Applied Cryptography

Lecture 1

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Our first encounter with secrecy:
Secret-Sharing

Secrecy

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  - Access to learning and/or influencing information

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  - Access to learning and/or influencing information
- One of the aspects of access control is secrecy

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- Other ideas?

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- Note: any one share can be chosen before knowing the message [why?]

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    - i.e., for all possible values of the message, the view is distributed the same way

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    - Leakage resilience ...

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    - our previous example: (2,2) secret-sharing

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Additive Secret-Sharing

PROOF

Additive Secret-Sharing: Proof

- Share(M):
  - Pick $s_1,\ldots,s_{n-1}$ uniformly at random from G
  - Let $s_n = M - (s_1 + \ldots + s_{n-1})$
- Reconstruct($s_1,\ldots,s_n$): $M = s_1 + \ldots + s_n$
- Claim: Upto n-1 shares give no information about M
- Proof: Let $T \subseteq \{1,\ldots,n\}$, $|T| = n-1$. We shall show that $\{ s_i \}_{i\in T}$ is distributed the same way (in fact, uniformly) irrespective of what M is.
  - For concreteness consider $T = \{2,\ldots,n\}$. Fix any (n-1)-tuple of elements in G, $(g_1,\ldots,g_{n-1}) \in G^{n-1}$. To prove $\Pr[(s_2,\ldots,s_n)=(g_1,\ldots,g_{n-1})]$ is independent of M.
  - Fix any M.
  - $(s_2,\ldots,s_n) = (g_1,\ldots,g_{n-1}) \Leftrightarrow (s_2,\ldots,s_{n-1}) = (g_1,\ldots,g_{n-2})$ and $s_1 = M-(g_1+\ldots+g_{n-1})$.
  - So $\Pr[(s_2,\ldots,s_n)=(g_1,\ldots,g_{n-1})] = \Pr[(s_1,\ldots,s_{n-1})=(M-(g_1+\ldots+g_{n-1}), g_1,\ldots,g_{n-2})]$
  - But $\Pr[(s_1,\ldots,s_{n-1})=(M-(g_1+\ldots+g_{n-1}), g_1,\ldots,g_{n-2})] = 1/|G|^{n-1}$, since $(s_1,\ldots,s_{n-1})$ are picked uniformly at random
  - Hence $\Pr[(s_2,\ldots,s_n)=(g_1,\ldots,g_{n-1})] = 1/|G|^{n-1}$, irrespective of M. □

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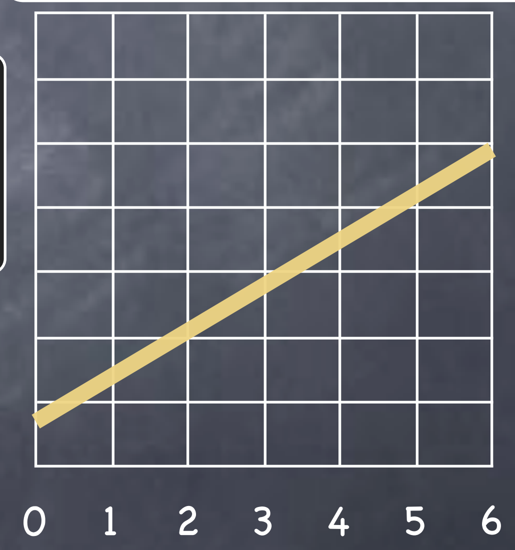

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    - Sharing picks a random "line" $y = f(x)$, such that $f(0)=M$. Shares $s_i = f(i)$.

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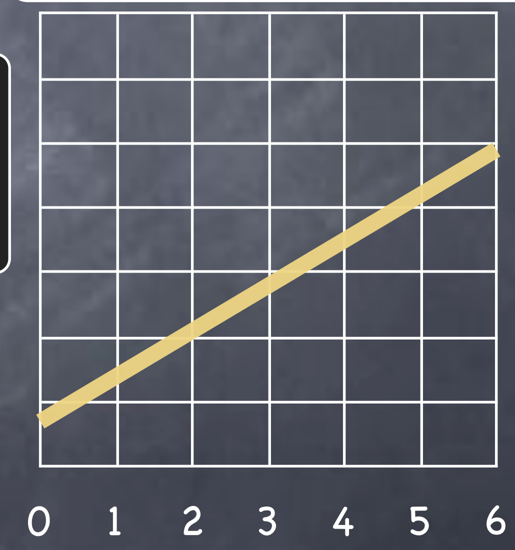

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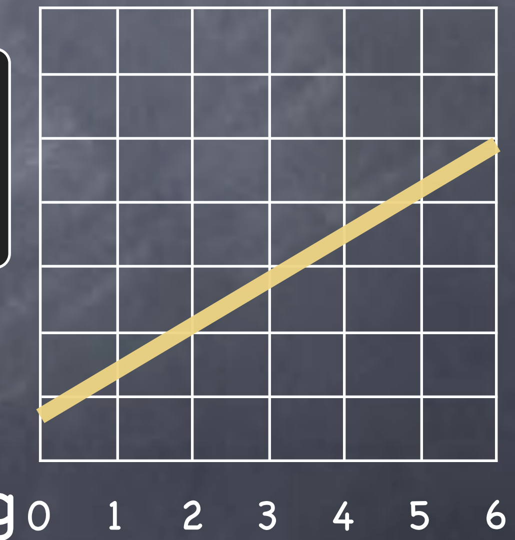

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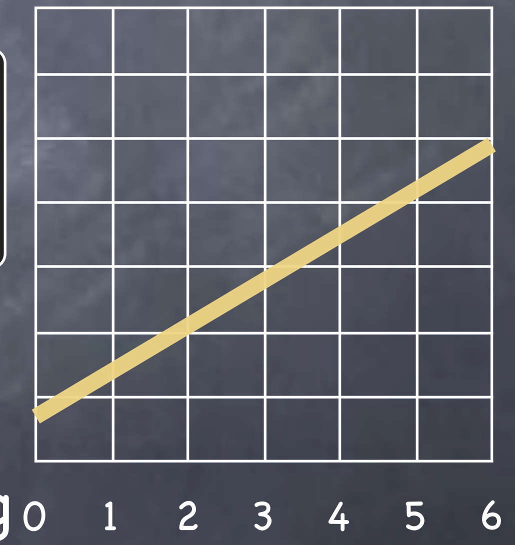

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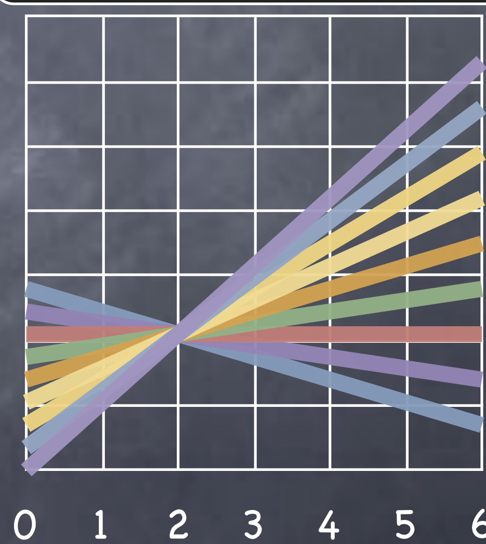

PROOF

(n,2) Secret-Sharing: Proof

- Share(M): pick random $r \leftarrow F$. Let $s_i = r \cdot i + M$ (for $i=1,\ldots,n < |F|$)
- Reconstruct(s_i, s_j): $r = (s_i - s_j)/(i-j)$; $M = s_i - r\,i$
- Claim: Any one share gives no information about M
- Proof: For any $i \in \{1,\ldots,n\}$ we shall show that s_i is distributed the same way (in fact, uniformly) irrespective of what M is.
- Consider any $g \in F$. We shall show that $\Pr[\ s_i = g\ ]$ is independent of M.
- Fix any M.
- For any $g \in F$, $s_i = g \Leftrightarrow r \cdot i + M = g \Leftrightarrow r = (g-M) \cdot i^{-1}$ (since $i \neq 0$)
- So, $\Pr[\ s_i = g\ ] = \Pr[\ r = (g-M) \cdot i^{-1}\ ] = 1/|F|$, since r is chosen uniformly at random □

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    - Reconstruct($s_1,\ldots,s_t$): Lagrange interpolation to find $M=c_0$
      - Need t points to reconstruct the polynomial. Given t-1 points, there is exactly one polynomial passing through (0,M') for each M'

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- Shamir's secret-sharing solves threshold secret-sharing. How about the others?

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  - More efficient schemes known for large classes of access structures

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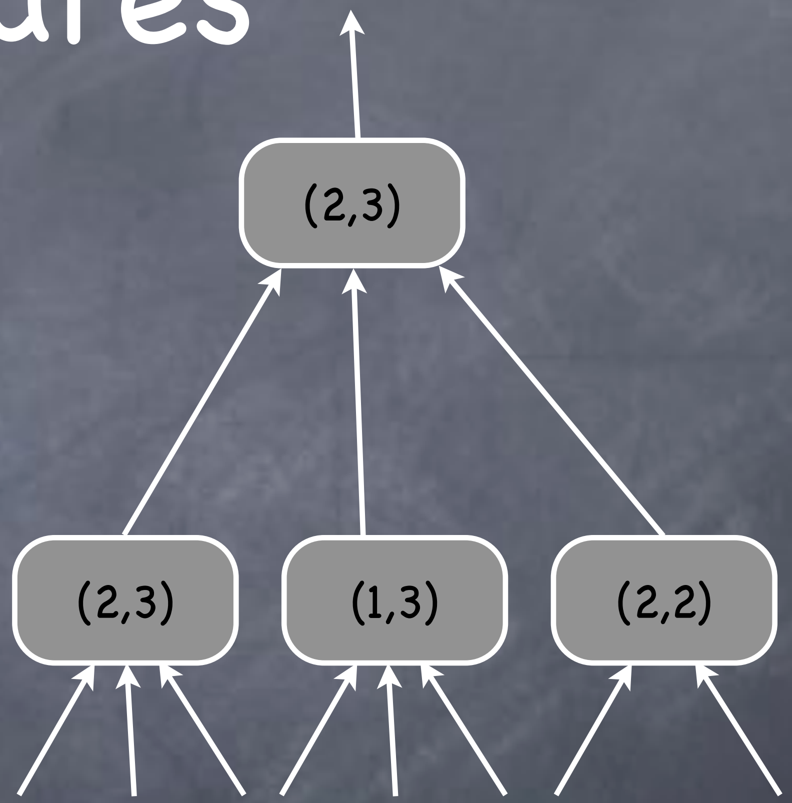

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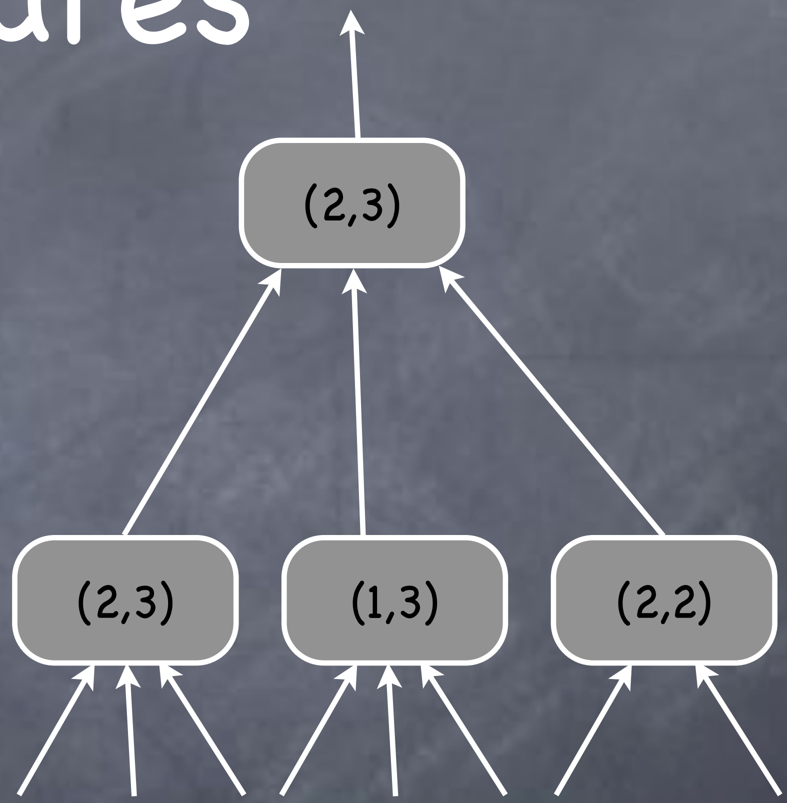

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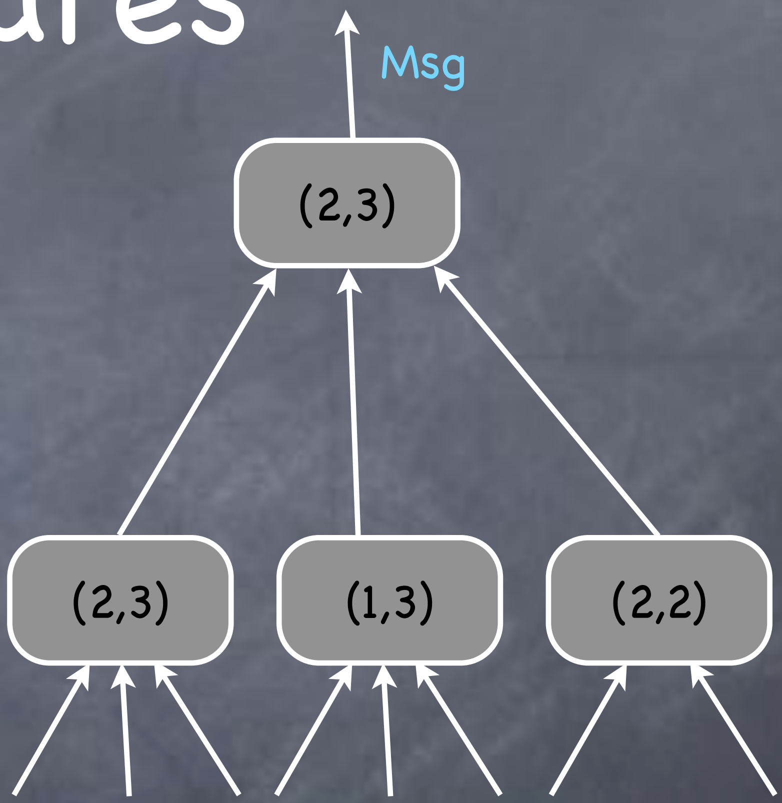

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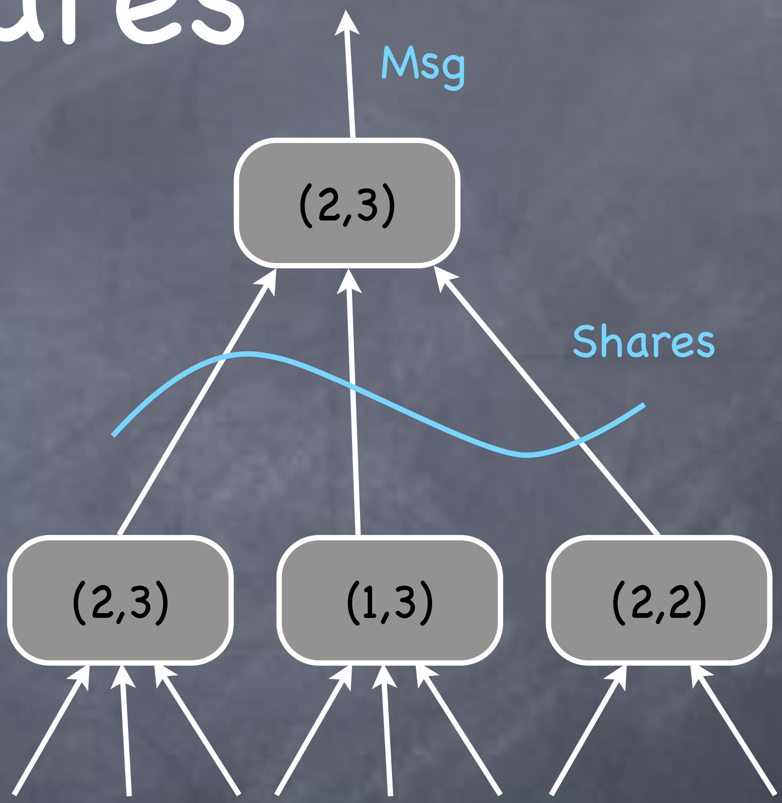

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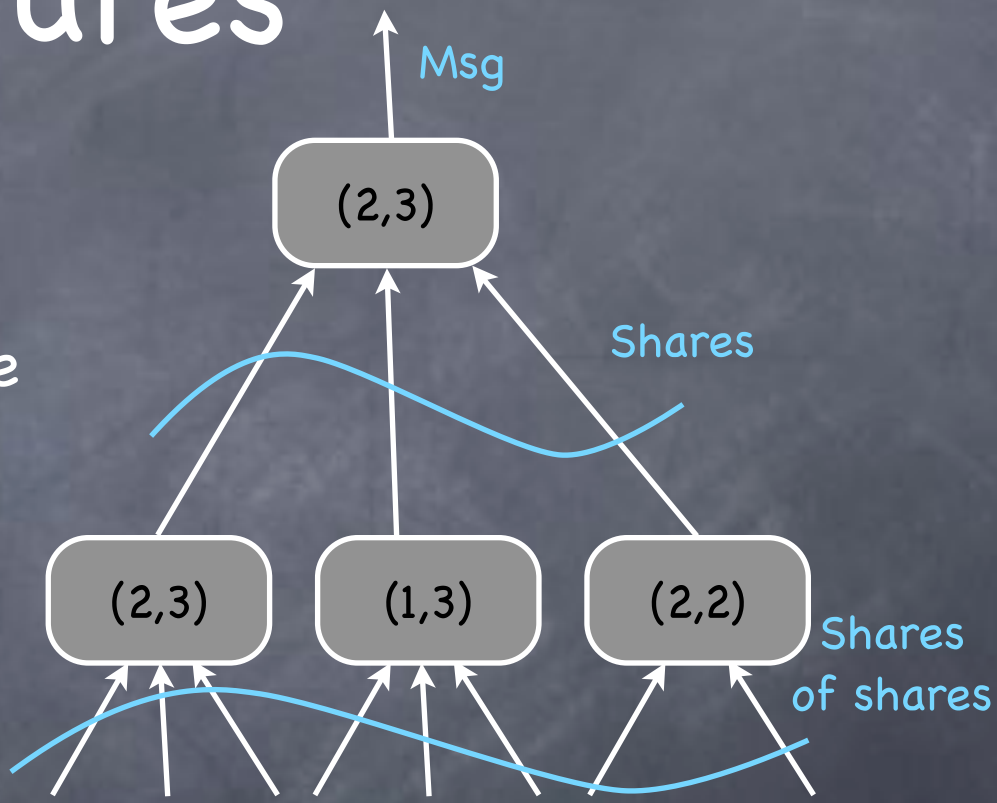

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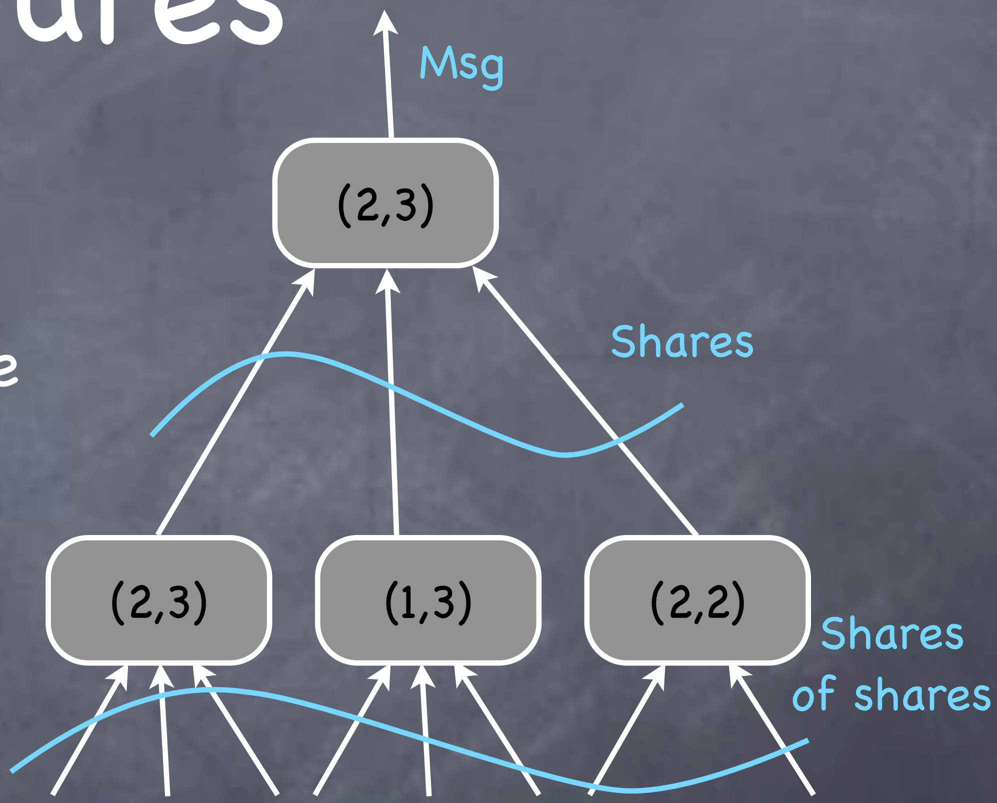

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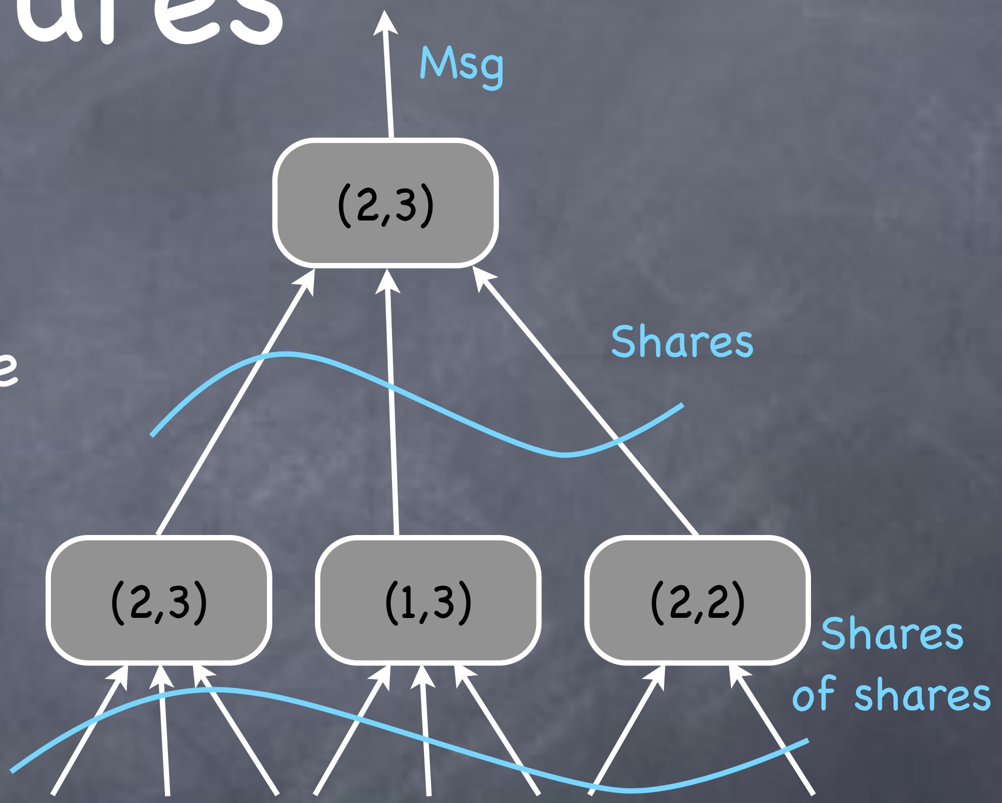

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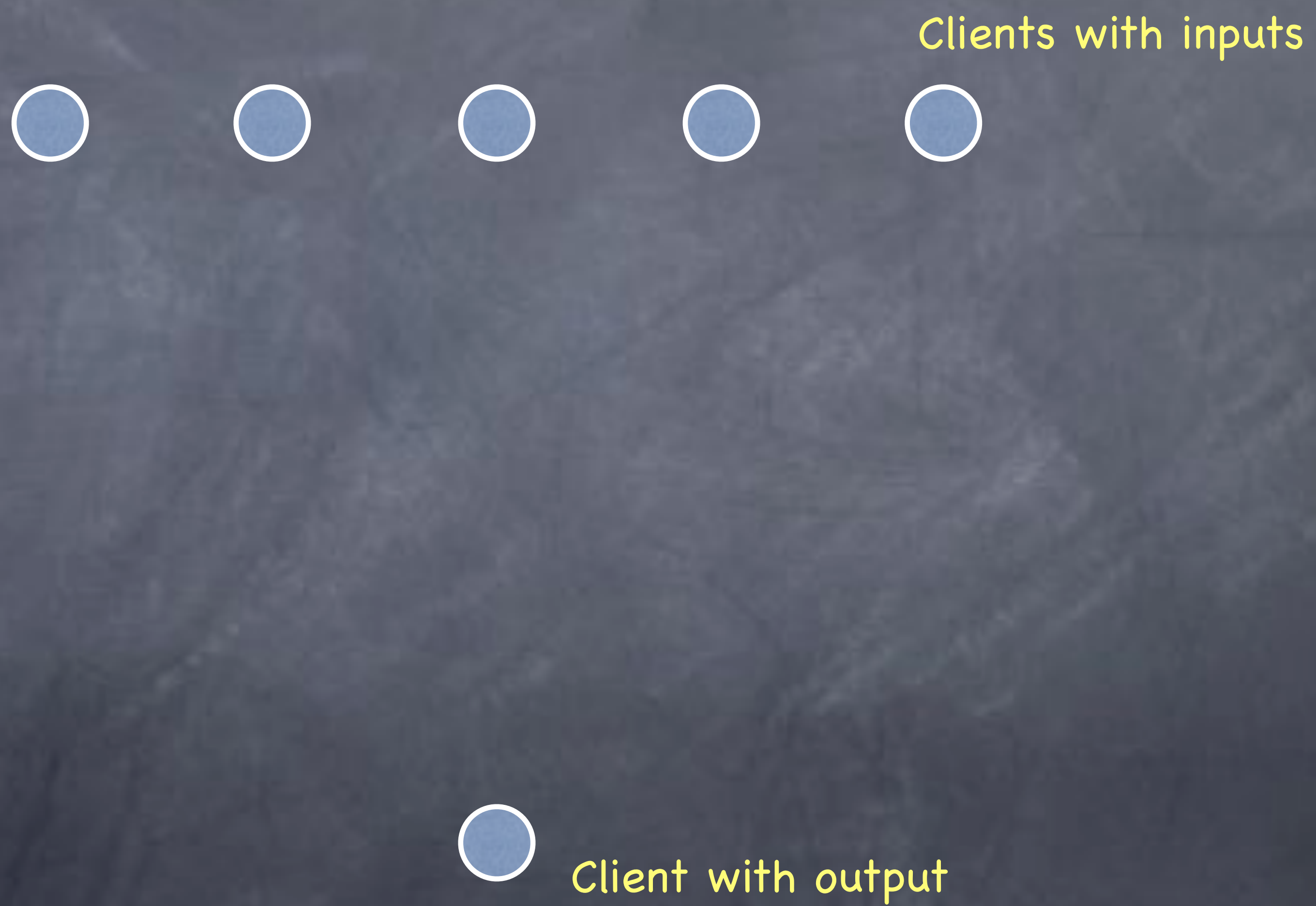

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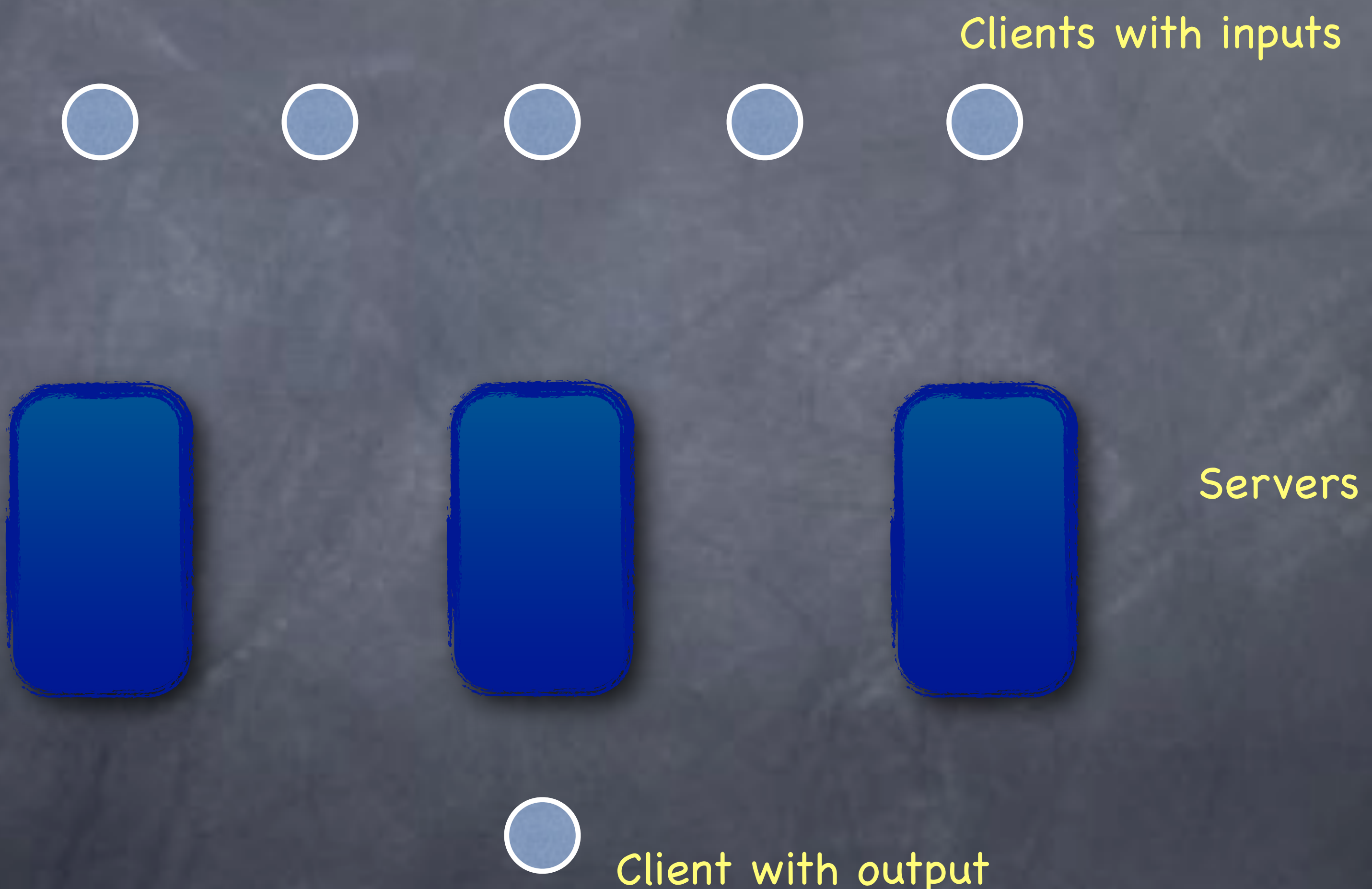

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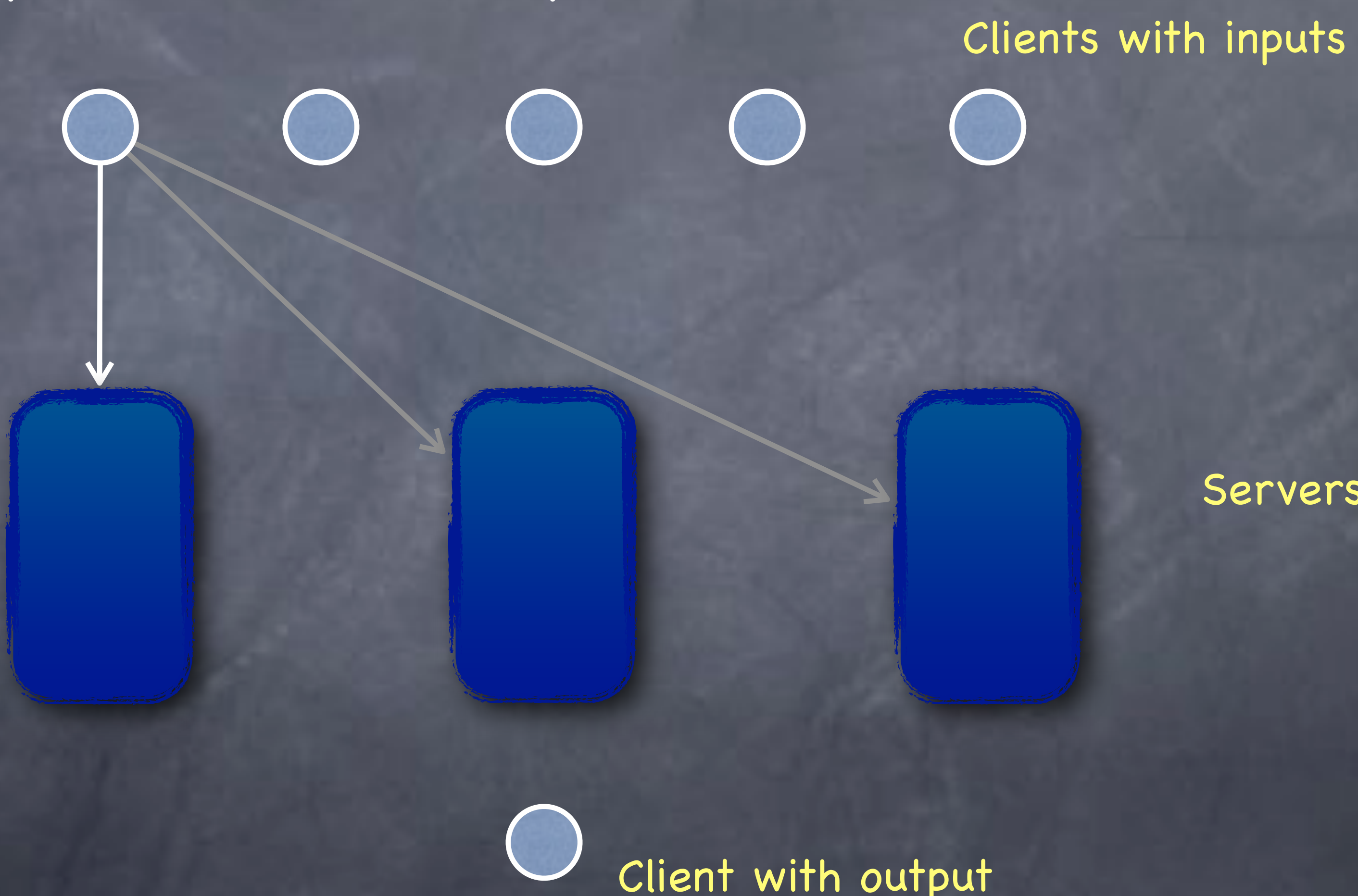

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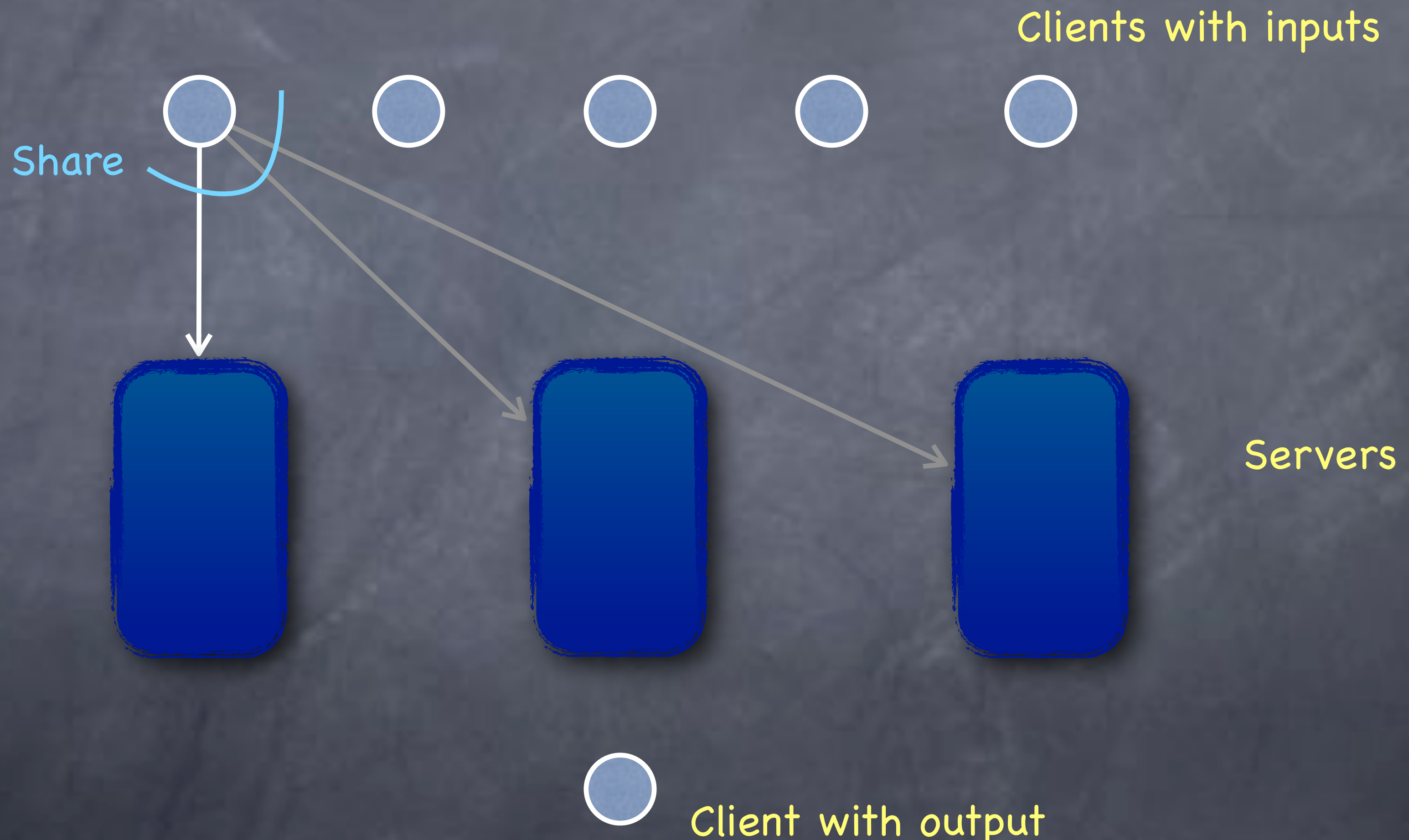

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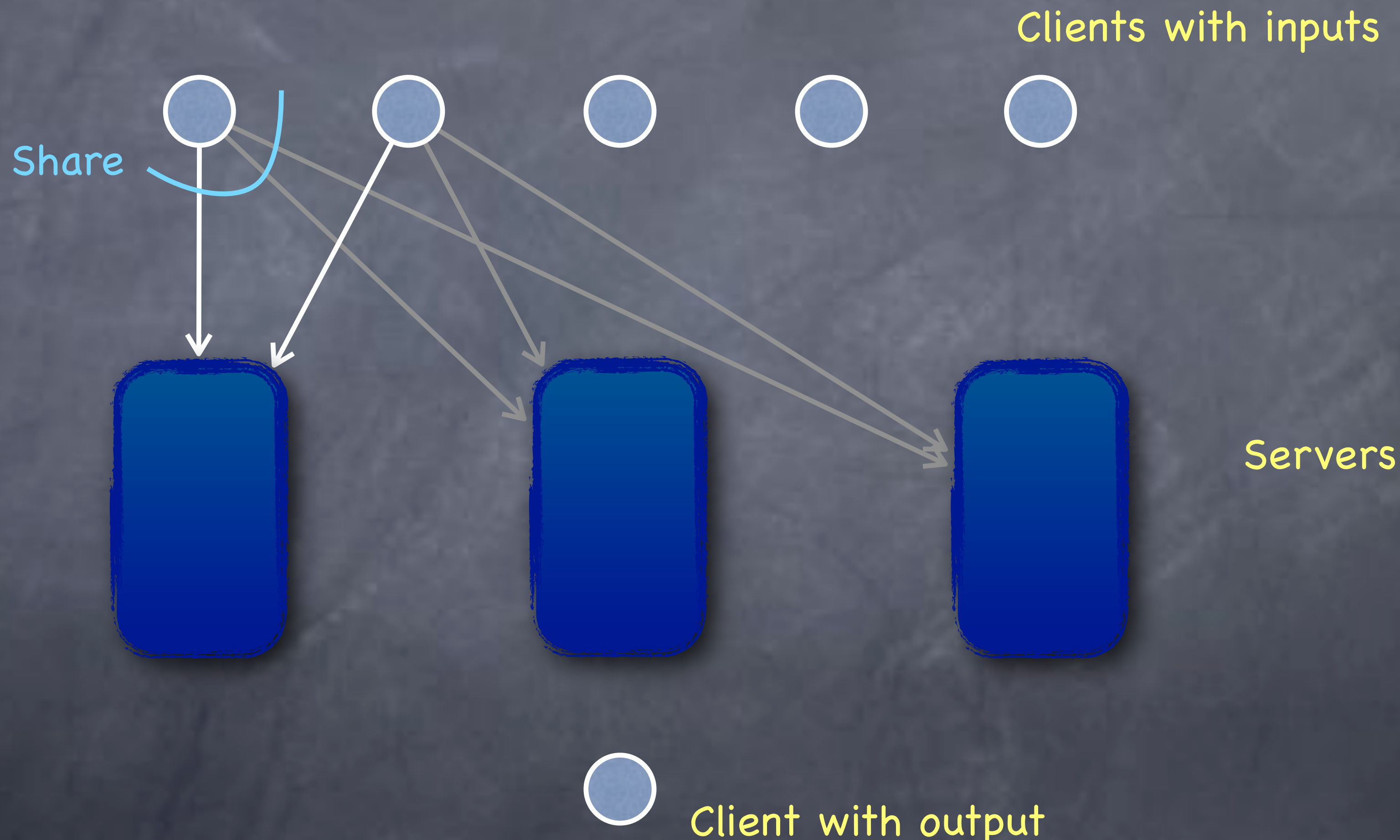

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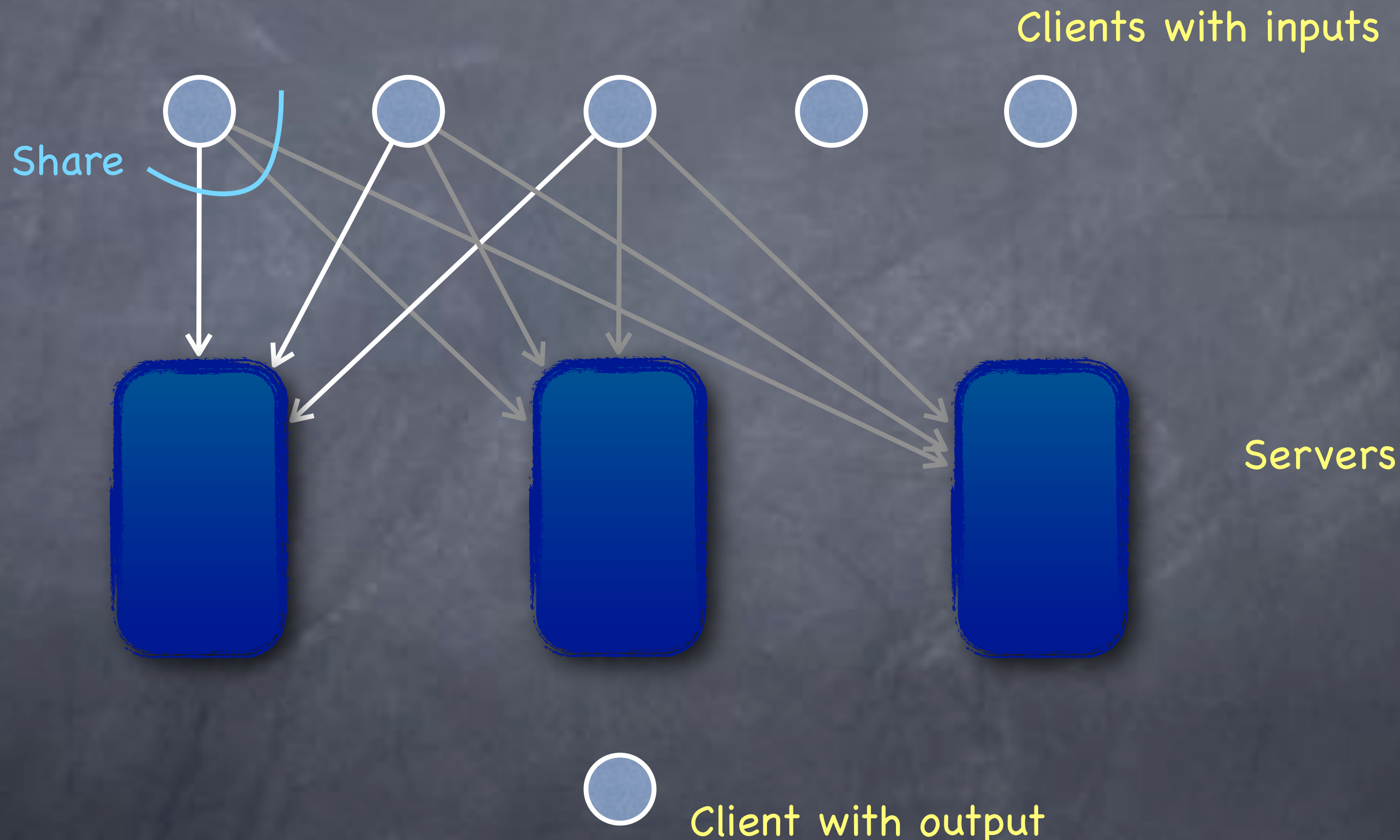

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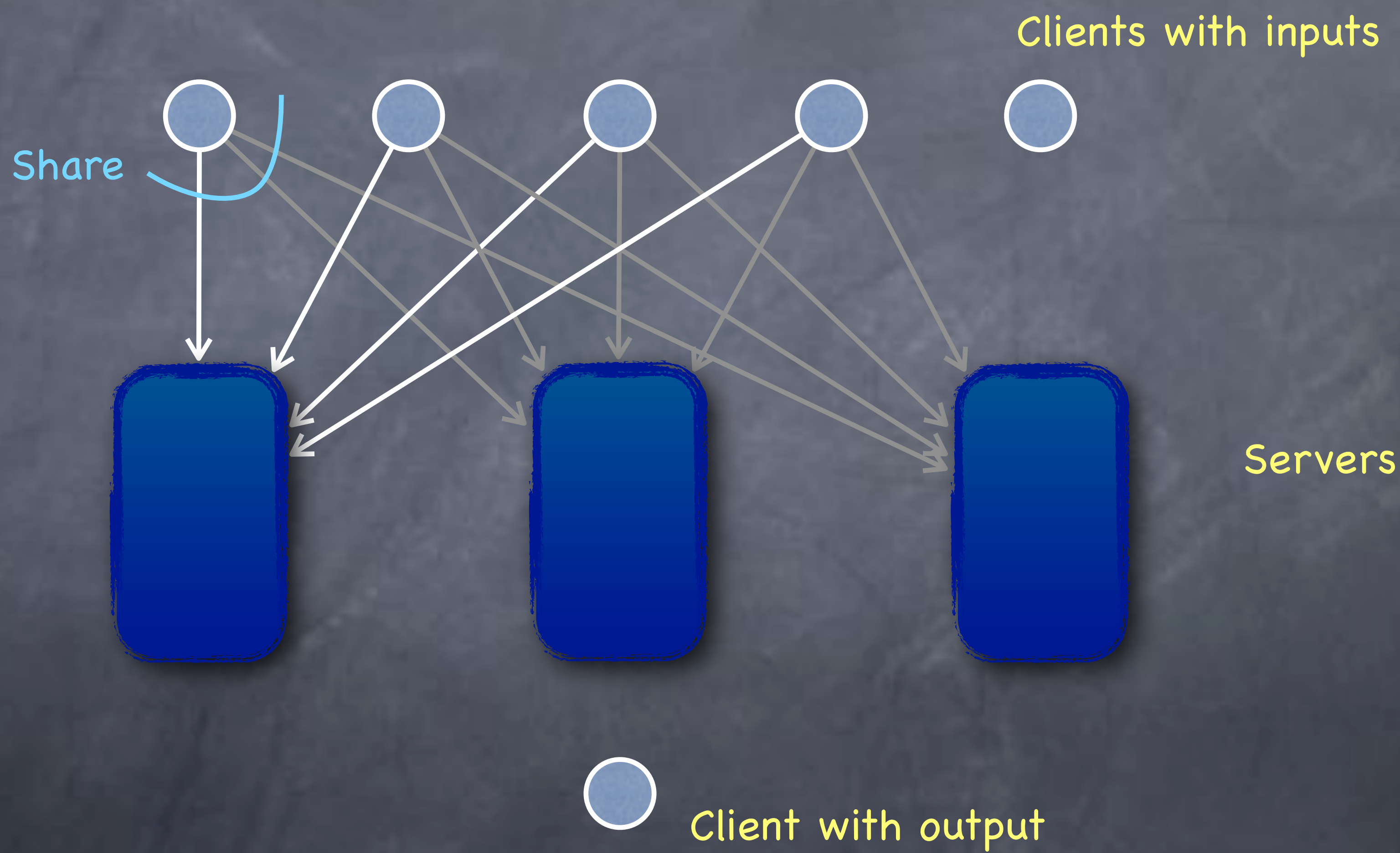

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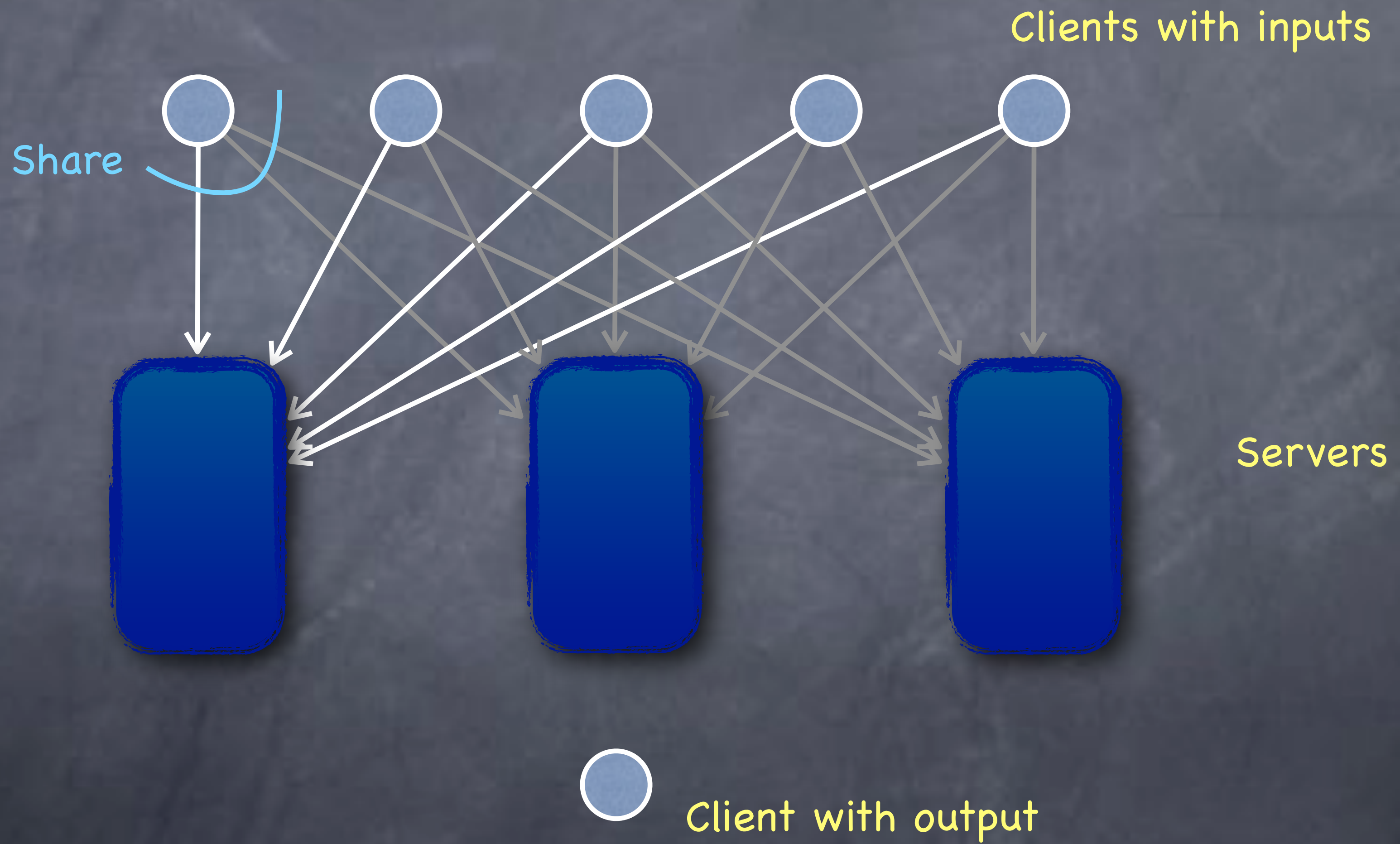

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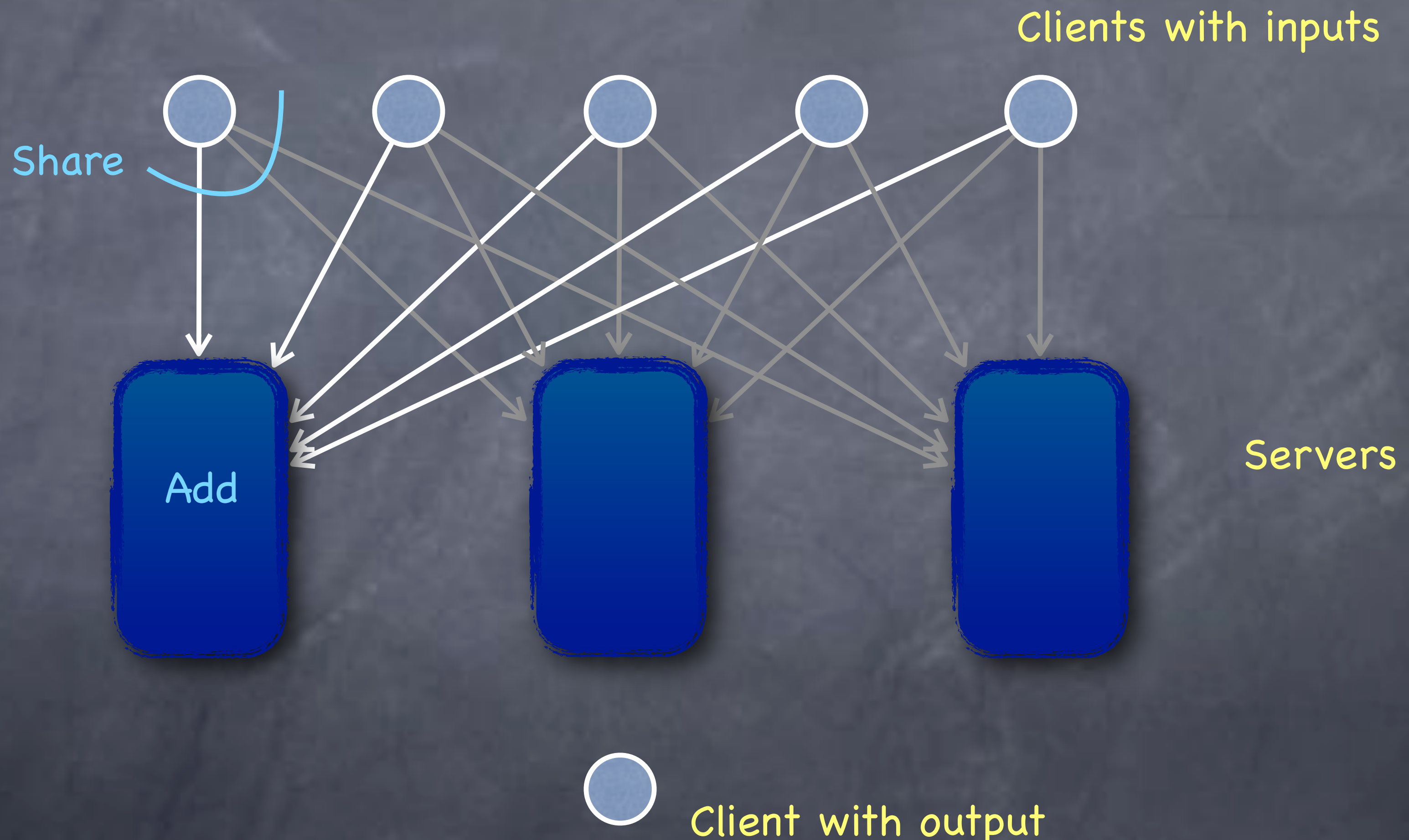

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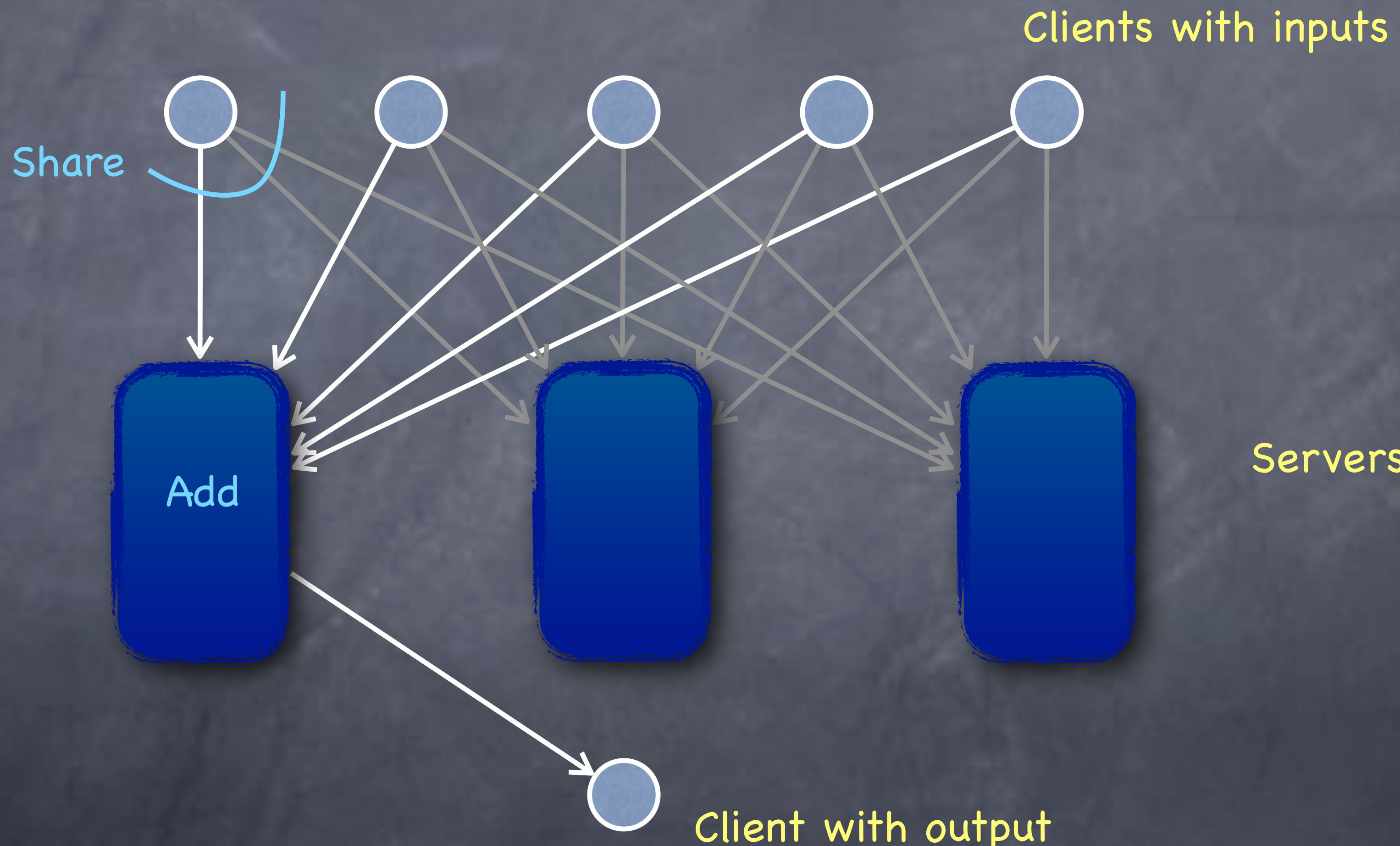

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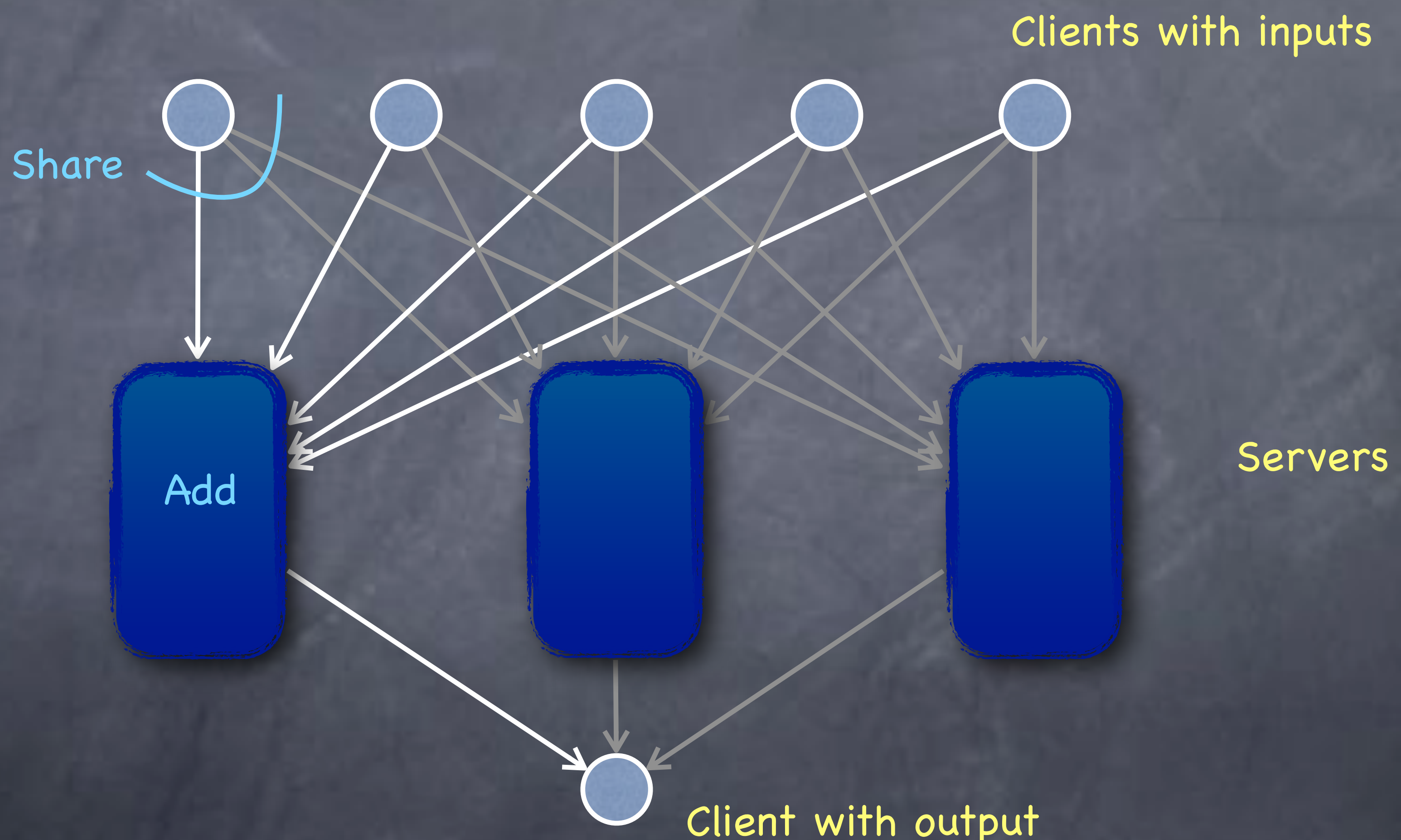

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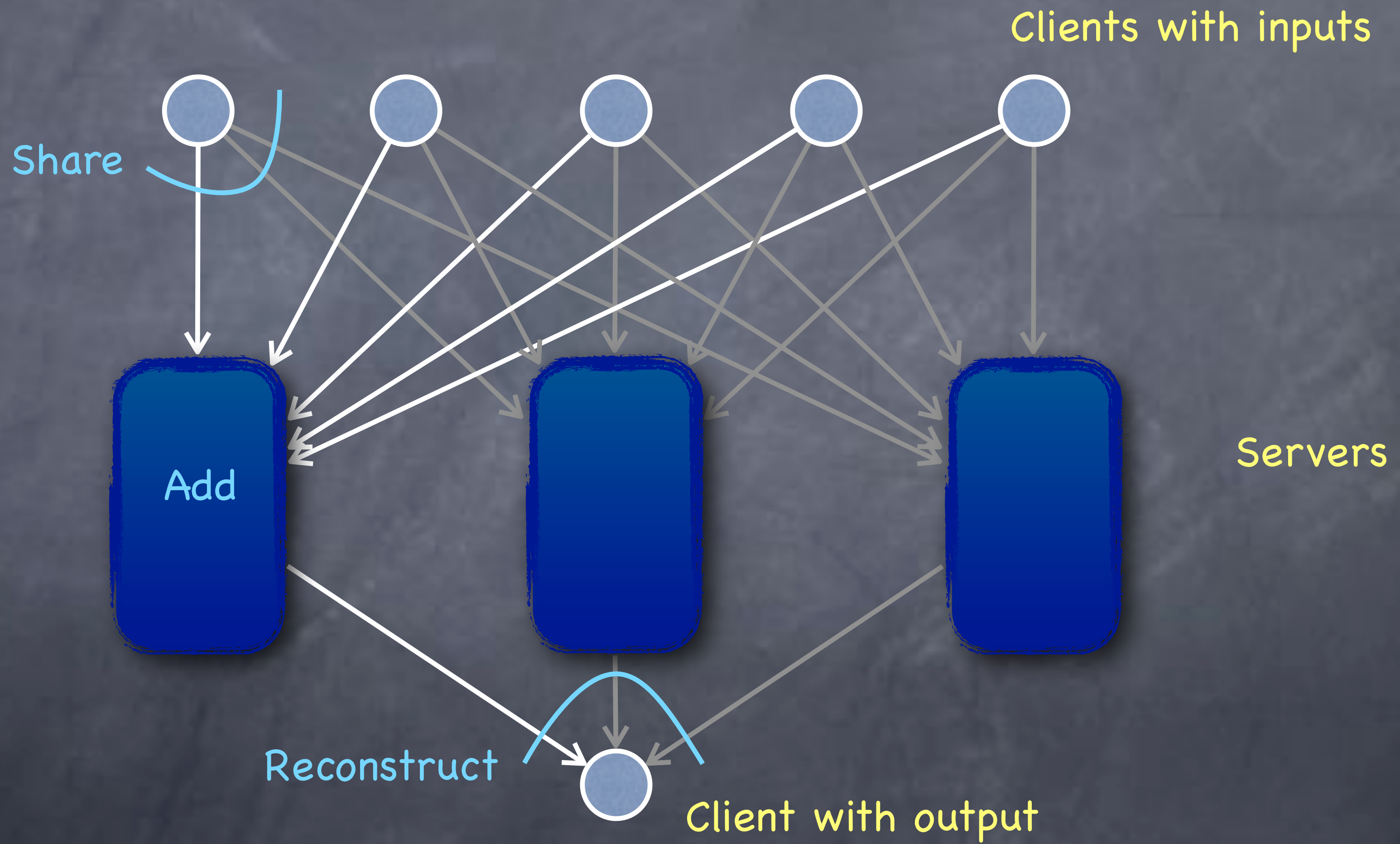

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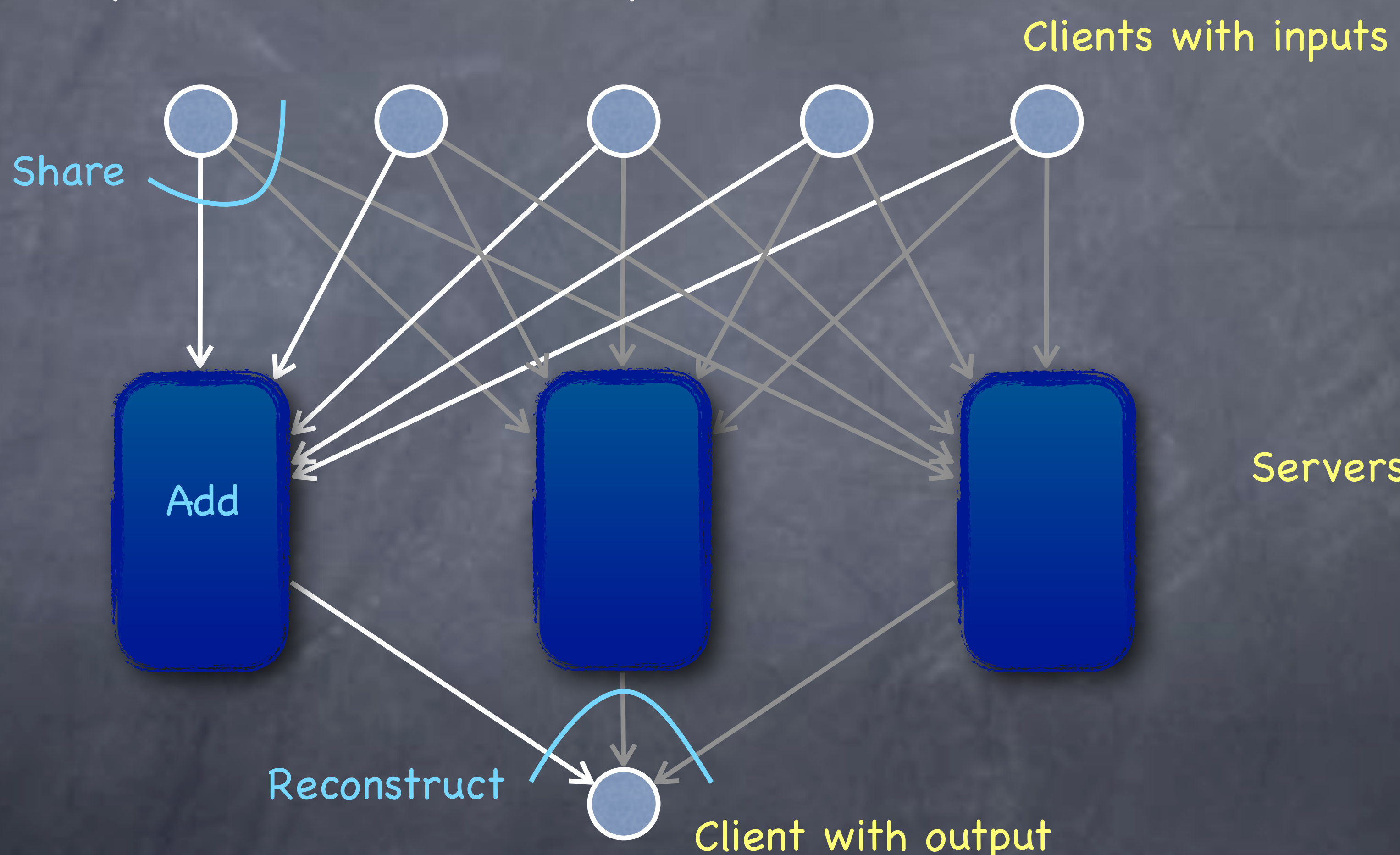

- Secure against passive corruption (no set of parties learn more than what they must) if at least one server is uncorrupted

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- Correctness: if dealer honest, reconstruction correct; even if dealer corrupt, a fixed consistent secret at the end of sharing

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    - Otherwise malicious players can cause denial-of-service

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