Zero-Knowledge Proofs

Lecture 15

Interactive Proofs

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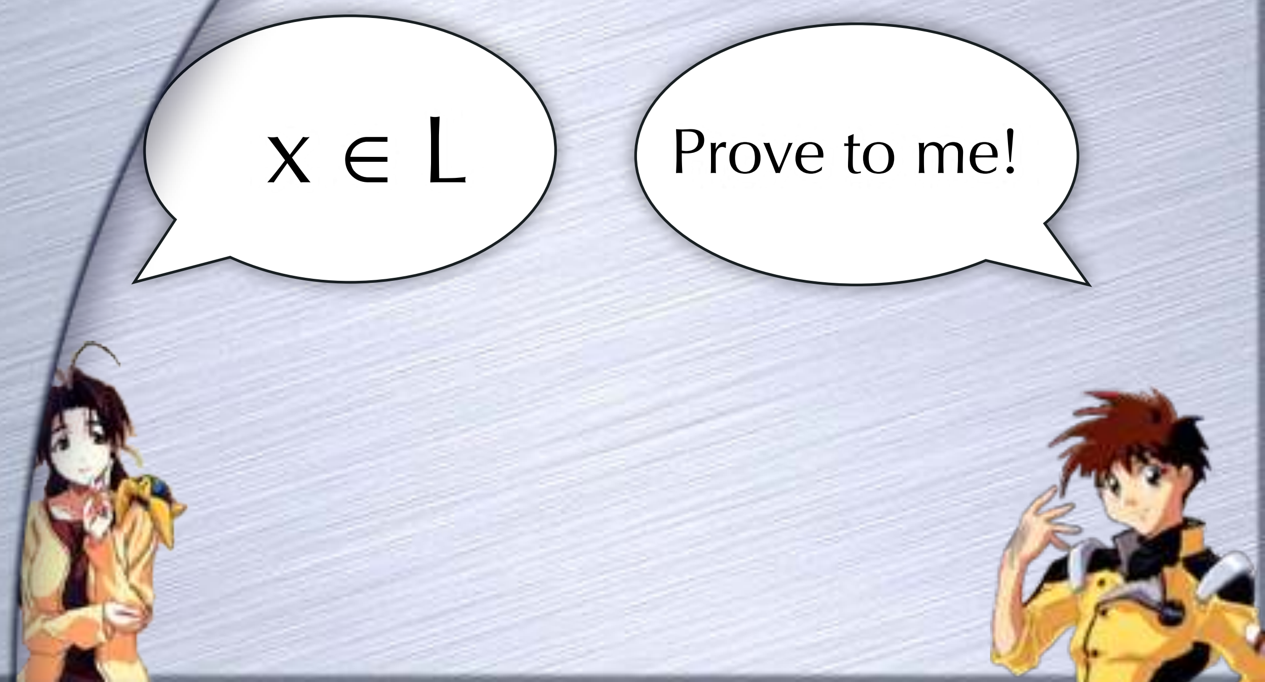

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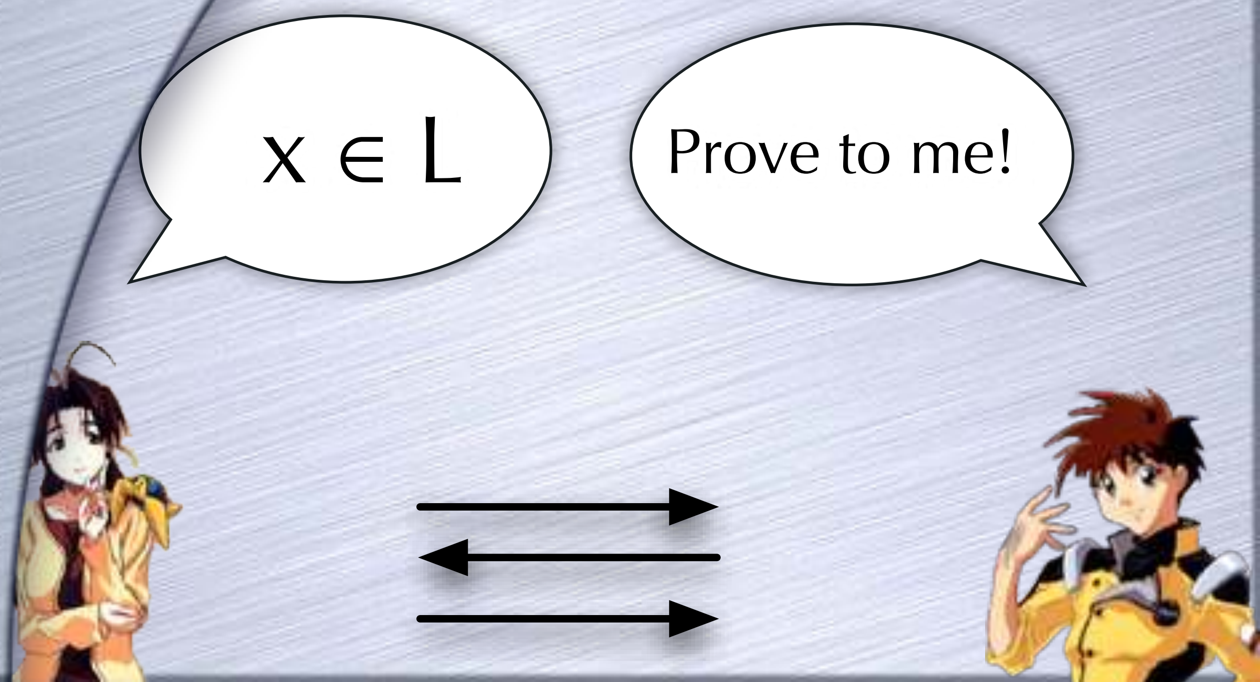

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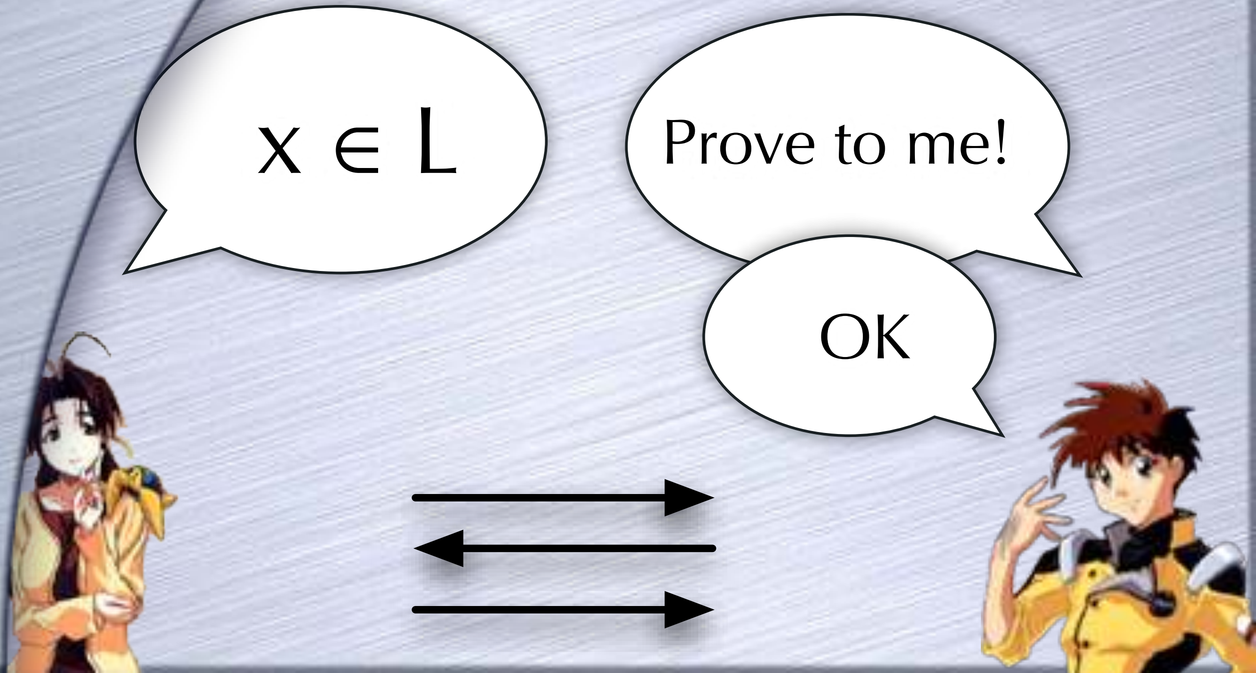
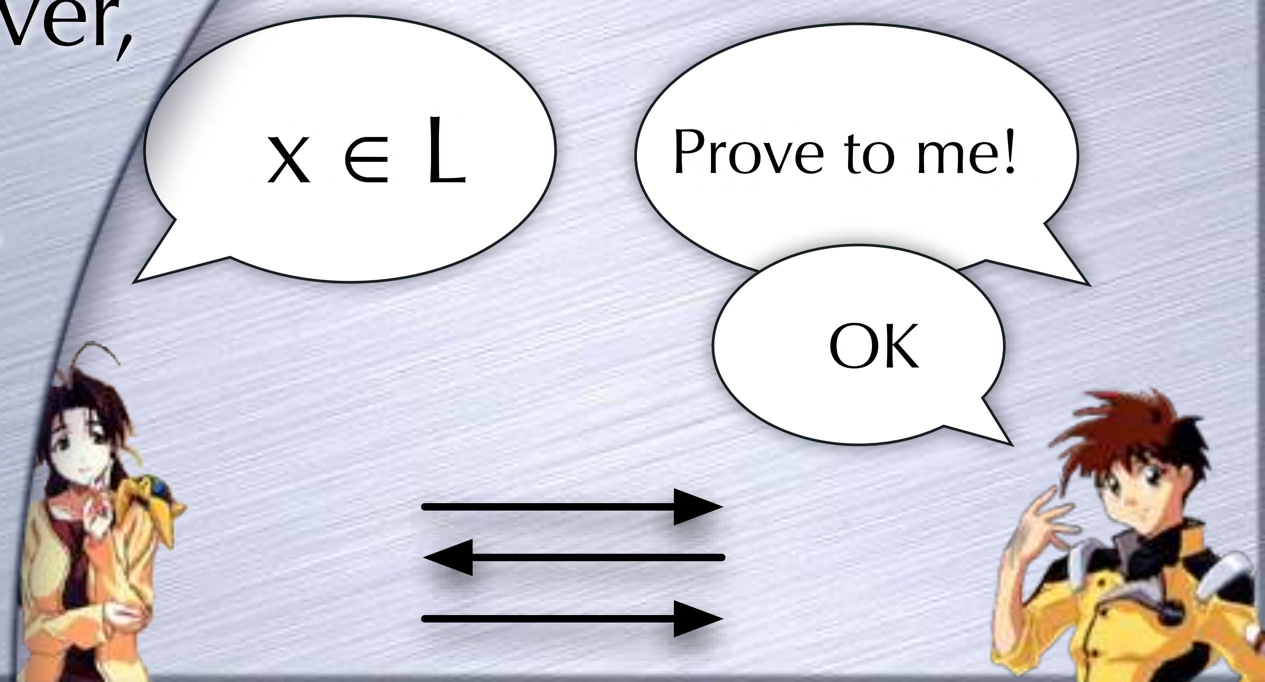


Interactive Proofs

- ***Prover*** wants to convince ***verifier*** that x has some property
  - i.e. x is in “language” L
- All powerful prover, computationally bounded verifier (for now)



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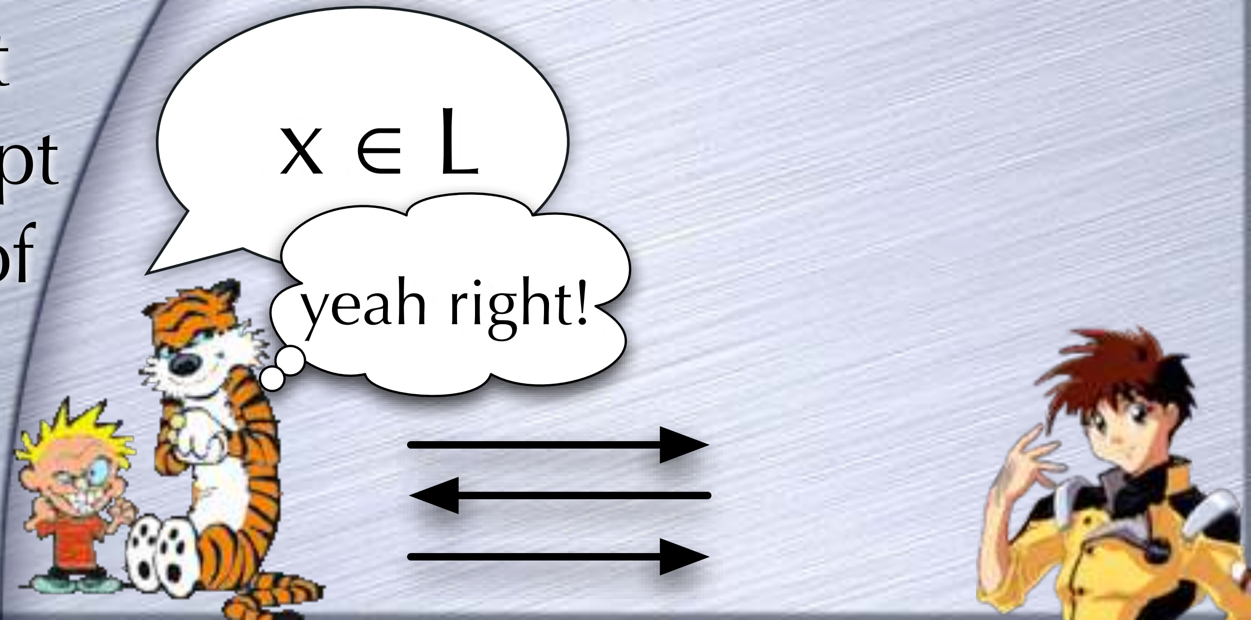

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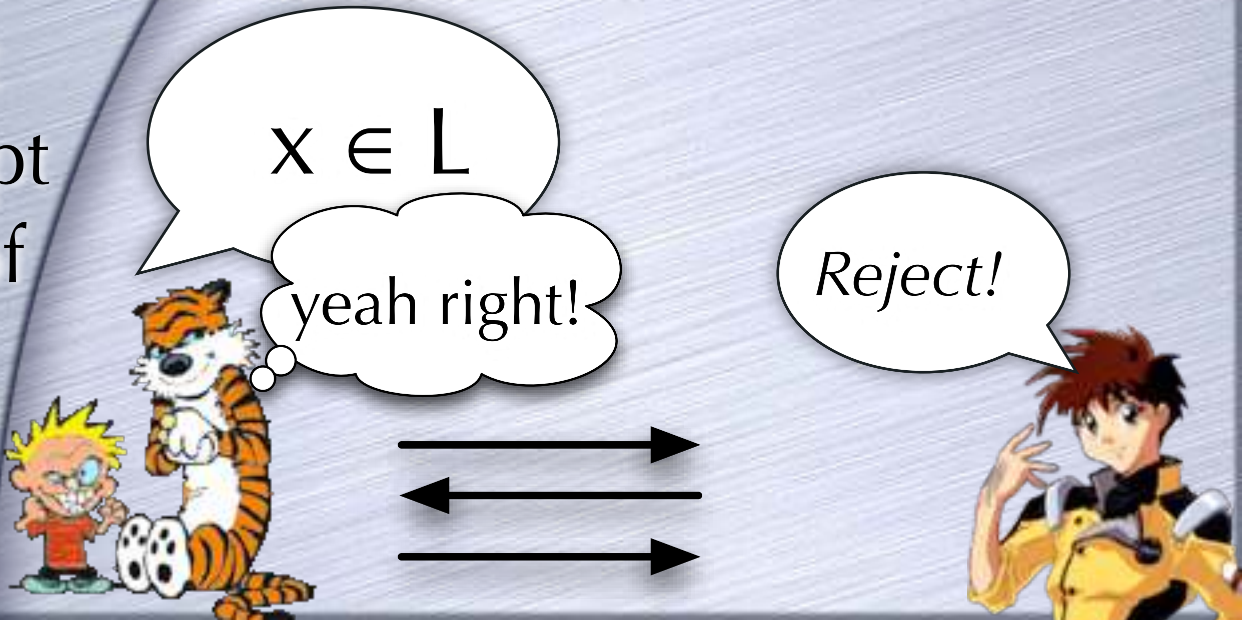

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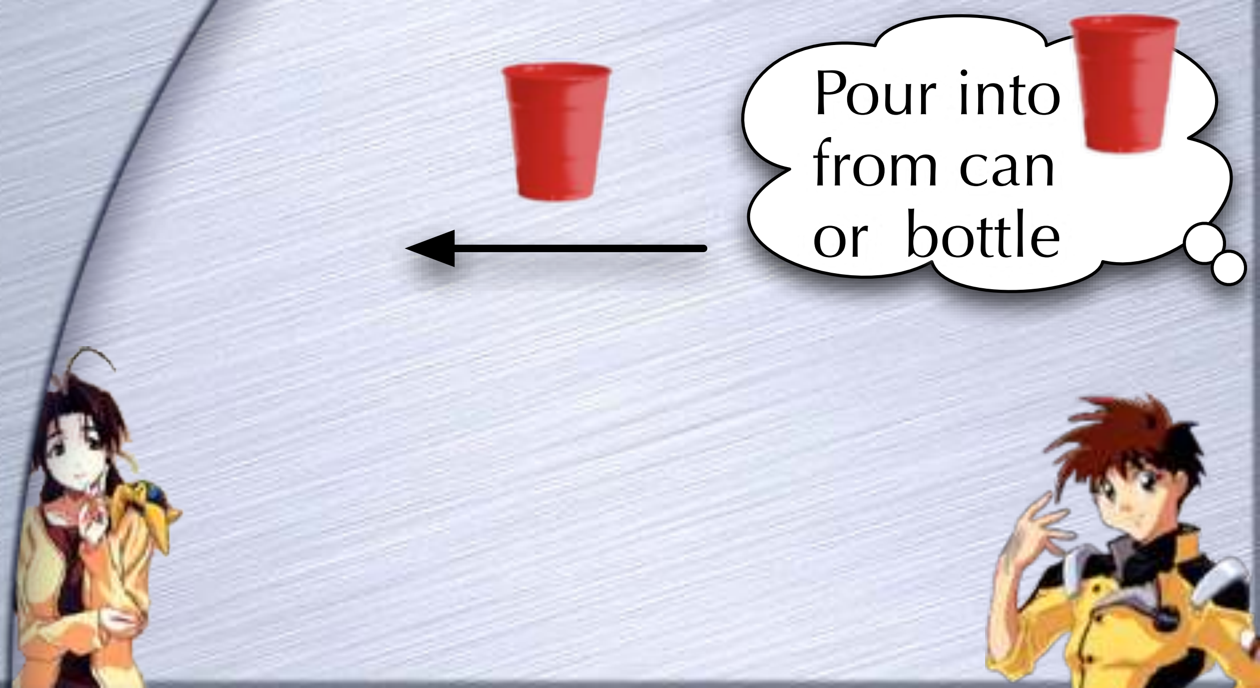

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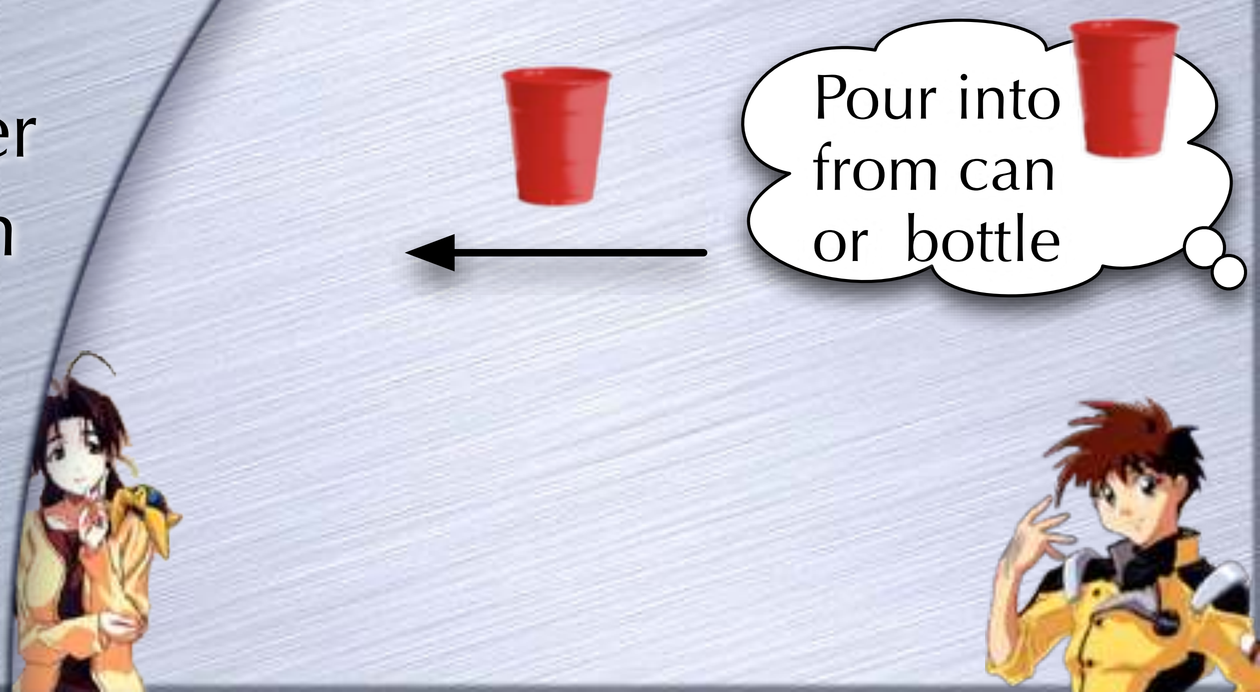

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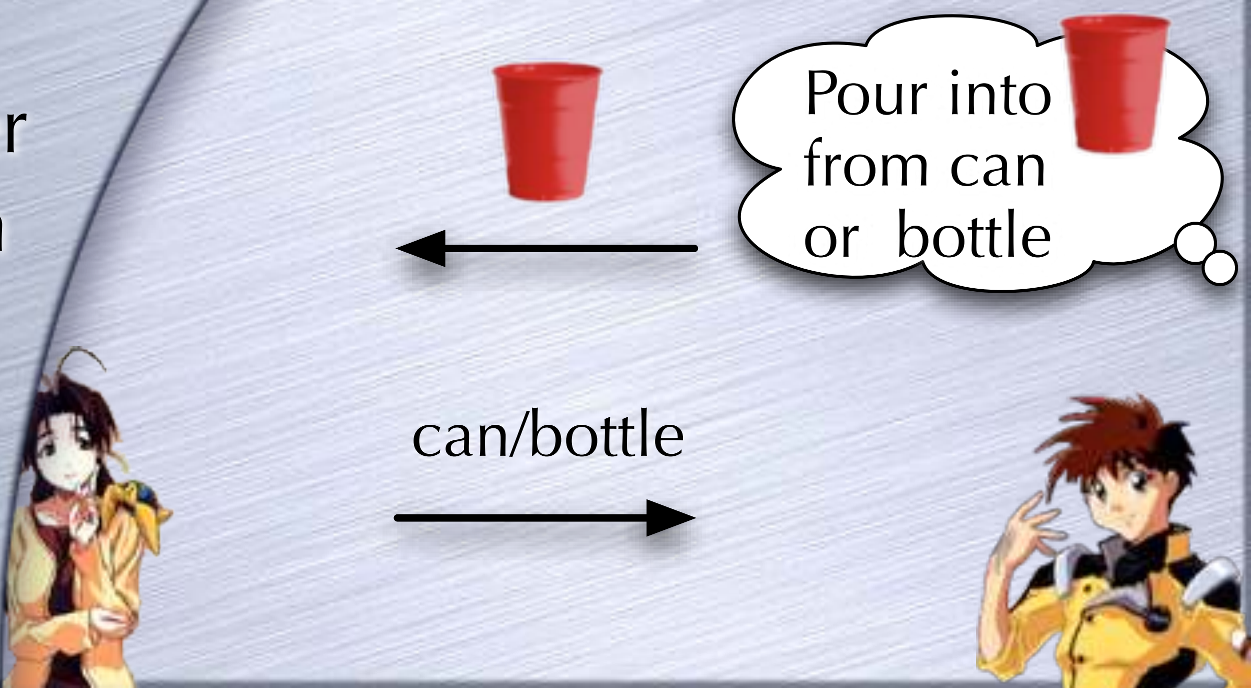

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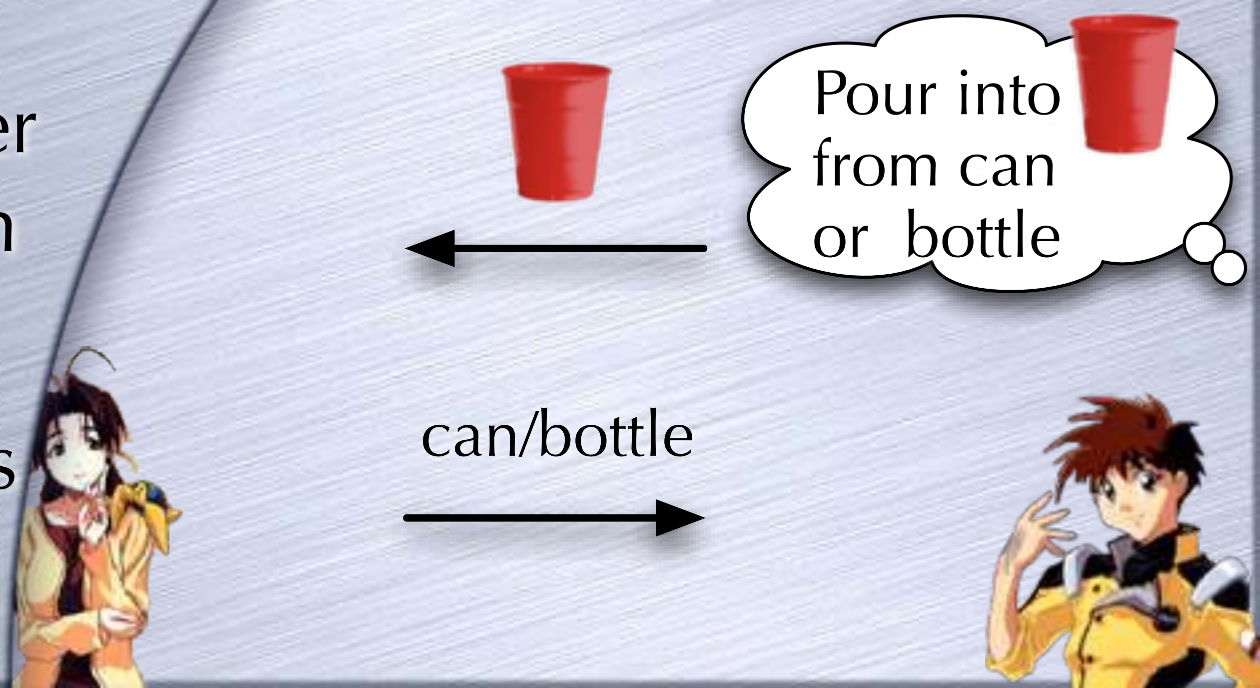

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- **Graph Non-Isomorphism**
  - Prover claims: G_0 ***not*** isomorphic to G_1
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Set G^* to be $\pi(G_0)$ or $\pi(G_1)$ (π random)

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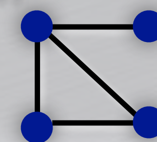
Isomorphism: Same graph can be represented as a matrix in different ways:

e.g., $G_0 = \begin{matrix} 0101 \\ 1001 \\ 0001 \\ 1110 \end{matrix}$ & $G_1 = \begin{matrix} 0101 \\ 1011 \\ 0100 \\ 1100 \end{matrix}$

both are isomorphic to the graph represented by the drawing



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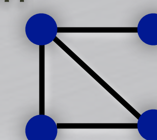

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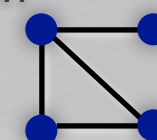
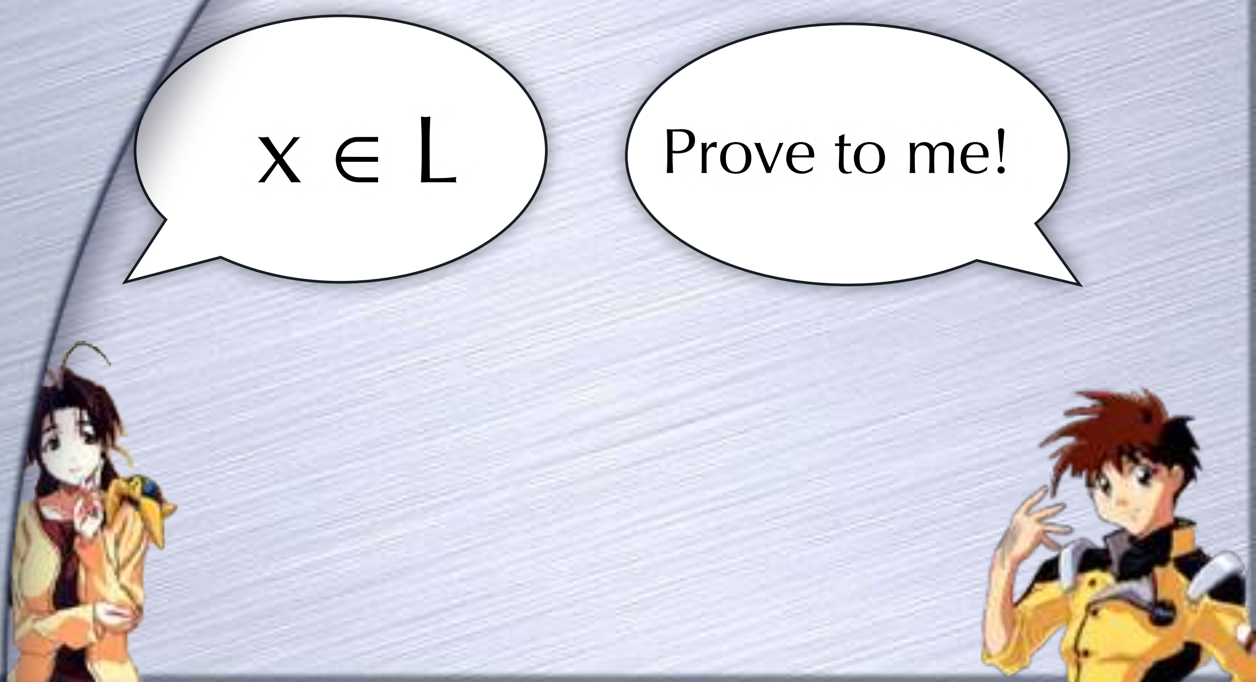


$\xleftarrow{G^*}$

Set G^* to be $\pi(G_0)$ or $\pi(G_1)$ (π random)

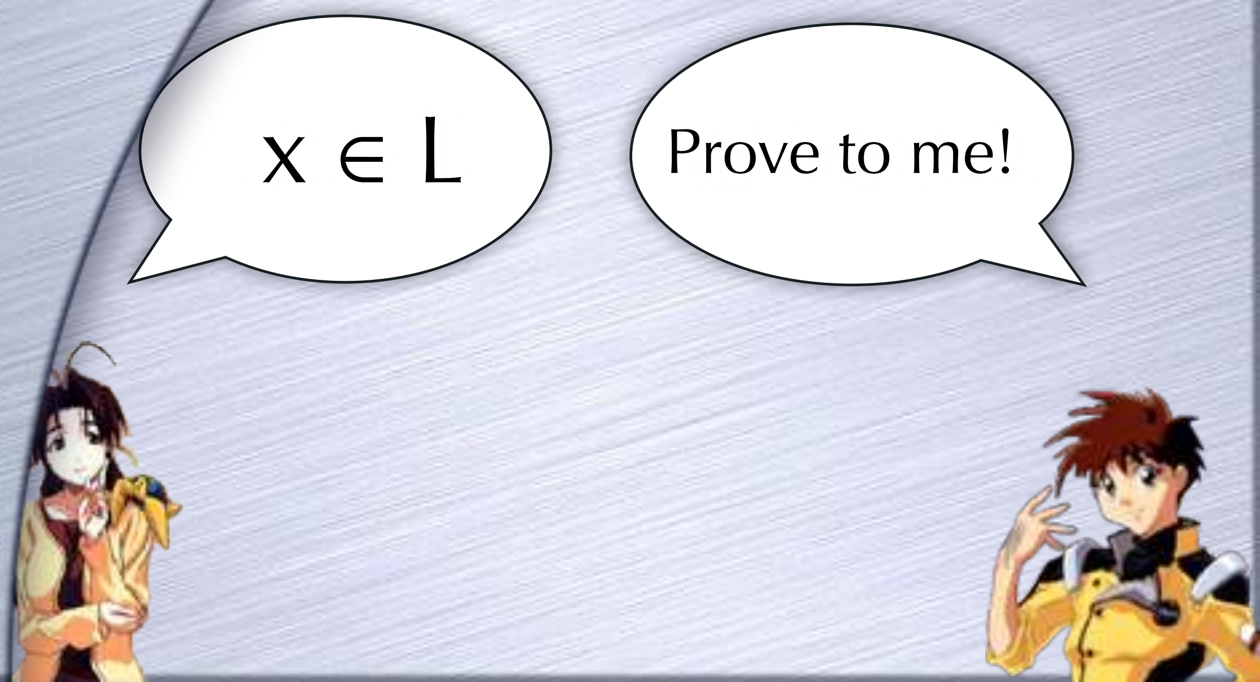
$\xrightarrow{G_0/G_1}$

Proofs for NP languages



Proofs for NP languages

- Proving membership in an **NP** language L



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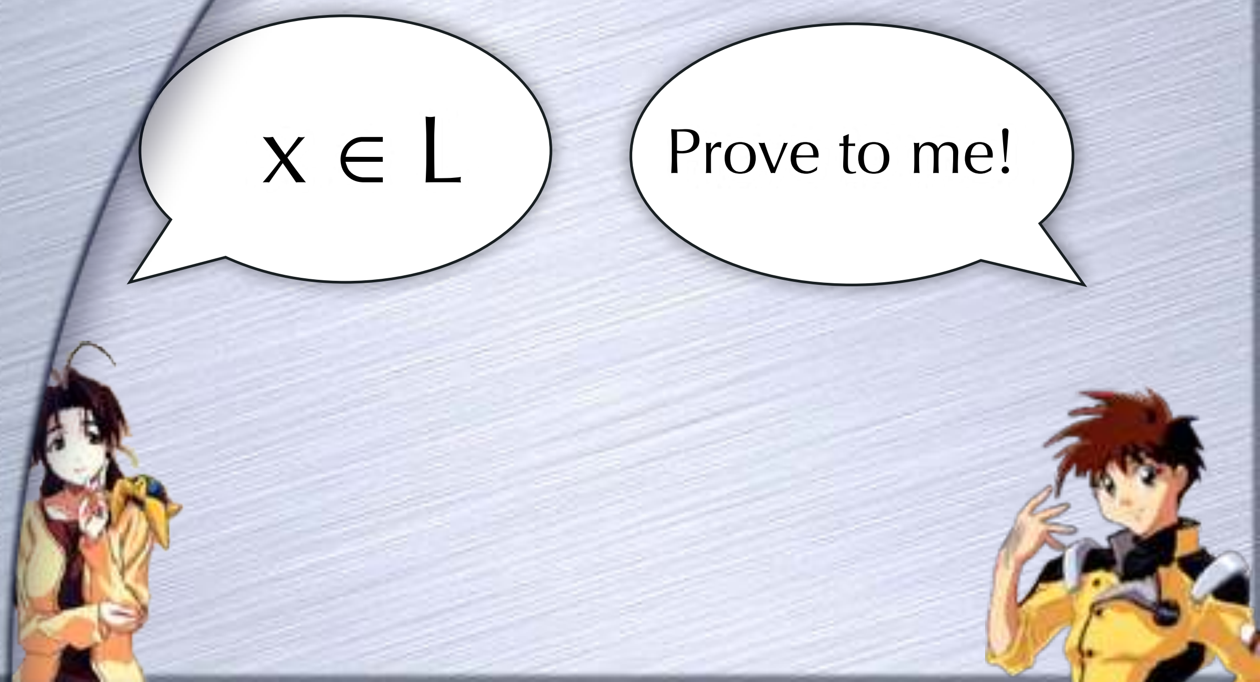
  - $x \in L$ iff $\exists w\ R(x,w)=1$ (for R in **P**)



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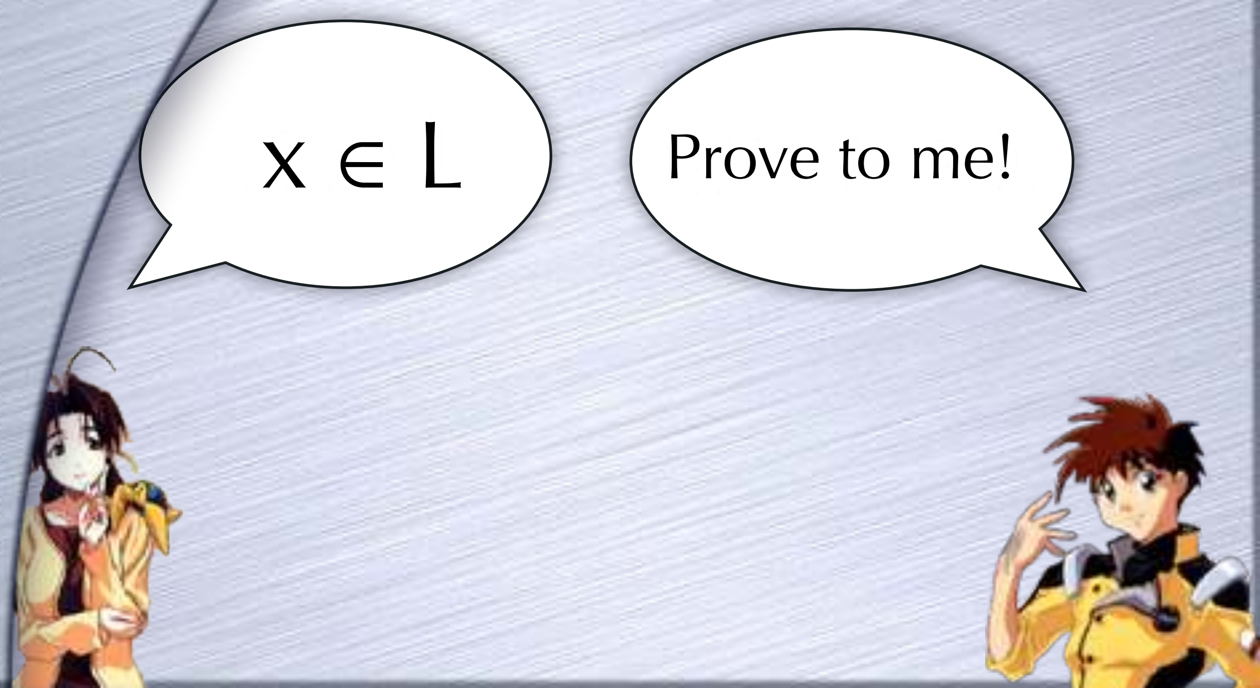

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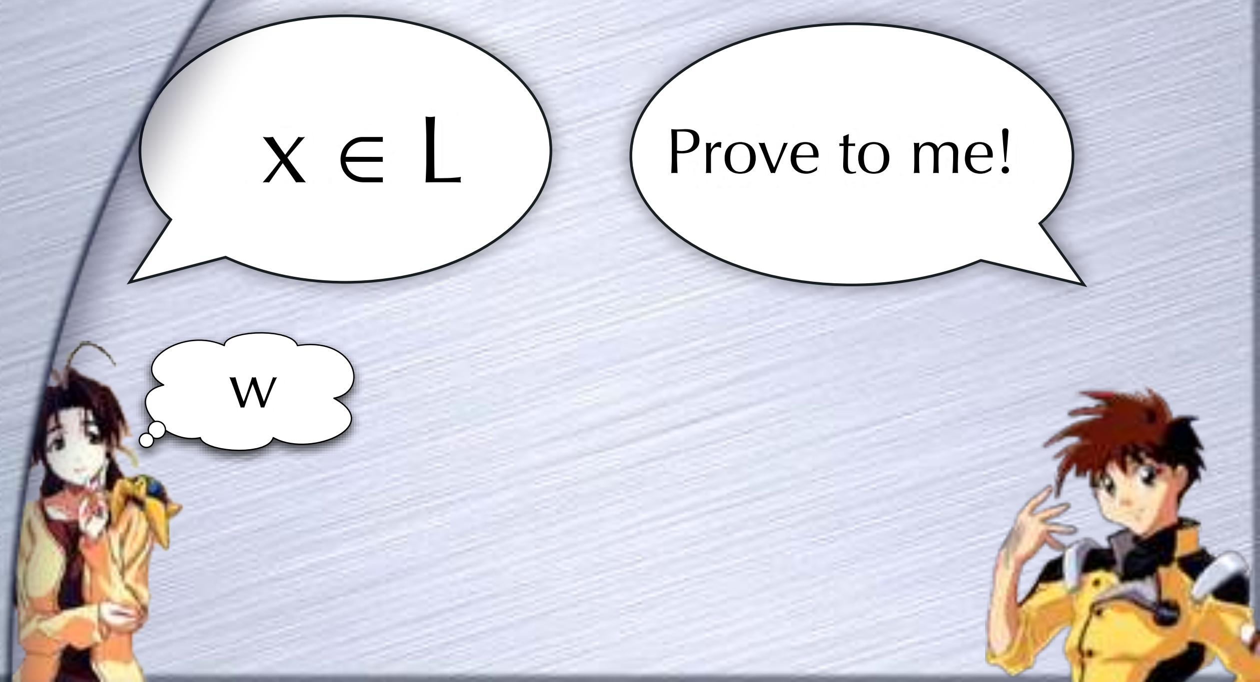

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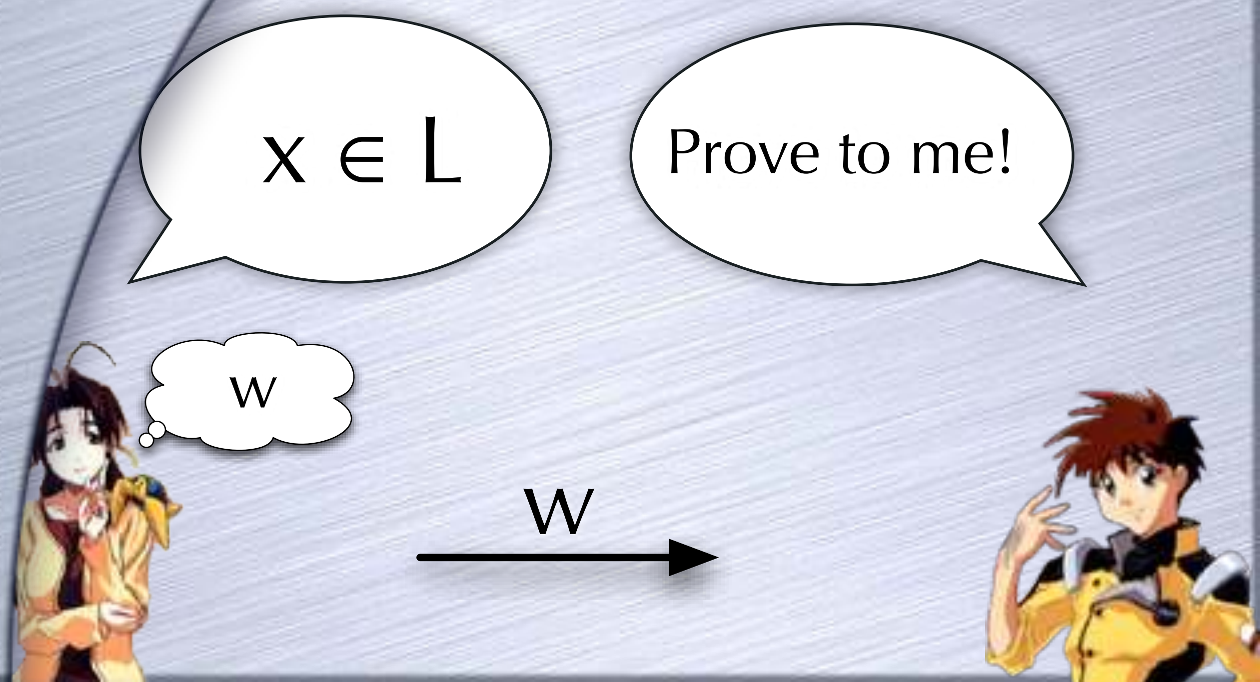

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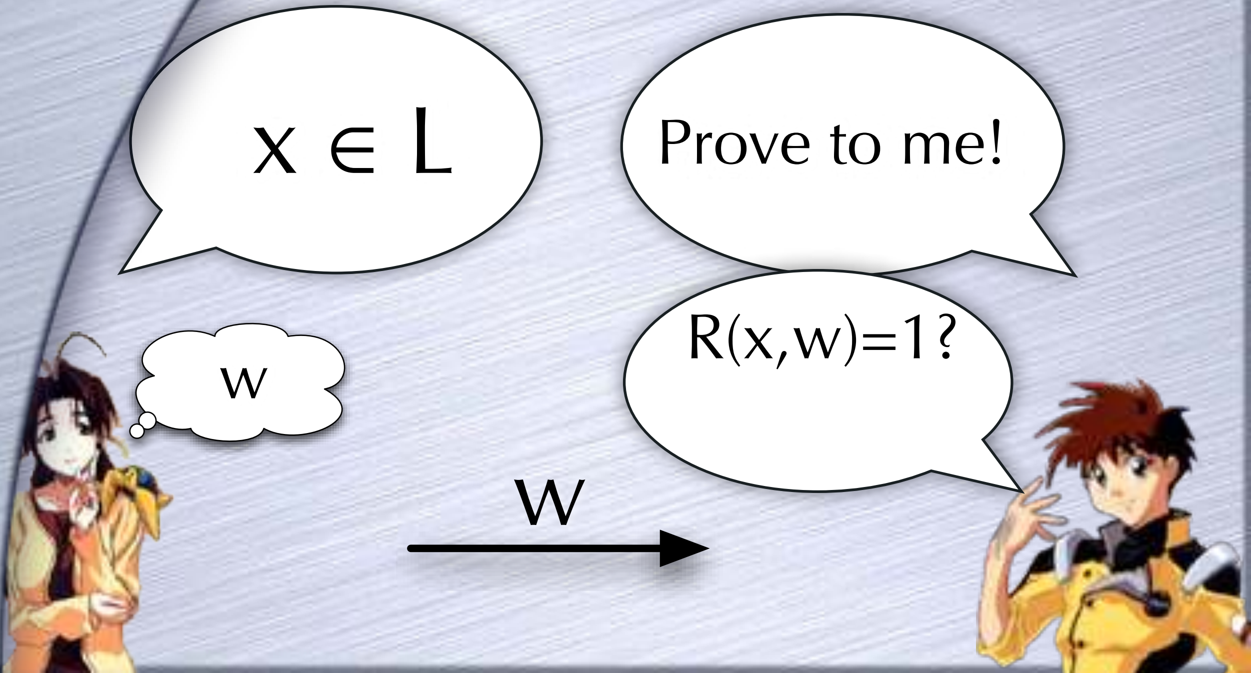

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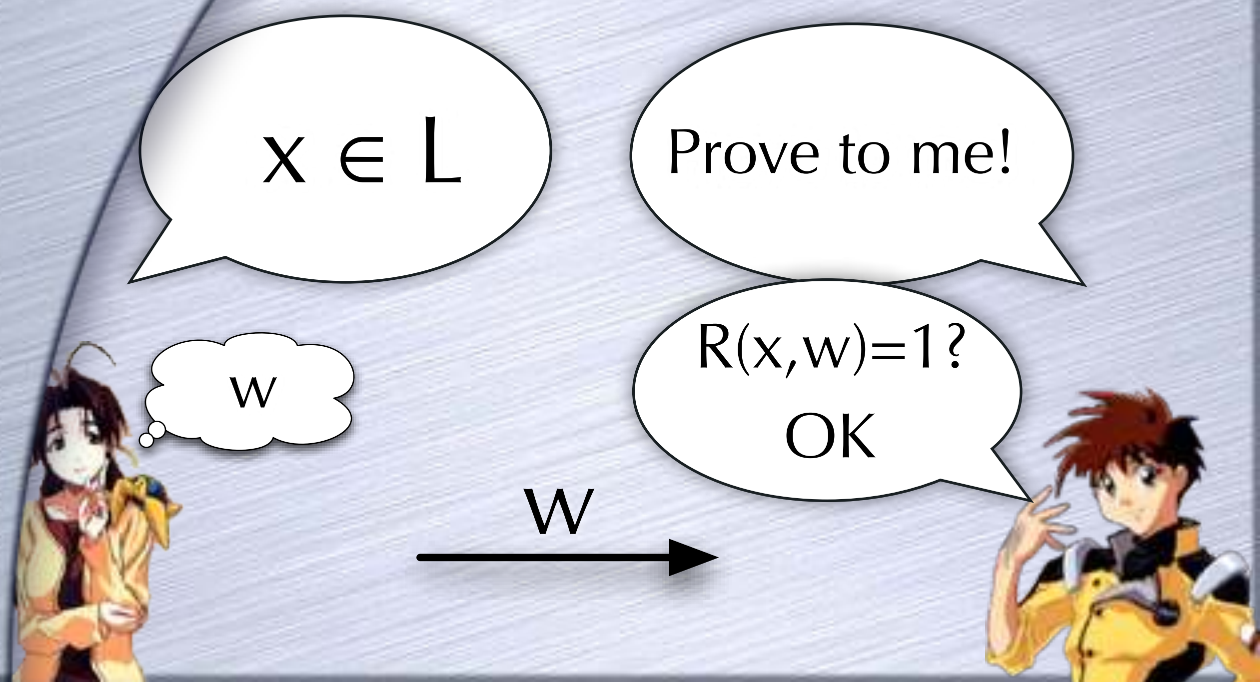

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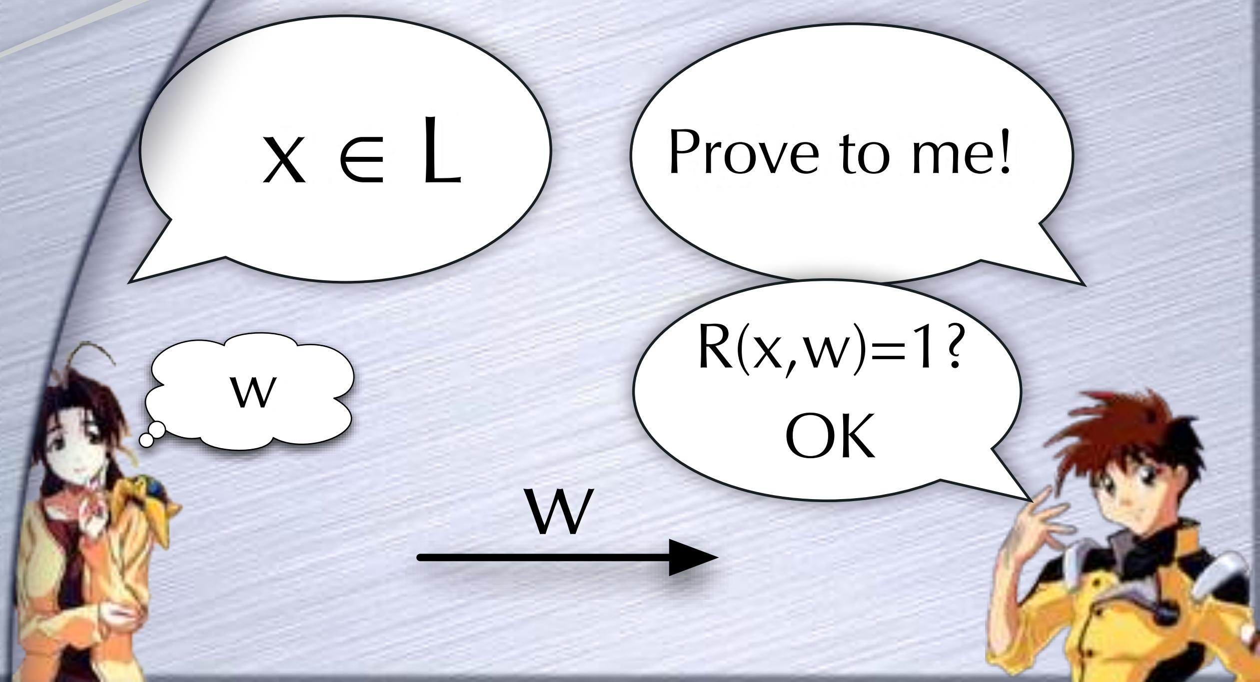
NP is the class of languages which have <u>non-interactive</u> and <u>deterministic</u> proof-systems



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- *What if prover doesn't want to reveal w?*

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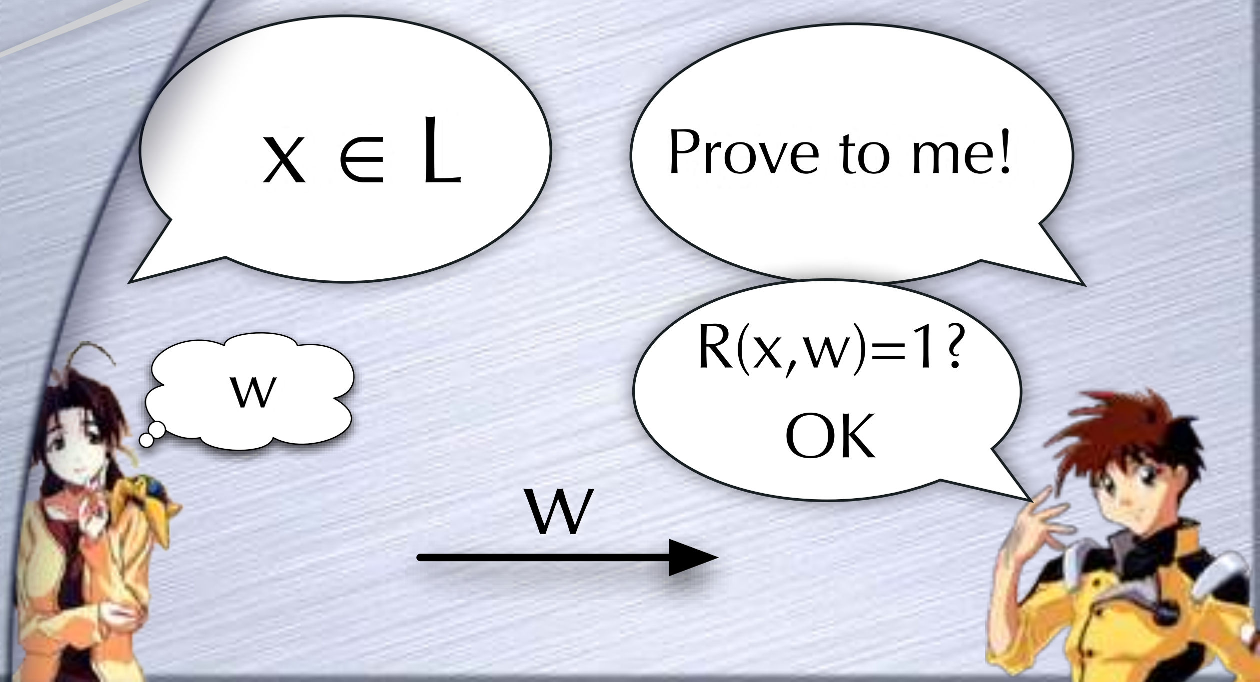

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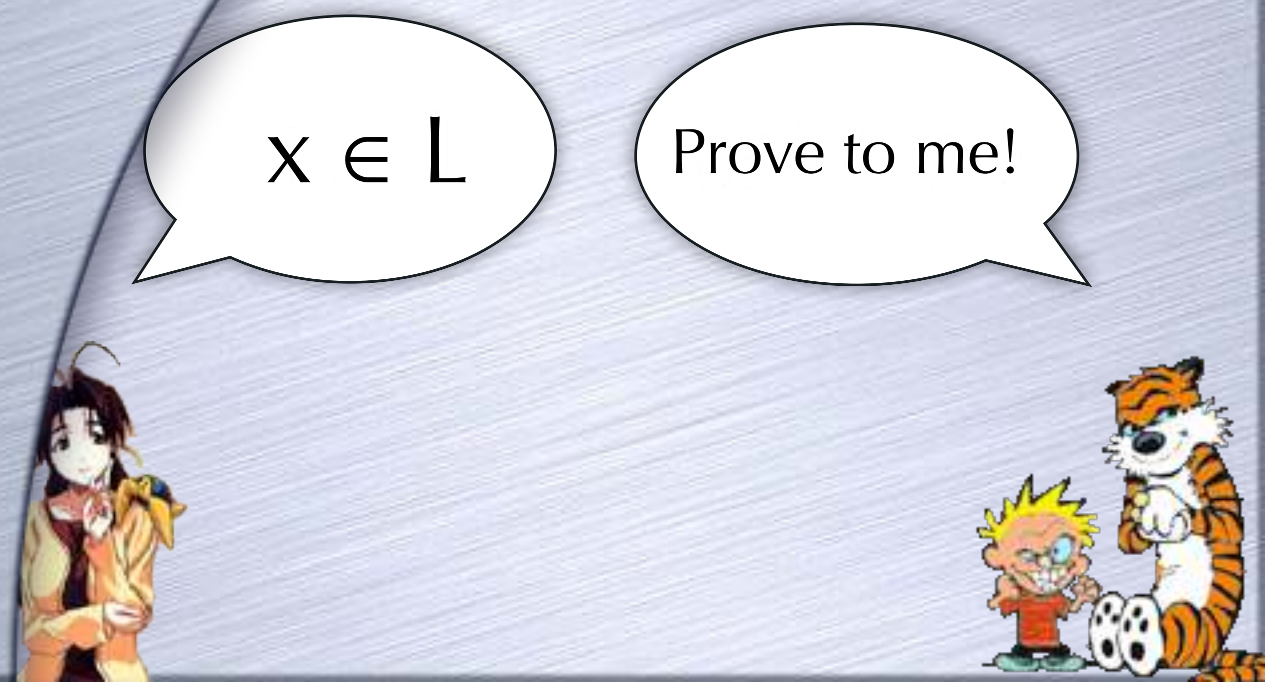

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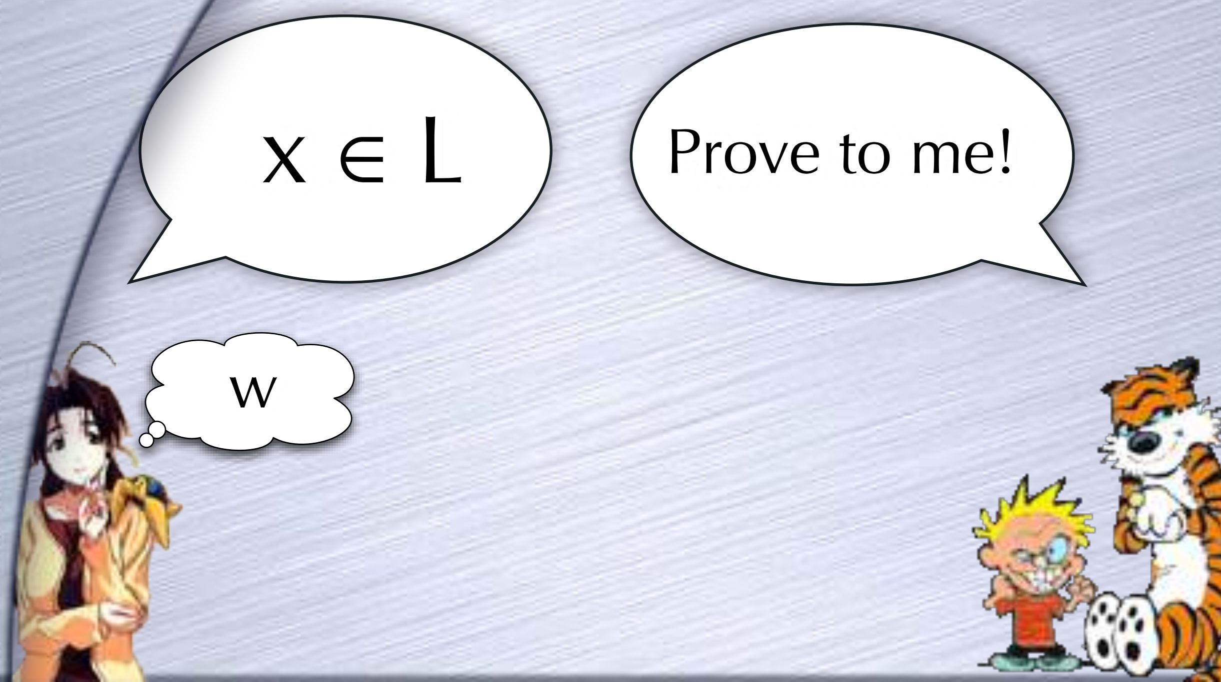

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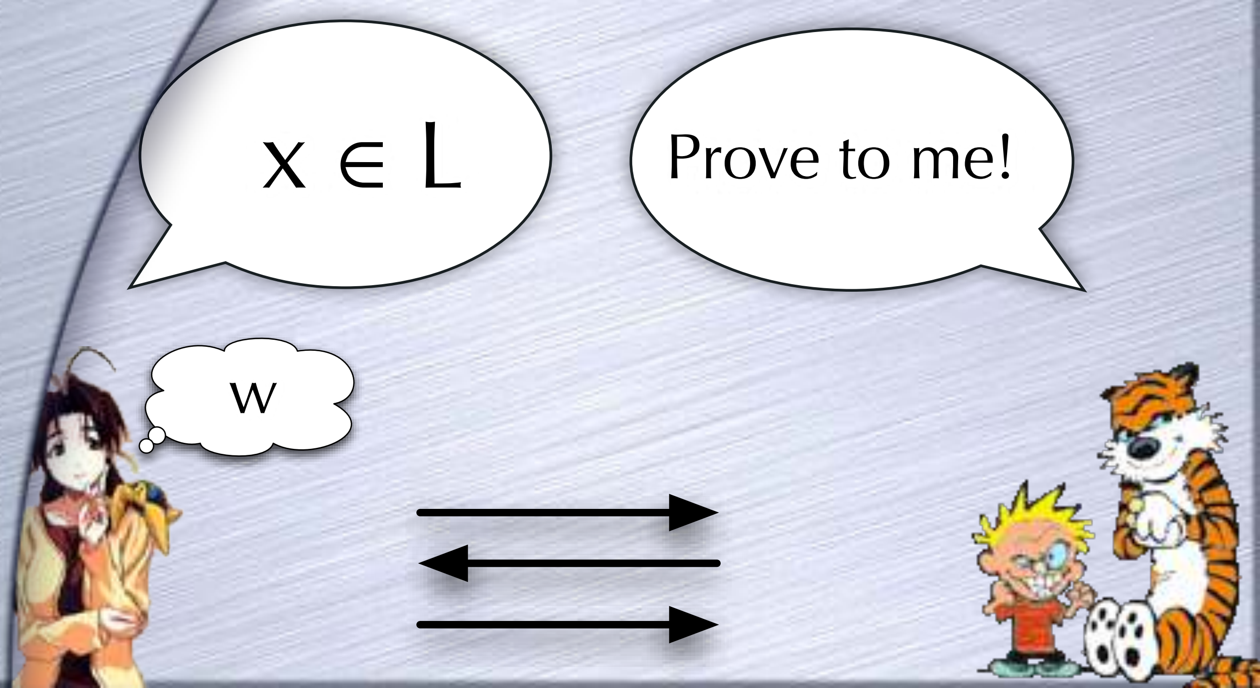

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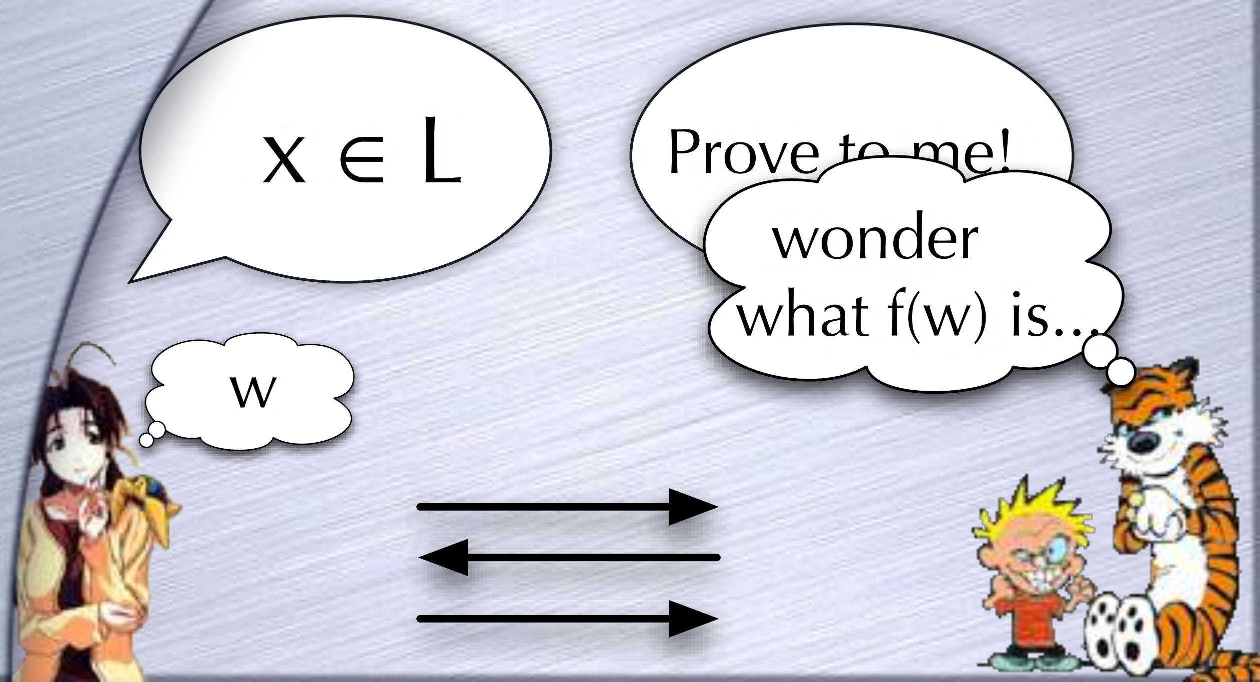

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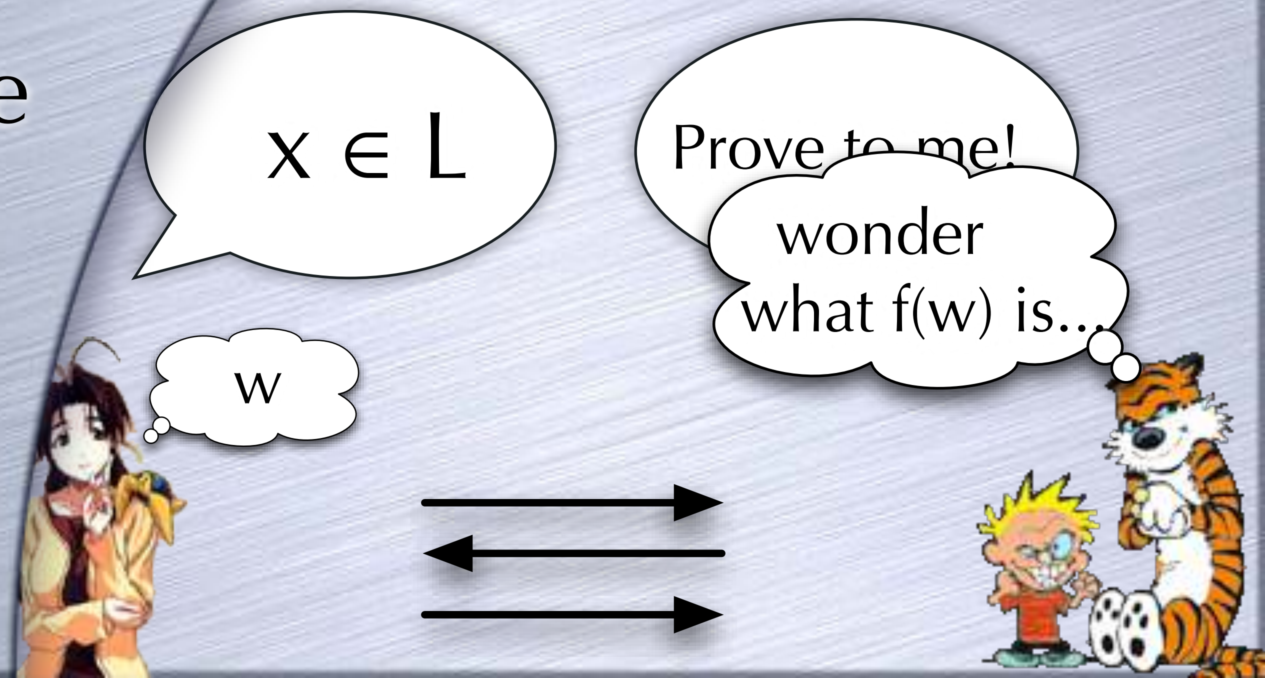

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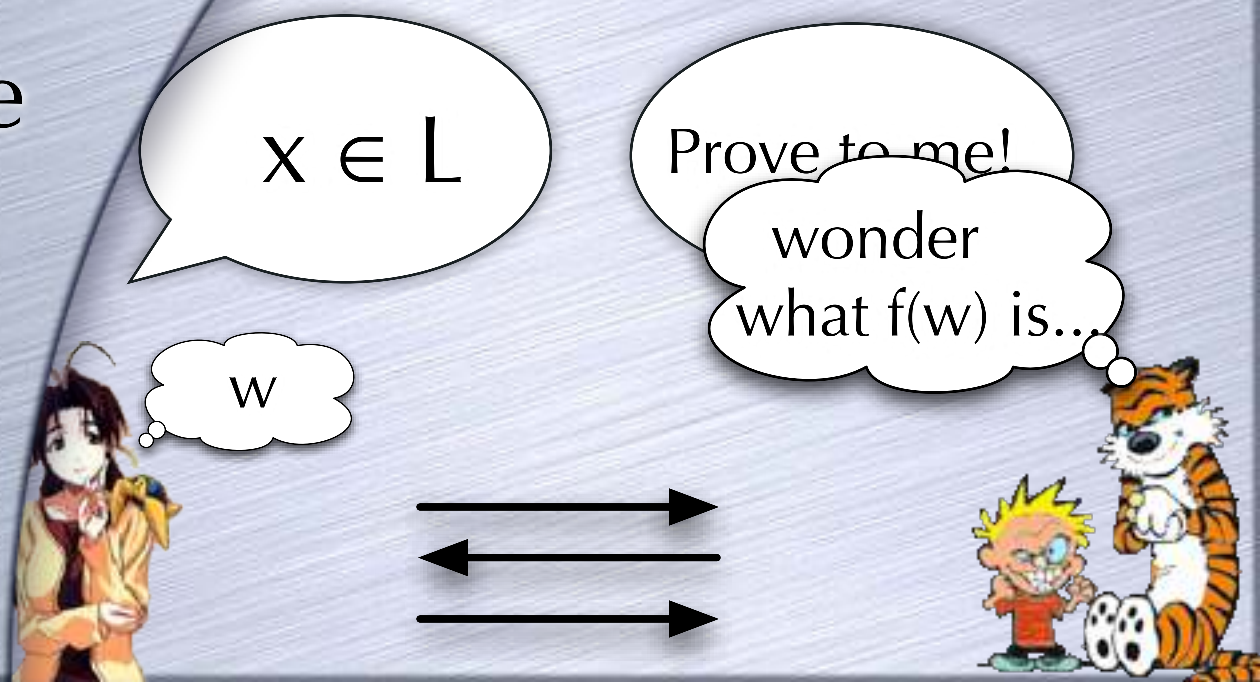
- How to formalize this?
  - Simulation!



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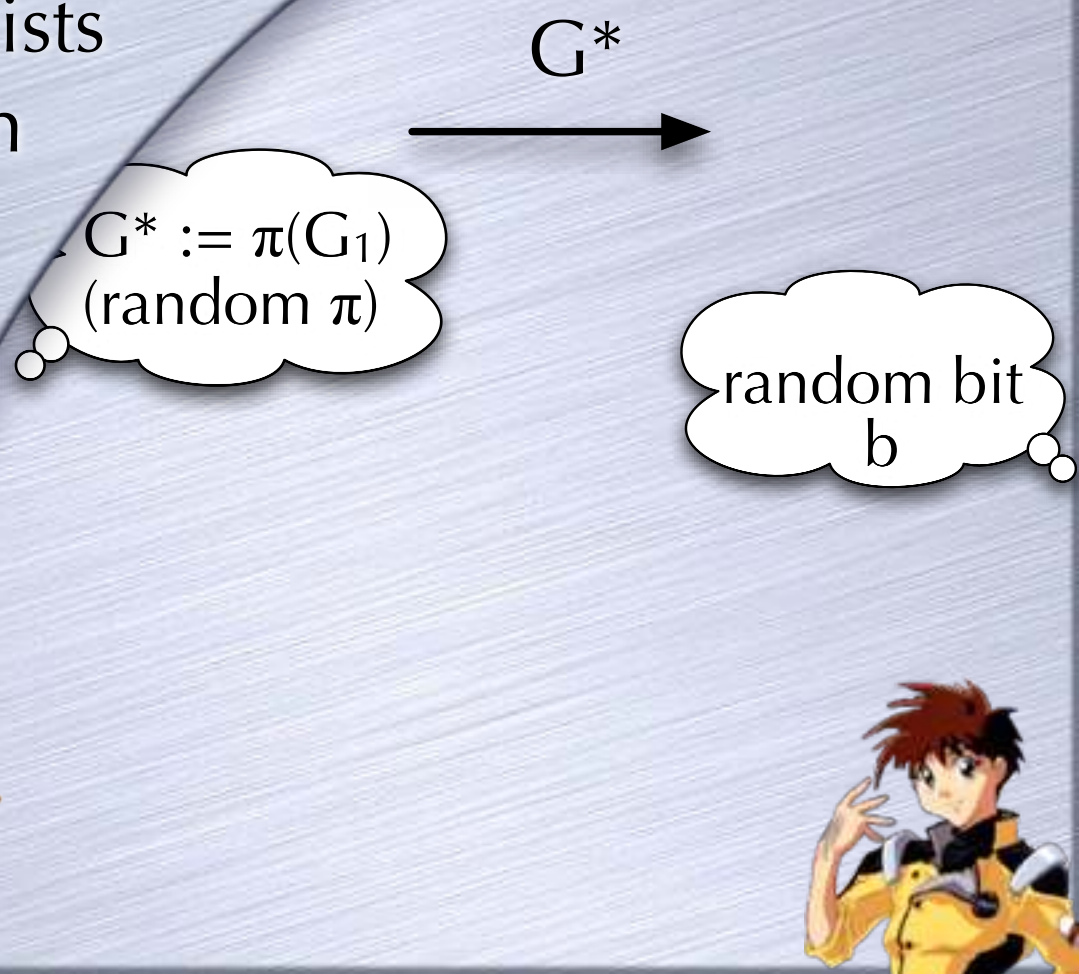

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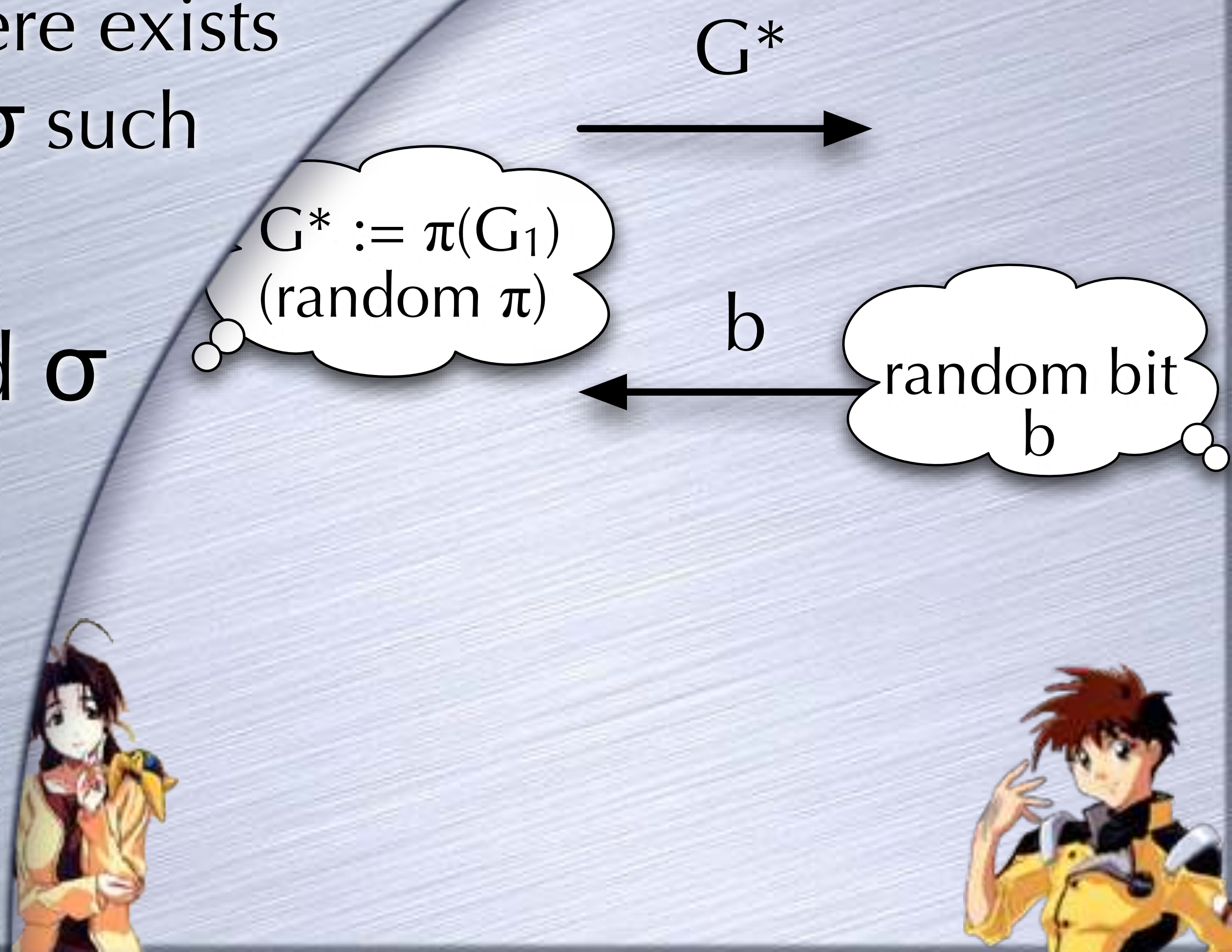

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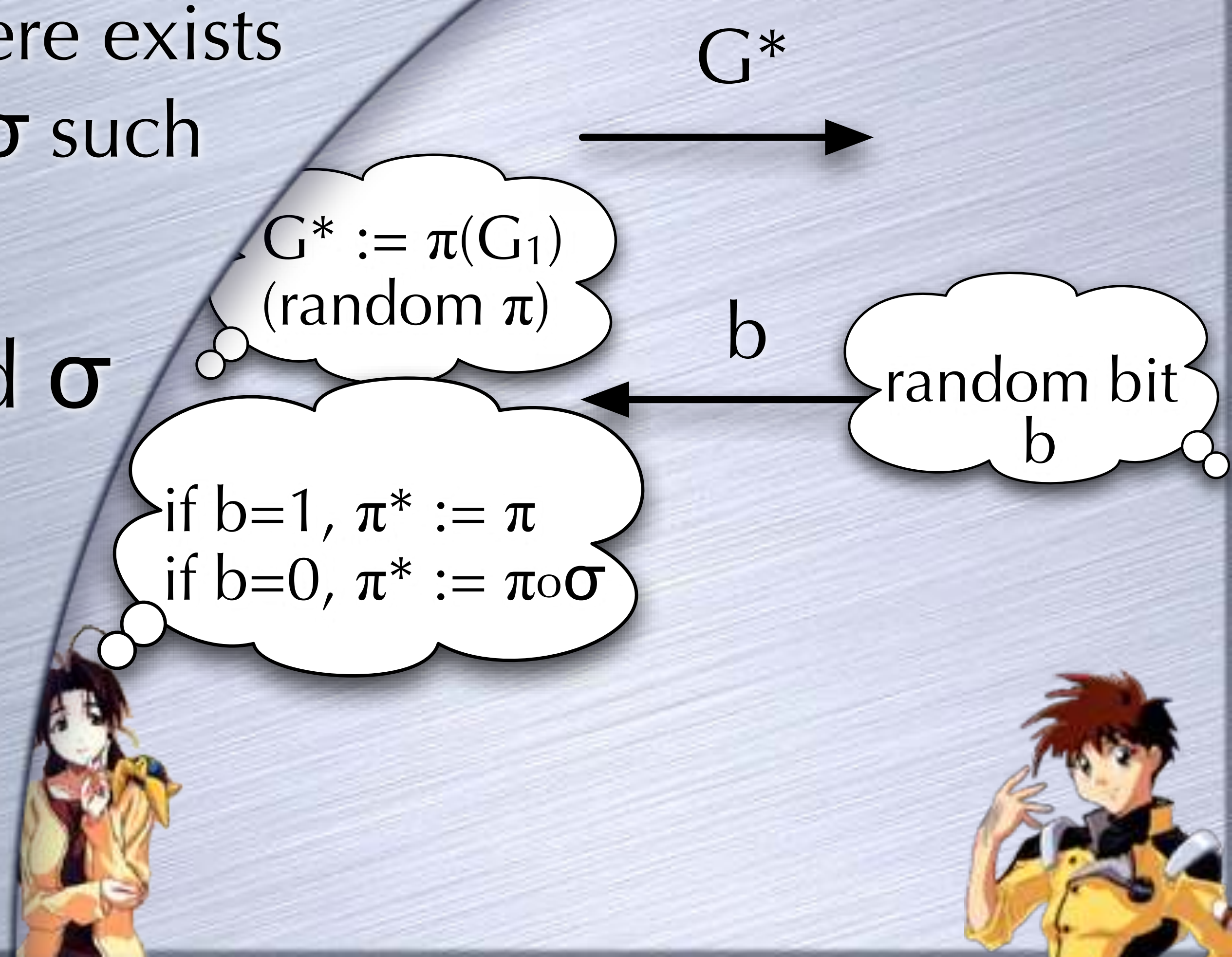

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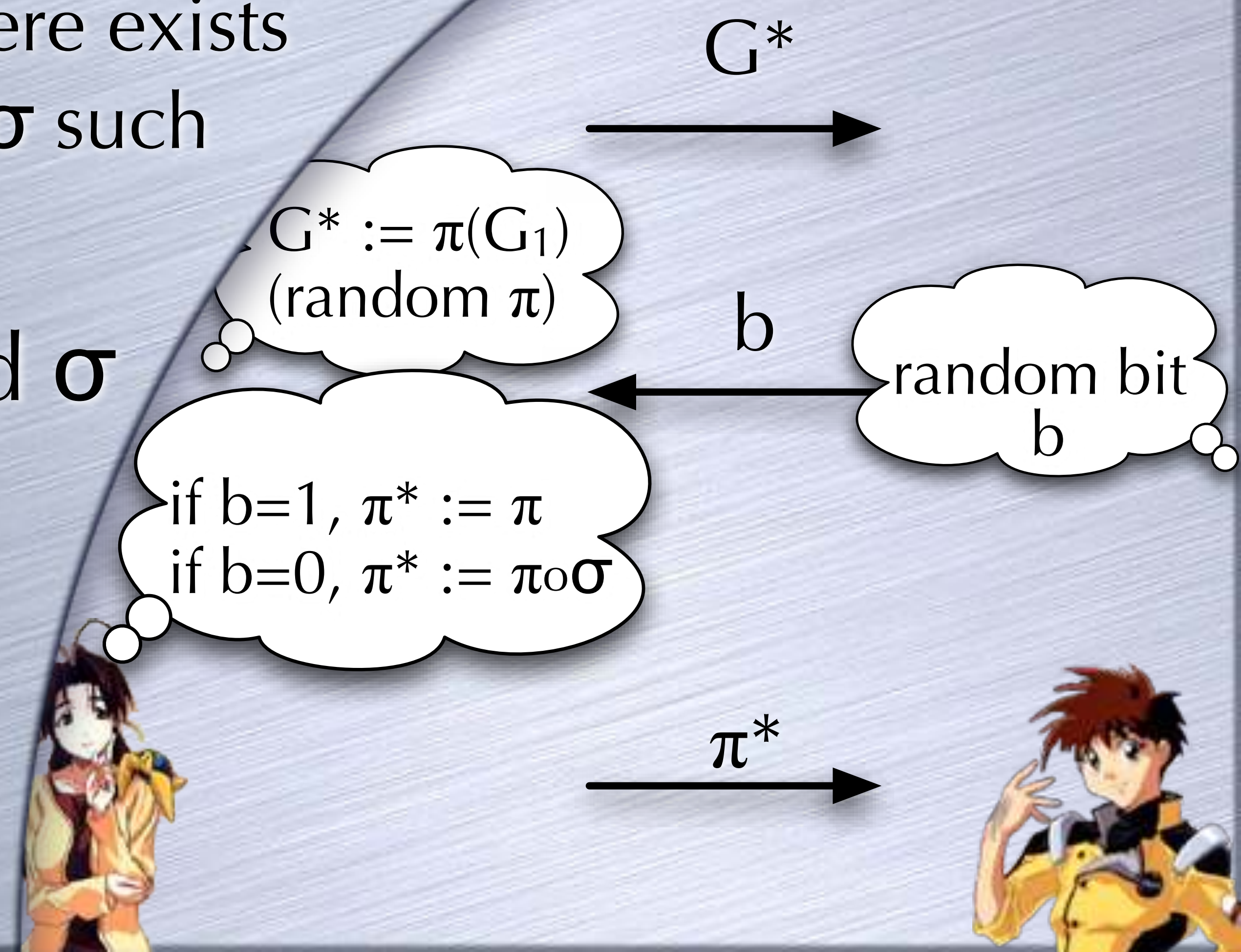

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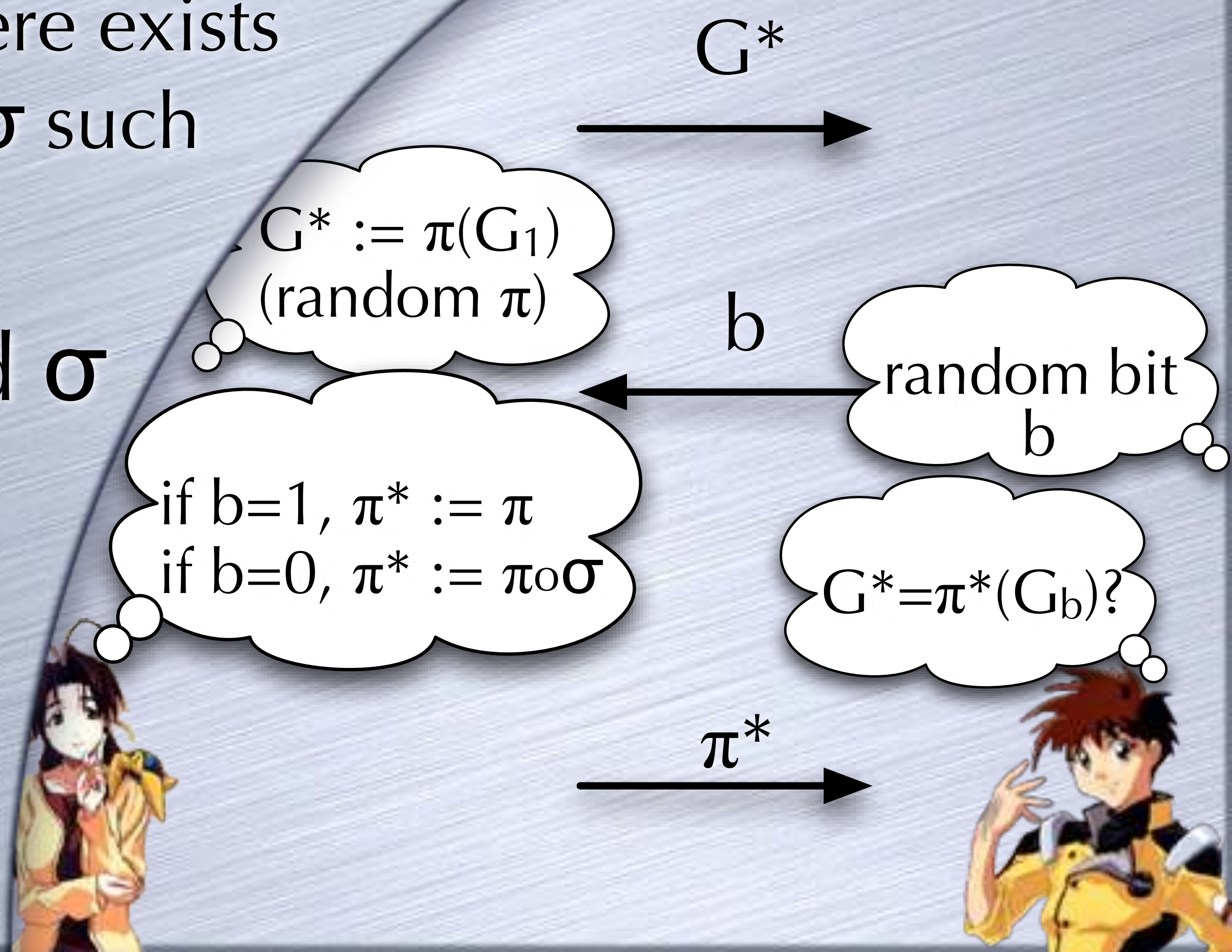

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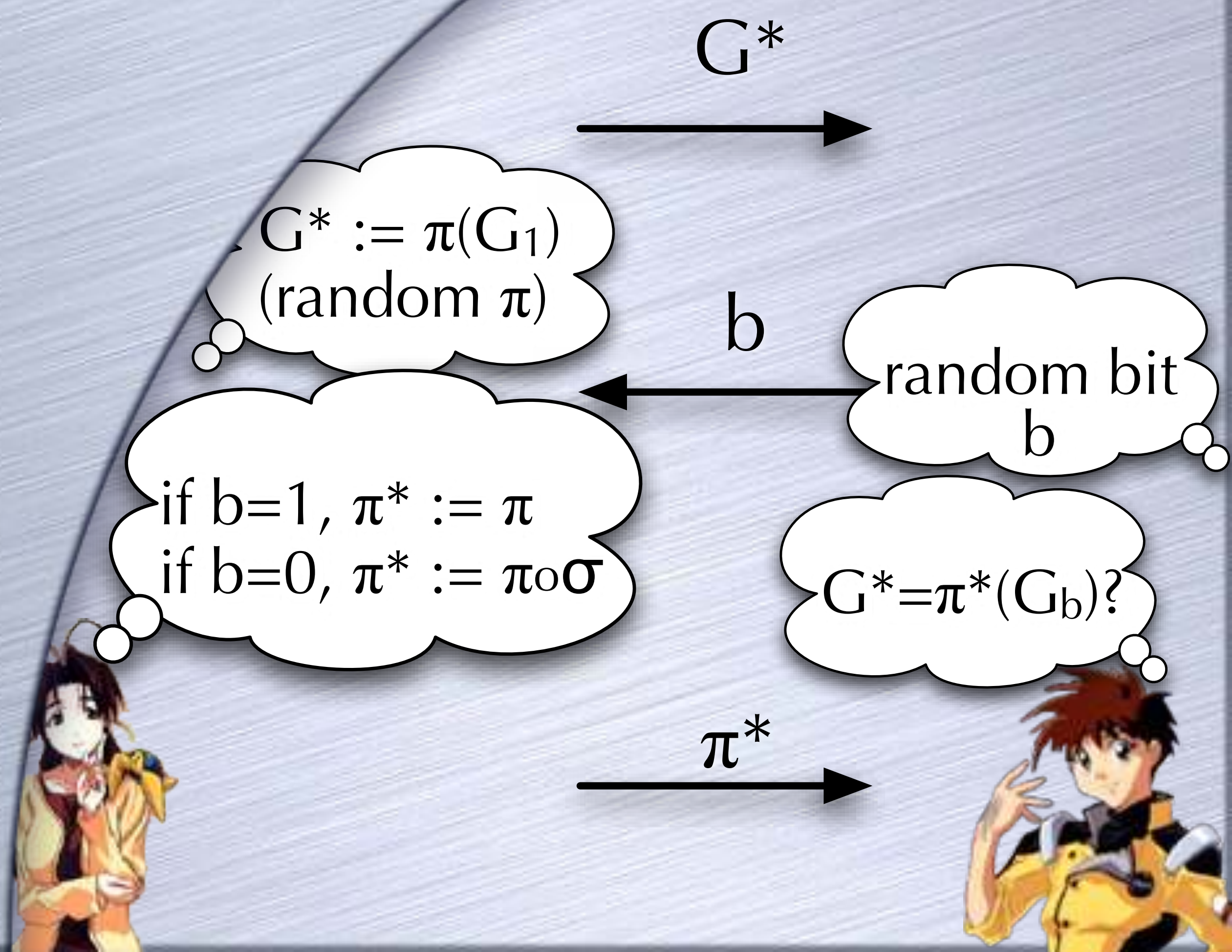

An Example

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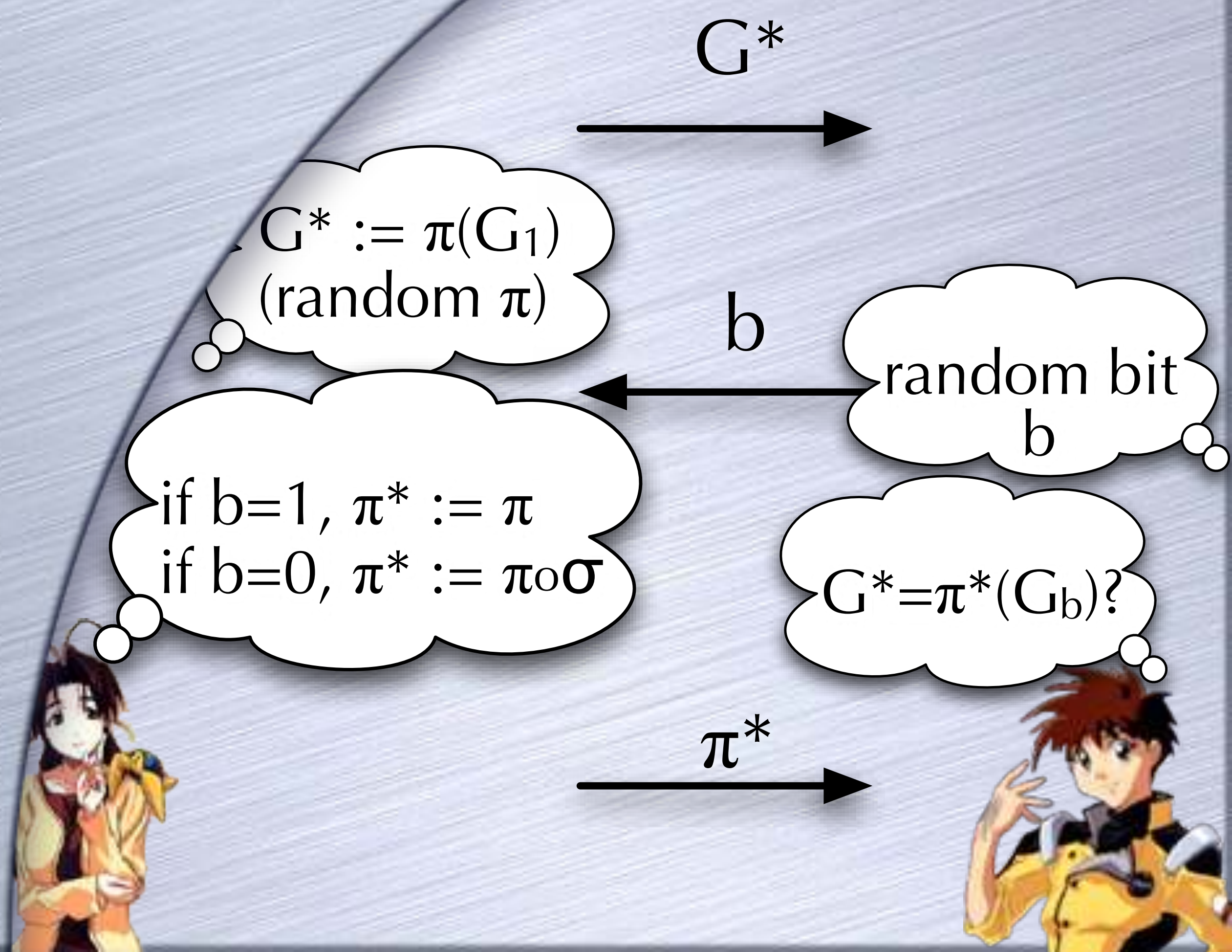

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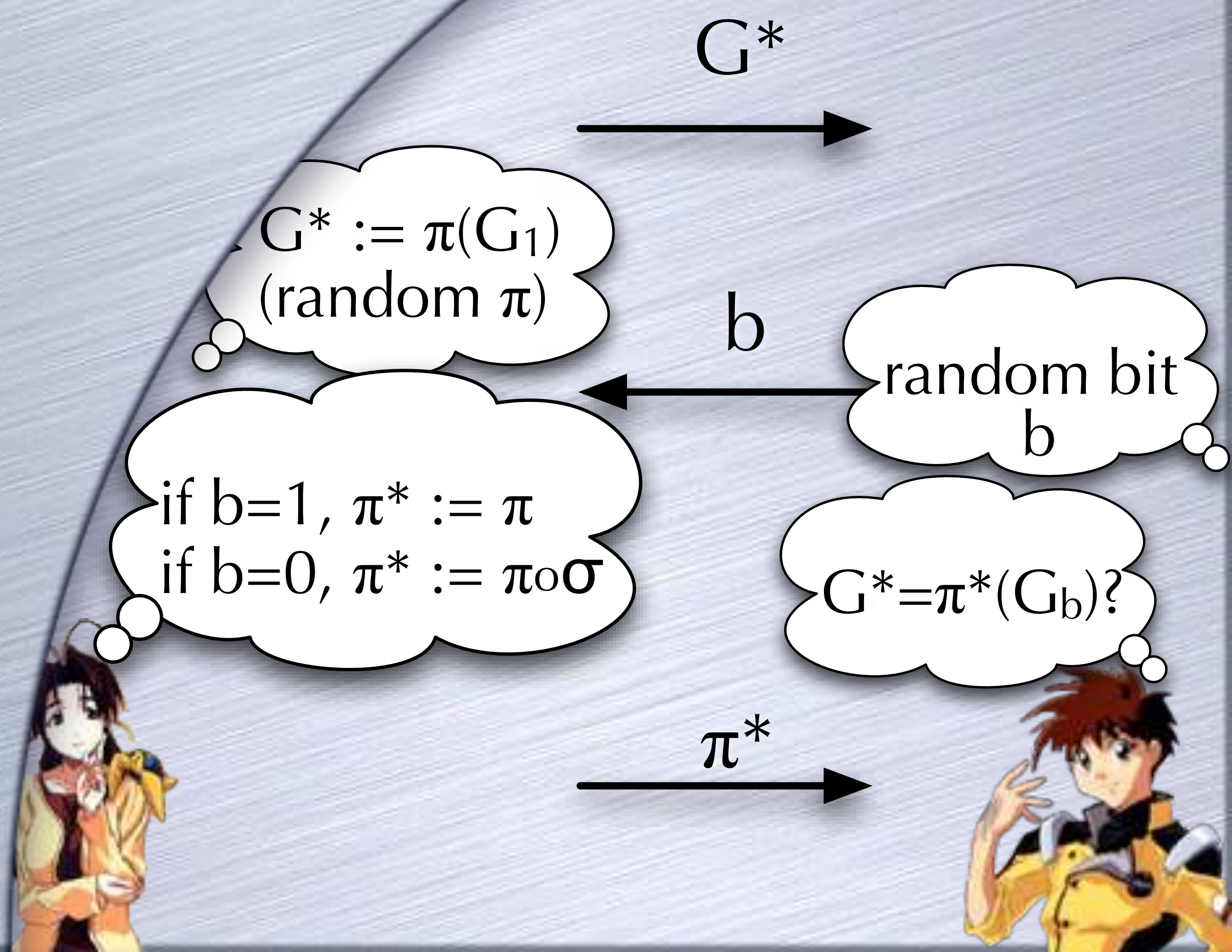

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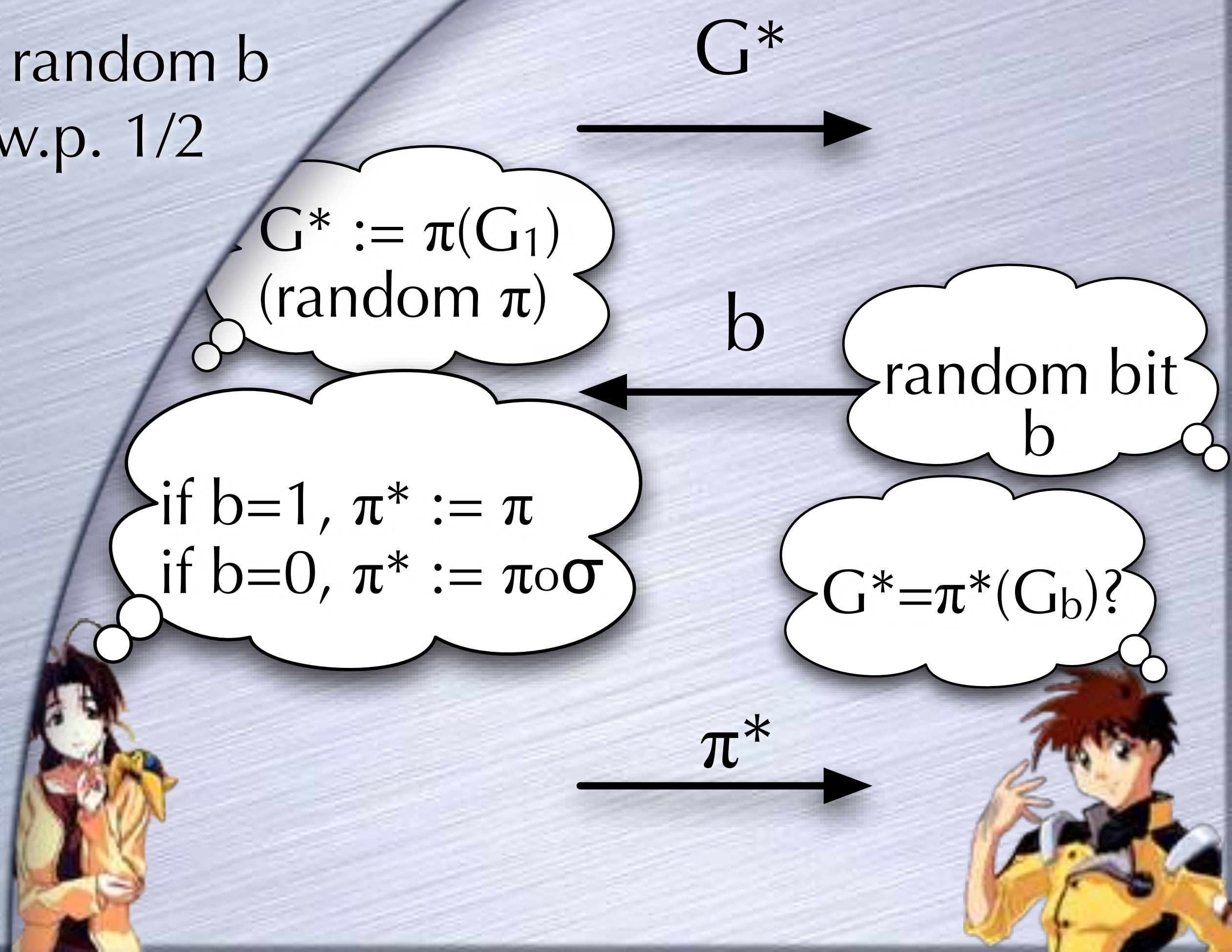

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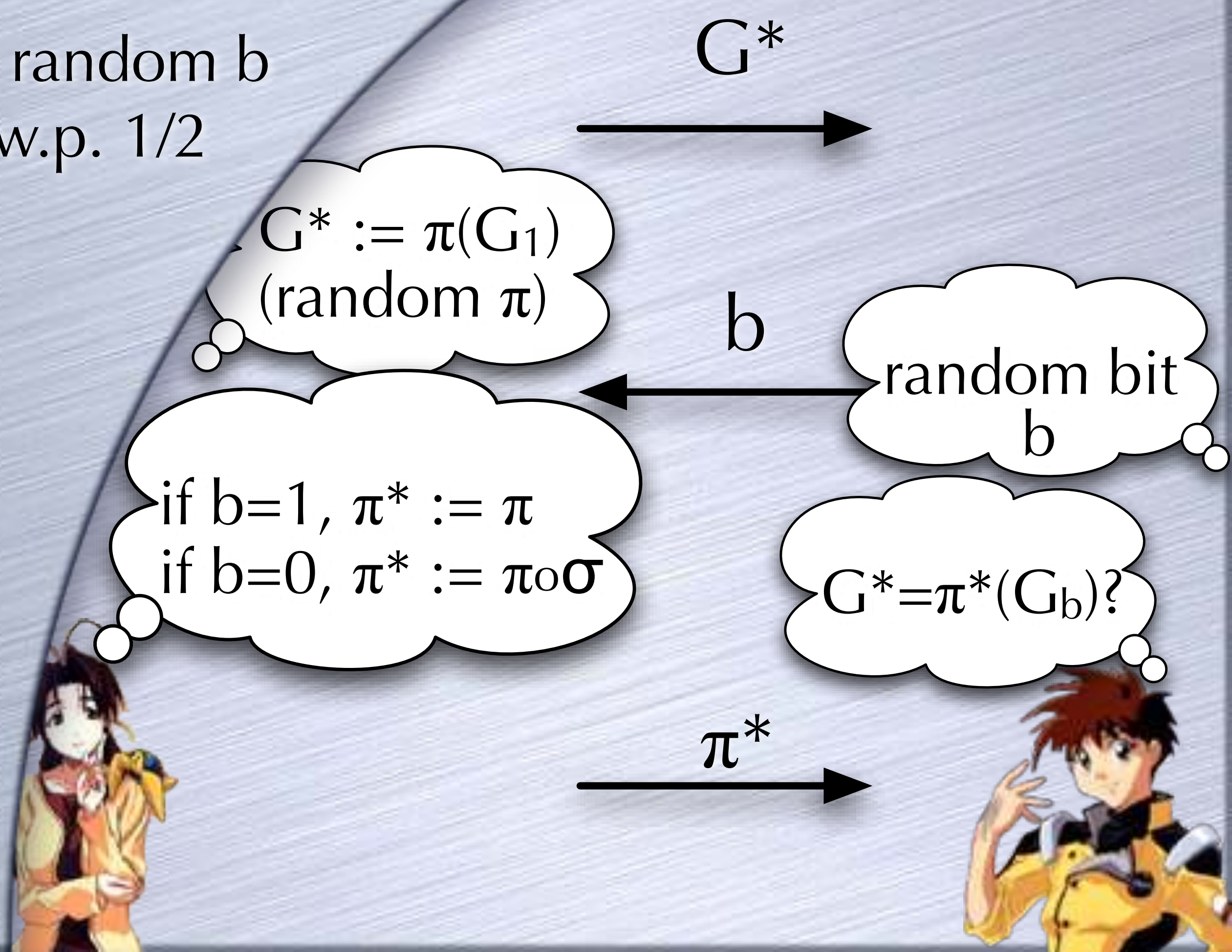

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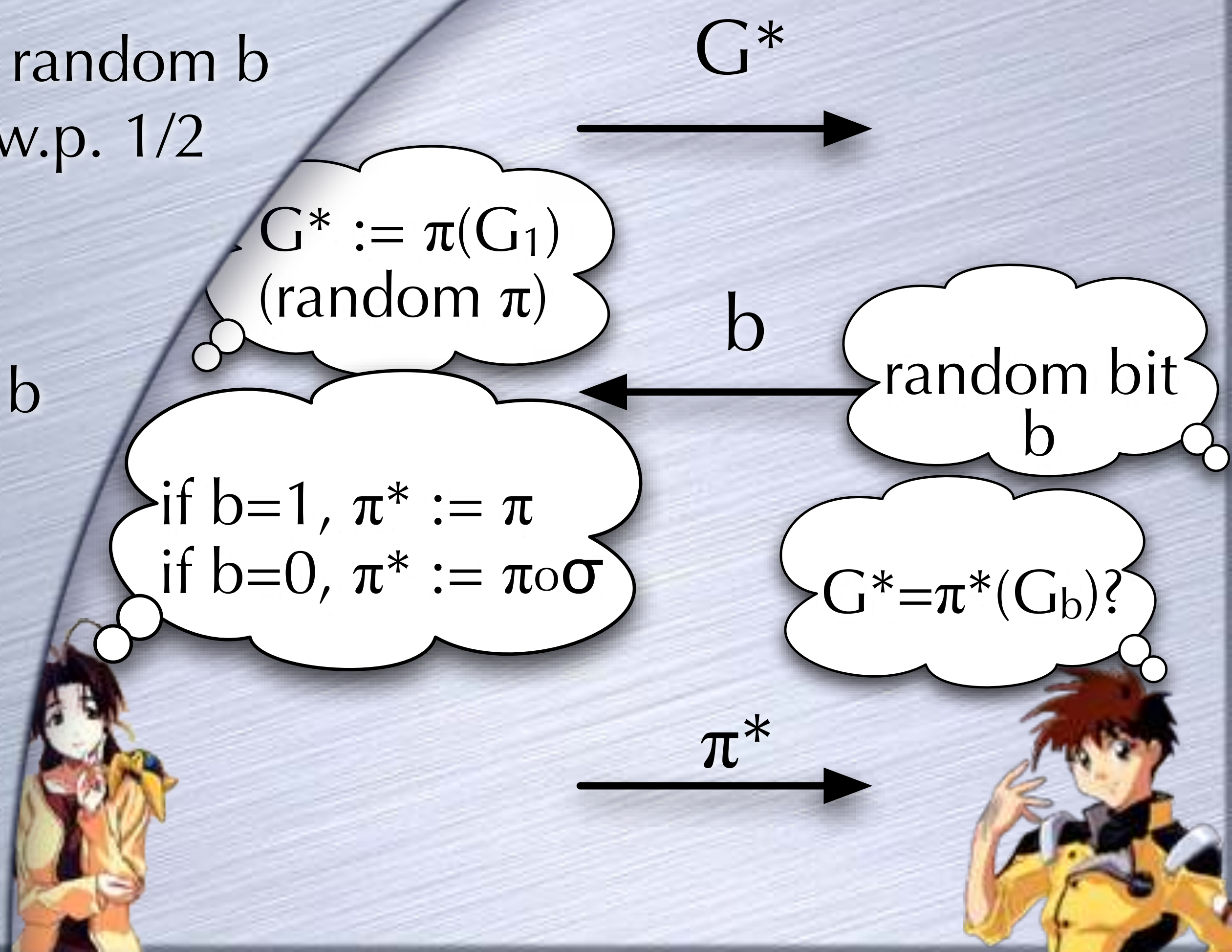

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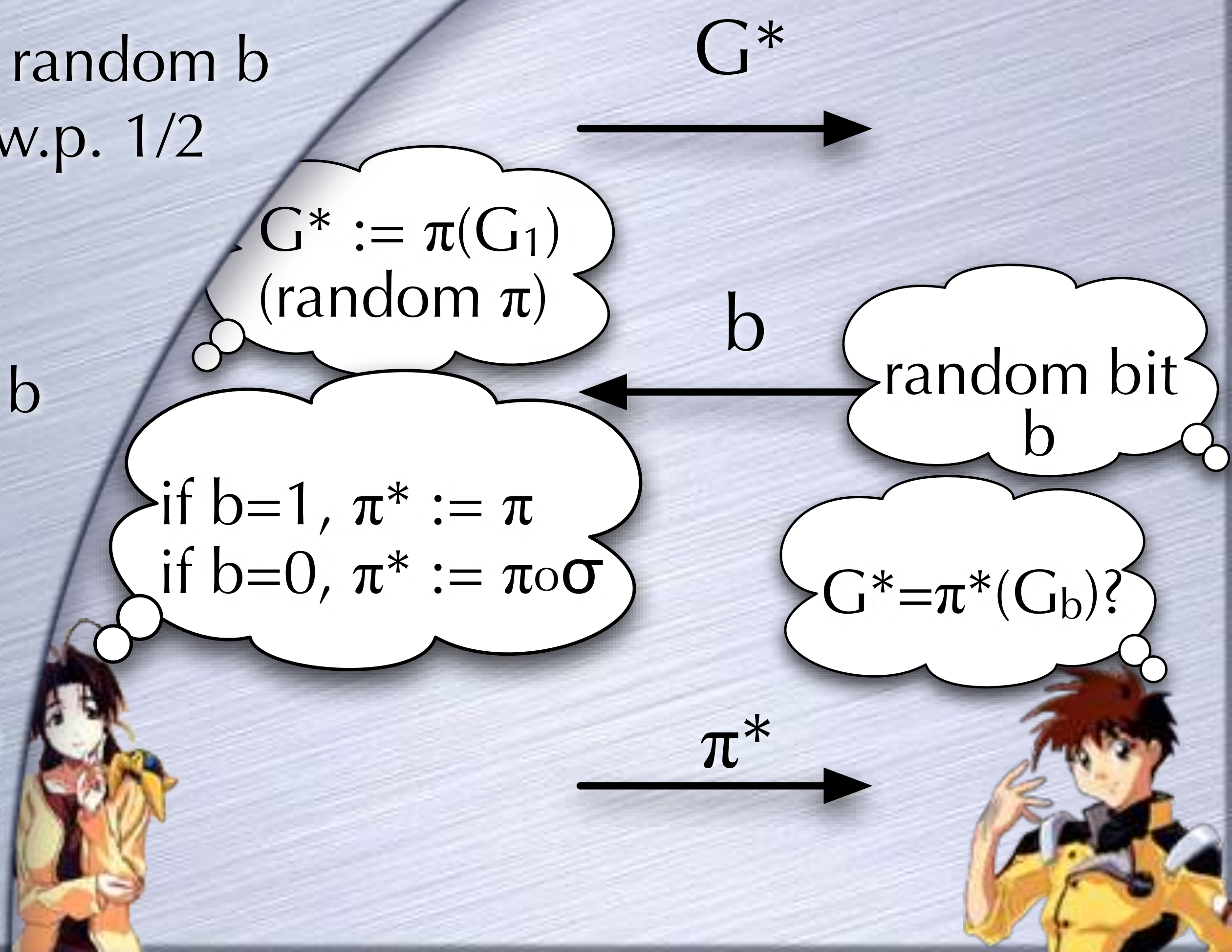
- **Why ZK?**
  - Verifier's view: random b and π^* s.t. $G^* = \pi^*(G_b)$
  - Which he could have generated by himself (whether $G_0 \sim G_1$ or not)



Zero-Knowledge Proofs

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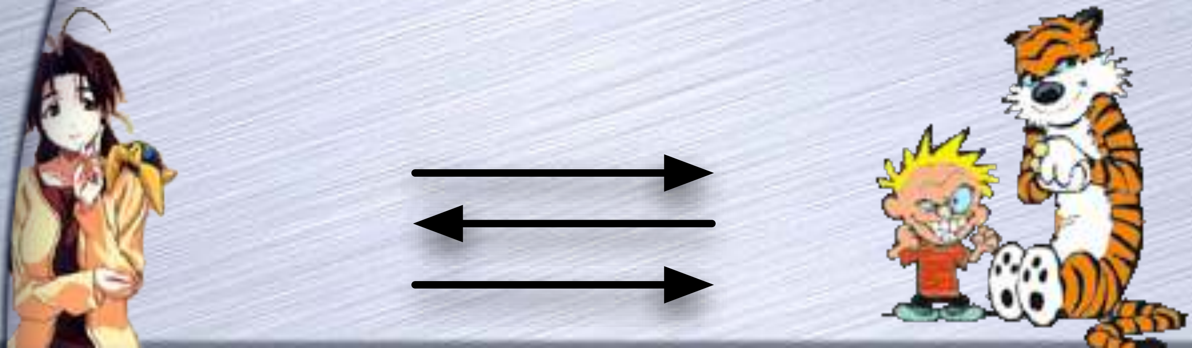

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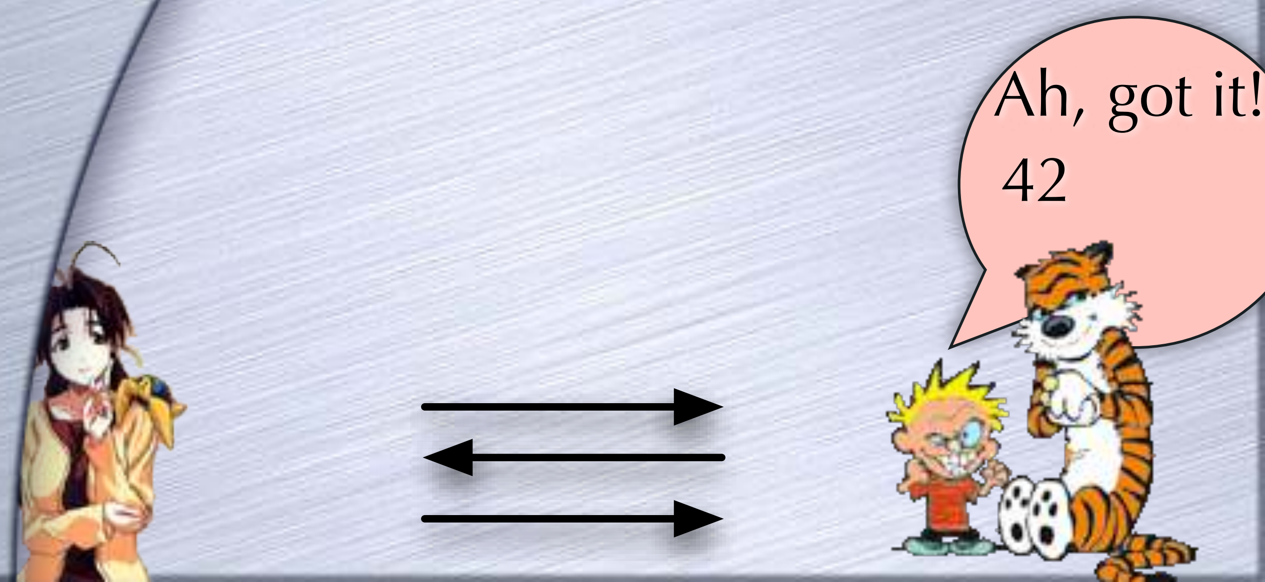

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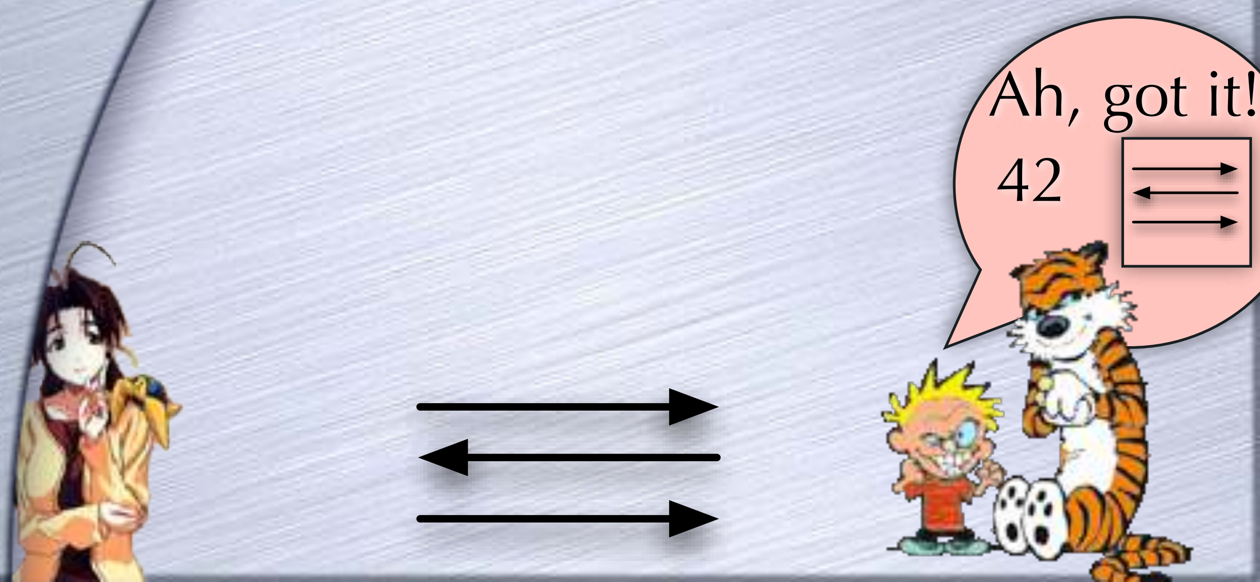

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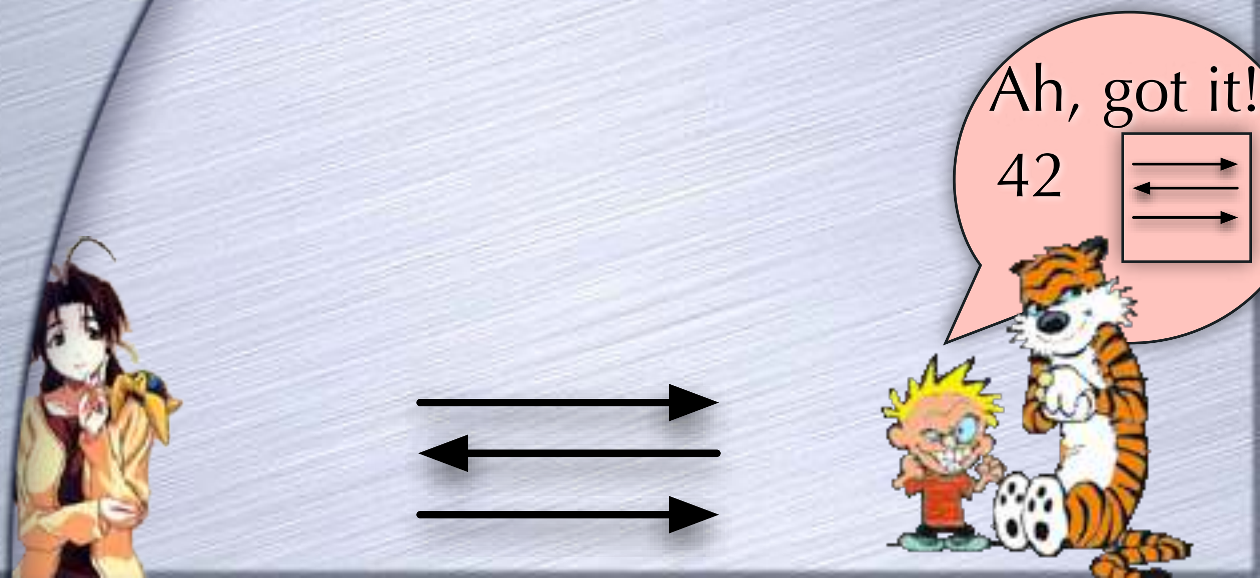

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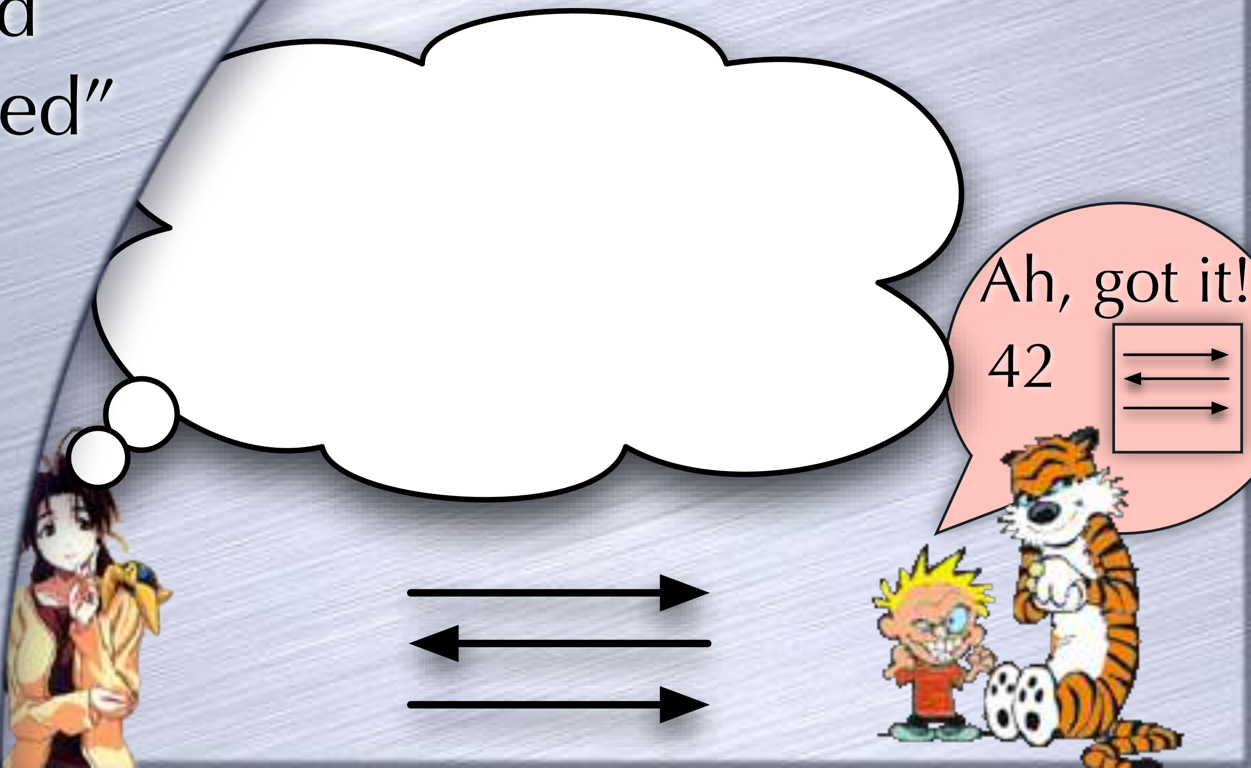

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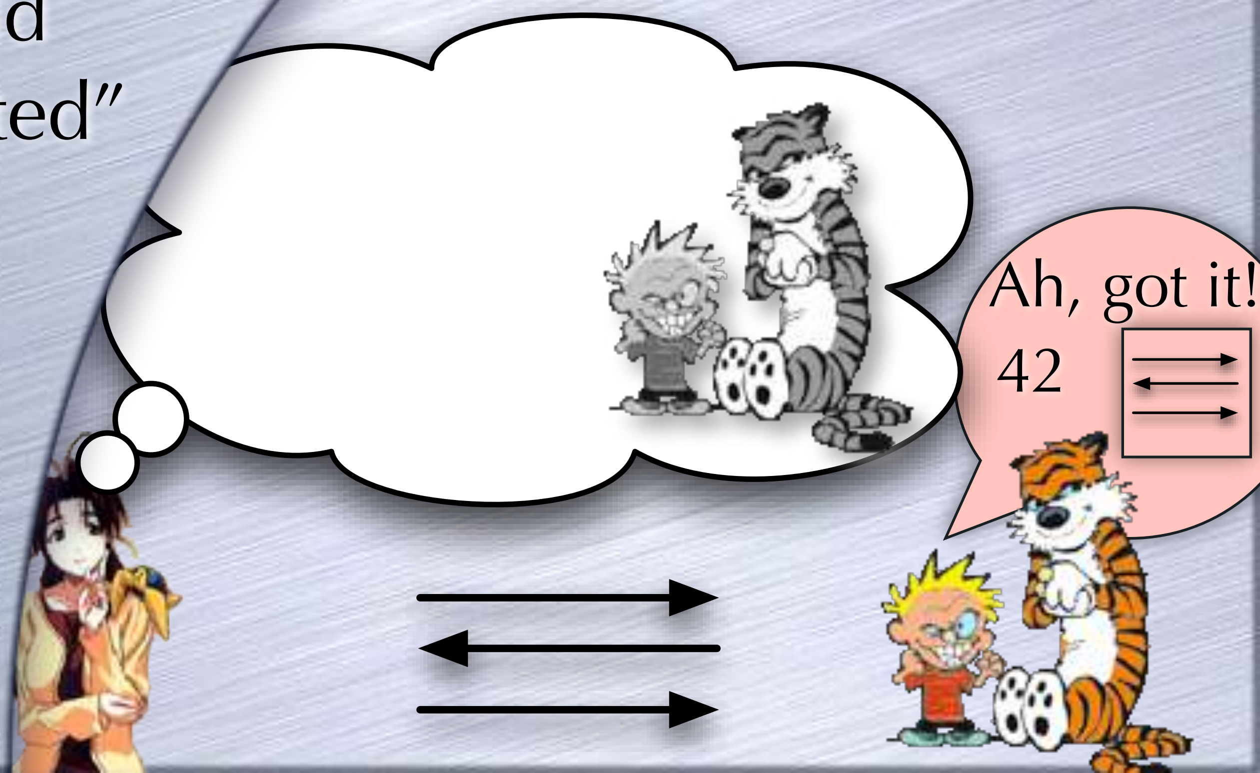

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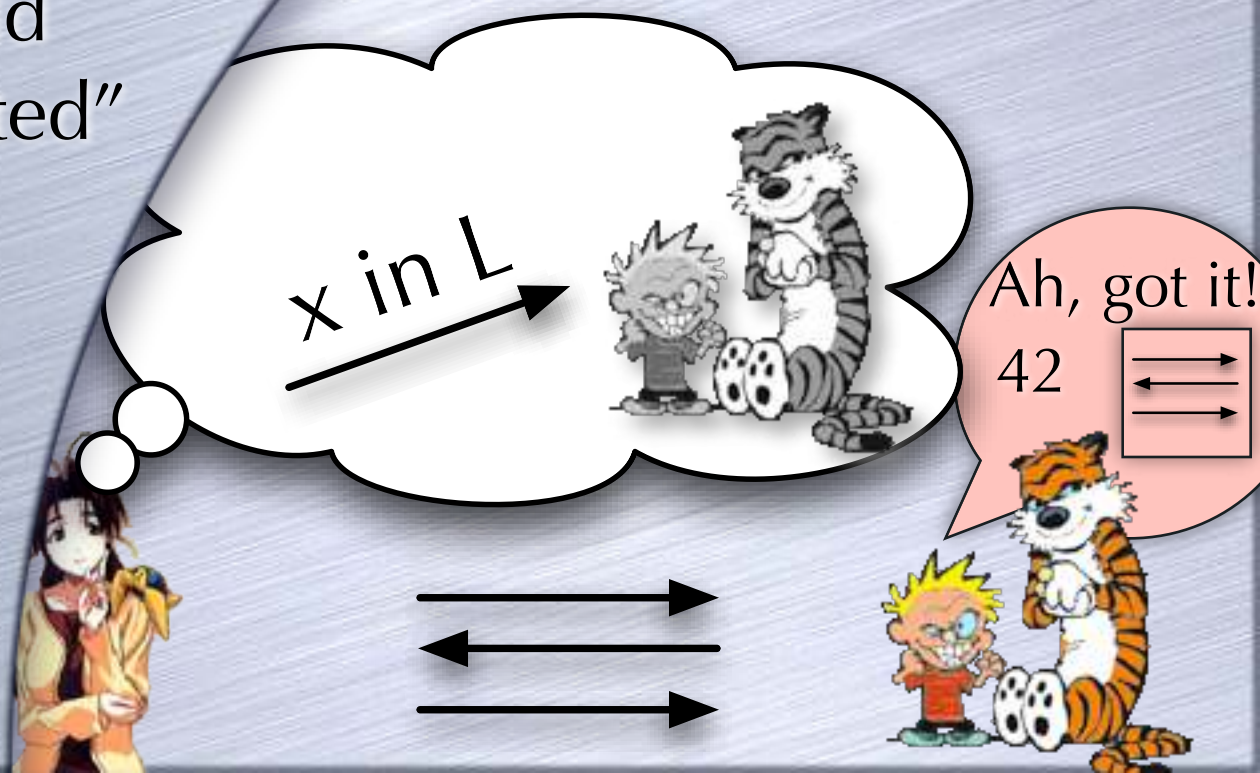

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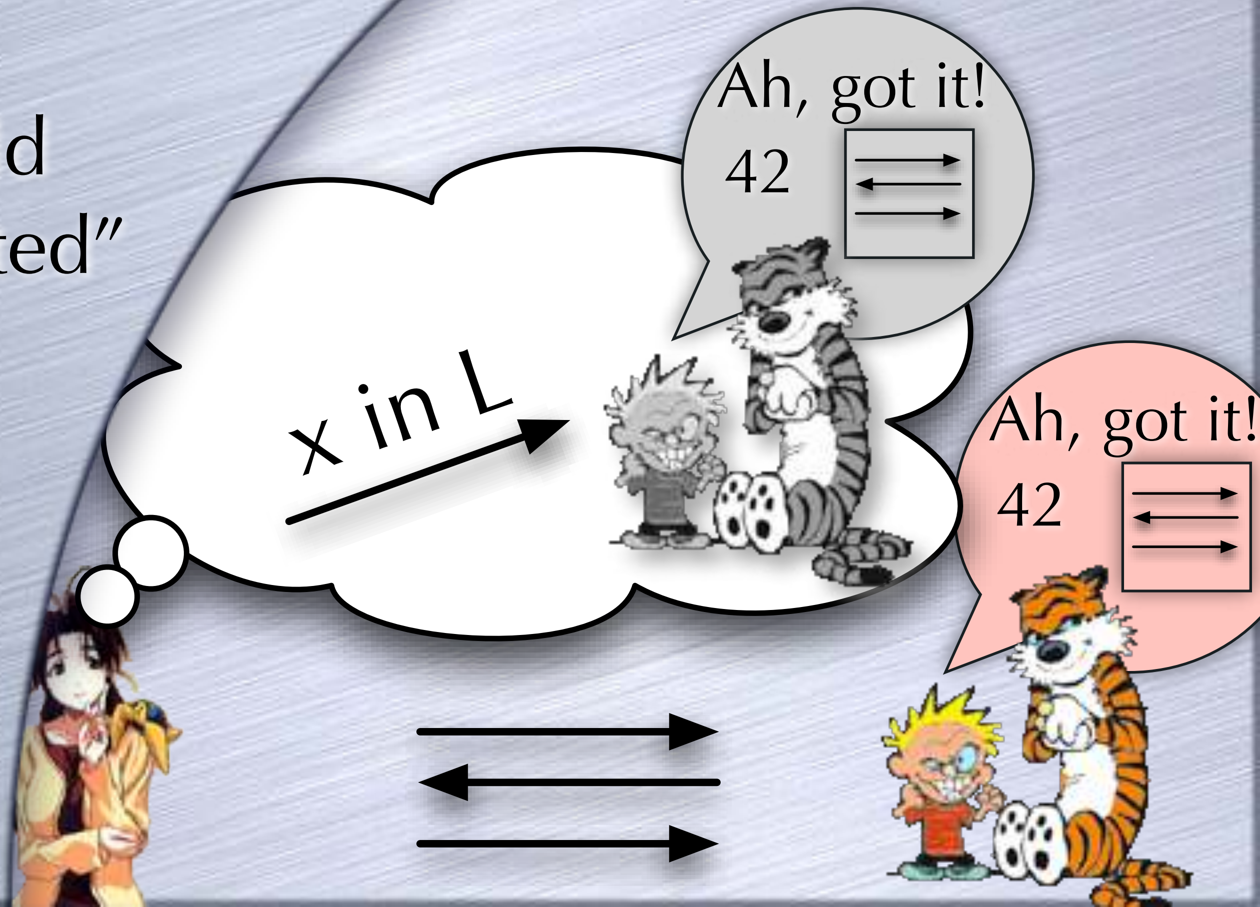

Zero-Knowledge Proofs

- Interactive Proof
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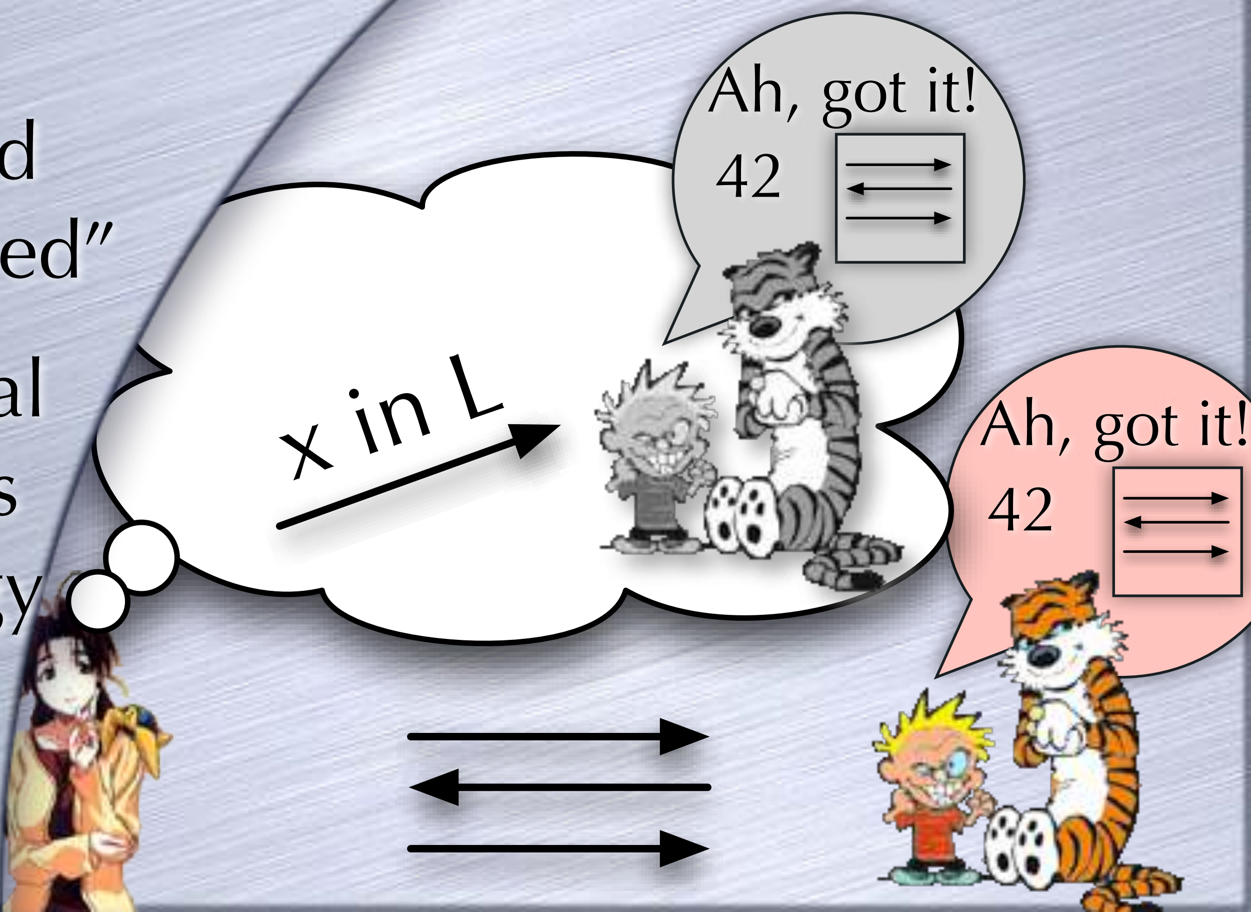
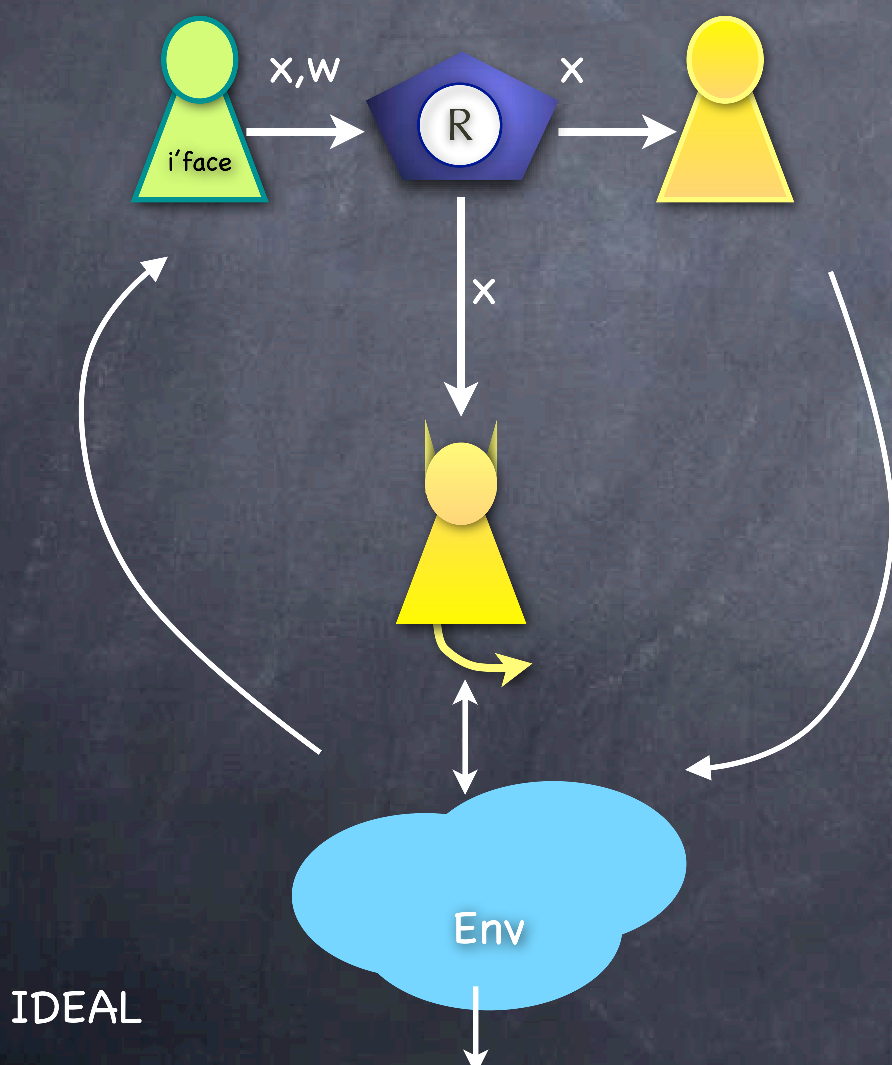
- ZK Property:
  - Verifier's view could have been "simulated"
  - For every adversarial strategy, there exists a simulation strategy



ZK Property (in other pict's)



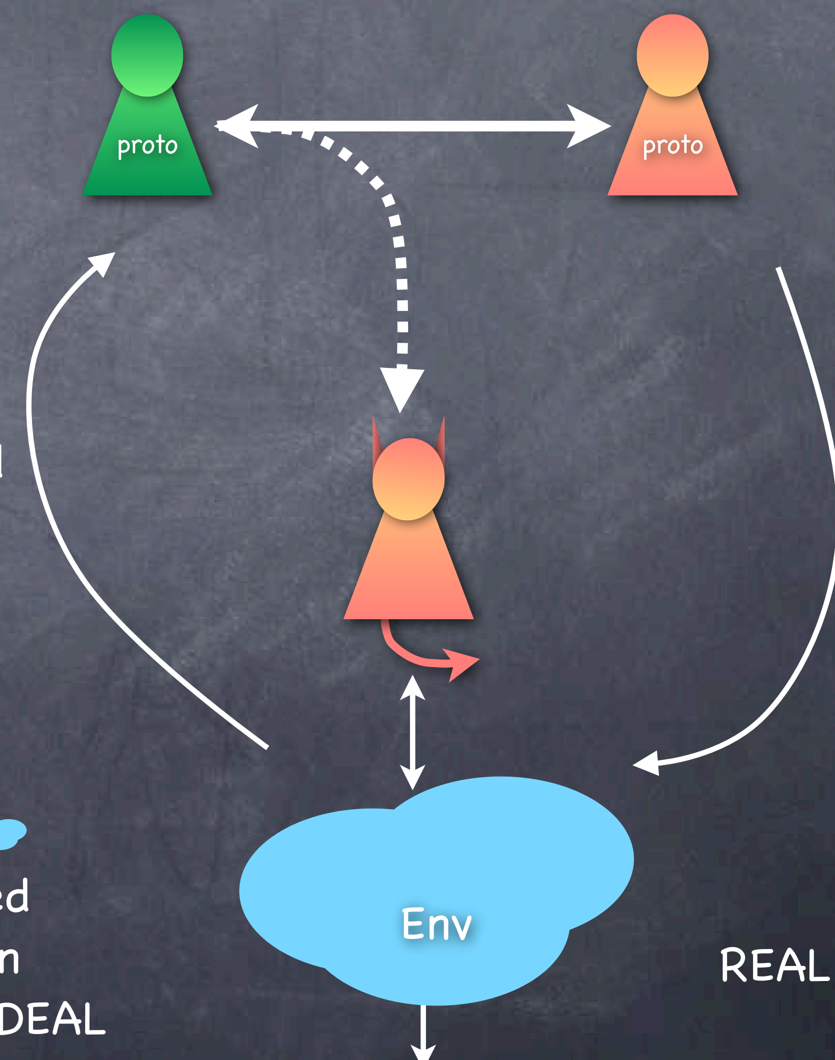
Secure (and correct) if:

∀ 

∃  s.t.

∀ 

output of  is distributed identically in REAL and IDEAL



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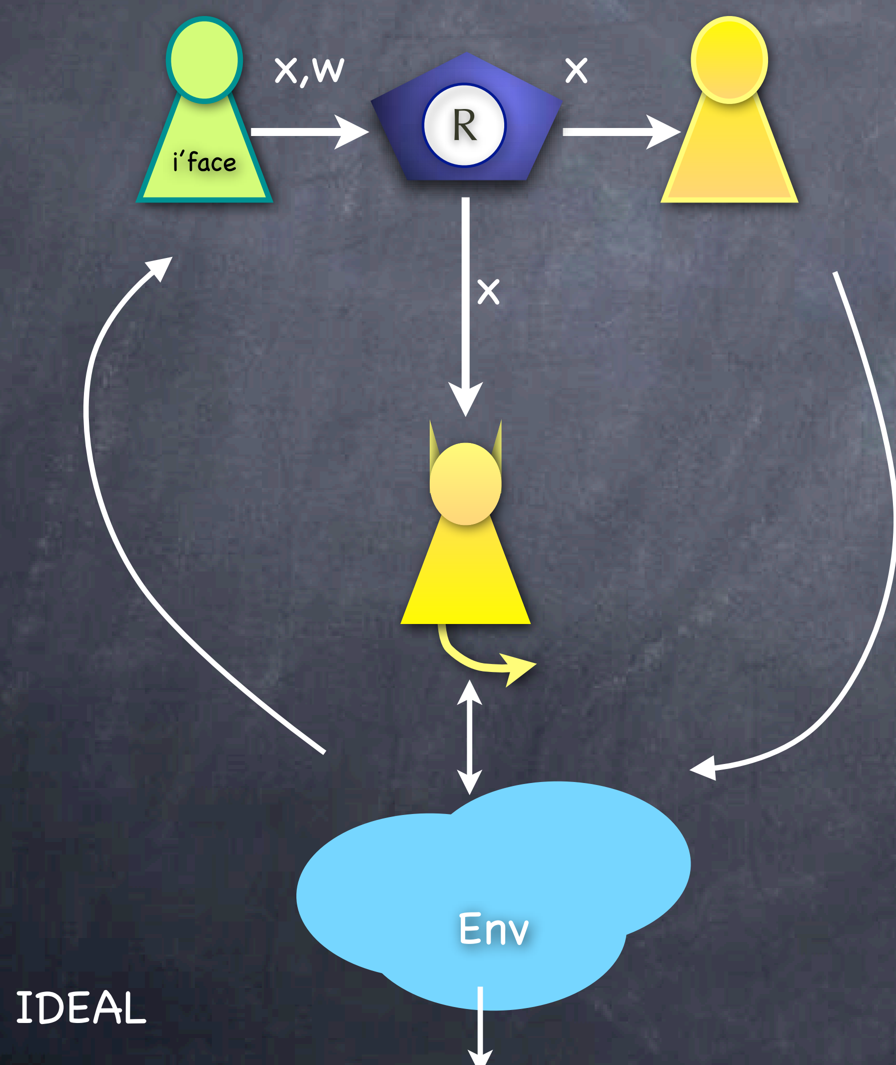

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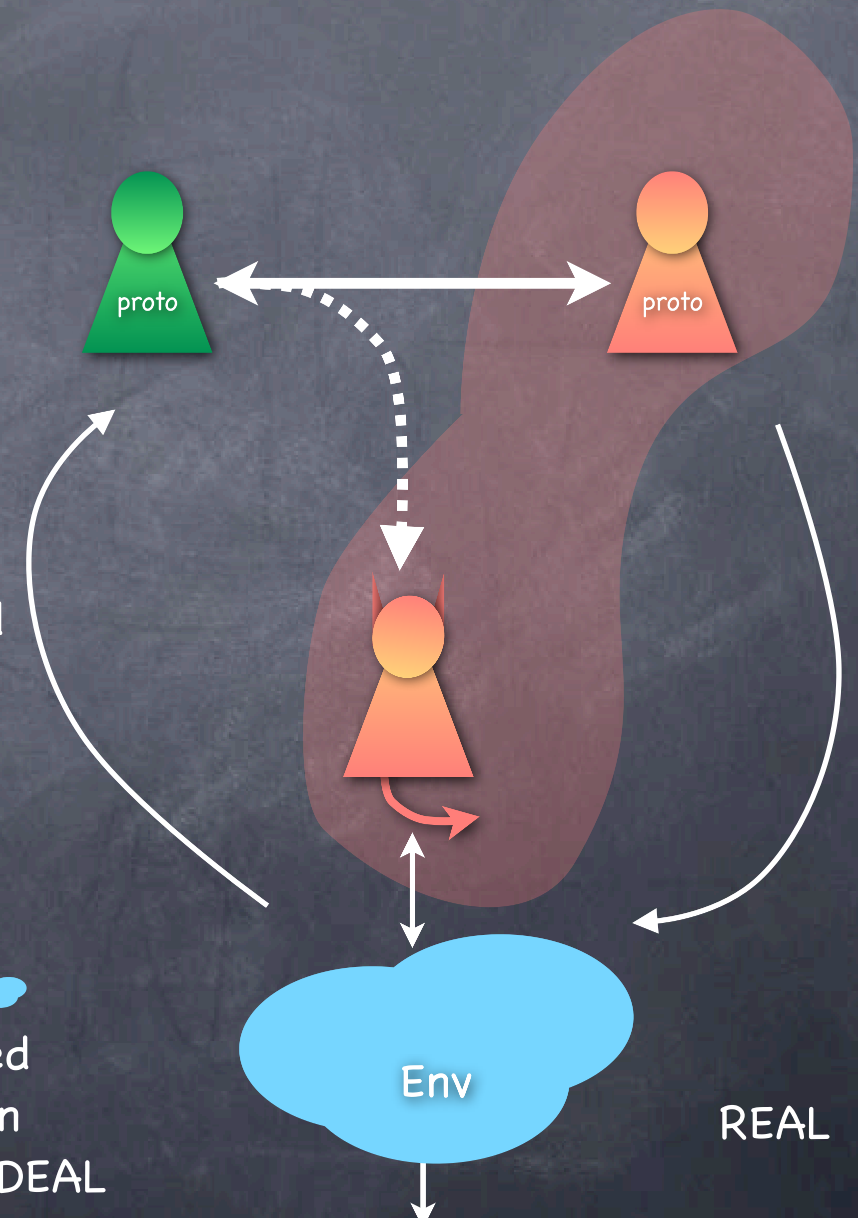

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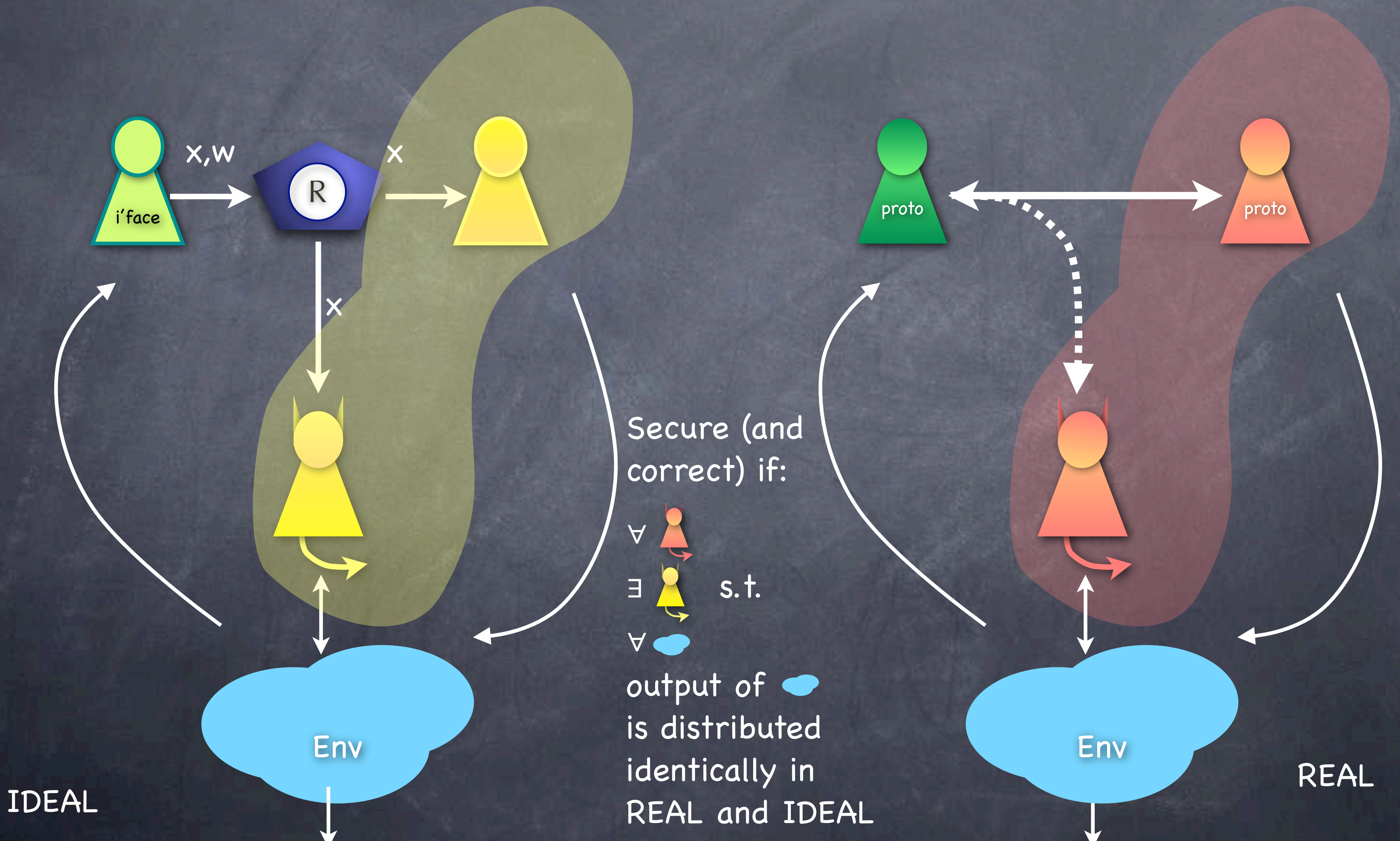

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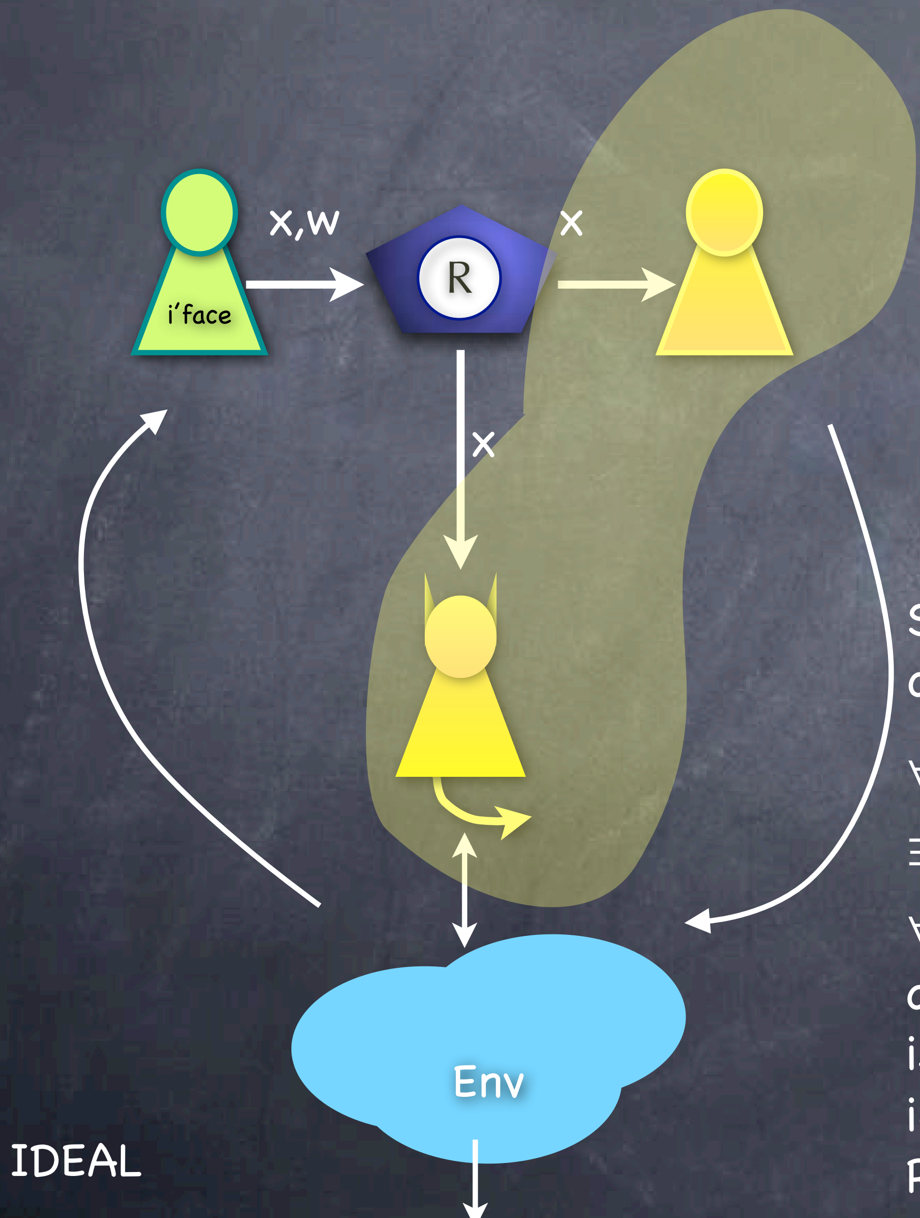
Classical definition uses simulation only for corrupt receiver;



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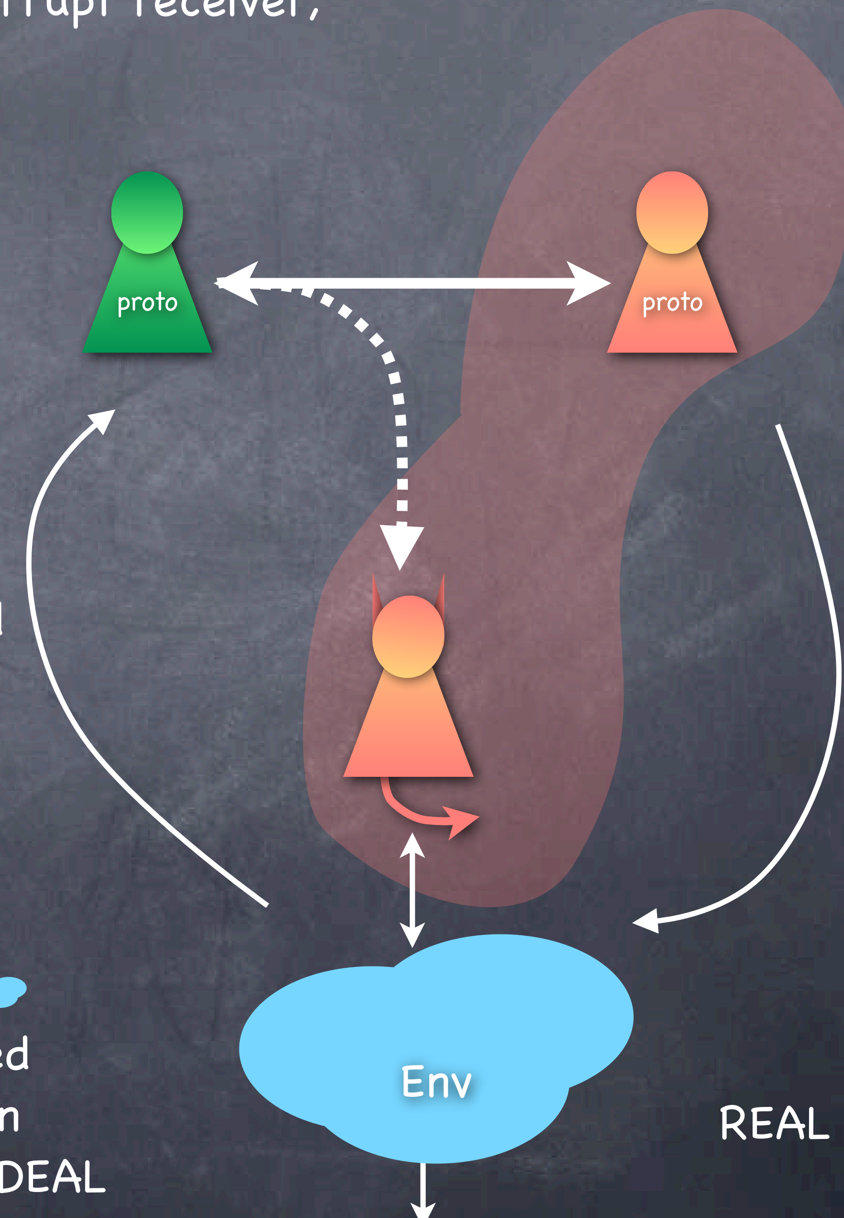
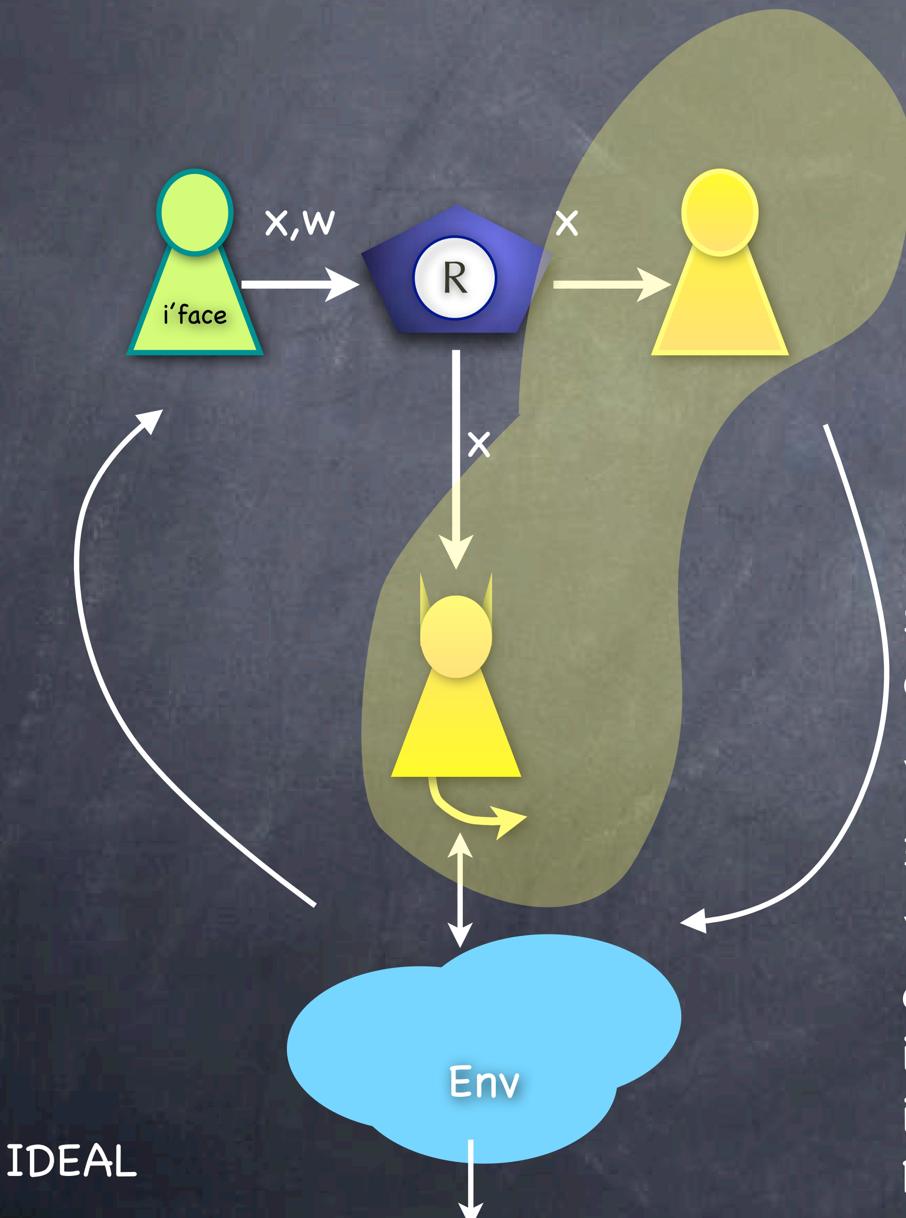


ZK Property (in other pict's)

Classical definition uses simulation only for corrupt receiver; and uses only standalone security: Environment gets only a transcript at the end



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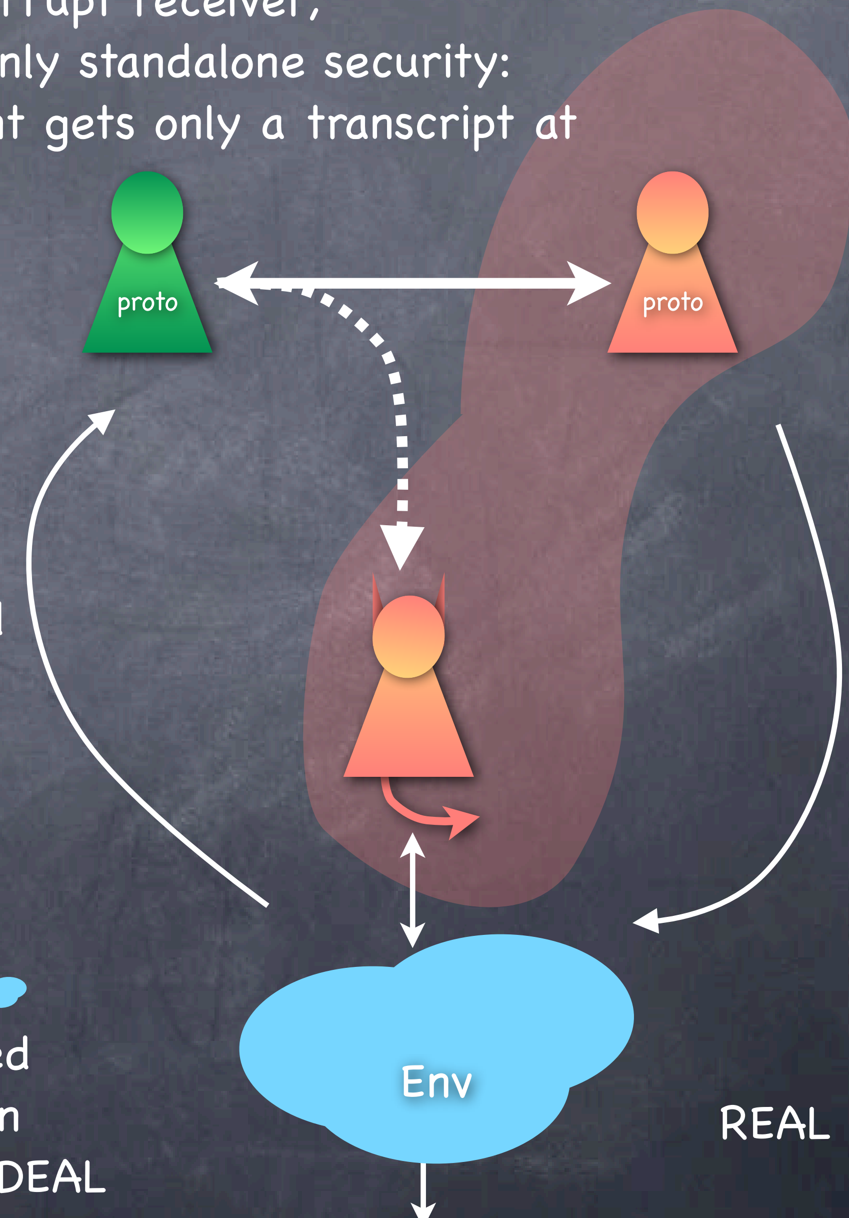

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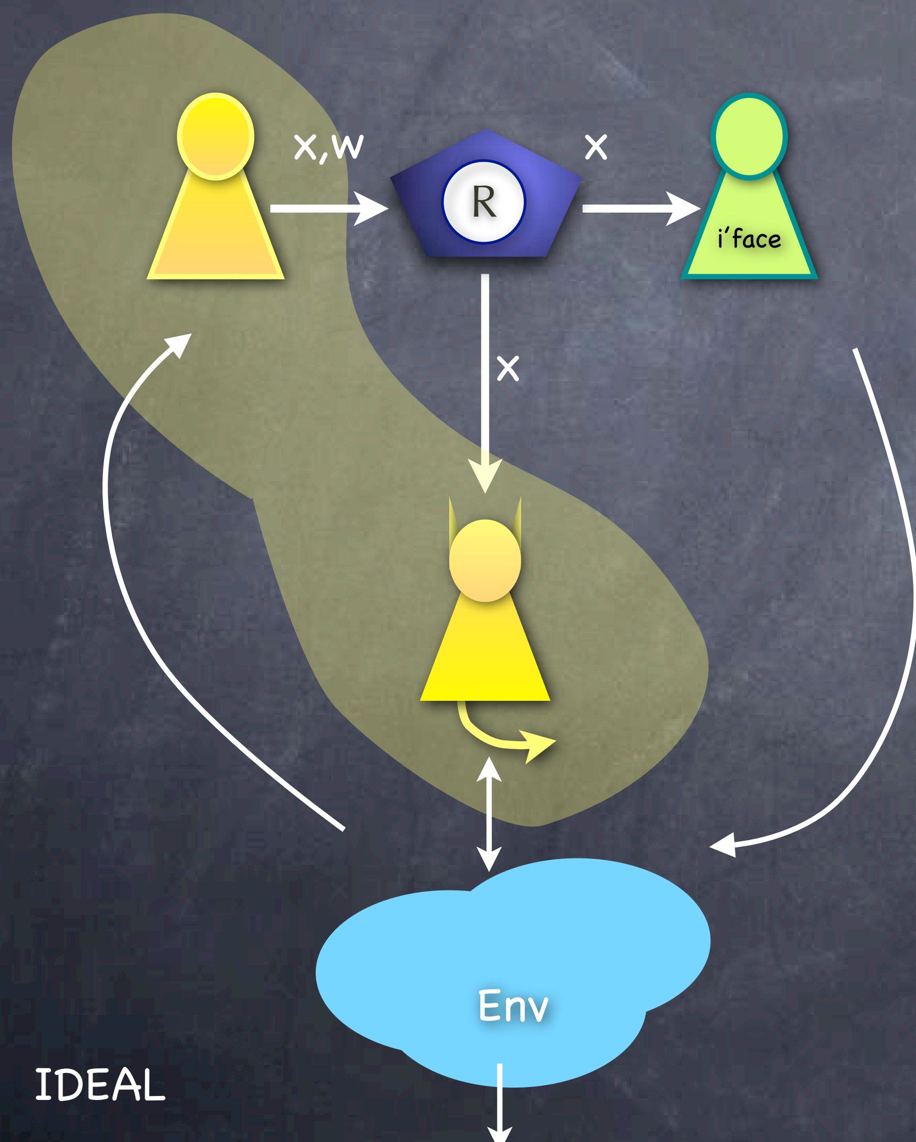

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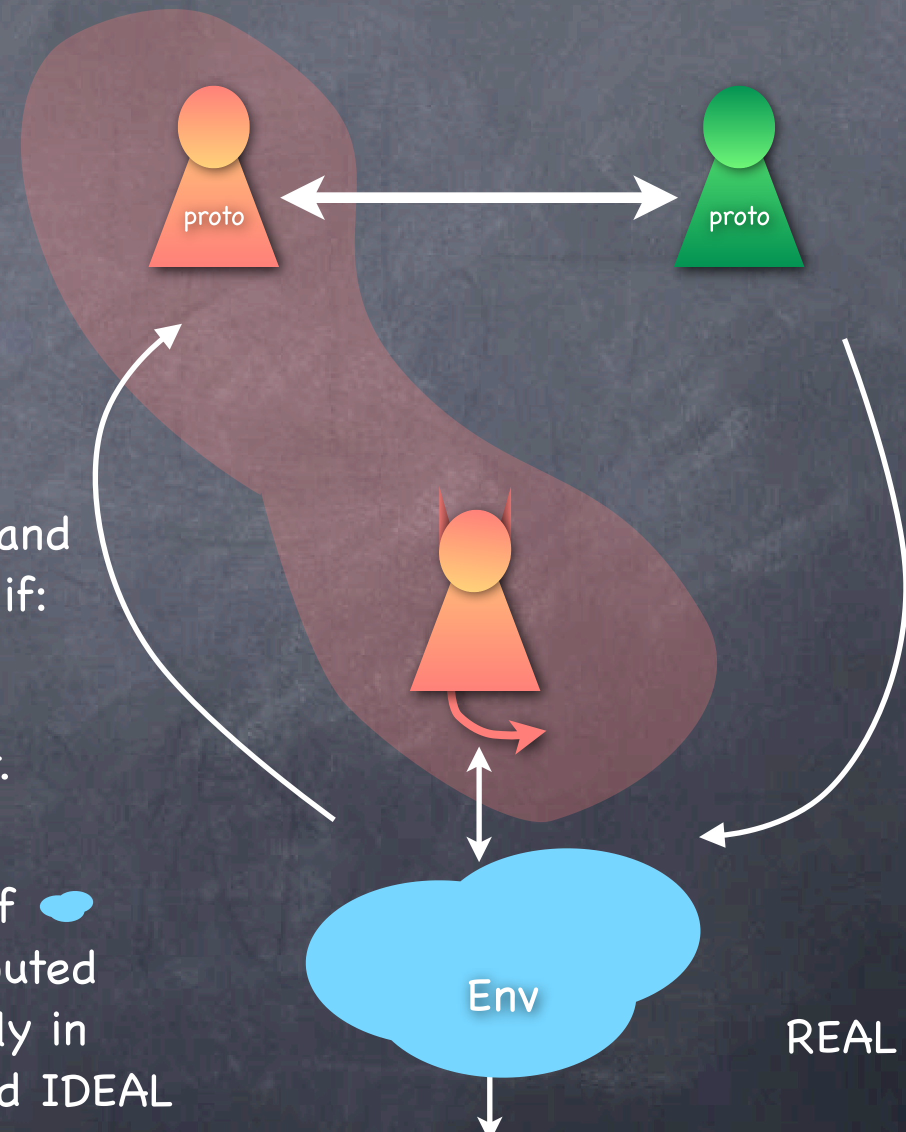
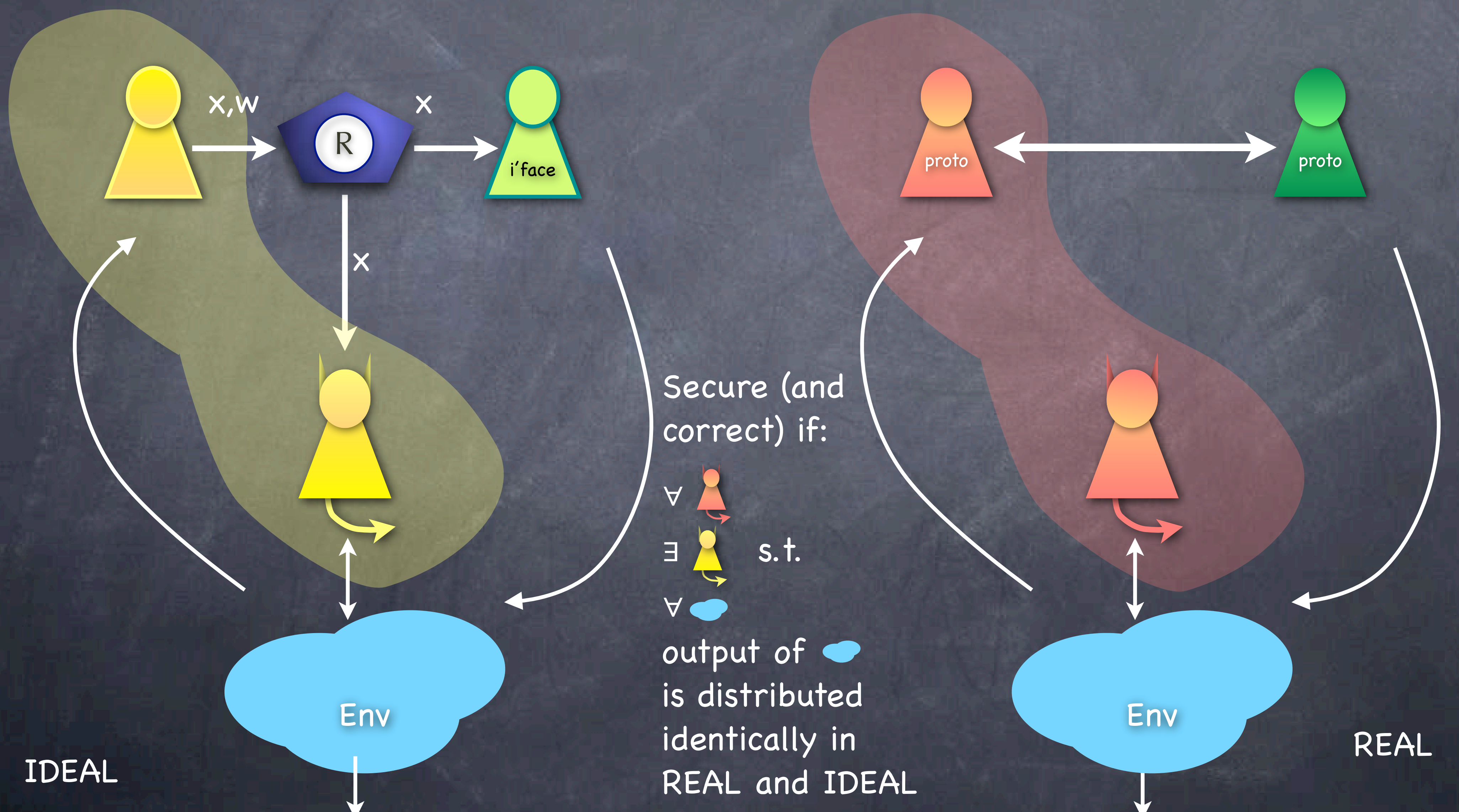


SIM ZK

- SIM-ZK would require simulation also when prover is corrupt



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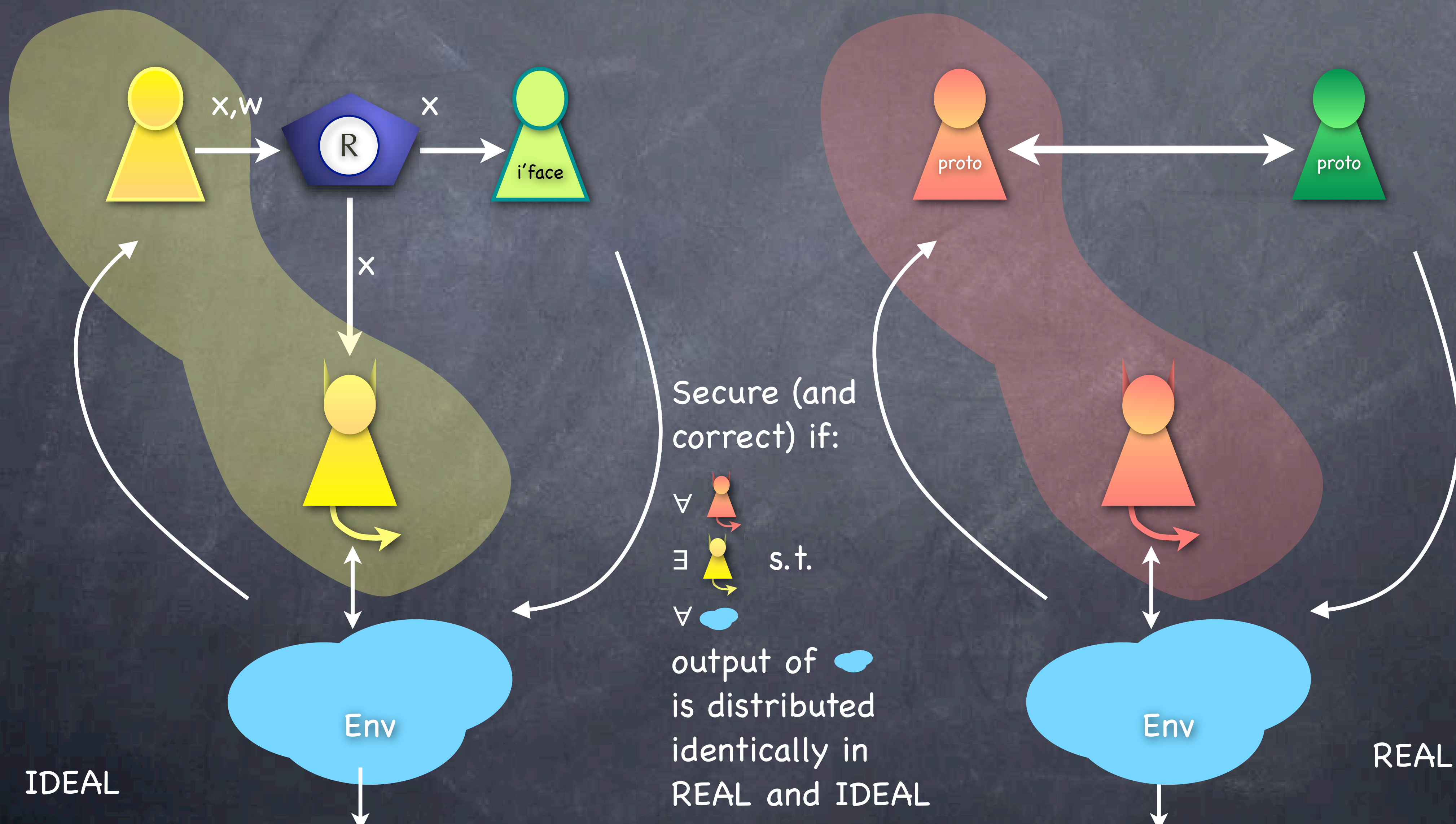
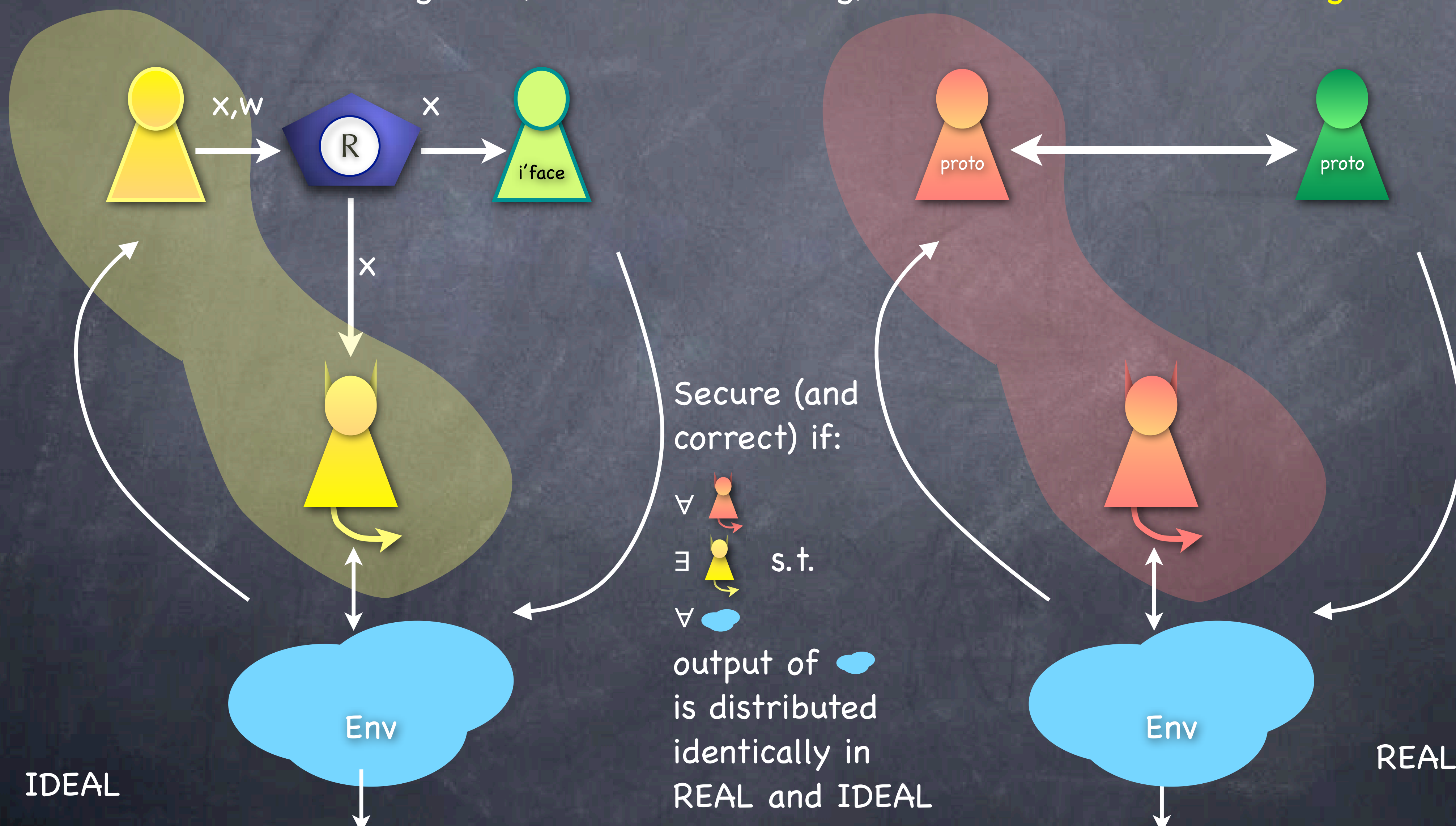


SIM ZK

- SIM-ZK would require simulation also when prover is corrupt
  - Then simulator is a witness extractor
- Adding this (in standalone setting) makes it a Proof of Knowledge



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A ZK Proof for Graph Colorability

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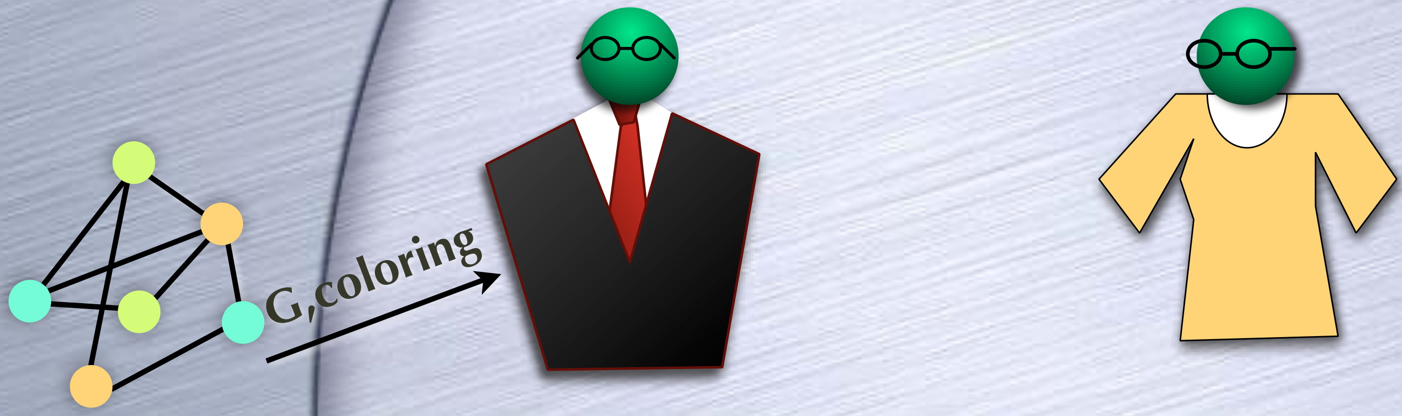
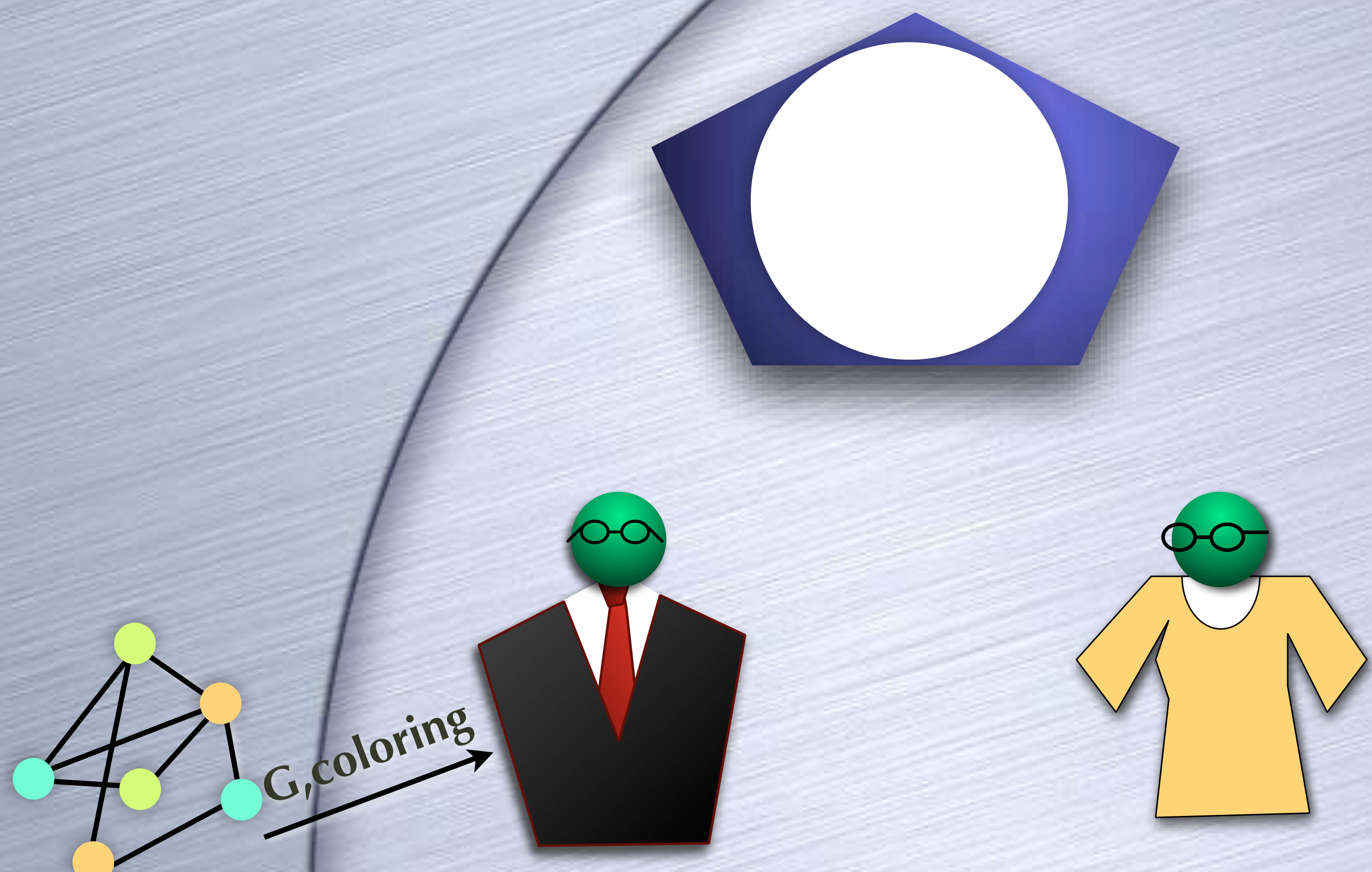


A ZK Proof for Graph Colorability

- Uses a commitment protocol as a subroutine



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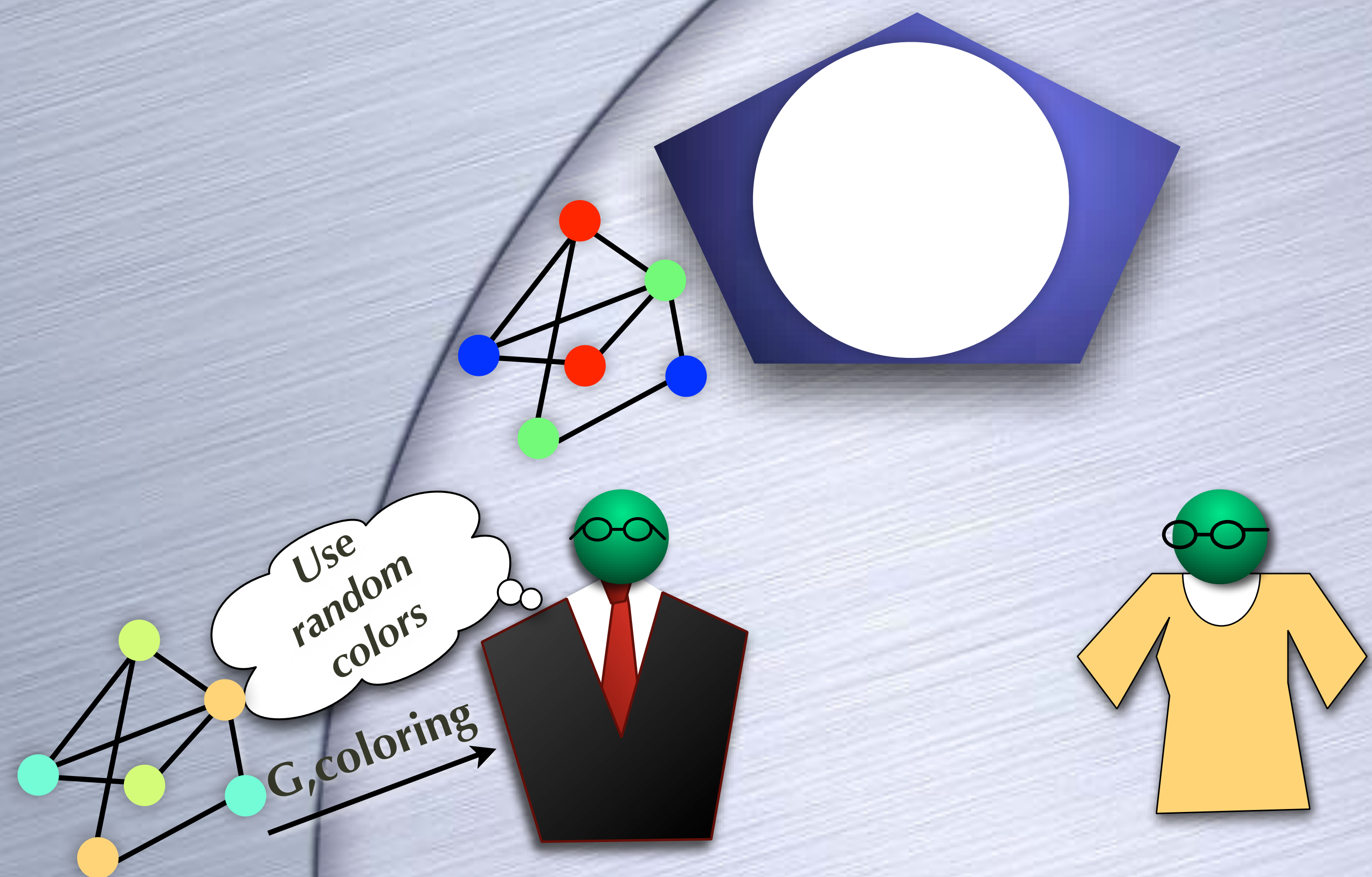

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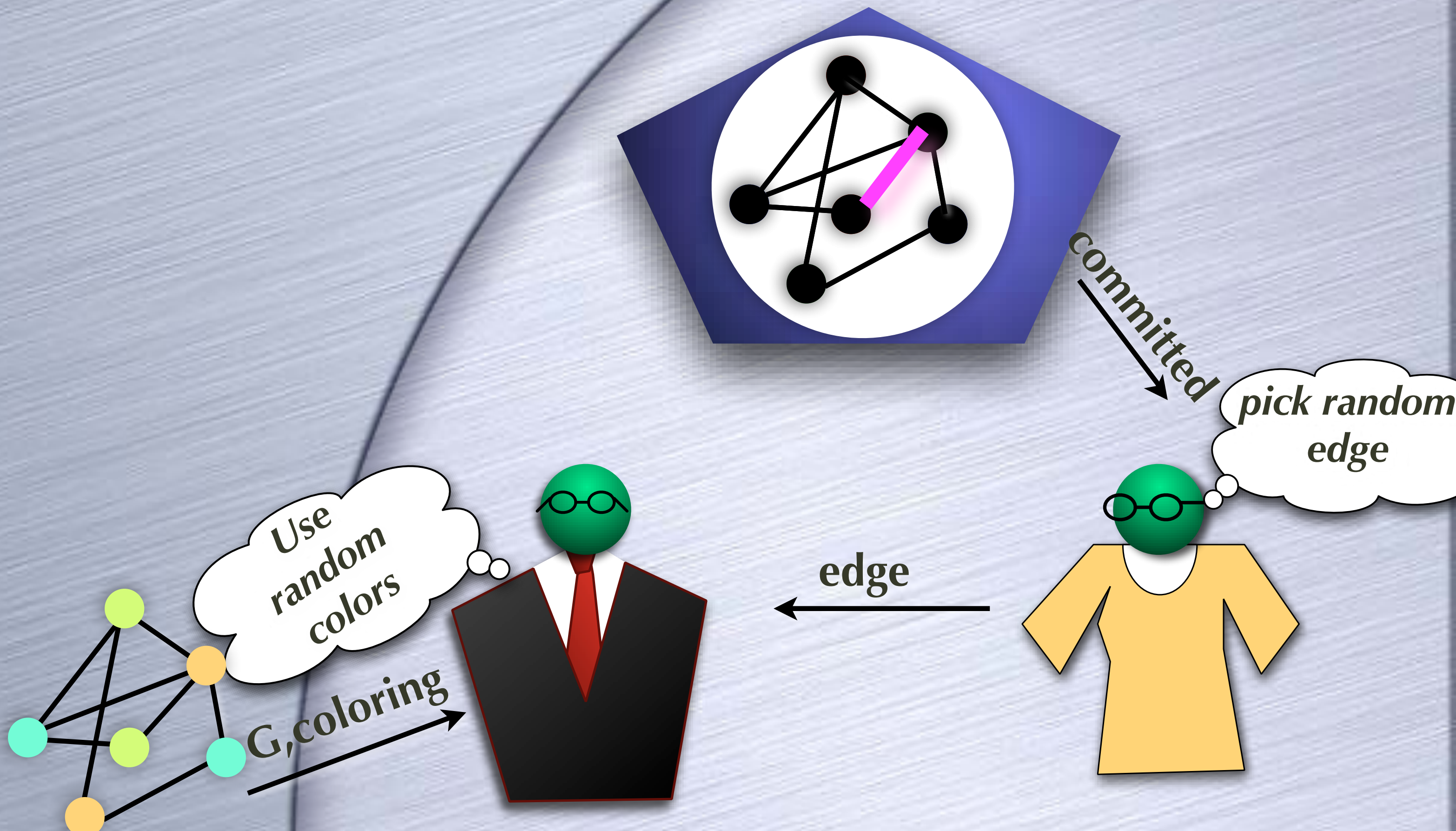

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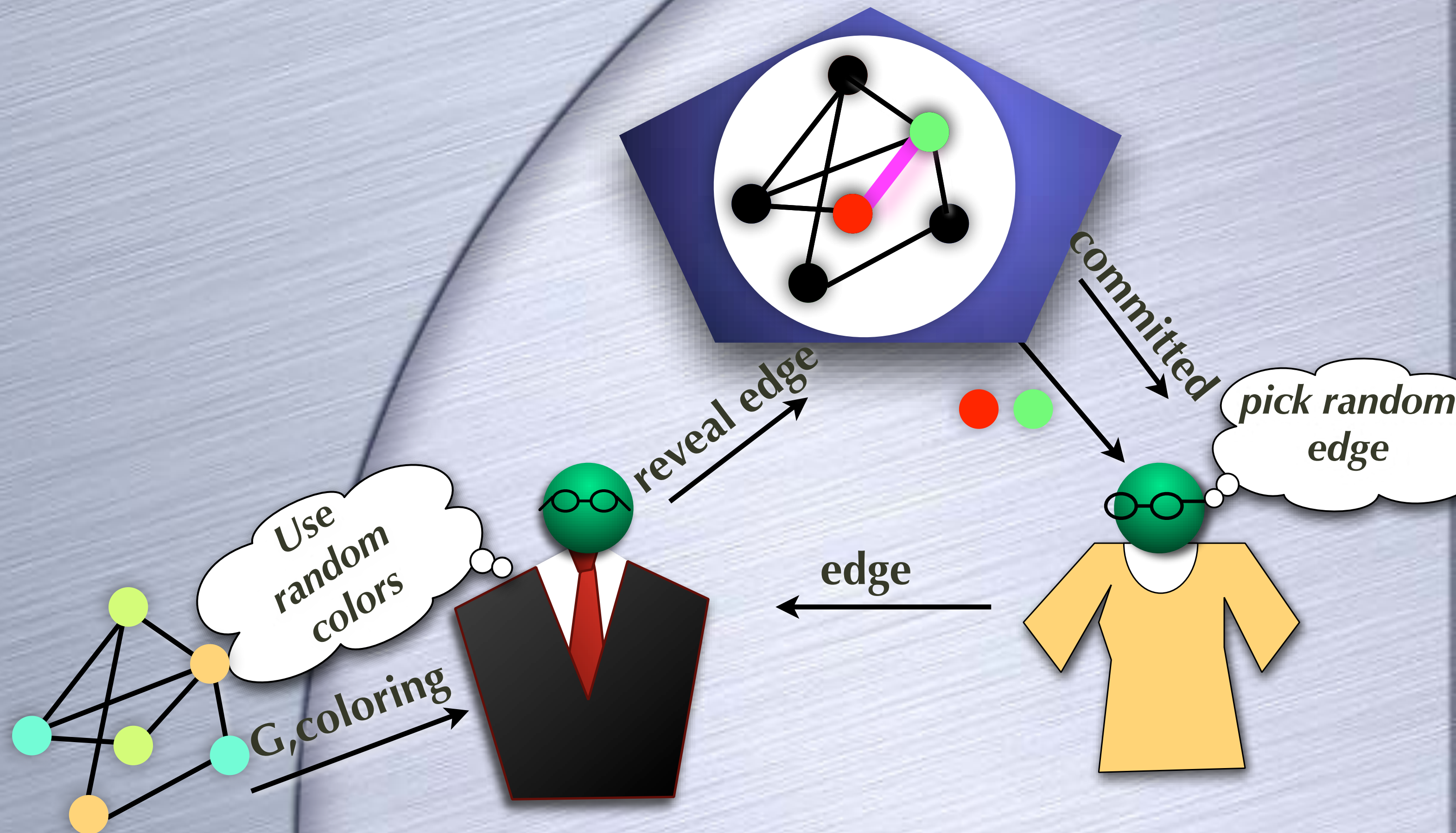

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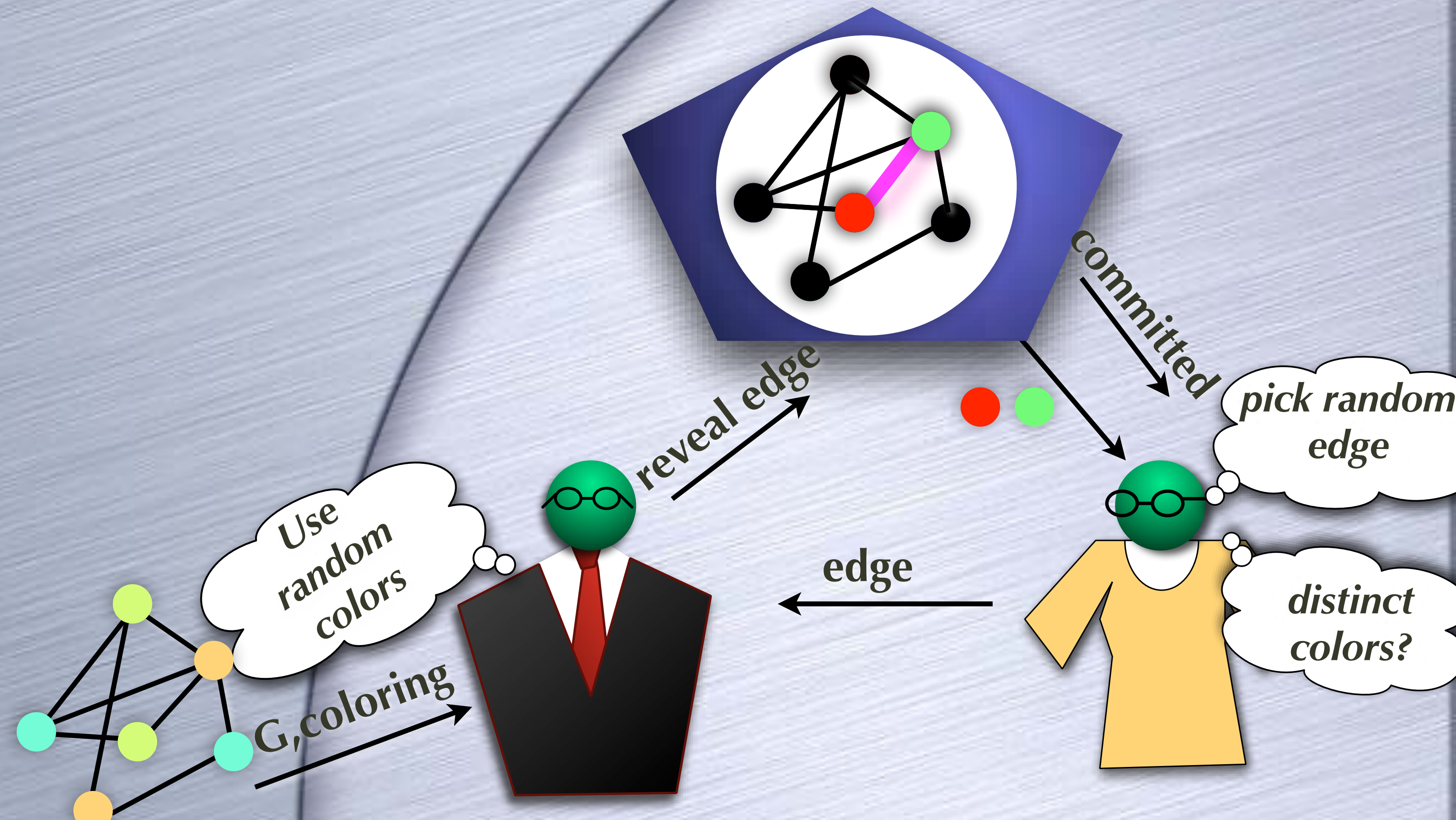

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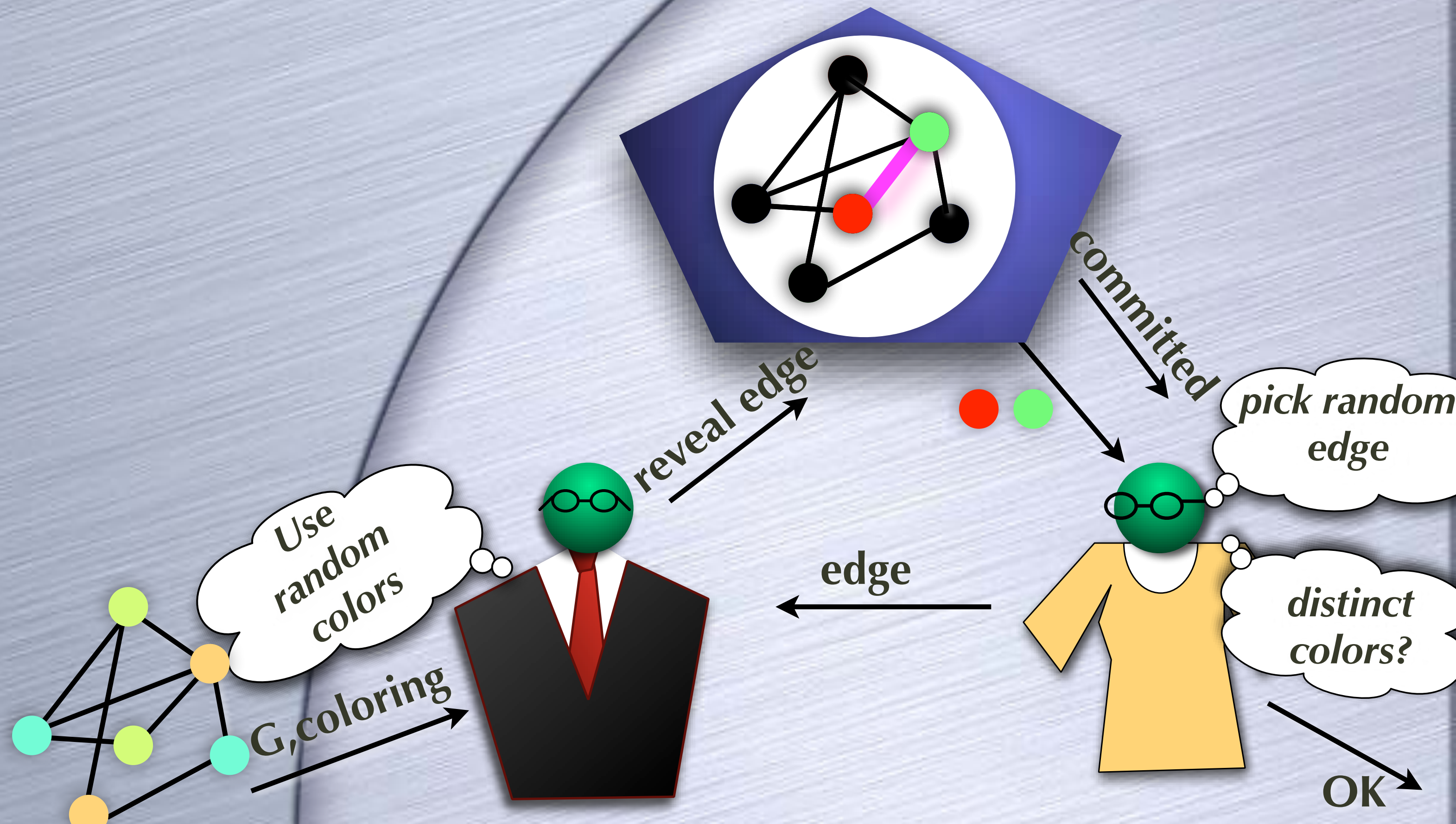

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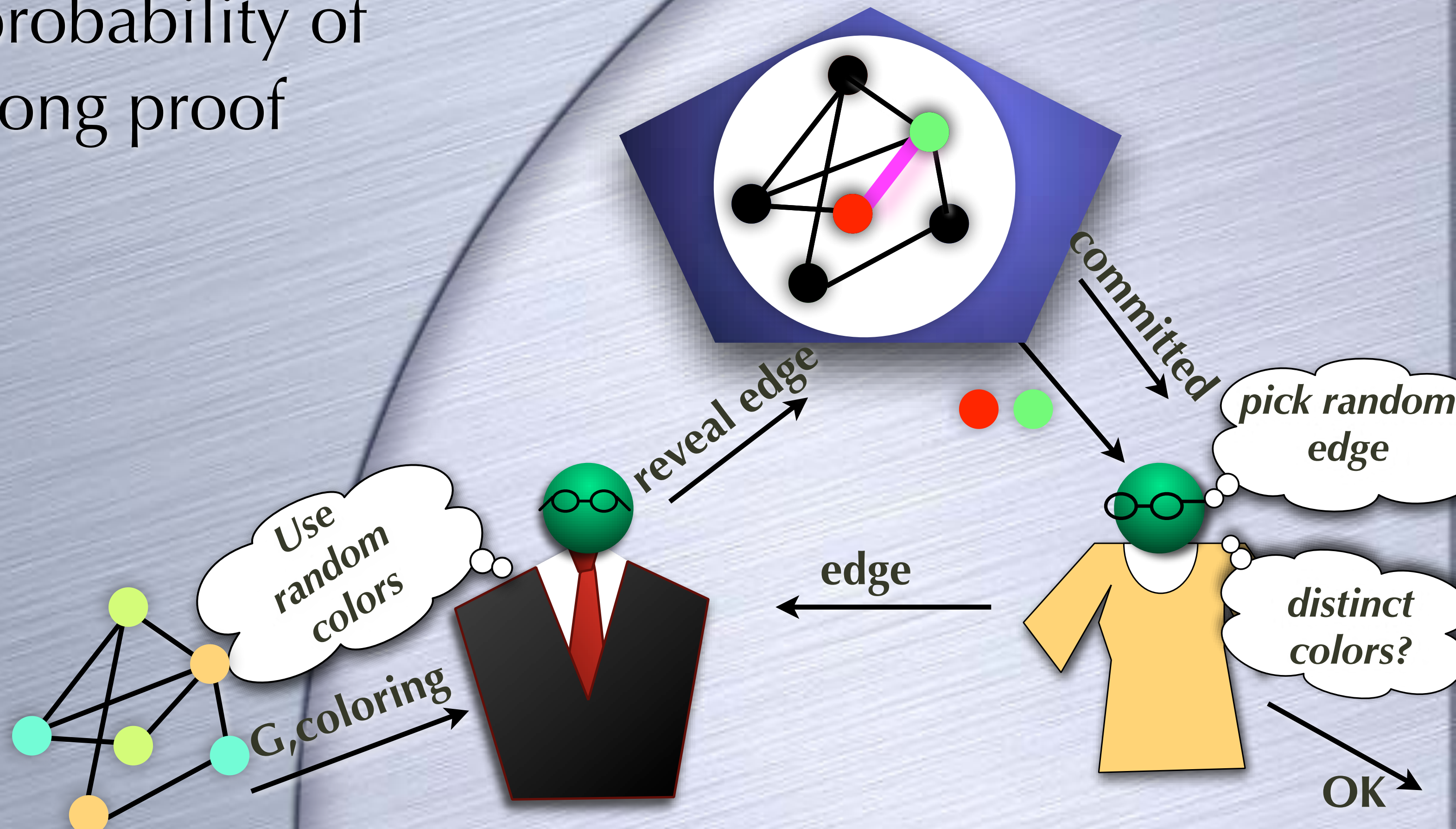
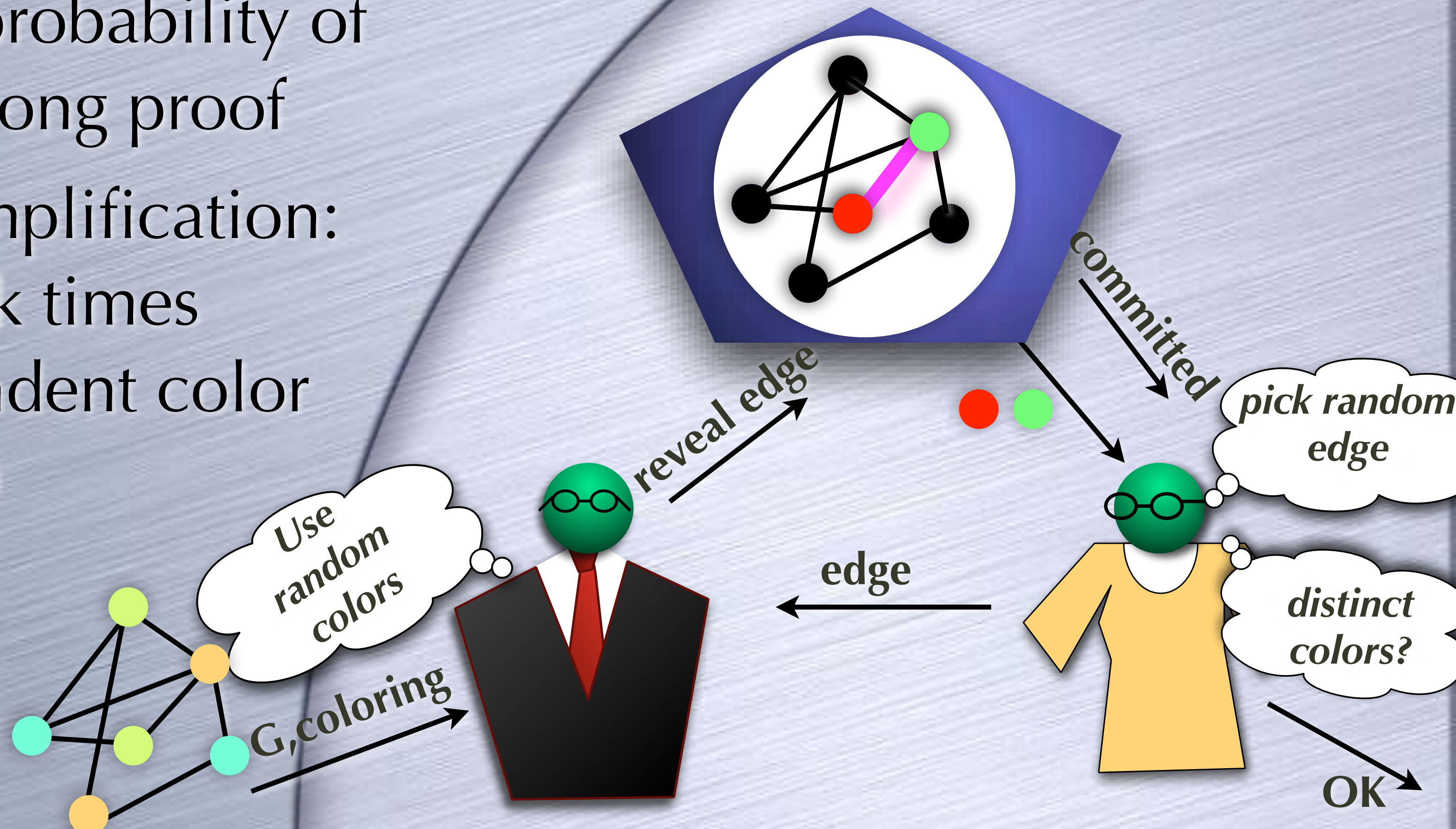


A ZK Proof for Graph Colorability

- Uses a commitment protocol as a subroutine
- At least 1/m probability of catching a wrong proof
- Soundness amplification: Repeat say mk times (with independent color permutations)



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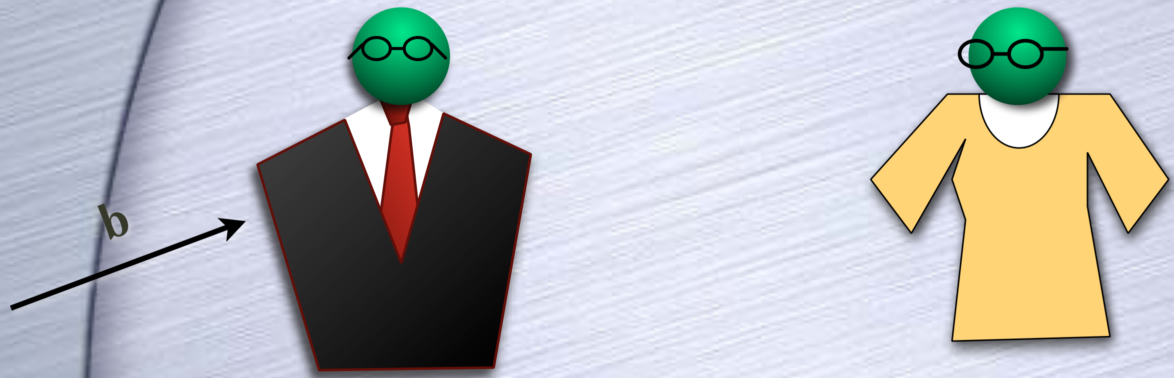

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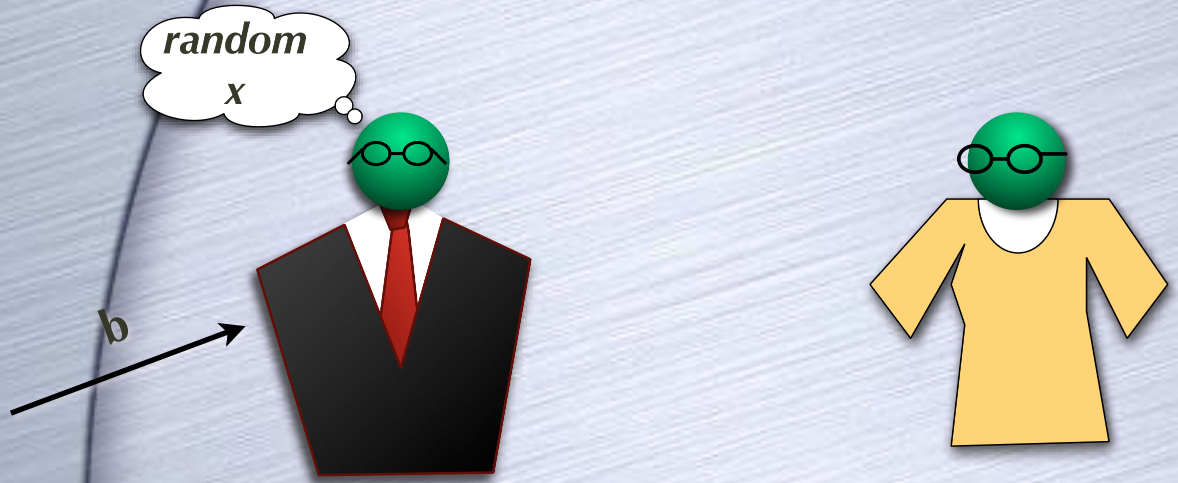

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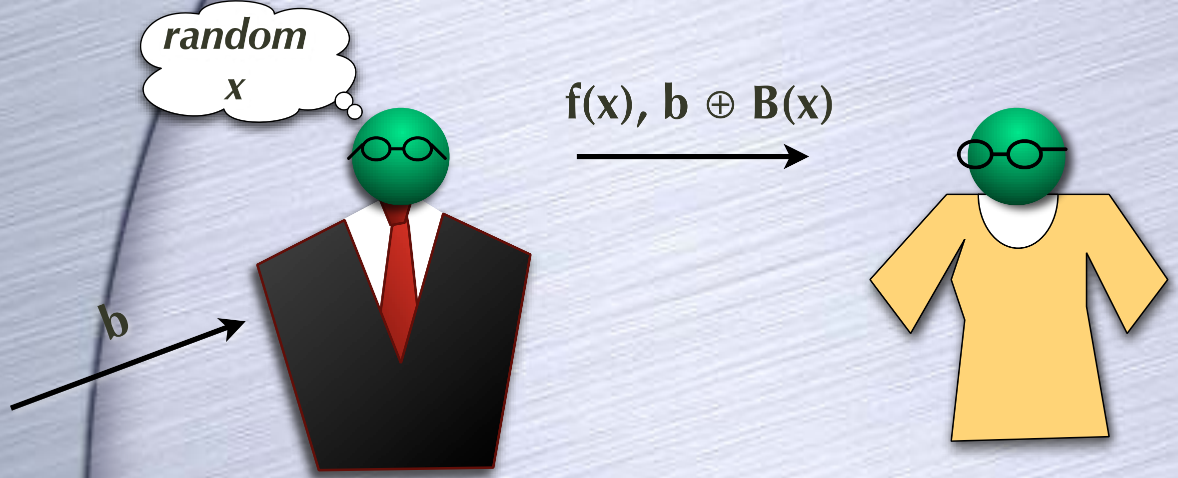

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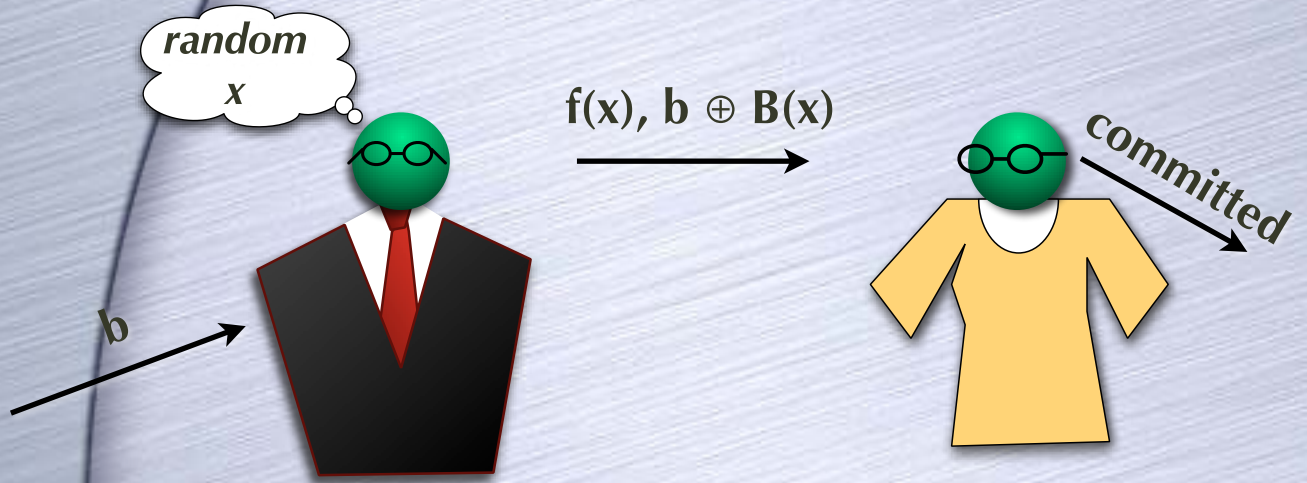

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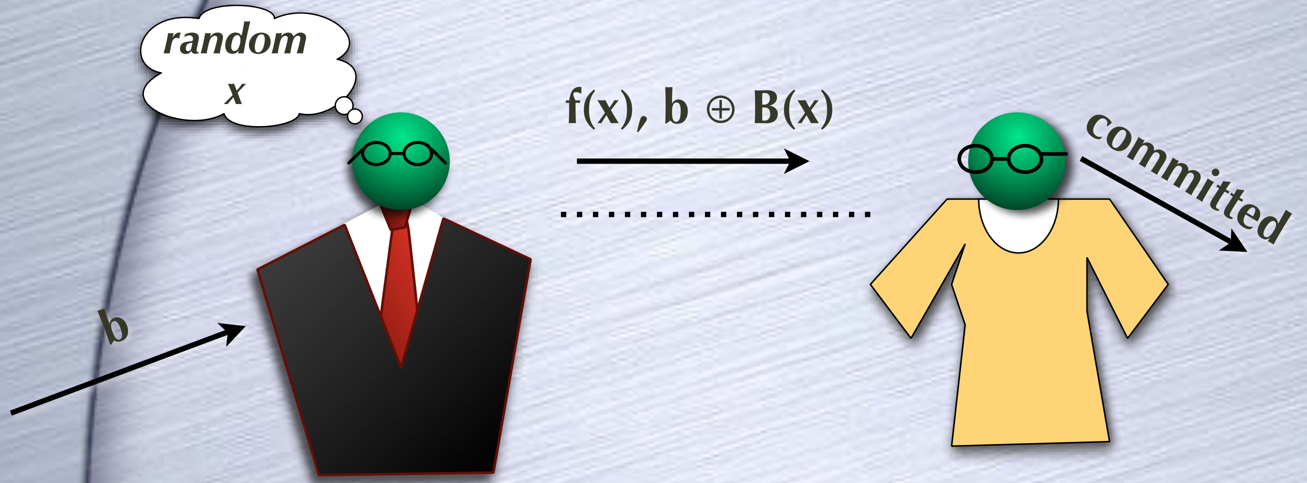

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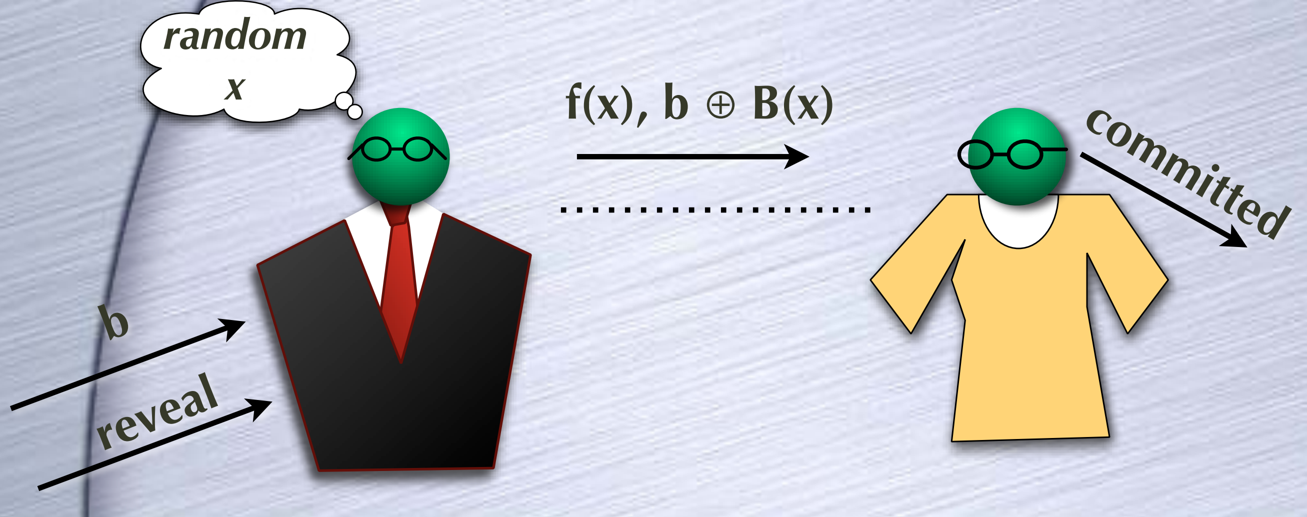

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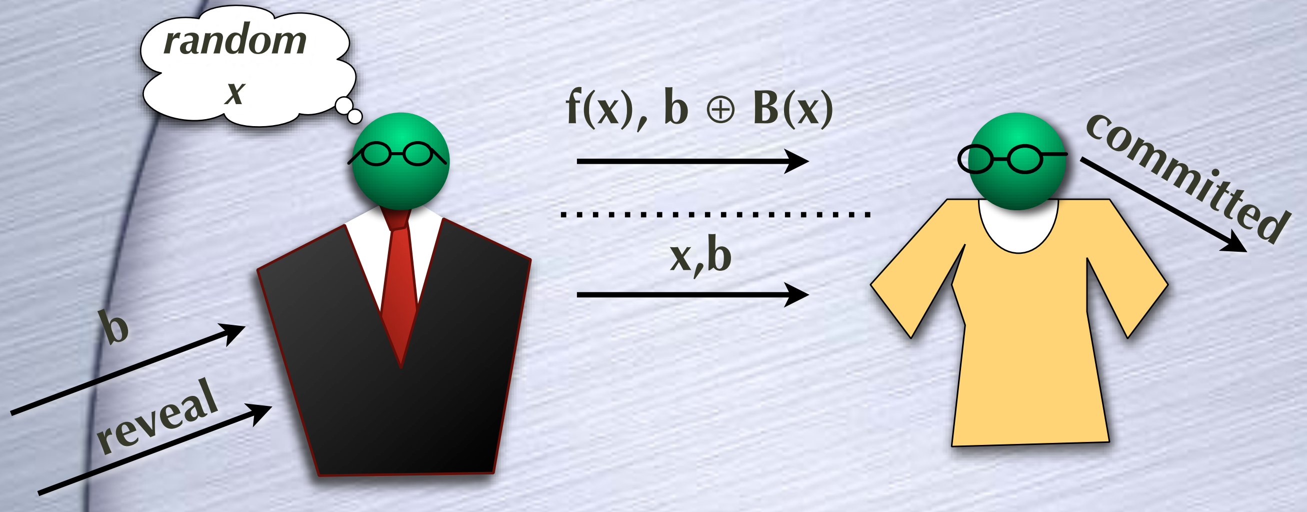

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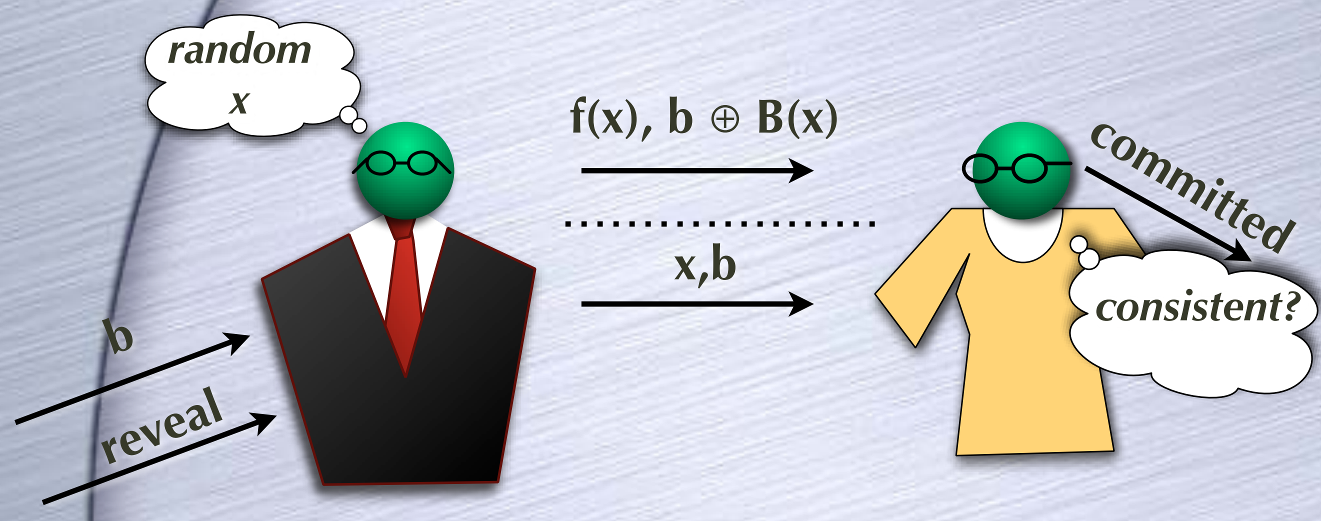

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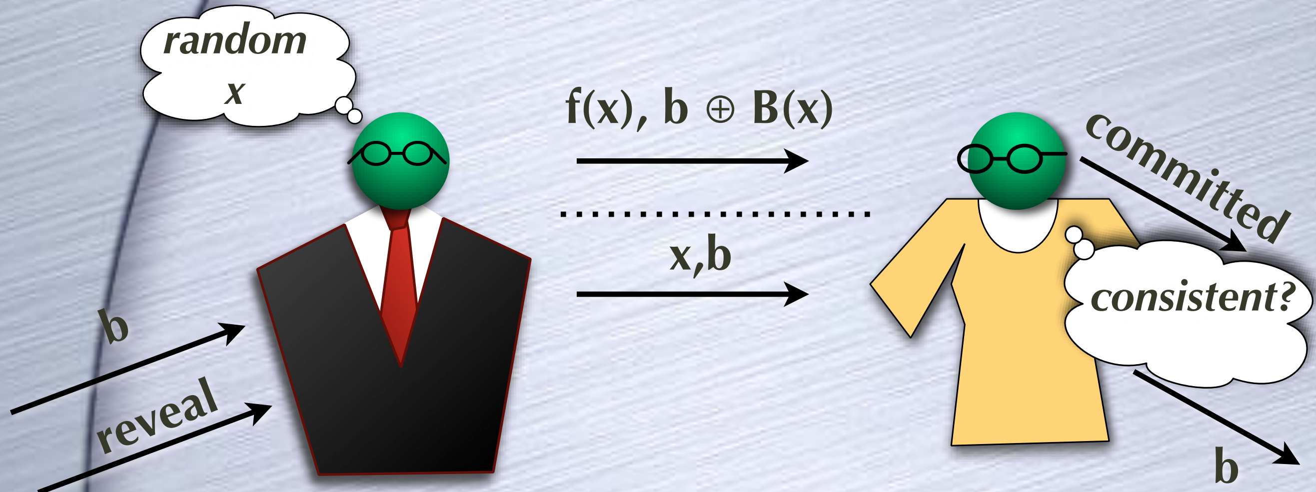

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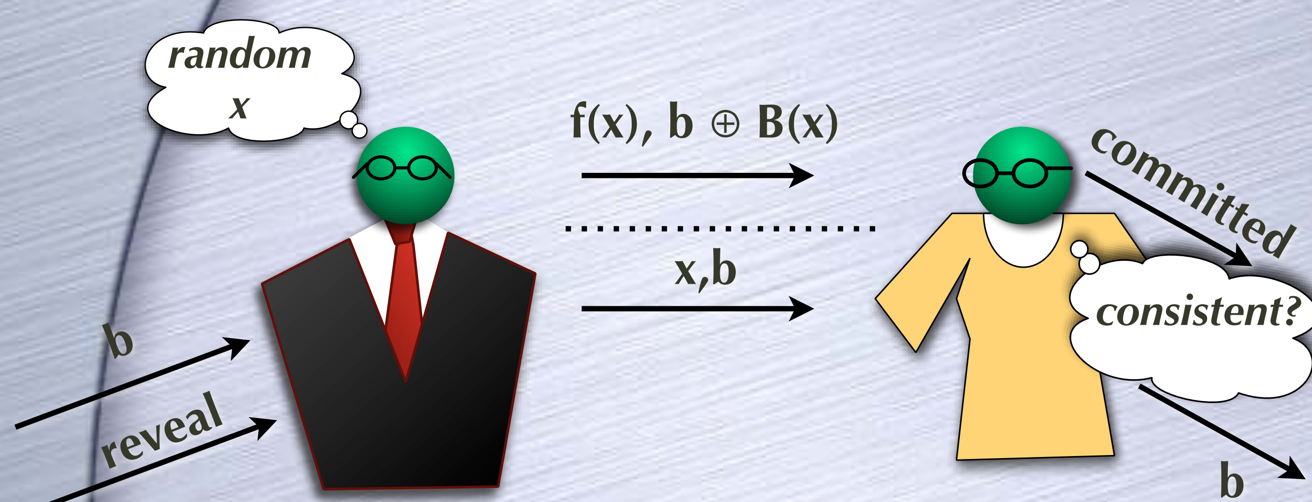

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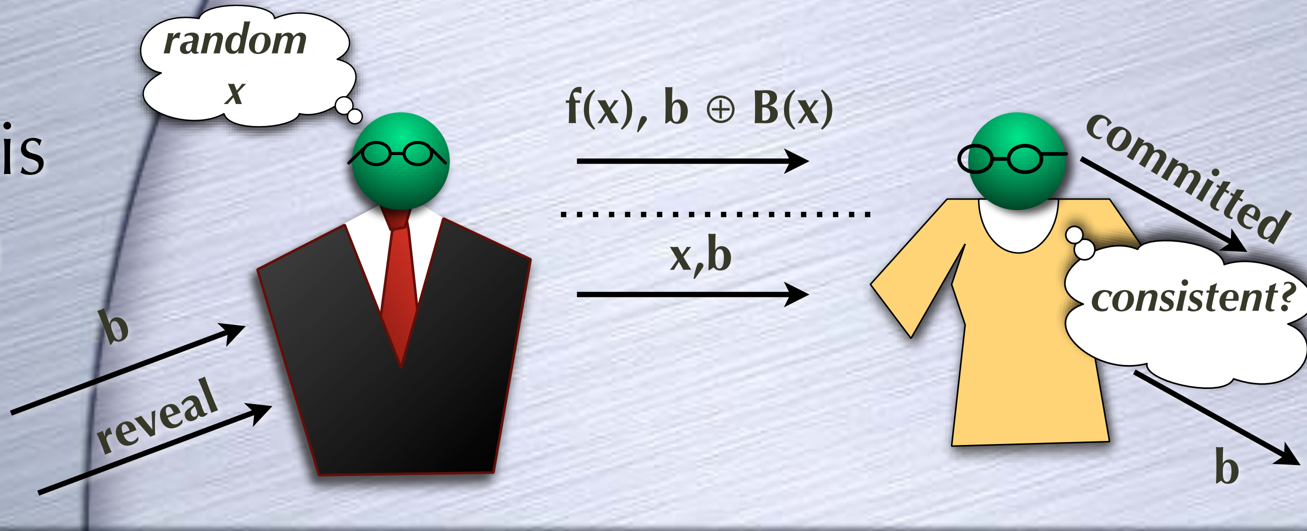
- Perfectly binding because f is a permutation
- Hiding because B(x) is pseudorandom given f(x)



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$\xrightarrow{x_1}$

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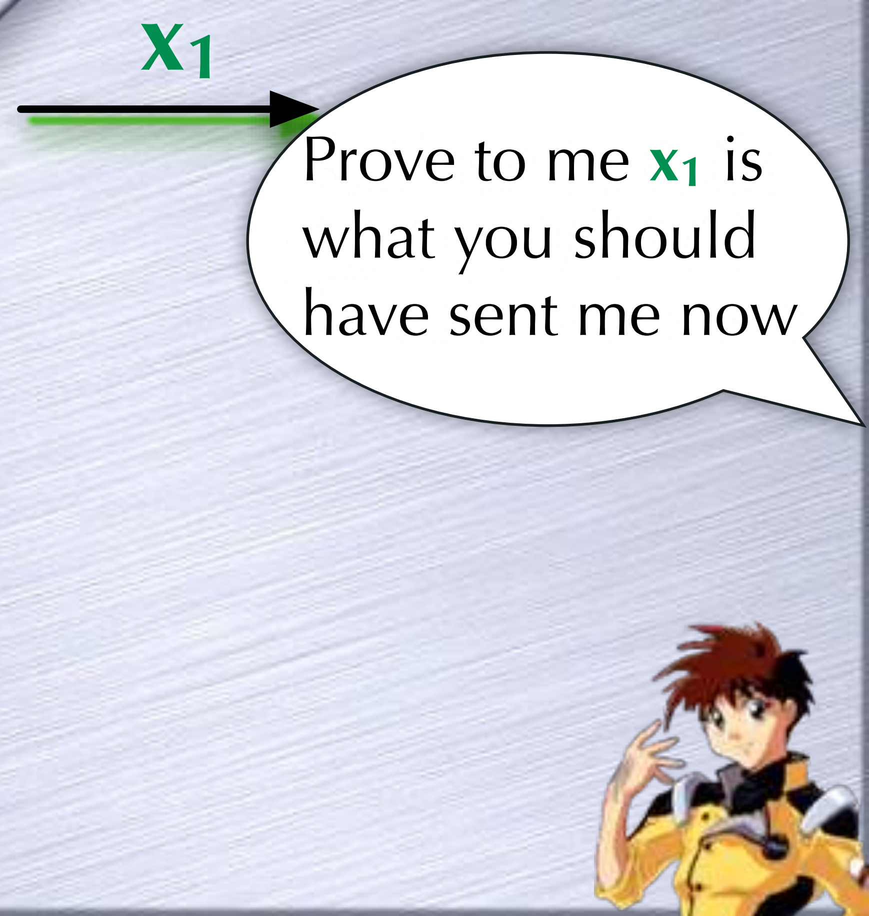

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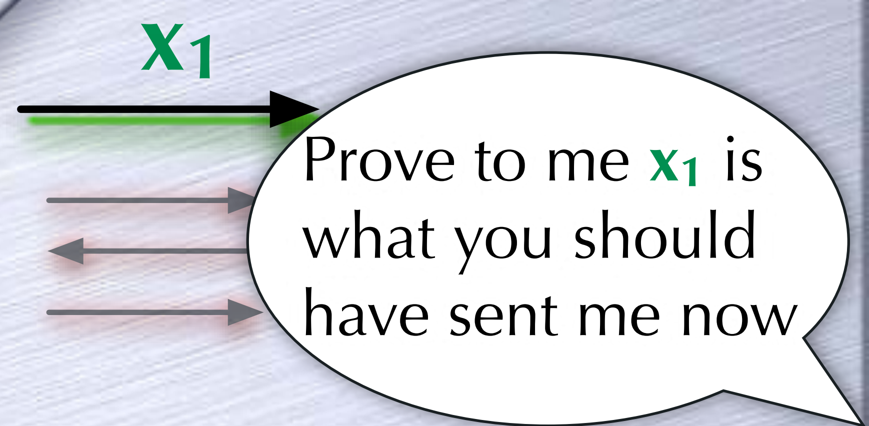

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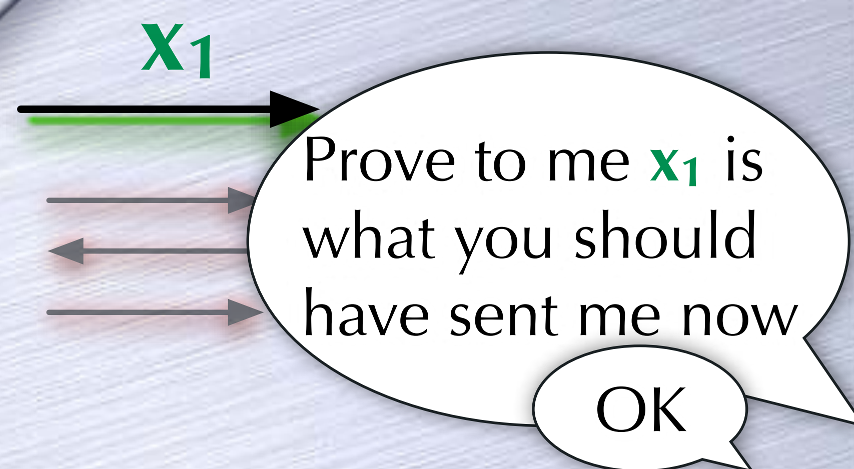

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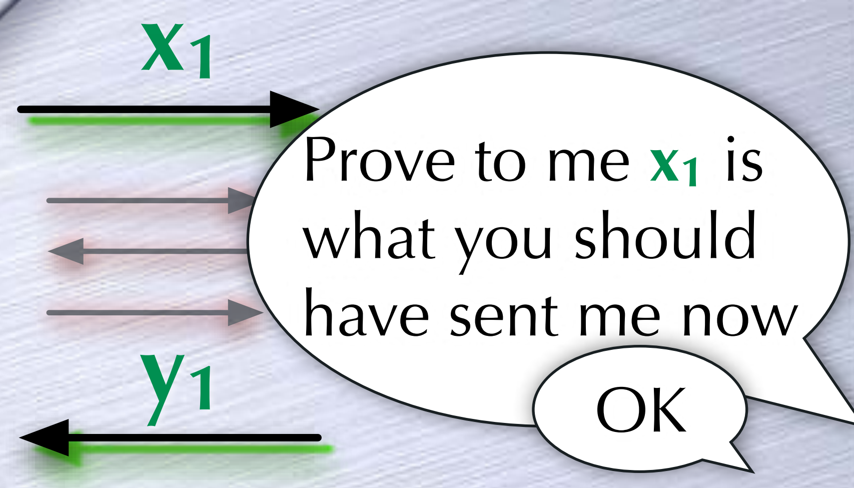

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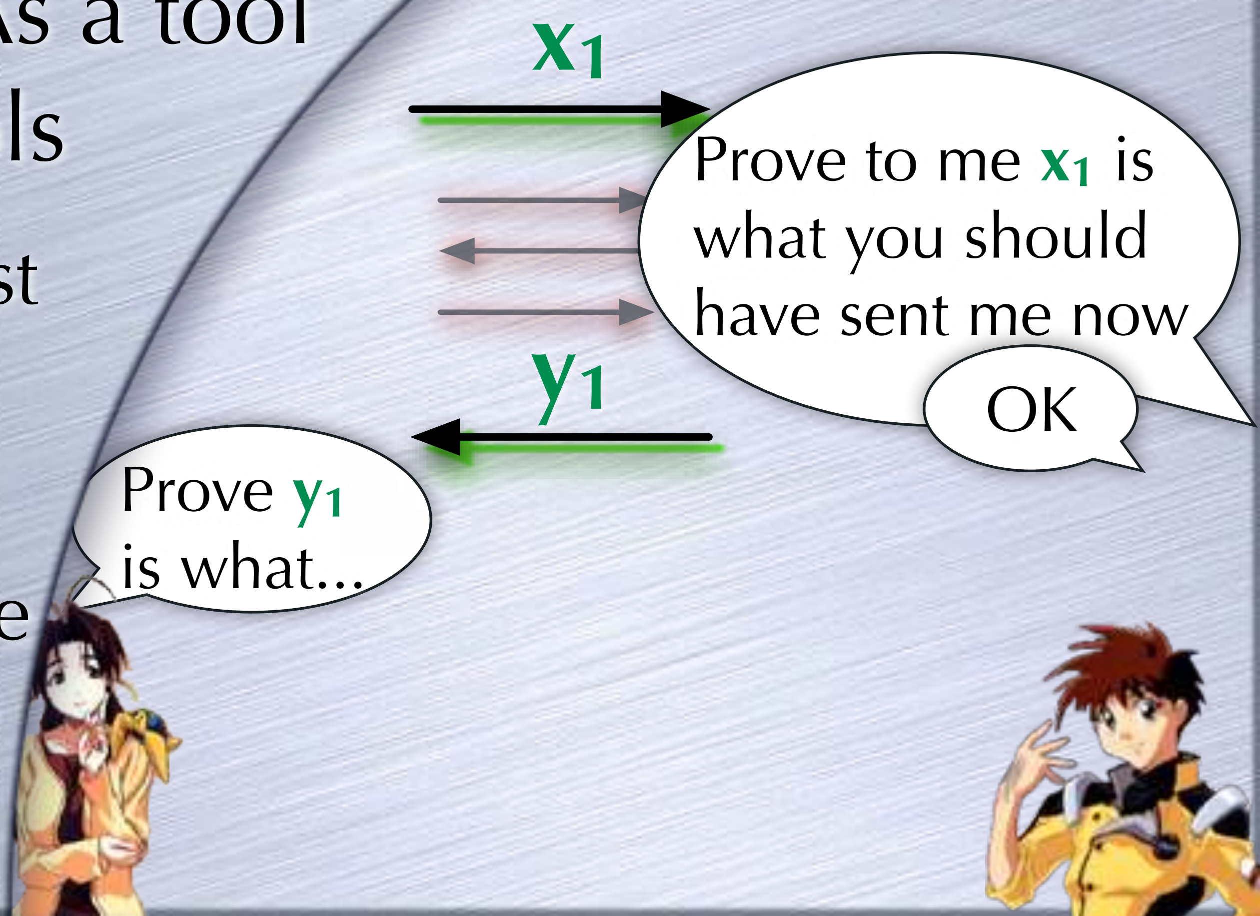

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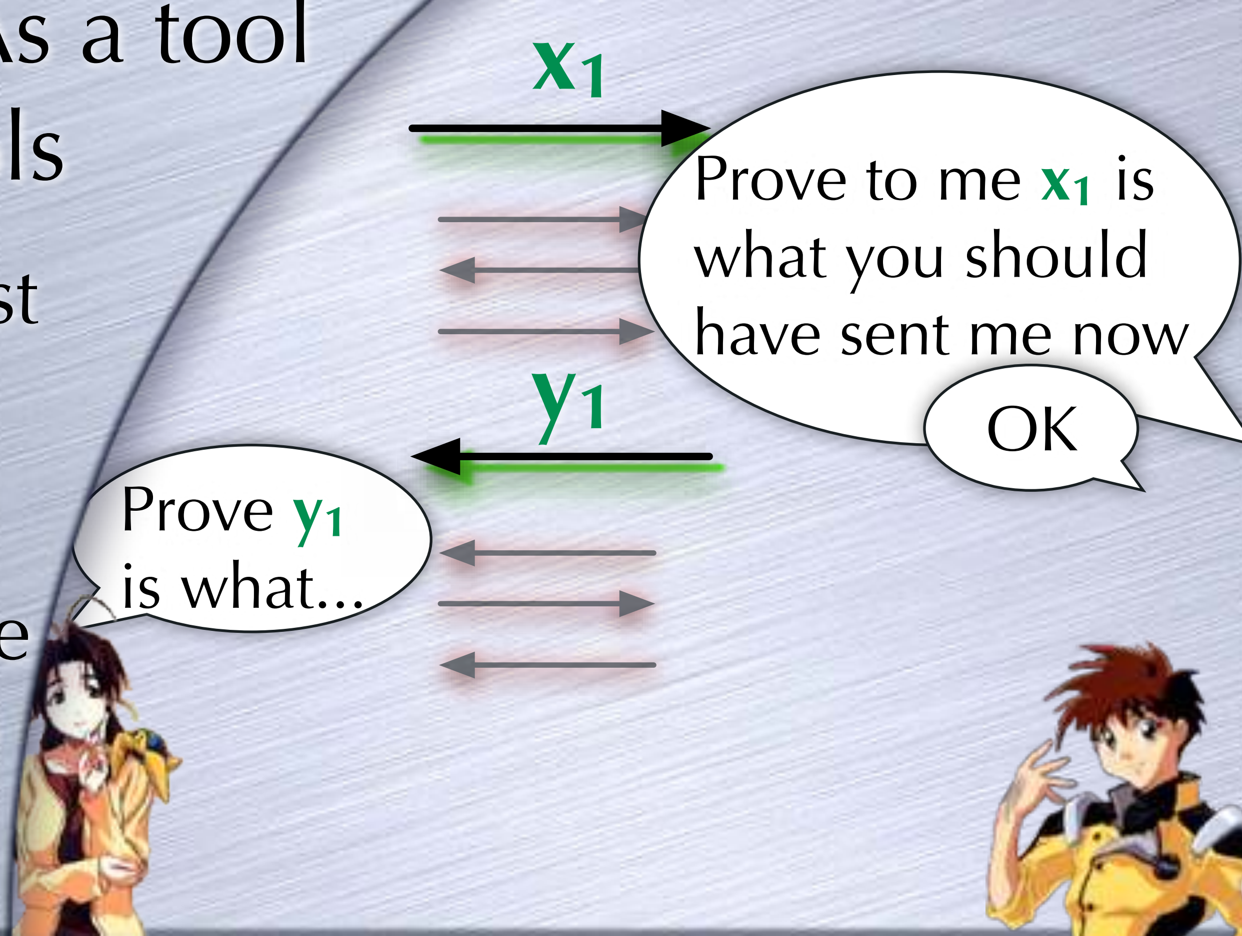

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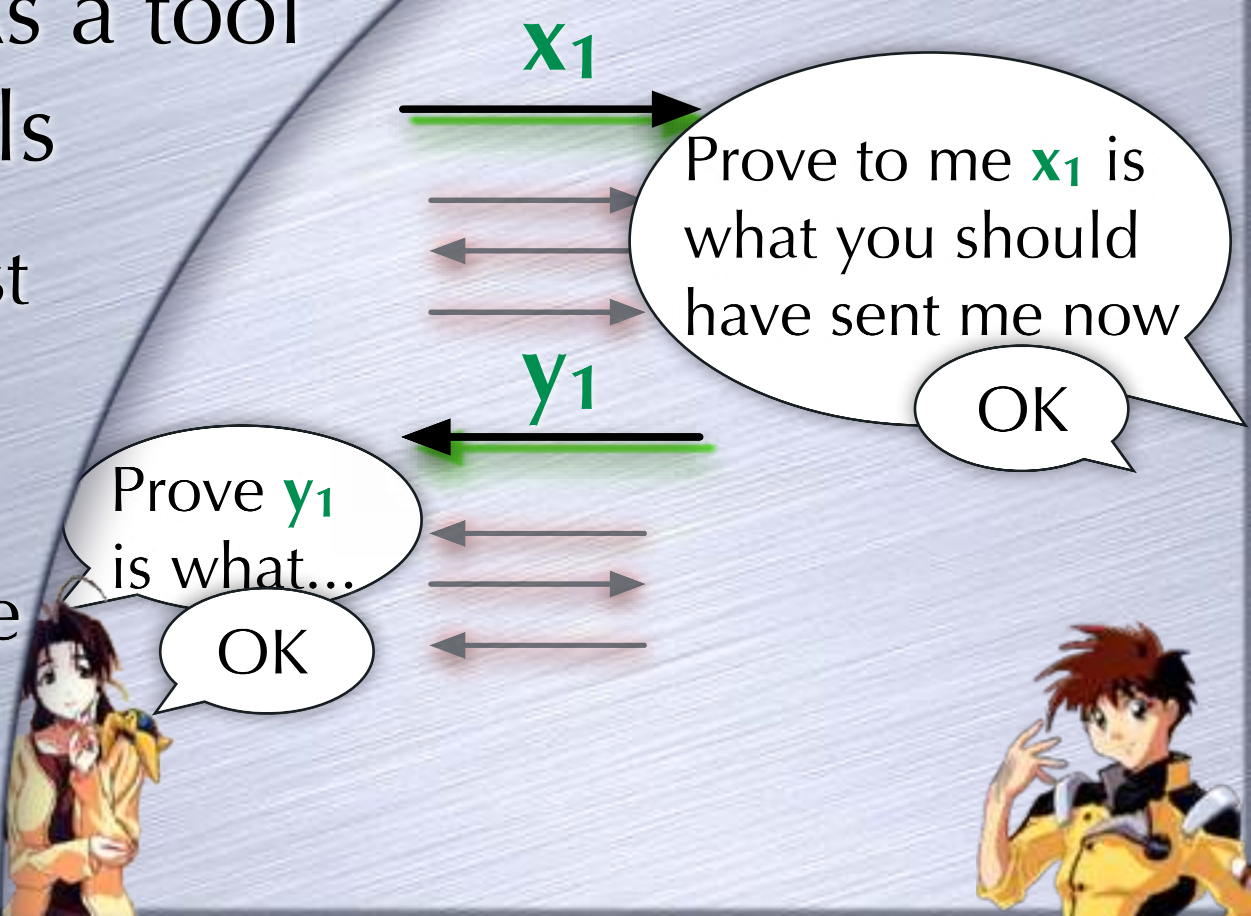

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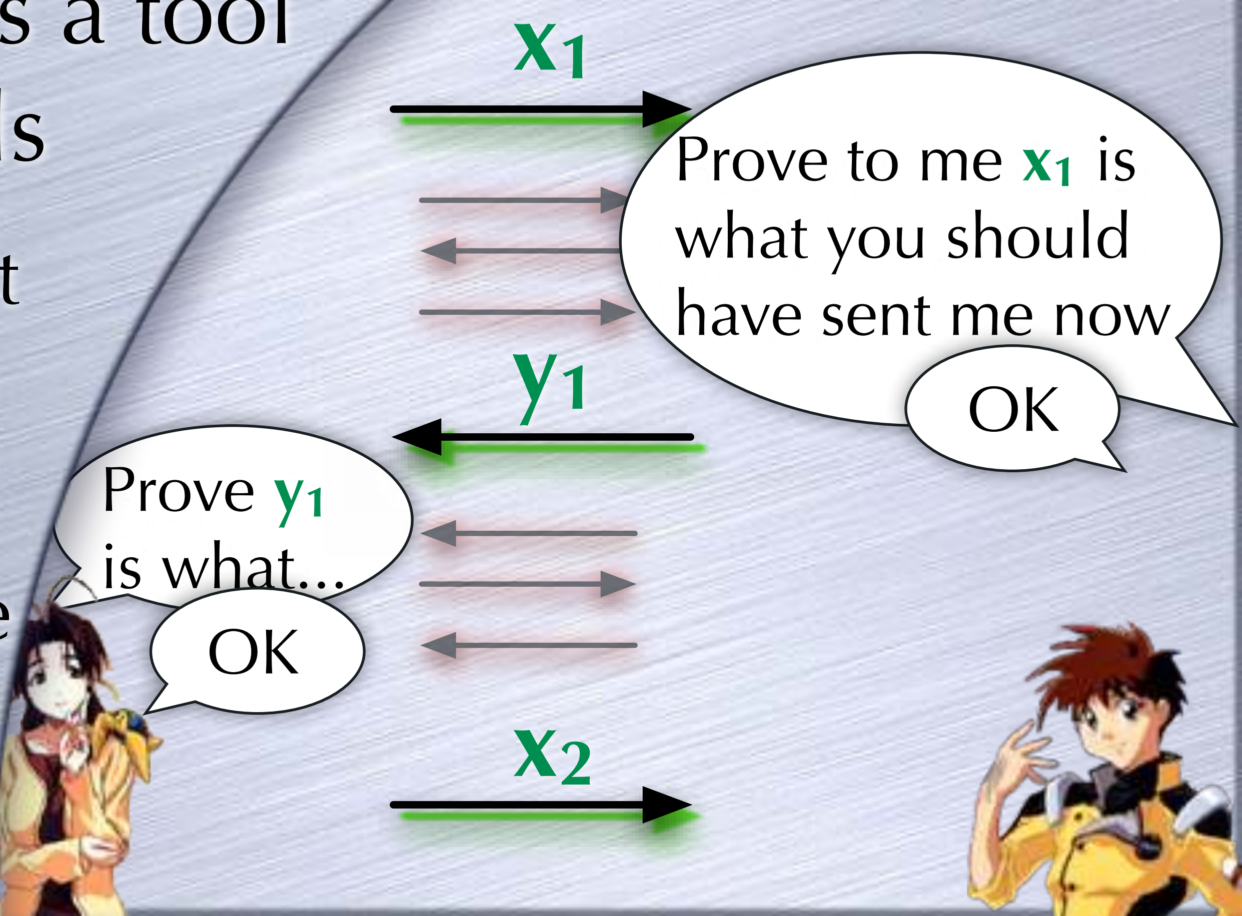

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