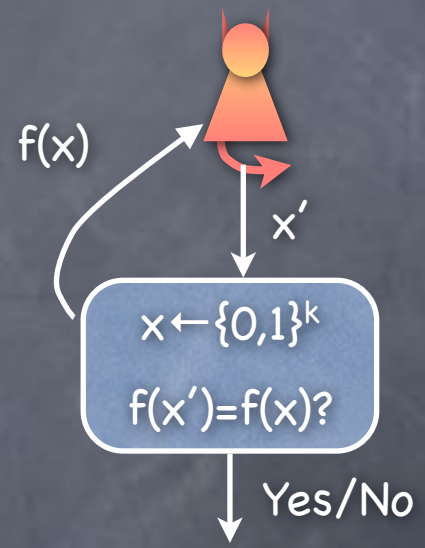

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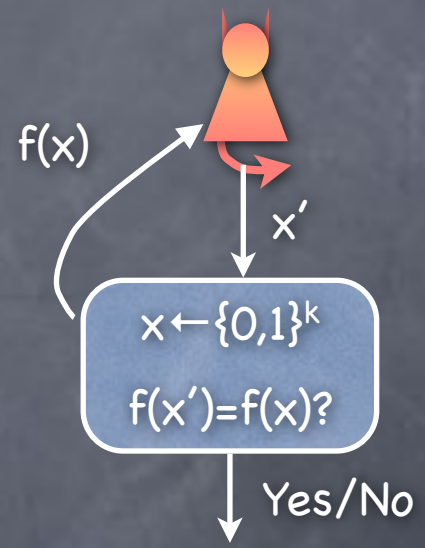

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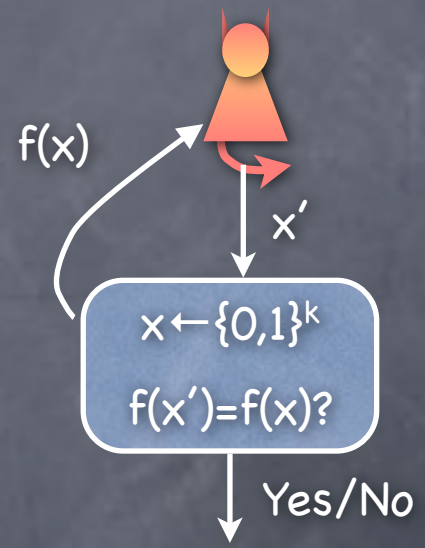

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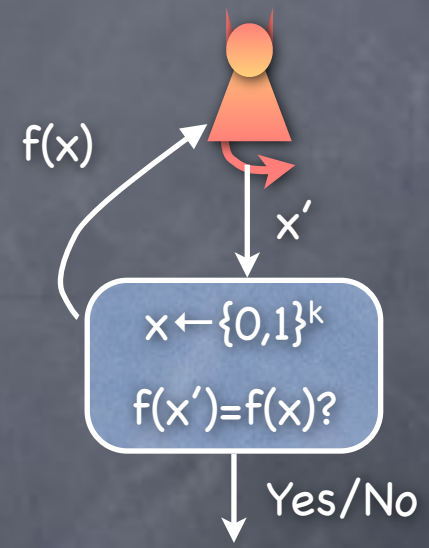

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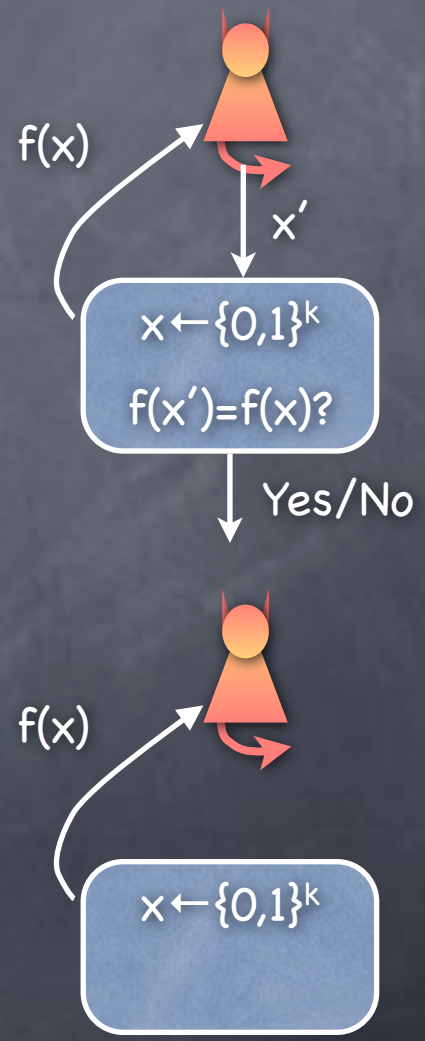

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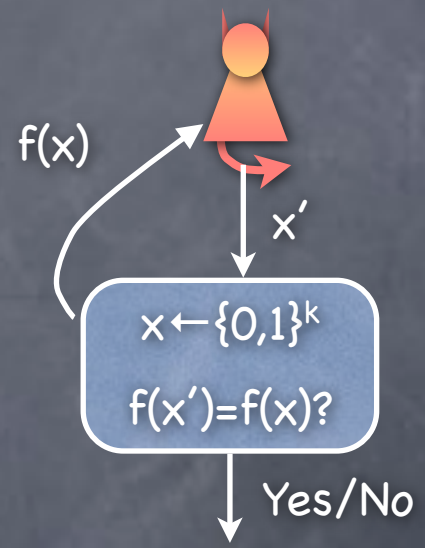

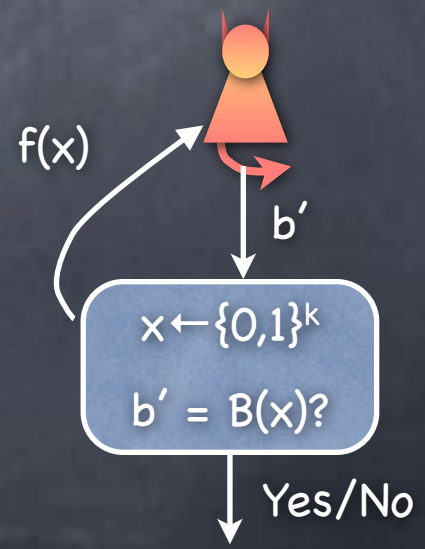

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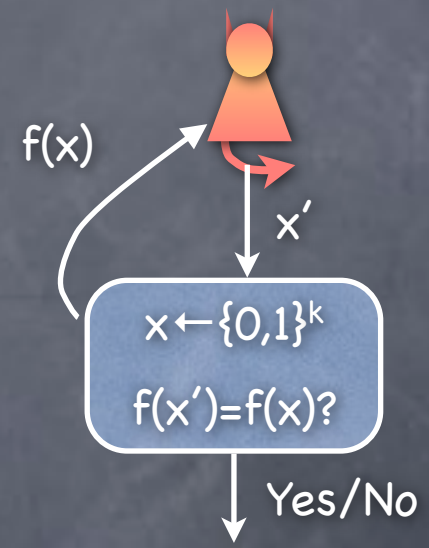
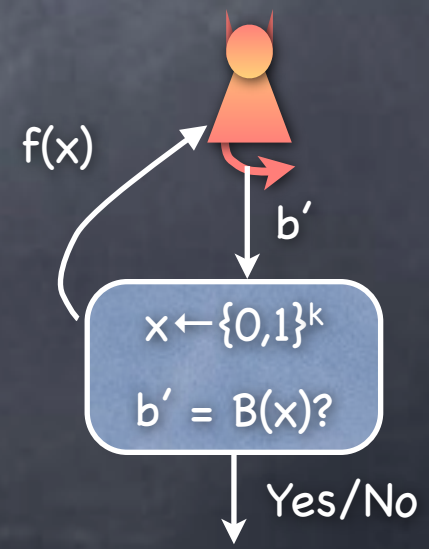
  - B(x) remains "completely" hidden, given f(x)





Next

Next

- Candidate OWFs

Next

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- Using OWF/Hardcore-predicates to build PRG and (CPA-secure) SKE