Defining Encryption

Lecture 2

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Secrecy when Computationally Bounded

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- Today: defining symmetric-key encryption

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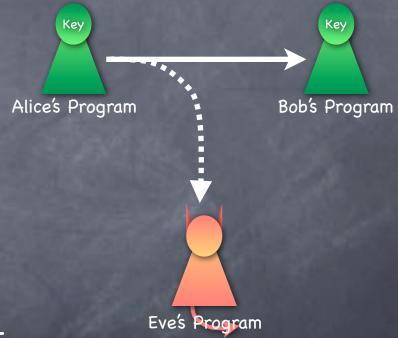




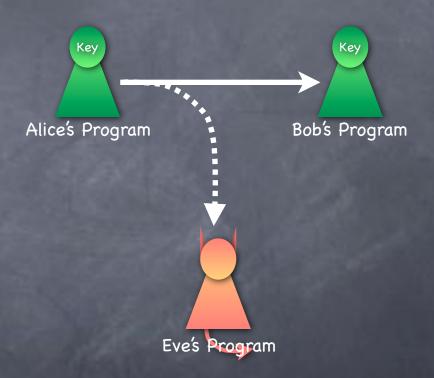
Alice wants Bob to learn a message, "without Eve learning it"



- Alice, Bob and Eve. Alice and Bob share a key (a bit string)
- Alice wants Bob to learn a message, "without Eve learning it"
- Alice can send out a bit string on the channel. Bob and Eve both get it

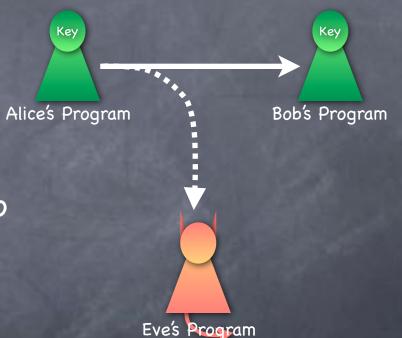


Encryption: Syntax



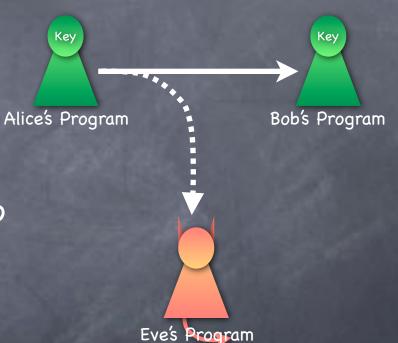
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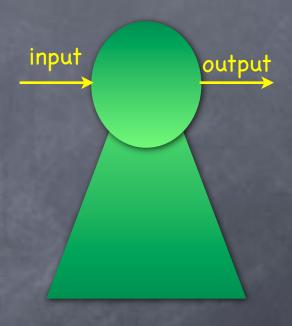
- Three algorithms
 - Key Generation: What Alice and Bob do a priori, for creating the shared secret key
 - Encryption: What Alice does with the message and the key to obtain a "ciphertext"
 - Decryption: What Bob does with the ciphertext and the key to get the message out of it



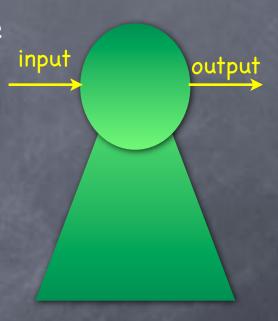
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- All of these are (probabilistic) computations

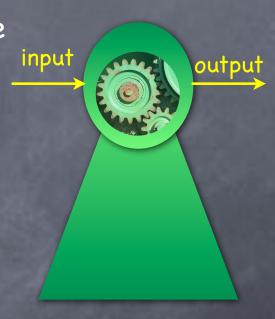




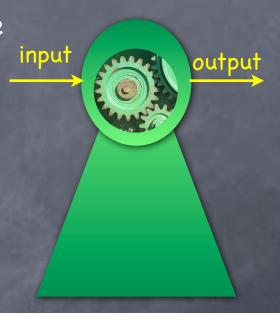
In our model (standard model) parties are programs (computations, say Turing Machines)



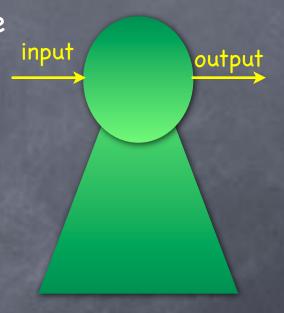
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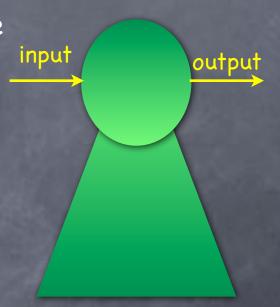
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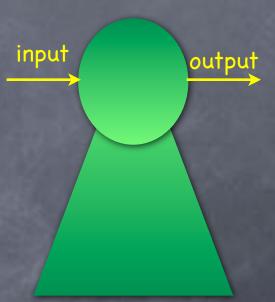
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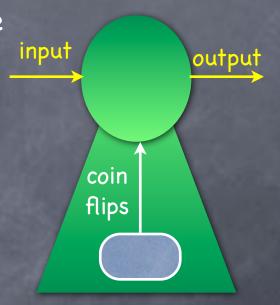
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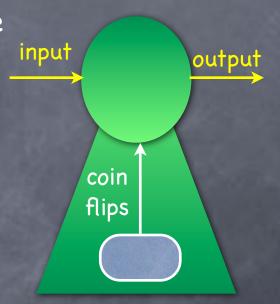
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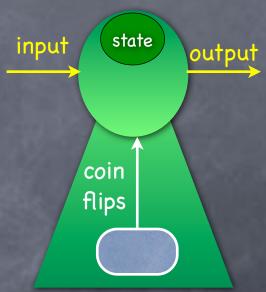


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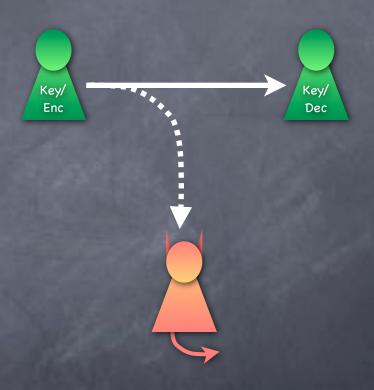


Ideal coin flips: If n coins flipped, each outcome has probability 2⁻ⁿ

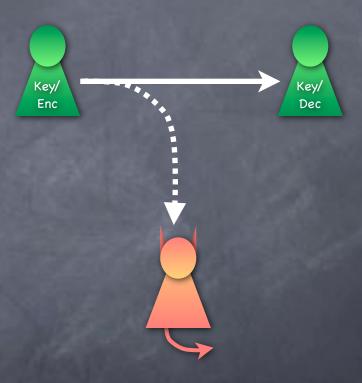
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 - Sometimes stateful



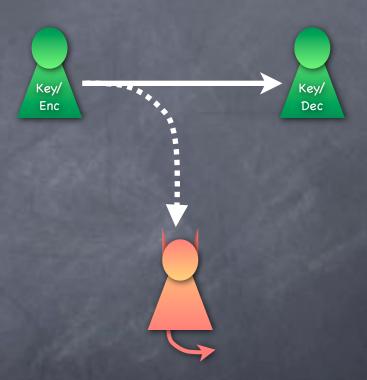
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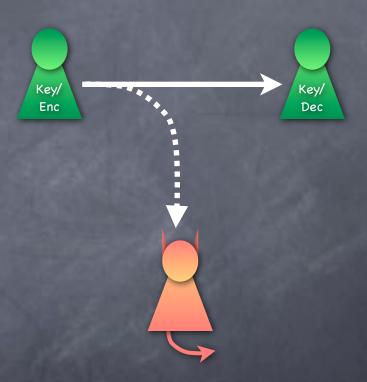
Where does the message come from?



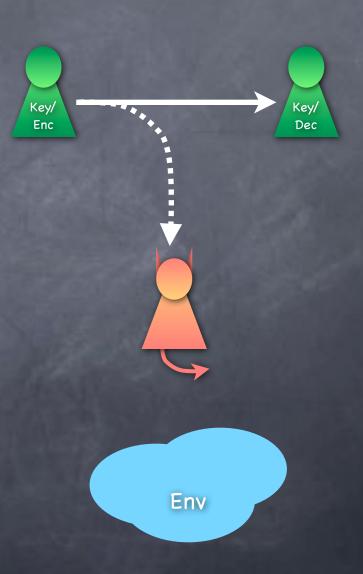
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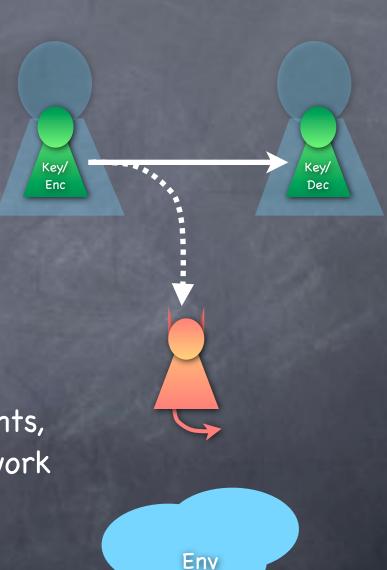
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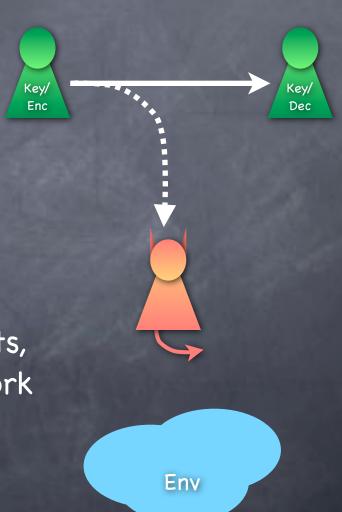
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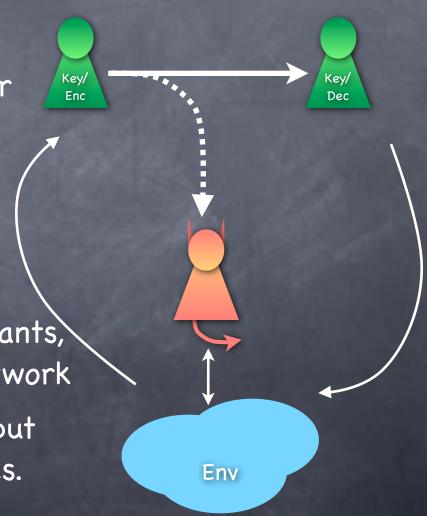
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 - Includes the operating systems and other programs run by the participants, as well as other parties, if in a network

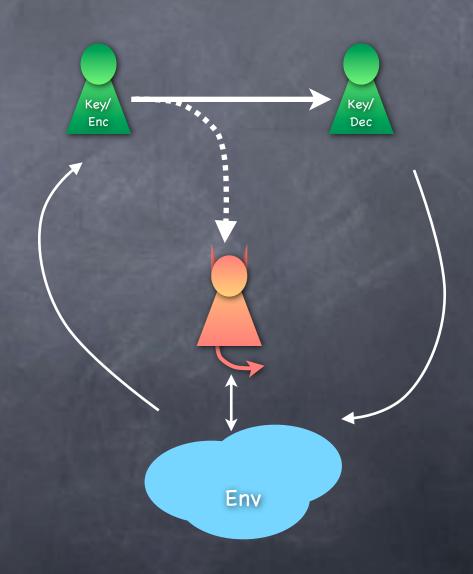


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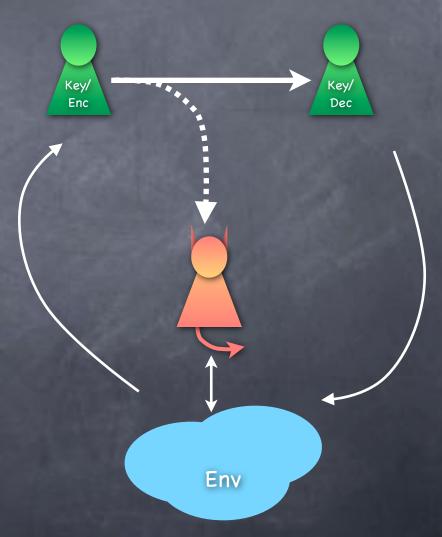


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 - Includes the operating systems and other programs run by the participants, as well as other parties, if in a network
 - Abstract entity from which the input comes and to which the output goes. Arbitrarily influenced by Eve

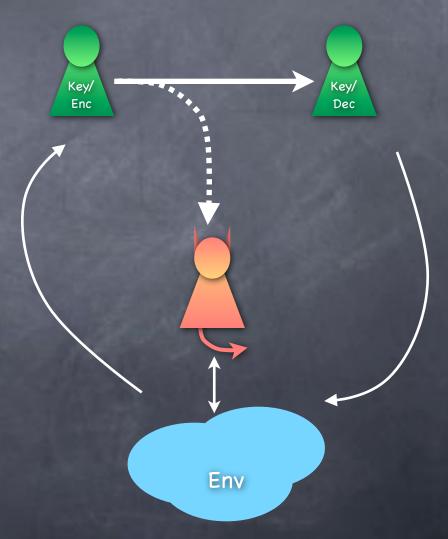




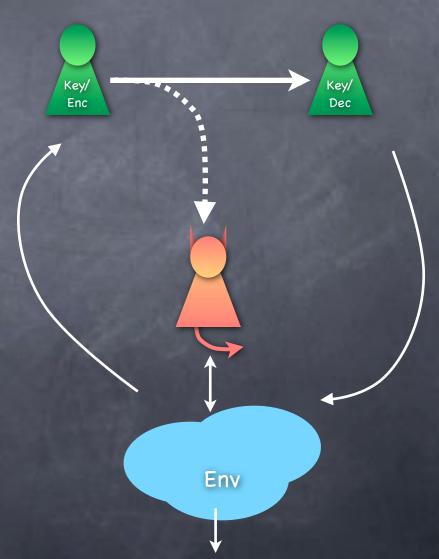
Eve shouldn't be able to produce any "bad effects" in any environment



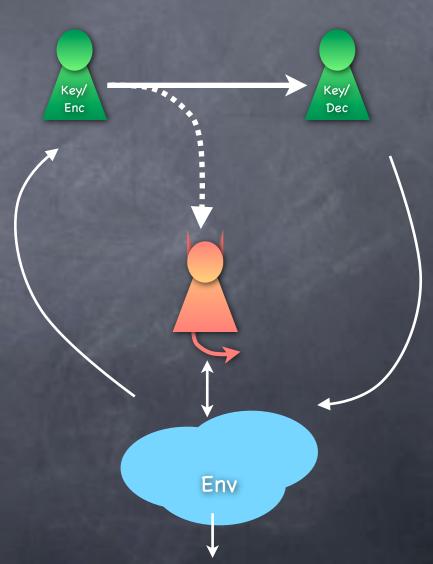
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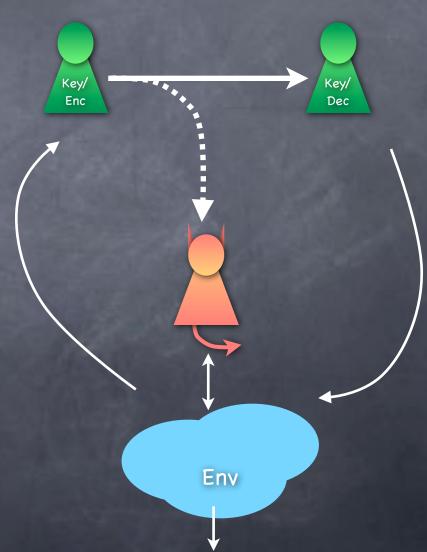
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- Effects in the environment: modeled as a bit in the environment (called the output bit)

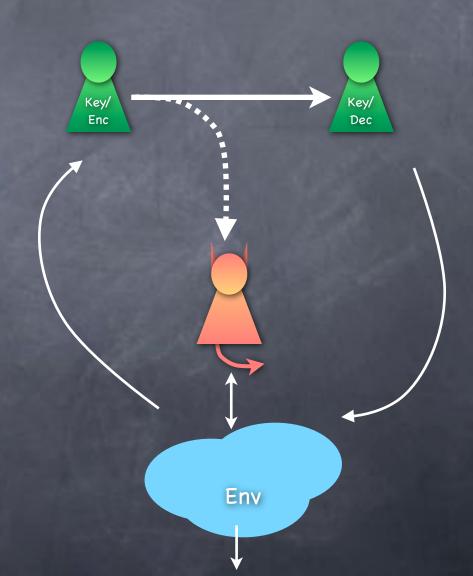


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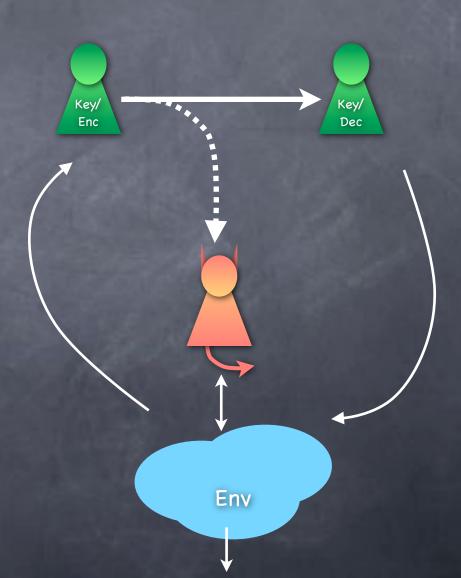
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- What is bad?
 - Anything that Eve couldn't have caused if an "ideal channel" was used





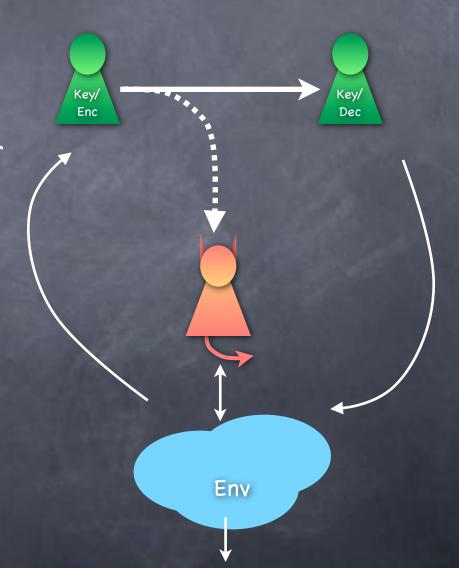
Defining Security The REAL/IDEAL Paradigm

Eve shouldn't produce any more effects than she could have in the ideal world



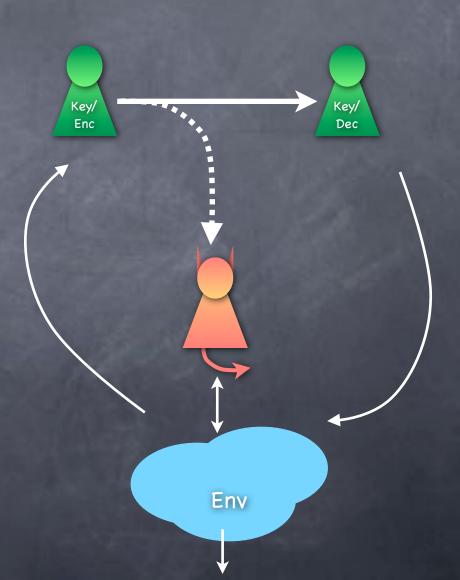
Defining Security The REAL/IDEAL Paradigm

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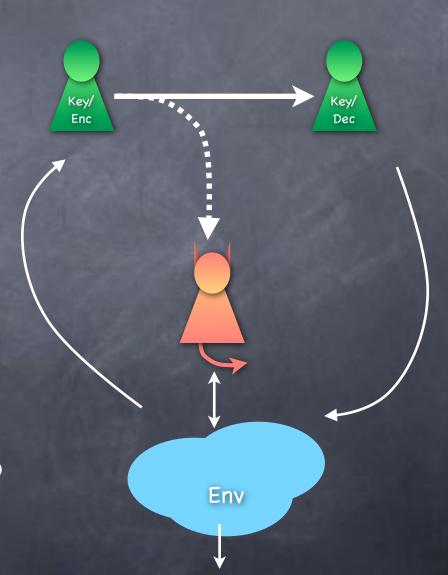


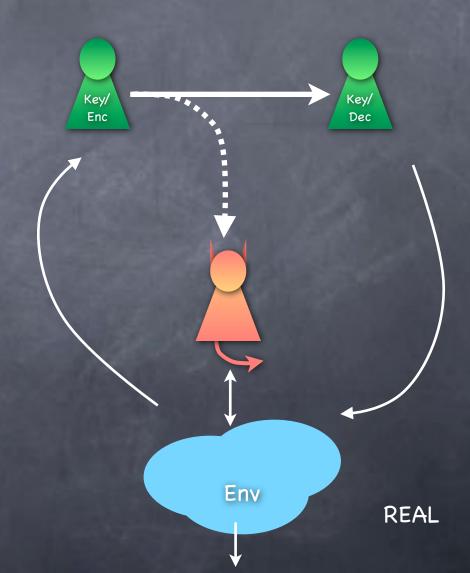
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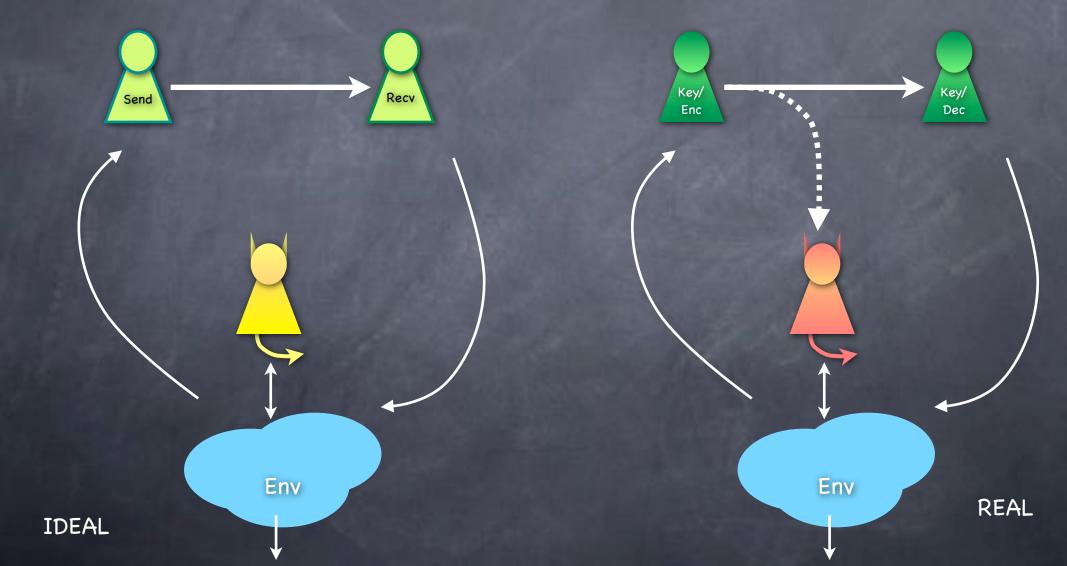
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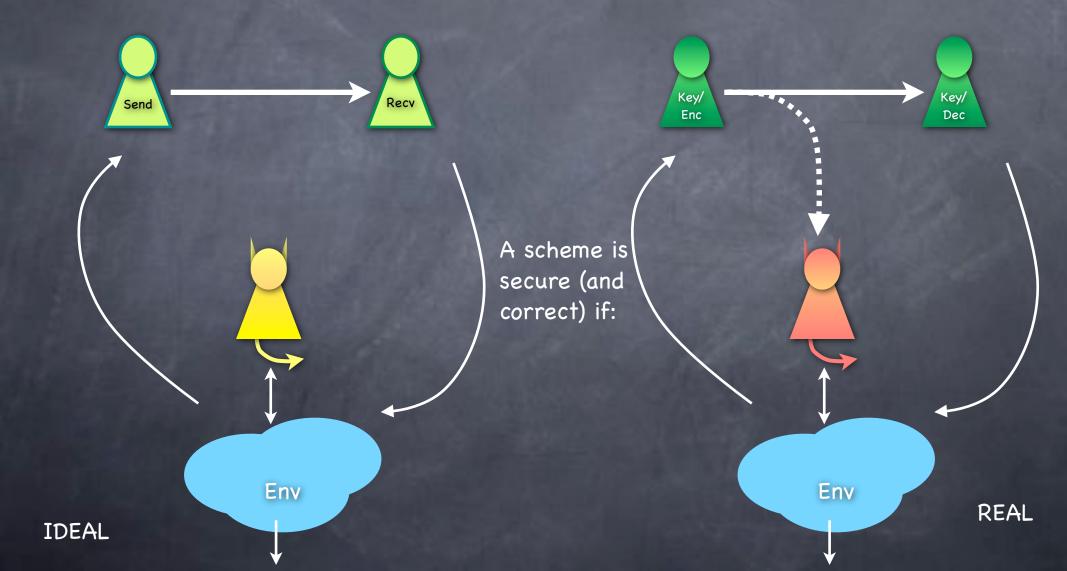


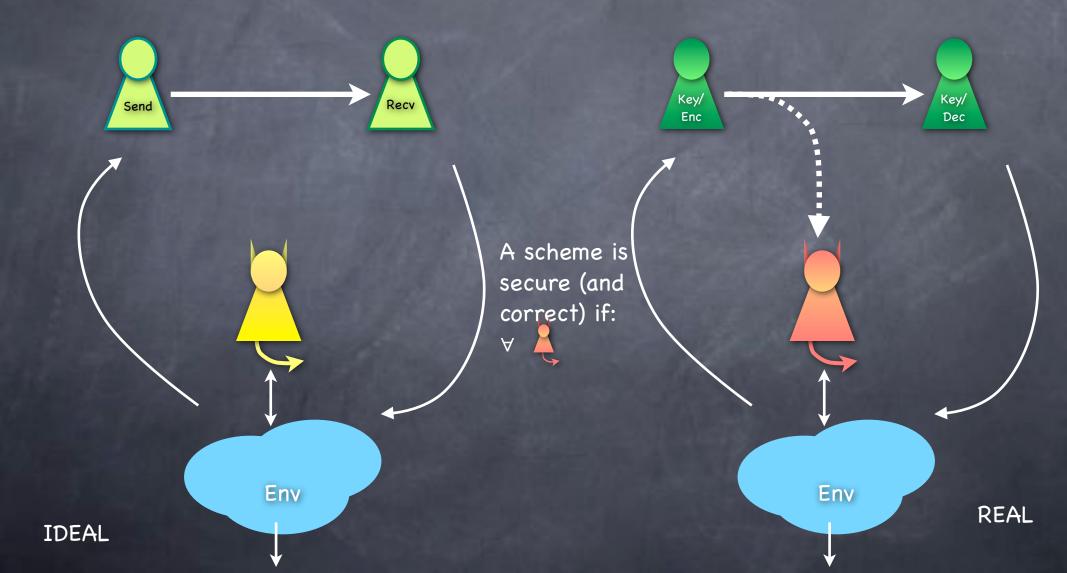
- Eve shouldn't produce any more effects than she could have in the ideal world
 - IDEAL world: Message sent over a (physically) secure channel. No encryption in this world.
 - REAL world: Using encryption
 - Encryption is secure if whatever Eve can do in the REAL world (using some strategy), she can do in the IDEAL world too (using an appropriate strategy)

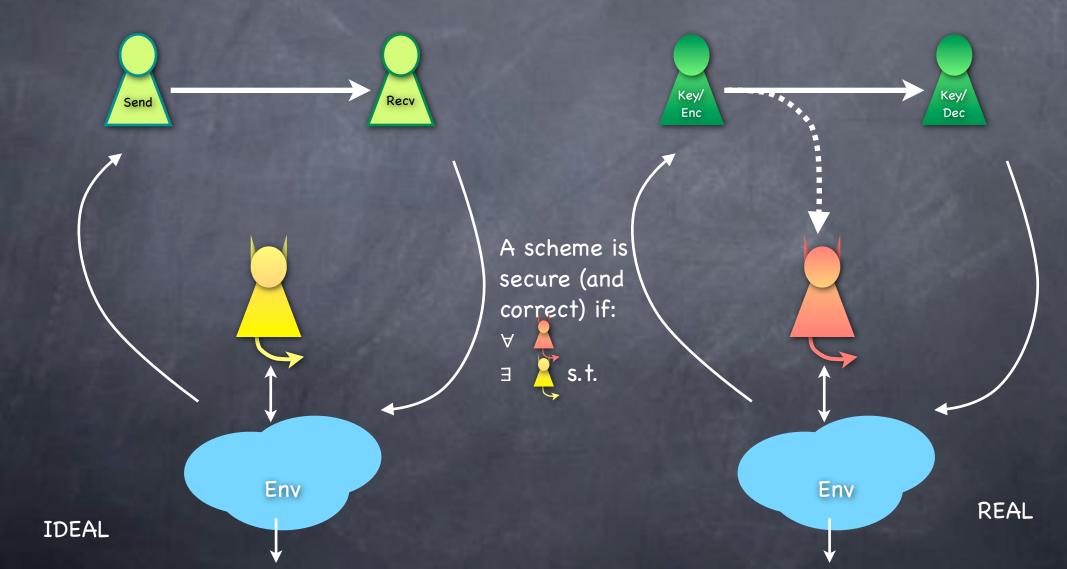


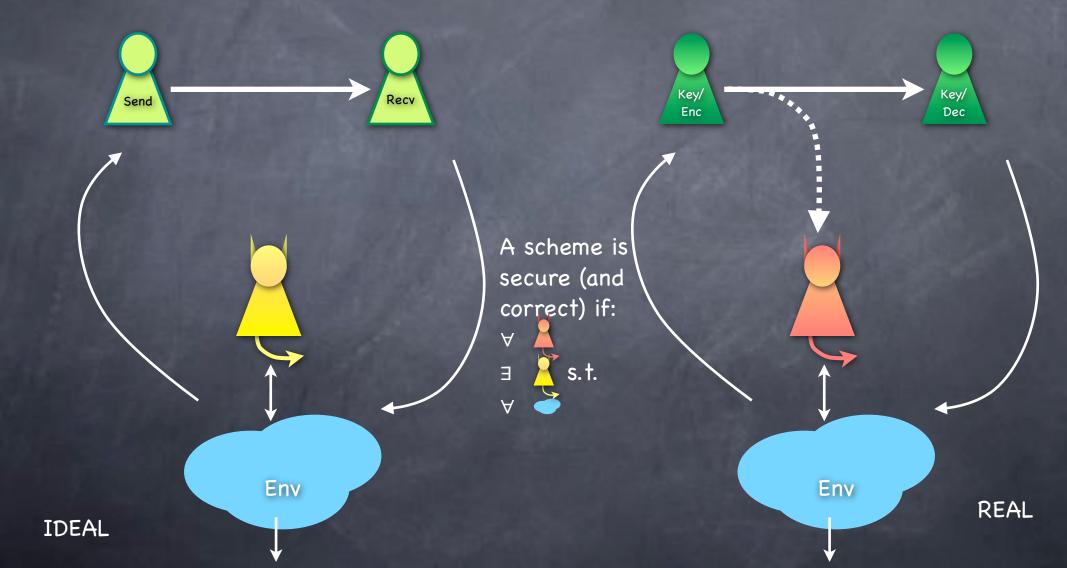


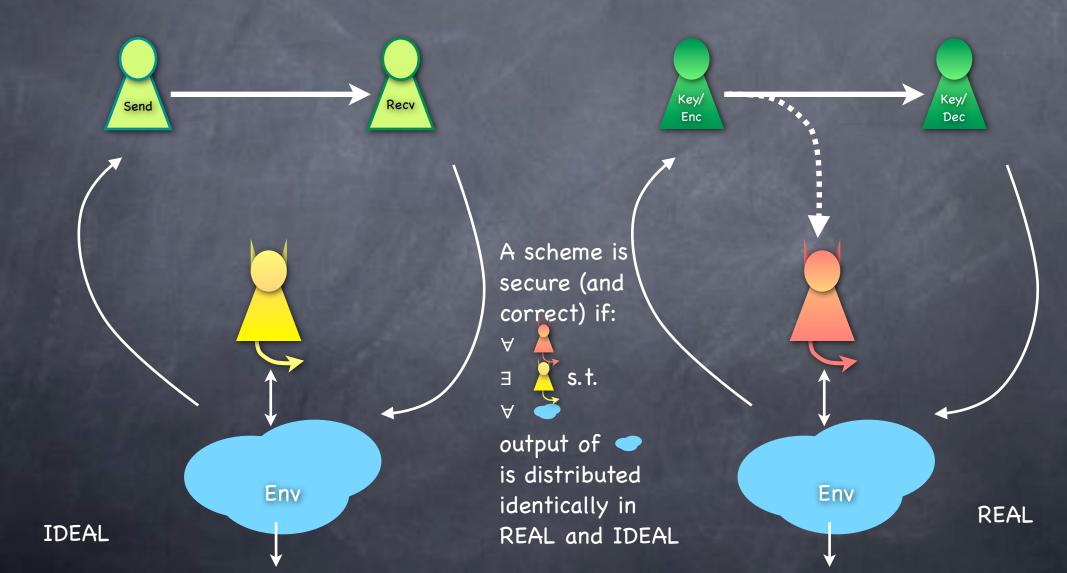












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- REAL/IDEAL (a.k.a simulation-based) security forms the basic template for a large variety of security definitions
- We will see three definitions of symmetric-key encryption
 - Security of "one-time encryption"
 - Security of (muti-message) encryption
 - Security against "active attacks"
- Will also see alternate (but essentially equivalent) security definitions

Onetime Encryption The Syntax

- Shared-key (Private-key) Encryption
 - Key Generation: Randomized
 - $_{\odot}$ K \leftarrow %, uniformly randomly drawn from the key-space (or according to a key-distribution)
 - Encryption: Deterministic
 - Enc: $\mathcal{M} \times \mathcal{H} \rightarrow \mathcal{C}$
 - Decryption: Deterministic
 - \bullet Dec: $C \times \mathcal{H} \rightarrow \mathcal{M}$

Perfect Secrecy



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Distribution of the ciphertext

Perfect Secrecy

```
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```

Distribution of the ciphertext

N M	0	1	2	3
а	X	У	У	Z
b	У	X	Z	У

Perfect Secrecy

- Perfect secrecy: ∀ m, m' ∈ M
- Distribution of the ciphertext is defined by the randomness in the key

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- Perfect secrecy: ∀ m, m' ∈ M
 - {Enc(m,K)}_{K←KeyGen} = {Enc(m',K)}_{K←KeyGen}
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Assuming K uniformly drawn from %

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Assuming K uniformly drawn from \mathcal{K} Pr[Enc(a,K)=x] = $\frac{1}{4}$,

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Same for Enc(b,K).

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- Perfect secrecy: ∀ m, m' ∈ M
- Distribution of the ciphertext is defined by the randomness in the key
- In addition, require correctness
 - ∀ m, K, Dec(Enc(m,K), K) = m

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Perfect Secrecy

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 - Ø ∀ m, K, Dec(Enc(m,K), K) = m
- E.g. One-time pad: $\mathcal{M} = \mathcal{K} = \mathcal{C} = \{0,1\}^n$ and $Enc(m,K) = m \oplus K$, $Dec(c,K) = c \oplus K$

M K	0	1	2	3
a	X	У	У	Z
b	У	X	Z	У

Assuming K uniformly drawn from %Pr[Enc(a,K)=x] = $\frac{1}{4}$,

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 - More generally $\mathcal{M} = \mathcal{K} = \mathcal{C} = \mathcal{C}$ (a finite group) and Enc(m,K) = m+K, Dec(c,K) = c-K

M K	0	1	2	3
a	X	У	У	Z
Ь	У	X	Z	У

Assuming K uniformly drawn from K

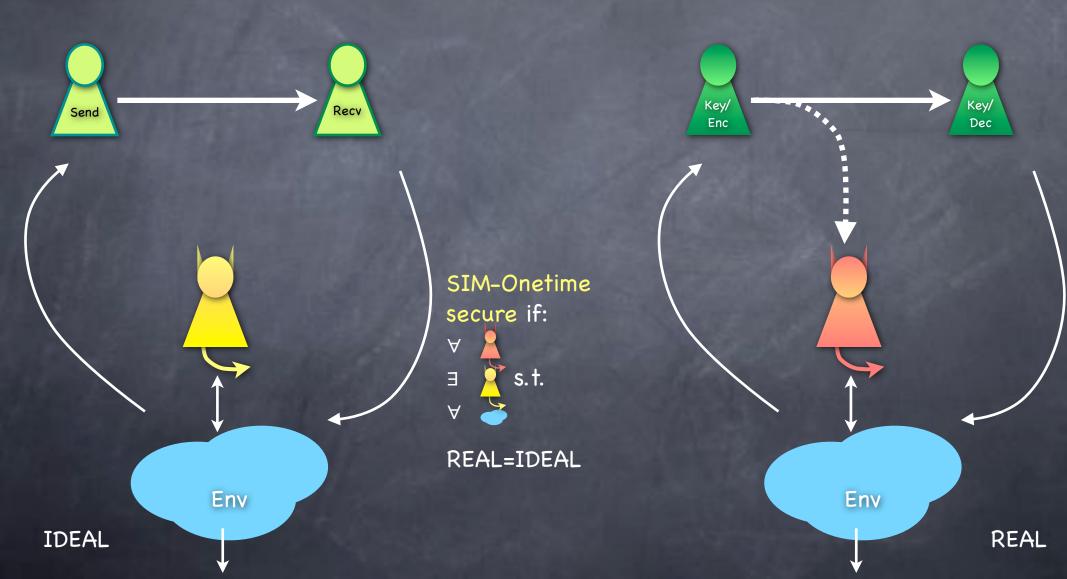
Pr[Enc(a,K)=x] = ½,

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Pr[Enc(a,K)=z] = ¼

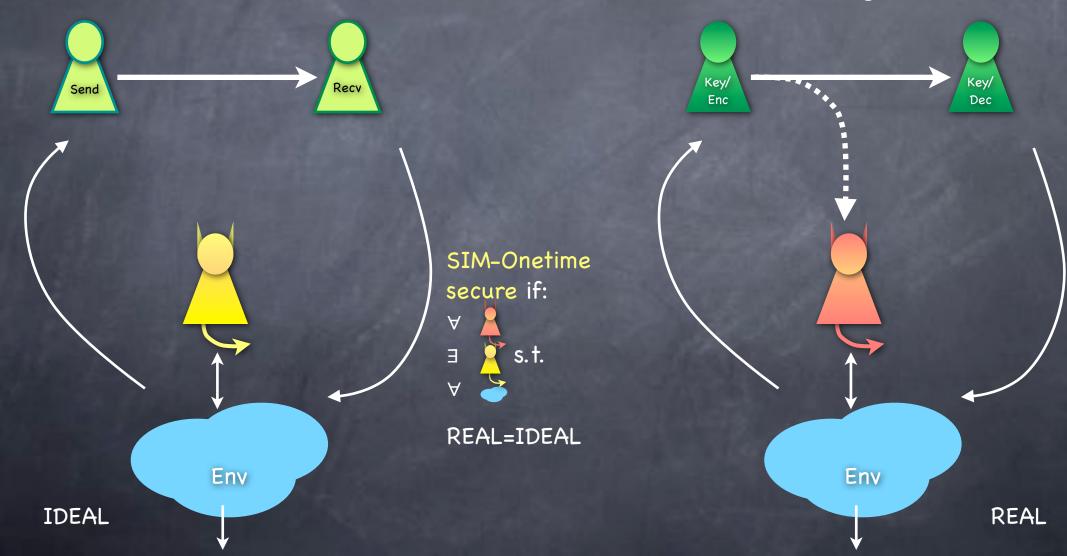
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SIM-Onetime Security



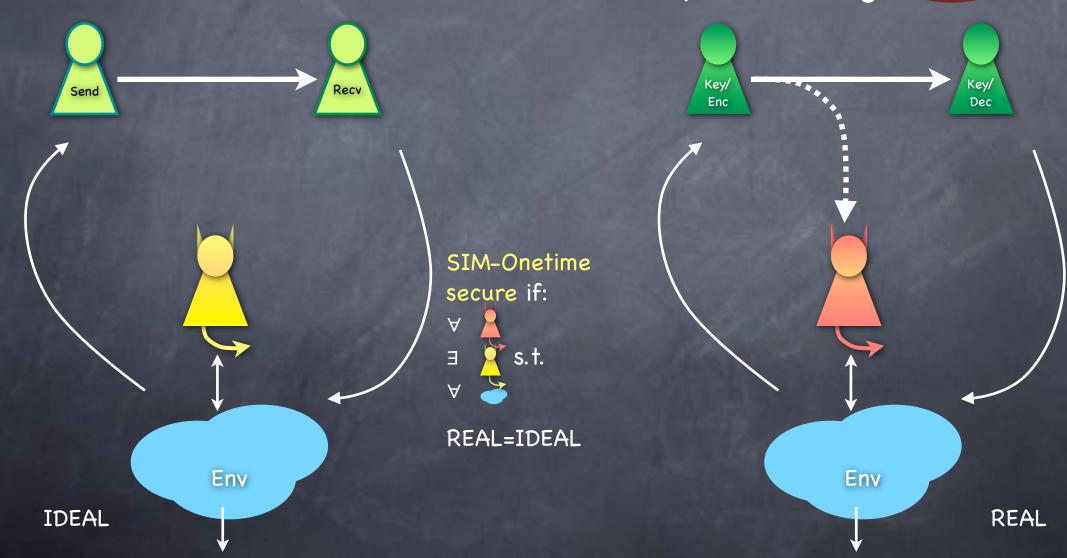
Onetime Encryption SIM-Onetime Security

Class of environments which send only one message

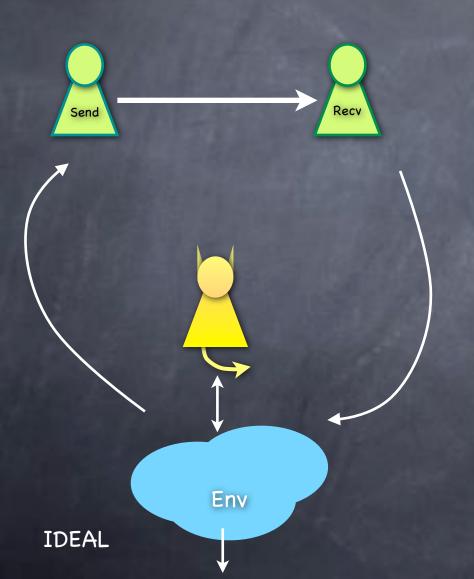


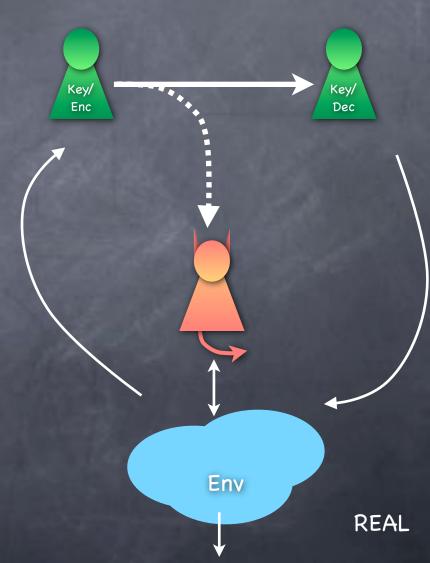
Onetime Encryption Equivalent to SIM-Onetime Security Equivalent to perfect secrecy + correctness

Class of environments which send only one message

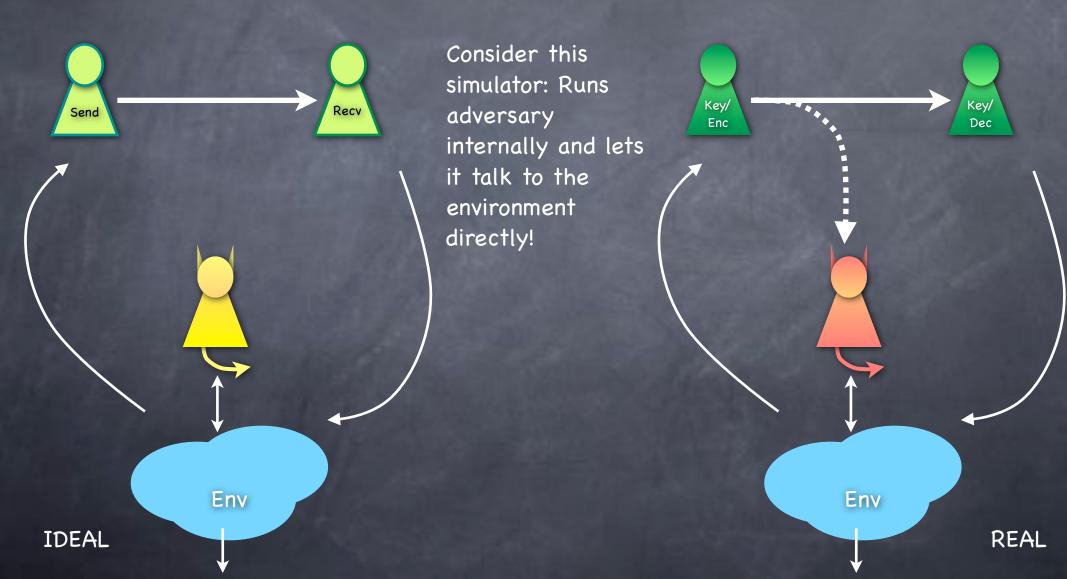


Perfect Secrecy + Correctness ⇒ SIM-Onetime Security

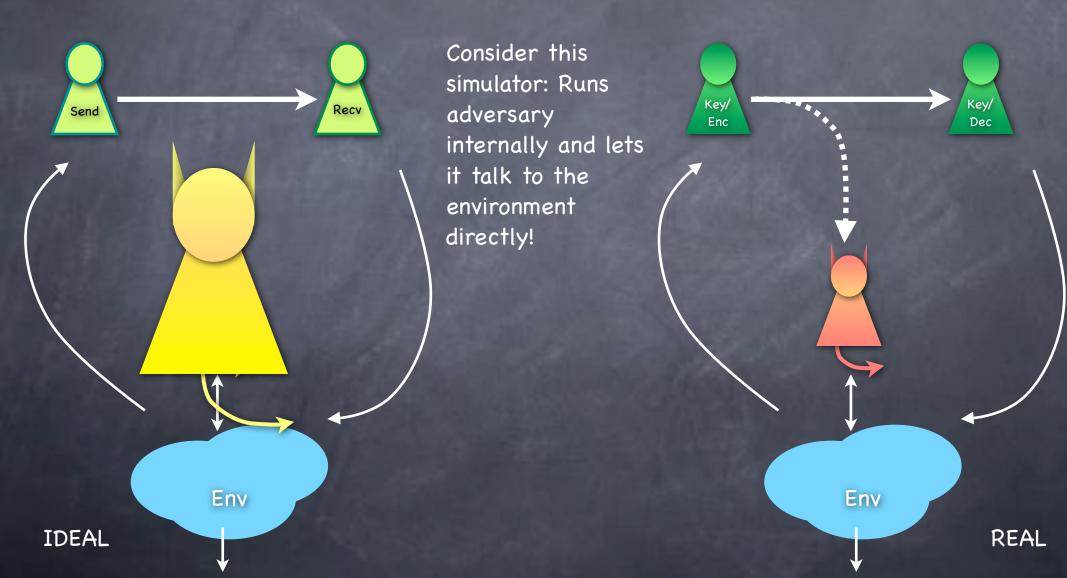




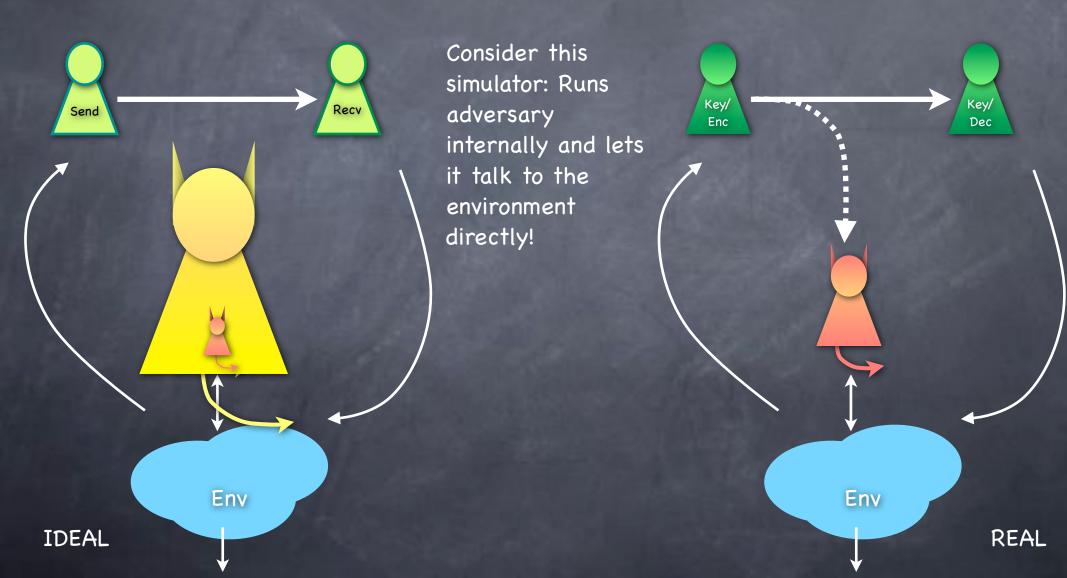
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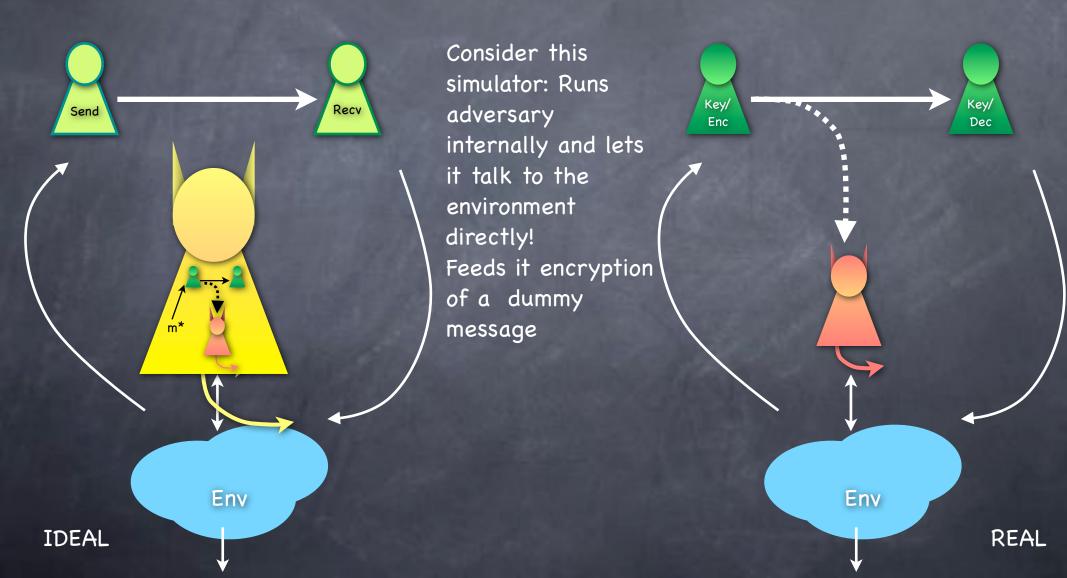
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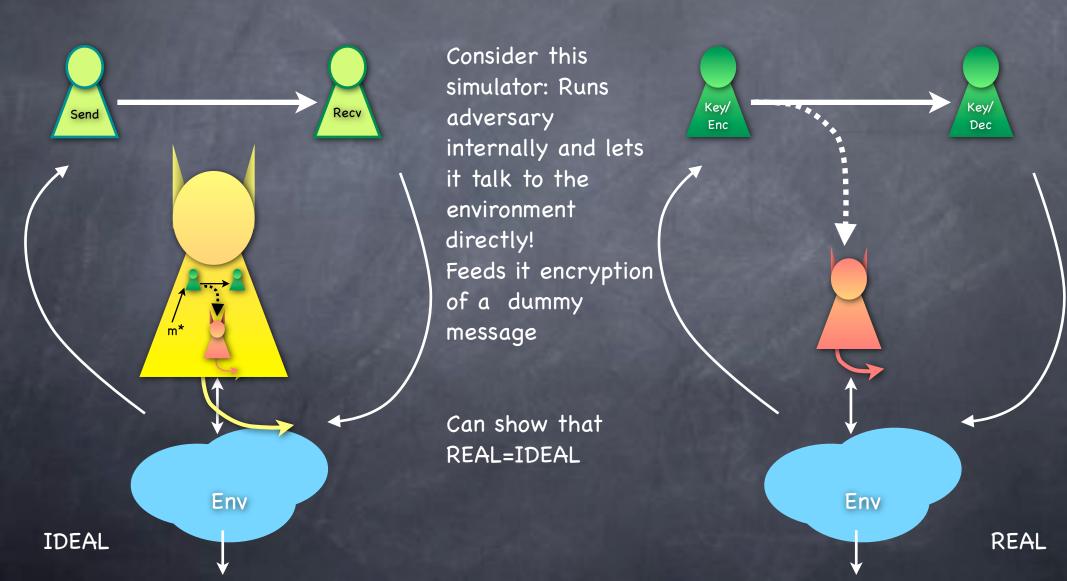
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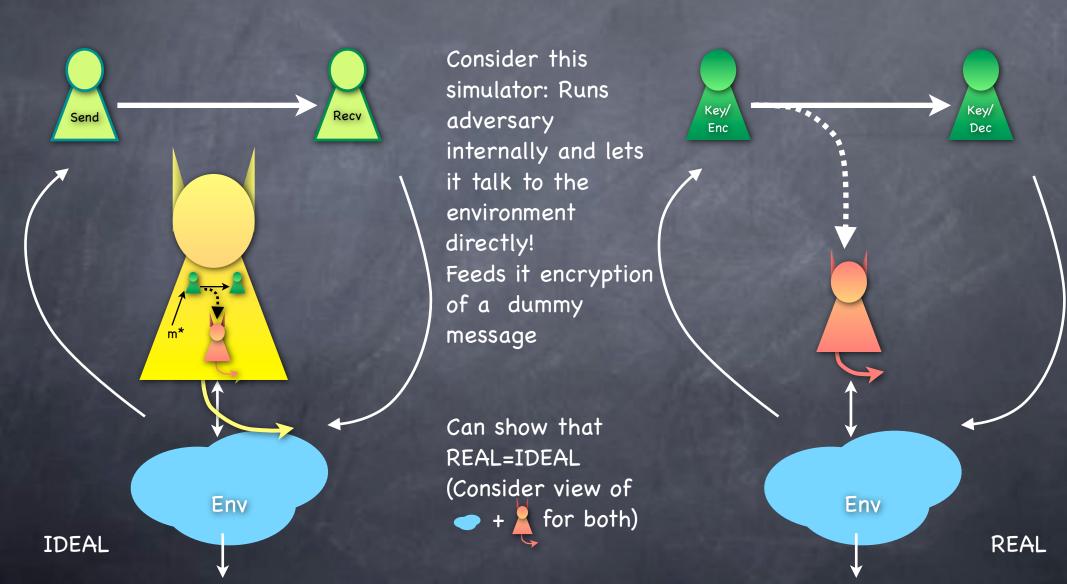
Perfect Secrecy + Correctness ⇒ SIM-Onetime Security



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Random coins used by the encryption scheme is kept private within the programs of the scheme (KeyGen, Enc, Dec)

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 - Also, Eve' allowed to learn the fact that a message is sent

IND-Onetime Security

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IND-Onetime Experiment

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IND-Onetime Experiment





Onetime Encryption IND-Onetime Security

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IND-Onetime Security

IND-Onetime Experiment



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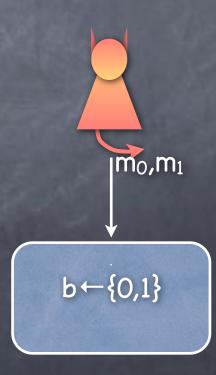


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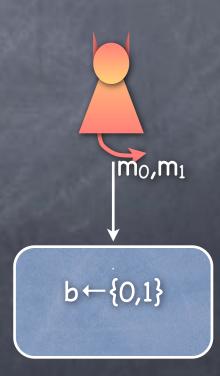




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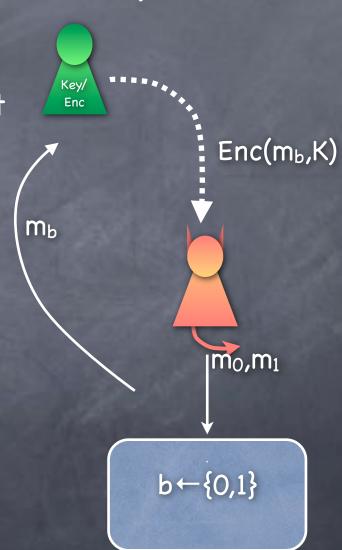
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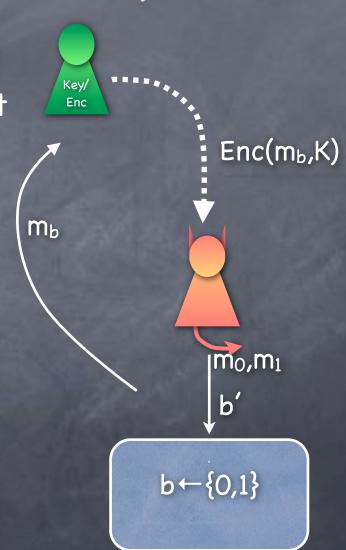


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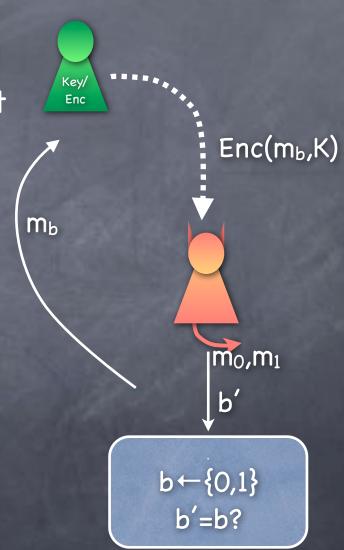
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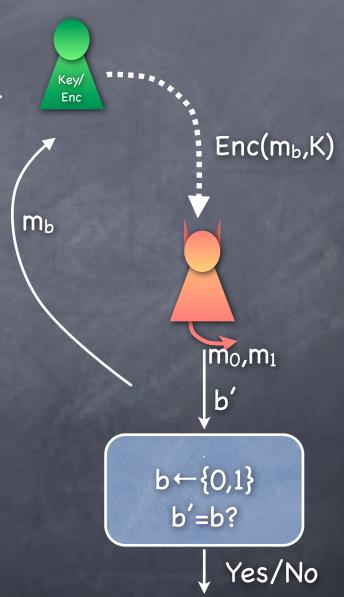
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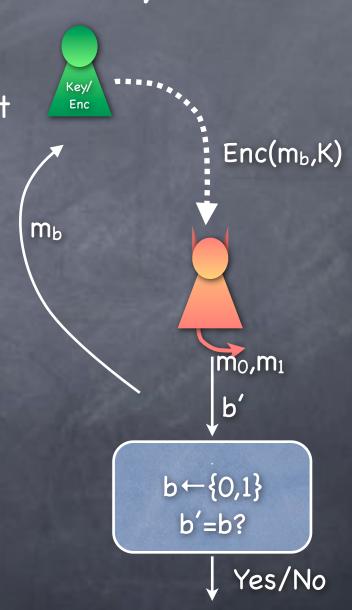
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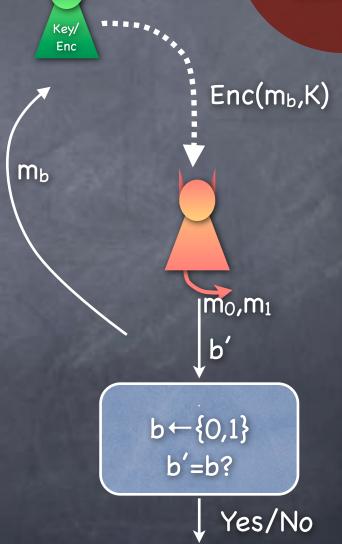
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Equivalent to perfect secrecy



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- IND- definitions tend to be technical: more low-level details, but may not make the big picture clear. Could have "weaknesses"
- SIM- definitions give the big picture, but may not give details of what is involved in satisfying it. Could be "too strong"
- Best of both worlds when they are equivalent: use IND- definition while say, proving security of a construction; use SIM- definition when low-level details are not important