ZK Proofs (cntd.)
Composition

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Composition

Lecture 16

RECALL

An Example

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An Example

- **Graph Isomorphism**

RECALL

An Example

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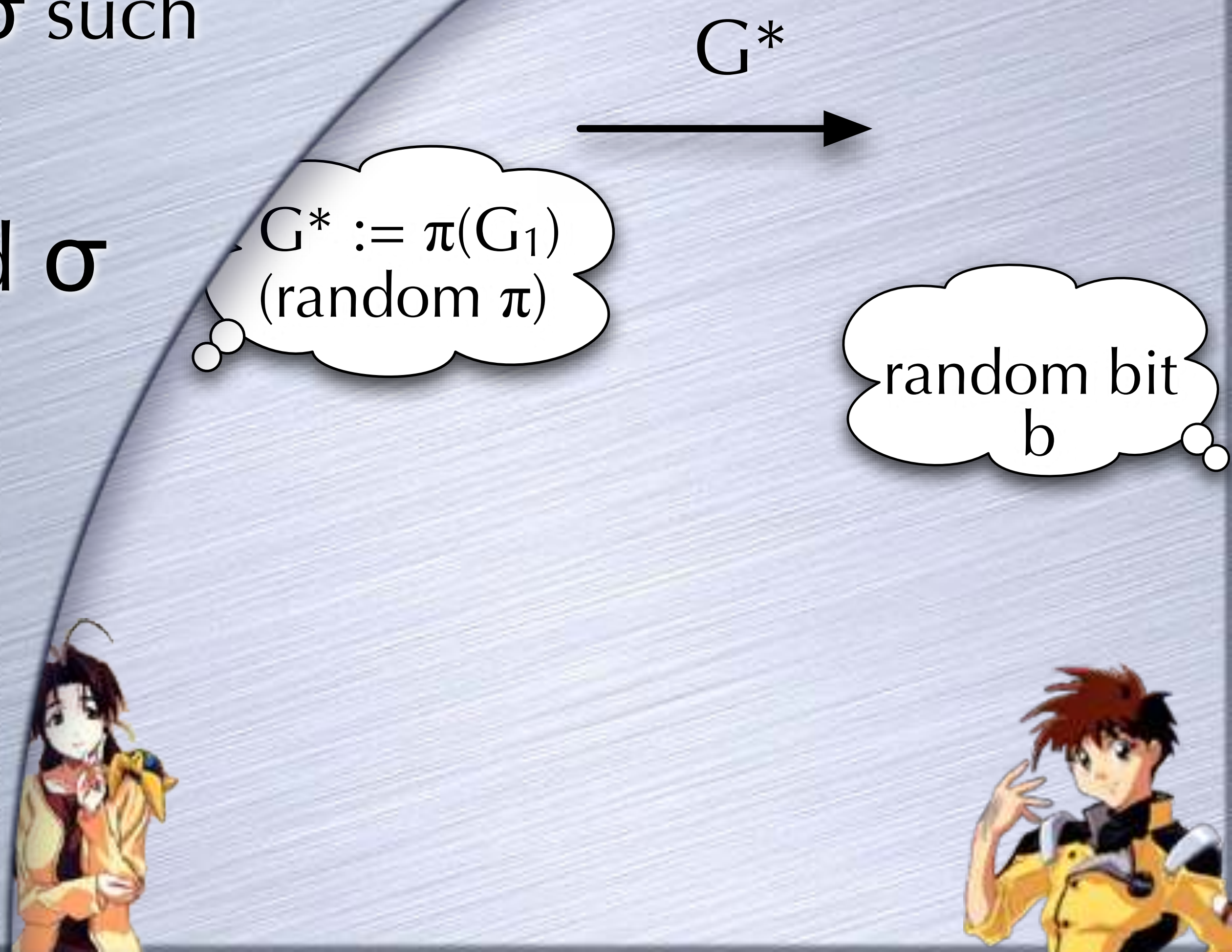

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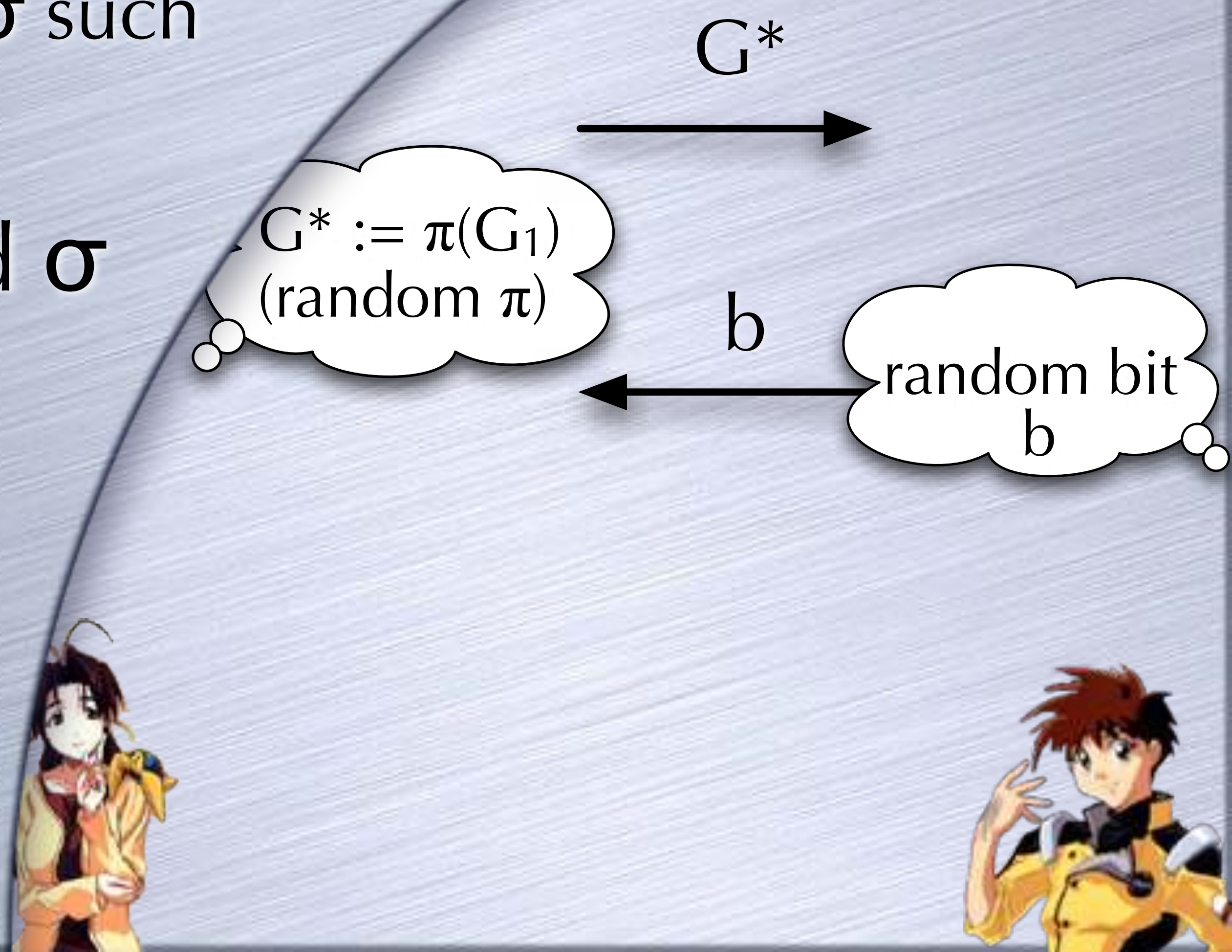

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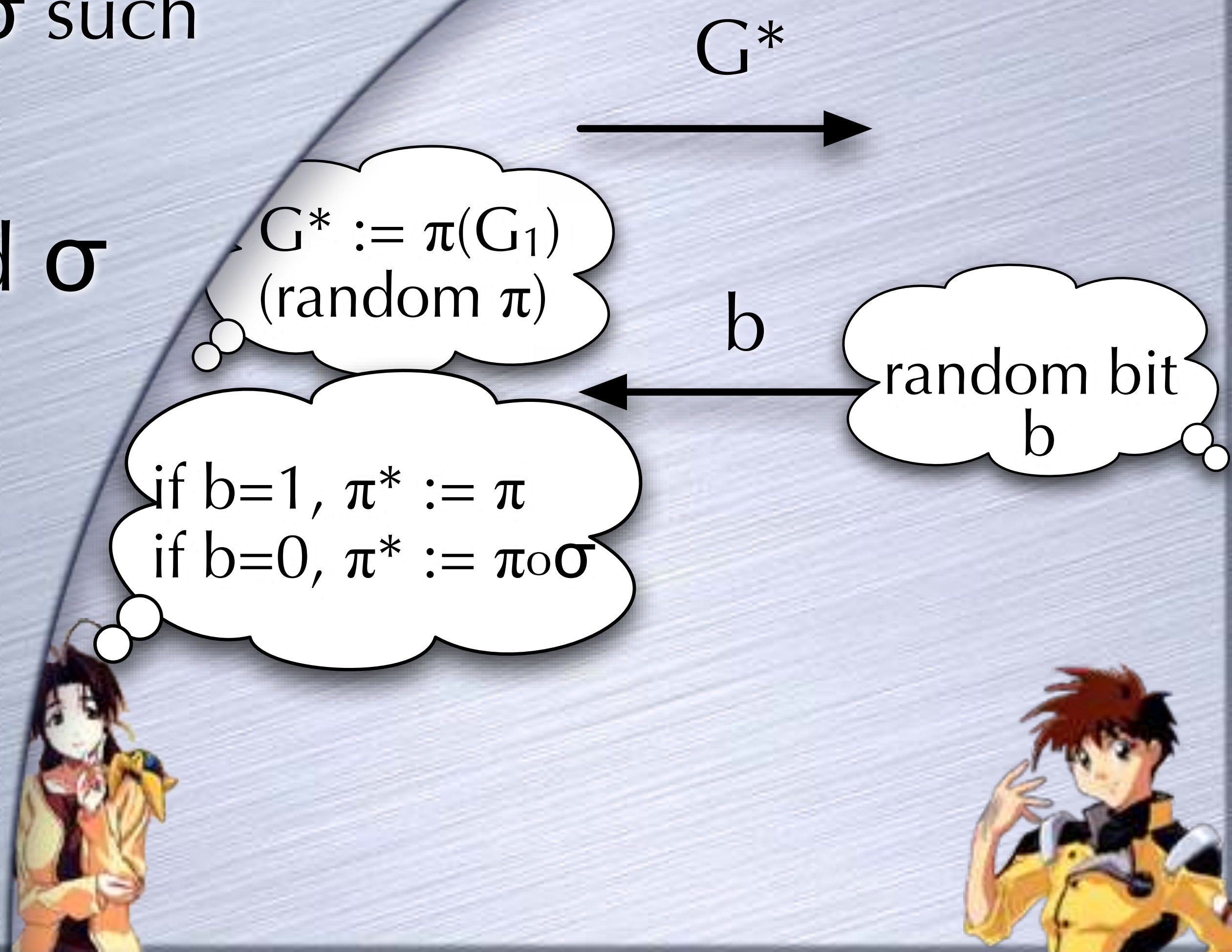

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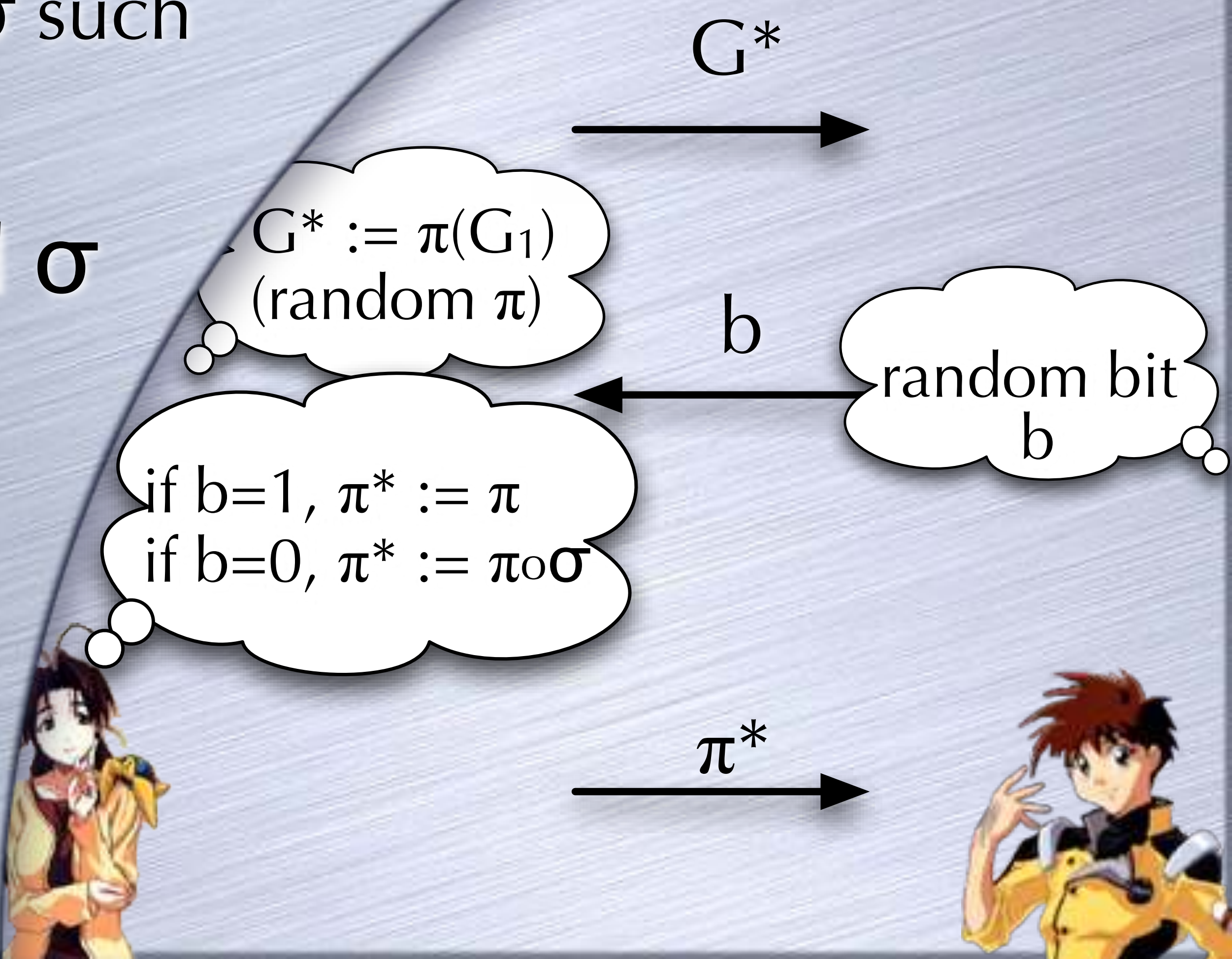

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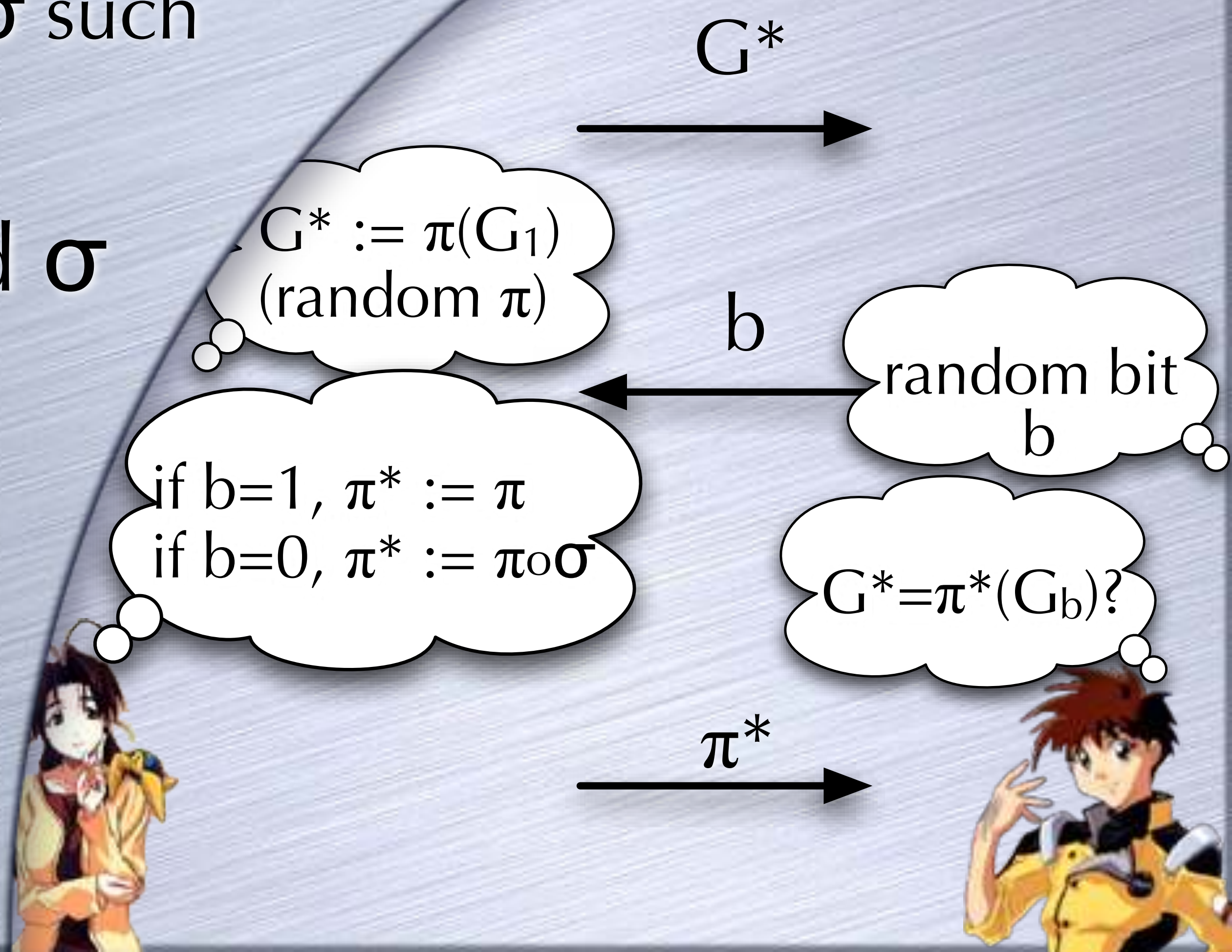

The Legend of William Tell

A Side Story

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Bob: *William Tell is a great marksman!*

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Bob: *William Tell is a great marksman!*

Charlie: *How do you know?*

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Bob: But I picked b at random and she had no trouble answering me...

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  - Complete and Sound

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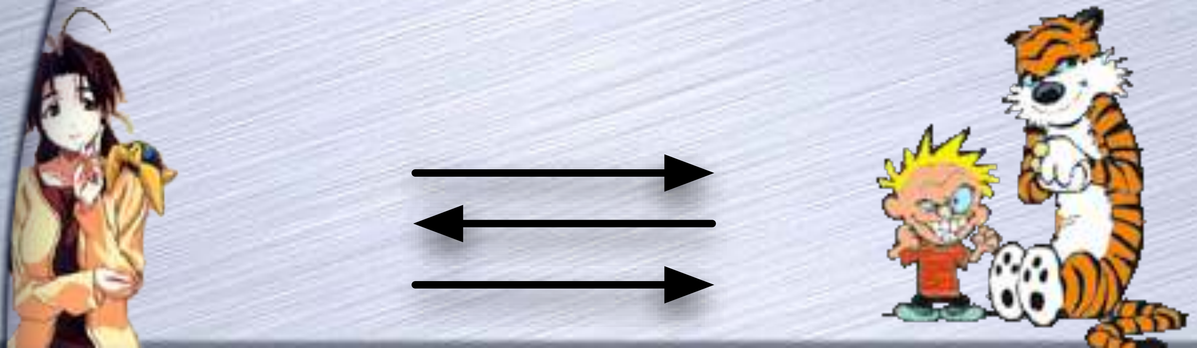

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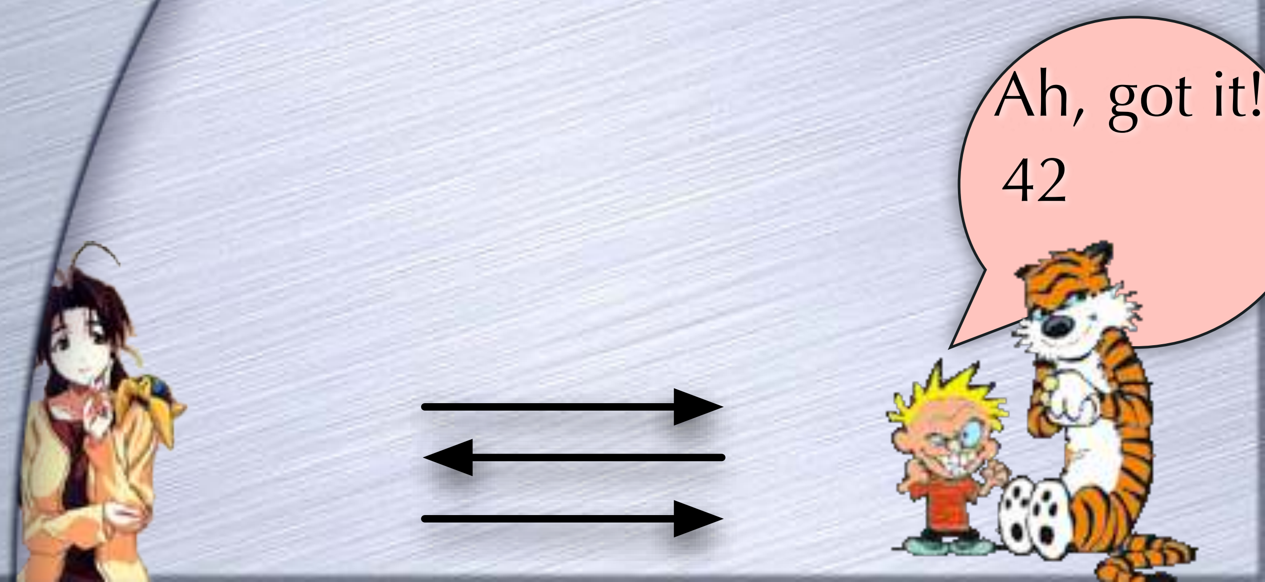

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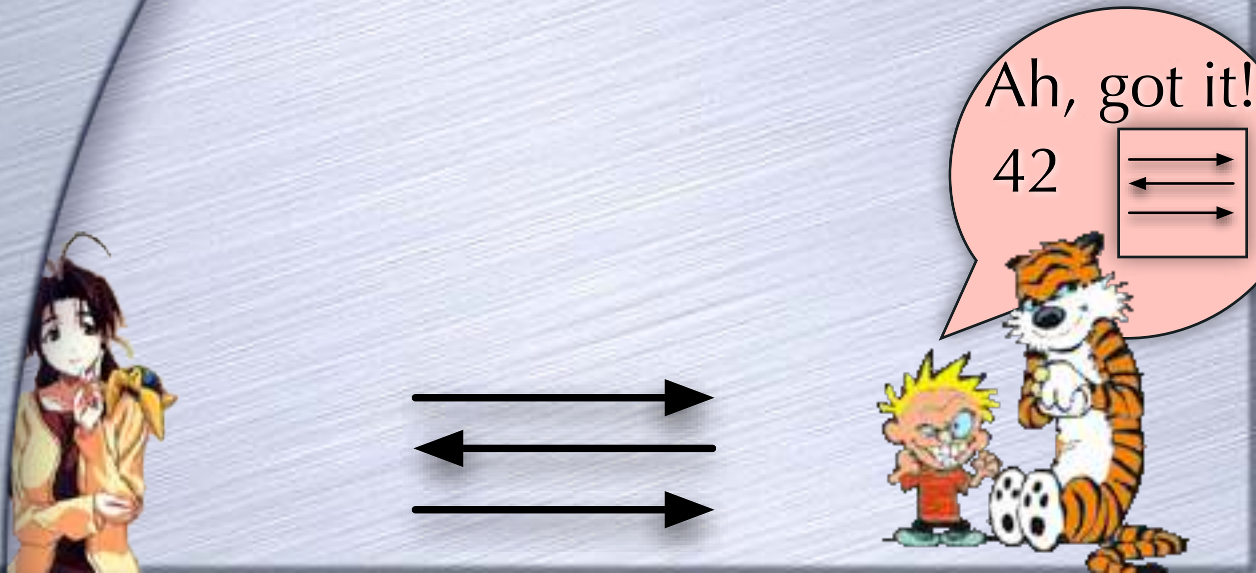

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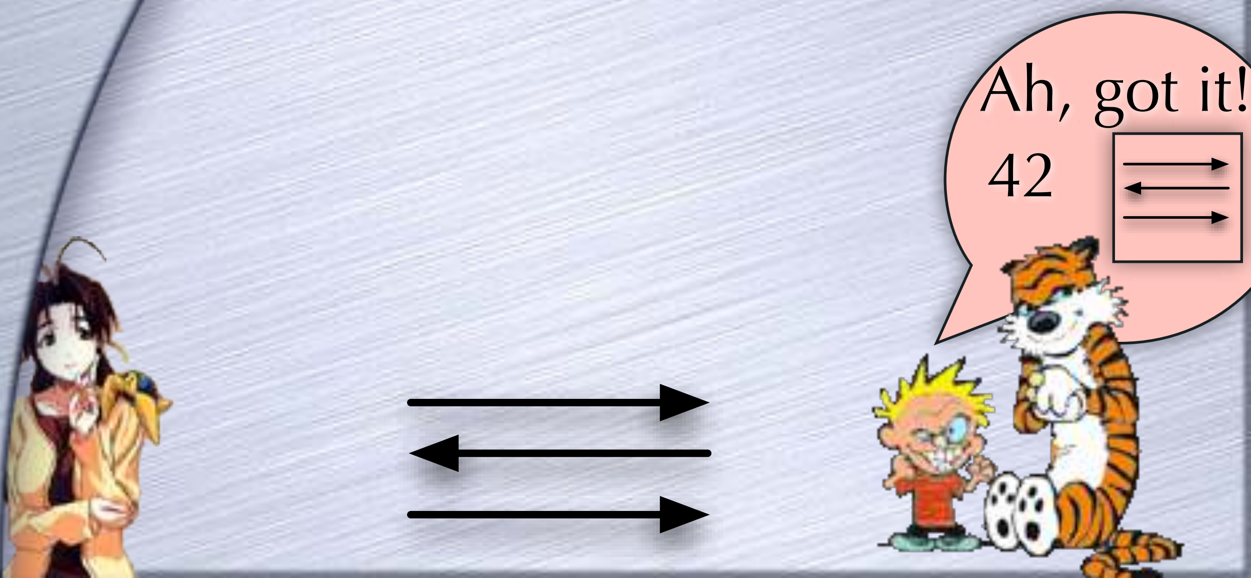

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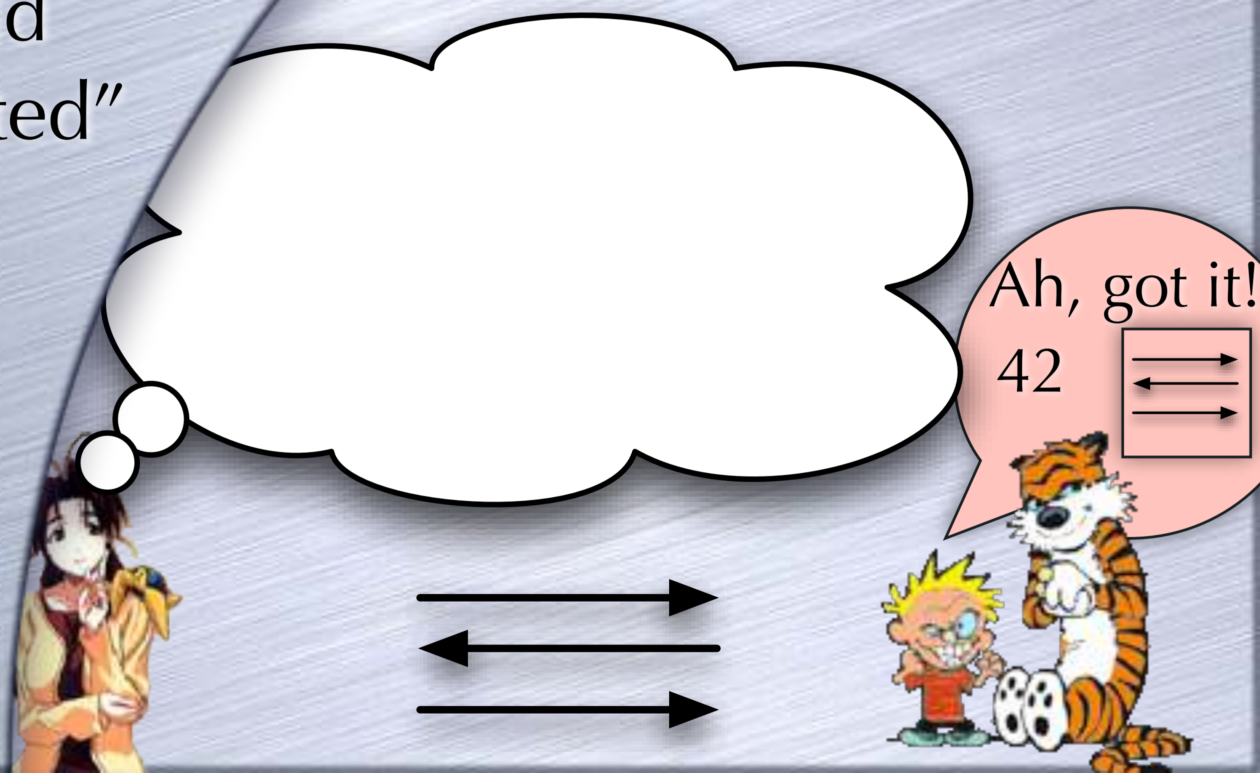

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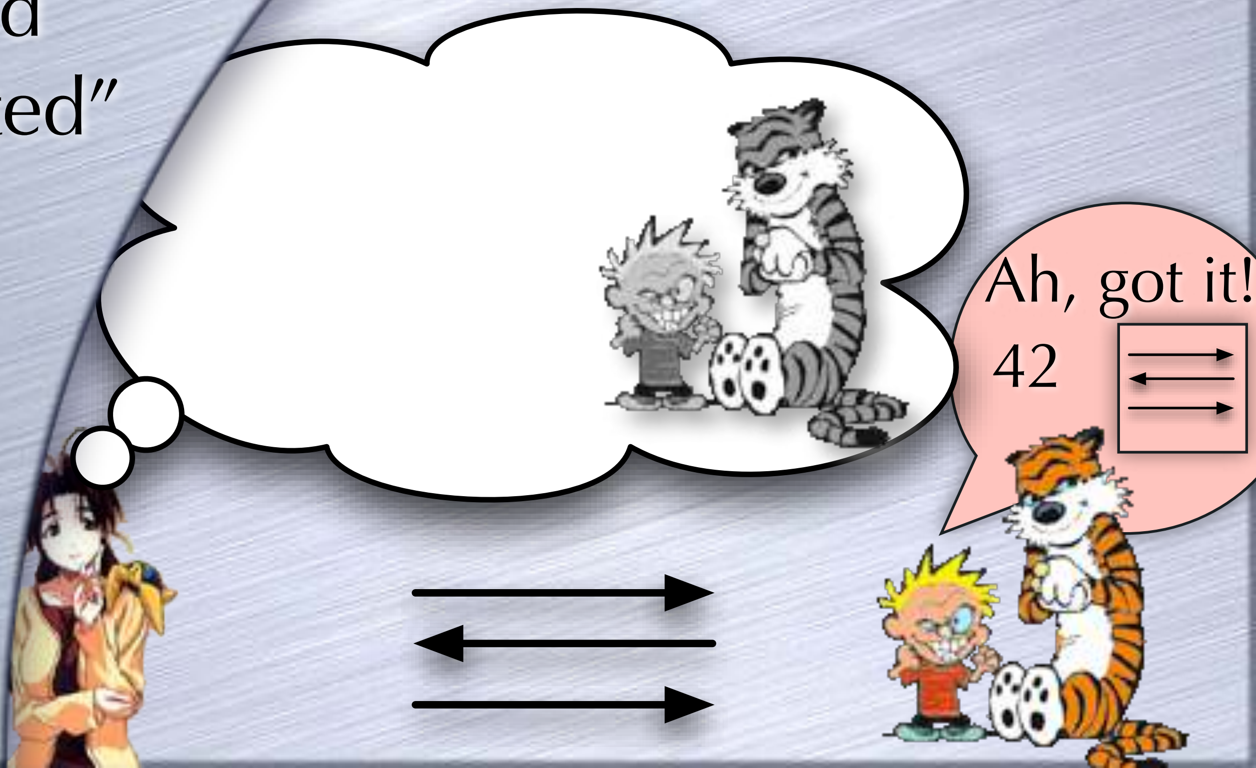

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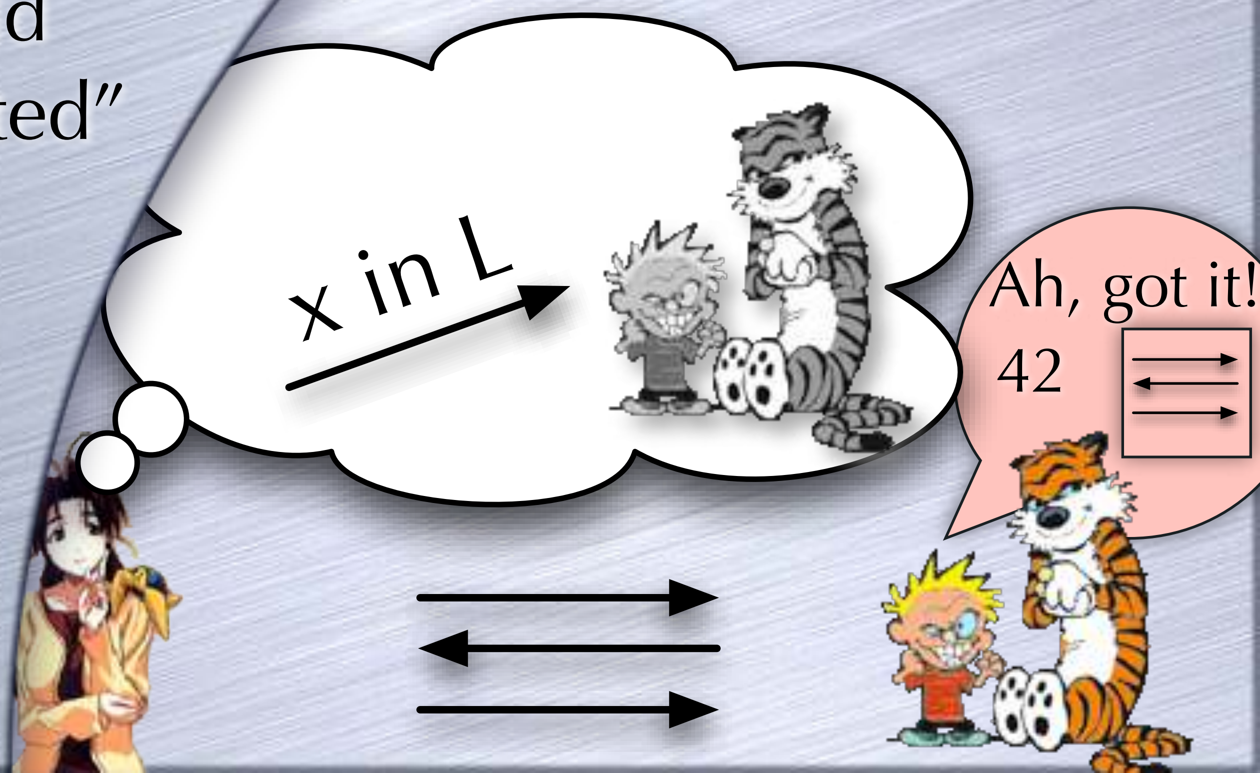

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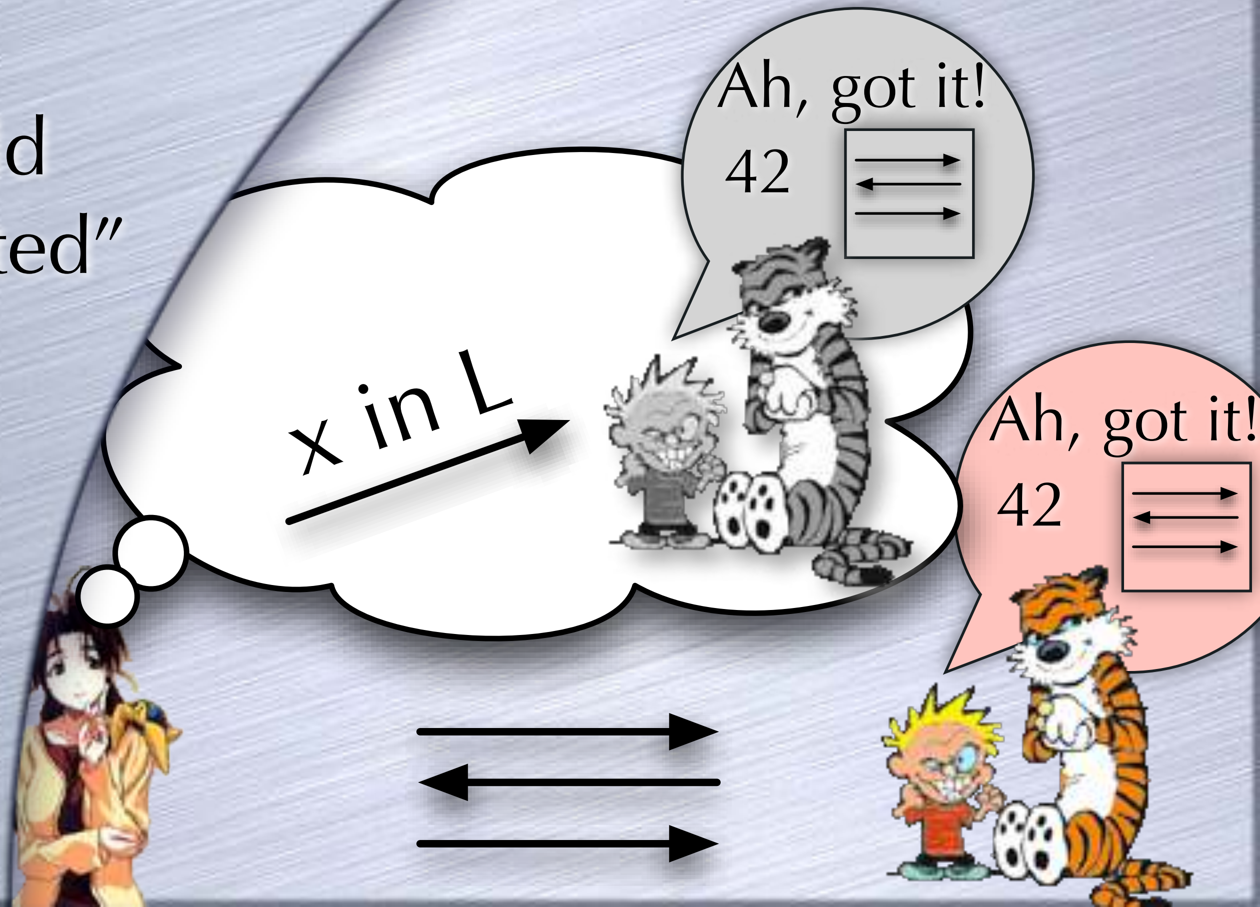

Zero-Knowledge Proofs

- Interactive Proof
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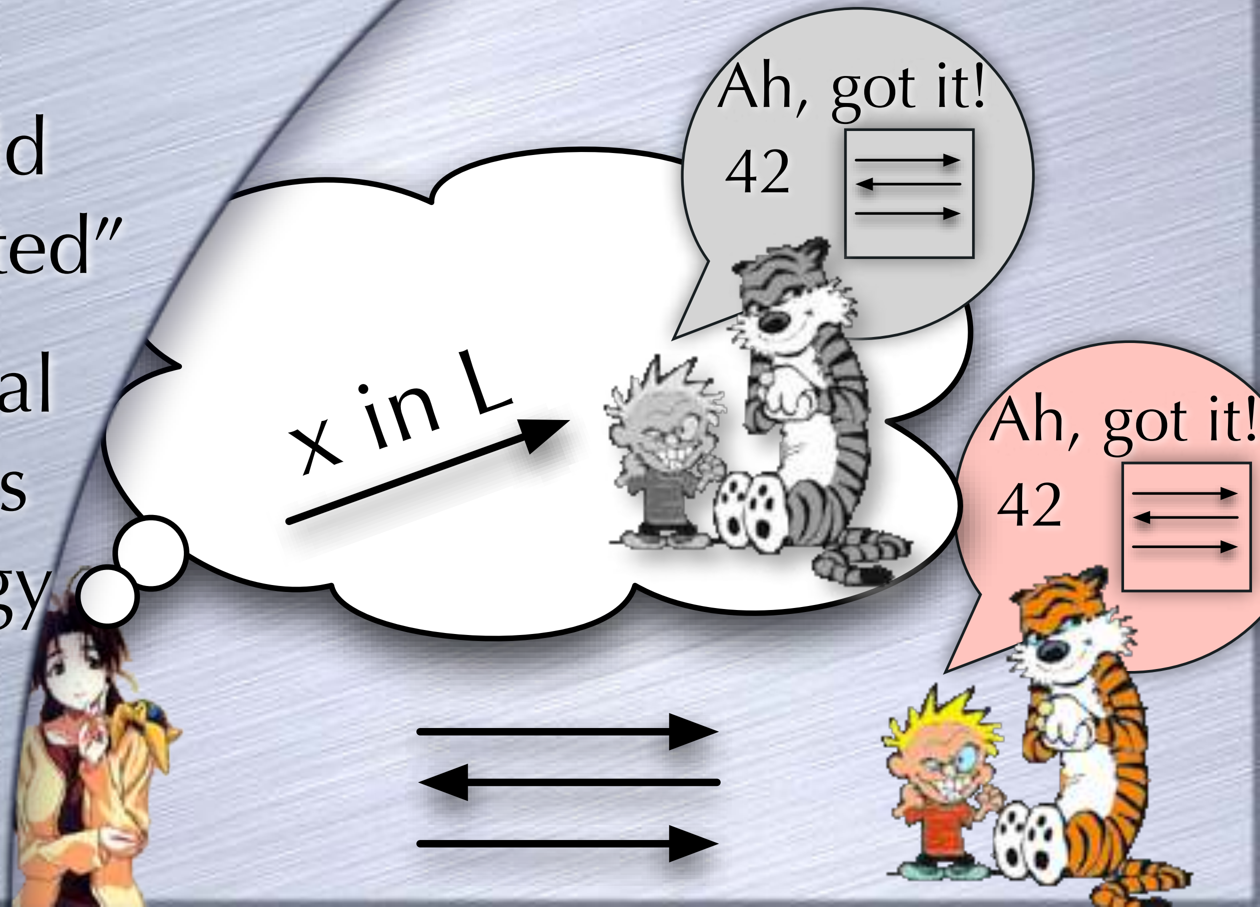
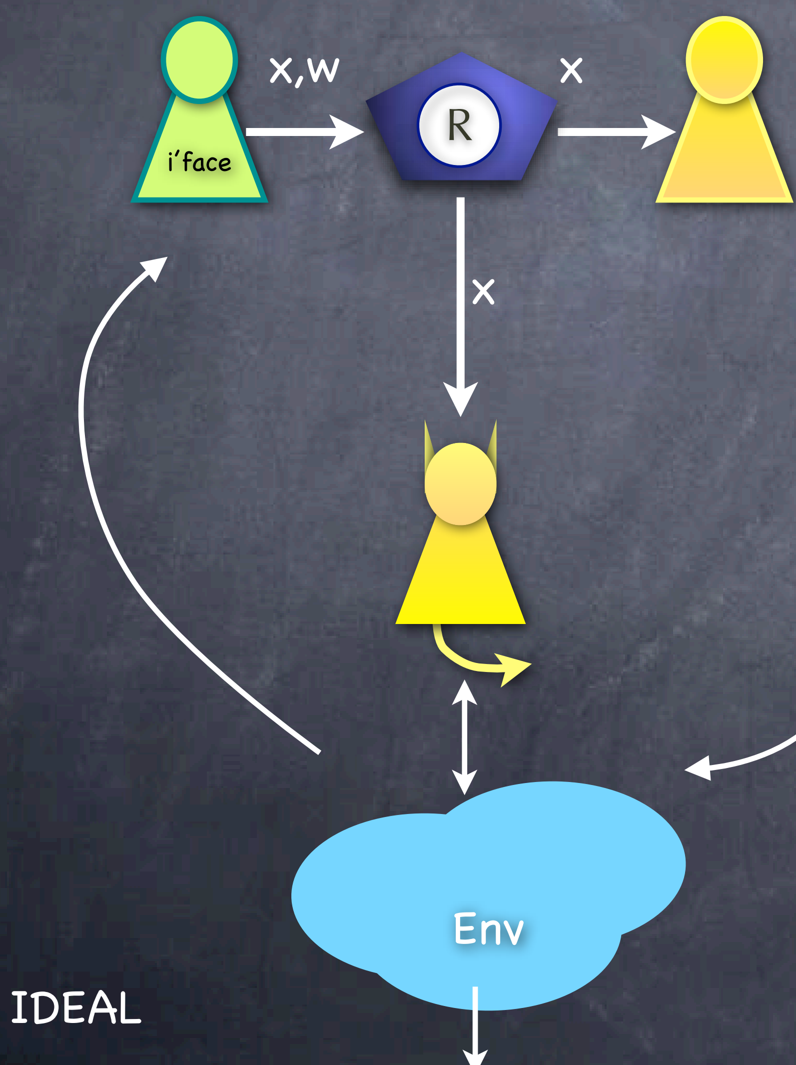
- ZK Property:
  - Verifier's view could have been "simulated"
  - For every adversarial strategy, there exists a simulation strategy



ZK Property (in other pict's)



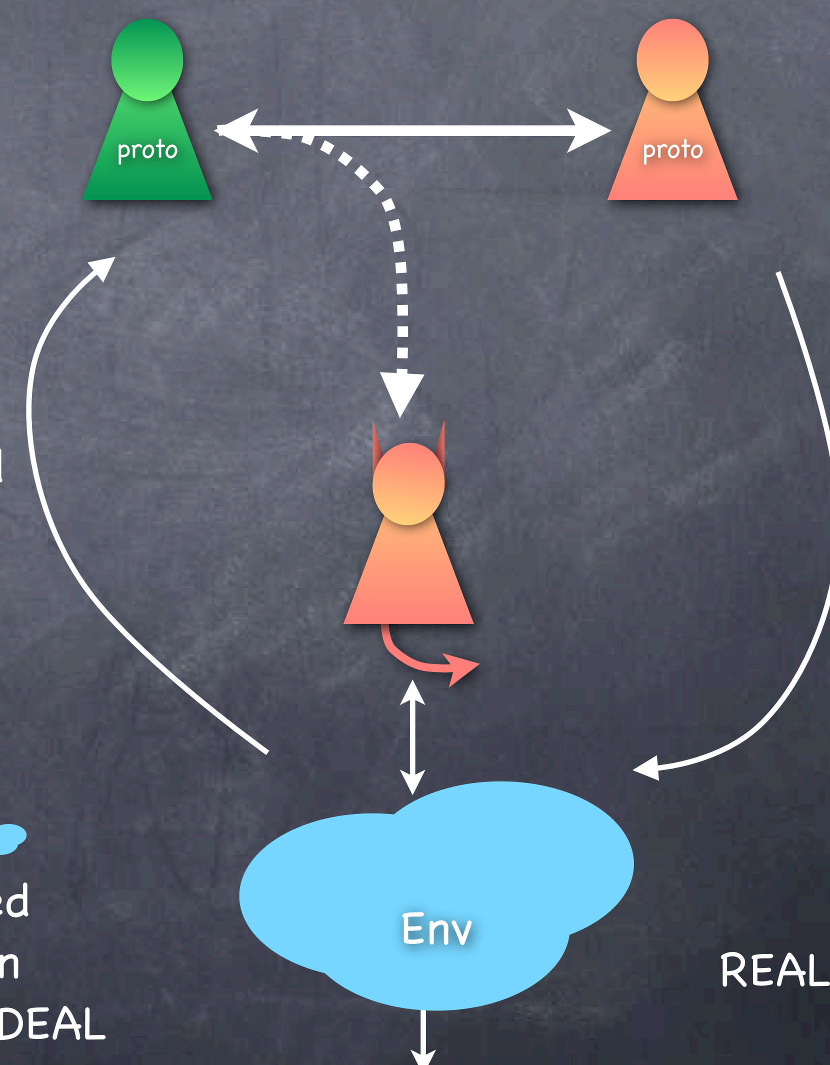
Secure (and correct) if:

∀

∃ s.t.

∀

output of is distributed identically in REAL and IDEAL



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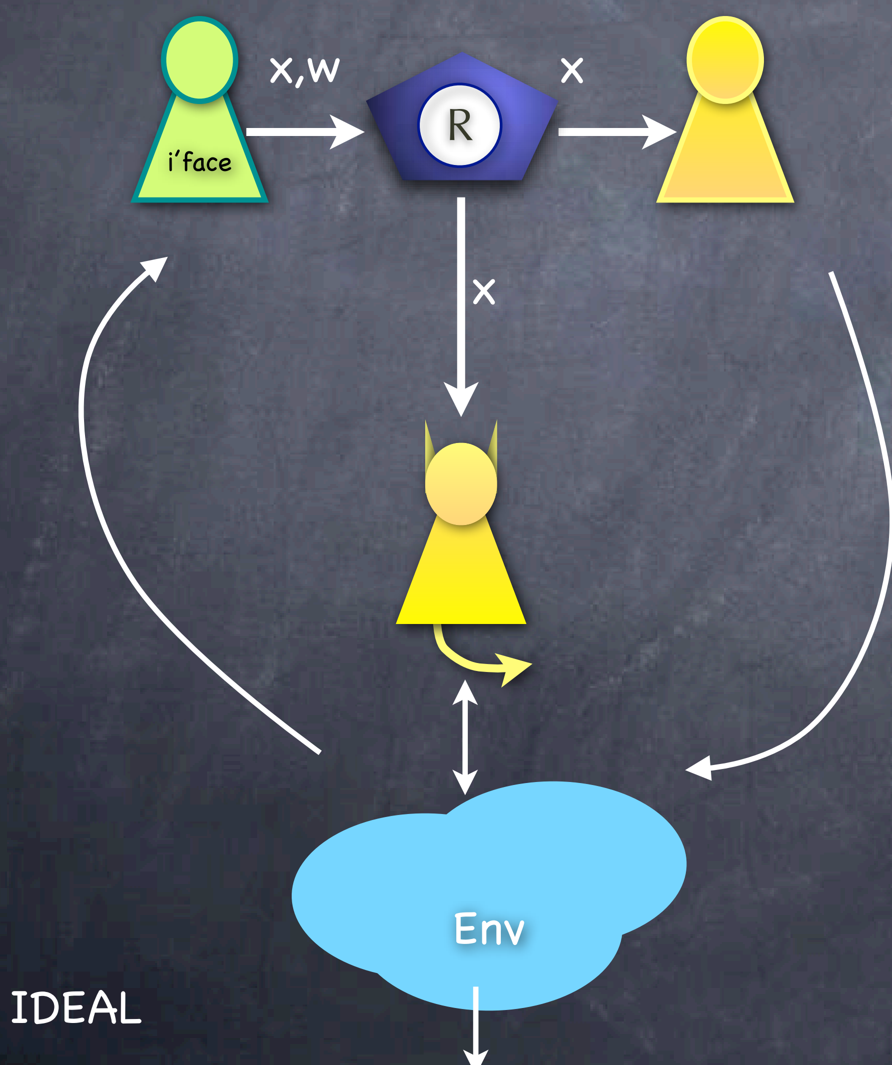

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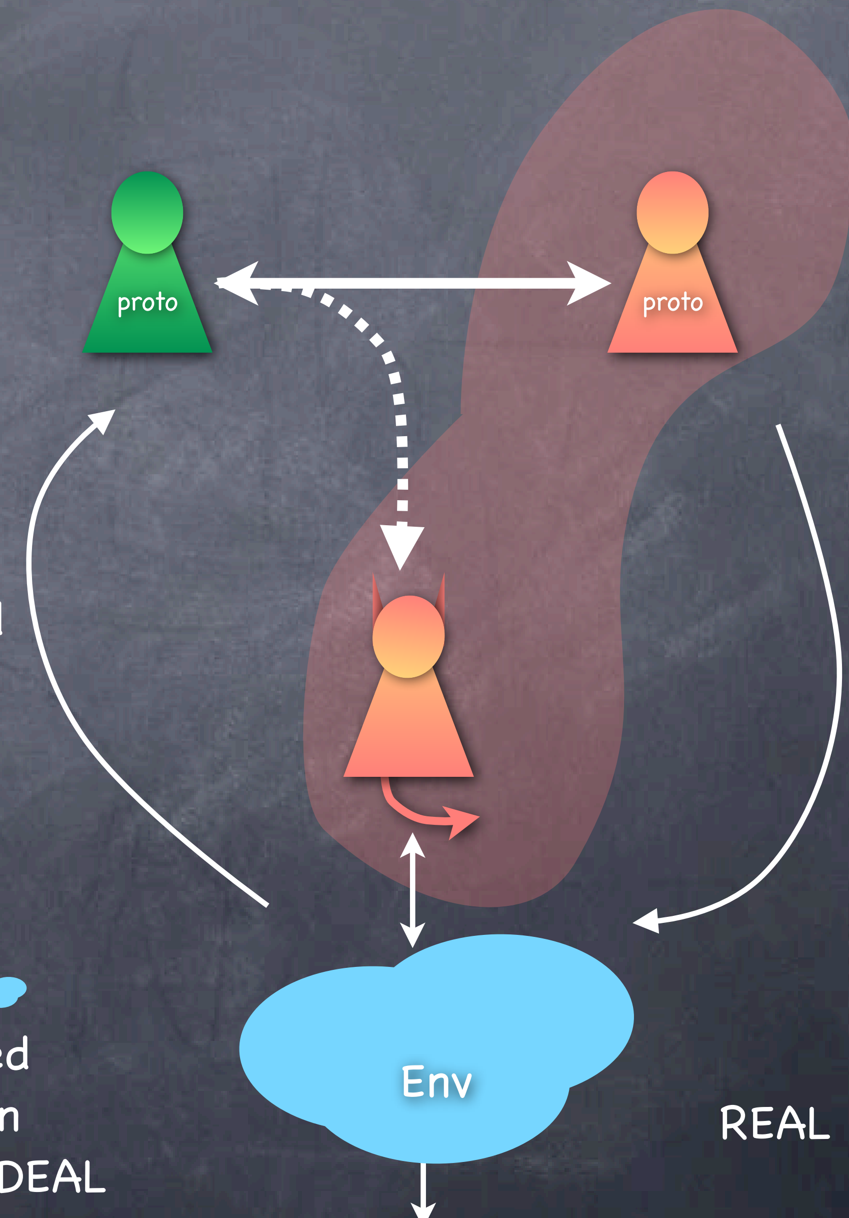

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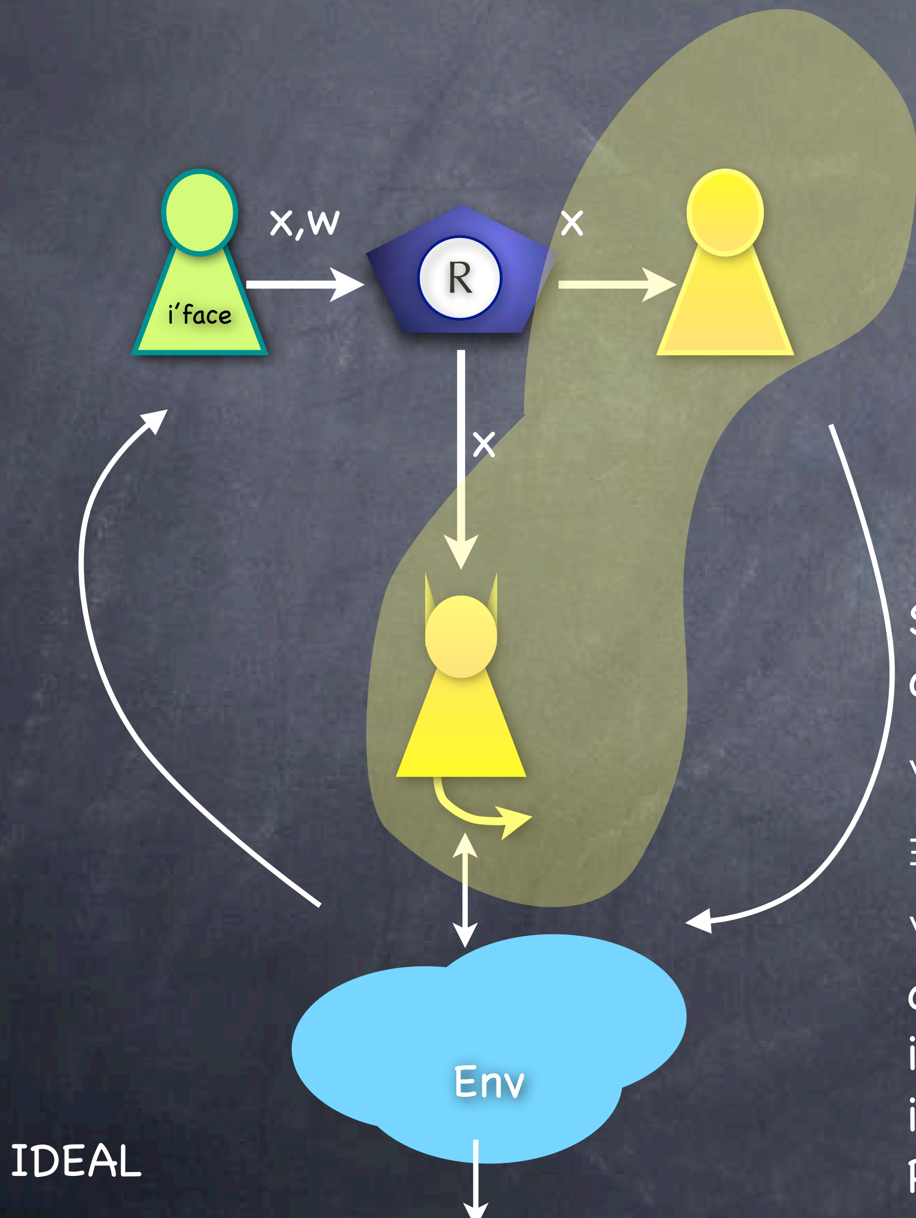

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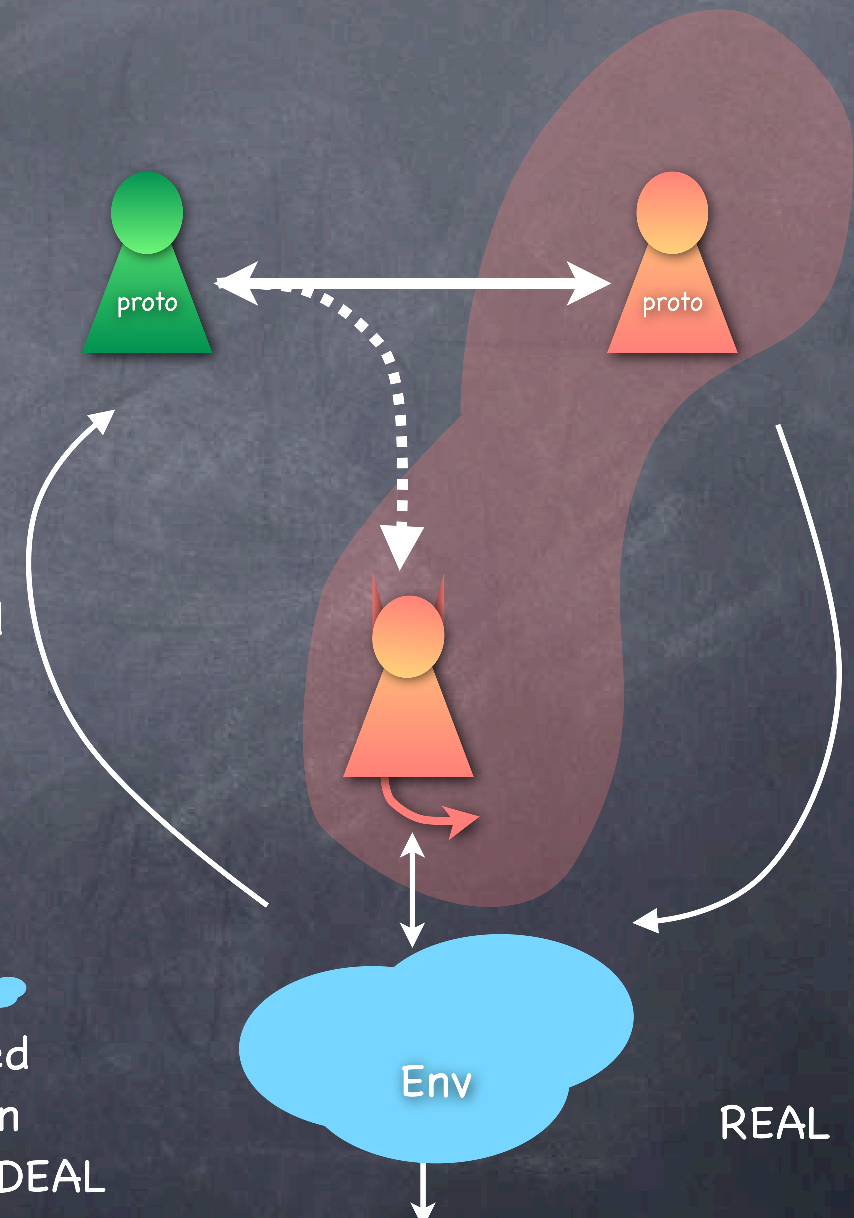
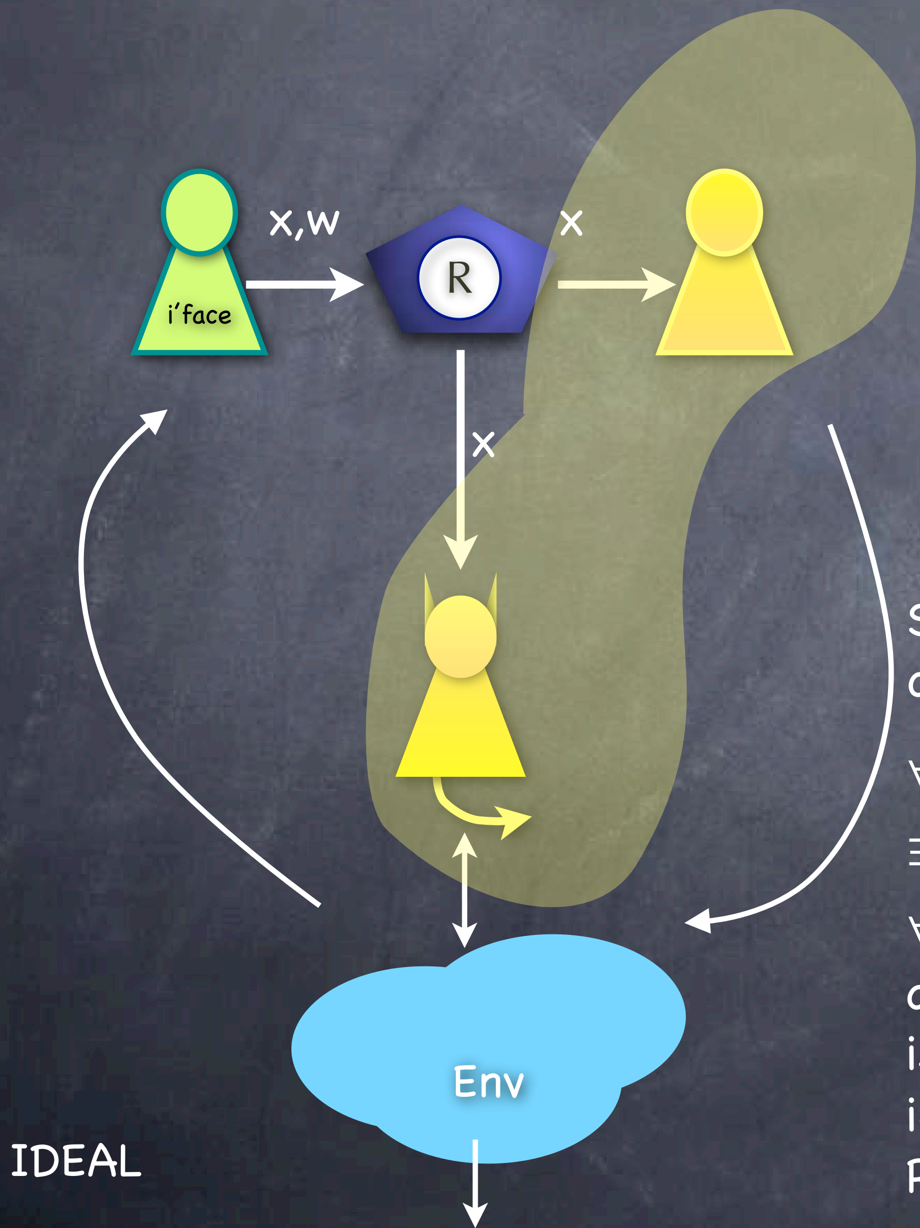


ZK Property (in other pict's)

Classical definition uses simulation only for corrupt receiver;



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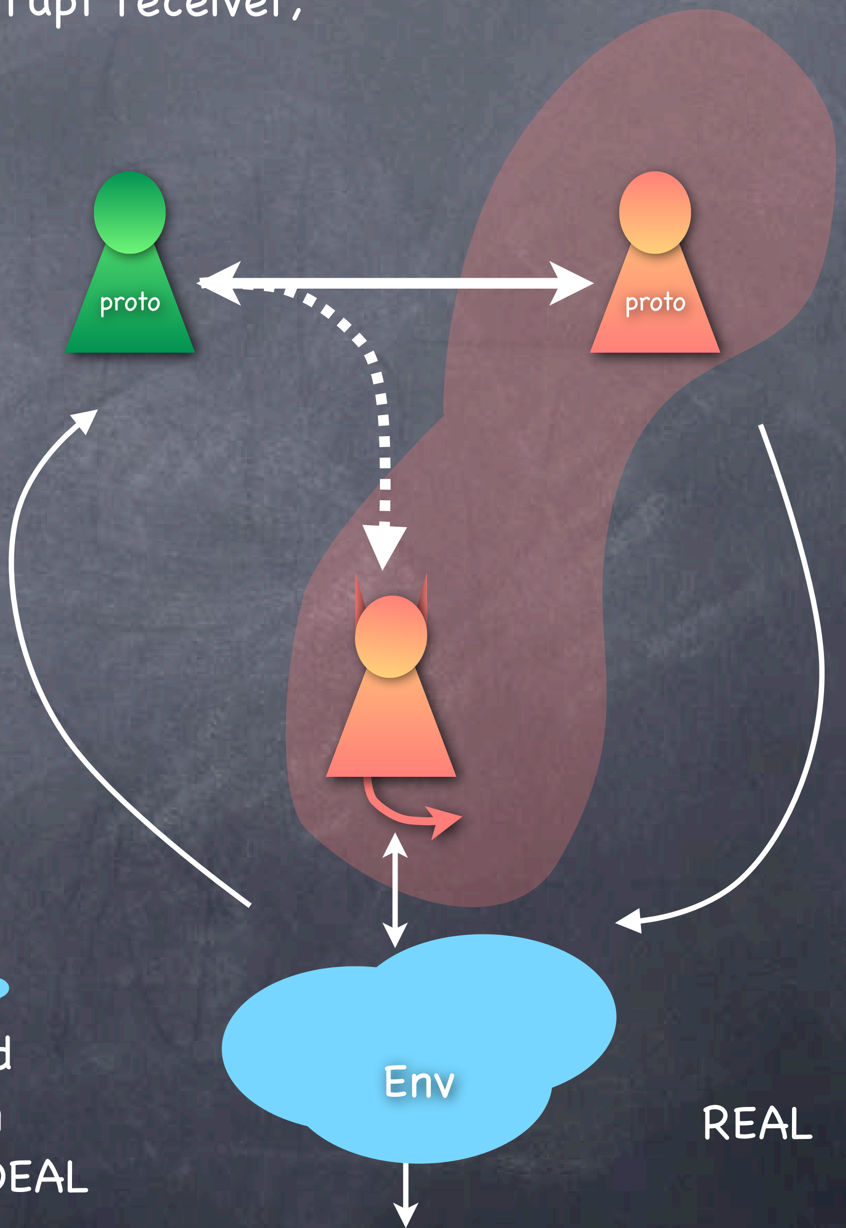
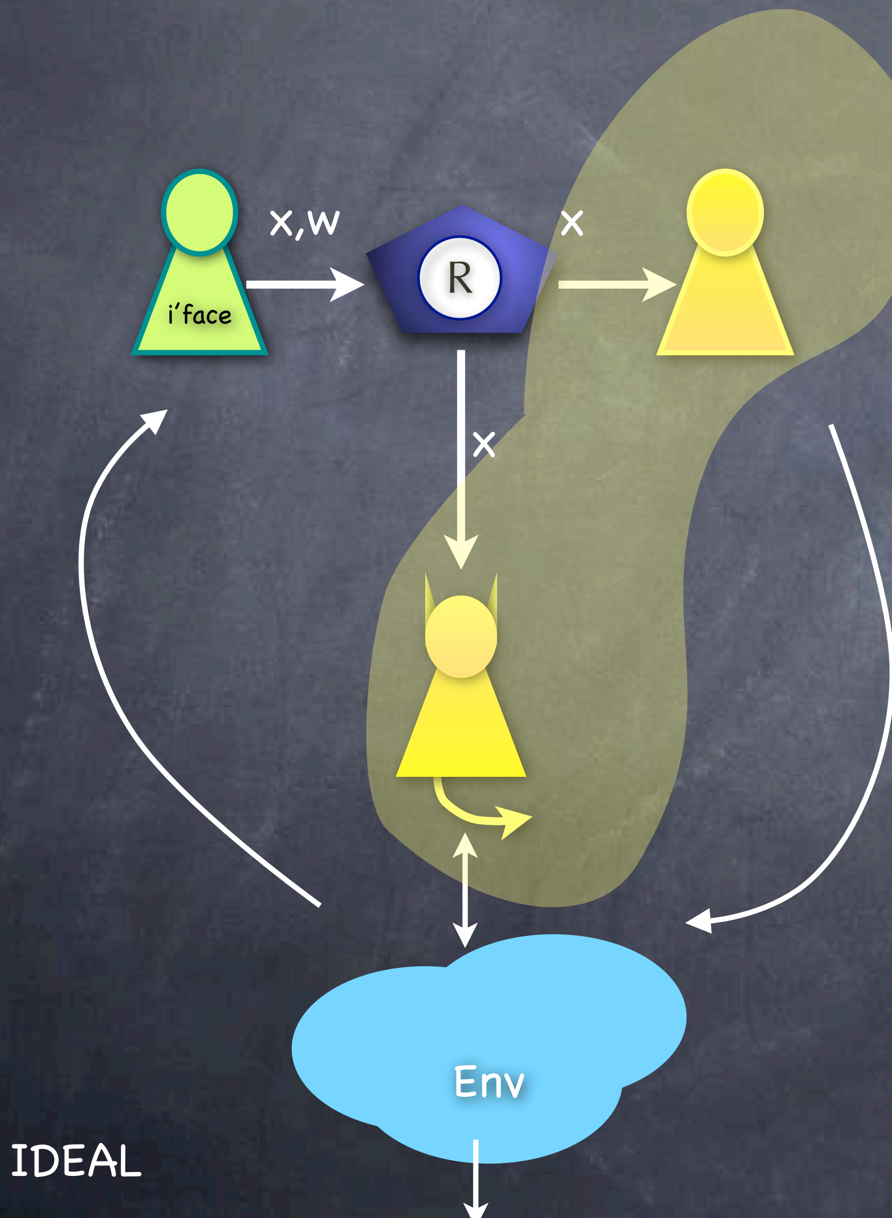


ZK Property (in other pict's)

Classical definition uses simulation only for corrupt receiver; and uses only standalone security: Environment gets only a transcript at the end



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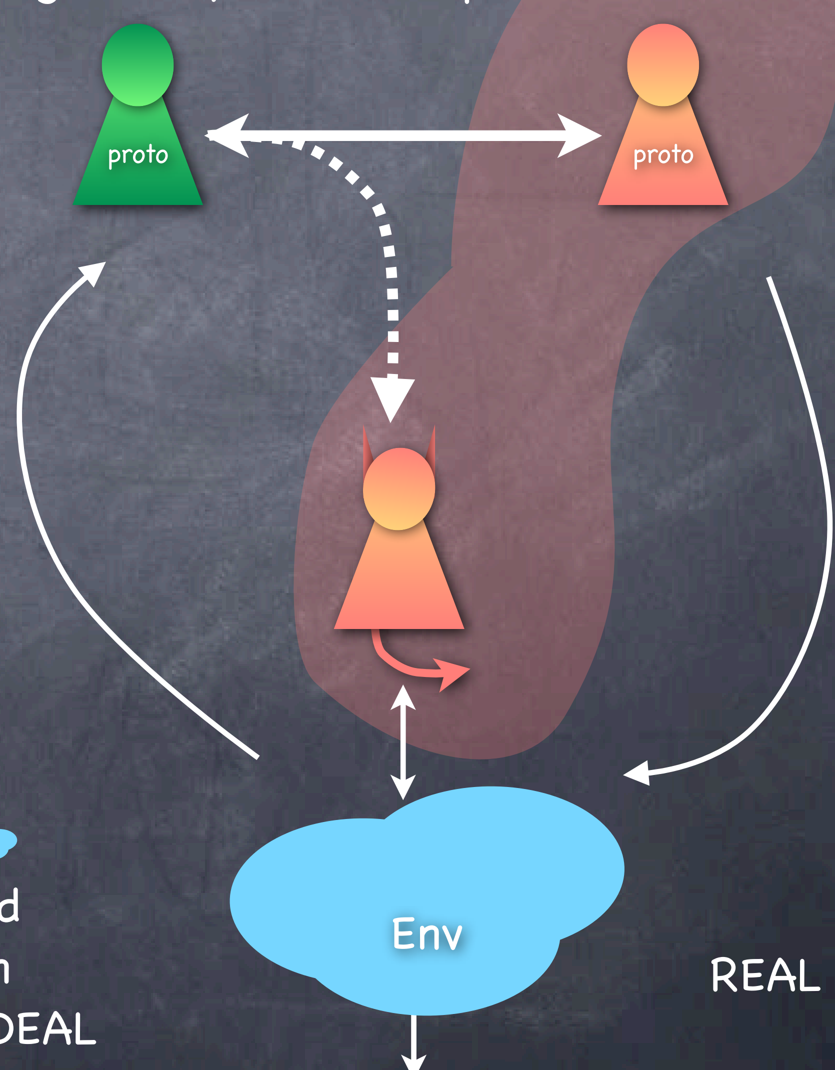
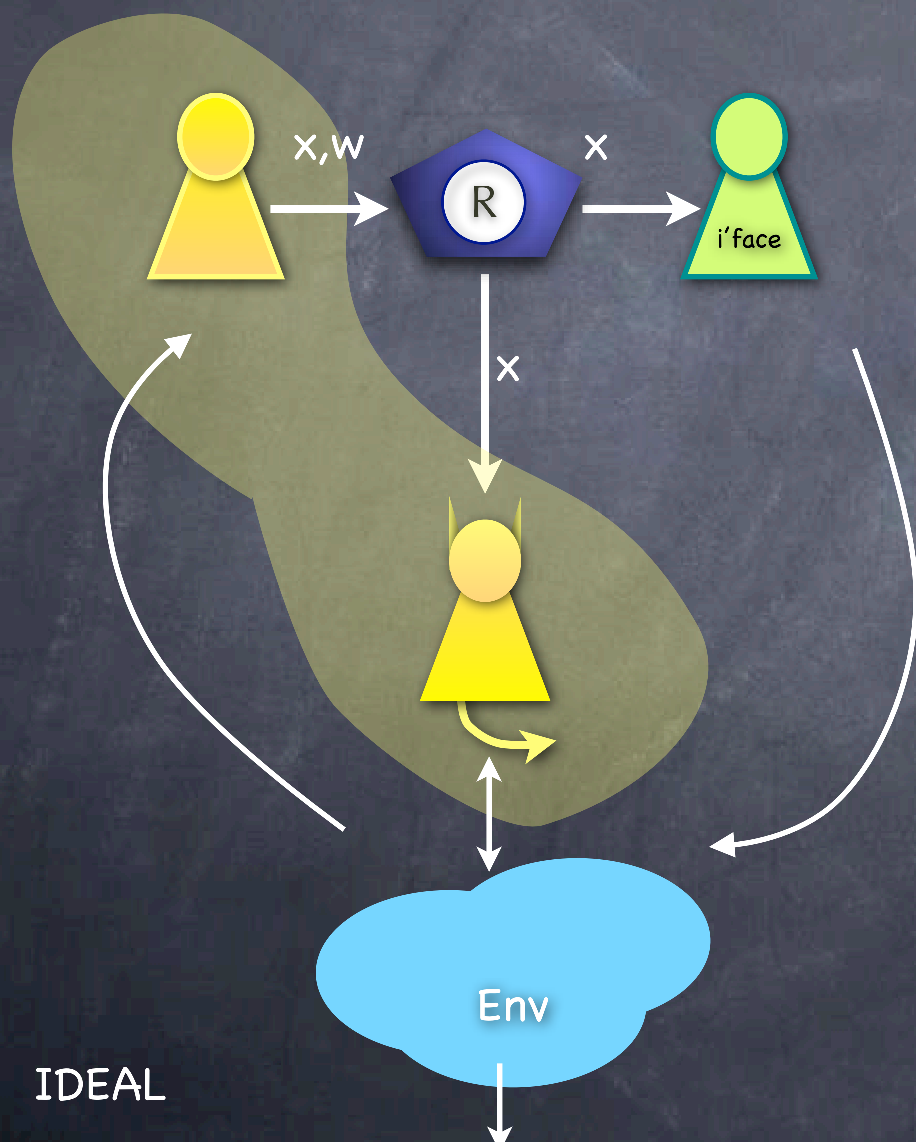


SIM ZK



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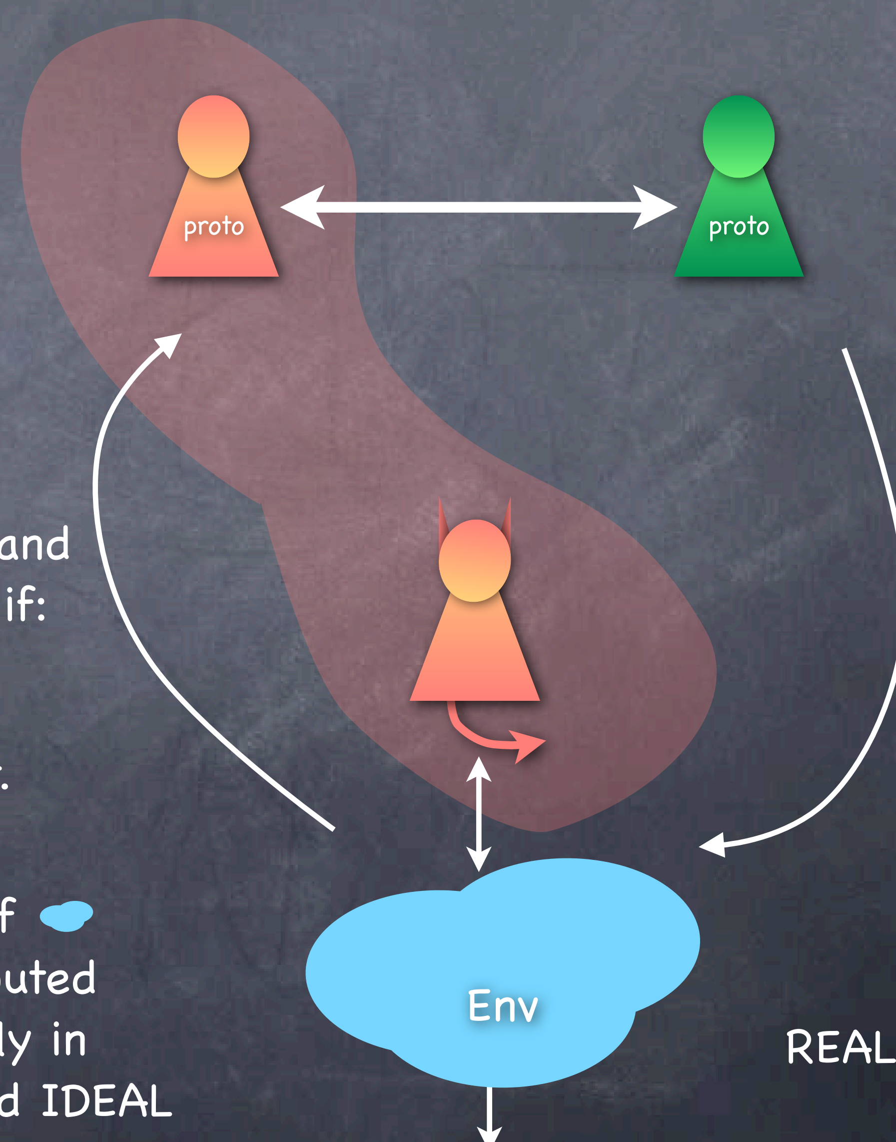
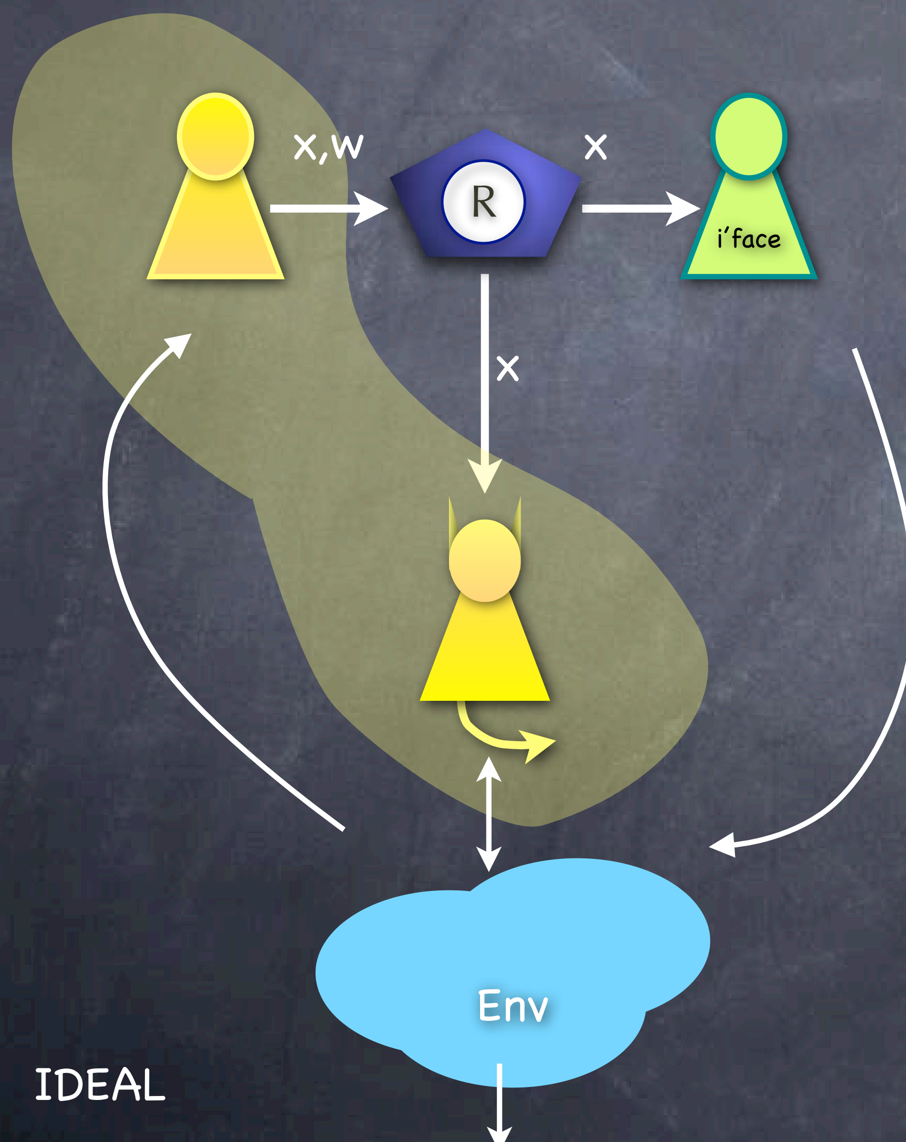


SIM ZK

- SIM-ZK would require simulation also when prover is corrupt



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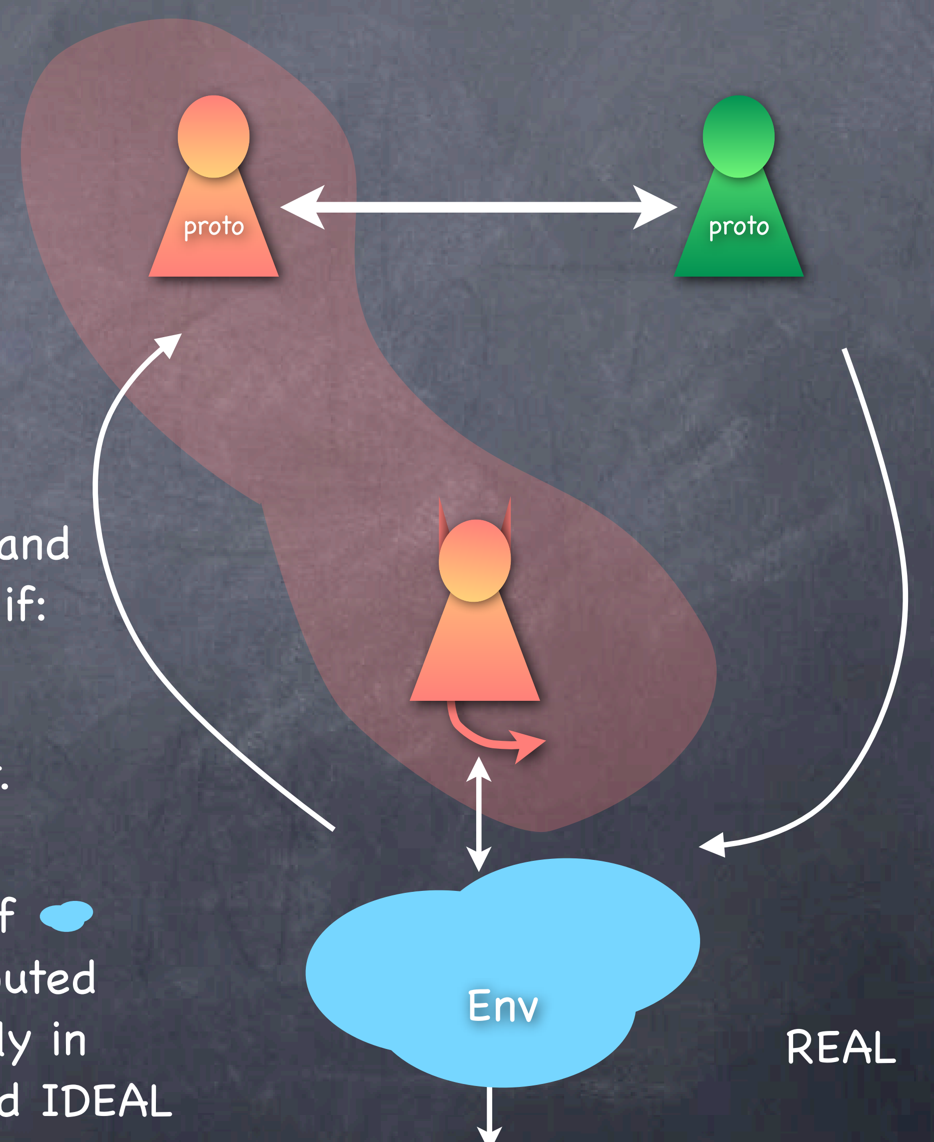

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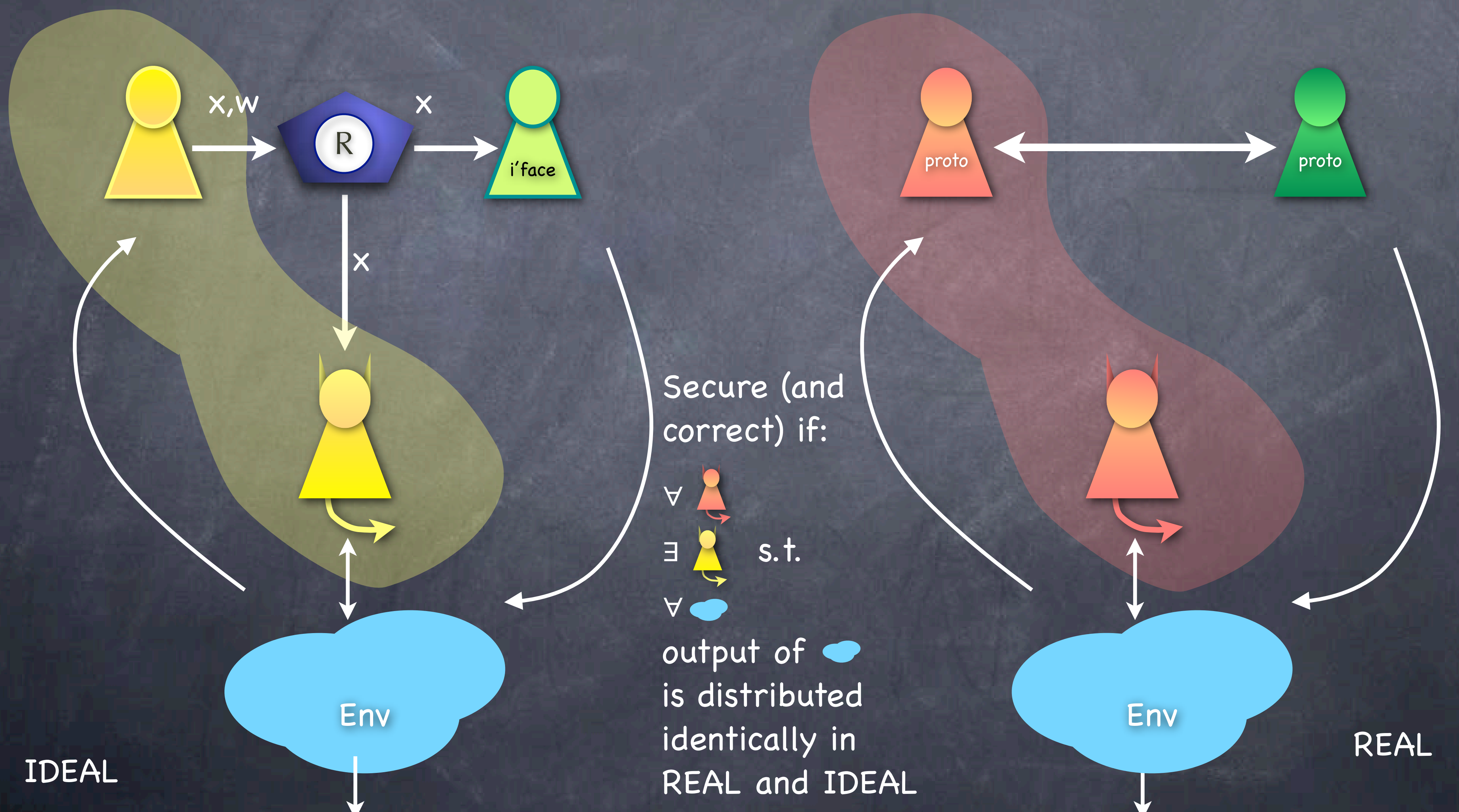
  - Then simulator is a witness extractor



SIM ZK

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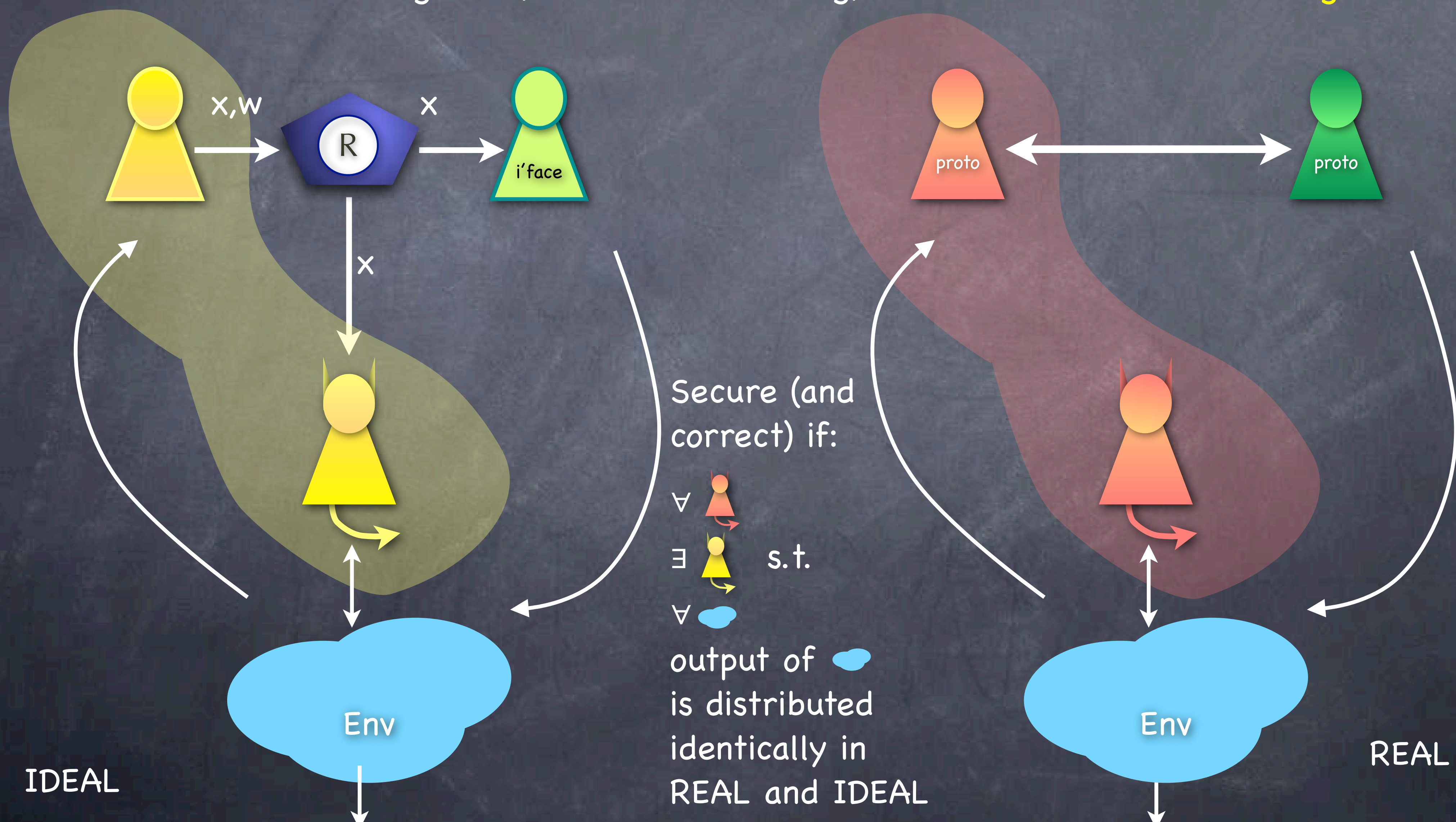
  - Then simulator is a witness extractor
- Adding this (in standalone setting) makes it a Proof of Knowledge



A ZK Proof for Graph Colorability

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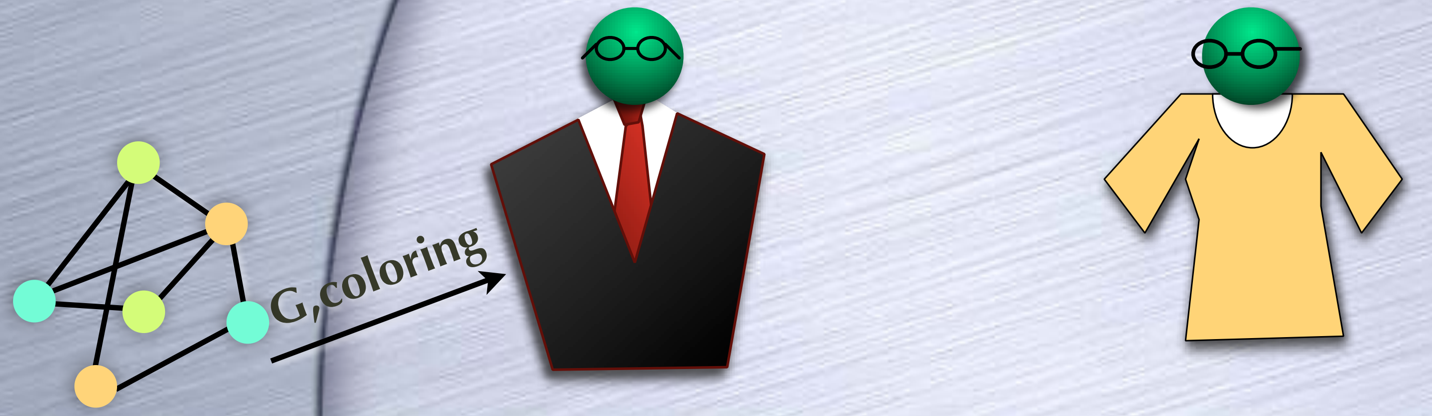
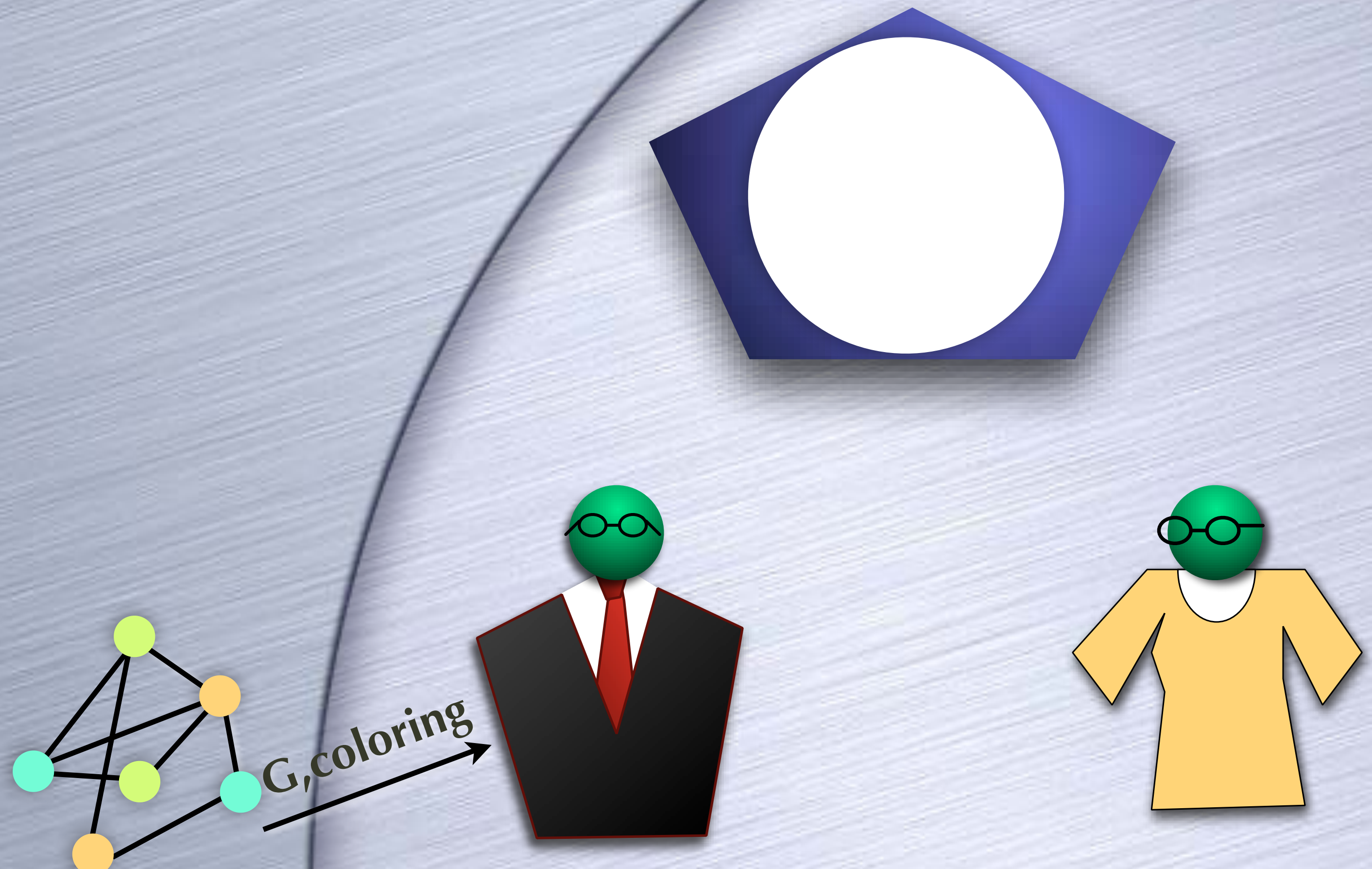


A ZK Proof for Graph Colorability

- Uses a commitment protocol as a subroutine



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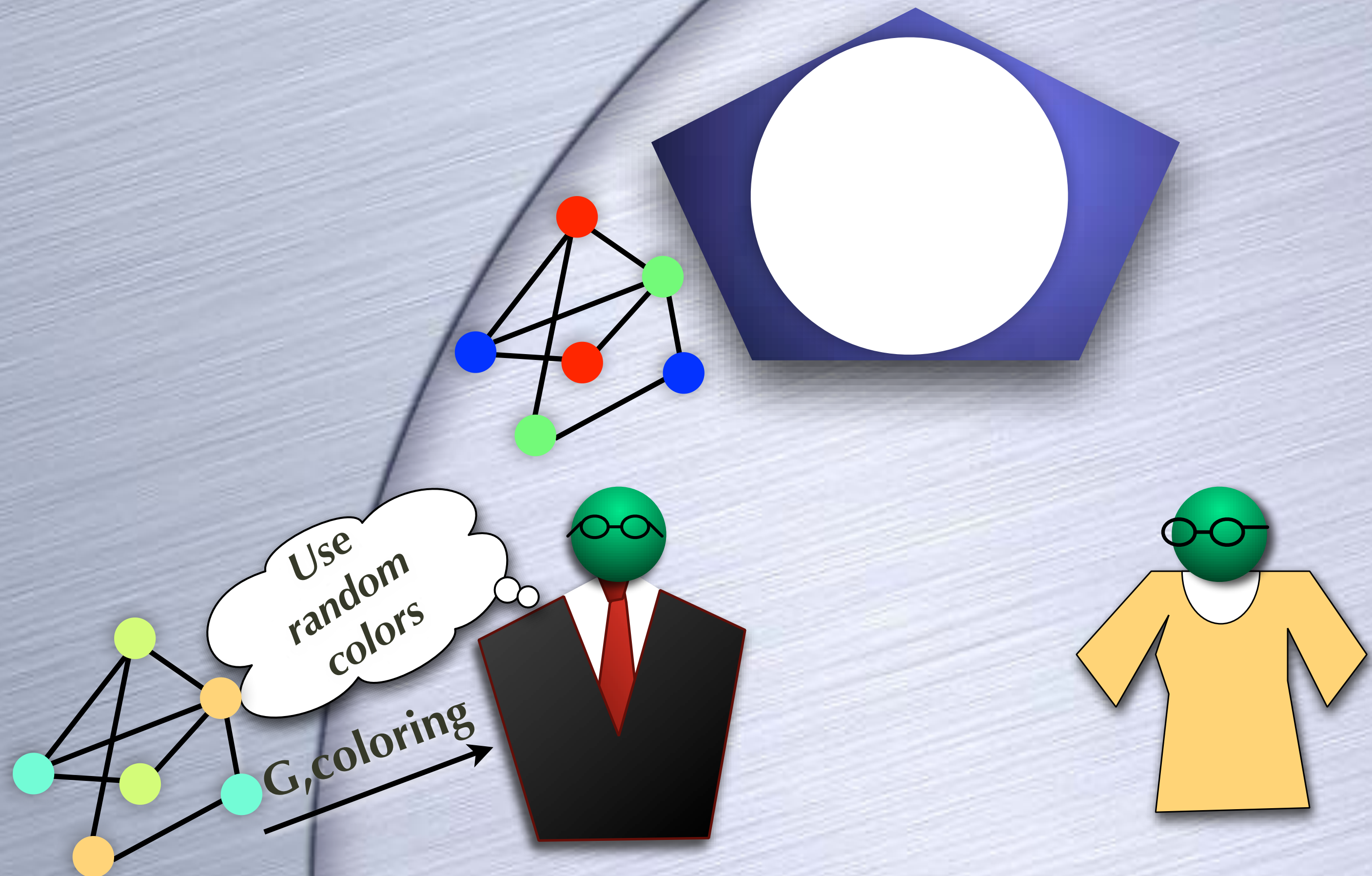

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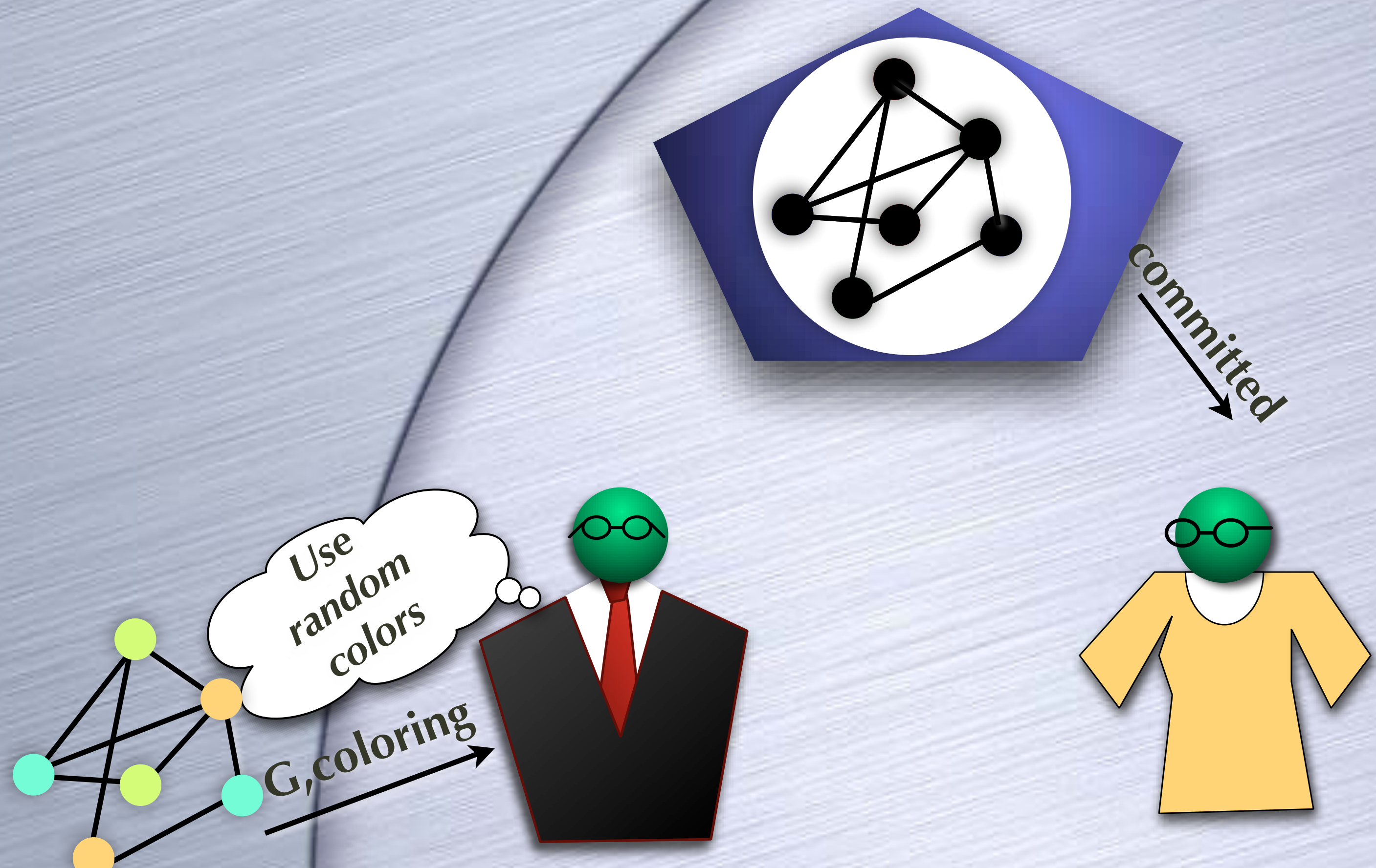

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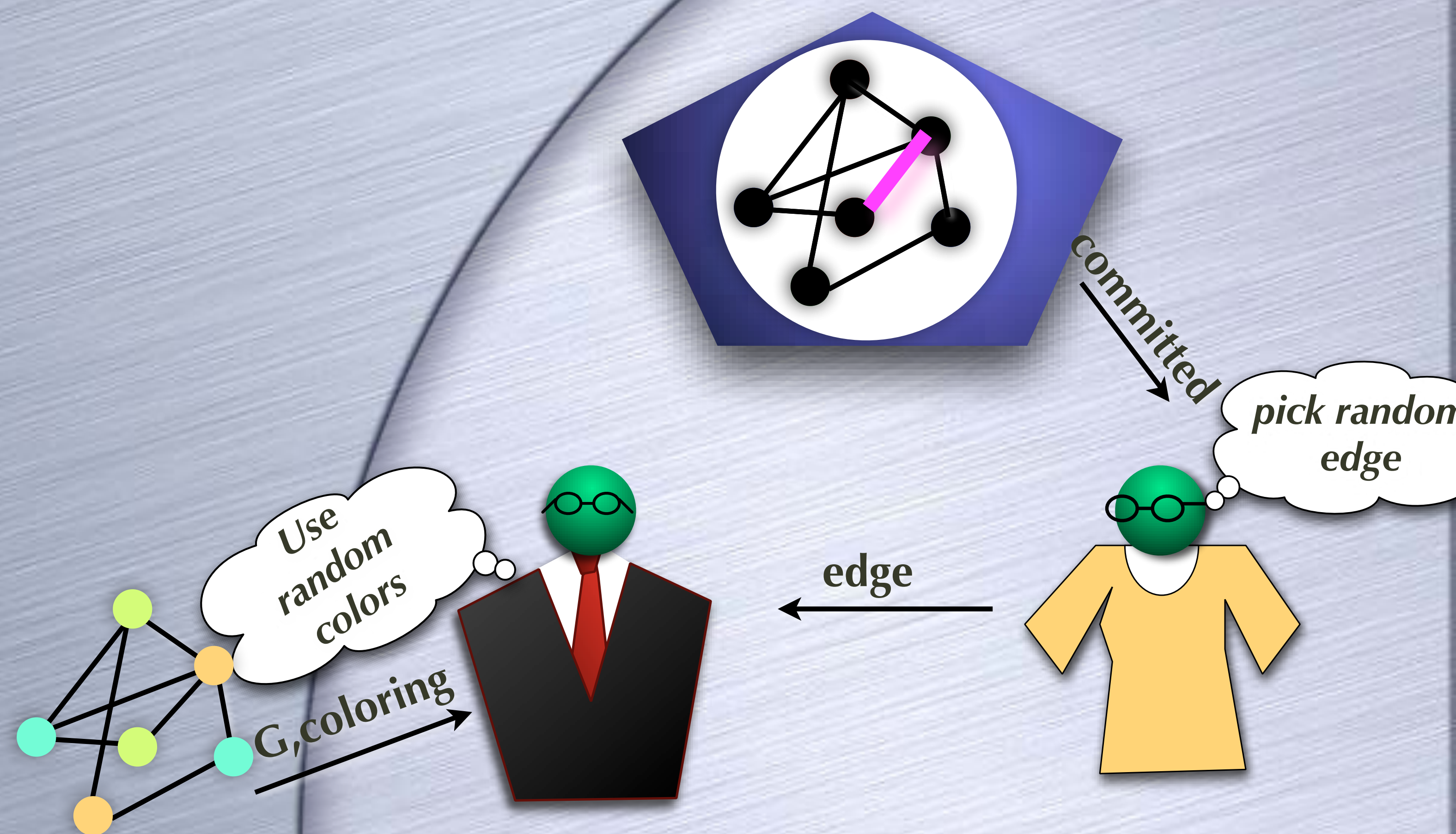

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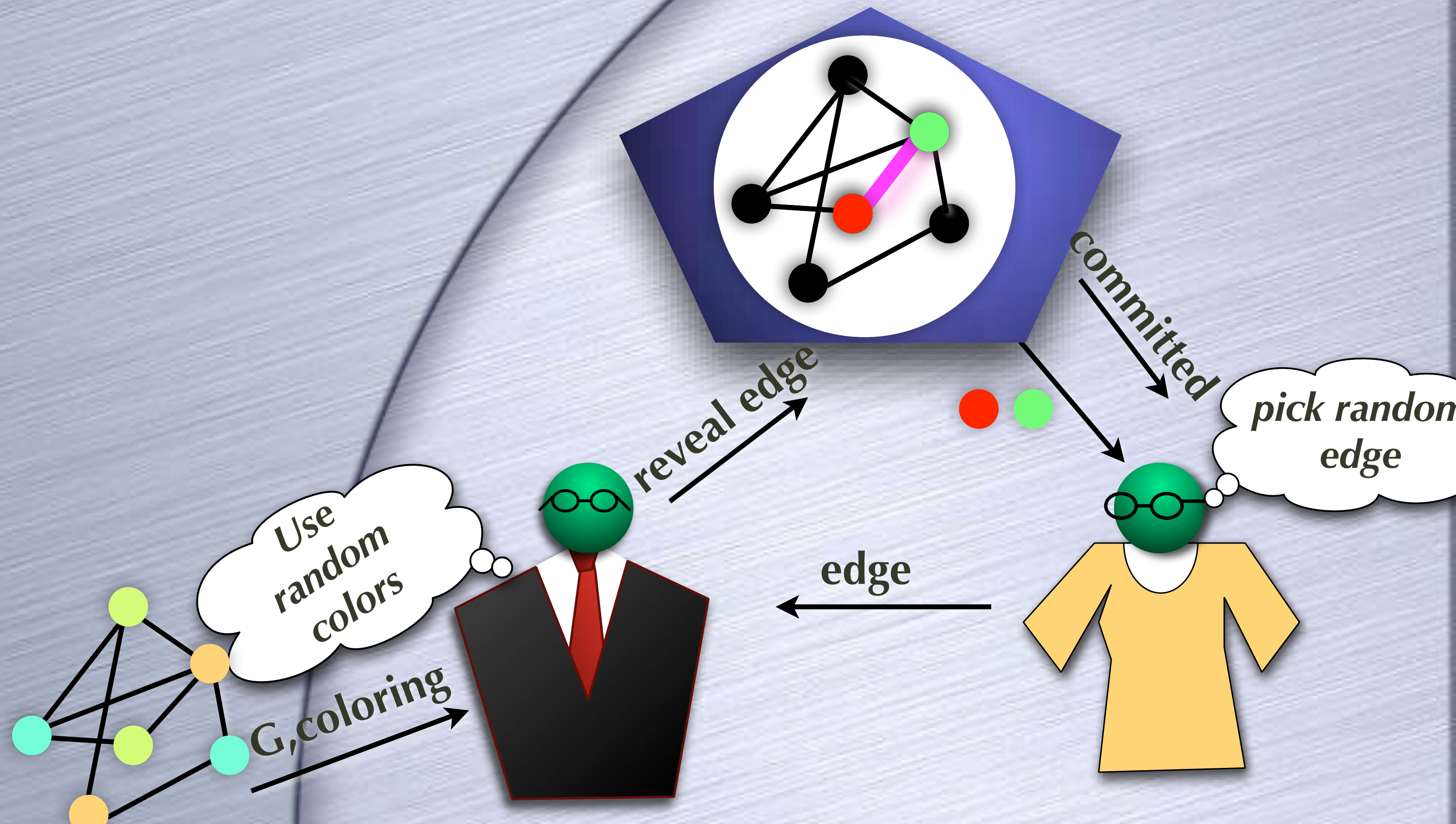

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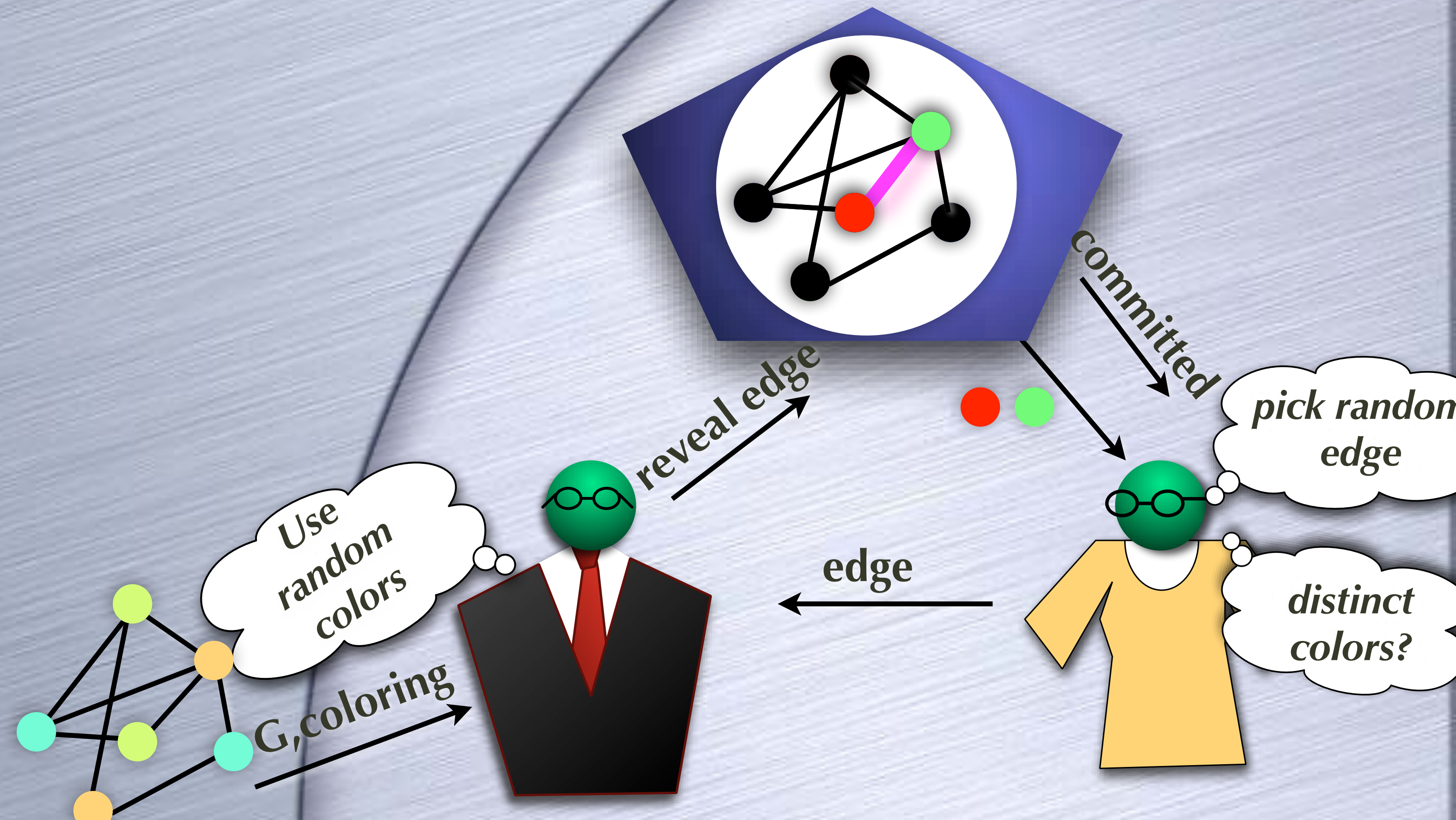

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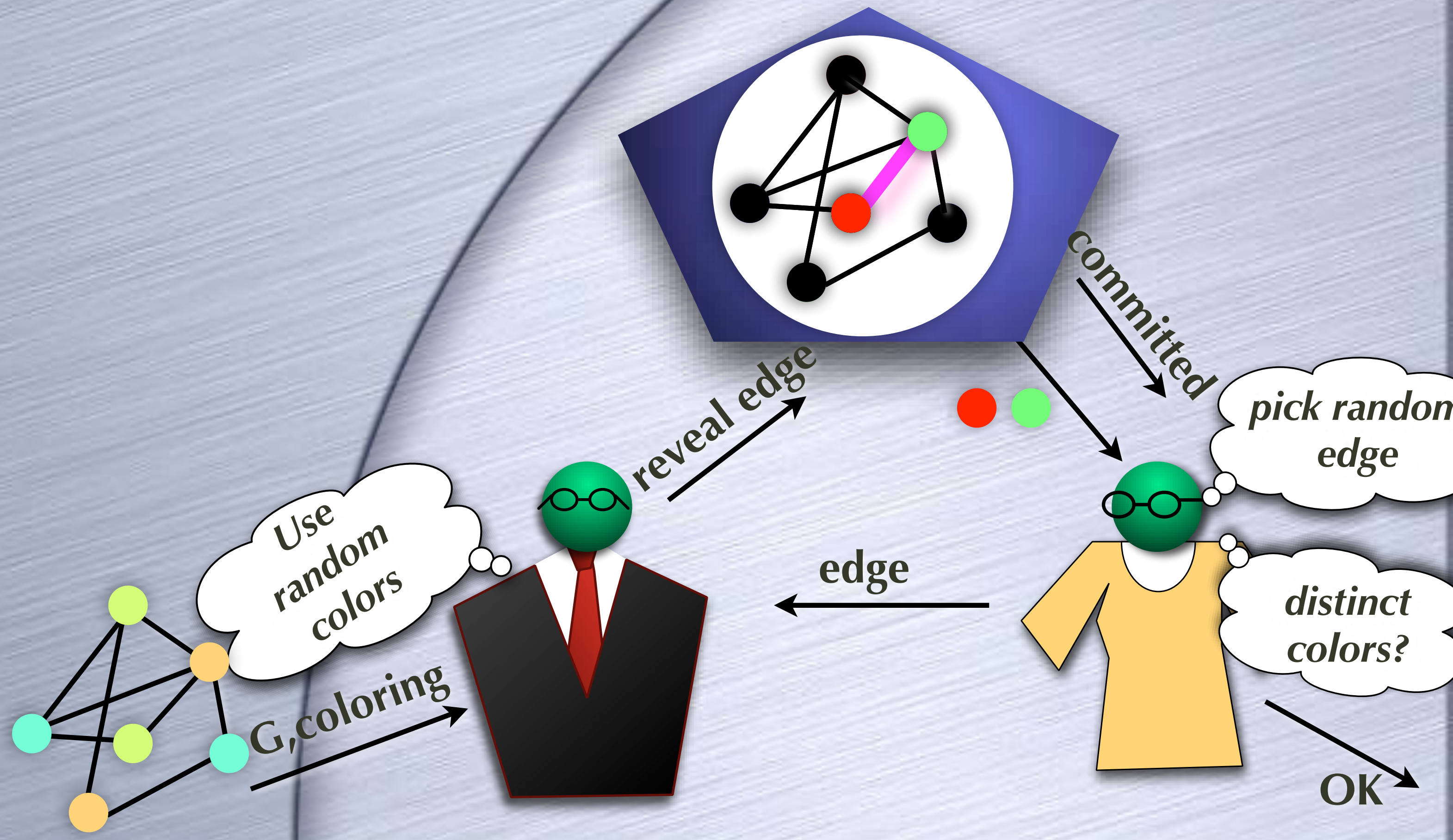
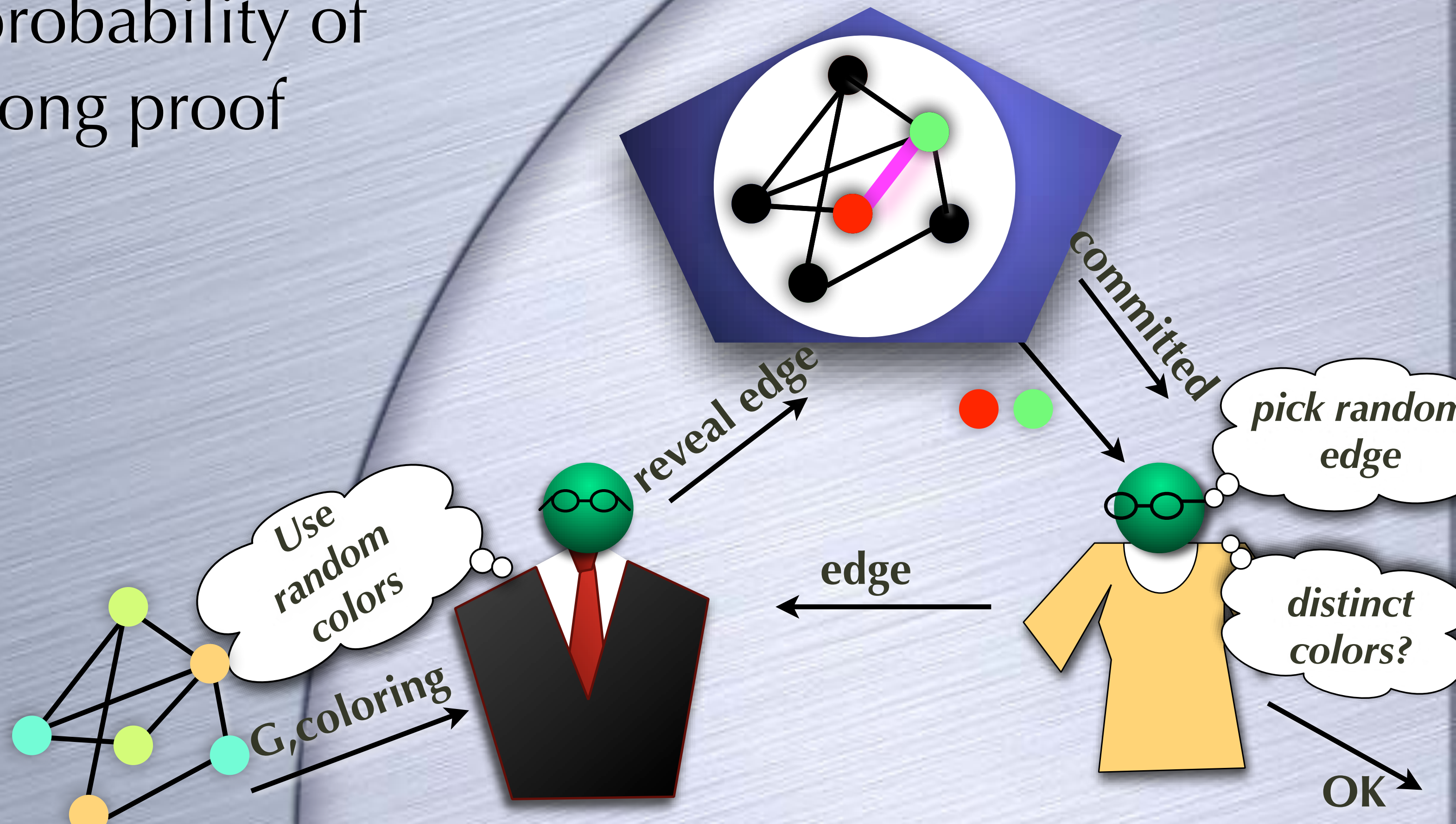


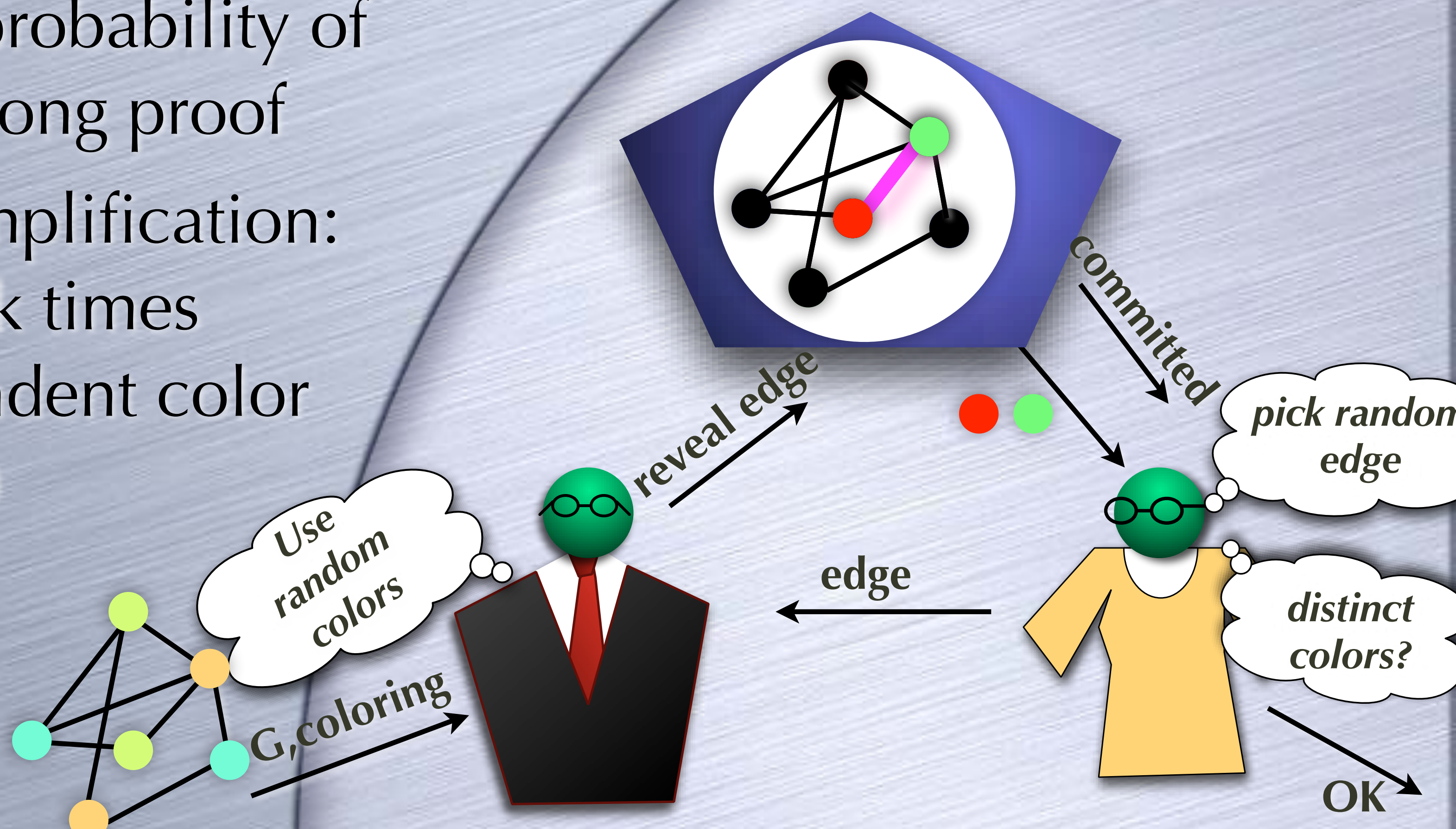
A ZK Proof for Graph Colorability

- Uses a commitment protocol as a subroutine
- At least 1/m probability of catching a wrong proof



A ZK Proof for Graph Colorability

- Uses a commitment protocol as a subroutine
- At least 1/m probability of catching a wrong proof
- Soundness amplification: Repeat say mk times (with independent color permutations)



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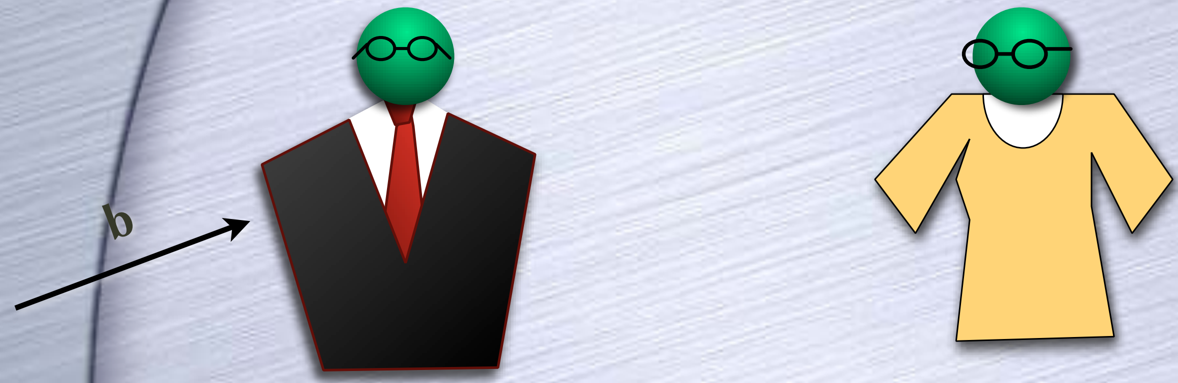

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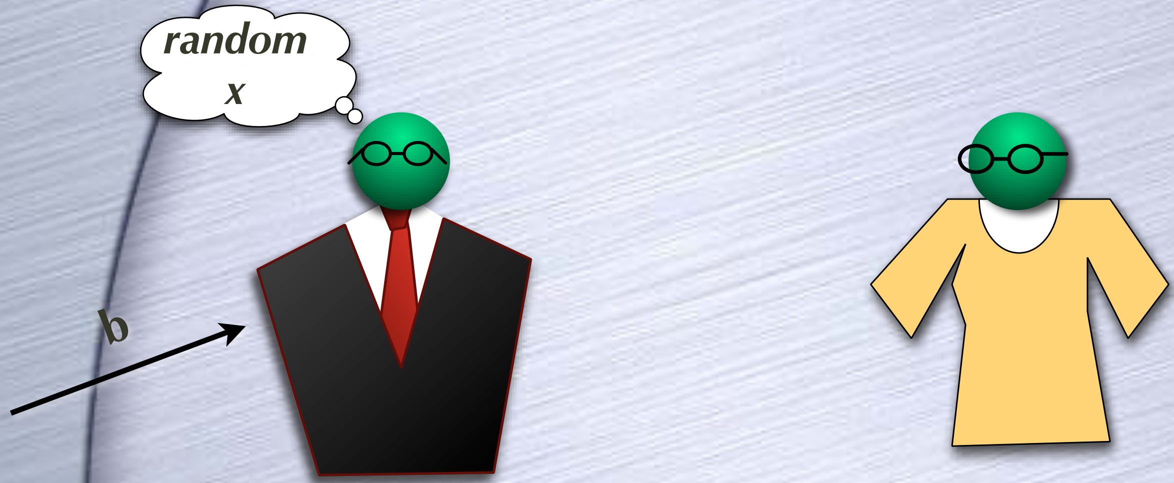

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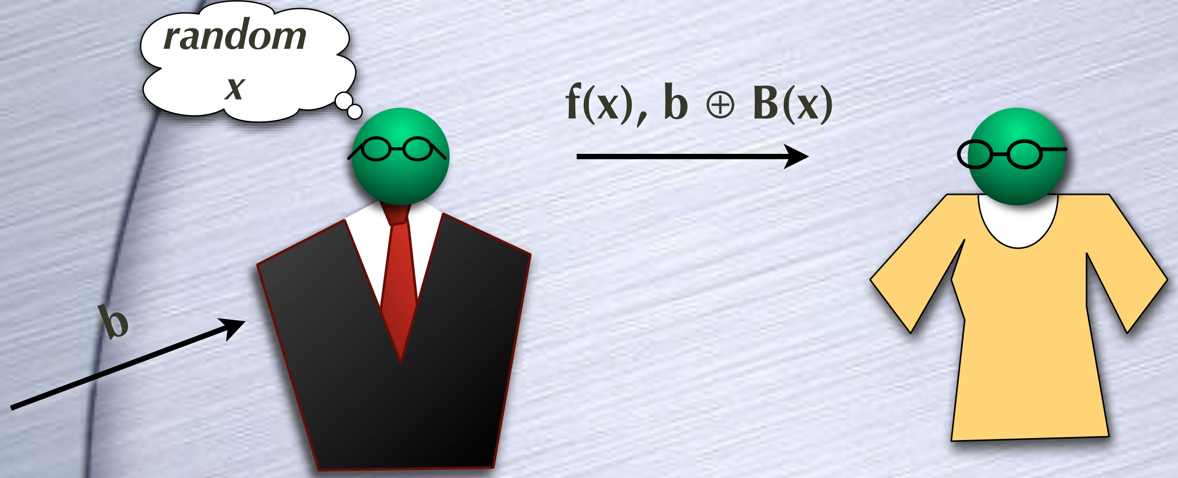

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- Using a OWP f and a hardcore predicate for it B
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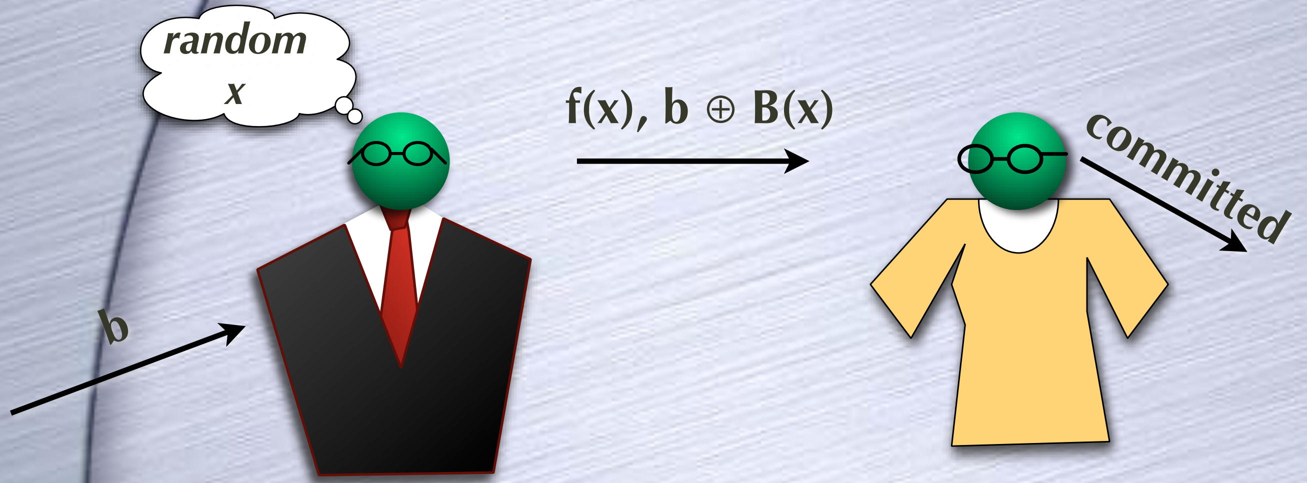

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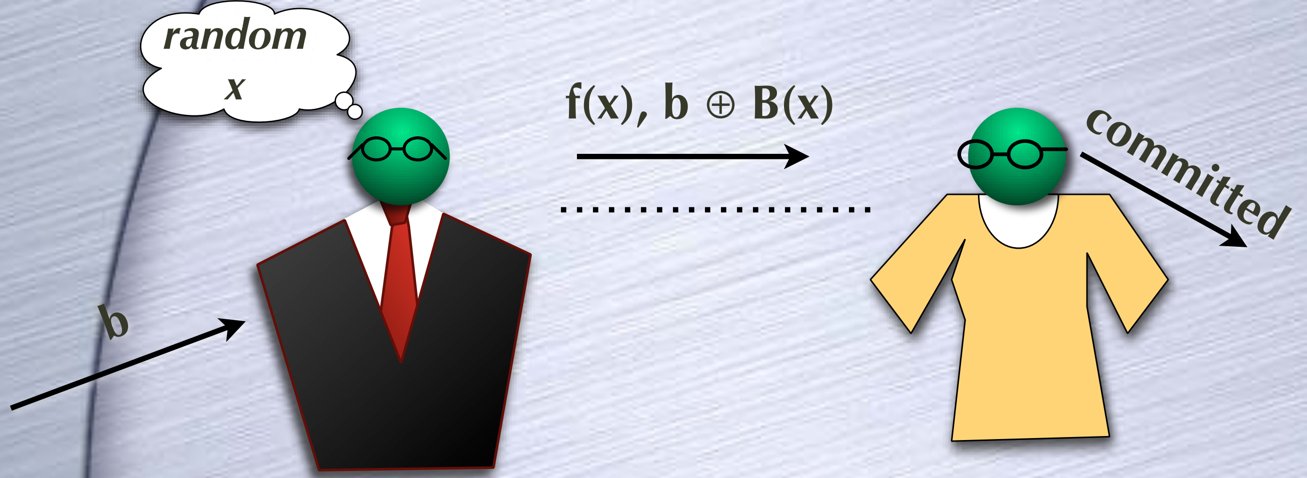

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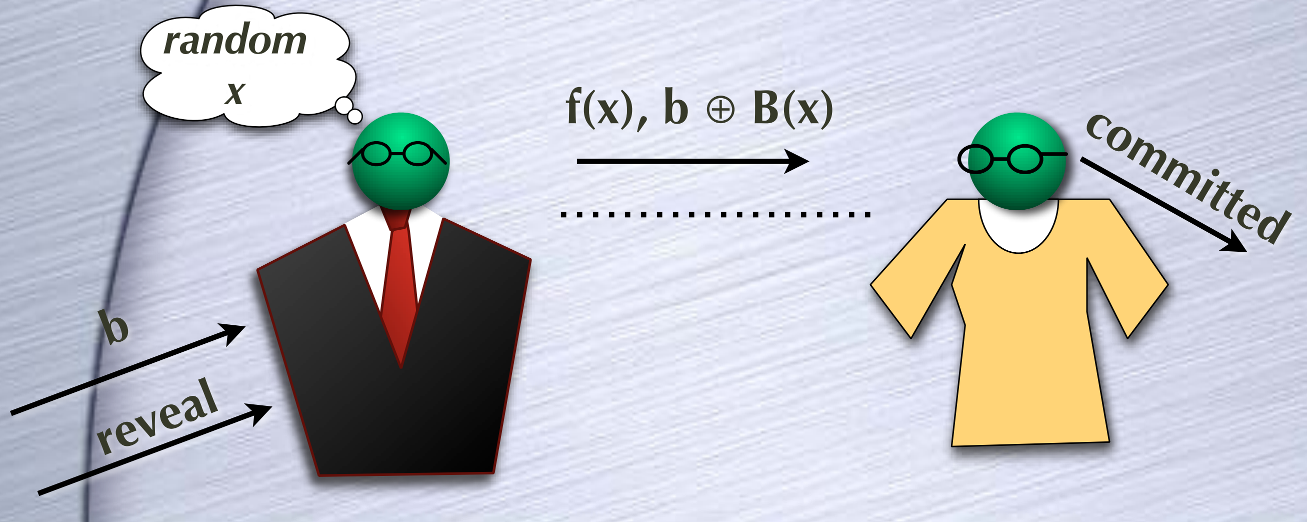

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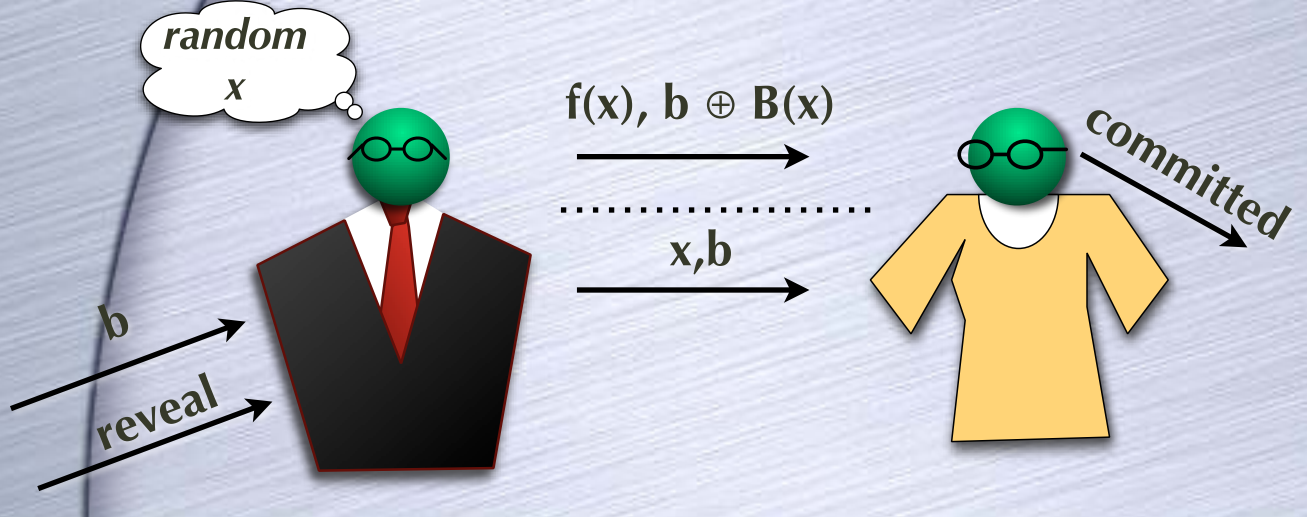

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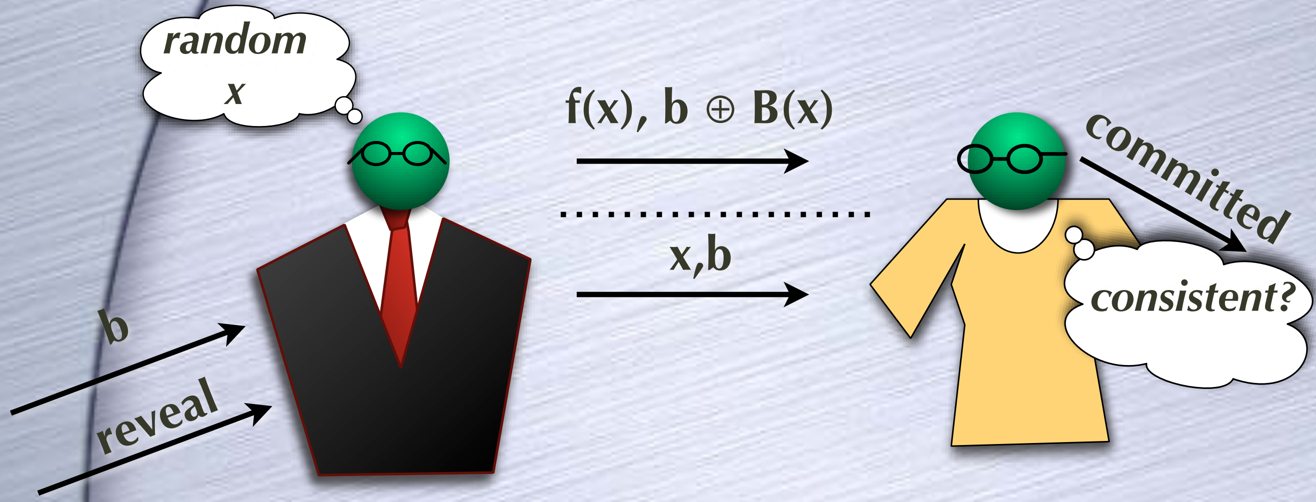

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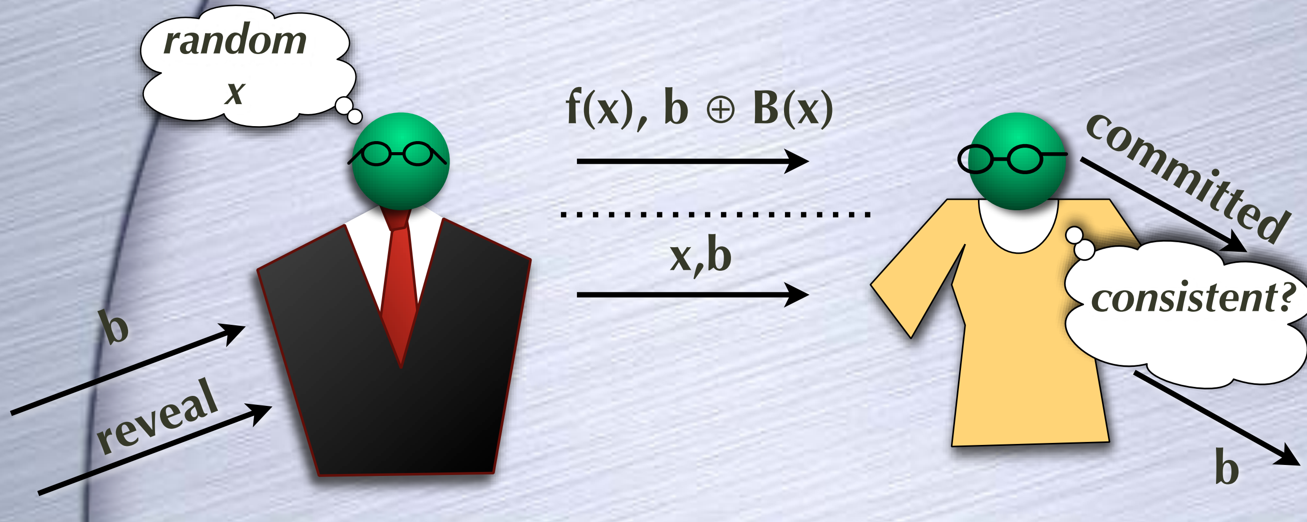

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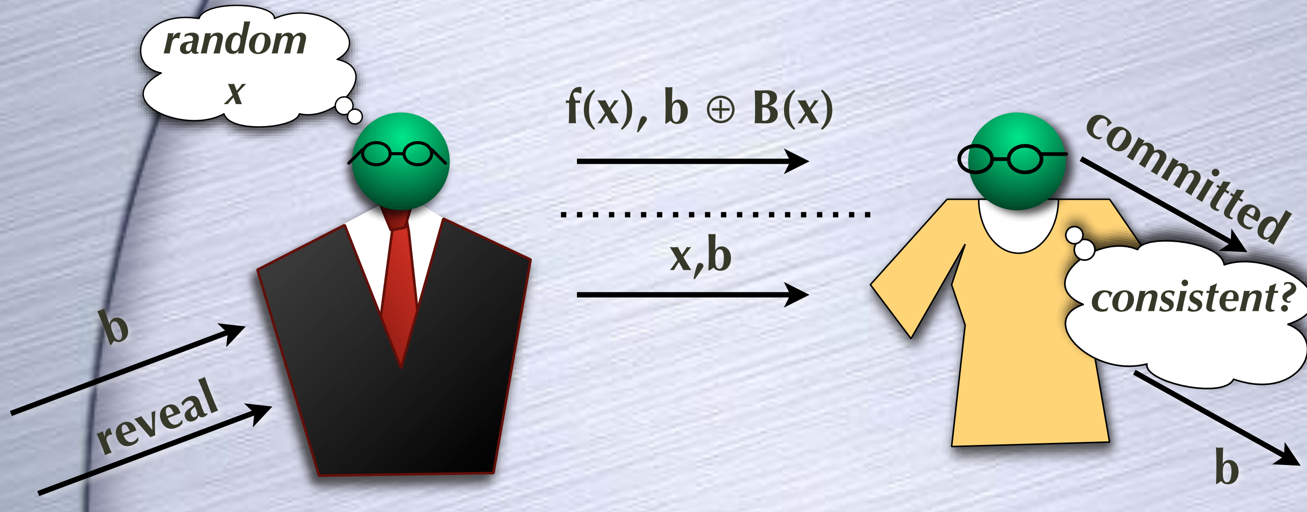

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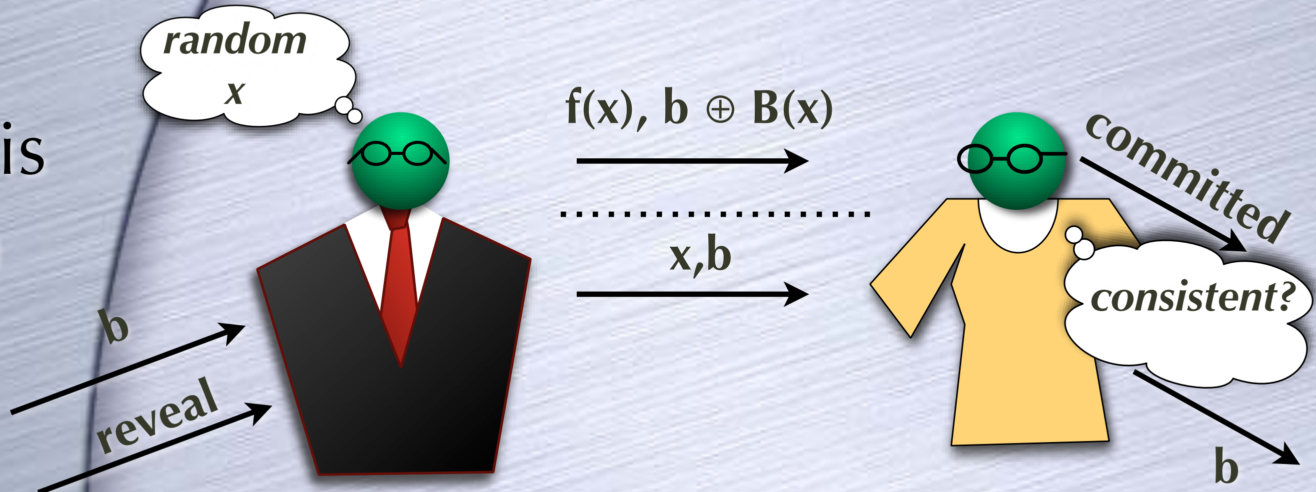

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  - ZKPoK, Statistical ZK Arguments, Non-Interactive ZK (using a common random string), Witness-Indistinguishable Proofs, ...

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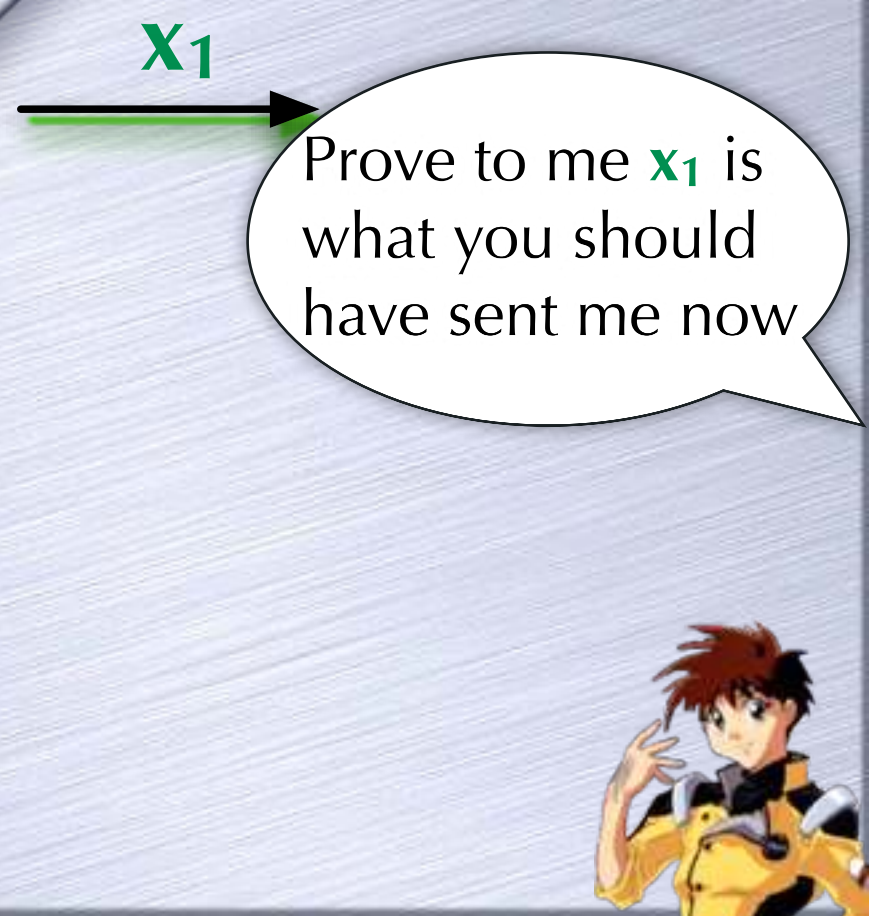

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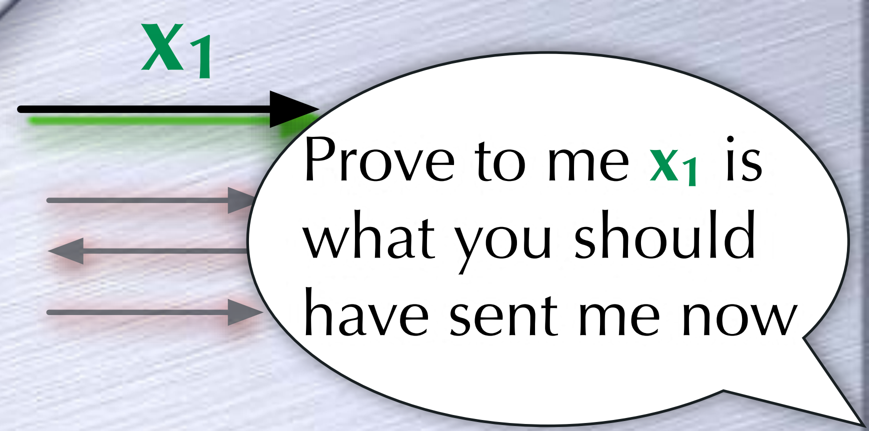

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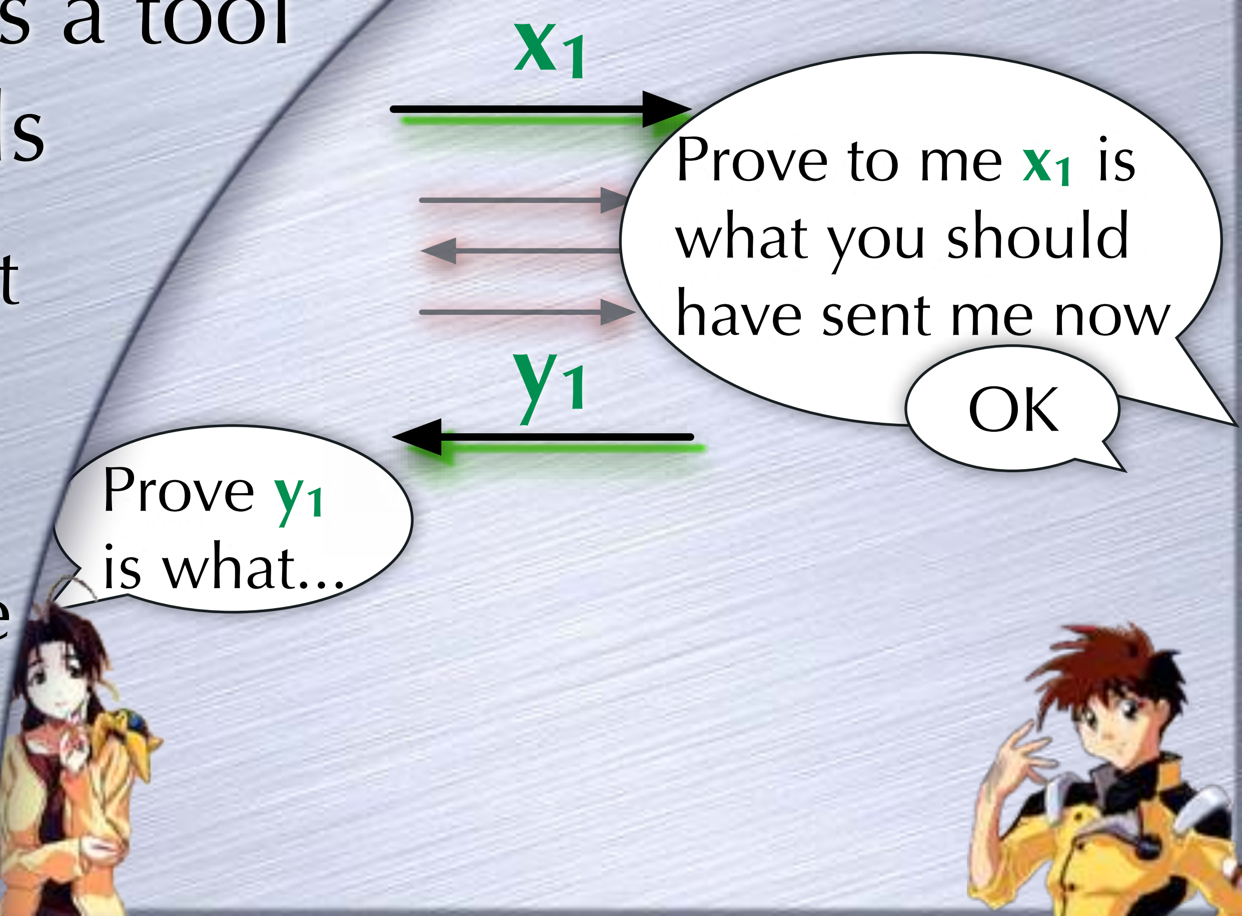

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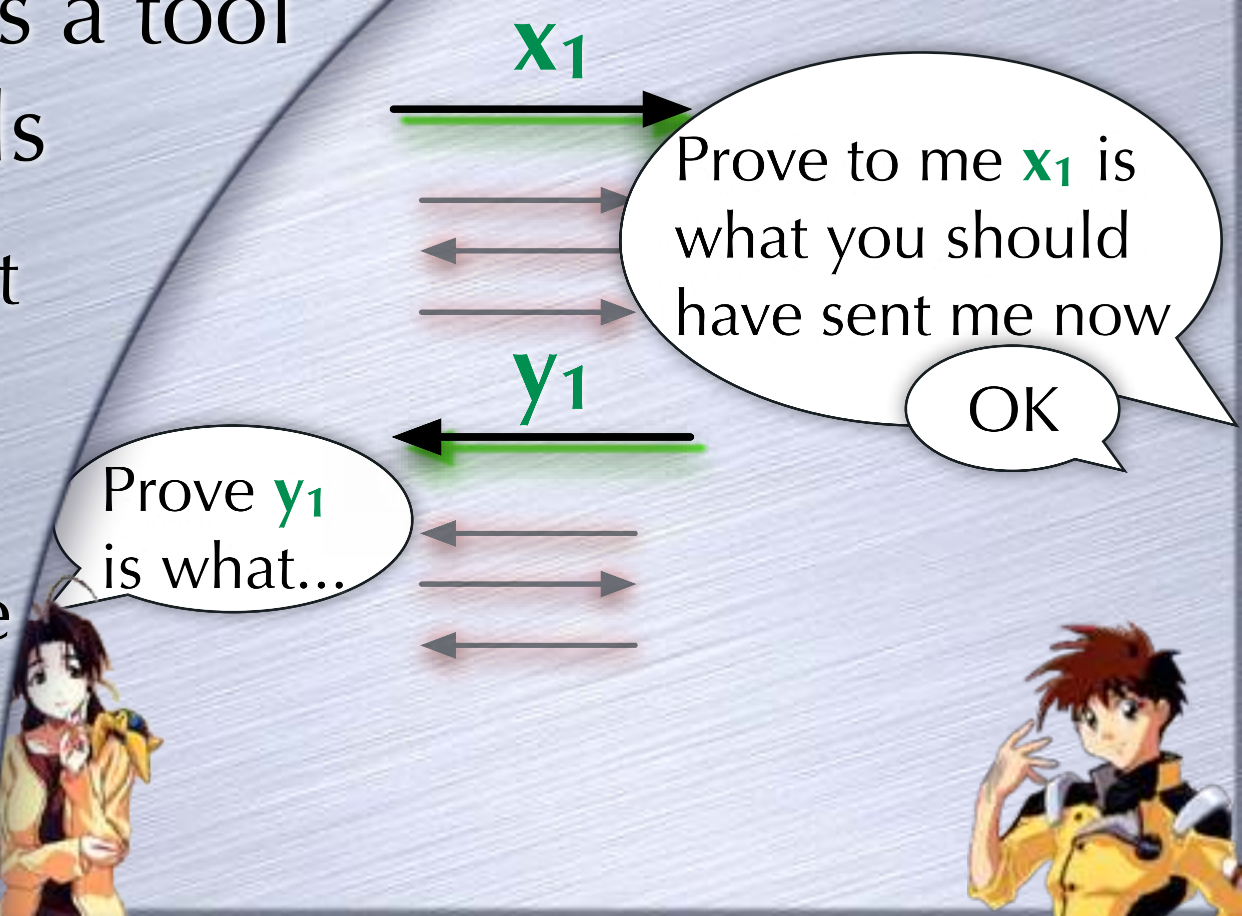

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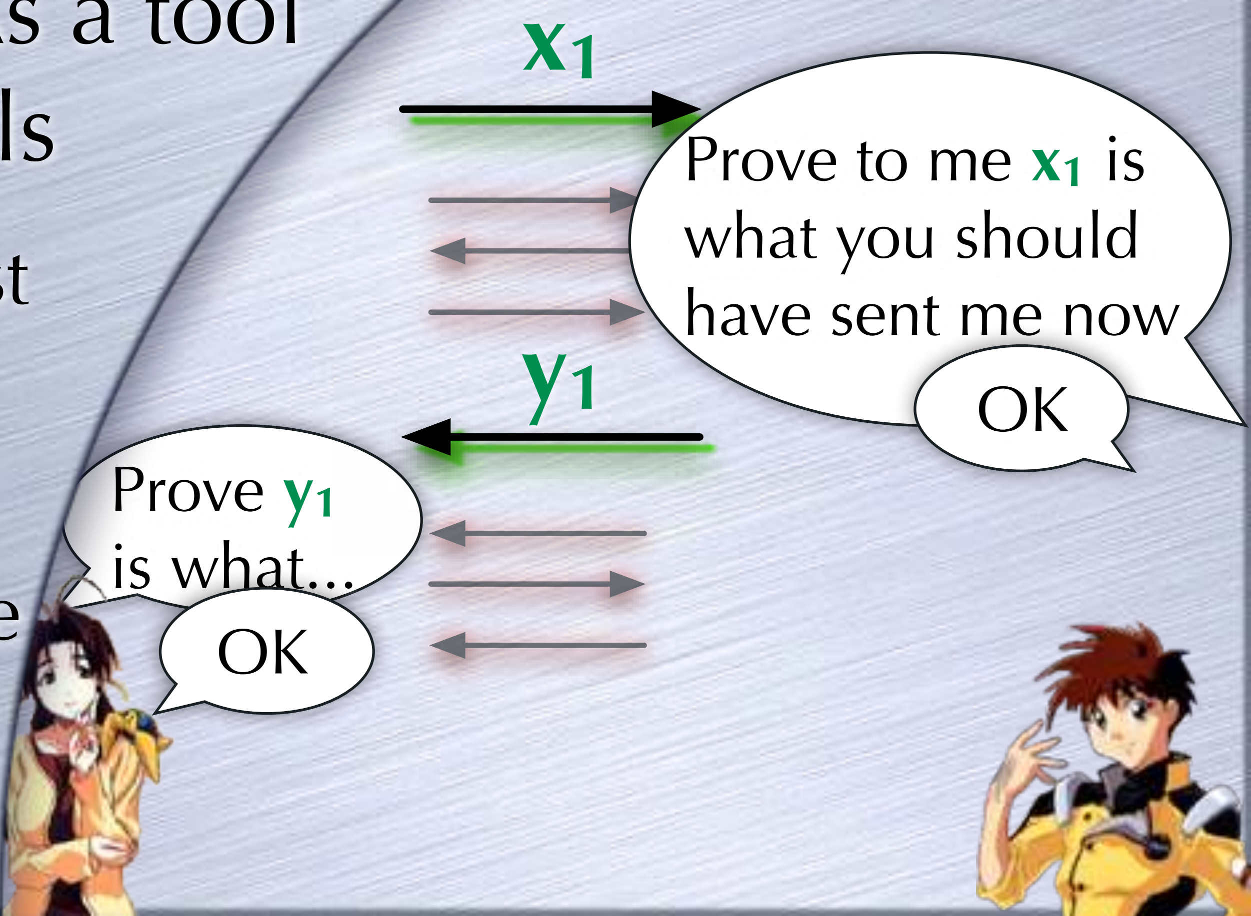

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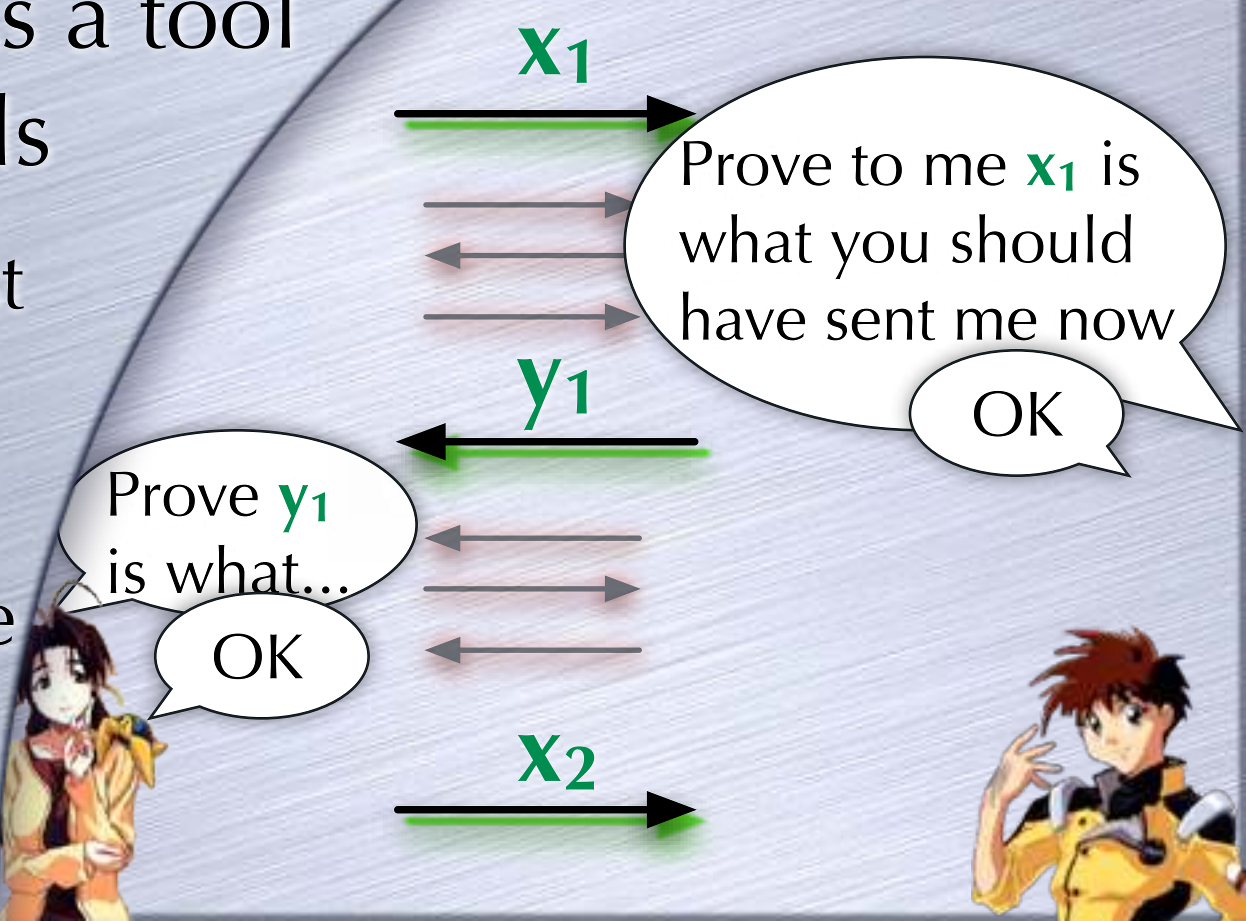

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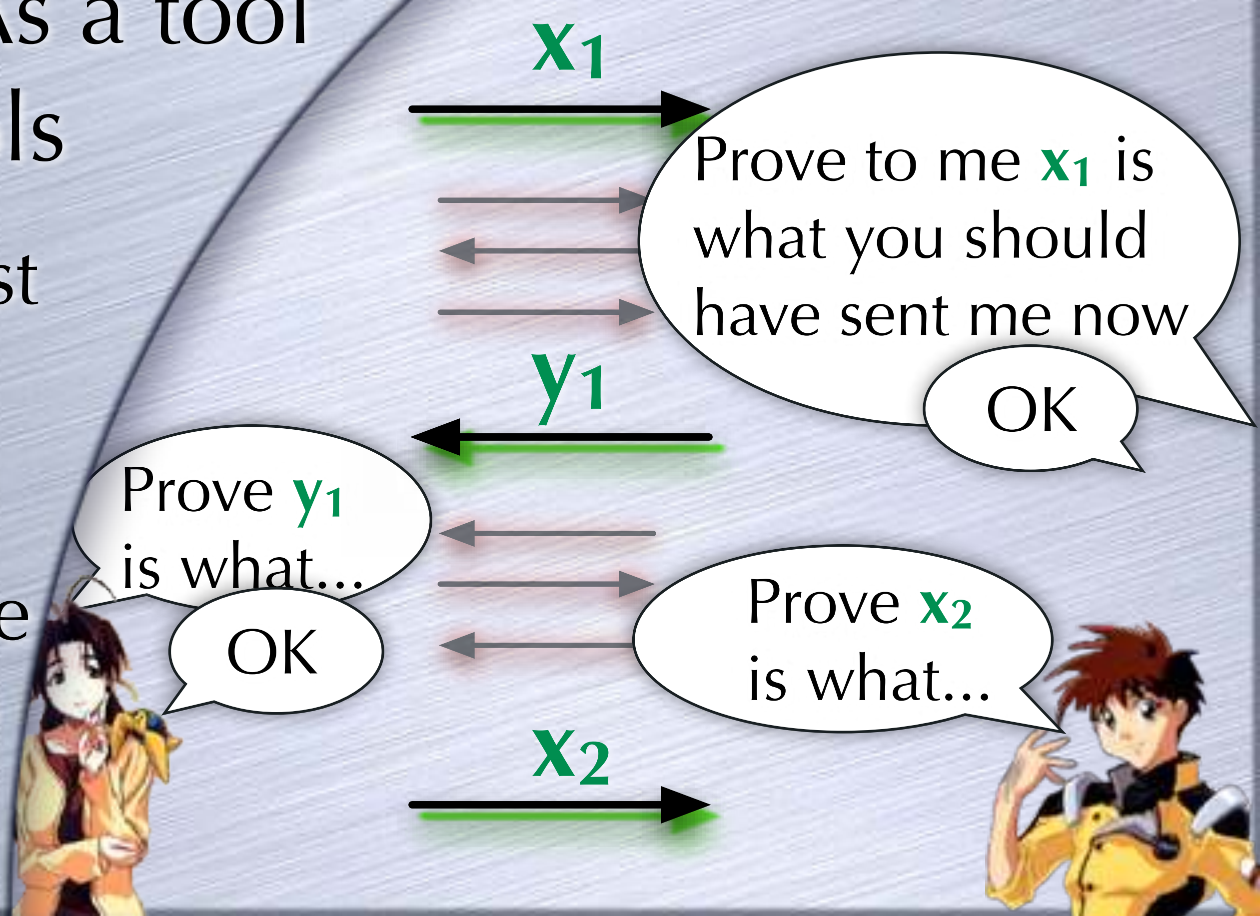

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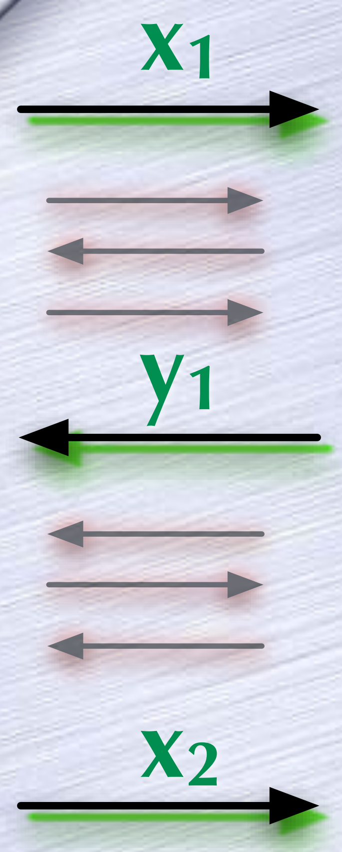

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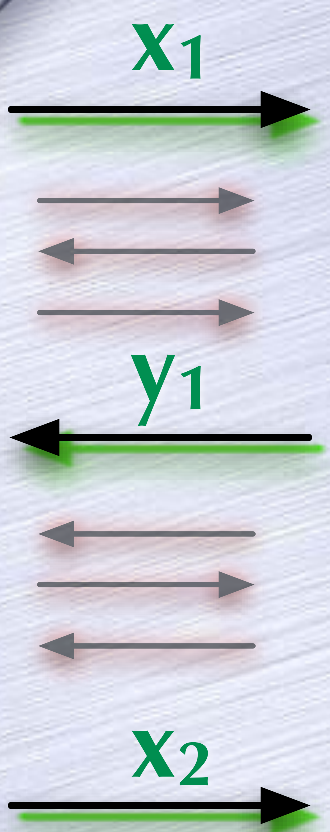

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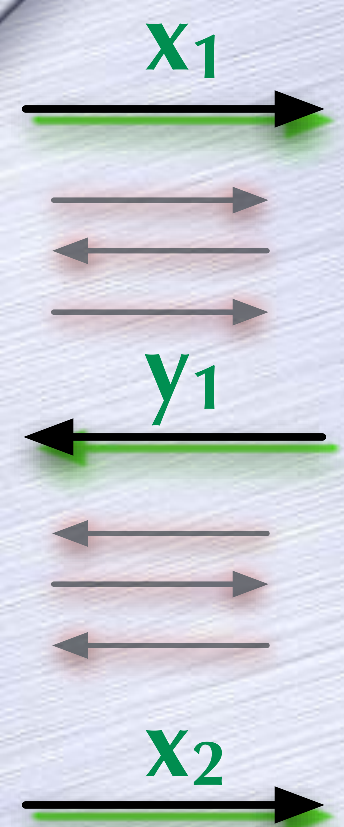

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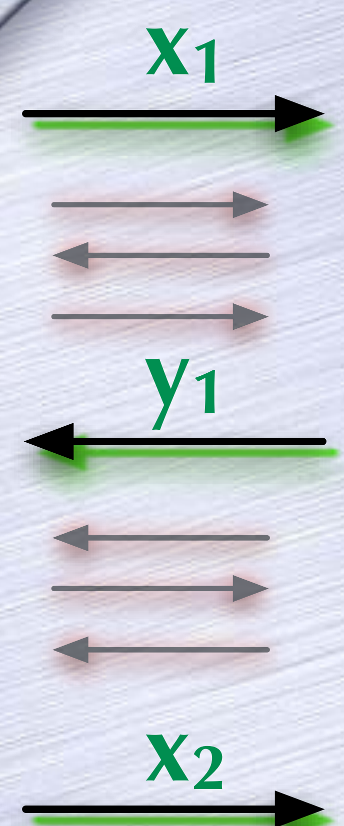

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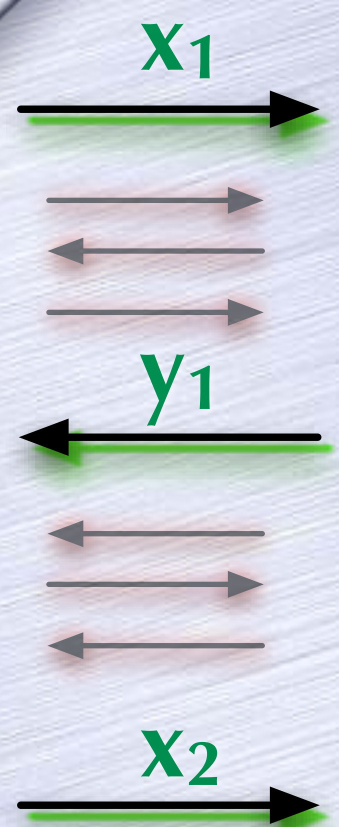
  - In general, to allow composition more complicated protocols



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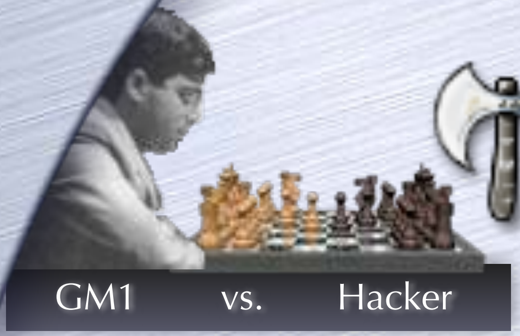

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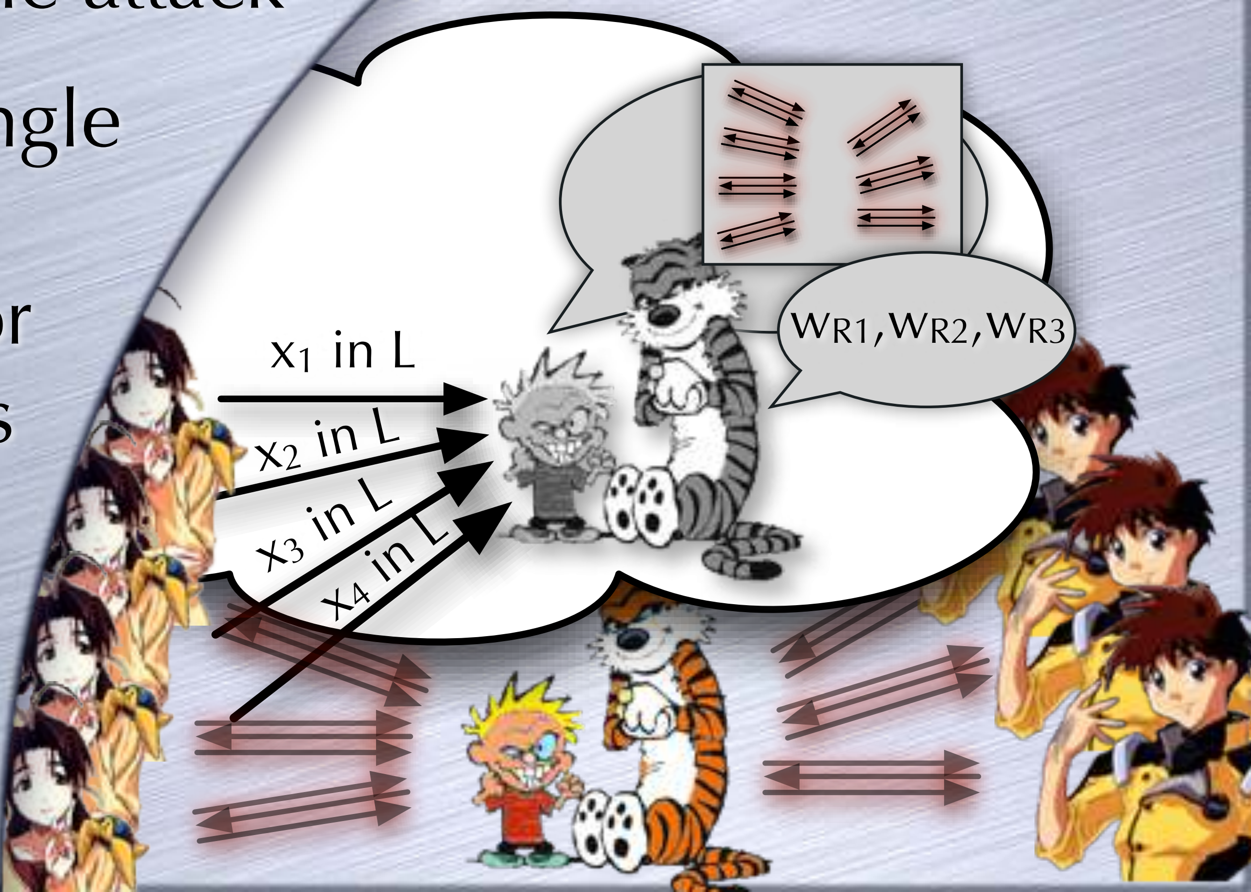

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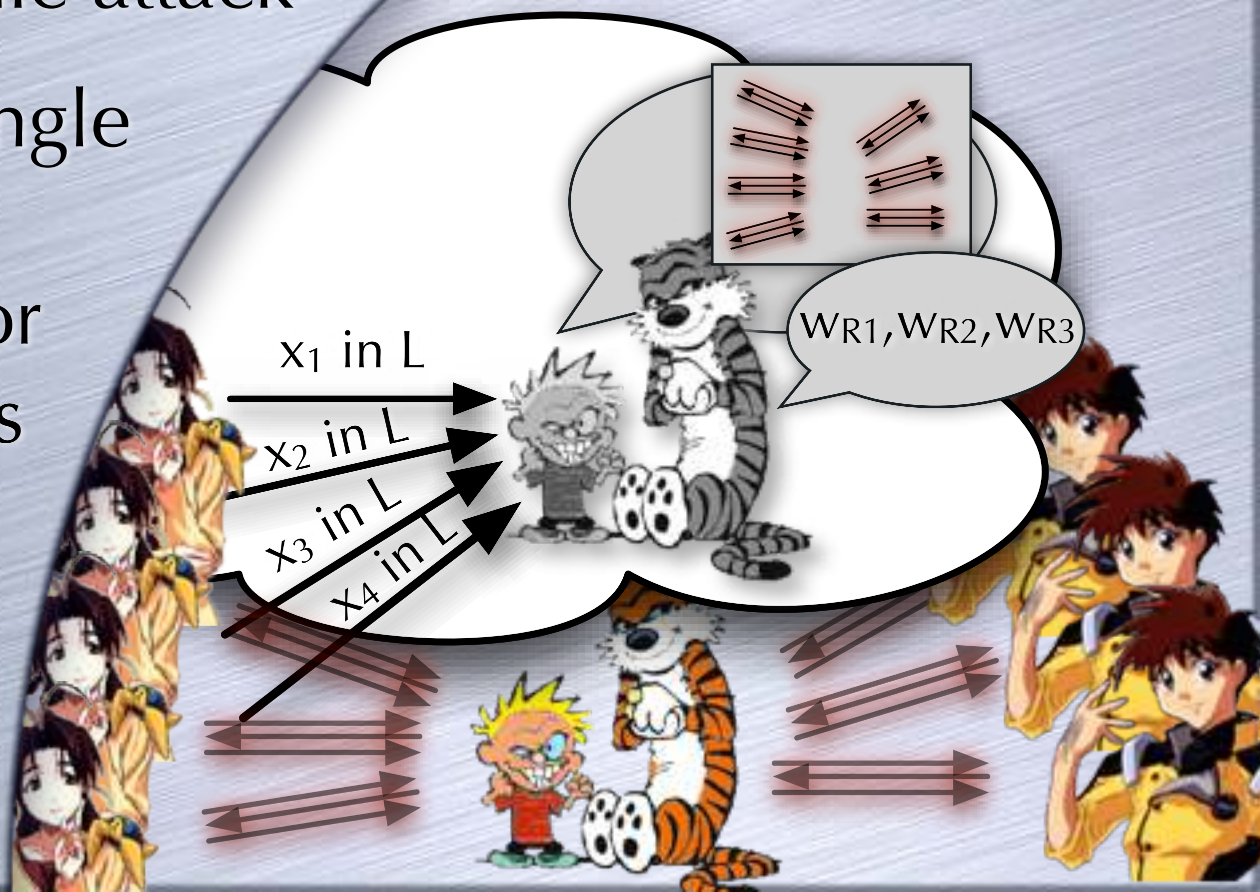
    - Or when run along with other protocols



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