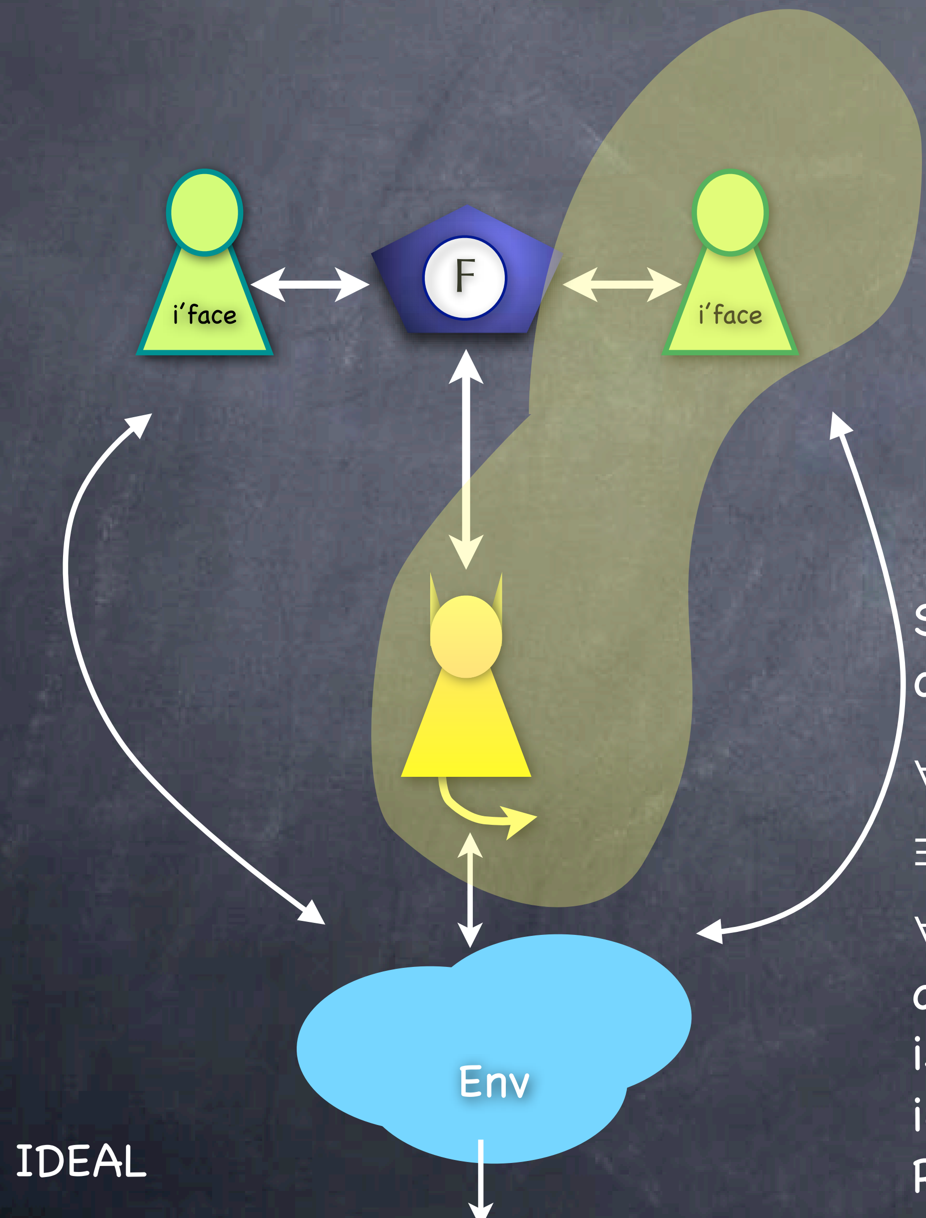
Secure
2-Party Computation

Lecture 14

Yao's Garbled Circuit

RECALL

SIM-Secure MPC



Secure (and correct) if:

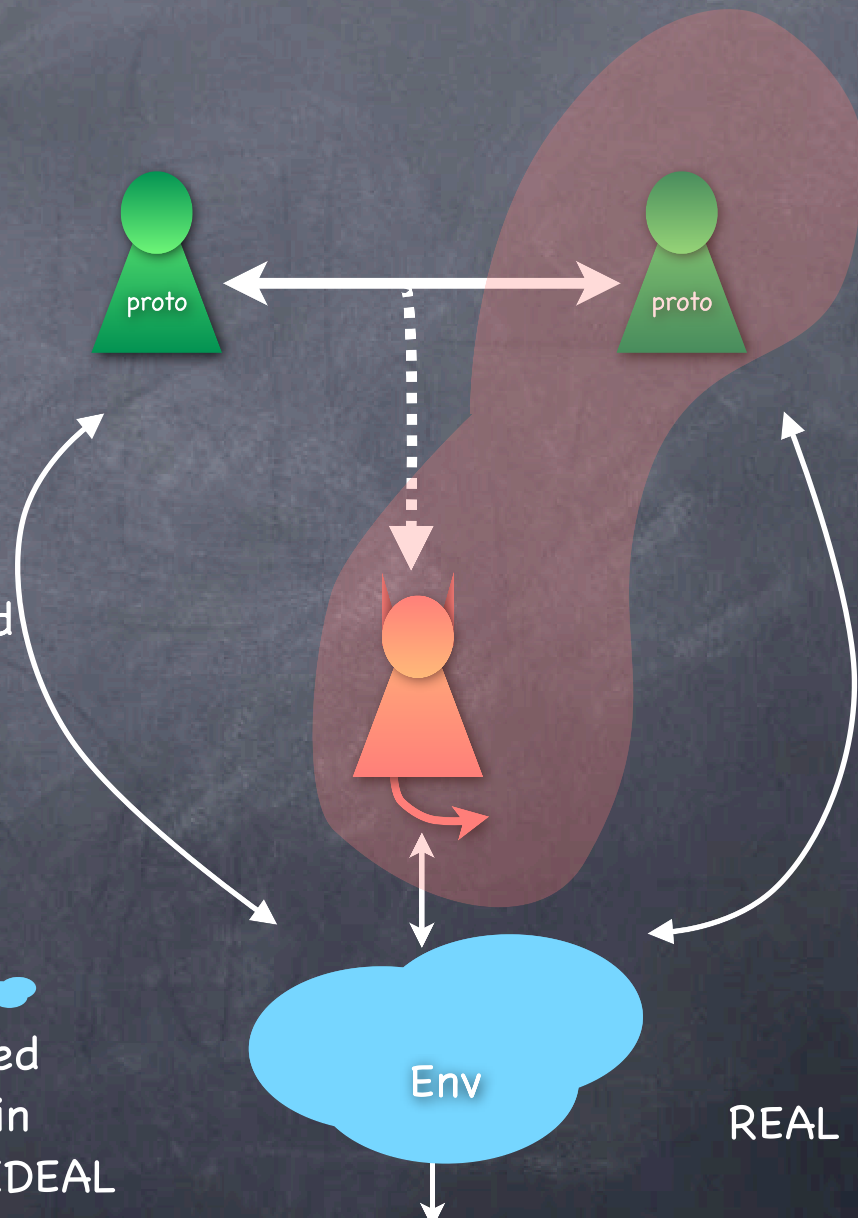
∀

∃ s.t.

∀

output of

is distributed identically in REAL and IDEAL



RECALL

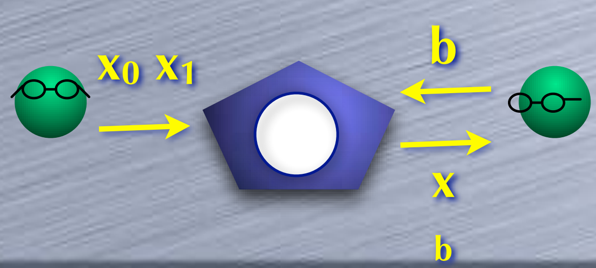
Passive Adversary

- Gets only read access to the internal state of the corrupted players (and can use that information in talking to environment)
  - Also called "Honest-But-Curious" adversary
  - Will require that simulator also corrupts passively
- Simplifies several cases
  - e.g. coin-tossing [why?], commitment [coming up]
- Oddly, sometimes security against a passive adversary is more demanding than against an active adversary
  - Active adversary: too pessimistic about what guarantee is available even in the IDEAL world
  - e.g. 2-party SFE for OR, with output going to only one party (trivial against active adversary; impossible without computational assumptions against passive adversary)

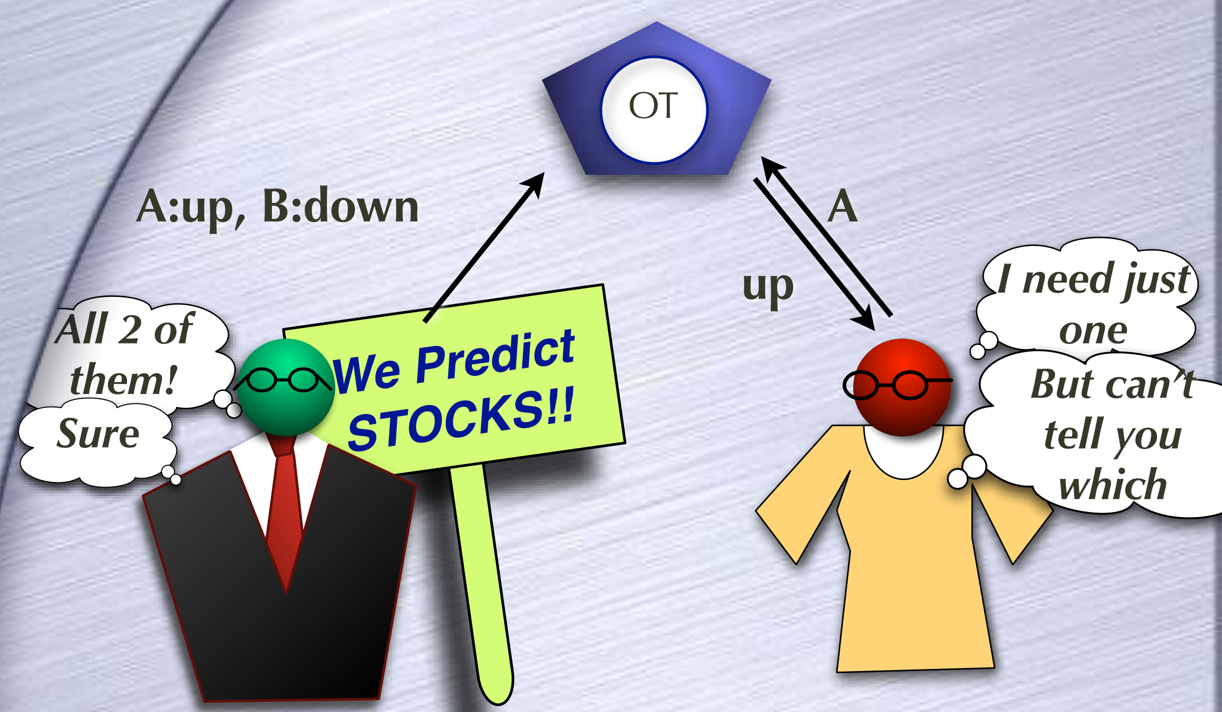
RECALL

Oblivious Transfer

- Pick one out of two, without revealing which
  - Intuitive property: transfer partial information "obliviously"



IDEAL World



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- So, for passive security, enough to consider one-sided SFE

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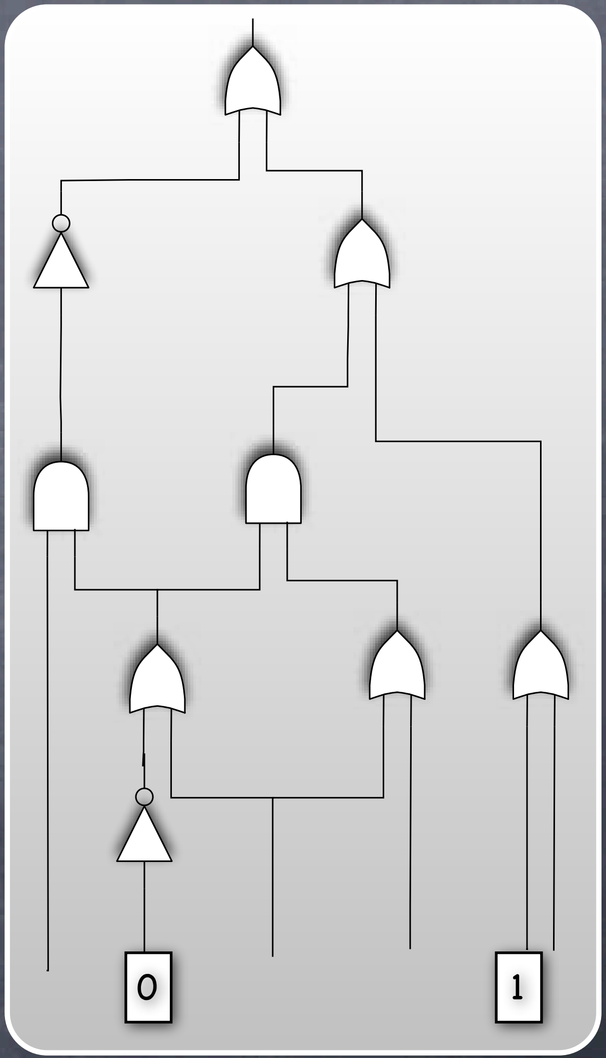
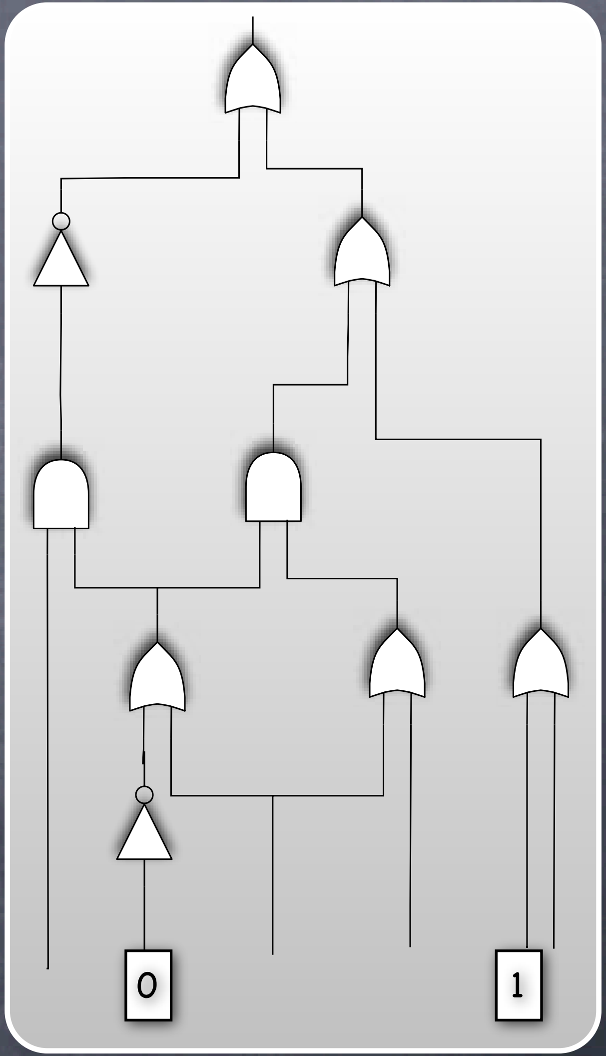
- Can we do "general" deterministic, one-sided SFE (i.e., for all functions)?

Boolean Circuits



Boolean Circuits

- Directed acyclic graph



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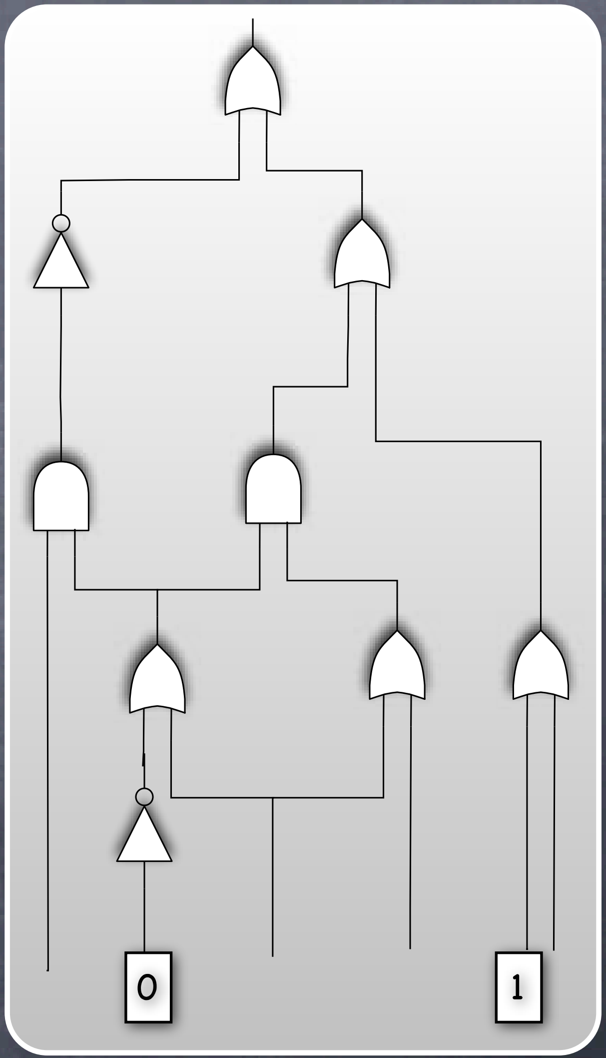

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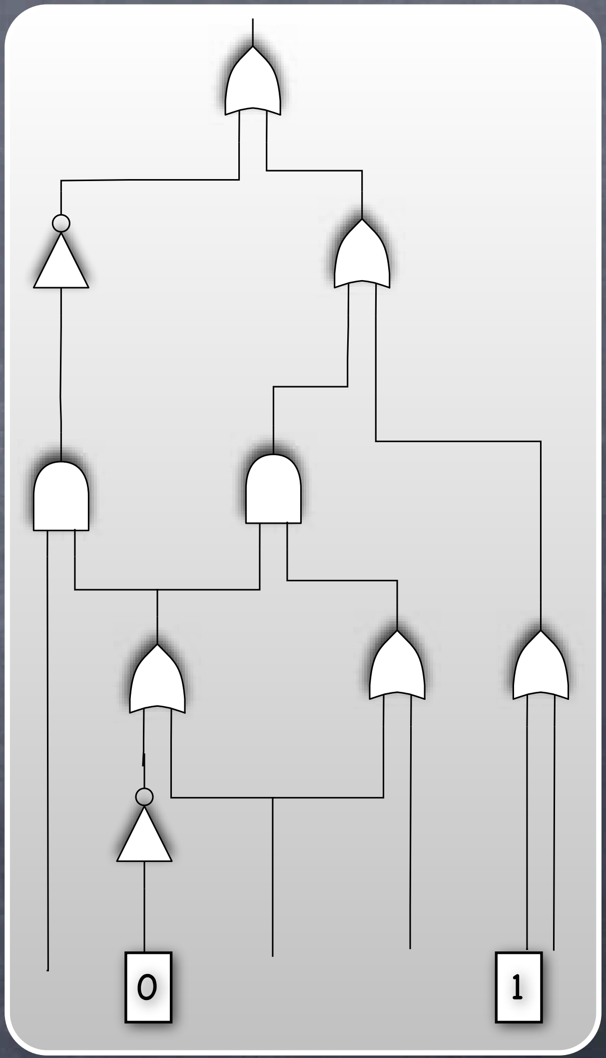

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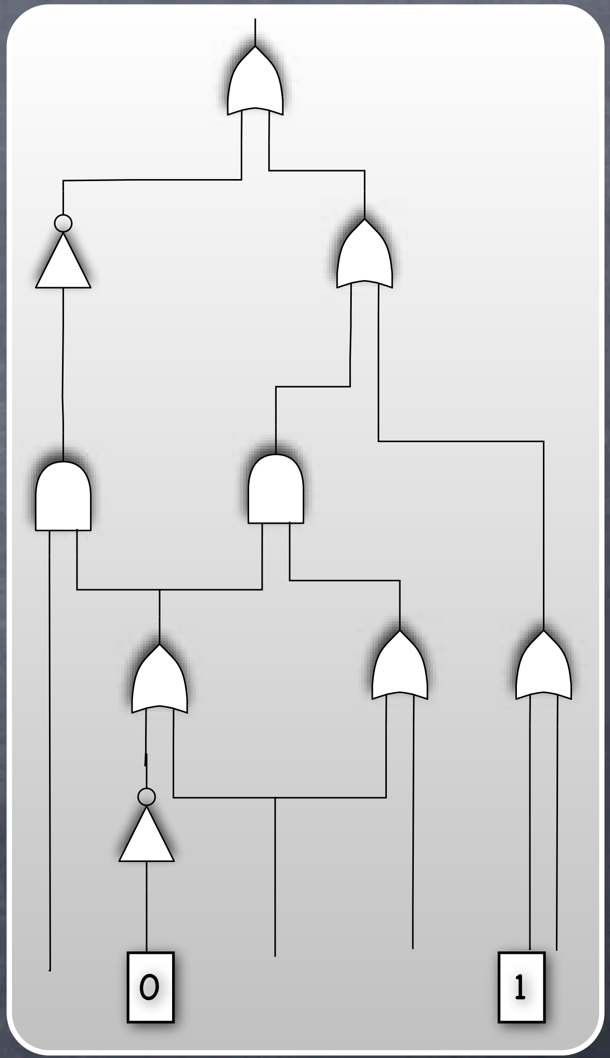

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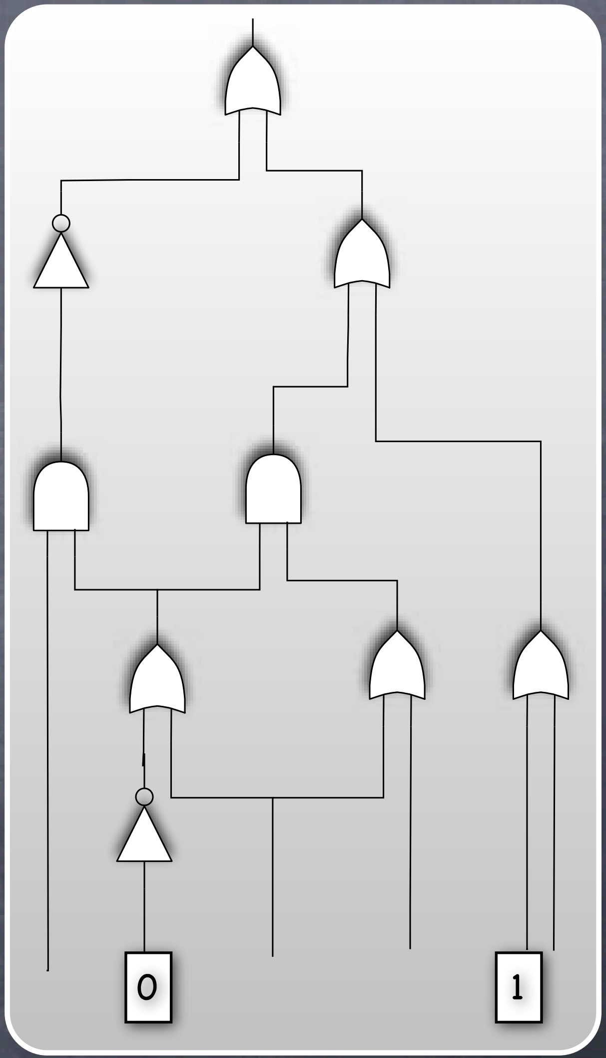

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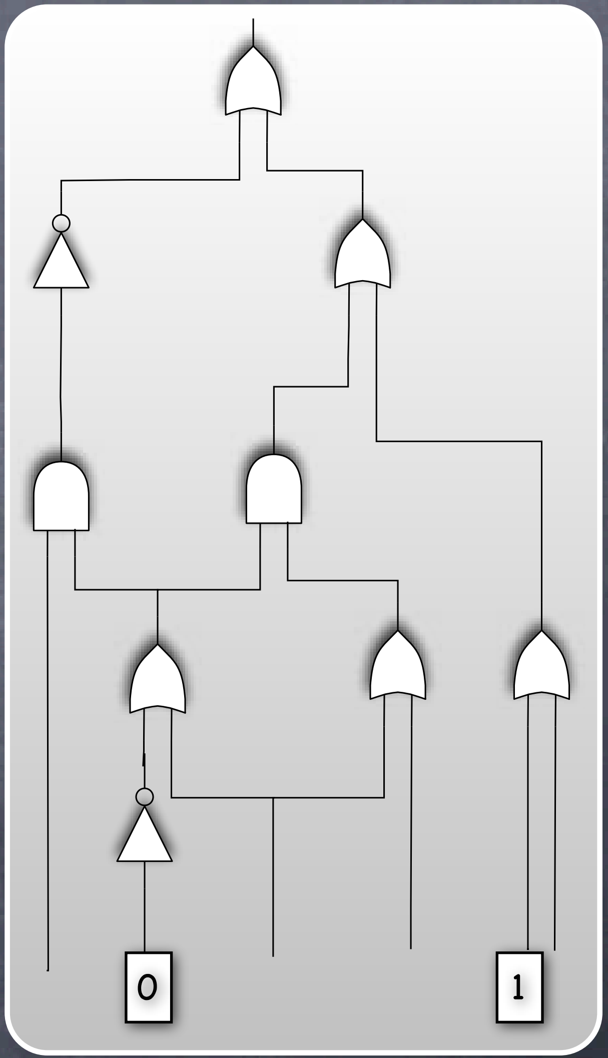

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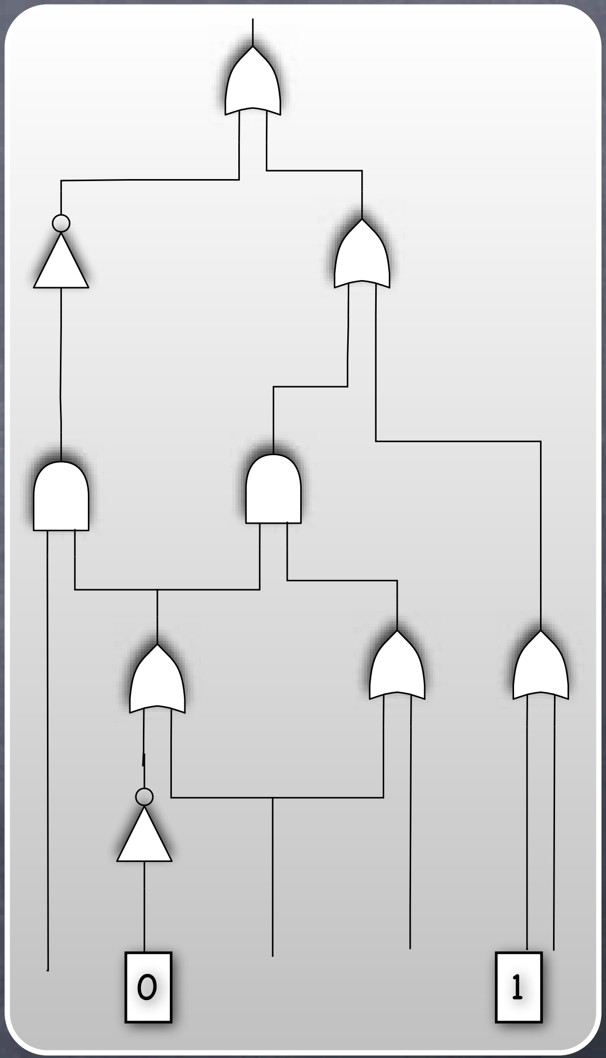
    - Note: no memory gates



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  - Often problems already described as succinct programs/circuits

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  - Any ideas?

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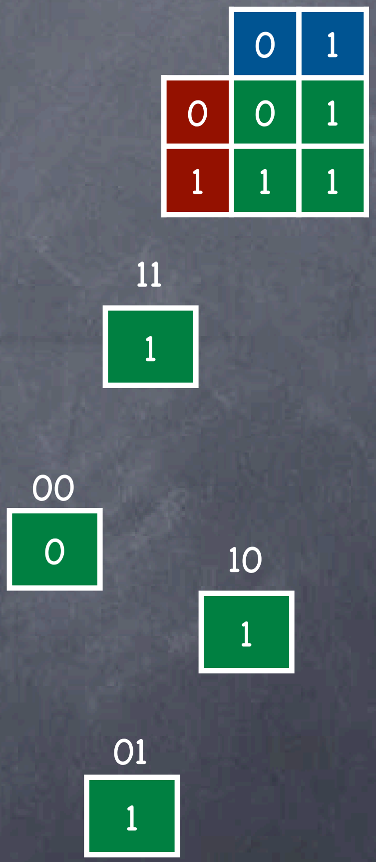

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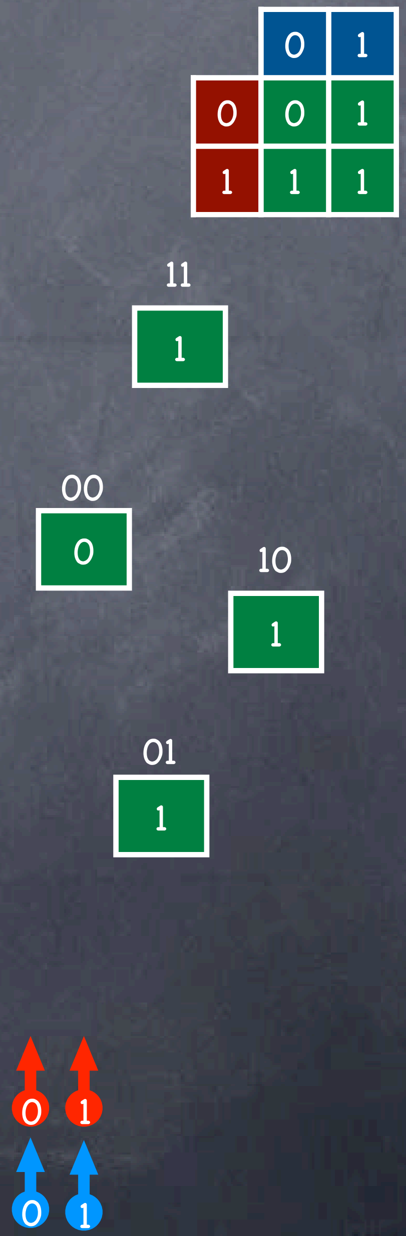

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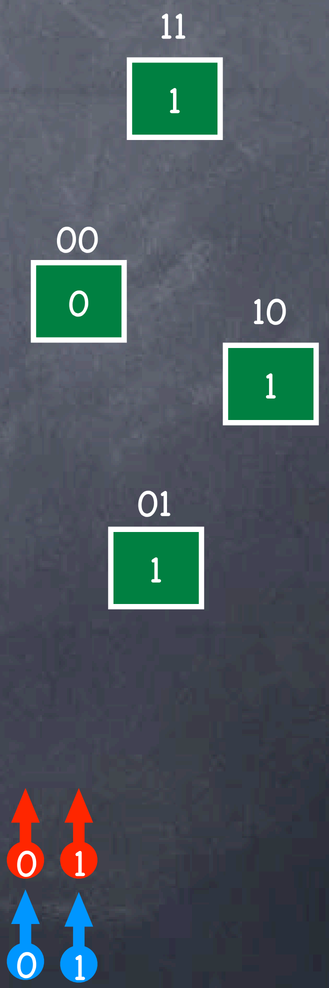

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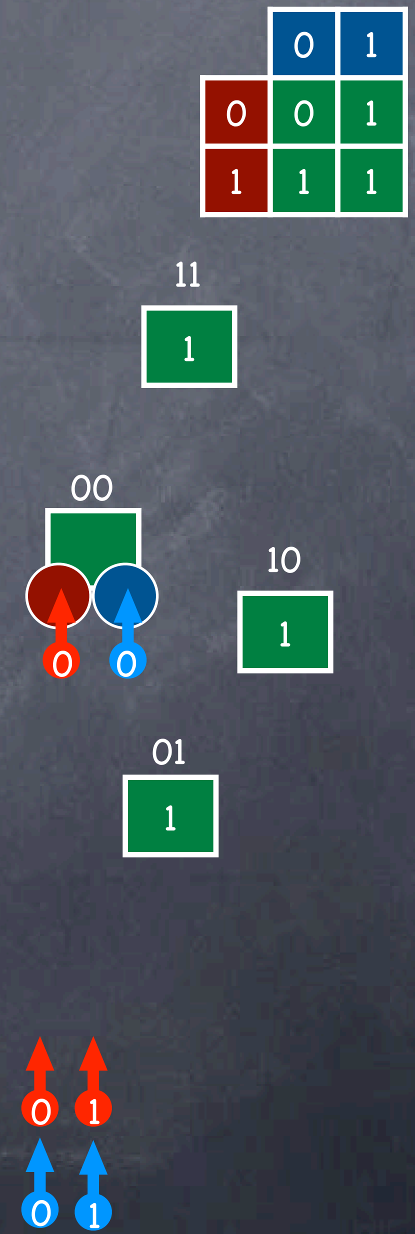

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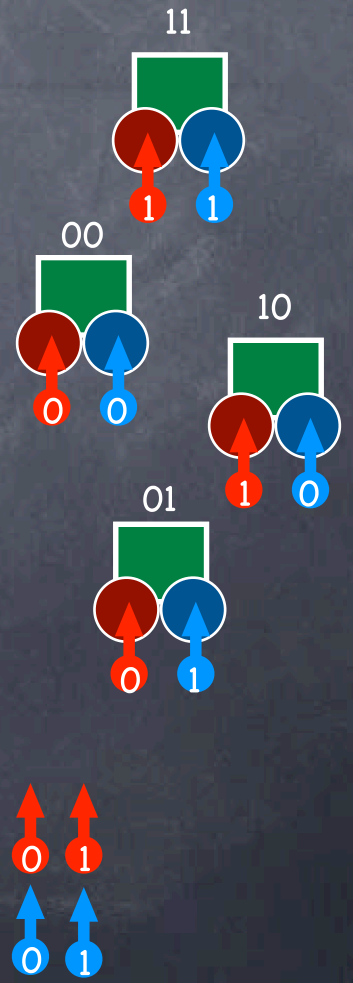

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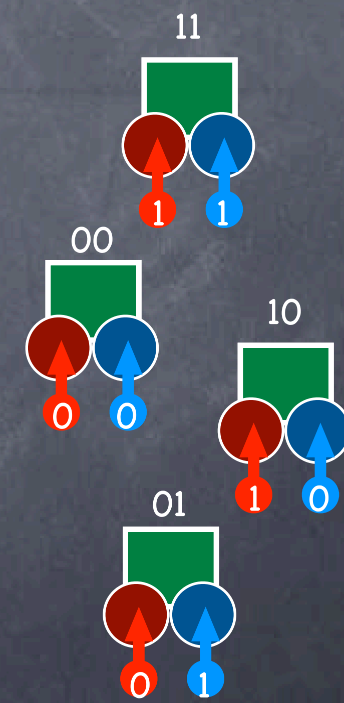

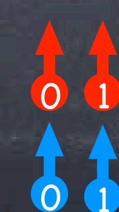

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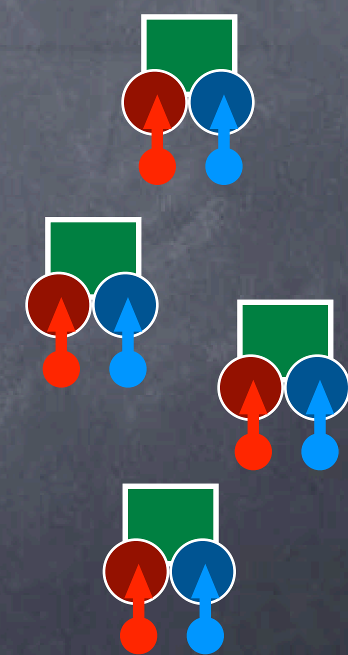

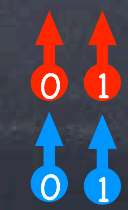

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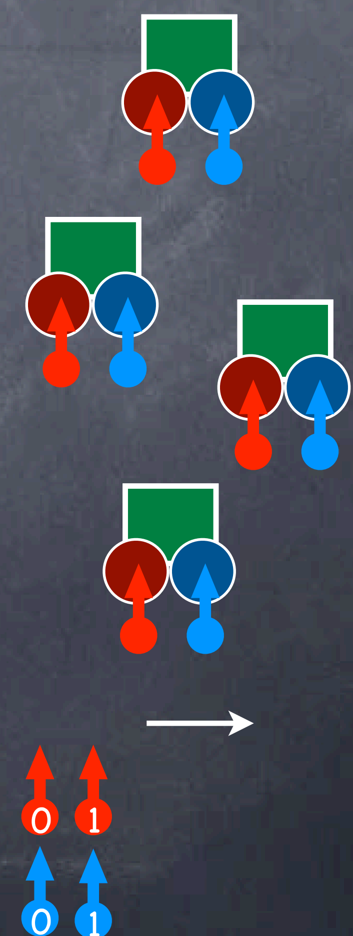

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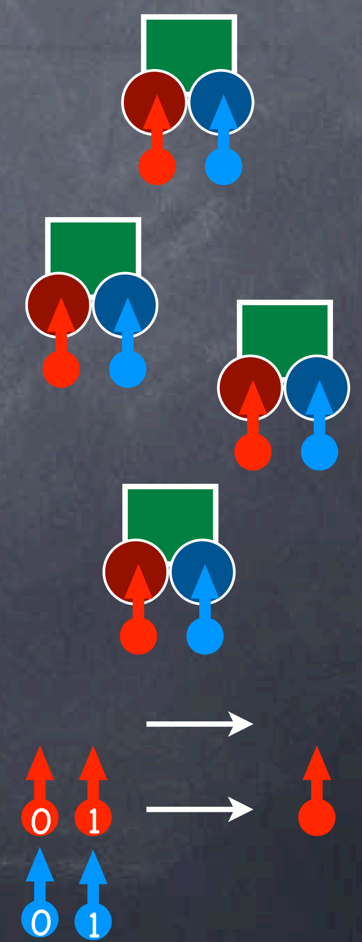
- She un-labels the four boxes and sends them in random order to Bob. Also sends the key $K_{x=a}$ (labeled only as K_x).

	0	1
0	0	1
1	1	1



A Physical Protocol

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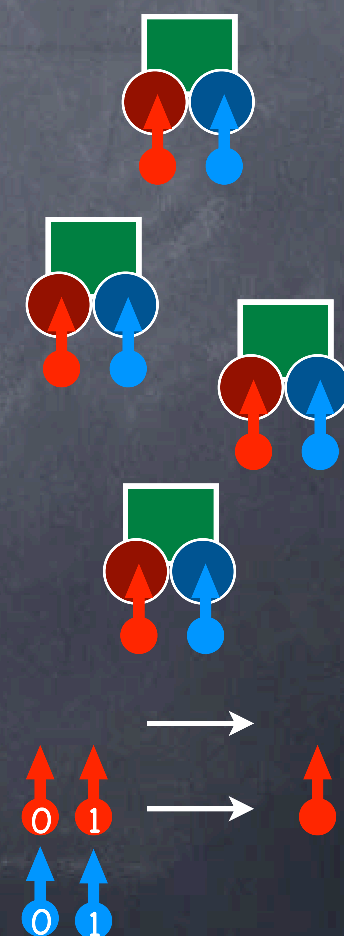
    - So far Bob gets no information



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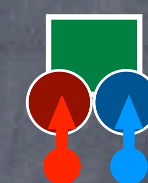

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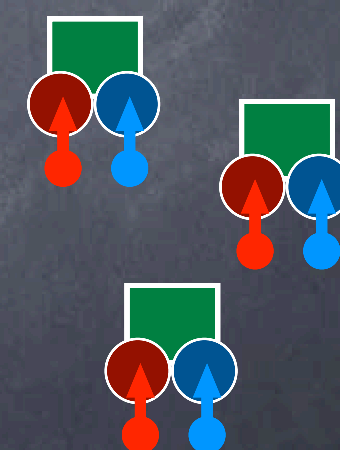
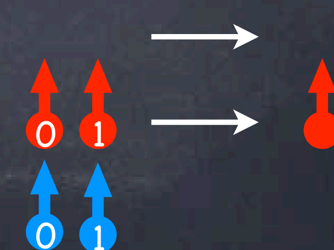


- Bob "obliviously picks up" $K_{y=b}$, and tries the two keys K_x,$K_{y=b}$ on the four boxes. For one box both locks open and he gets the output.



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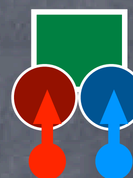

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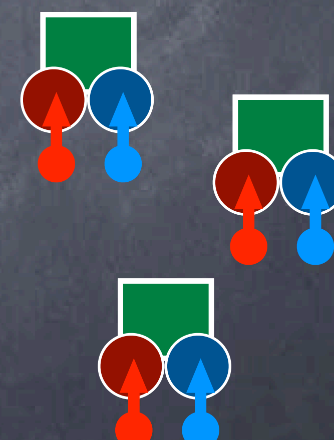

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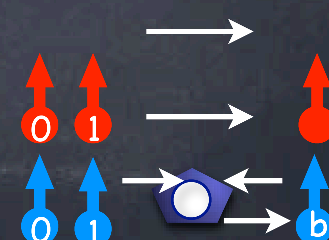
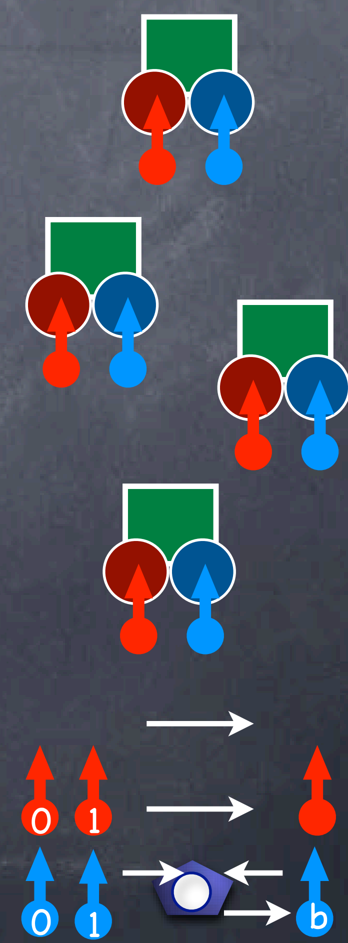


A Physical Protocol

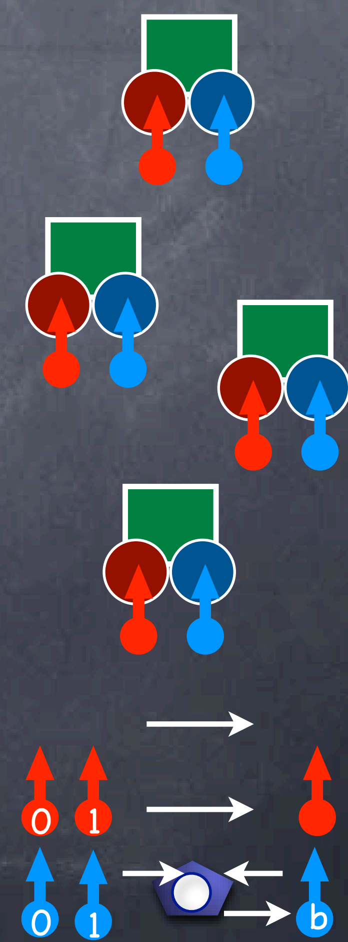
	0	1
0	0	1
1	1	1



A Physical Protocol

- Secure?

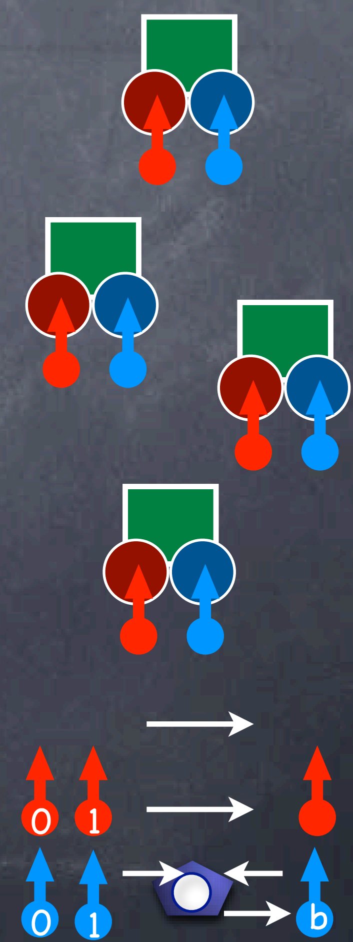
	0	1
0	0	1
1	1	1



A Physical Protocol

- Secure?
- For curious Alice: only influence from Bob is when he picks up his key $K_{y=b}$

	0	1
0	0	1
1	1	1



A Physical Protocol

- Secure?
- For curious Alice: only influence from Bob is when he picks up his key $K_{y=b}$
  - But this is done "obliviously", and so she learns nothing

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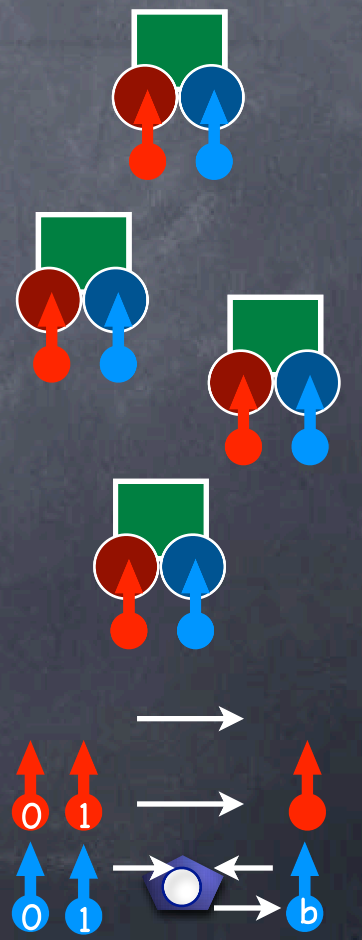

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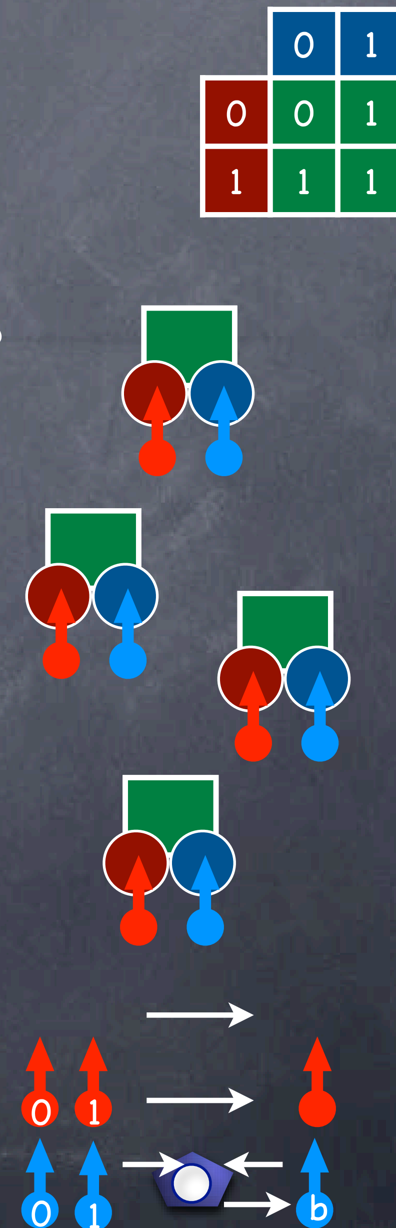

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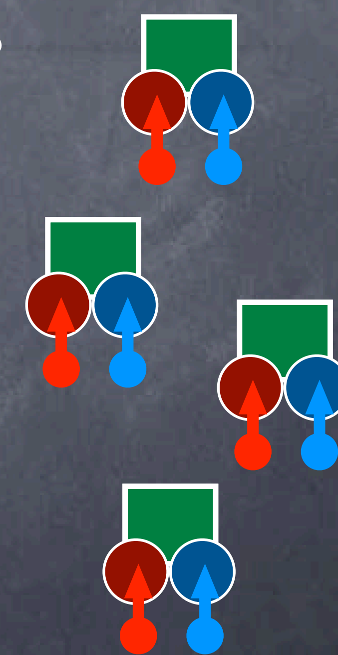
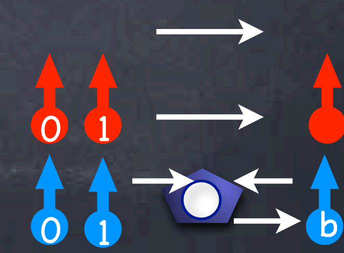
  - What Bob sees: K_y opens a lock in two boxes, K_x opens a lock in two boxes; only one random box fully opens. It has the outcome.

	0	1
0	0	1
1	1	1





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	0	1
0	0	1
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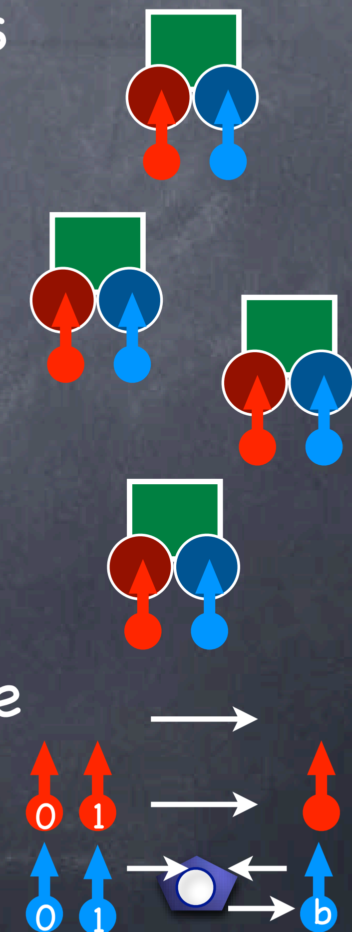

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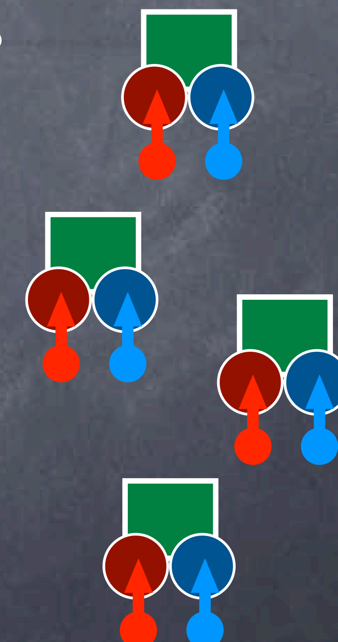
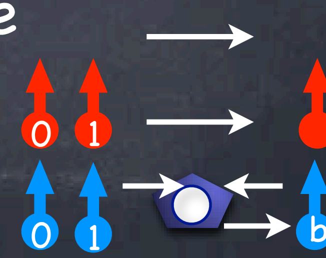
  - Note when y=1, cases x=0 and x=1 appear same
  - Formally, easy to simulate (can stuff unopenable boxes arbitrarily)

	0	1
0	0	1
1	1	1





Larger Circuits

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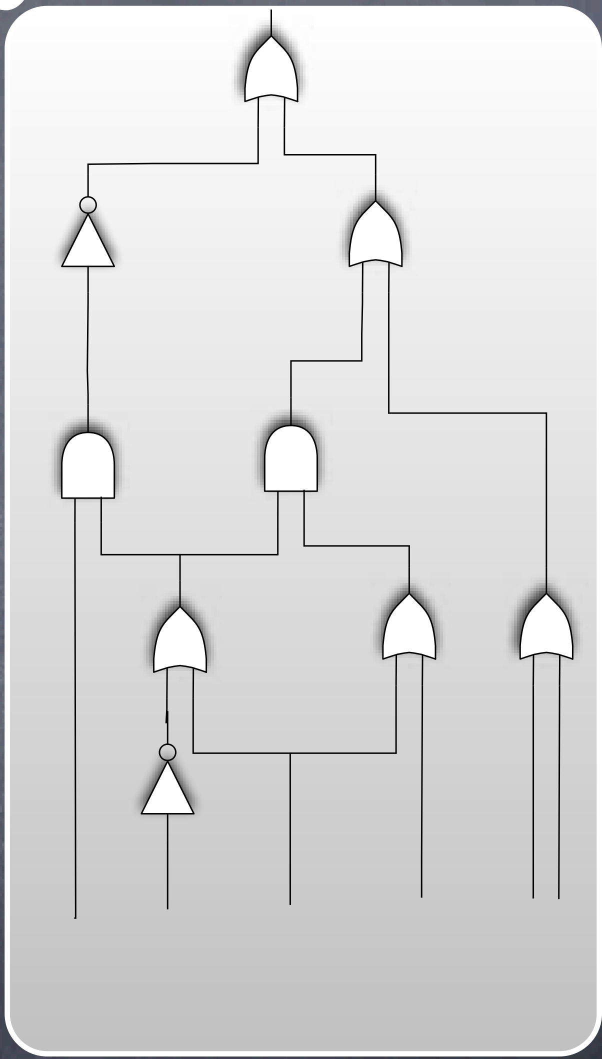
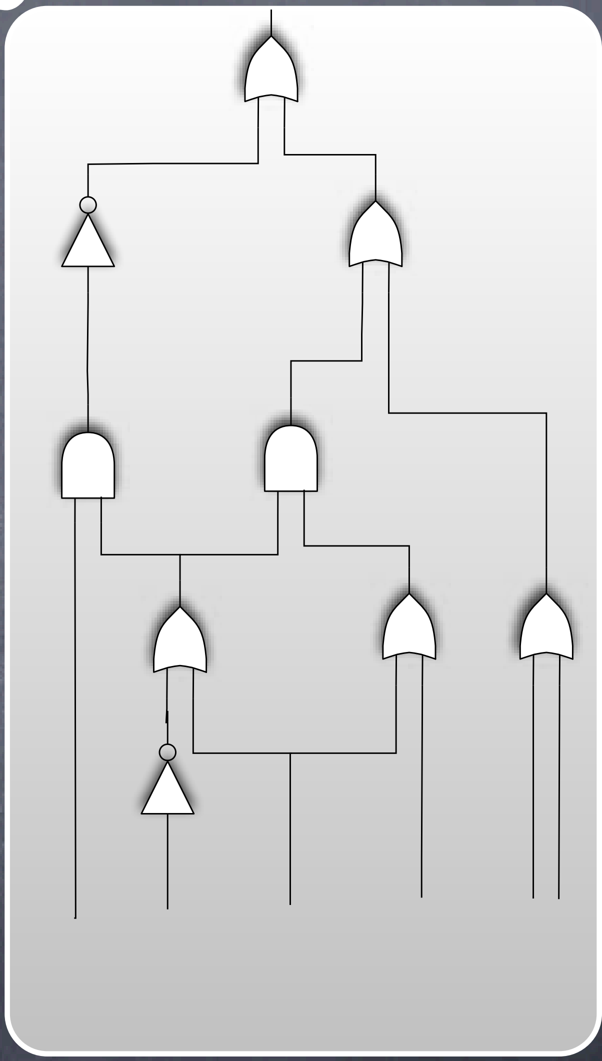


Larger Circuits

- Idea: For each gate in the circuit Alice will prepare locked boxes, but will use it to keep keys for the next gate



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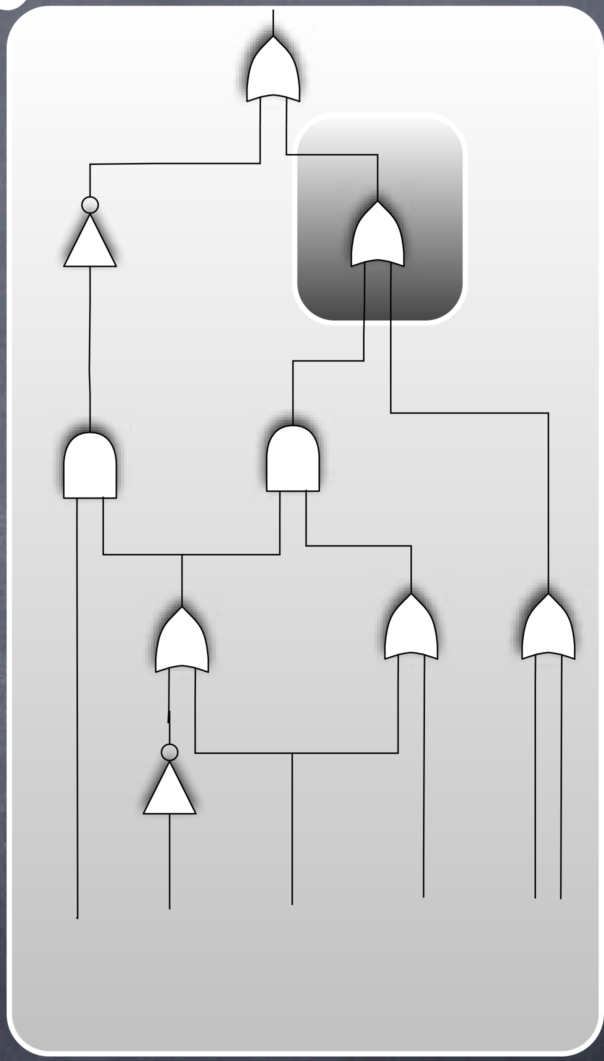

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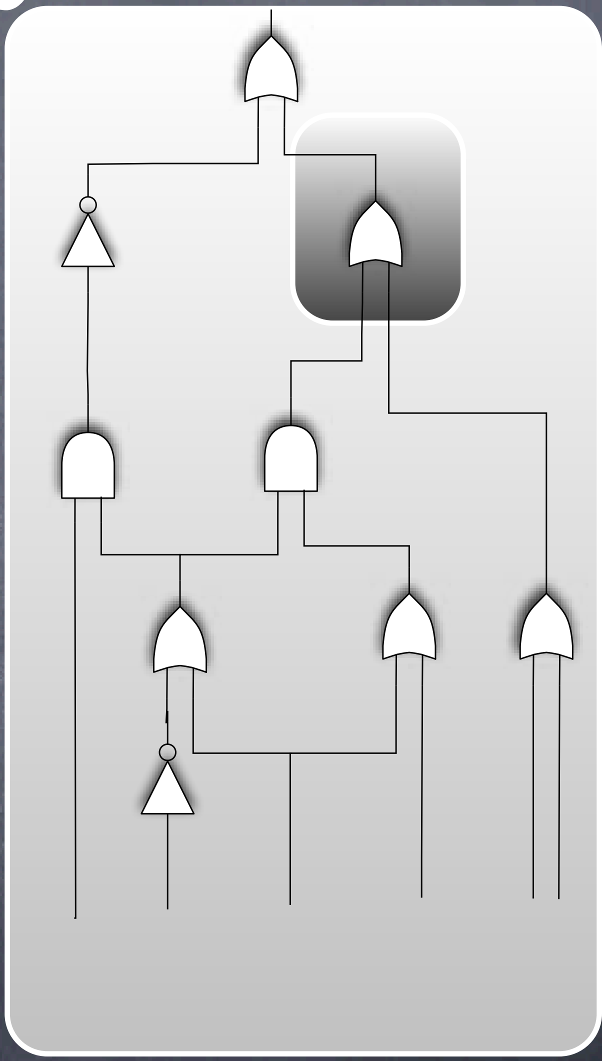
  - For each wire w in the circuit (i.e., input wires, or output of a gate) pick 2 keys $K_{w=0}$ and $K_{w=1}$



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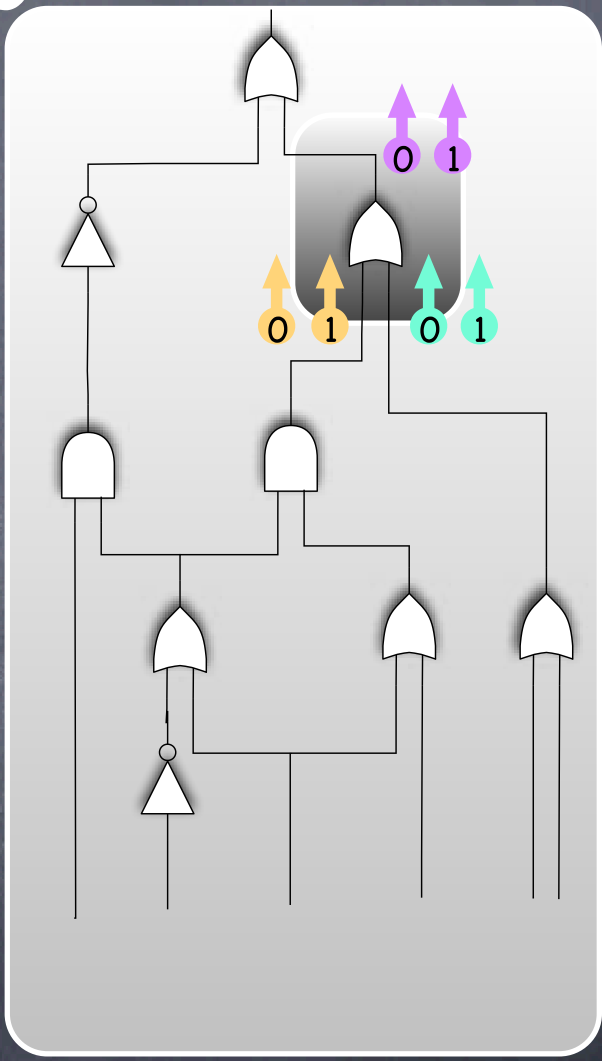

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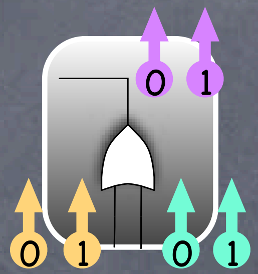

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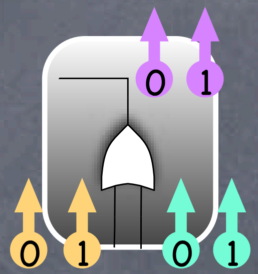

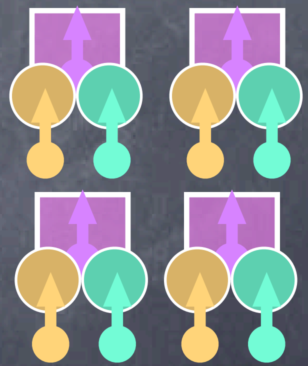

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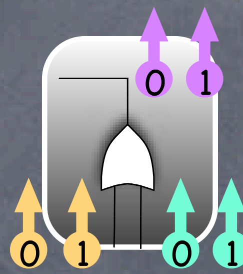

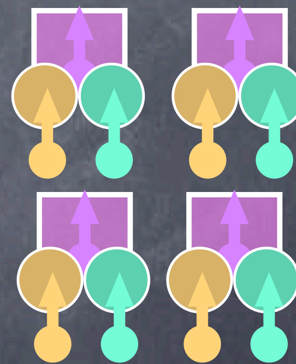

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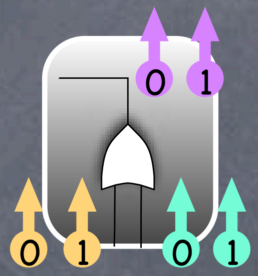

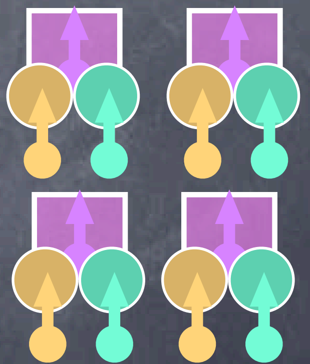

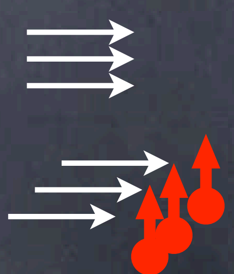

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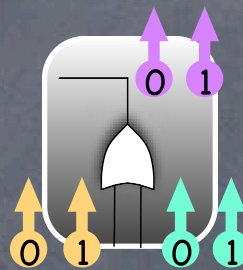

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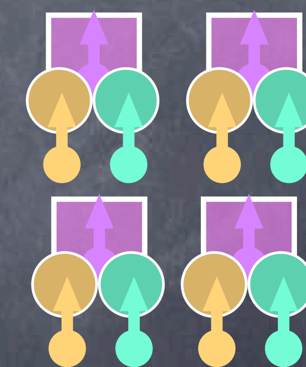

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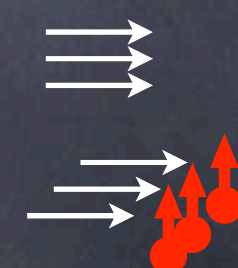

  - Give to Bob: Boxes for each gate, one key for each of Alice's input wires
    - Obliviously: one key for each of Bob's input wires

Larger Circuits

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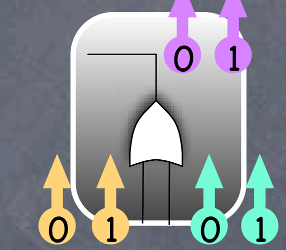

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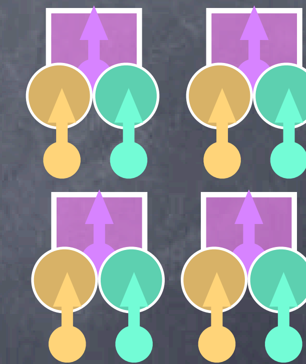

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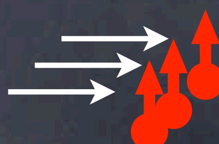
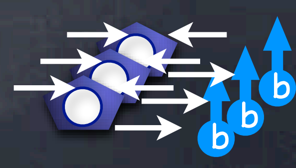
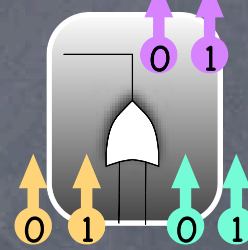


    - Obliviously: one key for each of Bob's input wires



Larger Circuits

- Idea: For each gate in the circuit Alice will prepare locked boxes, but will use it to keep keys for the next gate



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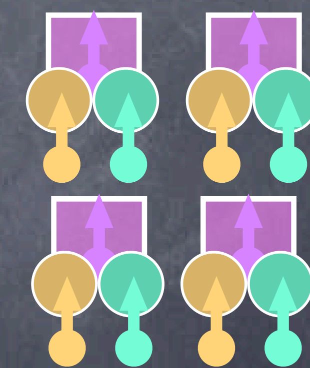

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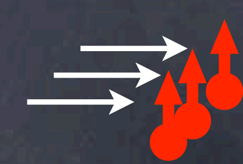

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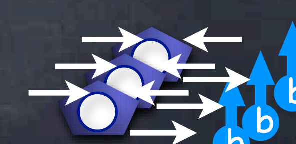
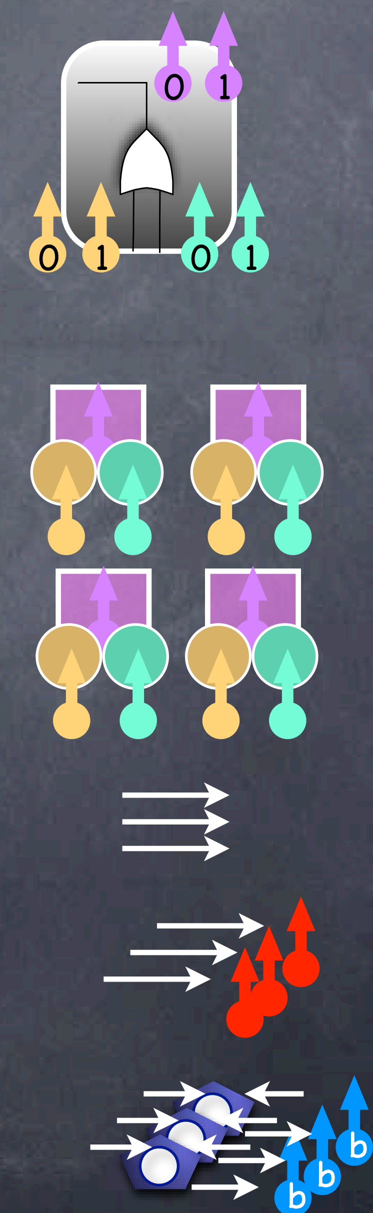


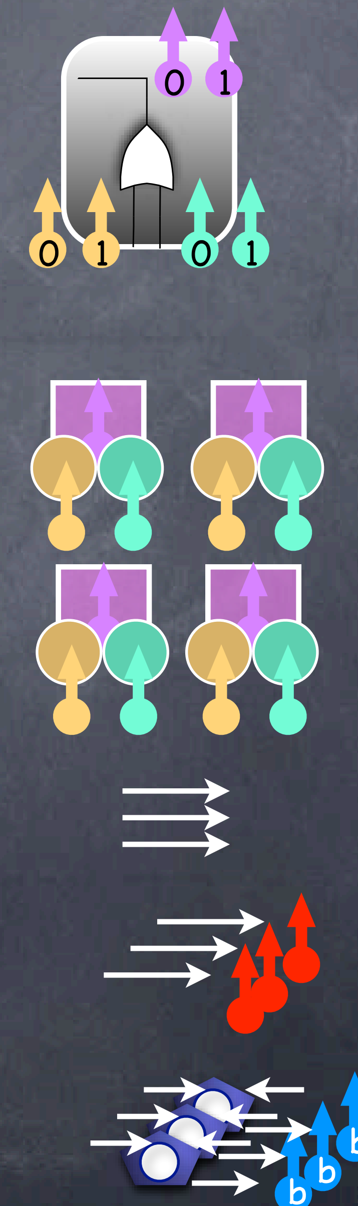
  - Boxes for output gates have values instead of keys

Larger Circuits



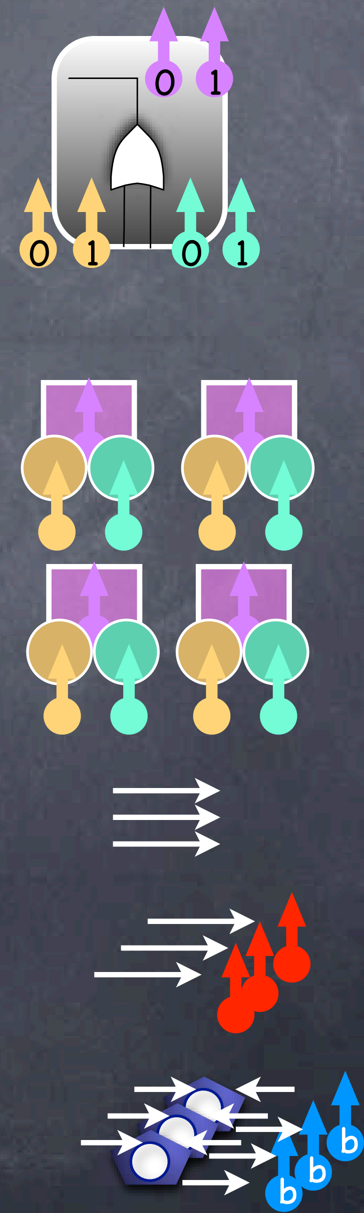
Larger Circuits

- Evaluation: Bob gets one key for each input wire of a gate, opens one box for the gate, gets one key for the output wire, and proceeds



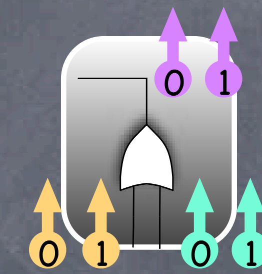
Larger Circuits

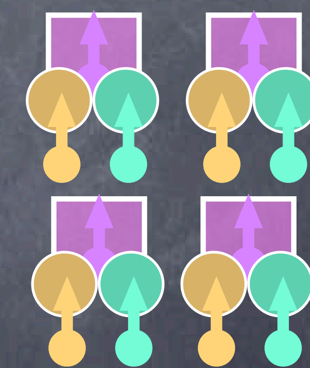
- Evaluation: Bob gets one key for each input wire of a gate, opens one box for the gate, gets one key for the output wire, and proceeds
  - Gets output from a box in the output gate

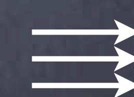


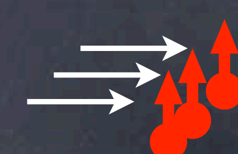
Larger Circuits

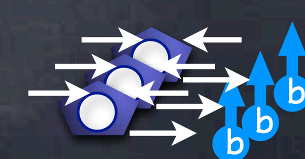
- Evaluation: Bob gets one key for each input wire of a gate, opens one box for the gate, gets one key for the output wire, and proceeds
    - Gets output from a box in the output gate
- Security similar to before











Larger Circuits

- Evaluation: Bob gets one key for each input wire of a gate, opens one box for the gate, gets one key for the output wire, and proceeds
  - Gets output from a box in the output gate
- Security similar to before
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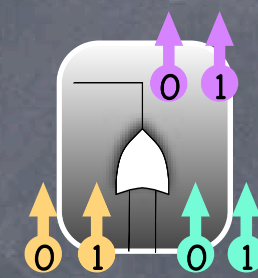

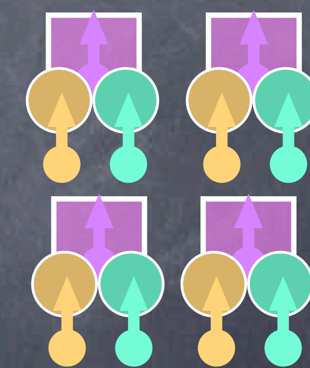

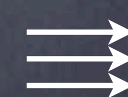

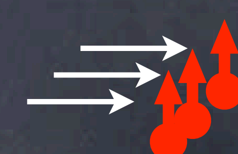

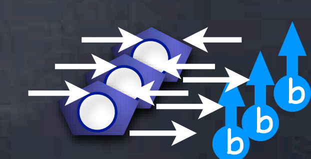

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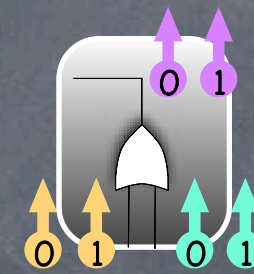

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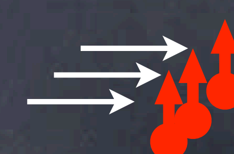
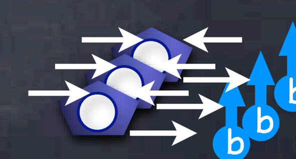
  - Everything is simulatable for curious Bob given final output: Bob could prepare boxes and keys (stuffing unopenable boxes arbitrarily); for an output gate, place the output bit in the box that opens





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  - Can we really <u>compose</u>? Yes, for passive security.

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- Coming up: Zero-Knowledge proofs and general multi-party computation, more protocols (for different settings). Universal Composition